Inventory Optimization

Dinesh K. Sharma
Madhu Jain *Editors*

Data Analytics and Artificial Intelligence for Inventory and Supply Chain Management

Springer

Inventory Optimization

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Inventory management is a very tedious task faced by all the organizations in any sector of the economy. It makes decisions for policies, activities and procedures in order to make sure that the right amount of each item is held in stock at any time. Many industries suffer from indiscipline in ordering and production mismatch. Providing best policy to control such mismatch would be invaluable to them.

The primary objective of this book series is to explore various effective methods for inventory control and management using optimization techniques. The series will facilitate many potential authors to become the editors or author in this book series. The series focuses on an aspect of Operations Research which does not get the importance it deserves. Most researchers working on inventory management are publishing under different topics like decision making, computational techniques and optimization techniques, production engineering etc. The series will provide the much needed platform for them to publish and reach the correct audience.

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This book series will publish volumes of books which will be edited and reviewed by the reputed researcher of inventory optimization area. The beginner and experienced researchers both can publish their innovative research work in the form of edited chapters in the books of this series by getting in touch with the contact person. Practitioners and industrialist can share their real time experience bolstered with case studies. The objective is to provide a platform to the practitioners, educators, researchers and industrialist to publish their valuable work in the area of inventory optimization.

This series will be beneficial for practitioners, educators and researchers. It will also be helpful for retailers/managers for improving business functions and making more accurate and realistic decisions.

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Editors

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Dedicated to
Our Beloved Mothers
Dinesh K. Sharma
Madhu Jain

Foreword

Businesses and organizations that deal with the inventory, supply chain and logistics primarily rely on the inventory management, space optimization, coping with erroneous forecasts, customer satisfaction, and their vast network of suppliers and partners to keep things moving effectively. This book offers an overview of state-of-the-art, some perspectives and implementation of data analytics, optimization and AI techniques for deterministic as well as stochastic modeling of inventory and supply chains.

The chapters of this book contain the practical inventory and supply chain models to ensure adoption of advanced techniques, new solutions, data analytics, intelligent computing and nature-inspired optimization algorithms. These models lead to better decision-making required in many industries and business organizations for demand forecasting, planning and digital-execution tracking. The inventory and supply chain models presented in each chapter provide valuable insights into how things are produced, manufactured, and supplied to the customers by using soft computing, data analytic and AI-driven tools.

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Preface

Intelligent computing can be used in supply chain management to tackle huge volumes of data, understand relationships of demand vs supply, optimize the earnings before interest, taxes, depreciation, and amortization for better decision-making in an organization as an integrated end to end supply chain. Artificial intelligence (AI)-driven tools can provide valuable insights for the inventory, logistics, warehouse efficiency, on time delivery, forecasting of supply and demand. AI-based solution-agnostic assessment and strategy will support the companies for better alignment and inventory control, and capabilities to create a strategic intelligent road map for the supply chain and logistics.

This book contains various inventory and supply chain models to tackle logistics and optimization problems. Recent developments in the areas of nature-inspired optimization and AI approaches to manage an end to end inventory and supply chains for implementation and system integration have been presented. The models and performance analysis presented facilitated valuable insights for the retailers and managers to improve business operations and make more realistic and better decisions. This book offers a number of smartly designed strategies related to inventory control and supply chain management for the optimal decision.

This book consists of 15 chapters that bring interesting aspects of modeling and applications of artificial intelligent computing and optimization techniques in inventory and supply chain management. The subject presented in the book will be beneficial for academicians, researchers and practitioners to get ideas about recent advances in the modeling and data analytics for the optimal decision of inventory, logistics and supply chain. In different chapters of the book statistical, optimization and AI techniques for the stochastic inventory control and supply chain management are presented. Chapter 1 deals with Markov decision processes for a supply chain (SC) model based on a two-tier queueing-inventory system that delivers packages of fixed-size items from its inventory stored in a distribution center to a retail shopping mall (RSM). Chapter 2 presents a state of art and literature survey on the inventory models with imperfect production systems. The prominent works related to nature-inspired optimization algorithms and their applications in inventory control are presented. Chapter 3 is concerned with multi-objective mathematical model for socially responsible supply chain inventory planning by framing four objectives related to cost, local development, steadiness in employment and investment in green technology. The proposed model has the capability to obtain the optimal number of products to be manufactured and re-manufactured, number of inventories, number of employees to recruit and lay-off within a certain region in each quarter of the year, decision on time to invest in green technology. Chapter 4 facilitates an overview of critical areas of supply chain where AI can help in improving the flexibility of nature-inspired optimization techniques, assuring delivery to the final mile, providing personalized solutions to the stakeholders in upstream and downstream supply chains and many more. The inventory model with price-dependent demand and imperfect production has been studied in Chap. 5. The markdown policy is adopted to lower the on-hand inventory and to enhance the total revenue or the rate of sale of goods. Chapter 6 is devoted to study the inspection process for the inventory queueing system with retrial orbit, balking and defective items. Chapter 7 investigates how a poultry farmer can manage the farm so that any type of viral flu can be tackled along with profit-maximizing, order quantity, etc.

In Chap. 8, the inventory model is developed to analyze the issues of pavement cracks based on Deep Learning to detect and classify five types of road problems including longitudinal, traverse, block, alligator cracks and potholes. In Chap. 9, Economic Production Quantity model is developed to propose an optimal strategy to decarbonize the environment through reusing policy of waste plastic from a dump yard in a rhino brick industry where each unit of production contains some proportion of plastic material. The cost analysis of supply chain for deteriorating inventory items with shortages in fuzzy environment is done in Chap. 10. Chapter 11 deals with the inventory problem in multi-echelon setup for scrutinizing the execution of a single manufacturer and multiple retailers' supply chain. The impacts of product type on inventory cost and lot size on the lead time are studied. Chapter 12 describes a production inventory model with flexible production system under online payment and preorder discount facility. Chapter 13 discusses a joint effect of preservation technology investment and order cost reduction under different carbon emission regulation policies by developing a sustainable inventory model in an inflationary environment. Chapter 14 examines the impact of corporate credibility on inventory management decisions using the concepts of AI. Chapter 15 is devoted to analyze a bidirectional neural network dynamic inventory control model for reservoir operation. Both forward and backward dynamic inventory control operations based on the previous states of the reservoir and the current states of the input are considered. The subject matter presented in the book will enrich the knowledge in the direction of optimal control and future design of the inventory systems and supply chains that will be of great importance not only from theoretical point of view but will strongly reflect the practical and managerial implementations in the concerned commercial/industrial organizations, production/manufacturing systems, business/service sectors and many more areas.

Princess Anne, USA — Dinesh K. Sharma
Roorkee, India — Dr. Madhu Jain

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About This Book

This book is the collection of contributors with a focus on the use of optimization and AI techniques in the area of inventory and supply chain management. The main aim of this book is to address the inventory control and management aspects to real-world supply chain and logistics that retailers, practitioners, educators and scholars may find useful. The main focus is to discuss a number of optimal control policy and AI techniques for the appropriate ordering strategy by employing the use of advanced techniques. It will provide the theoretical and applicable subject matters for the undergraduate and graduate students, researchers, practitioners and professionals in the area of artificial intelligent computing and its applications in inventory control and supply chain management and logistics.

Contents

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Dinesh K. Sharma is a professor of Quantitative Methods and Computer Applications in the Department of Business, Management and Accounting at the University of Maryland Eastern Shore, USA. He earned his M.S. in Mathematics, M.S. in Computer Science, Ph.D. in Operations Research, and a second Ph.D. in Management. Professor Sharma has over 28 years of teaching experience and served in several committees to supervise Ph.D. students and also acts as an external Ph.D. thesis examiner for several universities in India. Dr. Sharma's research interests include mathematical programming, artificial intelligence and machine learning techniques, inventory and supply chain management, healthcare management, and portfolio management. He has published over 250 refereed journal articles and conference proceedings and has also won 14 best paper awards. Professor Sharma has collaborated on a number of funded research grants. Professor Sharma is editor of the *Journal of Global Information Technology* and *Review of Business and Technology Research* and is on the editorial board of several journals and a paper reviewer for a number of additional journals and conferences. Additionally, he is a member of Decision Sciences (USA), a life member of the Operational Research Society of India, and served as a program chair and coordinator of several international conferences in many countries

Madhu Jain is presently working as an associate professor in the Department of Mathematics, Indian Institute of Technology Roorkee, India. Before joining IIT Roorkee, she served as reader/associate professor/professor in the Department of Mathematics in Dr. B. R. A. University, Agra, for 10 years and as visiting faculty in Indian Institute of Technology, Delhi. She has published over 450 research papers in International Journals, over 100 papers in Conference Proceeding/edited books, and 20 books and 6 edited books. She was conferred Young Scientist Award and SERC visiting fellow of the Department of Science and Technology (DST), India, and Career Award of University Grant Commission (UGC), India. She is the recipient of and Vigyan Gaurav award and Distinguish Service Award of Vijnana Parisad of India for her outstanding contributions to Mathematics. She was visiting fellow of Isaac Newton Institute of Mathematical Sciences, Cambridge, UK, for during summer in 2010, 2011, and 2014. At present, Dr. Madhu Jain is holding the post of president of Global Society of Mathematical and Allied Sciences. She has participated over 150 International/National Conferences in India and Abroad and visited many reputed universities/Institutes in USA, Canada, Australia, UK, Germany, France, Holland, Belgium, Singapore, Mauritius, Taiwan, UAE. Her current research interest includes stochastic modeling, inventory and supply chain management, queueing theory, software and hardware reliability, bio-informatics, data analytics, soft computing, etc.

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Chapter 1
Markov Decision Processes of a Two-Tier Supply Chain Inventory System

R. Sivasamy, Dinesh K. Sharma, and Keamogetse Setlhare

Abstract Our supply chain (SC) model is based on a two-tier queueing-inventory system. This facility delivers packages of Q > 0 fixed-size items from its inventory stored in a distribution centre (DC) to a retail shopping mall (RSM). This RSM processes customer requirements through an $M/M_1 + M_2/1/K$-rule service facility. Requests to the RSM node follow the Poisson process with the parameter (λ). The inventory guideline applied at node RSM is of type (0, Q), and the order of most items Q > 0 is placed on the DC when inventory decreases from Q to level 0 with a threshold guideline called f-policy. Thus, at the end of the delay, the DC output of the quantity ordered by RSM is exponentially distributed with the parameter γ ($\gamma > 0$). For each request to the RSM node, an article is only available after the exponential service time with the parameter μ ($\mu > 0$). The waiting room at the commercial node has unlimited capacity. This SC management (SCM) is called a queue inventory system with a f_1-policy. The common stochastic process $\mathbf{Z}(t) = (N(t), I(t))$ to link the number of customers demands available at time 't' by N(t) and the on-hand inventory level by I(t) is formulated. Because of the properties of the exponential distributions and of the f_1-policy, this pair-sequence $\mathbf{Z}(t)$ forms a Markov decision process (MDP) that changes its states with the passage of times 't'. The Z-systems' stationary properties are checked at service completion. We also use the linear function of total expected cost to determine the optimal order quantity and optimal order patterns. Using the same methodology, the analysis is extended by assuming that the two processes are independent under a new policy f_2. Numerical illustrations are also provided to support the results obtained from the proposed MDP approach.

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1.1 Markov Decision Processes of a Supply Chain

A supply chain (SC) is a system that monitors the constant flow of products (cars, televisions, etc.) between different wholesale and retail vendors and within their sub-departments and collects revenue from each of them. The client or customer is an integral part of the SC everywhere. The main objective is to satisfy the customer's needs and therefore generate the maximum profit for our SC.

The key task for success is the integration of sales branches into a supply chain that would improve customer satisfaction and loyalty economically. Several SC researchers (Dey and Giri 2014; Hsien-Jen 2013; Lin 2009) have used integrated policy to study the effects of integration between a seller and a buyer, as it helps reduce lead times and inventory costs. This task not only helps identify problem areas and inform wise decisions but also reduces the costs and improves the final expected revenues.

This book chapter discusses an integration method between inventory and queue models that allow requirements to occur during the inventory period that are only processed after the ordered items have arrived.

An example is a wholesale sales office (for example headquarters) selling cars electronics consumer products (TVs refrigerators etc.) and providing orders for one or more middle retail sales centres. After that each shopping centre can serve customer requirements using some queue discipline like the FCFS (First Come First Served) and inventory policy like the (s, S), (r, Q) etc., to maximize their ends.

Markov decision making focuses on sequential decision making under conditions of uncertainty. It would also represent a discrete stochastic process under the partial control of an outside observer. During each transition period, the state of use of the process is observed and based on this observation the controller selects an action "a" that affects the use of the system in the next moment. In addition, the observer receives a reward r (a, s) or costs c (a, s) for each step, depending on the selected action "a" and the state of the system at "s". The key components of this model are states and transition probabilities in the action space.

The main objective is how the observer should act to maximize the reward that may materialize subject to certain restrictions on the permissible states of the system. The objective is, therefore, to find decision rules that specify the action to be carried out at a given moment, as well as guidelines that consist of a series of decision rules.

A simple approach is to focus on the properties of the MDP, whose future states and reward or cost depend on the number of available shares and the transition probabilities in each period from the original state to the current state. To answer the question on "When is the optimal policy?", We will build and propose the optimal criteria and the shape of the base model.

Several estimation methods will be used to find service providers in mathematical models, including service queue interactions and inventory control. For example, Berman and Kim (1999) studied the behaviour of service systems associated with an inventory. Inventory templates for ongoing review have become popular as computerized inventory control systems have evolved in recent years. Customers arriving during the inventory period contribute to the constant evaluation of inventory samples, leading to loss of revenue (Daduna and Szekli 2006).

Many retail businesses inevitably experience stagnation as stiff competition can force customers to change brands or locations. To support this type of switching phenomenon, a pure inventory model is described (Mohebbi and Posner 1998).

Further sections of this chapter are organized as follows: Sect. 1.2 deals with a DC attached to a RSM via an $M/M_1 + M_2/1$ /K-rule facility operating under a deterministic (0, Q) inventory policy and obtained steady state characteristics of the inventory cum queuing management. We present in Sect. 1.3 the analysis of the problem concerning performance mmeasures and compute numerical results for the proposed system and thus form the core of the chapter. The results for M/M/1/FCFS branch under the same f-policy are deduced in Sects. 1.4 and 1.6 provides conclusions and future scope.

1.2 $M/M_1 + M_2/1$ /K-Rule/f_1-policy Queues + Inventory Model with Backorders

We present a SC system that integrates $M/M_1 + M_2/1$/K-rule queue under the FCFS discipline with an inventory management, continuous review, and exponential lead time evaluation results from a prior M/M/1 system, for which Schwarz and Daduna (2006) examined under a deterministic (0, Q) order policy with a threshold ‘1’. In the proposed SC model, we have integrated a two-tier inventory system: a distribution centre (DC) that delivers $Q > 0$ items from the stocked warehouse to the second linked home, i.e., a retail shopping mall (RSM) that operates with an $M/M_1 + M_2/1$/K-rule equipment.

An inventory policy is applied on this RSM node of type (0, Q) where $Q > 0$ item orders are placed with DC when inventory levels go from Q level to zero while there is at least one request in the queue waiting. The delay time in DC for the size ordered by RSM is exponentially distributed with the parameter ($\gamma > 0$). Whenever the server is free to serve a customer and inventory is zero, this service will start when the next replenishment arrives while the customers who arrive during this replenishment phase are added to the backorder list. As the higher the number of items backordered, the higher the demand for the item and so such backorders are an important factor in this integrated SCM analysis.

Suppose that a non-queue observer monitors our SC system and ensures that the service times are exponentially distributed with a mean service time of ($1/\mu$) and the queue length range is (0, K]. Alternately, if the queue length range is [(K

$+ 1), \infty)$, a temporary server is installed to join with the permanent one to boost the exponential service rate to $(\mu + \mu_1)$, $\mu_1 > 0$. Each arriving customer demands exactly one item from the inventory which passes through a homogenous Poisson process with the demand rate λ. We assume that sequences of service times, inter-arrival times, and lead times are mutually independent, and all random times within those sequences are identically distributed.

Our primary focus is on the bi-variate sequence $\mathbf{Z}(t) = (N(t), I(t))$, where N(t) = queue length and I(t) = on hand inventory level at time 't'. It can be shown that the sequence $\mathbf{Z}(t)$ forms a Markov process under the continuous review inventory policy (0, Q) with a threshold level $(N(t) = n > 0, I(t) = 0)$ for placing an order. Here, the number of items is sold out at the current service rate (if available) while the customer is waiting for service only.

Thus, this book chapter provides a solution to the problem of finding performance metrics and the optimal order quantity Q* for the SC model under consideration.

Main Issues: How does inventory 'f' policy react to customer queues at the service centre? How are traditional queue length, waiting times, etc., influenced by attached inventory policy?, and vice versa.

Inventory policy "f": The inventory policy for the current study is the policy '$f_1 = (0, Q)$' with a queue length threshold of "1" as the decision variable. There can be at most one order, with 0 being the order point, and exactly a bulk of Q units arrive with each replenishment. According to guidelines of the 'f_1' policy subject to its threshold value 1, an order is triggered when inventory is empty and there is at least one customer in the system. For more details on the same type of inventory management associated with a queue system that depletes inventory at the server's service rate (and higher than the customer demand rate) during non-empty queue time, see (Schwarz and Daduna 2006).

$\mathbf{E_f}$ state space: Let $\mathbf{N}_0 = \{0, 1, 2\ldots\ldots\}$ and $\mathbf{Q}_0 = \{0,1, 2, \ldots, Q\}$ be the state spaces of the queue length process N(t) and that of the Inventory process I(t) respectively. If inventory policy 'f_1' and queue policy 'K' work together, the queue-inventory ($\mathbf{N}(t)$, $\mathbf{I}(t)$) does not reach the state (0, Q) because the replenishment order is not triggered if there are no clients on the server. However, to simplify the notation, we use the Cartesian product state space $E_f = \mathbf{N}_0 \times \mathbf{Q}_0$ and find that all formulas for this state (0, Q) must specify exactly zero stationary accessibility.

Service and ordering methods: Whenever the server is free to serve a customer and inventory is zero, this service will start when the next replenishment arrives. This means that customers who arrive during this replenishment phase when inventory is empty are added to the backorder list. More recent studies in integrated models have addressed the problem of how classical performance measures (e.g., queue length, waiting time, etc.) are influenced by the management of attached inventory and vice versa.

1.2.1 The Stable M/M_1 + M/1 /K-rule/f_1-policy Model

The time dependent Markov process (N (t), I (t)) serves as a Markov Decision process on the state space E_f in the sequel. Let the random vector of the stationary state process be denoted by $Z = (N, I) = \lim_{t\to\infty}(N(t), I(t))$. We denote this integrated type of queueing-inventory system by $M/M_1 + M_2/1$/K-rule/f_1-policy.

1.2.1.1 Generator Matrix of the Inventory Process I When K > Q

The standard probabilistic arguments can establish the limiting process **Z** be distributed according to the equilibrium of the $M/M_1 + M_2/1$/K-rule/f_1-policy. Since we assume that **Q < ∞, it** can be shown that the process Z is regular and irreducible in the space [E_f - (0, Q)]. Therefore, **Z** is a quasi-birth- and-death (QBD) process with a generating matrix between levels A of order (Q + 1) x (Q + 1):

$$\boldsymbol{A} = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ Q \end{array} \begin{pmatrix} -\gamma & 0 & 0 & \cdots & \cdots & \gamma \\ \mu & -\mu & 0 & 0 & \cdots & 0 \\ 0 & \mu & -\mu & 0 & \cdots & 0 \\ 0 & 0 & \mu & -\mu & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \mu & -\mu \end{pmatrix} \tag{1.1}$$

We see that the matrix **A** serves as the infinitesimal generator of a finite, irreducible, continuous time Markov process $\mathbf{I} = \lim_{t\to\infty}\mathbf{I}(t)$ governing the transition states/changes of the DC if $\mu, \gamma > 0$ with stationary distribution $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \ldots, \alpha_Q)$, where $\alpha_0 + \alpha_1 + \ldots + \alpha_Q = 1$. Denote the unit vector of size (Q + 1) by $\mathbf{e} = (1, 1, \ldots, 1)$. Then,

$$\left(\alpha_0, \alpha_1, \ldots, \alpha_Q\right)\boldsymbol{A} = \mathbf{0}, \left(\alpha_0, \alpha_1, \ldots, \alpha_Q\right)\mathbf{e} = 1 \tag{1.2}$$

The unique solution for $\boldsymbol{\alpha}$ of (1.2) is given in (1.3):

$$\alpha_0 = \frac{\mu}{\mu + Q\gamma}, \text{ and } \alpha_j = \frac{\gamma}{\mu + Q\gamma} \text{ for } j = 1, \ldots, Q \tag{1.3}$$

By replacing the service rate μ by $\mu_2 = (\mu + \mu_1)$ in the matrix generator A, the DC can obtain the inter-level distribution, say $\boldsymbol{\alpha}^* = (\alpha_0^*, \alpha_1^*, \ldots, \alpha_Q^*)$ to deliver goods to the RSM when the queue length '$N \in \{(K + 1), (K + 2), \ldots\}$' at any time of the completion of the service. By solving for the vector of stationary distributions α_j for $j = 1, \ldots, Q$, and for $\boldsymbol{\alpha}^* = (\alpha_0^*, \alpha_1^*, \ldots, \alpha_Q^*)$ we get

$$\alpha_0^* = \frac{(\mu + \mu_1)}{(\mu + \mu_1) + Q\gamma}, \quad \text{and} \quad \alpha_j^* = \frac{\gamma}{(\mu + \mu_1) + Q\gamma} \quad \text{for} \quad j = 1, \ldots, \mathrm{Q} \tag{1.4}$$

We want to find out how the two alternating distributions $\{\alpha_j\}$ and $\{\alpha_j^*\}$ play a role in finding the optimal value for Q (i.e. the optimal order quantity Q*) in our discussions.

1.2.1.2 Generator Matrix of the QBD Process Z When K > Q

Since the **Z** process is a level-dependent QBD process, using the properties of the QBD in terms of the K-policy and the f-policy of the inventory process, we get an infinitesimal generating matrix $\mathbf{R} = (\mathbf{r}_{(\mathrm{Li},\,\mathrm{Lj})})$, where $\mathrm{L_i} = ((\mathrm{i},0), (\mathrm{i},1), \ldots(\mathrm{i},\mathrm{Q}))$ is called the i[th] level for $\mathrm{i} \in \mathrm{N_0}$, and $\bigcup_{i=0}^{\infty} L_i = E_f$ as follows, where each entry is a square matrix of order (Q + 1):

$$\mathbf{R} = \begin{array}{c} \\ \\ L_1 \\ L_2 \\ \vdots \\ L_{K-1} \\ L_K \\ L_{K+1} \\ L_{K+2} \\ \vdots \\ \vdots \\ \vdots \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccccccccc} L_0 & L_1 & L_2 & \cdots & L_{K-1} & L_K & L_{K+1} & L_{K+2} & L_{K+3} & \cdots \end{array} \\ \begin{pmatrix} -\boldsymbol{A}_0 & \boldsymbol{A}_0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots \\ \boldsymbol{A}_2 & \boldsymbol{A}_1 & \boldsymbol{A}_0 & \ddots & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \boldsymbol{A}_2 & \boldsymbol{A}_1 & \ddots & 0 & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & \mathbf{A}_1 & \boldsymbol{A}_0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & \boldsymbol{A}_2 & \boldsymbol{A}_1 & \boldsymbol{A}_0 & 0 & 0 & \ddots \\ 0 & 0 & 0 & \cdots & 0 & \boldsymbol{A}_2 & \boldsymbol{A}_1 & \mathbf{A}_0 & 0 & \ddots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \mathbf{A}_4 & \mathbf{A}_3 & \mathbf{A}_0 & \ddots \\ \vdots & \vdots & \ddots & \ddots & 0 & 0 & 0 & \mathbf{A}_4 & \mathbf{A}_3 & \ddots \\ \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 & 0 & \mathbf{A}_4 & \ddots \\ \vdots & \vdots & \vdots & \cdots & 0 & 0 & 0 & 0 & \ddots & \ddots \end{pmatrix} \end{array} \tag{1.5}$$

Here, explicit forms of all non-zero square matrices of order (Q + 1) of the generator R are given: $\mathrm{A_0} = \mathrm{diag}(\lambda)$ i.e. a diagonal matrix,

$$\boldsymbol{A}_0 = \begin{pmatrix} \lambda & 0 & \cdots & 0 & 0 \\ 0 & \lambda & \cdots & 0 & 0 \\ 0 & 0 & \ddots & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \lambda \end{pmatrix}, \boldsymbol{A}_2^{(\mu)} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \mu & 0 & \cdots & 0 & 0 \\ 0 & \mu & 0 & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \mu & 0 \end{pmatrix} \tag{1.6}$$

$$A_1^{(\mu)} = \begin{pmatrix} -(\gamma+\lambda) & 0 & 0 & \cdots & \gamma \\ 0 & -(\mu+\lambda) & 0 & \cdots & 0 \\ 0 & 0 & -(\mu+\lambda) & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & -(\mu+\lambda) \end{pmatrix} \tag{1.7}$$

Here, $\mathbf{A_3}$=$A_1^{(\mu+\mu_1)}$ and $\mathbf{A_4}$=$A_2^{(\mu+\mu_1)}$ i.e., matrix $\mathbf{A}_3$ ($\mathbf{A}_4$) is obtained from $\mathbf{A}_1$ ($\mathbf{A}_2$) by replacing μ by ($\mu_2 = \mu + \mu_1$) and keeping other elements as such in their respective positions. It is noticed that inter-level generator matrix A = $\mathbf{A}_0 + \mathbf{A}_3 + \mathbf{A}_4$, which is not a function of λ**.**

Clearly, the QBD process **Z** is an extension of the standard birth-and-death process whose state space consists only of the level L_i. When the transitions of a QBD process are independent of the level L_i, it is termed a *homogeneous* or *level independent* QBD process. The level independent QBD version has been studied extensively by several researchers and is given a very lucid treatment in the excellent text by (Latouche and Ramaswami 1999; Neuts 1981). Sivasamy (2021) investigated a state-dependent queueing model M]M1+M2/1/K-policy and obtained closed-form analytic solutions for queue length and waiting time variables. Sivasamy et al. (2019) studied the stationary characteristics of the QBD process representing the queue length of a two-server service system. This proposed research is on an integrated queueing inventory system (IQIS) considers the same queueing model as Sivasamy (2021) but is attached to a retailer vendor (RV) supplier with an underlying two-dimensional QBD process 'Z' to represent both queue length and the inventory levels. In addition, the proposed analysis employs a similar type of method applied by Sivasamy et al. (2019) for obtaining the distribution of the Z process and other metrics of interest.

Steady State Distribution: Denote by $\mathbf{\Pi} = (\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \ldots)$, the steady state distribution of **Z** and by $\boldsymbol{\pi}_n$ the row vector $\boldsymbol{\pi}_n = (\pi(n, 0),\ldots, \pi(n, Q))$ associated to level $n \in N_0$. Let **e**= (1,1, …) be a column vector of unit elements. As the state (0, Q) cannot be accessible from any other state (0, j) for j = 0, 1,…, (Q-1), we must set π(0, Q) to a constant value zero.

We have that if the inter-level generator matrix A is irreducible, then **Z** is positive recurrent if and only if $\rho_2 = (\alpha^* \mathbf{A}_0\, \mathbf{e})/(\alpha^* \mathbf{A}_4\mathbf{e}) = \frac{\lambda(\mu_2+\gamma Q)}{\mu_2 Q\gamma} < 1$,

$$\rho_2 = (\alpha * \boldsymbol{A}_0)/(\alpha * \boldsymbol{A}_4) = \frac{\lambda(\mu_2+\gamma Q)}{\mu_2\, \mathrm{Q}\gamma} = \left(\frac{\lambda}{\mu_2}\right)\left(\frac{1}{1-\alpha_0^*}\right) < 1 \tag{1.8}$$

where $\mu_2 = \mu + \mu_1$. We now assume $\rho_2 < 1$ or $\frac{1}{Q\gamma} + \frac{1}{\mu_2} < \frac{1}{\lambda}$ for the rest of the analysis which ensures that process Z is stable or in equilibrium under the inventory policy 'f'. We also assume that

$$\rho_1 = (\alpha \boldsymbol{A}_0)/(\alpha \boldsymbol{A_2}) = \frac{\lambda(\mu_1+\gamma Q)}{\mu_1\, \mathrm{Q}\gamma} = \left(\frac{\lambda}{\mu_1}\right)\left(\frac{1}{1-\alpha_0}\right) < 1 \tag{1.9}$$

1.2.1.3 Steady State Characteristics of the Inventory Cum Queueing Management

We determine the optimal policy for the inventory system with the underlying **Z** process that has the infinitesimal generator (1.5) subject to the stability condition attained in the above section. Berman and Kim (1999, 2001) have shown that the ordering policy ($r \geq 0$, Q) is only suboptimal. They have further proved that optimal policies play crucial role as decision variable in the queue length. Such optimal policies are of monotone structure. Let $\rho_2 = \left(\frac{\lambda}{\mu_2}\right)\left(\frac{1}{1-\alpha_0^*}\right)$ be denoted as ρ.

Theorem 1.1 *The steady state distribution* $\mathbf{\Pi}$ *of the process* $\mathbf{Z}$ *is given by* (1.10):

$$\pi_n = \begin{cases} \left(\frac{\beta}{1-\alpha_0}\right)\left(\alpha_1, \alpha_2, \ldots, \alpha_Q, 0\right), \text{ n} = 0 \\ \beta\rho_1^n\alpha \; for \; n \; = \; 1, 2 \ldots, (K+1) \\ \beta\rho_1^{K+1}\rho^{[n-(K+1)]}\alpha^* \; for \; n = (K+2), (K+3), \ldots \end{cases}$$
$$\beta = \frac{(1-\rho)(1-\rho_1)}{(1-\rho)\left(1-\rho_1^{(K+1)}\right) + (1-\rho_1)\rho_1^{(K+1)}} \tag{1.10}$$

Proof Solving the matrix equations $\mathbf{\Pi}$ $\mathbf{R} = \mathbf{0}$, $\mathbf{\Pi}$ $\mathbf{e} = \mathbf{1}$ we obtain the following system of equations in terms of the sub-matrices of **R** and sub-vectors of $\mathbf{\Pi}$:

$$\begin{aligned} &-\pi_0 A_0 + \pi_1 A_2 = 0 \\ &\pi_{n-1}A_0 + \pi_n A_1 + \pi_{n+1}A_2 = 0 \text{ for n} = 1, 2, \ldots \text{K} \\ &\pi_K A_0 + \pi_{K+1}A_1 + \pi_{K+2}A_4 = 0, \quad \text{for n} = \text{K} + 1 \\ &\pi_{n-1}A_0 + \pi_n A_3 + \pi_{n+1}A_4 = 0 \text{ for n} = (\text{K}+2), (\text{K}+3), \ldots \end{aligned} \tag{1.11}$$

Adding all equations of (1.11) for n $\in \mathbf{N}_0$, we obtain

$$\sum_{n=1}^{K+1} \boldsymbol{\pi}_n(\boldsymbol{A}_0+\boldsymbol{A}_1+\boldsymbol{A}_2 = \boldsymbol{A}) + \sum_{n=K+2}^{\infty} \boldsymbol{\pi}_n(\boldsymbol{A}_0+\boldsymbol{A}_3+\boldsymbol{A}_4 = \boldsymbol{A}*) = 0$$

This implies that $\boldsymbol{\pi}_n\boldsymbol{A} = 0$ for n = 1, 2, …, (K + 1) and $\Rightarrow \boldsymbol{\pi}_n\boldsymbol{A}^* = 0$ for n = (K + 2), (K + 3), ….

Thus, it is observed that the non-negative vector $\boldsymbol{\pi}_n$ is proportional to the vector $\boldsymbol{\alpha}$ for n = 1, 2, …, (K + 1) and that the non-negative vector $\boldsymbol{\pi}_n$ is proportional to the vector α^* for n = (K + 2), (K + 3),…. We let $\boldsymbol{\pi}_n = \beta\boldsymbol{\alpha}\boldsymbol{R}^{\boldsymbol{n}}$ in (1.11) for n = 1, 2, … (K + 1), β >0 and **R** is a rate matrix. This substitution indicates that

$$\begin{aligned} &\beta\rho_1^{(K+1)}\boldsymbol{\alpha}\boldsymbol{R}^{n-1}\left(\boldsymbol{A}_0 + \boldsymbol{R}\boldsymbol{A}_1 + \boldsymbol{R}^2\boldsymbol{A}_2\right) = \mathbf{0} \\ &\Rightarrow \boldsymbol{A}_0 + \boldsymbol{R}\boldsymbol{A}_1 + \boldsymbol{R}^2\boldsymbol{A}_2 = \mathbf{0} \end{aligned} \tag{1.12}$$

Add **R A2** on both sides of (1.12), we get

$$A_0 + \mathbf{R}[A_2 + (\mathbf{I} + \mathbf{R})2_4] \ = \mathbf{RA}_4$$

$$\Rightarrow A_0 + \mathbf{R}\left\{(\mathbf{I} - \mathbf{R})^{-1}[(\mathbf{I} - \mathbf{R})A_1 + (\mathbf{I} - \mathbf{R})(\mathbf{I} + \mathbf{R})A_2]\right\} = \mathbf{R}A_2$$

$$\Rightarrow A_0 + \mathbf{R}\{(\mathbf{I} - \mathbf{R})^{-1}[(A_1 + A_2) - (\mathbf{RA}_1 + \mathbf{R}^2 A_2)]\} = \mathbf{R}A_2$$

$$\Rightarrow A_0 e + \mathbf{R}\{(\mathbf{I} - \mathbf{R})^{-1}[(A_0 + A_1 + A_2) - \left(A_0 + \mathbf{R}A_1 + \mathbf{R}^2 A_2\right)]\}e = \mathbf{R}A_2 e$$
$$\Rightarrow A_0 e + \mathbf{R}\left\{(\mathbf{I} - \mathbf{R})^{-1}[0 - 0]\right\}e = \mathbf{R}A_4 e$$

$$\Rightarrow A_0 \mathbf{e} = \mathbf{R}A_4 \mathbf{e}$$

$$A_0 e = \mathbf{R}A_2 e \Rightarrow \alpha A_0 e = \alpha \mathbf{R}A_2 e \tag{1.13}$$

Since $\rho_1 = (\alpha \mathrm{A}_0\ \mathrm{e})/(\alpha \mathrm{A}_2\ \mathrm{e})$ of (1.9), the equality statement in (1.13) is satisfied if and only if the rate matrix **R** is a diagonal matrix with each diagonal element is ρ_1 i.e., $\mathbf{R} = \mathrm{diag}(\rho_1)$.

Now, we let $\boldsymbol{\pi}_{n+K} = (\boldsymbol{\pi}_{1+K})e\beta\boldsymbol{\alpha}^{*}\mathrm{R}^{\mathrm{n}-1}$ in (1.11) for n = 1, 2, … (K + 1), $\beta > 0$ and **R** is another rate matrix. This substitution indicates that

$$\beta\rho_1^{(K+1)}\boldsymbol{\alpha}^{*}\mathbf{R}^{n-1}\left(A_0 + \mathbf{R}A_3 + \mathbf{R}^2 A_4\right) = 0$$
$$\Rightarrow A_0 + \mathbf{R}A_3 + \mathbf{R}^2 A_4 = 0 \tag{1.14}$$

As before after some manipulations we can show that

$$\boldsymbol{\alpha}^{*} A_0 e = \boldsymbol{\alpha}^{*}\mathbf{R}A_4 e \Rightarrow \mathbf{R} = \mathrm{diag}(\rho_2) \tag{1.15}$$

Now substituting the value of $\boldsymbol{\pi}_1 = \beta\boldsymbol{\alpha}\mathbf{R}$ to $-\boldsymbol{\pi}_0 A_0 + \boldsymbol{\pi}_1 A_2 = 0$ and simplifying we get

$$\boldsymbol{\pi}_0 A_0 = \boldsymbol{\pi}_1 A_2 \Rightarrow \boldsymbol{\pi}_0 = \boldsymbol{\pi}_1 A_2 (A_0)^{-1} \Rightarrow \boldsymbol{\pi}_0 = \boldsymbol{\pi}_1 \frac{1}{\lambda} A_2$$
$$\Rightarrow \boldsymbol{\pi}_0 = \beta\left(\frac{\lambda}{\mu(1-\alpha_0)}\right)\frac{1}{\lambda}\boldsymbol{\alpha}A_2 = \beta\left(\frac{\lambda}{\mu(1-\alpha_0)}\right)\frac{1}{\lambda}\left(\mu\alpha_1, \mu\alpha_2, \ \ldots, \mu\alpha_Q, 0\right)$$
$$= \beta\left(\alpha_1, \alpha_2, \ \ldots, \alpha_Q, 0\right)/(1-\alpha_0)$$

From the normalizing equation, we can find the value of β. Thus,

$$\sum_{n=0}^{K} \boldsymbol{\pi}_n \boldsymbol{e} + \sum_{n=K+1}^{\infty} \boldsymbol{\pi}_n \boldsymbol{e} = \boldsymbol{\pi}_0 \boldsymbol{e} + \sum_{n=1}^{K} \beta \rho_1^n \boldsymbol{\alpha e}$$
$$+ \sum_{n=(K+1)}^{\infty} \beta \rho_1^{K+1} \rho^{[n-(K+1)]} \boldsymbol{\alpha} * \boldsymbol{e} = 1$$
$$\Rightarrow \beta + \beta \left[\frac{\rho_1 \left(1 - \rho_1^{(K+1)}\right)}{1 - \rho_1} + \frac{\rho_1^{(K+1)}}{1 - \rho} \right] = 1$$
$$\Rightarrow \beta \left[\frac{\left(1 - \rho_1^{(K+1)}\right)}{1 - \rho_1} + \frac{\rho_1^{(K+1)}}{1 - \rho}) \right] = 1$$
$$\Rightarrow \beta = \frac{(1 - \rho)(1 - \rho_1)}{(1 - \rho)\left(1 - \rho_1^{(K+1)}\right) + (1 - \rho_1)\rho_1^{(K+1)}}$$

Hence the theorem.

Lemma 1.1 *The third equation of* (1.11) *is also satisfied by the values of* (1.10) *stated in the Theorem* 1.1.

Proof Substitute for $\pi_n = \beta \rho_1^n \alpha$ for n = K, and(K + 1) and $\pi_{K+2} = \beta \rho_1^{K+1} \rho \alpha^*$ in the third equation of (1.11) which is now given in (1.16).

$$\pi_K A_0 + \pi_{K+1} A_1 + \pi_{K+2} A_4 = 0 \tag{1.16}$$

We see that, on using $\mathbf{A}_0 + \rho_1 \mathbf{A}_1 + \rho_1^2 \mathbf{A}_2 = 0$,

$$\begin{aligned}
&\beta \rho_1^K \alpha (\boldsymbol{A}_0 + \rho_1 \boldsymbol{A}_1) + \beta \rho_1^{K+1} \rho \alpha^* \boldsymbol{A}_4 = 0 \\
&\Rightarrow \beta \rho_1^K \alpha (\boldsymbol{A}_0 + \rho_1 \boldsymbol{A}_1 + \rho_1^2 \boldsymbol{A}_2) - \beta \rho_1^{K+2} \alpha \boldsymbol{A}_2 + \beta \rho_1^{K+1} \rho \alpha^* \boldsymbol{A}_4 = 0 \\
&\Rightarrow \beta \rho_1^{K+1} \left(\rho \alpha^* \boldsymbol{A}_4 - \alpha \rho_1 \boldsymbol{A}_2\right) = 0 \\
&\Rightarrow \left(\rho \alpha^* \boldsymbol{A}_4 - \alpha \rho_1 \boldsymbol{A}_2\right) = 0 \\
&\Rightarrow \rho \left(\alpha^* \boldsymbol{A} e\right) = \alpha (\rho_1 \boldsymbol{A}_2 e)
\end{aligned} \tag{1.17}$$

which implies that $\rho_1 = (\alpha \mathbf{A}_0 \mathbf{e})/(\alpha \mathbf{A_2 e})$ **and** $(\rho_2 =)\rho = (\alpha^* \mathbf{A}_0 \mathbf{e})/(\alpha^* \mathbf{A}_4 \mathbf{e})$**.**

Therefore, all the equations given by (1.11) are now solved to have a unique solution Π as stated in (1.10) of Theorem 1.1. As the result $\mathbf{\Pi} > 0$ and $\mathbf{\Pi}$e = 1 it sets that the bi-variate sequence **Z** process is an ergodic process on the state space $\mathbf{E}_f$-{(0 Q)}. This type of stationary distribution $\mathbf{\Pi} > 0$ and $\mathbf{\Pi}$e = 1 is first derived for the M/M$_1$ + M$_2$/1/K-rule/f$_1$-policy model.

1.3 Performance Measures for the Inventory-Queueing System

Consider the results derived from the integration of quieting inventory system. Denote the conditional average inventory levels: by $\bar{\mathrm{I}}(0)$ for N = 0, by $\bar{\mathrm{I}}(1)$ for N = 1, 2, …, K and by $\bar{\mathrm{I}}(2)$ given that N = (K + 1), (K + 2), …. Using the results of the preceding inter-play and the stable distributions, we get, using $\boldsymbol{\Pi} > 0$, and $\boldsymbol{\Pi}\mathrm{e} = 1$:

$$
\begin{aligned}
\overline{I}(0) &= \sum_{j=0}^{\mathrm{Q}-1} j\pi(0, j) = \beta\frac{(\mathrm{Q}-1)}{2} \\
\overline{I}(1) &= \sum_{n=0}^{K}\sum_{j=0}^{Q} j\pi(n, j) = \beta(1-\alpha_0)\frac{(\mathrm{Q}+1)}{2}\left[\frac{\rho_1\left(1-\rho_1^K\right)}{1-\rho_1}\right] \\
\overline{I}(2) &= \sum_{n=K+1}^{\infty}\sum_{j=0}^{Q} j\boldsymbol{\pi}(n, j) = \beta\frac{\rho_1^{(K+1)}}{1-\rho}\sum_{j=0}^{Q} j\alpha_j^* \\
&= \beta\frac{\rho_1^{(K+1)}}{1-\rho}\left(1-\alpha_0^*\right)\left(\frac{(\mathrm{Q}+1)}{2}\right)
\end{aligned} \tag{1.18}
$$

Thus, the unconditional average inventory is $\overline{I}\,(>1) = \bar{\mathrm{I}}(1) + \bar{\mathrm{I}}(2)$ is given by

$$
\overline{I}(>1) = \beta\frac{(\mathrm{Q}+1)}{2}\left[(1-\alpha_0)\left(\frac{\rho_1\left(1-\rho_1^K\right)}{1-\rho_1}\right) + \frac{\left(1-\alpha_0^*\right)\rho_1^{(K+1)}}{(1-\rho)}\right] \tag{1.19}
$$

The marginal mean queue length in the system (queue + service) L is given by

$$
\begin{aligned}
\mathrm{L} &= \sum_{n=0}^{K} \mathrm{n}\,\boldsymbol{\pi}_n\boldsymbol{e} + \sum_{n=K+1}^{\infty} \mathrm{n}\,\boldsymbol{\pi}_n\boldsymbol{e} \\
&= \beta\rho_1\left[\frac{1-\rho_1^{K+1}-(K+1)(1-\rho_1)\rho_1^K}{(1-\rho_1)^2}\right] \\
&\quad + \beta\rho_1\left[\frac{\rho\rho_1^K + (K+1)(1-\rho)\rho_1^K}{(1-\rho)^2}\right]
\end{aligned} \tag{1.20}
$$

The marginal mean queue length in the queue (queue only) $\mathrm{L_q}$ is given by

$$
\mathrm{L_q} = \sum_{\mathrm{n}=1}^{\mathrm{K}}(\mathrm{n}-1)(\boldsymbol{\pi}_{\mathrm{n}}\boldsymbol{e}) + \sum_{\mathrm{n}=\mathrm{K}+1}^{\infty}(\mathrm{n}-1)\boldsymbol{\pi}_{\mathrm{n}}\boldsymbol{e} = \mathrm{L} - (1-\pi_0\boldsymbol{e}) = \mathrm{L} - (\mathbf{1}-\beta) \tag{1.21}
$$

The Little's Law: The random variable "W" represents the total system waiting time (service queue) and "W_q" represents the total waiting time of the queue (excluding service time). Then regardless of the number of servers, queuing rules or other specific cases, the conservation equation, also known as Little's Law, is valid for almost any system of the Markovian type or not.

$$\mathrm{L} = \lambda\ \mathrm{W}\ or\ \mathrm{L}_q = \lambda\ \mathrm{W}_q \tag{1.22}$$

Corollary 1.1 *If K < ∞, the steady state probability function P(N > 0, I = j) of the M/M/1/K/ f_1-policy model is given by* (1.23) *for j = 0,1,.., Q and N = 1, 2, …,K. Let's discuss this.*

$$\begin{aligned}
&\mathrm{P}(\mathrm{K} \geq \mathrm{N} > 0, \mathrm{I} = \mathrm{j}) = \beta \sum_{(n=1)}^{K} \rho_1^n \alpha_j = \beta \frac{\rho_1 (1 - \rho_1^K)}{(1 - \rho_1)} \alpha_j \\
&\mathrm{P}(\mathrm{N} \geq (\mathrm{K}+1), \mathrm{I} = \mathrm{j}) = \beta \rho_1^{K+1} \sum_{n=K=1}^{\infty} \rho^{[n-(K+1)]} \alpha_j^* = \beta \frac{\rho_1^{K+1})}{(1 - \rho)} \alpha_j^* \\
&\mathrm{P}\ (\mathrm{N} = 0,\ \mathrm{I} = \mathrm{j}) = \frac{\beta}{1 - \alpha_0} \alpha_{j+1} \quad \text{for j} = 0, 1, \ldots, (\mathrm{Q} - 1)
\end{aligned} \tag{1.23}$$

Corollary 1.2 *The mean reorder rate (MRR) is given by.*

$$\begin{aligned}
&\mathrm{MRR} = \beta \frac{(1 - \rho_1^K)}{(1 - \rho_1)} \left(\frac{\lambda \gamma}{\mu + Q\gamma} \right) + \beta \frac{\rho_1^{K+1}}{(1 - \rho)} \left(\frac{\lambda \gamma}{\mu + \mu_1 + Q\gamma} \right) \\
&\rightarrow \frac{\lambda \gamma}{\lambda + Q\gamma} \text{ as K} \rightarrow \infty, \rho = \rho_1 \\
&\text{where } \beta \rightarrow (1 - \rho) \text{ as K} \rightarrow \infty \\
&\mathrm{MRR} \rightarrow \frac{\lambda}{Q} \text{ as K} \rightarrow \infty \text{ and } \gamma \rightarrow \infty
\end{aligned} \tag{1.24}$$

Proof Since the reorder is triggered for the **Z**-process under the K-rule and f-policy just after reaching the state (n > 0, 0) by a service completion from the state (n + 1,1) at rate μ for n = 1, 2, …, K and for n = (K + 1), (K + 2), …, at the rate (μ + μ_1), MRR is computed for the **Z** process is computed by:

$$\mathrm{MRR} = \sum_{n=1}^{K} \mu \pi (n + 1, 1) + \sum_{n=K+1}^{\infty} (\mu + \mu_1) \pi (n + 1, 1) \tag{1.25}$$

Substituting for $\pi(n, 1)$ in (1.25), we can compute the results of (1.24).

Operating Costs: There are costs associated with operating the system, arising both from customer queuing and from goods held in the system. We have a fixed holding cost ‘h’ per item per unit time, a fixed inventory cost ‘g’ per order, and a cost ‘Vw’ per customer per unit waiting time. Revenue R is paid to the system each time customer service is completed. There is a policy in place to determine how much stock to order at each decision time. These decisive epochs are arrival and departure times, as well as order arrival times (Schwarz and Daduna 2006).

We always assume that there is at most one outstanding order. With these costs, the total expected cost (TEC) is:

$$\mathrm{TEC} = \overline{I}h + MRRg + \overline{W}_{\mathrm{q}}\,\mathrm{V_w} \tag{1.26}$$

Remark If $\nu = \infty$ the inventory on hand leaves state (1,0) immediately to the state (1,Q), so the state space is then given by $E_f = \{(n, j): n \in \{0, 1, \ldots, \}, 1 \le j \le \mathrm{Q}\}$. If in addition the order size $\mathrm{Q} < \infty$ is fixed then the inventory positions are uniformly distributed, *i.e.,* $\mathrm{P(I = j) = 1/Q}$ for j = 1, 2, …, Q.

Economic Order quantity: We now illustrate a numerical exercise by fixing the inventory holding cost h = \$125.25 per unit, g = \$700.2 and $\mathrm{V_w}$ = \$380.2 and the other input values are randomly selected as ($\lambda = 3.75$, $\mu = 5.0$, $\mu_1 = 0.5$, $\gamma = 4.2$, K = 4) such that these values satisfy the steady state condition of the Z-process under study for Q ∈{4,5,6,7,8,9,10,11,12}. It can be shown that the optimum order quantity Q* that minimizes the TEC is given by

$$\mathrm{Q}^* = \sqrt{\frac{2\lambda g}{h}} = \sqrt{\frac{(2)(3.75)(700.2)}{125.25}} = \sqrt{41.84} \simeq 6\,or\,7 \text{ nearest integer} \tag{1.27}$$

The corresponding minimum cost is

$$\begin{aligned} \mathrm{TEC1} = \overline{I},\ \mathrm{TEC2} = \mathrm{g},\ \mathrm{TEC3} = \overline{\mathrm{W}}_{\mathrm{q}}\,\mathrm{V}_w \\ \mathrm{TEC} = \min\,(\mathrm{TEC1} + \mathrm{TEC2} + \mathrm{TEC3}) \end{aligned} \tag{1.28}$$

The corresponding values of the TEC are computed and the numerical results of TEC1, TEC2 and TEC3 are drawn in a single Fig. 1.1.

Figure 1.1 shows the relationship between the various order quantity values considered from Q = 4 to Q = 12 and the different costs:

TEC1 = Holding cost which, generally, increases with increasing Q values.
TEC2 = Ordering cost, generally, decreases with increasing values of Q.
TEC3 = Waiting time cost, generally, decreases with increasing values of Q.

Further, it is noticed that the costs of TEC1 and TEC2 are almost equal at the position (Q = 6, TEC2 = 362.44 = TEC1). Similarly, the costs of TEC1 and TEC3 are almost equal at the position (Q = 7, TEC3 = 417.78 = TEC1). The SCM must know these costs for comparing with the optimum order quantity and the optimum TEC.

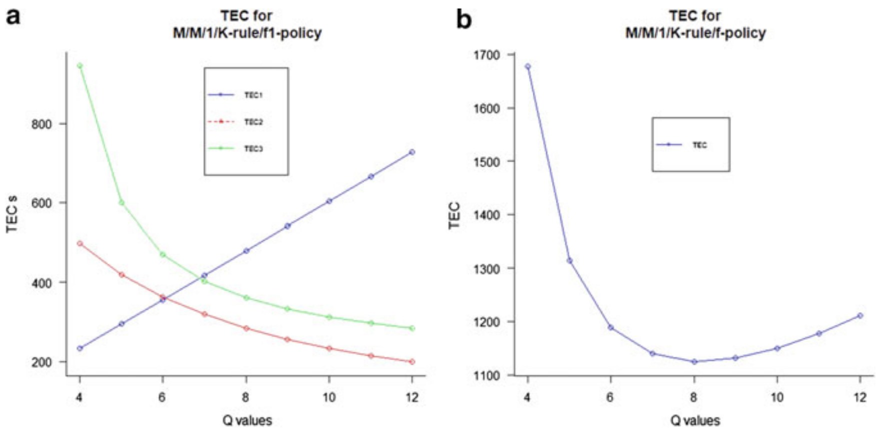

Fig. 1.1 Graph of Q versus TEC for h = \$125.25 per unit, g = \$700.2 and V_w = \$380.2 and, (λ = 3.75, μ=*5.0,* μ_1= 0.5, γ= 4.2, K = 4): Q = 4,5,6,7,8,9,10,11,12

From the Fig. 1.1, it is found the minimum total expected cost is \$1137.66 which occurs against the optimum order quantity is 8, i.e.,

$$\text{TEC}(Q* = 8) = \$1137.66.$$

Thus, the optimum TEC(Q* = 8) = \$1137.66 does not match with either the cost (Q = 6, TEC2 = 362.44 = TEC1) or of (Q = 7, TEC3 = 417.78 = TEC1).

1.4 Results for the SCM Attached to M/M/1 Queues with Zero Lead Time

Let us consider the case when $\gamma \to \infty$ i.e., the replenishment lead time vanishes. This indicates that the order we place for Q units when the **Z**-process reaches the state (n$\geq$ 1,0) immediately the ordered quantity Q (>0) is delivered instantaneously; this event forces the next state of the **Z**-process to move to the state (1, Q). Thus, the system cannot visit the state (0, 0) of the **Z**-process where $\alpha_0 = 0$ and $\lim\limits_{\gamma\to\infty} (\alpha_j = \frac{\gamma}{\mu+Q\gamma}) = \frac{1}{Q}$, for j = 1, 2,.., Q. This applies also for $\alpha^*_0 = 0$ and $\lim\limits_{\gamma\to\infty} (\alpha^*_j = \frac{\gamma}{\mu+\mu_1+Q\gamma}) = \frac{1}{Q}$ for j = 1, 2,.., Q.

- If Q = 1, and $\gamma \to \infty$ then from the results of Theorem 1.1, we observe that the steady state characteristics of the model $M/M_1 + M_2/1$/K-rule is ensured if $\rho = \lambda/(\mu + \mu_1) < 1$, irrespective of the value of $\rho_1 = \lambda/\mu < 1$ or not.
- If Q > 1, $\gamma < \infty$ and either $K \to \infty$, or $\mu_1 = 0$, then the K-policy rule cannot be implemented.

So, one can extract the results of M/M/1 queueing system attached to the inventory system under f_1-policy from the result of Theorem 1.1 by letting either $K \to \infty$, or $\mu_1 = 0$. Thus, expressions for the stationary probabilities $\boldsymbol{\pi}_n$, n = 0, 1, 2, ..., ∞, and α_j, j = 0, 1, 2, ..., Q are

$$\boldsymbol{\pi}_n = \beta\rho_1^n\boldsymbol{\alpha},\ \boldsymbol{\alpha} = (\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_Q), \alpha_0 = \frac{\mu}{Q\gamma + \mu}, \alpha_j = \frac{\gamma}{Q\gamma + \mu}$$
$$\text{where } \beta = 1 - \left(\frac{\lambda}{\mu(1-\alpha_0)}\right), \rho = \rho_1 = \left(\frac{\lambda}{\mu(1-\alpha_0)}\right)$$
$$\text{and } \boldsymbol{\pi}_0 = \frac{\beta}{1-\alpha_0}(\alpha_1, \alpha_2, \ldots, \alpha_Q, 0), \sum_{n=0}^{\infty}\boldsymbol{\pi}_n\boldsymbol{e} = 1 \tag{1.29}$$

Corollary 1.3 *As K→ ∞, the steady state probability function P(N > 0, I = j) of the M/M/1 queue under f-policy is given by, for j = 0, 1,.., Q*

$$P(N > 0,\ I = 0) = \frac{\lambda}{\lambda + Q\gamma} \text{ and } P(N > 0,\ I = j) = \frac{\rho\gamma}{\lambda + Q\gamma}$$
$$P\ (N\ = 0,\ I = j) = \frac{\beta}{1-\alpha_0}\frac{\gamma}{\mu + Q\gamma},\quad \text{for } j = 0, 1, 2, \ldots, (Q-1) \tag{1.30}$$

Proof Application of the limiting condition $K \to \infty$ in the results of Theorem 1.1 leads to.

$P(N > 0, I = 0) = \sum_{n=1}^{\infty}\boldsymbol{\pi}_n\boldsymbol{e}\alpha_0 = \beta\sum_{n=1}^{\infty}\rho_1^n\alpha_0 = (1-\rho)\left(\frac{\rho}{1-\rho}\right)\frac{\mu}{(\mu+Q\gamma)} = \frac{\lambda}{\lambda+Q\gamma}$, since.

$\beta = 1-\rho, \alpha_0 = \frac{\mu}{Q\gamma+\mu}, \alpha_j = \frac{\gamma}{Q\gamma+\mu}$.

Further, for j = 1, 2, …,Q

$$P(N > 0,\ I = j) = \sum_{n=1}^{\infty}\boldsymbol{\pi}_n\boldsymbol{e}\alpha_j = \beta\sum_{n=1}^{\infty}\rho_1^n\alpha_j = (1-\rho)\left(\frac{\rho}{1-\rho}\right)\frac{\gamma}{(\mu + Q\gamma)} = \frac{\rho\gamma}{\lambda + Q\gamma},$$
$$P\ (N\ = 0,\ I = j) = \frac{\beta}{1-\alpha_0}\frac{\gamma}{\mu + Q\gamma} = \frac{1-\rho}{1-\alpha_0}\frac{\gamma}{\mu + Q\gamma},\quad \text{for } j = 0, 1, 2, \ldots, (Q-1)$$

It is remarked that when $\mu_1 = 0$, we see that $\rho_1 = \rho$ and $\alpha_0 = \alpha_0^*$. Thus, we can obtain from the results of M/M/1/K-rule queue to that of the M/M/1 queue attached to the inventory system under f_1-policy discussed by (Schwarz and Daduna 2006):

$$\bar{I}(> 1) = \beta(1-\alpha_0)\left(\frac{\rho}{1-\rho}\right)\frac{(Q+1)}{2}, \beta = 1-\rho$$
$$\bar{I} = \beta\left[\frac{(Q-1)}{2} + (1-\alpha_0)\frac{(Q+1)}{2}\left(\frac{\rho}{1-\rho}\right)\right] \tag{1.31}$$

Recall that, as $\gamma \to \infty$, we have $\lim_{\nu\to\infty}\left(\alpha_0 = \frac{\mu}{\mu+Q\nu}\right) = 0$ and $\beta = (1-\rho)$. Thus

$$\lim_{\gamma\to\infty}\overline{I}(>1) = \rho\frac{(Q+1)}{2} \text{and} \lim_{\gamma\to\infty}\overline{I} = (1-\rho)\frac{(\mathrm{Q}-1)}{2} + \rho\frac{(\mathrm{Q}+1)}{2}$$
$$= \frac{(\mathrm{Q}\text{ - }1)}{2} + \rho$$

From (1.31) we see that $\lim_{\gamma\to\infty}\overline{I}(>1) = \rho = \lim_{\gamma\to\infty}\overline{I}$, if Q = 1 as expected.

Corollary 1.4 *As K → ∞, under the steady state conditions, the mean waiting time.* $\overline{W}_q$ *for customers of the M/M/1 queue under f-policy is given by*

$$\overline{W}_q = \frac{\mathrm{L}-\rho}{\lambda} = \frac{\rho}{\mu(1-\rho)}, \mathrm{L} = \frac{\beta\rho}{(1-\rho)^2} = \frac{\rho}{(1-\rho)} \tag{1.32}$$

We define the asymptotic total cost function as in Berman and Kim (1999, 2001) for the case of M/M/1 queue with attached inventory under f + -policy:

$$\mathrm{TEC} = \overline{I}\ \mathrm{h} + \mathrm{MRR\ A} + \overline{W}_q v_w = [(Q-1)/2+\rho]h + (\lambda/Q)A + [\rho/\mu(1-\rho)]V_w$$

The optimal order quantity is the classical economic order quantity.
$\mathrm{Q}* = \sqrt{\frac{2\lambda A}{h}}$
Total profit = λ R − TEC.

1.5 SCM Attached $M/M_1 + M_2/1/K$-rule/f_2-Policy with Dissatisfied Customers

This section deals with the same $M/M_1 + M_2/1/K$-rule/f_2-policy queueing-inventory system under a new f_2-policy. This f_2-policy retains all norms of the threshold version of the f_1-policy with an additional constraint over the states of the stationary **Z**-process.

f_2-policy

(i) f_2- policy retains all standards from the threshold version of f_1-policy
(ii) Customers are served independently of the warehouse inventory process. It ensures that the Stationary queue length distributions {P(**N** = n) = π_n: n = 0, 2, …} and the inventory level distribution {P(**I** = j) = α_j: j = 0,1, 2,…Q} are independent probability distributions.

We now address some of the expected issues by the action of the additional norm (ii) of the new f_2-policy. If the state of process **Z** changes to (**N** = n, **I** = j) at the time the service ends the next service will start immediately for every (n j) ∈ $\mathbf{E}_f$. Also, at the service completion point if the server moves to process state (**N** = n > 1, **I** = 0) not only is the order placed immediately but the front-line client also receives service in the meantime (i.e., the time it's out of stock) and leaving the system with no product for which the customers have been queued for some time before their service begins. We call this category of customers who leave the system without a unit of the inventory as dissatisfied customers.

Let $\rho_1 = \frac{\lambda}{\mu_1}$ and $\rho = \frac{\lambda}{\mu+\mu_1}$. We consider all these variations over the level independent sub-matrices of the infinitesimal generator $\mathbf{R}$ of the $\mathbf{Z}$-process:

$$\boldsymbol{A}_0 = \begin{pmatrix} \lambda & 0 & \cdots & 0 & 0 \\ 0 & \lambda & \cdots & 0 & 0 \\ 0 & 0 & \ddots & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \lambda \end{pmatrix}, \boldsymbol{A}_2^{(\mu)} = \begin{pmatrix} \mu & 0 & \cdots & 0 & 0 \\ \mu & 0 & \cdots & 0 & 0 \\ 0 & \mu & 0 & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \mu & 0 \end{pmatrix}$$

$$\boldsymbol{A}_1^{(\boldsymbol{\mu})} = \begin{pmatrix} -(\boldsymbol{\gamma}+\boldsymbol{\lambda}+\boldsymbol{\mu}) & 0 & 0 & \cdots & \boldsymbol{\gamma} \\ 0 & -(\boldsymbol{\mu}+\boldsymbol{\lambda}) & 0 & \cdots & 0 \\ 0 & 0 & -(\boldsymbol{\mu}+\boldsymbol{\lambda}) & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & -(\boldsymbol{\mu}+\boldsymbol{\lambda}) \end{pmatrix},$$

and

$$\mathbf{R} = \begin{array}{c} \\ \\ L_1 \\ L_2 \\ \vdots \\ L_{K-1} \\ L_K \\ L_{K+1} \\ L_{K+2} \\ \vdots \\ \vdots \\ \vdots \end{array} \begin{array}{c} \begin{array}{cccccccccc} L_0 & L_1 & L_2 & \cdots & L_{K-1} & L_K & L_{K+1} & L_{K+2} & L_{K+3} & \cdots \end{array} \\ \begin{pmatrix} (\boldsymbol{A}_1+\boldsymbol{A}_2) & \boldsymbol{A}_0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots \\ \boldsymbol{A}_2 & \boldsymbol{A}_1 & \boldsymbol{A}_0 & \ddots & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \boldsymbol{A}_2 & \boldsymbol{A}_1 & \ddots & 0 & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & \mathbf{A}_1 & \boldsymbol{A}_0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & \boldsymbol{A}_2 & \boldsymbol{A}_1 & \boldsymbol{A}_0 & 0 & 0 & \ddots \\ 0 & 0 & 0 & \cdots & 0 & \boldsymbol{A}_2 & \boldsymbol{A}_1 & \mathbf{A}_0 & 0 & \ddots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \mathbf{A}_4 & \mathbf{A}_3 & \mathbf{A}_0 & \ddots \\ \vdots & \vdots & \ddots & \ddots & 0 & 0 & 0 & \mathbf{A}_4 & \mathbf{A}_3 & \ddots \\ \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 & 0 & \mathbf{A}_4 & \ddots \\ \vdots & \vdots & \vdots & \cdots & 0 & 0 & 0 & 0 & \ddots & \ddots \end{pmatrix} \end{array}$$

This means that we now deal with the same $M/M_1 + M_2/1/K$-rule based queueing-inventory system under the new f_2-policy. We have same the inter-level generator matrix $\mathbf{A}$ as before. Thus,

$$\alpha_0 = \frac{\mu}{\mu + Q\gamma}, \quad \text{and} \quad \alpha_j \frac{\gamma}{\mu + Q\gamma} \quad \text{for} \quad j = 1, \ldots, \mathrm{Q}$$
$$\alpha_0^* = \frac{\mu + \mu_1}{\mu + \mu_1 + Q\gamma}, \quad \text{and} \quad \alpha_j^* = \frac{\gamma}{\mu + \mu_1 + Q\gamma} \quad \text{for} \quad j = 1, \ldots, \mathrm{Q}$$

So, the **Z** process is positive recurrent under the new threshold type f_2-policy if and only if

$$\theta = (\alpha * \boldsymbol{A}_0\boldsymbol{e})/(\alpha * \boldsymbol{A}_4\boldsymbol{e}) = \frac{\lambda}{\mu_2} < 1, \quad \theta < 1 \Rightarrow \lambda < \mu_2 \tag{1.33}$$

where $\mu_2 = \mu + \mu_1$. We also assume $\theta_1 = \frac{\lambda}{\mu_1} < 1$ for further analysis.

Theorem 1.2 *The steady state distribution* $\boldsymbol{\Pi}$ *of Z (now the new under* f_2*-policy) is given.*

$$\boldsymbol{\pi}_\mathrm{n} = \begin{cases} \beta\theta_1^{\mathrm{n}}\boldsymbol{\alpha}, \boldsymbol{\alpha} = \left(\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_\mathrm{Q}\right) \text{ for n = 0,1, 2, } \ldots,\mathrm{K} \\ \beta\theta_1^{\mathrm{K}+1}\theta^{[\mathrm{n}-(\mathrm{K}+1)]}\boldsymbol{\alpha}*, \boldsymbol{\alpha}* = \left(\alpha_0^*, \alpha_0^*, \alpha_0^*, \ldots, \alpha_\mathrm{Q}^*\right) \text{ for n = (K + 1), (K + 2), } \ldots, \end{cases}$$
$$\beta = \frac{(1-\theta)(1-\theta_1)}{(1-\theta)\left(1-\theta_1^{(\mathrm{K}+1)}\right) + (1-\theta_1)\theta_1^{(\mathrm{K}+1)}} \tag{1.34}$$

Proof Solving $\boldsymbol{\Pi}\mathbf{R} = 0$, we obtain that.

$$\begin{aligned} &\pi_0(\mathbf{A}_1 + \mathbf{A}_2) + \pi_1\mathbf{A}_2 = 0 \\ &\pi_{n-1}\mathbf{A}_0 + \pi_n\mathbf{A}_1 + \pi_{n+1}\mathbf{A}_2 = 0 \text{ for n} = 1, 2, \ldots \mathrm{K} \\ &\pi_K\mathbf{A}_0 + \pi_{K+1}\mathbf{A}_1 + \pi_{K+2}\mathbf{A}_4 = 0, \text{ for n} = \mathrm{K} + 1 \\ &\pi_{n-1}\mathbf{A}_0 + \pi_n\mathbf{A}_3 + \pi_{n+1}\mathbf{A}_4 = 0 \text{ for n} = (\mathrm{K}+2), (\mathrm{K}+3), \ldots \end{aligned} \tag{1.35}$$

Adding all equations of (1.35) for n $\in \mathbf{N}_0$, we obtain

$$\sum_{n=0}^{K+1} \boldsymbol{\pi}_n(\boldsymbol{A}_0 + \boldsymbol{A}_1 + \boldsymbol{A}_2 = \boldsymbol{A}) + \sum_{n=K+2}^{\infty} \boldsymbol{\pi}_n(\boldsymbol{A}_0 + \boldsymbol{A}_3 + \boldsymbol{A}_4 = \boldsymbol{A}*) = 0$$

This implies that $\boldsymbol{\pi}_n\boldsymbol{A} = 0$ for n = 0, 1, 2, …, (K + 1) and $\boldsymbol{\pi}_n\boldsymbol{A}^* = 0$ for n = (K + 2), (K + 3), ….

Thus, it is observed that the non-negative vector $\boldsymbol{\pi}_n$ is proportional to the vector $\boldsymbol{\alpha}$ for n = 0, 1, 2, …, (K + 1) and that the non-negative vector $\boldsymbol{\pi}_n$ is proportional to the vector α^* for n = (K + 2), (K + 3), …. As before, we let $\boldsymbol{\pi}_n = \beta\boldsymbol{\alpha}\theta_1^n$ for n = 0, 1, 2, … (K + 1), $\boldsymbol{\pi}_n = \beta\alpha^*\boldsymbol{\theta}^{\boldsymbol{n}}$ β >0. From the normalizing equation, we can find the value of β. Thus, the results of (1.34) can be proved.

The marginal mean queue length in the system L and (queue only) L_q are given by

$$\begin{aligned}\text{L}=&\left(L_1=\sum_{n=0}^{K}\text{n}\,\boldsymbol{\pi}_n\boldsymbol{e}\right)+\left(L_2=\sum_{n=K+1}^{\infty}\text{n}\,\boldsymbol{\pi}_n\boldsymbol{e}\right)\\ =&\beta\theta_1\left[\frac{1-\theta_1^{K+1}-(K+1)(1-\theta_1)\theta_1^K}{(1-\theta_1)^2}\right]\\ &+\beta\theta_1\left[\frac{\theta\theta_1^K+(K+1)(1-\theta)\theta_1^K}{(1-\theta)^2}\right]\end{aligned}\tag{1.36}$$

$$L_q=\sum_{n=1}^{K}(\text{n}-1)\boldsymbol{\pi}_n\boldsymbol{e}+\sum_{n=K+1}^{\infty}(\text{n}-1)\boldsymbol{\pi}_n\boldsymbol{e}=\text{L}-(1-\beta)$$

The waiting time in the queue (excluding the service time) W_q is given by $L_q = \lambda W_q$.

Corollary 1.5 *The mean reorder rate (MRR) is given by.*

$$\begin{aligned}\text{MRR}=&\beta\left(\frac{\lambda\mu}{\mu+Q\gamma}\right)+\beta\frac{\left(1-\theta_1^K\right)}{(1-\theta_1)}\left(\frac{\lambda\gamma}{\mu+Q\gamma}\right)\\ &+\beta\frac{\theta_1^{K+1}}{(1-\theta)}\left(\frac{\lambda\gamma}{\mu+\mu_1+Q\gamma}\right)\end{aligned}\tag{1.37}$$

Proof Since the reorder is triggered for the Z-process under the K-rule and f_2-policy just after reaching the state (n > 0, 0) by a service completion from the state (n + 1,1) at rate μ for n = 1, 2, …, K and at rate ($\mu + \mu_1$), for n = (K + 1), (K + 2), …, and the state 1.0) can be reached at rate λ, the MRR is computed for the Z process as follows:

$$\text{MRR}=\lambda\pi(0,0)+\sum_{n=1}^{K}\mu\pi(n+1,1)+\sum_{n=K+1}^{\infty}(\mu+\mu_1)\pi(n+1,1)$$

which leads to the stated result of (1.37).

Let D = Expected value of the dissatisfied number of customers. Then, using the indicator function $P(I=0/n>0)=1$ and $P(I=j/n>0)=0$ for j = 1, 2,.., Q, we find that

$$\begin{aligned}D=\mu P(I=0/n>0)=&\mu\left[\sum_{n=1}^{\infty}\pi(n,0)\right]\\ =&\mu\left[\beta\alpha_0\frac{\left(1-\theta_1^{K+1}\right)}{1-\theta_1}+\beta\alpha_0^*\frac{\theta_1^{(K+1)}}{1-\theta}\right]\end{aligned}\tag{1.38}$$

Lemma 1.2 *The average on hand inventory, say* $\bar{\mathbf{I}}$*, is given by.*

$$\overline{\boldsymbol{I}} = \beta\alpha_1 \frac{\left(1-\theta_1^{K+1}\right)}{1-\theta_1} + \beta\alpha_1^* \frac{\theta_1^{(K+1)}}{1-\theta} \tag{1.39}$$

Proof The average on hand inventory is computed by.

$$\bar{\mathbf{I}} = \sum_{n=0}^{K}\sum_{j=1}^{Q} j\pi(n,j) + \sum\nolimits_{n=K+1}^{\infty}\sum_{j=1}^{Q} j\pi(n,j)$$

Substitution for $\pi(n, j)$, $(n, j) \in \mathbf{E}_f$ will lead to (1.39).

Total Expected Cost (TEC):

Let, h = fixed holding cost per item and time unit,
u = Fixed cost for each replenishment order.
$\mathbf{v_w}$ **=** Waiting cost per customer and time unit.
g = cost of lost sales per item and time unit,
With these costs, the total expected cost (TEC) is given

$$\text{TEC} = \overline{I}\ \text{h} + \text{MRR u} + \overline{W}_q v_w + gD \tag{1.40}$$

1.5.1 Optimum Order Quantity "Q*"

Let us find an optimum order quantity, say Q^*, which minimizes the TEC of (1.40). Assume that the order quantity Q increases over a period from Q = 4 to Q = 12. Let the input values of queueing-inventory system be $\lambda = 3.75$, $\mu = 5.0$, $\mu_1 = 0.5$, $\gamma = 4.2$ and the costs of the inventory system be h = 625.25, u = 700.2, Vw = 380.2 and g = 295.0.

As the Q increases from 4 to 12, we expect that.

TEC1 = $\overline{I}$ h increases, TEC2 = MRR u decreases,

TEC3 = $\overline{W}_q v_w$ neither increases nor decreases and TEC4 = gD decreases.

Figure 1.2 shows the decreasing or increasing properties between Q values and the total expected costs. In Fig. 1.2a, all four components of the TEC of (1.40) are represented while Q varies from 4 to 12. Inspecting Fig. 1.2a, we can suggest a local optimum value for Q.

(i) between TEC1 (Inventory holding cost) and TEC2 (Ordering cost) as Q* = 9
(ii) between TEC1 (Inventory holding cost) and TEC4 (dissatisfaction cost) as Q* = 5
(iii) between TEC3 (waiting cost) and TEC4 (Ordering cost) as Q* = 9

Inspecting Fig. 1.2b which shows relationship between the sums of (TEC1 + TEC3) and (TEC2 + TEC4) as Q increases. Inspecting Fig. 1.2b, we can trace a local optimum value for Q as Q* = 9.

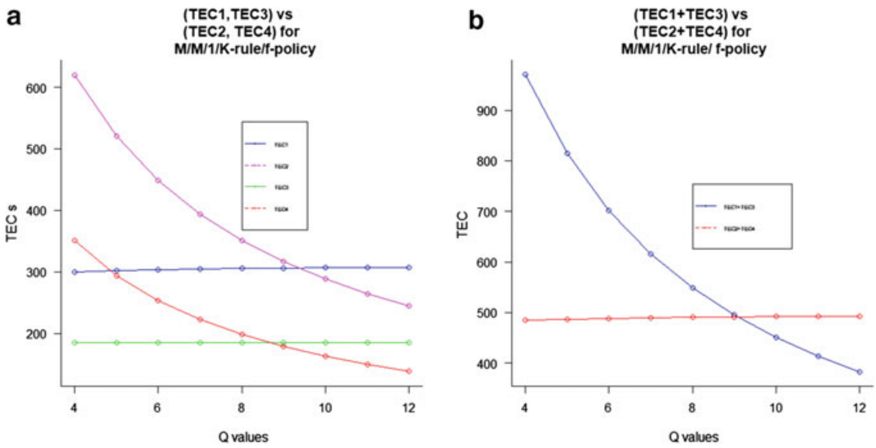

Fig. 1.2 Graph of Q versus TEC for $\lambda = 3.75$, $\mu = 5.0$, $\mu_1 = 0.5$, $\gamma = 4.2$, K = 4 and h = 625.25, u = 700.2, Vw = 380.2 and g = 295.0. a (TEC1, TEC3) vs (TEC2, TEC4) for M/M/1/K-rule/f-policy. b (TEC1 + TEC3) vs (TEC2 + TEC4) for M/M/1/K-rule/f-policy

Here the term 'optimum' value means the 'Q' value in Fig. 1.2 against which the values of (TEC1 + TEC2) and (TEC3 + TEC4) are almost equal. Similar type of an optimum is suggested by inspecting the Fig. 1.2a also.

1.6 Conclusion

This paper extended the investigations of queueing-inventory management due to (Schwarz, M, and Daduna, H, 2006), replacing the $M/M/1/\infty$/FCFS facility by an $M/M_1 + M_2/1/K$-rule/FCFS facility. A typical type of supply chain (SC) model was discussed as an integrated queueing-inventory system (IQIS). It has a distribution centre (DC) for delivering the orders to a retailer vendor (RV) that runs an $M/M_1 + M_2/1/K$-rule based service unit. The ordering policy is of threshold type i.e., (queue length, Inventory level) = ($n > 0$,0) for placing an order of Q units. This policy monitors the periodical changes of queue length of demands and the inventory status and thus fixing the reorder instant ($n > 0$, 0) as an ideal point during stock out periods. The three sequences of replenishment periods, inter-arrival times and service times are assumed to be mutually independent and exponential processes. The primary goal of this paper is to obtain the optimum order quantity Q* and the minimum total expected cost TEC(Q*).

Using matrix analysis, we generated a Markov process $\mathbf{Z} = (N, I)$ for supply chain management and generated its stationary distributions, that is, for the probability functions $P(N = n, I = j)$ explicit expressions are obtained. We can read more structural information and perform profitable analysis to achieve our goal through the obvious shape of the steady-state distribution.

Different performance metrics are computed for the stationary system, and the total expected cost is calculated for a given dataset with varying $Q \in \{4, 5, \ldots, 12\}$. The results are illustrated numerically indicating the optimum results, *i.e.,* Q^* and $TEC(Q^*)$. All these ideas were also discussed for a case of independent queueing system and inventory system in IQIS.

As a concluding remark we hope to apply the proposed methodology further for various advanced types of queueing-inventory systems of G/G/c or M/G/c types operating under (s, S) inventory policies.

References

Berman O, Kim E (1999) Stochastic models for inventory management at service facilities. Comm Stat Stoch Model 15(4):695–718

Berman O, Kim E (2001) Dynamic order replenishment policy in internet-based supply chains. Math Meth Oper Res 53:371–390

Daduna H, Szekli R (2006) M/M/1 Queueing systems with inventory. Queueing Syst 54:55–78

Dey O, Giri BC (2014) Optimal vendor investment for reducing defect rate in a vendor-buyer integrated system with imperfect production process. Int J Prod Econ 155:222–228

Hsien-Jen LIN (2013) An integrated supply chain inventory model with imperfect-quality items, controllable lead time and distribution-free demand. Yugos J Oper Res 23(1):87–109

Latouche G, Ramaswami V (1999) Introduction to matrix analytic methods in stochastic modeling. In: ASA-SIAM series on statistics and applied probability, SIAM, Philadelphia PA

Lin YJ (2009) An integrated vendor-buyer inventory model with backorder price discount and effective investment to reduce ordering cost. Comput Ind Eng 56(4):1597–1606

Mohebbi E, Posner MJM (1998) A continuous-review inventory system with lost sales and variable lead time. Nav Res Logist 45:259–278

Neuts MF (1981) Matrix geometric solutions in stochastic models—an algorithmic approach. John Hopkins University Press, Baltimore

Schwarz M, Daduna H (2006) Queueing systems with inventory management with random lead times and with backordering. Math Meth Oper Res 64:383–414

Sivasamy R (2021) An M/M(1)+M(2)/1 Queue with a state dependent K-policy. J xi'an Univ Archit Technol XIII(I):111–124

Sivasamy R, Thillaigovindan N, Paulraj G, Paranjothi N (2019) Quasi-birth and death processes of two server queues with stalling. Opsearch. https://doi.org/10.1007/s12597-019-00376-1

Chapter 2
Nature-Inspired Optimization for Inventory Models with Imperfect Production

Neetu Singh, Madhu Jain, and Praveendra Singh

Abstract Inventory management plays a crucial role in a production system, as such the system designer needs to design the inventory model which takes care of all aspects of the concerned system. It is evident that production systems unescapably face process deterioration, which is a significant concern for the system designers to ensure the proper planning of the concerned inventory systems. This paper presents a literature survey of studies on inventory models with imperfect production systems. A considerable amount of work has been done on this front as this is a realistic phenomenon encountered in the production systems. We have reviewed the prominent research on inventory models with imperfect production systems and nature-inspired optimization techniques. A brief discussion and review of literature on the nature-inspired optimization algorithms and their applications in inventory control are presented. The present article will be beneficial for the academicians and practitioners to get updated accounts of the works done on inventory models with imperfect production systems.

Keywords Imperfect production · Inventory management · Economic order quantity · Nature Inspired Optimization

2.1 Introduction

Inventory theory plays a crucial role in production and operations management due to its suppleness to supervising manufacturing systems and, other industries,

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and maximizing service level to the clients. Inventory models deal with the proper stock management to maximize the customer service. The revenue of any production system depends on various factors, including optimal inventory control. The total inventory cost needs to be minimized without affecting the customer service by maintaining proper inventory to fulfil the demand. For the balance between under stocking and over stocking, the appropriate planning of the concerned system is of great importance. The typical Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models remain of great importance. Still, their use in real-time systems is limited as the assumptions are very simple and performed under ideal conditions. In EOQ inventory models, we study the optimized inventory level that minimizes the total costs. However, in EPQ models, the optimal production lot size under specified manufacturing environments is obtained. The inventory control and maintenance are the main issues of the product manufacturers and have attracted many researchers to analyze the production inventory systems.

There is a considerable amount of work available on different inventory models for the production systems. Several models are coming up to deal with the inventory control of real-time production systems. In a real-time manufacturing/production organization, the units produced may be defectives owing to diverse issues in the system. The challenging domains of the current age of progressive production environment are machine failure, imperfect quality products, quality deterioration and some unavoidable issues like labour walkouts, natural calamities, machine failures, resources shortage and transport issues, etc. More advanced inventory models are required to design real-time production systems. The work on imperfect production scenarios is increasing as it is a real phenomenon which greatly affects the customers' service. Over the periods, frequent researches have been carried out to develop EPQ models which are more realistic by incorporating the imperfections of the process of production.

In this study, we have reviewed the research articles on inventory control of imperfect production systems by using nature-inspired optimization techniques. A brief introduction of some commonly used nature-inspired metaheuristics algorithms is provided. Some inventory studies which have used nature-inspired algorithms as an optimization tool are also presented. The current article is structured in different sections. An inventory model for the imperfect items is briefly described in Sect. 2.2. We have presented a survey of the literature on imperfect production by process deterioration in Sect. 2.3. A brief account of nature-inspired optimization (NIO) algorithms and some inventory studies via NIO algorithms are provided in Sects. 2.4 and 2.5, respectively. The important works on inventory management with imperfect production are reviewed in Sect. 2.6. At last, the highlights and scope of researches on the EPQ models are discussed in Sect. 2.7.

2.2 An Imperfect Production Inventory Model

In most of the researches on inventory, the units produced in a production system are perfect and can be stored for a long duration. In a real-time production system, a proportion of defectives generates during the production process, which is called an imperfect production system. Figure 2.1 shows the inventory level for a general imperfect production system (IPS). The produced items decay with a given deterioration rate. The production and demand rates are considered to be time dependent.

The notations used to formulate the stock level of the inventory are given below:

$P(t)$:	Production rate
α:	Fraction of perfect produced items
$D(t)$:	Demand rate
$\theta(t)$:	Deterioration rate
$r(t)$:	Rework rate
I_m:	Maximum inventory level

The differential equations governing the stock level are as follows:

$$\frac{dI(t)}{dt} = \alpha P(t) - D(t) - \theta(t)I(t); \; 0 \leq t \leq T_1 \tag{2.1}$$

$$\frac{dI(t)}{dt} = r(t) - D(t) - \theta(t)I(t); \; T_1 \leq t \leq T_2 \tag{2.2}$$

$$\frac{dI(t)}{dt} = -D(t) - \theta(t)I(t); \; T_2 \leq t \leq T_3 \tag{2.3}$$

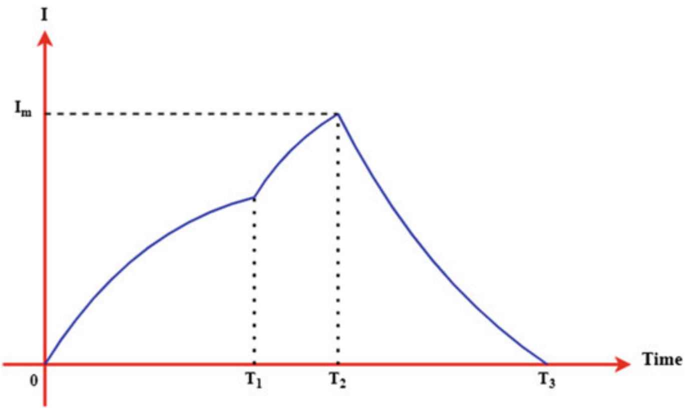

Fig. 2.1 Inventory level for an IPS with rework

subject to the initial and boundary conditions

$$I(0) = 0, I(T_2) = I_m, I(T_3) = 0 \tag{2.4}$$

The various cost components concerned with the imperfect production inventory model can be obtained by using the stock level function $I(t)$. A special case of the above model can be found in Widyadana and Wee (2012). The special case is obtained by considering $P(t) = P$, $\theta(t) = \theta$, $D(t) = d$, $r(t) = \alpha P$.

2.3 Inventory Production Systems with Process Reliability

In the present competitive situation, production business administrators need to provide their clients best services. However, the production facilities are subject to deterioration due to ageing. Process reliability is a significant aspect of imperfect production situations, and the production system costs and profits are highly affected by it. It is almost impossible to have a system which produces all the perfect items. The classical EPQ model assumes that all the manufactured items are faultless, which is inaccurate for real-time manufacturing situations.

It is worthwhile to have a look at the EPQ models by considering the production process reliability (*R*) and demand (D). The function for the total cost (TC) incurred in the production cycle is given by.

TC = Set-up cost + Holding cost + Production cost.

The production cost can be obtained as follows:

Production cost = $a(1\text{-}R)^b D^c$, where *a*, *b*, and *c* are non-negative real numbers.

A summary of EPQ models is presented in Table 2.1. Kim et al. (2001) considered process deterioration in the EPQ model. Goyal and Cardenas-Barron (2002) proposed a generalization of the study considered by Salameh and Jaber (2000). Ben-Daya (2002) gave a unified model to study the preventive maintenance of EPQ with imperfect production and increasing hazard rate. Chung and Hou (2003) investigated the inventory model with process deterioration and process shifts and found the optimal production run length. Jamal et al. (2004) considered process deterioration with two rework policies. Lin et al. (2003) analyzed inventory management for the imperfect manufacturing systems considering periodic inspection plans and assuming that the defective objects are reworked before or after a sale.

Hsieh and Lee (2005) investigated inventory models to study a production system which produces a proportion of imperfect items. Wang (2005) recommended product inspections for the imperfect production system. Contrary to other works with full inspection policy to eliminate all the defective items, the inspections were performed when the production run ends to avoid inspection costs of a complete inspection. Rahim and Al-Hajailan (2006) examined imperfect production situations, including shortages and a time-dependent defective rate on the optimal production run. They

Table 2.1 An overview on production inventory with process reliability

Authors	Contributions
Porteus (1986)	First considered EPQ control model
Rosenblatt and Lee (1986)	Suggested that for an imperfect manufacturing organization, the optimal manufacturing size is lesser as compared to a manufacturing organization having no defects
Lee and Rosenblatt (1987)	Examined the effect of scheduled inspection on the optimal production quantity
Lee and Rosenblatt (1989)	Investigated production planning and maintenance planning considering the inspection, process restoration and detection delay
Cheng (1989)	Studied process deterioration in economic production quantity model as an unconstrained geometric programming problem
Lin et al. (1991)	Modified the work of Lee and Rosenblatt (Porteus 1986; Rosenblatt and Lee 1986) for general case by including increasing failure rate
Lee and Park (1991)	Generalized the model presented by Rosenblatt and Lee (1986) with the inclusion of reworking cost and warranty cost. They considered the deteriorating production system with periodic inspection and process restoration and developed expected annual cost, and determined production cycle and inspection intervals
Cheng (1991a)	Studied the imperfect production processes and demand-dependent unit production cost by formulating the inventory model as a geometric program
Cheng (1991b)	Considered EPQ model with imperfect production where unit production cost depends on demand
Khouja and Mehrez (1994)	Developed an inventory model for deteriorating quality products with a variable production rate
Liou et al. (1994)	Investigated the Economic Manufacturing Quantity (EMQ) model with inspection errors
Tseng (1996)	Recommended preventive maintenance over inspection for the improvement of the system reliability
Makis (1998)	Investigated EMQ model for production process with random deterioration and inspections
Hariga and Ben-Daya (1998)	Developed EPQ model under the feature of imperfect production with time to process shift as general distribution
Kim and Hong (1999)	Proposed an EMQ model including process deterioration state shift time
Boone et al. (2000)	Studied the cost benefit analysis of EPQ with imperfections in the manufacturing process
Salameh and Jaber (2000)	Analyzed imperfect production situation considering that some imperfect quality units with minor issues are not defective and proposed to use them in some other less restrictive inventory situation/to sell in the market after 100% screening

mentioned that after out-of-control stage, the percentage of defective rate should increase. Hejazi et al. (2008) obtained the economic production lot size for the inventory system by considering the inspections. Chung et al. (2009) investigated inventory models with imperfect quality and two warehouses. Some other worth mentioning works in the imperfect production systems are due to Wang and Sheu (2001), Rahim and Ben-Daya (2001), Tripathy et al. (2003), Wang (2004), Rahim and Ohta (2005), Wee et al. (2005), Jaber (2006), Lin and Gong (2006), Rau and OuYang (2007), Jaber et al. (2008), Chakraborty et al. (2008), Chakraborty et al. (2009), Sarkar et al. (2010) and many others.

Sana (2010a) studied an EPQ model with imperfect units and provided optimality conditions. Sana (2010b) considered a deteriorating inventory model with imperfect manufactured items. They assumed that the imperfect products were reworked and returned to their actual worth and proposed a production inventory model to study a three-level supply chain for both kinds of items. Roy et al. (2011) considered an EOQ model consisting of a certain percentage of imperfect products with an inspection. Khan et al. (2011) used a similar approach as Salameh and Jaber (2000) for the EOQ and EPQ model with imperfect processes considering a faulty inspection. Tripathy et al. (2011) suggested the necessity to study reliability and instantaneous receipt in an imperfect production system. Tripathy et al. (2011) considered the optimal order quantity in a fuzzy environment for the imperfect manufacturing scenario by incorporating reliability. Chakraborty et al. (2012) developed an inventory study incorporating process deterioration and machine breakdown. Krishnamoorthi and Panayappan (2012) analyzed an EPQ model with imperfect production, inspection error and return of defective products. Sarkar (2012) analyzed an EMQ model in an imperfect manufacturing environment with inflation and rework. They considered that for increasing time, there is an increase in the production of imperfect quality objects, which can be reduced if the system is more reliable. Sana (2012a) developed an EOQ model with imperfect products considering that the imperfect products are traded at a lesser amount after a complete screening procedure. Later Sana (2012b) considered an inventory model for a deteriorating production with shortages and periodic maintenance, including free minimal repair warranty and minimizing the cost per unit product. Hsu and Hsu (2012, 2013a, b) developed inventory models for imperfect production considering different scenarios. Taleizadeh (2013) investigated the imperfect manufacturing scenario with multiple products and rework. Assuming that the remaining good quality items may also become imperfect after deterioration, Tai (2013) analyzed two EPQ models for the imperfect products with imperfect inspection and rework. Masud et al. (2014) analyzed an inventory control model for imperfect quality production to minimize the total inventory cost. Pal et al. (2014) gave an imperfect EPQ model to maximize the profit. Roul et al. (2015) investigated an imperfect production inventory management model with multiple products to improve reliability and maximize returns. Paul et al. (2015) investigated a disruption recovery model for an imperfect production system. Lin and Srivastava (2015) proposed an imperfect production inventory model, including quantity discounts, maintenance and a two-warehouse scenario.

Kang et al. (2016) conducted a mathematical study for the work-in-process inventory model with defective items following random distributions. Kim and Sarkar (2017) analyzed an imperfect manufacturing system with multiple stages with controllable lead time using a stochastic inventory model and used an analytical method to get the optimal solution. Cheikhrouhou et al. (2018) proposed an inventory model including inspection considering the return of imperfect objects. Shah et al. (2018) considered price and stock dependent demand for an imperfect EPQ model, including returned/rework inventories. Kim et al. (2018) proposed an upgraded method to analyze imperfect products in the long run. Malik and Sarkar (2019) investigated an inventory problem for a supply chain to control the lead-time variability by taking two dissimilar transportation modes for two reliability cases for the seller. Malik and Sarkar (2020) analyzed a real-time disruption management plan for an imperfect production system with multiple products. Mandal and Pal (2021) investigated an imperfect EPQ model subject to breakdowns, assuming that the client's demand is dependent on selling cost and advertisement. The production shift is of a random time following two parameter Weibull distribution. Giri and Das (2022) analyzed an imperfect production-inventory model with one manufacturer and a single retailer. They assumed that after receiving the order size from the manufacturer in unequal-sized batches, the retailer separates the defective items by performing a completely reliable screening and determine the optimal pricing, advertising and inventory decisions.

2.4 Nature-Inspired Optimization at a Glance

There has been a growing interest in using nature-inspired approaches to solve real-life decision problems for the last two decades. Genetic algorithm (1975), particle swarm optimization (1995), differential evolution (1995), Artificial bee colony (2007), grey-wolf optimization (2014), etc. are the most popular nature-inspired algorithms. These algorithms are based on natural entities' processes to obtain their goals. A brief outlines of some commonly used metaheuristics for the inventory control are as follows:

- ***Genetic algorithm (GA)*** works on the principle of natural evolution. In GA, the fittest individuals are selected in a generation, and some biological operators viz. crossover and mutation are performed on the selected individuals. Thus, a new generation of individual solutions is obtained. Various kinds of crossover, mutation, and selection operators are provided in the literature (2007). A brief pseudo-code for the genetic algorithm is given in Table 2.2.
- ***Differential evolution (DE)*** is an enhanced version of GA which also performs based on mutation and crossover operators. The exploration and exploitation capabilities of DE are more than that of GA. A detail of operators used in DE can be extracted from Mallipeddi et al. (2011).

Table 2.2 Pseudocode for GA

```
Generate random individuals
Evaluate fitness of each individual
While termination condition is false do

      Choose parent individuals
      Perform recombination
      Mutate resultant children
      Evaluate fitness of new individuals
      Select individuals for next generation
 end
```

- ***Particle swarm optimization (PSO)*** is a swarm intelligence-based metaheuristic approach. PSO mimics the movement of a swarm, in which the group of birds change their position and direction in order to locate the food source. The velocity and positions of particles are updated by evaluating the local best and global best particle positions. A brief pseudo code for implementation of PSO algorithm is provided in Table 2.3.

The velocity (v_i) and position (X_i) equations for the particles at ith generation are given as (Kennedy and Eberhart 1995):

$$v_i = v_{i-1} + A_1 r_1 (p_{best} - X_{i-1}) + A_2 r_2 (g_{best} - X_{i-1}) \tag{2.5}$$

Table 2.3 Pseudo code for PSO

```
Initialize particle's position and velocities
while termination condition is false do
      For i^th particle do
          Update velocity using (1)
          Update position using (2)
          Calculate fitness f(X_i)
           if f(X_i) < f(p_best)
               p_best = X_i
           end
           if f(X_i) < f(g_best)
               g_best = X_i
           end
      end
 end
```

$$X_i = X_{i-1} + v_i \tag{2.6}$$

where,

A_l:	Acceleration factors, $l = 1, 2$
r_l:	Random number generated from uniform distribution in $(0, 1)$; $l = 1, 2$
g_{best}:	Global best particle
p_{best}:	Personal best particle

- ***Artificial bee colony (ABC)*** and ***grey-wolf optimization (GWO)*** are also swarm intelligence algorithms and are used by many researchers in different contexts (Boussaïd et al. 2013; Yang 2020).
- There are many other nature-inspired algorithms which are also used to study complex inventory systems. Some of these algorithms are: dragonfly algorithm (DA) (Meraihi et al. 2020), bat algorithm (BA) (Yang and He 2013), ant colony optimization (ACO) (Dorigo et al. 2006), firefly algorithm (FA) (Kumar and Kumar 2021), harmony search (HS) (Manjarres et al. 2013), moth flame optimization (MFO) (Mirjalili 2015), and many others.

2.5 Important Contributions on Nature-Inspired Optimization for Inventory Control

Many researchers analyzed the optimal ordering policies for many complex inventory systems using nature-inspired algorithms. A list of abbreviations used to mention important contributions of eminent researchers are given in Table 2.4. The important contributions to inventory problems via GA and PSO are listed in Tables 2.5 and 2.6, respectively. Table 2.7 presents some inventory control articles using DE and ABC algorithms. Some other nature-inspired algorithms and their applications in inventory control are listed in Table 2.8.

2.6 Economic Production Quantity (EPQ) Models and NIO Algorithms

For the complex inventory problems arising in production systems, finding the analytical solution is difficult and time-consuming due to fact that the traditional mathematical methods have their limitations. For such situations, we have a class of meta-heuristic approaches known as nature-inspired intelligence (NII). Using GA, Pasandideh and Niaki (2008) investigated an EPQ model for multiple items. Pal et al.

Table 2.4 Some abbreviations (Abbr.) used throughout the article

Abbr.	Key term	Abbr.	Key term	Abbr.	Key term
AD	Advertisement Dependent	FP	Finite Planning	QPSO	Quantum-behaved PSO
AM	Amelioration	FS	Fuzzy Simulation	RC	Real Coded
AP	Advance Payment	HN	Hopfield Neural network	RR	Remanufacturing & Return
AU	All Unit discount	HY	Hybrid algorithm	RP	Random Planning
BC	Binary Coded	IC	Interval Compared	SC	Supply Chain
CBC	Chaotic Bee Colony	IN	Inflation	SD	Stock Dependent
CF	Constriction Factor	LS	Lost Sale	SN	Stackelberg–Nash equilibrium
CD	Constant Deterioration	LSA	Local Search Algorithm	SS	Supplier Selection
CM	Contractive Mapping	LT	Lead Time	TC	Trade Credit
CP	Constant Production	MI	Multi Items	TDE	Tournament DE
CPR	Cross Perishability	PB	Partial Backorder	TH	TS Holding cost
CR	Credit Dependent	PD	Price Dependent	TP	TS Purchasing cost
CS	Closed SC	PE	Promotional Efforts	TS	Time Sensitive
DL	Dynamic Lot-sizing	PR	Probabilistic Demand	TW	Two Warehouse
DP	Dynamic Production	PS	Pattern Search	VB	Vendor Buyer SC
DY	Dynamic Demand	PT	Preservation Technology	VF	Variation Free demand
FB	Full Backorder	QD	Quantity Discounts		
FM	Fuzzy Modelling	QM	Queueing model		

(2009) analyzed EPQ models with crisp and fuzzy parameters for freshly launched items with discounts and used GA to get the optimum solution. Dye and Hsieh (2010) extended the conventional EOQ model, including price and time-dependent demand and variable purchasing cost and obtained the optimal solution by particle swarm optimization (PSO) algorithm. Dye and Ouyang (2011) investigated the deterioration of the inventory model, including changing demand and trade credit financing using PSO. Bhunia et al. (2014) developed an EOQ inventory model with deteriorating items and demand depending on the stock level and proposed a discount strategy using different versions of PSO. Masud et al. (2014) developed a reliability-based model for the production inventory system and used metaheuristic approaches

Table 2.5 Optimization for inventory problems by using GA

Techniques	Authors	Decay	Demand	Shortages	Other features
RC	Pal et al. (2005)	CD	TS	PB	TW, FP
GA	Megala and Jawahar (2006)	×	VF	×	HN
RC	Maity et al. (2007)	CD	TS	×	MI, DP
IC	Moon et al. (2008)	×	VF	×	QD, MI, SS
FS	Das et al. (2010)	CD	VF	×	RP, CP, IN, FM
HY	Widyadana et al. (2010)	×	VF	×	MI, CP
BC	Shavandi et al. (2012)	CD	PD	PB	MI, CP
GA	Yang et al. (2012)	×	VF	FB	MI, SS, AU
IC	Guchhait et al. (2013)	×	TS	×	FM
BC	Lee et al. (2013)	×	DY	×	FP, QD
FS	Jana et al. (2014)	CD	SD	PB	MI, RP, IN, FM
LSA	Saracoglu et al. (2014)	×	PR	LS	MP, FP
CM	Chakraborty et al. (2015)	CD	SD	×	MI, FM, TC, VB
RC	Kumar and Kumar (2016)	TS	SD	×	TC, MS, IN
RC	Yang et al. (2019)	CD	VF	×	MI, PT, CPR
PS	Lashgari et al. (2019)	CD	DY	FB	MI, SS
GA	Klinmalee et al. (2020)	×	VF	×	MI, SS, LT
GA, PSO	Salehi Amiri et al. (2020)	×	PD	×	VB
GA	Fathi et al. (2021)	×	PR	LS	QM

viz., PSO and GA, and concluded that PSO results are superior over GA results and recommended PSO as a better choice for the inventory model under consideration.

Pirayesh and Poormoaied (2015) considered an EPQ model with multiple products, imperfect production and restricted production using a hybrid algorithm GPSO-LS created from GA and PSO. Nobil et al. (2016) solved a bi-level decision-making problem related to an EPQ model with imperfect production, multiple objects and consisting of several machines with varied production volumes. They proposed a hybrid genetic algorithm integrating genetic algorithm and derivatives method. The first-level solutions were obtained randomly, and the second-level solutions were obtained by applying the analytical method. Ruidas et al. (2017) dealt with an EOQ model consisting of imperfect quality units. The perfect and reworked items were used to fulfil the customer's demand, and the scrap items were sold

Table 2.6 Optimization for inventory problems by using PSO

Technique	Authors	Decay	Demand	Shortages	Other features
MO	Tsou (2008)	×	VF	×	LT
SO	Gaafar and Aly (2009)	×	DY	FB	DL
SO	Dye and Hsieh (2010)	CD	TS, PD	PB	FP, TP
SO	Yang and Lin (2010)	×	VF	×	SC
SO	Che (2012)	×	VF	×	SC, QD
CF	Dye (2012)	TS	TS, PD	PB	FP, TC
SO	Sue-Ann et al. (2012)	×	VF	×	VB
CF	Hsieh and Dye (2013)	CD	TS	×	FP, CP, PT
HY	Sadeghi et al. (2013)	×	VF	×	VB
QB	Bhunia et al. (2014)	CD	SD	PB	FM
MO	Mousavi et al. (2014a)	×	DY	PB	QD, FP, MI
HY	Sadeghi et al. (2014a)	×	VF	×	FM, VB
QB	Bhunia and Shaikh (2015)	CD	PD	PB	TW, TC
HY	Sadeghi et al. (2016)	×	VF	×	FM, VB
SO	Singh et al. (2016)	CD	TS	PB	IN
SO	Pakhira et al. (2017)	CD	TS	PB	PE, VB, FM
QB	Tiwari et al. (2017)	CD	SD	PB	TW, IN
HY	Beklari et al. (2018)	×	VF	×	MI, VB
QB	Mondal et al. (2019)	TS	PD, AD	×	AM, FM
QB	Shaikh et al. (2019a)	CD	PD	PB	AP, FM
QB	Shaikh et al. (2019b)	CD	SD	PB	FM, TW
QB	Das et al. (2020)	TS	PD	PB	PT
SO	Pakhira et al. (2020)	CD	SD, AD	PB	VB, TW, FM
QB	Rahman et al. (2020)	CD	CS	×	TC, FM, PT
QB	Shaikh et al. (2020)	CD	PD	PB	TC, FM
QB	Das et al. (2021)	TS	PD, SD	PB	TC, PT
CF	Singh and Kumar (2021)	CD	TS	×	TH

in the secondary market at salvage cost. They formulated the unconstrained inventory optimization problem and used the PSO technique for optimization. Al-Salamah (2018) investigated a production-inventory system with a single imperfect unreliable machine. The system was assumed to produce both perfect and imperfect items, and a rework procedure which was different from the main production process to restore the imperfect items. After the machine breakdown, it is sent for restoration, and the halted production is continued when the machine is fixed. As the concerned model becomes complex, they proposed a heuristic method viz. artificial bee colony algorithm to find an optimal solution. Bhunia et al. (2018) investigated the marketing strategies for the inventory system with single deteriorating item, finite storage capacity and shortages.

Table 2.7 Optimization for inventory problems by using DE and ABC

Technique	Authors	Decay	Demand	Shortages	Other features
DE	Lo (2010)	CD	VF	×	VB, FP
ABC	Taleizadeh et al. (2011a)	×	VF	×	QD, MI, FM
HY	Wang et al. (2012a)	×	VF	LS	MI, LT
FS	Wang et al. (2012b)	×	VF	PB	FM, LT
DE	Cui et al. (2014)	×	VF	×	LT, SS, RFID
CBC	Jain et al. (2015)	×	VF	×	AD, SC
DE	Parsopoulos et al. (2015)	×	DY	×	RR
HY ABC	Cui et al. (2017)	×	PR	×	CS
HY DE	Huang et al. (2019)	CD	VF	×	VB, FM
HY ABC	Pramanik and Maiti (2019)	CD	TS, PD	×	IN, TC, FP
HY ABC	Cui et al. (2020)	×	PR	LS	LT
DE	Mahmoodi (2020)	CD	PD	PB	SN
TDE	Manna et al. (2021)	CD	TS	PB	AU, TW

Table 2.8 Optimization for inventory problems by using miscellaneous nature-inspired algorithms

Technique	Authors	Decay	Demand	Shortages	Other features
HS	Taleizadeh et al. (2011b)	×	VF	×	MI, QD, FM
HS	Taleizadeh et al. (2011c)	×	PR	×	MI, VB,
HS	Mousavi et al. (2014b)	×	VF	PB	FM, FP
ACO	Roozbeh Nia (2014)	×	VF	FB	VB, FM, MI
BA	Sadeghi et al. (2014b)	×	VF	×	VB, Reliability
ACO	Chen (2018)	CD	PD	FB	VB
FA	Atabaki et al. (2019)	×	PD	×	CSC
FA	Khalifehzadeh and Fakhrzad (2019)	×	VF	FB	SC
GWO	Alejo-Reyes et al. (2020)	×	VF	×	SS, MI
MFO	Ai et al. (2021)	CD	VF	×	AU, MI, SS
BDA	Vahdani et al. (2021)	CD	DY	FB	FP, CP

They investigated different scenarios and formulated the concerned problem, which was solved using PSO.

Das et al. (2020) considered quantum-behaved PSO (QPSO) to solve inventory model with deterioration, preservation and shortages. The related optimization problem was nonlinear for which an analytical solution was not possible. Malik and

Sarkar (2020) proposed GA and pattern search techniques for solving an imperfect production system and compared all randomly generated test results. A warranty period and selling price-sensitive demand rate was assumed to develop a mathematical model. The shortages are partially backlogged in this study. Harifi (2021) investigated three inventory models using the Emperor Penguins Colony (EPC) algorithm because of the high computational complexity of the models under consideration. They also compared the results using other prevalent metaheuristic algorithms and concluded that the planned procedure for the inventory models has improved solutions, lesser cost and lesser CPU usage than other algorithms. Maiti (2021) analyzed an inventory system for imperfect production, including varying production and obtained the optimal decision parameters using a dominance-based PSO algorithm.

2.7 Conclusions

In real-time production systems, disruptions due to certain causes are common, so studying the imperfect production systems is crucial. For production/manufacturing units, the system designer needs to make an optimal policy to tackle such situations and hence can improve the performance of such systems. In this chapter, we have summarised the prominent research works on inventory models for the production system wherein produced items may be imperfect. We have discussed the inventory models with process deterioration and machine breakdown. The nature-inspired optimization techniques have been briefly outlined. A review of inventory models using nature-inspired optimization techniques is presented. GA and PSO are frequently used nature-inspired algorithms for inventory management. Some other metaheuristic optimization techniques can also be used in the field of inventory control. The disadvantage of using nature-inspired techniques is that these algorithms provide a nearly optimal solution, not an optimal one. A comparison by using other techniques should be undertaken to ensure the optimal solution. Hybrid algorithms can also be developed to combine features of two or more nature-inspired optimization algorithms.

References

Ai X, Yue Y, Xu H, Deng X (2021) Optimizing multi-supplier multi-item joint replenishment problem for non-instantaneous deteriorating items with quantity discounts. PLoS ONE 16(2):e0246035. https://doi.org/10.1371/journal.pone.0246035

Alejo-Reyes A, Cuevas E, Rodríguez A, Mendoza A, Olivares-Benitez E (2020) An improved grey wolf optimizer for a supplier selection and order quantity allocation problem. Mathematics 8(9):1457. https://doi.org/10.3390/math8091457

Al-Salamah M (2018) Economic production quantity with the presence of imperfect quality and random machine breakdown and repair based on the artificial bee colony heuristic. Appl Math Model 63:68–83

Atabaki MS, Khamseh AA, Mohammadi M (2019) A priority-based firefly algorithm for network design of a closed-loop supply chain with price-sensitive demand. Comput Ind Eng 135:814–837. https://doi.org/10.1016/j.cie.2019.06.054

Beklari A, Nikabadi MS, Farsijani H, Mohtashami A (2018) A hybrid algorithm for solving vendors managed inventory (VMI) model with the goal of maximizing inventory turnover in producer warehouse. Ind Eng Manag Syst 17(3):570–587. https://doi.org/10.7232/iems.2018.17.3.570

Ben-Daya M (2002) The economic production lot-sizing problem with imperfect production processes and imperfect maintenance. Int J Prod Econ 76(3):257–264

Bhunia AK, Shaikh AA (2015) An application of PSO in a two-warehouse inventory model for deteriorating item under permissible delay in payment with different inventory policies. Appl Math Comput 256:831–850. https://doi.org/10.1016/j.amc.2014.12.137

Bhunia AK, Shaikh A, Dhaka V, Sarla P, Cárdenas-Barrón LE (2018) An application of genetic algorithm and PSO in an inventory model for single deteriorating item with variable demand dependent on marketing strategy and displayed stock level. Scientia Iranica 25(3):1641–1655

Bhunia AK, Mahato SK, Shaikh AA, Jaggi CK (2014) A deteriorating inventory model with displayed stock-level-dependent demand and partially backlogged shortages with all unit discount facilities via particle swarm optimization. Int J Syst Sci Oper Logist 1(3):164-180. https://doi.org/10.1080/23302674.2014.946981

Boone T, Ganeshan R, Guo Y, Ord JK (2000) The impact of imperfect processes on production run times. Decis Sci 31(4):773–787

Boussaïd I, Lepagnot J, Siarry P (2013) A survey on optimization metaheuristics. Inf Sci 237:82–117. https://doi.org/10.1016/j.ins.2013.02.041

Chakraborty T, Giri BC (2012) Joint determination of optimal safety stocks and production policy for an imperfect production system. Appl Math Model 36:712–722

Chakraborty T, Giri BC, Chaudhuri KS (2008) Production lot sizing with process deterioration and machine breakdown. Eur J Oper Res 185:606–618

Chakraborty T, Giri BC, Chaudhuri KS (2009) Production lot sizing with process deterioration and machine breakdown under inspection schedule. Omega 37:257–271

Chakraborty D, Jana DK, Roy TK (2015) Multi-item integrated supply chain model for deteriorating items with stock dependent demand under fuzzy random and bifuzzy environments. Comput Ind Eng 88:166–180. https://doi.org/10.1016/j.cie.2015.06.022

Che ZH (2012) A particle swarm optimization algorithm for solving unbalanced supply chain planning problems. Appl Soft Comput 12(4):1279–1287. https://doi.org/10.1016/j.asoc.2011.12.006

Cheikhrouhou N, Sarkar B, Ganguly B, Malik AI, Batista R, Lee YH (2018) Optimization of sample size and order size in an inventory model with quality inspection and return of defective items. Ann Oper Res 271(2):445–467

Chen Z (2018) Optimal inventory replenishment and pricing for a single-manufacturer and multi-retailer system of deteriorating items. Int J Oper Res 31(1):112–139. https://doi.org/10.1504/IJOR.2018.088559

Cheng TCE (1989) An economic production quantity model with flexibility and reliability considerations. Eur J Oper Res 39(2):174–179

Cheng TCE (1991a) EPQ with process capability and quality assurance considerations. J Oper Res Soc 42(8):713–720

Cheng TCE (1991b) An economic order quantity model with demand-dependent unit production cost and imperfect production processes. IIE Trans 23(1):23–28

Chung KJ, Hou KL (2003) An optimal production run time with imperfect production processes and allowable shortages’. Comput Oper Res 30(4):483–490

Chung KJ, Her CC, Lin SD (2009) A two-warehouse inventory model with imperfect quality production processes. Comput Ind Eng 56(1):193–197

Cui L, Wang L, Deng J (2014) RFID technology investment evaluation model for the stochastic joint replenishment and delivery problem. Expert Syst Appl 41(4):1792–1805. https://doi.org/10.1016/j.eswa.2013.08.078

Cui YY, Guan Z, Saif U, Zhang L, Zhang F, Mirza J (2017) Close loop supply chain network problem with uncertainty in demand and returned products: genetic artificial bee colony algorithm approach. J Clean Prod 162:717–742. https://doi.org/10.1016/j.jclepro.2017.06.079
Cui L, Deng J, Zhang Y, Tang G, Xu M (2020) Hybrid differential artificial bee colony algorithm for multi-item replenishment-distribution problem with stochastic lead-time and demands. J Clean Prod 254:119873. https://doi.org/10.1016/j.jclepro.2019.119873
Das D, Roy A, Kar S (2010) Improving production policy for a deteriorating item under permissible delay in payments with stock-dependent demand rate. Comput Math Appl 60(7):1973–1985. https://doi.org/10.1016/j.camwa.2010.07.031
Das S, Zidan AM, Manna A (2020) An application of preservation technology in inventory control system with price dependent demand and partial backlogging. Alex Eng J 59:1359–1369
Das SC, Manna AK, Rahman MS, Shaikh AA, Bhunia AK (2021) An inventory model for non-instantaneous deteriorating items with preservation technology and multiple credit periods-based trade credit financing via particle swarm optimization. Soft Comput 25(7):5365–5384. https://doi.org/10.1007/s00500-020-05535-x
Deep K, Thakur M (2007) A new crossover operator for real coded genetic algorithms. Appl Math Comput 188(1):895–911. https://doi.org/10.1016/j.amc.2006.10.047
Dorigo M, Birattari M, Stutzle T (2006) Ant colony optimisation. IEEE Comput Intell Mag 1(4):28–39. https://doi.org/10.1109/MCI.2006.329691
Dye C-Y (2012) A finite horizon deteriorating inventory model with two-phase pricing and time-varying demand and cost under trade credit financing using particle swarm optimization. Swarm Evol Comput 5:37–53. https://doi.org/10.1016/j.swevo.2012.03.002
Dye C, Hsieh T (2010) A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and unit purchasing cost. Comput Math Appl 60:1895–1907
Dye CY, Ouyang LY (2011) A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade credit financing. Comput Ind Eng 60(1):127–137
Fathi M, Khakifirooz M, Diabat A, Chen H (2021) An integrated queuing-stochastic optimization hybrid genetic algorithm for a location-inventory supply chain network. Int J Prod Econ 237:108139. https://doi.org/10.1016/j.ijpe.2021.108139
Gaafar LK, Aly AS (2009) Applying particle swarm optimization to dynamic lot sizing with batch ordering. Int J Prod Res 47(12):3345–3361. https://doi.org/10.1080/00207540701581783
Giri BC, Dash A (2022) Optimal batch shipment policy for an imperfect production system under price-, advertisement- and green-sensitive demand. J Manag Anal 9(1):86–119
Goyal SK, Cardenas-Barron LE (2002) Note on: economic production quantity model for items with imperfect quality–a practical approach. Int J Prod Econ 77:85–87
Guchhait P, Maiti MK, Maiti M (2013) A production inventory model with fuzzy production and demand using fuzzy differential equation: an interval compared genetic algorithm approach. Eng Appl Artif Intell 26(2):766–778. https://doi.org/10.1016/j.engappai.2012.10.017
Harifi S, Khalilian M, Mohammadzadeh J, Ebrahimnejad S (2021) Optimization in solving inventory control problem using nature inspired Emperor Penguins Colony algorithm. J Intell Manuf 32:1361–1375. https://doi.org/10.1007/s10845-020-01616-8
Hariga M, Ben-Daya M (1998) Economic Manufacturing lot sizing problem with imperfect manufacturing processes: bounds and optimal solutions. Nav Res Logist 45:423–433
Hejazi SR, Tsou JC, Barzoki MR (2008) Optimal lot size of EPQ model considering imperfect and defective products. J Ind Eng Int 4(7):59–68
Holland JH (1975) Adaptation in natural and artificial systems. University of Michigan Press, Ann Arbor, MI, USA
Hsieh T-P, Dye C-Y (2013) A production–inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. J Comput Appl Math 239:25–36. https://doi.org/10.1016/j.cam.2012.09.016
Hsieh CC, Lee ZZ (2005) Joint determination of production run length and number of standbys in a deteriorating production process. Eur J Oper Res 162(2):359–371

Hsu J, Hsu L (2012) An integrated single-vendor single-buyer production-inventory model for items with imperfect quality and inspection errors. Int J Ind Eng Comput 3(5):703–720
Hsu J, Hsu L (2013a) An integrated vendor–buyer cooperative inventory model in an imperfect production process with shortage backordering. Int J Adv Manuf Technol 65:493–505
Hsu J, Hsu L (2013b) Two EPQ models with imperfect production processes, inspection errors, planned backorders, and sales returns. Comput Ind Eng 64(1):389–402
Huang Y-D, Wee H-M, Praharsi Y, Lo CC (2019) An integrated vendor–buyer replenishment policy for deteriorating items with fuzzy environment and resource constraint. INFOR: Inf Syst Oper Res 57(4):667–691. https://doi.org/10.1080/03155986.2019.1624472
Jaber MY (2006) Lot sizing for an imperfect production process with quality corrective interruptions and improvements, and reduction in setups. Comput Ind Eng 51(4):781–790
Jaber MY, Goyal S, Imran M (2008) Economic production quantity model for items with imperfect quality subject to learning effects. Int J Prod Econ 115:143–150
Jain V, Kundu A, Chan FTS, Patel M (2015) A chaotic bee colony approach for supplier selection-order allocation with different discounting policies in a coopetitive multi-echelon supply chain. J Intell Manuf 26(6):1131–1144. https://doi.org/10.1007/s10845-013-0845-8
Jamal AM, Sarker BR, Mondal S (2004) Optimal manufacturing batch size with rework process at a single-stage production system. Comput Ind Eng 47(1):77–89
Jana DK, Das B, Maiti M (2014) Multi-item partial backlogging inventory models over random planning horizon in random fuzzy environment. Appl Soft Comput 21:12–27. https://doi.org/10.1016/j.asoc.2014.02.021
Kang CW, Ullah M, Sarkar B, Hussain I, Akhtar R (2016) Impact of random defective rate on lot size focusing work-in-process inventory in manufacturing system. Int J Prod Res 55(6):1748–1766. https://doi.org/10.1080/00207543.2016.1235295
Karaboga D, Basturk B (2007) A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm. J Global Optim 39(3):459–471. https://doi.org/10.1007/s10898-007-9149-x
Kennedy J, Eberhart R (1995) Particle swarm optimization. In: Proceedings of ICNN'95-international conference on neural networks 4:1942–1948. https://doi.org/10.1109/ICNN.1995.488968
Khalifehzadeh S, Fakhrzad MB (2019) A modified firefly algorithm for optimizing a multi stage supply chain network with stochastic demand and fuzzy production capacity. Comput Ind Eng 133:42–56. https://doi.org/10.1016/j.cie.2019.04.048
Khan M, Jaber MY, Bonney M (2011) An economic order quantity (EOQ) for items with imperfect quality and inspection errors. Int J Prod Econ 133(1):113–118
Khouja M, Mehrez A (1994) Economic production lot size model with variable production rate and imperfect quality. J Oper Res Soc 45:1405–1417
Kim CH, Hong Y (1999) An optimal production run length in deteriorating production processes. Int J Prod Econ 58:183–189
Kim MS, Sarkar B (2017) Multi-stage cleaner production process with quality improvement and lead time dependent ordering cost. J Clean Prod 144:572–590
Kim CH, Hong Y, Chang CH (2001) Optimal production run length and inspection schedules in a deteriorating production process. IIE Trans 33:421–426
Kim MS, Kim JS, Sarkar B, Sarkar M, Iqbal MW (2018) An improved way to calculate imperfect items during long-run production in an integrated inventory model with backorders. J Manuf Syst 47:153–167
Klinmalee S, Naenna T, Woarawichai C (2020) Application of a genetic algorithm for multi-item inventory lot-sizing with supplier selection under quantity discount and lead time. Int J Oper Res 38(3):403. https://doi.org/10.1504/IJOR.2020.107540
Krishnamoorthi C, Panayappan S (2012) An EPQ model with imperfect production systems with rework of regular production and sales return. Am J Oper Res 2:225–234

Kumar S, Kumar N (2016) An inventory model for deteriorating items under inflation and permissible delay in payments by genetic algorithm. Cogent Bus Manag 3(1):1239605. https://doi.org/10.1080/23311975.2016.1239605

Kumar V, Kumar D (2021) A systematic review on firefly algorithm: past, present, and future. Arch Comput Methods Eng 28(4):3269–3291. https://doi.org/10.1007/s11831-020-09498-y

Lashgari M, Sadjadi SJ, Sahihi M (2019) A multi-product, multi-period model to select supplier for deteriorating products while considering uncertainty as well as backorder. J Ind Eng Int 15(1):93–101. https://doi.org/10.1007/s40092-018-0266-6

Lee JS, Park KS (1991) Joint determination of production cycle and inspection intervals in a deteriorating production system. J Oper Res Soc 42:775–783

Lee HL, Rosenblatt MJ (1987) Simultaneous determination of production cycle and inspection schedules in a production system. Manage Sci 33:1125–1136

Lee HL, Rosenblatt MJ (1989) A production and maintenance planning model with restoration cot depending on detection delay. IIE Trans 21:368–375

Lee AHI, Kang H-Y, Lai C-M (2013) Solving lot-sizing problem with quantity discount and transportation cost. Int J Syst Sci 44(4):760–774. https://doi.org/10.1080/00207721.2011.618644

Lin GC, Gong DC (2006) On a production–inventory system of deteriorating items subject to random machine breakdowns with a fixed repair time. Math Comput Model 43(7–8):920–932

Lin T, Srivastava HM (2015) A two-warehouse inventory model with quantity discounts and maintenance actions under imperfect production processes. Appl Math Inf Sci 2505(5):2493–2505

Lin TM, Tseng ST, Liou MJ (1991) Optimal inspection schedule in the imperfect production system under general shift distribution. J Chin Inst Ind Eng 8(2):73–81

Lin CS, Chen CH, Kroll DE (2003) Integrated production-inventory models for imperfect production processes under inspection schedules. Comput Ind Eng 44(4):633–650

Liou MJ, Tseng ST, Lin TM (1994) The effects of inspection errors to the imperfect EMQ model. IIE Trans 26:42–51

Lo CC (2010) A fuzzy integrated vendor-buyer inventory policy of deteriorating items under credibility measure. In: 2010 IEEE international conference on industrial engineering and engineering management. https://doi.org/10.1109/IEEM.2010.5674589

Mahmoodi A (2020) Stackelberg-Nash equilibrium of pricing and inventory decisions in duopoly supply chains using a nested evolutionary algorithm. Appl Soft Comput 86:105922. https://doi.org/10.1016/j.asoc.2019.105922

Maiti AK (2021) Cloudy fuzzy inventory model under imperfect production process with demand dependent production rate. J Manag Anal 8(4):741–763. https://doi.org/10.1080/23270012.2020.1866696

Maity AK, Maity K, Mondal S, Maiti M (2007) A Chebyshev approximation for solving the optimal production inventory problem of deteriorating multi-item. Math Comput Model 45(1–2):149–161. https://doi.org/10.1016/j.mcm.2006.04.011

Makis V (1998) Optimal lot sizing and inspections policy for an EMQ model with imperfect inspections. Nav Res Logist 45:165–186

Malik AI, Sarkar B (2019) Coordinating supply-chain management under stochastic fuzzy environment and lead-time reduction. Mathematics 7(5):480. https://doi.org/10.3390/math7050480

Malik AI, Sarkar B (2020) Disruption management in a constrained multi-product imperfect production system. J Manuf Syst 56:227–240

Mallipeddi R, Suganthan PN, Pan QK, Tasgetiren MF (2011) Differential evolution algorithm with ensemble of parameters and mutation strategies. Appl Soft Comput 11(2):1679–1696. https://doi.org/10.1016/j.asoc.2010.04.024

Mandal A, Pal B (2021) Optimizing profit for pricing and advertisement sensitive demand under unreliable production system. Int J Syst Sci Oper Logist 8(2):99–118

Manjarres D, Landa-Torres I, Gil-Lopez S, Del Ser J, Bilbao MN, Salcedo-Sanz S, Geem ZW (2013) A survey on applications of the harmony search algorithm. Eng Appl Artif Intell 26(8):1818–1831. https://doi.org/10.1016/j.engappai.2013.05.008

Manna AK, Akhtar M, Shaikh AA, Bhunia AK (2021) Optimization of a deteriorated two-warehouse inventory problem with all-unit discount and shortages via tournament differential evolution. Appl Soft Comput 107:107388. https://doi.org/10.1016/j.asoc.2021.107388

Masud MAA, Paul SK, Azeem A (2014) Optimization of a production inventory model with reliability considerations. Int J Logist Syst Manag 17(1):22–45

Megala N, Jawahar N (2006) Genetic algorithm and Hopfield neural network for a dynamic lot sizing problem. Int J Adv Manuf Technol 27(11–12):1178–1191. https://doi.org/10.1007/s00170-004-2306-1

Meraihi Y, Ramdane-Cherif A, Acheli D, Mahseur M (2020) Dragonfly algorithm: a comprehensive review and applications. Neural Comput Appl 32(21):16625–16646. https://doi.org/10.1007/s00521-020-04866-y

Mirjalili S (2015) Moth-flame optimization algorithm: a novel nature-inspired heuristic paradigm. Knowl-Based Syst 89:228–249. https://doi.org/10.1016/j.knosys.2015.07.006

Mirjalili S, Mirjalili SM, Lewis A (2014) Grey wolf optimizer. Adv Eng Softw 69:46–61. https://doi.org/10.1016/j.advengsoft.2013.12.007

Mondal R, Shaikh AA, Bhunia AK (2019) Crisp and interval inventory models for ameliorating item with Weibull distributed amelioration and deterioration via different variants of quantum behaved particle swarm optimization-based techniques. Math Comput Model Dyn Syst 25(6):602–626. https://doi.org/10.1080/13873954.2019.1692226

Moon IK, Goyal SK, Cha BC (2008) The joint replenishment problem involving multiple suppliers offering quantity discounts. Int J Syst Sci 39(6):629–637. https://doi.org/10.1080/00207720701832614

Mousavi SM, Sadeghi J, Niaki STA, Alikar N, Bahreininejad A, Metselaar HSC (2014b) Two parameter-tuned meta-heuristics for a discounted inventory control problem in a fuzzy environment. Inf Sci 276:42–62. https://doi.org/10.1016/j.ins.2014.02.046

Mousavi SM, Niaki STA, Bahreininejad A, Musa SN (2014a) Multi-item multiperiodic inventory control problem with variable demand and discounts: a particle swarm optimization algorithm. Sci World J 1–16. https://doi.org/10.1155/2014a/136047

Nia AR, Far MH, Niaki STA (2014) A fuzzy vendor managed inventory of multi-item economic order quantity model under shortage: an ant colony optimization algorithm. Int J Prod Econ 155:259–271. https://doi.org/10.1016/j.ijpe.2013.07.017

Nobil AH, Sedigh AHA, Cárdenas-Barrón LE (2016) A multi-machine multi-product EPQ problem for an imperfect manufacturing system considering utilization and allocation decisions. Expert Syst Appl 56:310–319

Pakhira N, Maiti MK, Maiti M (2017) Two-level supply chain of a seasonal deteriorating item with time, price, and promotional cost dependent demand under finite time horizon. Am J Math Manag Sci 36(4):292–315. https://doi.org/10.1080/01966324.2017.1334605

Pakhira N, Maiti K (2020) Systems MM-I J of F & 2020, undefined. (Two-level supply chain for a deteriorating item with stock and promotional cost dependent demand under shortages. Ijfs Usb Ac Ir 17(1):29–52. https://ijfs.usb.ac.ir/article_5109.html

Pal S, Maiti MK, Maiti M (2009) An EPQ model with price discounted promotional demand in an imprecise planning horizon via genetic algorithm. Comput Ind Eng 57(1):181–187

Pal B, Sana SS, Chaudhuri K (2014) Joint pricing and ordering policy for two echelon imperfect production inventory model with two cycles. Int J Prod Econ 155:229–238

Pal P, Das CB, Panda A, Bhunia AK (2005) An application of real-coded genetic algorithm (for mixed integer nonlinear programming in an optimal two-warehouse inventory policy for deteriorating items with a linear trend in demand and a fixed planning horizon). Int J Comput Math 82(2):163–175. https://doi.org/10.1080/00207160412331296733

Parsopoulos KE, Konstantaras I, Skouri K (2015) Metaheuristic optimization for the single-item dynamic lot sizing problem with returns and remanufacturing. Comput Ind Eng 83:307–315. https://doi.org/10.1016/j.cie.2015.02.014

Pasandideh SHR, Niaki STA (2008) A genetic algorithm approach to optimize a multi-products EPQ model with discrete delivery orders and constrained space. Appl Math Comput 195(2):506–514

Paul SK, Sarker R, Essam D (2015) Managing disruption in an imperfect production–inventory system. Comput Ind Eng 84:101–112

Pirayesh M, Poormoaied S (2015) GPSO-LS algorithm for a multi-item EPQ model with production capacity restriction. Appl Math Model 39(17):5011–5032

Porteus EL (1986) Optimal lot sizing, process quality improvement and setup cost reduction. Oper Res 34:137–144

Pramanik P, Maiti MK (2019) An inventory model for deteriorating items with inflation induced variable demand under two level partial trade credit: a hybrid ABC-GA approach. Eng Appl Artif Intell 85:194–207. https://doi.org/10.1016/j.engappai.2019.06.013

Rahim M, Al-Hajailan W (2006) An optimal production run for an imperfect production process with allowable shortages and time-varying fraction defective rate. Int J Adv Manuf Technol 27:1170–1177

Rahim MA, Ben-Daya M (2001) Joint determination of production quantity, inspection schedule and quality control for an imperfect process with deteriorating products. J Oper Res Soc 52:1370–1378

Rahim MA, Ohta H (2005) An integrated economic model for inventory and quality control problems. Eng Optim 37:65–81

Rahman MS, Manna AK, Shaikh AA, Bhunia AK (2020) An application of interval differential equation on a production inventory model with interval-valued demand via center-radius optimization technique and particle swarm optimization. Int J Intell Syst 35(8):163–175. https://doi.org/10.1002/int.22254

Rau H, OuYang BC (2007) A general and optimal approach for three inventory models with a linear trend in demand. Comput Ind Eng 52(4):521–532

Rosenblatt MJ, Lee HL (1986) Economic production cycles with imperfect production process. IIE Trans 18:48–55

Roul JN, Maity K, Kar S, Maiti M (2015) Multi-item reliability dependent imperfect production inventory optimal control models with dynamic demand under uncertain resource constraint. Int J Prod Res 53(16):4993–5016

Roy DM, Sana SS, Chaudhuri K (2011) An economic order quantity model of imperfect quality items with partial backlogging. Int J Syst Sci 42(8):1409–1419

Ruidas S, Selkh MR, Nayak PK, Pal M (2017) An interval valued EPQ model in imperfect production system with rework of regular production, shortages and sales return via particle swarm optimization. Int J Pure Appl Math 113(6):375–384

Sadeghi J, Mousavi SM, Niaki STA, Sadeghi S (2013) Optimizing a multi-vendor multi-retailer vendor managed inventory problem: two tuned meta-heuristic algorithms. Knowl-Based Syst 50:159–170. https://doi.org/10.1016/j.knosys.2013.06.006

Sadeghi J, Sadeghi S, Niaki STA (2014a) Optimizing a hybrid vendor-managed inventory and transportation problem with fuzzy demand: an improved particle swarm optimization algorithm. Inf Sci 272:126–144. https://doi.org/10.1016/j.ins.2014.02.075

Sadeghi J, Mousavi SM, Niaki STA, Sadeghi S (2014b) Optimizing a bi-objective inventory model of a three-echelon supply chain using a tuned hybrid bat algorithm. Transp Res Part E Logist Transp Rev 70:274–292. https://doi.org/10.1016/j.tre.2014.07.007

Sadeghi J, Mousavi SM, Niaki STA (2016) Optimizing an inventory model with fuzzy demand, backordering, and discount using a hybrid imperialist competitive algorithm. Appl Math Model 40(15–16):7318–7335. https://doi.org/10.1016/j.apm.2016.03.013

Salameh MK, Jaber MY (2000) Economic production quantity model for items with imperfect quality. Int J Prod Econ 64:59–64

Salehi Amiri SAH, Zahedi A, Kazemi M, Soroor J, Hajiaghaei-Keshteli M (2020) Determination of the optimal sales level of perishable goods in a two-echelon supply chain network. Comput Ind Eng 139:106156. https://doi.org/10.1016/j.cie.2019.106156

Sana SS (2010a) A production–inventory model in an imperfect production process. Eur J Oper Res 200(2):451–464

Sana SS (2010b) An economic production lot size model in an imperfect production system. Eur J Oper Res 201:158–170

Sana SS (2012a) An economic order quantity model for nonconforming quality products. Serv Sci 4(4):331–348

Sana SS (2012b) Preventive maintenance and optimal buffer inventory for products sold with warranty in an imperfect production system. Int J Prod Res 50(23):6763–6774

Saracoglu I, Topaloglu S, Keskinturk T (2014) A genetic algorithm approach for multi-product multi-period continuous review inventory models. Expert Syst Appl 41(18):8189–8202. https://doi.org/10.1016/j.eswa.2014.07.003

Sarkar B (2012) An inventory model with reliability in an imperfect production process. Appl Math Comput 218(9):4881–4891

Sarkar B, Sana SS, Chaudhuri K (2010) Optimal reliability, production lot size and safety stock in an imperfect production system. Int J Math Oper Res 2(4):467–490

Shah NH, Patel DG, Shah DB (2018) EPQ model for returned/reworked inventories during imperfect production process under price-sensitive stock-dependent demand. Oper Res Int J 18(2):343–359

Shaikh AA, Das SC, Bhunia AK, Panda GC, Al-Amin Khan M (2019a) A two-warehouse EOQ model with interval-valued inventory cost and advance payment for deteriorating item under particle swarm optimization. Soft Comput 23(24):13531–13546. https://doi.org/10.1007/s00500-019-03890-y

Shaikh AA, Cárdenas-Barrón LE, Tiwari S (2019b) A two-warehouse inventory model for non-instantaneous deteriorating items with interval-valued inventory costs and stock-dependent demand under inflationary conditions. Neural Comput Appl 31(6):1931–1948. https://doi.org/10.1007/S00521-017-3168-4

Shaikh AA, Tiwari S, Cárdenas-Barrón LE (2020) An Economic Order Quantity (EOQ) inventory model for a deteriorating item with interval-valued inventory costs, price-dependent demand, two-level credit policy, and shortages. Optimization and inventory management. Asset Analytics. Springer, Singapore. https://doi.org/10.1007/978-981-13-9698-4_2

Shavandi H, Mahlooji H, Nosratian NE (2012) A constrained multi-product pricing and inventory control problem. Appl Soft Comput 12(8):2454-2461. https://doi.org/10.1016/j.asoc.2012.03.036

Singh RK, Yadav AS, Swami A (2016) A two-warehouse model for deteriorating items with holding cost under particle swarm optimization. Int J Adv Eng Manag Sci (6)

Singh P, Kumar A (2021) Inventory control of non-instantaneous deteriorating items with time-sensitive holding cost and demand. Global J Model Intell Comput (GJMIC) 1(1):13–22

Storn R, Price K (1995) Differential evolution-a simple and efficient adaptive scheme for global optimization over continuous spaces, vol 11. Technical Report, International Computer Science Institute

Sue-Ann G, Ponnambalam SG, Jawahar N (2012) Evolutionary algorithms for optimal operating parameters of vendor managed inventory systems in a two-echelon supply chain. Adv Eng Softw 52:47–54. https://doi.org/10.1016/j.advengsoft.2012.06.003

Tai AH (2013) Economic production quantity models for deteriorating/imperfect products and service with rework. Comput Ind Eng 66(4):879–888

Taleizadeh AA, Barzinpour F, Wee H-M (2011a) Meta-heuristic algorithms for solving a fuzzy single-period problem. Math Comput Model 54(5–6):1273–1285. https://doi.org/10.1016/j.mcm.2011.03.038

Taleizadeh AA, Niaki STA, Barzinpour F (2011b) Multiple-buyer multiple-vendor multi-product multi-constraint supply chain problem with stochastic demand and variable lead-time: a harmony search algorithm. Appl Math Comput 217(22):9234–9253. https://doi.org/10.1016/j.amc.2011.04.001

Taleizadeh AA, Niaki STA, Nikousokhan R (2011c) Constraint multiproduct joint-replenishment inventory control problem using uncertain programming. Appl Soft Comput 11(8):5143–5154. https://doi.org/10.1016/j.asoc.2011.05.045

Taleizadeh AA, Jalali-Naini SG, Wee H-M, Kuo T-C (2013) An imperfect multi-product production system with rework. Sci Iran 20(3):811–823

Tiwari S, Jaggi CK, Bhunia AK, Shaikh AA, Goh M (2017) Two-warehouse inventory model for non-instantaneous deteriorating items with stock-dependent demand and inflation using particle swarm optimization. Ann Oper Res 254(1–2). https://doi.org/10.1007/s10479-017-2492-5

Tripathy PK, Pattnaik M (2011) Optimal inventory policy with reliability consideration and instantaneous receipt under imperfect production process. Int J Manag Sci Eng Manag 6:413–420

Tripathy PK, Wee W, Majhi PR (2003) An EOQ model with process reliability considerations. J Oper Res Soc 54:549–554

Tripathy PK, Tripathy P, Pattnaik M (2011) A fuzzy EOQ model with reliability and demand-dependent unit cost. Int J Contemp Math Sci 6(30):1467–1482

Tseng ST (1996) Optimal preventive maintenance policy for deteriorating production system. IIE Trans 28:687–694

Tsou C (2008) Multi-objective inventory planning using MOPSO and TOPSIS. Expert Syst Appl 35(1–2):136–142. https://doi.org/10.1016/j.eswa.2007.06.009

Vahdani M, Sazvar Z, Govindan K (2021) An integrated economic disposal and lot-sizing problem for perishable inventories with batch production and corrupt stock-dependent holding cost. Ann Oper Res 315(2):2135–2167. https://doi.org/10.1007/s10479-021-04110-3

Wang CH (2004) The impact of a free-repair warranty on EMQ model for imperfect production systems. Comput Oper Res 31(12):2021–2035

Wang CH (2005) Integrated production and product inspection policy. Int J Prod Econ 95:123–134

Wang CH, Sheu SH (2001) Simultaneous determination of the optimal production–inventory and product inspection policies for a deteriorating production system. Comput Oper Res 28:1093–1110

Wang L, Dun C-X, Bi W-J, Zeng Y-R (2012a) An effective and efficient differential evolution algorithm for the integrated stochastic joint replenishment and delivery model. Knowl-Based Syst 36:104–114. https://doi.org/10.1016/j.knosys.2012.06.007

Wang L, Fu Q-L, Zeng Y-R (2012b) Continuous review inventory models with a mixture of backorders and lost sales under fuzzy demand and different decision situations. Expert Syst Appl 39(4):4181–4189. https://doi.org/10.1016/j.eswa.2011.09.116

Wee HM, Yu J, Lo C-C, Chen KL (2005) Multiple replenishment inventory model for reusable product with imperfect quality. J Manag Syst 12(3):89–107

Widyadana GA, Wee HM (2012) An economic production quantity model for deteriorating items with multiple production setups and rework. Int J Prod Econ 138(1):62–67. https://doi.org/10.1016/j.ijpe.2012.02.025

Widyadana GA, Wee HM, Chang J-Y (2010) Determining the optimal number of Kanban in multi-products supply chain system. Int J Syst Sci 41(2):189–201. https://doi.org/10.1080/00207720903042996

Yang X-S (2020) Nature-inspired optimization algorithms: challenges and open problems. J Comput Sci 46:101104. https://doi.org/10.1016/j.jocs.2020.101104

Yang X-S, He X (2013) Bat algorithm: literature review and applications. Int J Bio-Inspired Comput 5(3):141–149. https://doi.org/10.1504/IJBIC.2013.055093

Yang M-F, Lin Y (2010) Applying the linear particle swarm optimization to a serial multi-echelon inventory model. Expert Syst Appl 37(3):2599–2608. https://doi.org/10.1016/j.eswa.2009.08.021

Yang W, Chan FTS, Kumar V (2012) Optimizing replenishment polices using Genetic Algorithm for single-warehouse multi-retailer system. Expert Syst Appl 39(3):3081–3086. https://doi.org/10.1016/j.eswa.2011.08.171

Yang Y, Chi H, Tang O, Zhou W, Fan T (2019) Cross perishable effect on optimal inventory preservation control. Eur J Oper Res 276(3):998–1012. https://doi.org/10.1016/j.ejor.2019.01.069

Chapter 3
A Multi-objective Mathematical Model for Socially Responsible Supply Chain Inventory Planning

Apoorav Dhingra, Vijaypal Poonia, and Rakhee Kulshrestha

Abstract The supply chain inventory planning is well studied and analyzed in the literature considering various costs and other factors. However, social factor is largely overlooked although, it is important like cost factors which generally come from sourcing, production, storage and distribution. In the current work, a mathematical model for supply chain inventory was developed with four objectives (i.e., cost, local development, steadiness in employment and investment in green technology). The first objective is related to the total cost, which includes mostly inventory, manufacturing, and re-manufacturing related costs. The other two objectives are related to social aspects which focus on improvement in local development and steadiness in employment. The fourth objective is related to environmental aspect which focuses on the right time to invest in environmentally friendly machinery and technologies. The proposed model has the capability to obtain the optimal number of products to be manufactured and re-manufactured, number of inventories, number of employees to recruit and lay-off within a certain region in each quarter of the year, and decision on time to invest in green technology or pay govt. penalty, considering the upcoming demand in each quarter. The weighted sum method is applied to solve the multi-objective optimization problem.

Keywords Supply chain inventory planning · Multi-objective mathematical model · Re-manufacturing · Sustainability

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3.1 Introduction

Sustainability, defined as the combination of environmental, social, and economical factors, has become a popular word among researchers and the business world. Many efforts have been made by researchers and corporations to create sustainability, such as an industry work on embedding sustainability as attitude/habits/work culture (Galpin et al. 2015) and according to Bocken et al. (2014), business strategies need a re-evaluation. It has been clear that organizations must address sustainability concerns not only within the organization, but also look for sustainable retailer-supplier as discussed by Berning and Venter (2015).

There are various factors motivating researchers to explore the sustainable goal. A few of them are: Government strict rules and regulations imposed on organizations to maintain ecological balance and preserve the environment, and due to this new jobs are also introduced in the market, the 3R-rule which is also gaining importance in industries and most important is customer's inclination towards eco-friendly products. There are three main factors of sustainability (economic, social and environmental), but few studies have taken a social factor into account in the design of an SSC. Furthermore, less attention has been on sustainable supply chain inventory planning with local employee benefit factors, which is an important aspect of sustainability.

3.2 Literature Survey

Despite the fact that a significant quantity of research has already been done on supply chain architecture, the need to incorporate environmental indicators into supply chain activities is increased as explained by Validi et al. (2015). However, implementing the supply chains in a sustainable means is crucial in the businesses of different sizes with a wide range of products. As a result, many researchers in the last decade considered sustainable supply chain (SSC), Craig and Easton (2011), Ghadimi et al. (2016). SSC is a concept that emerged from the integration of supply chain and sustainability perspectives (Seuring and Müller 2008). The government legislation and corporate social responsibility (CSR) also working to spread the awareness regarding environmental and social responsibilities among industries (Zarbakhshnia et al. 2020). Pagell and Shevchenko (2014), an SSC generates "no harm on social or environmental systems while maintaining economic viability". Sustainable practices such as green purchasing, environmental procuring, logistics and social responsibility are all required for an SSC (Agrawal et al. 2015; Sarkis and Zhu 2018; Ghadimi et al. 2017). Forecasting, production planning and inventory control are only a few of the strategic and operational components of SSC that have been studied in the past decade. There has been a review of articles published on closed-loop supply chains (CLSC) by Kannan and Solemani (2017) while Brandenburg et al. (2014), provided a literature review of SSC. Also, Dakov and Novkov (2008), Ghadge et al. (2012), Ghadimi et al. (2019) reviewed future directions, challenges, and opportunities for SSC management. A review for the SSC models considering the number of objective functions as well as the character of objective functions, number of products, time horizon, resources, conservation and different costs considered is presented by Seuring (2013). Saffari et al. (2015) proposed a multi-criterion robust optimization model for a socially aware CLSC, which was solved using a non-dominated sorting GA. Fahimnia et al. (2015) proposed a multidimensional mixed-integer nonlinear programming (MINLP) model for the strategic green supply chain management. Jindal and Sangwan (2017) presented the MILP model, which determined the number of products to be re-manufactured and number of parts to be purchased from supplier, as well as allocation at facility centres multi-product, single time. Zarbakhshnia et al. (2019) developed multi-objective linear model including economic function and environmental function, to optimize the number of machines and obtain the Pareto optimal solution by using the ε-constraint method.

The decisions on inventory and warehousing material handling with a focus on reducing environmental and social impacts without affecting profit are considered as sustainable inventory management (SIM). Many researchers have worked in the area of SIM, the details of some work are discussed here. Chiu et al. (2016) solved a mathematical model to minimize the sum of production, delivery and inventory costs for multiple items by using differential calculus, they also solved the same model using an algebraic approach in Chiu et al. (2015). Chen et al. (2016) considered the retailer point of view to model a problem of cost minimization, with two decision variables, replenishment quantity and number of returned used products per unit time. An economic order quantity (EOQ) and economic production quantity (EPO) with achieving sustainable goals for a multi-product supply chain was studied by Soleymanfar et al. (2022). Becerra et al. (2022) complied with SIM in supply chain, and also discussed the trends and future scope.

The novelty of the present work lies in structuring the explored research gaps and answering the research question through a mathematical model of sustainable supply chain inventory planning that includes local employee benefit factors. The proposed model is to maintain a more efficient supply chain than a responsive supply chain which focuses on the cost of making and delivering the product to the customer. One can understand difference between the two from the characteristics mentioned in Table 3.1.

In this paper, the effort is on developing an SSC mathematical model including inventory planning from company's point of view which not only takes care about the cost factor involved in the production process, keeping goods as an inventory, re-processing of goods which are manufactured with a fault but also focused on local development by hiring employees which are living nearby to the organization. It also takes care to maintain steadiness in employment by hiring and lay-off employees depending on need (this process is carried out at regular intervals, but we are managing the number of employees only by predicting one-year demand). It will

Table 3.1 Comparison of efficient and responsive supply chains

	Efficient	Responsive
Main moto	To keep cost as low as possible	To respond to customer as soon as possible, even at expense of high cost
Designing of manufacturing product	Should be cost effective, quality can be compromised to certain extent	May take long time in designing phase but customer's requirement should be met
Decision on price of product	Price of product is such that company will have lower profit margin (as it produced in bulk). So that product can be easily sold	Price of product is such that company will have higher profit margin. Mostly people who focus on quality products gets benefitted
From production point of view	All sources should be utilized to its fullest	Flexible in capacity
Keeping inventory	Low inventory is kept. Try to focus on full-truck load condition	Usually, buffer inventory is kept so that customer can be satisfied at faster pace
About lead time	Should be less but not at expense of high cost	Aggressively reduce even if costs are significant
Decision on choice of supplier	Supplier should provide good quality at low cost	Supplier has to be flexible and can provide superior quality goods at faster pace
Carrying out transportation	Greater dependence on low cost modes	Greater dependence on responsive (fast) modes

also help one understand whether it is time to invest in eco-friendly technologies or to just pay penalty imposed by the government on the organization, which is also a crucial decision at this point of time.

3.3 Assumptions and Notation

In this paper, a problem of determining the no: of manufactured/re-manufactured units, their distribution to warehouse hiring and lay-off plane for workers are considered. Also, to reduce the CO_{2-eq} effect on the environment, there is needed to upgrade technology. Therefore, we also compare the cost of new technology versus effect of new technology is considered. In this proposed model the economic objective, we have considered the total cost, which includes inventory, manufacturing, and re-manufacturing related costs. The social objectives consider the daily to and fro distance traveled by employees from home to company and the number of hire/lay-off are considered in order to improve local development and steadiness in employment. The environmental objective is considered as CO_{2-eq} generated in the manufacturing and re-manufacture process.

To take best decisions during the planning phase of the SSC the following points need to be deliberated:

- Producing plan: Determine the optimal number of products to be manufactured in each period.
- Re-producing plan: Determine the optimal number of products to be re-manufactured in each period which came back from the consumer end.
- Distribution plan: Determine the optimal number of final products that should be kept in the warehouse in each period (in both cases of manufacturing and re-manufacturing).
- Hiring and lay-off plan: Determine the optimal number of employees to recruit and lay-off within a certain region in each period.
- Environmental investment plan: Depending on investment in technology and government-imposed penalty it will help us to determine better option which company should take.

The assumption to construct the model are given below:

- Holding cost for the item to be manufactured is less than holding cost for returned items (or items to be re-manufactured) considering the fact that if the returned item is kept for a longer time, it will affect the company's reputation and in-turn will affect its market in terms of lost customer and the extra incentives provided to the customer.
- Upcoming demand and cost parameters are known.
- Sufficient number of employees are available within a certain region for recruit process.
- Transportation cost for employees within a certain region is known for providing allowances.
- All employees are having same skill set.
- We know the Life cycle assessment (LCA) of our product in both manufacturing and re-manufacturing processes.
- We are focusing of CO_{2-eq} emission in production process only.

Parameters	
$S_r(t)$	Set-up cost of returned items in time t
$S_m(t)$	Set-up cost of manufactured items in time t
$D(t)$	Demand in time t
$H_s(t)$	Holding cost of manufactured item in time t
$H_r(t)$	Holding cost of returned item. in time t
T	Time horizon over which decision is to be taken
R	Region horizon over which decision is to be taken
$M(t)$	Demand after period t
$N(t)$	Number of returned items in time t

(continued)

(continued)

$P(re, t)$	Number of primary employees living in the region re and working in company during period ‘t = 0’
$w(re, t)$	Employees willing to work in region re during period t
PM	Time requires to manufacture one product by employee
MWT	Maximum time an employee can work in period t
PRM	Time requires to re-manufacture one product by employee
$p(re, t)$	Employee salary including allowances in period t and residing within certain radius re (corresponding to distance between region re and the company)
$h(t)$	Salary difference between old employee and new hiring in period t
$s(t)$	Salary of the old employee in period t
$\delta_r(t)$	Dirac-delta function corresponding to re-manufacturing process which can take value either 0 or 1
$\delta_m(t)$	Dirac-delta function corresponding to manufacturing process which can take value either 0 or 1
$\delta_i(t)$	Dirac-delta function corresponding to environmental process which can take value either 0 or 1
$\delta(t)$	Dirac-delta function which can take value either 0 or 1
$E_1(t)$	CO_{2-eq}/Kg- manufacturing product in period t
$E_2(t)$	CO_{2-eq}/Kg- re-manufactured product in period t
$l(t)$	Limit set by government of total CO_{2-eq} in period t
$f(t)$	Fraction/Percentage of products to be re-manufactured in period t
$PP(t)$	Price penalty which government should impose if environmental norms are not met in period *t*
$TI(t)$	Investment in technology required to improve environmental concerns in period *t*
Decision variables	
$x_m(t)$	Amount to be manufactured in time *t*
$x_r(t)$	Amount to be re-manufactured in time *t*
$I_s(t)$	Finished goods manufactured and is in the process to be dispersed on requirement basis
$I_r(t)$	Finished goods yet to be re-manufactured from the returns
$n(re, t)$	Employees living in region re and are currently working in company in period t
$LH(re, t)$	Employees living in region re and are newly hired in company in period *t*
$LF(re, t)$	Employees living in region re and are laid-off by the company in period *t*

3.4 Multi-objective Model

On the basis of assumptions discussed in the previous section, the mathematical model is developed for multi-objective, multi-period supply chain inventory planning.

3.4.1 Objective Functions

The construction of four objectives of the proposed model are as:

$$\text{Minimize OF 1}: \sum_{t=1}^{T} S_r(t)\delta_r(t) + S_m(t)\delta_m(t) + H_r(t)I_r(t) + H_s(t)I_s(t)$$

$$\text{Minimize OF 2}: \sum_{t=1}^{T}\sum_{r=1}^{R} p(re,t)n(re,t)$$

$$\text{Minimize OF 3}: \sum_{t=1}^{T}\sum_{r=1}^{R}(-h(t)LH(re,t) + s(t)LF(re,t))$$

$$\text{Minimize OF 4}: \sum_{t=1}^{T}\delta_i(t)\{[(x_m(t)E_1(t)) + (x_r(t)E_2(t)) - l(t)]PP(t)\} + \sum_{t=1}^{T}\{(1-\delta_i(t))TI(t)\}$$

Objective function OF 1 is to minimize the total cost of production, which includes separate manufacturing and re-manufacturing set-up costs, and inventory costs.

Objective function OF 2 focuses on minimization of total travel distance of employees from the company in terms of allowances offered by the company (more allowances will be given to employees who are at a farther distance from the company) in order to improve local development.

Objective function OF 3 targets on minimization of the total number of hiring and lay-off in order to establish consistency in the employment process and is expressed in a way that if there is any hiring then the hired employee is having less salary than the old working employee and in case of any lay-off the laid-off employee will be leaving by taking salary for some time period.

Objective function OF 4 emphasizes on minimization of environmental related cost which is imposed on the company either in terms of penalty by the government or in terms of the investment in technology which reduces air emissions here, in terms of CO_{2-eq}.

3.4.2 Constraints Sets

(i) Inventory level constraints: Constraints (3.1) and (3.2) are demand balance equations which balances the inventory from the previous period and the production from the current period in both manufacturing and re-manufacturing lines which can be used to meet demand in this period or to build up inventory to satisfy later demand.

Constraint (3.3) states that we need to re-manufacture certain fraction of products available as re-manufacture inventory of the previous period.

$$I_r(t-1) + N(t) - x_r(t) = I_r(t) \tag{3.1}$$

$$I_s(t-1) + x_r(t) + x_m(t) - D(t) = I_s(t) \tag{3.2}$$

$$x_r(t) \geq f(t) I_r(t-1) \tag{3.3}$$

(ii) Fixed-charge constraint: Constraint (3.4) indicates that whatsoever the total production takes place in the manufacturing and re-manufacturing process will not exceed the demand till the time horizon. Constraint (3.5) ensures that $M(t)$ is sufficiently large for use in Constraint (3.4). Constraint (3.6) specifies that manufactured products will always be more than re-manufactured ones.

$$x_r(t) + x_m(t) \leq M(t)\delta(t) \tag{3.4}$$

$$M(t) = \sum_{i=t}^{T} D(i) \tag{3.5}$$

$$x_m(t) \geq x_r(t) \tag{3.6}$$

(iii) Social Constraints: Constraints (3.7) and (3.8) states that the total number of employees to be laid-off does not exceed the number of employees employed during the previous period.

$$LF(re, t = 1) \leq P(re, t) \tag{3.7}$$

$$LF(re, t) \leq n(re, t-1) \qquad t > 1 \tag{3.8}$$

Constraints (3.9) and (3.10) corresponds to the total number of employees living in region '*re*' and are employed in the company during period '*t*'.

$$n(re, t = 1) = P(re, t) + LH(re, t = 1) - LF(re, t = 1) \tag{3.9}$$

$$n(re, t) = n(re, t-1) + LH(re, t) - LF(re, t) \qquad t > 1 \tag{3.10}$$

Constraint (3.11) states that the number of employees living in region '*re*' and are employed in the company during period '*t*' should not be more than the number of employees who are willing to work in that region.

$$n(re, t) \leq w(re, t) \tag{3.11}$$

Constraint (3.12) states that the number of employees hired from region '*re'* and during period '*t'* should not be more than the number of employees that are willing to work in that region.

$$LH(re, t) \leq w(re, t) \tag{3.12}$$

Constraint (3.13) states that the number of employees needed to work in region '*re*' and at period *t* should be sufficient enough to meet the manufacturing and re-manufacturing process.

$$n(re, t) \geq \left(\frac{PM}{MWT}\right)x_m(t) + \left(\frac{PRM}{MWT}\right)x_r(t) \tag{3.13}$$

Constraints (3.16), (3.17), and (3.18) considering lay-off is more in the farther region and total employees and total hiring's more from the nearest region and it is also taken into account that (*re*-1)th is the nearest region than *re*th region.

$$n(re - 1, t) \geq n(re, t) \tag{3.14}$$

$$LH(re - 1, t) \geq LH(re, t) \tag{3.15}$$

$$LF(re - 1, t) \geq LF(re, t) \tag{3.16}$$

(iv) Environmental Constraint: Constraint (3.14) states that company's air emissions are always more than the limit specified by the government.

$$x_m(t)E_1(t) + x_r(t)E_2(t) - l(t) \geq 0 \tag{3.17}$$

(v) Decision variables constraint: Constraint (3.15) ensures that the following decision variables are non-negative and are integers.

$$x_r(t), x_m(t), I_r(t), I_s(t), n(re, t), LH(re, t), LF(re, t) \geq 0 \text{ and are integers}$$

$$t \in \{1, 2, \ldots, T\}, re \in \{1, 2, \ldots, R\} \tag{3.18}$$

3.5 Solution Methodology and Result

To solve the proposed multi-objective mathematical model, weighted sum technique is used to combine four objectives to a single objective function. The results are shown in the following tables for ten different cases.

We have considered production in four quarters of a year. Some input parameters in each quarter are given in Table 3.2, considering notations as mentioned in the Sect. 3.4, where $t = 0$ means the final parameter in previous year production or in the beginning of the analysis.

Table 3.2 (Input): considering these initial parameters

$I_r(0)$	150	T	4	$h(1)$	5000	$E_1(3)$	350
$I_s(0)$	250	R	2	$h(2)$	4500	$E_1(4)$	400
$S_r(1)$	100	$M(1)$	190,000	$h(3)$	3000	$E_2(1)$	310
$S_r(2)$	110	$M(2)$	160,000	$h(4)$	4000	$E_2(2)$	461
$S_r(3)$	100	$M(3)$	105,000	$s(1)$	20,000	$E_2(3)$	360
$S_r(4)$	100	$M(4)$	60,000	$s(2)$	20,000	$E_2(4)$	405
$S_m(1)$	100	$N(1)$	100,000	$s(3)$	20,000	$l(1)$	450
$S_m(2)$	20	$N(2)$	100,000	$s(4)$	20,000	$l(2)$	450
$S_m(3)$	35	$N(3)$	200,000	$\delta_r(1)$	1	$l(3)$	450
$S_m(4)$	25	$N(4)$	150,000	$\delta_r(2)$	1	$l(4)$	450
$D(1)$	30,000	$P(r, t)$	100	$\delta_r(3)$	1	$f(1)$	0.001
$D(2)$	55,000	$w(1, 1)$	100	$\delta_r(4)$	1	$f(2)$	0.05
$D(3)$	45,000	$w(1, 2)$	180	$\delta_m(1)$	1	$f(3)$	0.0002
$D(4)$	60,000	$w(1, 3)$	160	$\delta_m(2)$	1	$f(4)$	0.00001
$H_s(1)$	16	$w(1, 4)$	170	$\delta_m(3)$	1	$PP(1)$	60,000
$H_s(2)$	21	PM	1	$\delta_m(4)$	1	$PP(2)$	65,000
$H_s(3)$	15	MWT	480	$\delta_i(1)$	0	$PP(3)$	35,000
$H_s(4)$	10	PRM	1.5	$\delta_i(2)$	0	$PP(4)$	50,000
$H_r(1)$	10	$p(1, 1)$	30,000	$\delta_i(3)$	0	$TI(1)$	160,000
$H_r(2)$	10	$p(1, 2)$	30,000	$\delta_i(4)$	0	$TI(2)$	113,600
$H_r(3)$	10	$p(1, 3)$	30,000	$E_1(1)$	300	$TI(3)$	108,000
$H_r(4)$	12	$p(1, 4)$	30,000	$E_1(2)$	450	$TI(4)$	98,000
$\delta(1)$	1	$w(2, 1)$	50	$p(2, 1)$	30,200		
$\delta(2)$	1	$w(2, 2)$	25	$p(2, 2)$	30,200		
$\delta(3)$	1	$w(2, 3)$	37	$p(2, 3)$	30,200		
$\delta(4)$	1	$w(2, 1)$	50	$p(2, 4)$	30,200		

Table 3.3 predicts the output in each quarter of a year, which includes the number of employees to hire or lay-off and at what time and from which region, number of serviceable and return stock to be held, amount to manufacture and re-manufacture products to minimize the total cost of the company considering variation in demands in each quarter.

Figure 3.1 is representing the manufactured, re-manufactured, and finished good and yet to re-manufacture products in the inventory. It is clear that in production yet to finish re-manufacture products are higher in 4th quarter. Figure 3.2 demonstrates the number of employees living, or hired within 1 km or beyond 1 km radius of the production house. From these results, interference drawn are

1. Whenever we are investing in technology (in whatsoever quarter) to benefit the environment we are getting costs.
2. Because of lack of full-truck load situations (i.e., considering more demand and limited transportation facilities), our warehouse is being filled up with products.
3. Our process of recruitment (depending on constraints) involves more employees from nearest region and less from farther region.
4. However, lay-off employees are less but they are more from farther region.

3.6 Conclusions

The multi-objective inventory planning sustainable supply chains problem is studied in this paper. A comprehensive approach based on three main factors (i.e., economy, society, and environment) of sustainability is less. A multi-objective mixed-integer linear mathematical model is developed. A weighed sum approach is used to solve the proposed model. The results obtained based on expected parameters help us in taking the important decisions to invest in the technology or not, hiring and lay-off, hiring and lay-off from which region. Our proposed model is also beneficial to the company, environment as well local employment personnel. Further, this model could be extended reverse logistics (mainly reuse, recycling and disposal).

Table 3.3 (Output): tabular form of 10 cases (taking different values of $\delta_i(t)$)

1	$\delta_i(1)$	1											
	$\delta_i(2)$	0											
	$\delta_i(3)$	1											
	$\delta_i(4)$	1											
	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	
	1	30,000	29,740	10	0	100,140	62	0	0	0	0	0	1.36786E+12
	2	55,000	87,049	5007	37,056	195,133	172	25	211	0	38	38	
	3	45,000	67,896	40	59,992	395,093	160	37	0	0	0	0	
	4	60,000	4	4	0	545,089	99	99	1	0	0	0	
2	$\delta_i(1)$	0											
	$\delta_i(2)$	1											
	$\delta_i(3)$	0											
	$\delta_i(4)$	1											
	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	
	1	30,000	71,997	2	42,249	100,148	100	50	58	50	58	50	3.777E+11
	2	55,000	6376	6375	0	193,773	17	17	0	0	0	0	
	3	45,000	52,496	52,496	59,992	341,277	17	17	0	0	0	0	
	4	60,000	4	4	0	491,273	17	17	0	0	0	0	
3	$\delta_i(1)$	1											
	$\delta_i(2)$	0											

(continued)

Table 3.3 (continued)

	$\delta_i(3)$	0											
	$\delta_i(4)$	1											
	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	
	1	30,000	29,749	1	0	100,149	62	0	0	0	0	0	5.356E+11
	2	55,000	49,800	5200	0	194,949	95	25	134	0	38	38	
	3	45,000	52,496	52,496	59,992	342,453	83	37	0	0	0	0	
	4	60,000	4	4	0	492,449	60	60	0	0	0	0	
4	$\delta_i(1)$	1											
	$\delta_i(2)$	0											
	$\delta_i(3)$	1											
	$\delta_i(4)$	0											
	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	
	1	30,000	29,740	10	0	100,140	62	0	0	0	0	0	6.328E+11
	2	55,000	87,049	5007	37,056	195,133	172	25	211	0	38	38	
	3	45,000	7904	40	0	395,093	160	37	0	0	0	0	
	4	60,000	59,996	4	0	545,089	99	99	0	0	0	0	
5	$\delta_i(1)$	1											
	$\delta_i(2)$	1											
	$\delta_i(3)$	0											
	$\delta_i(4)$	0											

(continued)

Table 3.3 (continued)

	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	
	1	30,000	71,998	1	42,249	100,149	100	50	58	50	0	0	1.673E+12
	2	55,000	6376	6375	0	193,774	17	17	0	0	58	58	
	3	45,000	22,500	22,500	0	371,274	17	17	0	0	0	0	
	4	60,000	59,996	4	0	521,270	17	17	0	0	0	0	
6	$\delta_i(1)$	0											
	$\delta_i(2)$	0											
	$\delta_i(3)$	0											
	$\delta_i(4)$	1											
	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	
	1	30,000	28,642	28,642	27,534	71,508	100	50	25	25	0	0	120,444,138
	2	55,000	19,008	19,008	10,550	152,500	74	25	0	0	25	26	
	3	45,000	52,500	52,500	70,550	300,000	62	37	0	0	0	0	
	4	60,000	3	3	10,556	449,997	50	50	1	0	0	0	
7	$\delta_i(1)$	1											
	$\delta_i(2)$	0											
	$\delta_i(3)$	0											
	$\delta_i(4)$	0											
	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	

(continued)

Table 3.3 (continued)

	1	30,000	29,749	1	0	100,149	62	0	0	0	0	0	5.355E+11
	2	55,000	49,800	5200	0	194,949	95	25	134	0	38	38	
	3	45,000	22,500	22,500	0	372,449	83	37	0	0	0	0	
	4	60,000	59,996	4	0	522,445	60	60	0	0	0	0	
8	$\delta_i(1)$	0											
	$\delta_i(2)$	1											
	$\delta_i(3)$	0											
	$\delta_i(4)$	0											
	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	
	1	30,000	71,997	2	42,249	100,148	100	50	58	50	0	0	3.775E+11
	2	55,000	6376	6375	0	193,773	17	17	0	0	58	58	
	3	45,000	22,500	22,500	0	371,273	17	17	0	0	0	0	
	4	60,000	59,996	4	0	521,269	17	17	0	0	0	0	
9	$\delta_i(1)$	0											
	$\delta_i(2)$	0											
	$\delta_i(3)$	1											
	$\delta_i(4)$	0											
	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	
	1	30,000	28,670	28,670	27,590	71,480	100	50	0	0	0	0	683,838,022
	2	55,000	36,480	36,480	45,550	135,000	180	10	40	0	0	0	

(continued)

Table 3.3 (continued)

	3	45,000	27	27	604	334,973	153	37	0	0	0	0	
	4	60,000	56,444	2952	0	482,021	95	95	0	0	0	0	
10	$\delta_i(1)$	0											
	$\delta_i(2)$	0											
	$\delta_i(3)$	0											
	$\delta_i(4)$	0											
	Quarter(t)	Demand	$x_m(t)$	$x_r(t)$	$I_s(t)$	$I_r(t)$	$n(re, t)$		$LH(re, t)$		$LF(re, t)$		Total cost
							$re = 1$	$re = 2$	$re = 1$	$re = 2$	$re = 1$	$re = 2$	
	1	30,000	28,800	28,800	27,850	71,350	100	50	44	44	0	0	19,590,450
	2	55,000	21,930	5220	0	166,130	37	25	0	0	44	44	
	3	45,000	22,500	22,500	0	343,630	31	31	0	0	0	0	
	4	60,000	30,000	30,000	0	463,630	31	31	0	0	0	0	

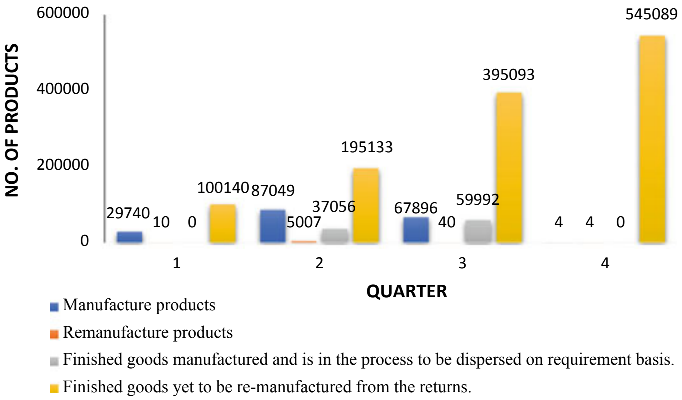

Fig. 3.1 Number of products in the different region in each quarter

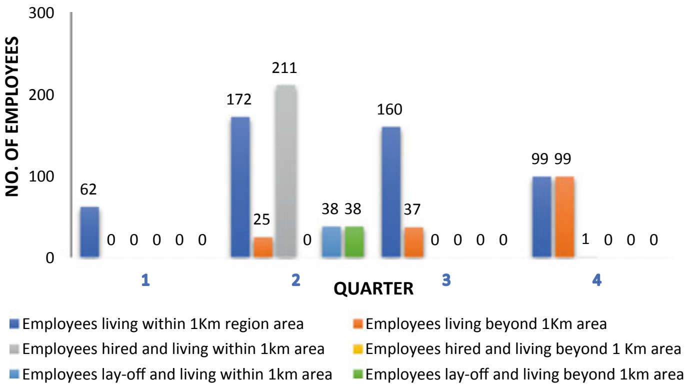

Fig. 3.2 Number of hired employees in the different region in each quarter

References

Agrawal S, Singh RK, Murtaza Q (2015) A literature review and perspectives in reverse logistics. Resour Conserv Recycl 97:76–92

Becerra P, Mula J, Sanchis R (2022) Sustainable inventory management in supply chains: trends and further research. Sustainability 14:2613

Berning A, Venter C (2015) Sustainable supply chain engagement in a retail environment. Sustainability 7(5):6246–6263

Bocken N, Short SW, Rana P, Evans S (2014) A literature and practice review to develop sustainable business model archetypes. J Clean Prod 65:42–56

Brandenburg M, Govindan K, Sarkis J, Seuring S (2014) Quantitative models for sustainable supply chain management: developments and directions. Eur J Oper Res 233(2):299–312

Chen CK, Weng TC, Lo CC (2016) Optimal replenishment quantity for new products and return rate of used products for a retailer. Appl Math Model 40(23–24):9754–9766

Chiu SW, Tseng C-T, Wu M-F, Sung P-C (2016) Multi-item EPQ model with scrap, rework and multi-delivery using common cycle policy. J Appl Res Technol 12:615–622

Chiu Y-SP, Wu M-F, Chiu SW, Chang H-H (2015) A simplified approach to the multi-item economic production quantity model with scrap, rework and multi-delivery. J Appl Res Technol 13:472–476
Craig RC, Easton PL (2011) Sustainable supply chain management: evolution and future directions. Int J Phys Distrib Logist Manag 41(1):46–62
Dakov I, Novkov S (2008) Sustainable supply chain management–scope, activities and interrelations with other concepts. In: Proceeding of 5th international conference on business and management 2008, pp 16–17
Fahimnia B, Sarkis J, Eshragh A (2015) A tradeoff model for green supply chain planning: a leanness-versus-greenness analysis. Omega 54:173–190
Galpin T, Whittington JL, Bell G (2015) Is your sustainability strategy sustainable? Creating a culture of sustainability. Corp Gov 15(1):1–17
Ghadge A, Dani S, Kalawsky R (2012) Supply chain risk management: present and future scope. Int J Logist Manag 23(3):313–339
Ghadimi P, Azadnia AH, Heavey C, Dolgui A, Can B (2016) A review on the buyer–supplier dyad relationships in sustainable procurement context: past, present and future. Int J Prod Res 54(5):1443–1462
Ghadimi P, Dargi A, Heavey C (2017) Making sustainable sourcing decisions: practical evidence from the automotive industry. Int J Logist 20(4):297–321
Ghadimi P, Wang C, Lim MK (2019) Sustainable supply chain modeling and analysis: Past debate, present problems and future challenges. Resour Conserv Recycl 140:72–84
Jindal A, Sangwan KS (2017) Multi-objective fuzzy mathematical modelling of closed-loop supply chain considering economical and environmental factors. Ann Oper Res 257(1):95–120
Kannan G, Soleimani H (2017) A review of reverse logistics and closed-loop supply chains: a journal of cleaner production focus. J Clean Prod 142:371–384
Pagell M, Shevchenko A (2014) Why research in sustainable supply chain management should have no future. J Supply Chain Manag 50(1):44–55
Saffari H, Makui A, Mahmoodian V, Pishvaee MS (2015) Multi-objective robust optimization model for social responsible closed-loop supply chain solved by nondominated sorting genetic algorithm. J Ind Syst Eng 8(3):42–58
Sarkis J, Zhu Q (2018) Environmental sustainability and production: taking the road less travelled. Int J Prod Res 56(1–2):743–759
Seuring S, Müller M (2008) From a literature review to a conceptual framework for sustainable supply chain management. J Clean Prod 16(15):1699–1710
Seuring S (2013) A review of modeling approaches for sustainable supply chain management. Decis Support Syst 54(4):1513–1520
Soleymanfar VR, Makui A, Taleizadeh AA, Tavakkoli-Moghaddam R (2022) Sustainable EOQ and EPQ models for a two-echelon multi-product supply chain with return policy. Environ Dev Sustain 24:5317–5343
Validi S, Bhattacharya A, Byrne PJ (2015) A solution method for a two-layer sustainable supply chain distribution model. Comput Oper Res 54:204–217
Zarbakhshnia N, Kannan D, Mavi RK, Soleimani H (2020) A novel sustainable multi-objective optimization model for forward and reverse logistics system under demand uncertainty. Ann Oper Res 295(2):843–880
Zarbakhshnia N, Soleimani H, Goh M, Razavi SS (2019) A novel multi-objective model for green forward and reverse logistics network design. J Clean Prod 208:1304–1316

Chapter 4
Artificial Intelligence Computing and Nature-Inspired Optimization Techniques for Effective Supply Chain Management

Madhu Jain, Dinesh K. Sharma, and Nidhi Sharma

Abstract The artificial intelligence (AI) and nature-inspired optimization (NIO) techniques can be used to reduce the impact of supply chain disruptions. Nowadays, AI is one of the highly demanded techniques that may be leveraged to increase supply chain and inventory resilience. The present study facilitates the overview of many critical areas of supply chain where AI can help in improving the flexibility of nature-inspired optimization techniques, assuring delivery to the final mile, providing personalized solutions to the stakeholders in upstream and downstream supply chains and many more. The basic AI-based models for supply chain as well as inventory system have been outlined to present state of art of the concerned topic. The usefulness of AI and NIO to speed up the availability of items just in time, optimal delivery process for logistic of items and future scope have been highlighted.

Keywords Supply chain management · Logistic · Artificial intelligence · Nature-inspired optimization

4.1 Introduction

Artificial intelligence (AI) is the process of making a robot, a thinking machine or a product to imagine what a person thinks. Artificial intelligence is the art of making machines that execute function that would require intelligence if performed by humans. The concept of intelligence, or intelligent conduct, must be examined in order to comprehend modern technology. Intelligence is evaluable by one's ability

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to respond quickly and adapt fully to new situations. AI study began in the mid-1950s, sparked by academics' interest in the now-famous Turing Imitation Game. The Turing Test was designed to see if a machine could deceive a human in an interrogation game. Artificial intelligence is a branch of study that has found several applications in addressing problems in a variety of fields.

The field of mathematical optimization has been at the forefront of debate, development, and deployment as industry experts and academics develop and deploy solutions to many challenges. Nature-inspired optimization (NIO) algorithms are a significant study area in the fields of computational intelligence, soft computing, and optimization in general. The development of several new algorithms based on natural processes such as natural selection, foraging for food, rules of physics, movement of a group, and other natural models has greatly expanded this field of study. Especially valuable about these algorithms is that they require no mathematical criteria to work. They are powerful tools for dealing with supply chain management (SCM) that cannot be dealt with conventional mathematical methods.

In recent years, wide global events have compelled major organizations to reconsider their supply chains, as well as their stability and dependability in the face of an uncertain future. Many other externalities and government activities throughout the globe need smooth and efficient management of supply chains. According to the research cited, for the most efficient use of emerging AI and NIO approaches, the rest of the contents are arranged section-wise. We discuss a number of approaches of AI including the applications of diverse area in Sect. 4.2. In Sect. 4.3, we focus on NIO techniques. Some symbols have been described in Table 4.1 related to the NIO algorithms. In Sect. 4.4, we described the basic concepts of SCM with the help of mathematical model. In Sect. 4.5, we highlight the uses of AI, SCM and NIO algorithms to maintain the supply chain. Section 4.6 presents an adapted model related to SCM. The acronyms' used are listed in literature works on AI, NIO, and SCM along with noble contributions are summarized in Sect. 4.7 with Tables 4.2 and 4.3. Section 4.8 is devoted to highlight the scope and future prospects of SCM using AI and NIO techniques.

4.2 Basic Concepts of AI

Artificial intelligence is a broad word that covers a wide range of technology. AI is concerned with computational hardware and software think intelligently in the same way that people do. According to "John McCarthy," recognized as the "Father of AI," AI is intelligent machine science and engineering, particularly intelligent computer programs. A topic of computer science called artificial intelligence deals with, "having robots accomplish tasks that humans would require intelligence to perform." Dirican (2015) explained AI as one of the most well-known of these technologies, which includes blockchain, IoT, cloud computing, and so on. In Fig. 4.1, we have outlined a variety of AI situations in which we are either deliberately or

Table 4.1 The common symbols of NIO algorithm

Conceptions	Symbols	Descriptions
Space dimension	$D, 0 < d \leq D$	The problem space description
Population size	$M, 0 < i \leq M$	Individual quantity
Iteration times	$N, 0 < t \leq N$	Algorithm termination condition
Individual position	$x_i(t) = (x_{i,1}(t), \ldots, x_{i,d}(t), \ldots, x_{i,D}(t))$	The expression of the i^{th} solution on the i^{th} iteration, also used to represent the i^{th} individual
Local best solution	$p_i(t) = (p_{i,1}(t), \ldots, p_{i,d}(t), \ldots, p_{i,D}(t))$	Local best solution of the i^{th} individual on the i^{th} iteration
Global best solution	$p_g(t) = (p_{g,1}(t), \ldots, p_{g,d}(t), \ldots, p_{g,D}(t))$	Global best solution of the whole population on the i^{th} iteration
Fitness function	$f(.)$	Unique standard to evaluate solutions
Precision threshold	δ	Algorithm termination condition

inadvertently involved. AI can be divided into three categories as we have shown in Fig. 4.2.

4.2.1 *Categorization of AI*

In Artificial Intelligence, various categories have been identified based on a machine's ability to make judgments based on past knowledge, memory, and self-awareness.

I. **Machine Learning (ML)**:—ML is one of the AI applications in which robots are not explicitly designed to perform certain tasks, but instead learn and improve via error and trial. Data mining is the process of discovering patterns from data and extracting useful information from using a combination of statistical techniques and rules. It is the reasoning that underpins a machine learning model. The linear regression algorithm is an example of a ML algorithm. With the aid of example data and prior expertise, machines can be taught to solve problems. ML is now used in some form or another in several embedded systems and software on the Internet. Machine learning has become so prevalent that businesses turn to it for a variety of reasons.

II. **Natural Language Processing (NLP)**:—A language-based interaction between computers and humans, whereby the computers are designed to process natural language. ML is a reliable technique in Natural Language Processing for extracting meaning from human language. In NLP, the computer collects the audio of a human conversation. After that, there is an audio-to-text interaction, during which the text is analyzed and the data are transformed into audio. The audio is then used by the machine to reply to people. Natural Language Processing (NLP) is used to analyze the quality of grammar in text in call centers

Table 4.2 Important contributions on SCM using AI and NIO

S. no	Authors	Contributions
1	Tang et al. (2000)	The authors have represented a multiproduct planning and scheduling using GA approach
2	Jeong et al. (2002)	This study described a computerized method for carrying out the forecasting tasks necessary in SCM for developing a generic forecasting model applicable to SCM with GA
3	Maiti et al. (2006)	They used the real coded GA to solve a multi-item inventory problem with the discount policy with two storage systems
4	Costa et al. (2010)	They designed a SCN with GA
5	Min (2010)	This article looked at numerous AI-related areas that face the actual SCM difficulties
6	Jaggi and Khanna (2010)	They studied a SC for degrading commodities by including a stock-dependent consumption rate, shortages under inflation, and a payment delay
7	Martínez-López and Casillas (2013)	The authors of this research conducted a literature assessment of AI-based marketing systems throughout several decades by focusing on applications to industrial marketing
8	Zhang et al. (2017)	A multi-layer graph is used to simulate the SC network
9	Jarrahi (2018)	This article led to a better understanding of how AI may assist and improve human decision-making rather than replace it
10	Elkhechafi et al. (2018)	They developed a model for supply chain optimization by using firefly algorithm
11	Giri and Sarker (2019)	They developed a three-tier SC model that included a single raw material source, a single manufacturer, and a single retailer
12	Giri and Sarker (2019)	A three-tier SC model is developed that included a single raw material source, a single manufacturer, and a single retailer
13	Canhoto and Clear (2020)	These studies offered a new methodology for AI and identified and managed the value-destroying scope of AI and machine learning for enterprises
14	Islam et al. (2020)	They created a green integrated inventory model for a three-tier agricultural product SC
15	Song et al. (2021)	Uncertain demand is defined that leads to an unsatisfactory match between supply and demand, resulting in an excess or lack of inventory for components and parts

(continued)

Table 4.2 (continued)

S. no	Authors	Contributions
16	Xie et al. (2021)	They proposed a robust rolling horizon model with multiple supply networks as sources
17	Jana et al. (2022)	They explained the supply of emergency relief materials through a differential evolution approach

with Interactive Voice Response (IVR) applications, language translation tools like Google Translate, and word processors like Microsoft Word.

III. **Automation and Robotics**:—With automation, machines are able to perform boring and repetitive tasks, resulting in an increased income and more efficient and cost-effective methods of production. Automation is carried out by several organizations using machine learning, neural networks, and graphs. Such automation, when combined with CAPTCHA technology, can help to prevent fraud during online financial transactions. Robotic process automation (RPA) is intended to carry out high-volume, repetitive tasks while adjusting to changing conditions.

IV. **Fuzzy Logic**:—In the actual world, we occasionally encounter situations in which it is difficult to determine if a condition is true or false; their fuzzy logic allows for significant flexibility in thinking, which leads to mistakes and uncertainties in every situation. The rules of fuzzy logic are written in plain language, which is a benefit for formalizing human thinking. Fuzzy logic has commercial and practical applications. It has the ability to control machinery and consumer goods. It may not provide exact logic, but it does provide adequate reasoning. Engineers can use fuzzy logic to deal with ambiguity.

V. **Expert Systems**:—The expert system's efficacy is entirely dependent on the expert's knowledge, which is stored in a knowledge base. The more information collected in it, the more efficient the system becomes. In Google Search Engine, for example, the expert system makes suggestions for spelling and mistakes. Expert systems were regarded as one of the earliest effective AI software models. They were developed for the first time in the 1970s, and then modified in the 1980s.

4.3 Nature-Inspired Optimization (NIO)

NIO is a collection of unique problem-solving techniques and approaches inspired by natural phenomena. Several real-world issues of industries as well as day to day life can be solved using NIO algorithms. Popular techniques of NIO algorithm include the GA, PSO, Cukcoo search, ant colony optimization and many others. These algorithms are extremely effective in locating optimal solutions to multi-dimensional and multi-modal issues. We shall cover a few algorithms in this section. NIO techniques

Table 4.3 Literature survey on AI, NIO, and SCM

S. no	Authors	Article	Artificial intelligence techniques	Nature-inspired optimization	Supply chain management
1	Modgil et al. (2021)	Impact of AI on SC resilience	√	X	√
2	Agbehadji et al. (2020)	Big data, AI and NI computing models	√	√	X
3	Szmelter-jarosz and Ghahremani-nahr (2021)	A neutrosophic fuzzy optimization model	√	X	√
4	Shadkam (2021)	Cuckoo optimization algorithm in reverse logistics	X	√	√
5	Mittal et al. (2019)	A NIO method for SCM problem	√	√	√
6	Soni et al. (2019)	Swarm intelligence approaches in SCM	X	√	√
7	Luan et al. (2019)	A novel method to solve supplier selection problem	√	√	√
8	Szmelter-jarosz and Ghahremani-nahr (2021)	Optimal sustainable closed-loop SC network	√	X	√
9	Sadrnia et al. (2014)	Green SC problems	√	√	√
10	Farooq et al. (2021)	SC operations management in pandemics	X	X	√

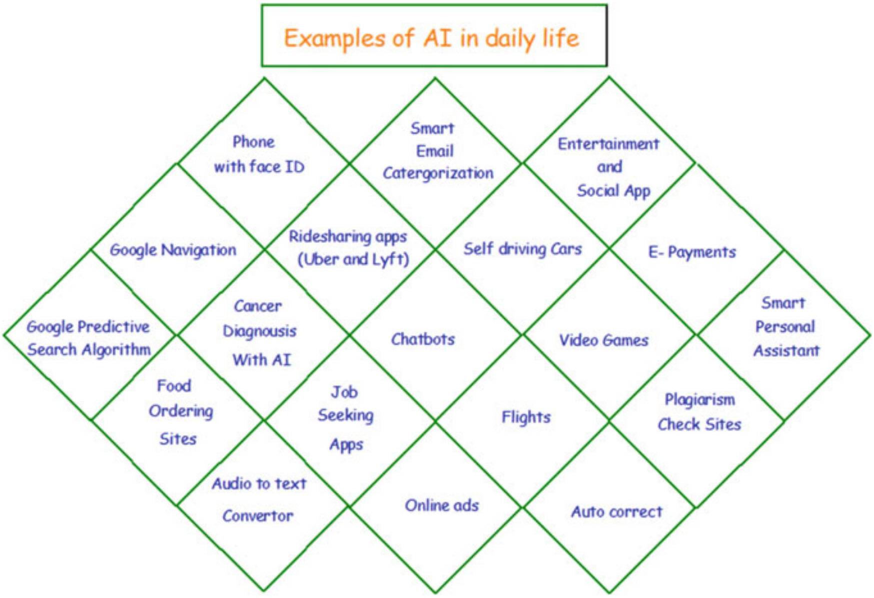


Fig. 4.1 Examples of AI in daily life

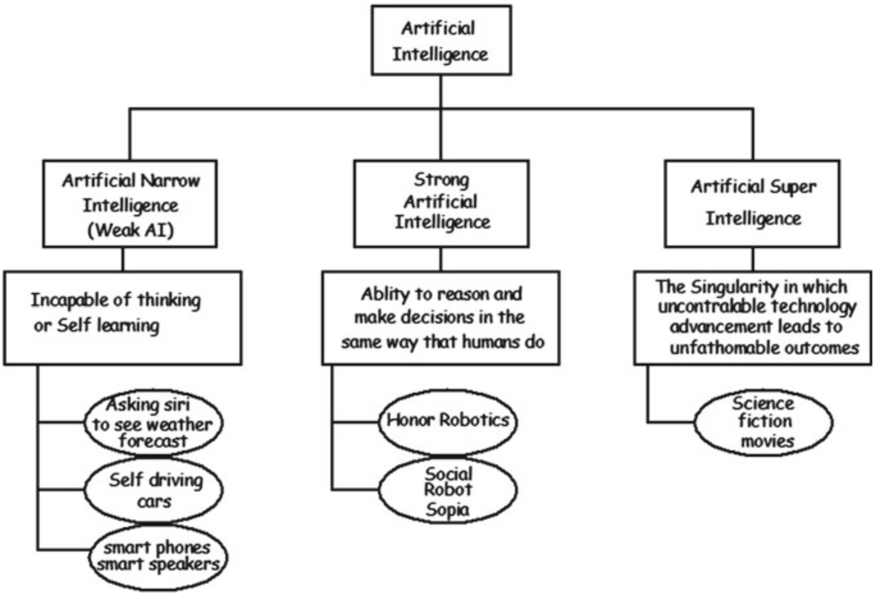


Fig. 4.2 Different categories of artificial intelligence

have applications in digital filter designing, image processing, machine-learning and differentiator designing, face-recognition, etc. Table 4.1 summarizes the common symbols used NIOA.

Let's now examine the iterative schemes used in some NIO algorithms.

- **Differential evolution (DE)**: In differential evolution (Miranda and Alves 2013), the main mutation scheme is

$$x_i^{t+1} = x_i^t + F\left(x_j^t - x_k^t\right) \tag{4.1}$$

  where x_i^t, x_j^t, and x_k^t are three distinct solution vectors from the population. The parameter $F \in (0, 2)$ controls the mutation strength.
- **Particle swarm optimization (PSO)**: PSO is a technique introduced by Kennedy and Eberhart (1995) for maximizing the number of particles in a swarm. The swarming behavior of birds and fish is the fundamental motivation for PSO. The particle's position and velocity at any iteration or pseudo-time can be modified repeatedly using

$$\left.\begin{aligned} v_i^{t+1} &= v_i^t + \Delta v_i^t \\ x_i^{t+1} &= x_i^t + \Delta x_i^t \end{aligned}\right\} \tag{4.2}$$

  with $\Delta x_i^t = v_i^{t+1}\Delta t$ and $\Delta v_i^t = \alpha\varepsilon_1\left[g^* - x_i^t\right] + \beta\varepsilon_2[x_i^* - x_i^t]$. Here ε_1 and ε_2 lie in [0,1].
- **Firefly algorithm (FA)**: The attraction and flashing behavior of tropical fireflies are the major characteristics of FA. The location vector x_i of firefly i is updated using

$$\left.\begin{aligned} x_i^{t+1} &= x_i^t + \Delta x_i^t \\ \Delta x_i^t &= \beta_0 e^{-\gamma r_{ij}^2}\left(x_j^t - x_i^t\right) + \alpha\varepsilon_i^t \end{aligned}\right\} \tag{4.3}$$

  where $\beta_0 > 0$ is the attractiveness at zero distance, that is $r_{ij} = 0$. Firefly visibility is controlled by the scale-dependent parameter γ, while it is essentially the strength of randomization that determines the visibility of fireflies by α.

4.4 Supply Chain Management

Many businesses are being compelled to raise their worldwide market to achieve their desired goals in today's environment. Simultaneously, these businesses must protect their home market share against competition. The problem is figuring out how to develop the global logistics and distribution network so that items may be delivered to customers through a dynamic and quickly changing variety of channels. The long-term competitiveness is thus determined by how effectively a firm can match client

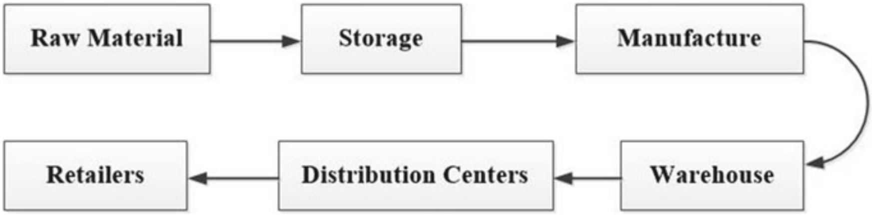


Fig. 4.3 Supply chain management

preferences in terms of service, pricing, quality, and flexibility by developing an appropriate supply chain.

There are three essential aspects of SCM that vary depending on market requirements: product, demand, and lead time characteristics. A pictorial view of SCM is shown in Fig. 4.3.

4.4.1 Two Echelon Supply Chain Inventory Model

A two-echelon supply chain, consisting of one producer and one retailer. Considering the fluctuating customer demand (D) and order quantity (Q), the manufacturer's pace of production (P), the model is developed under following assumptions:

Assumptions

1. No shortages are permitted.
2. The pace of demand is constant.
3. The cost of idle time is assessed at the manufacturer and store levels.

Model Formulation

At manufacturer inventory level (Fig. 4.4a), the manufacturing process begins with a raw material with order of lot size Q. At this level, there are two scenarios. The first is when the raw material quantity falls with the production rate P and approaches zero at time t_1. The second is when the finished goods accumulate at a pace of P. The following are the governing differential equations:

$$\frac{dI_{mr}}{\mathrm{dt}} = -\mathrm{P}, 0 \leq \mathrm{t} \leq \mathrm{t}_1 \tag{4.4}$$

with boundary conditions, $I_{mr}(0) = Q$, $I_{mr}(\mathrm{t}_1) = 0$.

For the finished products

$$\frac{dI_{mf}}{\mathrm{dt}} = \mathrm{P}, 0 \leq \mathrm{t} \leq \mathrm{t}_1, \tag{4.5}$$

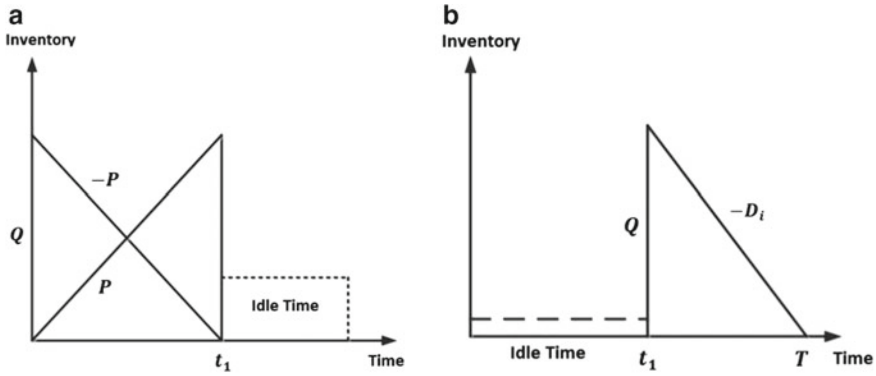


Fig. 4.4 **a** Manufacturer's inventory system. **b** Retailer's inventory system

with boundary conditions, $I_{mf}(0) = 0$, $I_{mr}(\mathrm{t}_1) = Q$.

At retailer's inventory level (Fig. 4.4b), the differential equation is

$$\frac{dI_r}{\mathrm{dt}} = -\mathrm{D}, \mathrm{t}_1 \le \mathrm{t} \le T, \tag{4.6}$$

with boundary conditions, $I_r(\mathrm{t}_1) = Q$, $I_{mr}(T) = 0$.

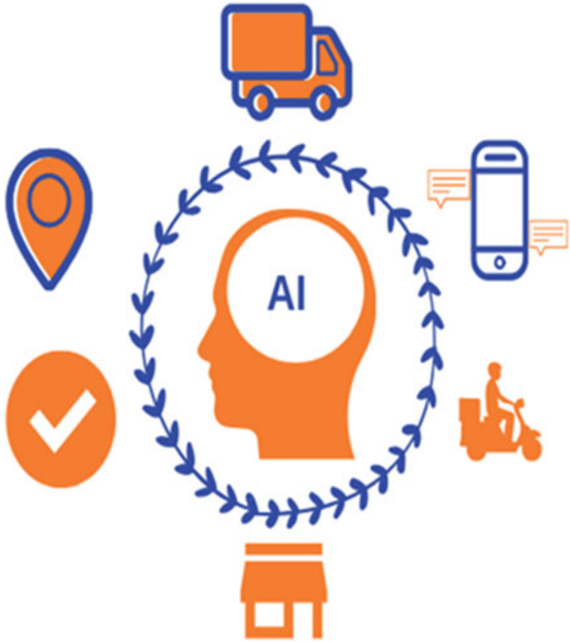
The average total cost (TC) of the model, which is the sum of manufacturer's cost (TC_m), and retailer's cost (TC_r).

$$\text{Total cost } (TC) = TC_m + TC_r \tag{4.7}$$

4.5 Role of AI and NIO Algorithm in SCM

Optimization, expert systems, planning, scheduling, simulations, and modeling are all examples of how AI is applied in decision-making of supply chain. A value chain is defined as "the upstream and downstream movement of products, services, finances, and/or information from a source to a client" by Mentzer et al. (2001). AI solutions have been used in various pioneering projects in the field of SCM. Inventory management, buying, site planning, freight consolidation, and routing/scheduling concerns have all become more common as expert systems and GAs have become more prominent. We look at a few SCM sectors where AI has been used, identify AI subfields that have been effective in enhancing SC judgments, and evaluate how they impact SC decision-making in this part (see Fig. 4.5).

Fig. 4.5 AI in supply chain management



4.5.1 Artificial Neural Network

An artificial neural network (ANN) is a parallel distributed information processing structure made up of a number of nonlinear processing units known as neurons. The majority of businesses use the economic order quantity (EOQ) model to calculate the maximum level of inventory or ordering lot size that they should keep for their business (Ali et al. 2011). However, the EOQ model has certain flaws, such as demand being constant in unit time, lead time being deterministic, purchase price of an item being constant, no lack of raw materials, and immediate material receipt, all of which are unrealistic assumptions. Some researchers used AI approaches to assess inventory levels and supply chain in order to circumvent these restrictions.

4.5.2 Adaptive Neuro-Fuzzy Inference System (ANFIS)

ANFIS is a straightforward data learning approach that converts a given input into a desired output using a fuzzy inference system model. It was developed by Jang (1993). The membership functions, fuzzy logic operators, and if–then logic are all used in this prediction. Mamdani and Sugeno models are two different kinds of fuzzy systems. In ANFIS operation, there are five basic processing phases: input fuzzification, fuzzy operator application, application method, output aggregation, and defuzzification.

ANFIS is based totally on Takagi–Sugeno-Kang version (TSK), or Sugeno fuzzy version where a rule R_k can be represented as:

$$R_k : IF \mu_{Ai}(x) \text{ and } \mu_{Bi}(y) \text{ then } f = p_k x + q_k y + r_k \tag{4.8}$$

In the antecedent section of the rule, A_i and B_i are n fuzzy membership functions of any form μ, such as Gaussian, triangular, trapezoidal, etc., where k is the number of rules. The subsequent sections of the kth rule's linear parameters are R_k and p_k, q_k, r_k. The parameters of the membership functions and the ensuing portion of the rule are adjusted during the training phase.

4.5.3 Inventory Control and Planning

Inventory planning enables businesses to purchase the appropriate quantity of goods and determine how frequently they should replenish. Inventory planning lowers the cost of keeping products in stock while also ensuring that there is adequate stock to create and sell items. Inventory management is a crucial component of supply chain management. An expert system might be integrated into the material requirement planning system, which would record previous master production plans and order patterns and then construct lot-sizing algorithms to anticipate optimal orders and inventory replenishment timing in the future. Teodorovic et al. (2002) used fuzzy logic to assess whether to accept or reject any client request for seat arrangement of airline seat stock control. They have considered a new use of AI in inventory management and planning. Teodorovic et al. (2002) created fuzzy logic algorithms to assess online, intelligent airline seat stock control choices in an application of artificial intelligence to inventory management and planning. Teodorovic et al. (2002) suggested fuzzy logic techniques for making online, intelligent airline seat inventory control decisions on whether to accept or reject any client. Several researchers have used one or two metaheuristic algorithms to deal with inventory control issues. PSO and GA are widely used metaheuristics for SCM.

4.5.4 Transportation Network Design

Intelligent Transportation System is one area where AI applications have made rapid progress (ITS). These frameworks aim to drive engagement forward by utilizing a variety of innovations and communication frameworks. Several attempts have been made to identify the time, place, and severity of an accident in order to aid traffic management in minimizing congestion. These approaches range from handwritten reporting to Neural Networks algorithms that are automated. The "Network Traffic

Design Problem (NTDP)" has an underlying problem called the "Origin-Destiny Traffic Assignment Problem (ODTAP)." Optimization via Simulation (OvS) is a unique method for solving this problem. A simulator can be used to assess the quality of OvS solutions generated by heuristics or metaheuristics.

4.5.5 *Purchasing and Supply Management*

Artificial intelligence (AI) enables procurement firms to use sophisticated computer algorithms to tackle complicated challenges more efficiently and effectively. From expenditure analysis to contract administration and strategic sourcing, AI may be implemented in a variety of software applications. To categorize procurement spend into categories and subcategories, machine learning techniques are used. Determining ordering periods and product order numbers are two strategic decisions in inventory control problems, with the goal of minimizing overall costs or maximizing total revenues. In literature, numerous available models include deterministic single products, deterministic multi-products, and stochastic single products. Nature-inspired intelligent algorithms have been shown to be capable of discovering near-optimal solutions for high-complexity issues in a reasonable amount of time.

4.5.6 *e-Synchronized SCM*

A well-defined supply chain strategy; information visibility throughout the whole supply chain; speed, cost, quality, and customer service; and tighter supply chain integration are all variables that determine the success of an e-SCM. Large organizations utilize electronic data exchange (EDI) as the primary instrument for facilitating supply chain linkages. Many businesses are moving away from traditional EDI and toward Internet-based EDI.

4.6 Adapted Model Related to Operational Decisions in Supply Chain (SC) Network

This model combines pandemic dynamics, SC design, and operational production-inventory control tactics, which are rarely encountered together in operations management literature. This study is carried out for two- and three-stage SC, as well as several pandemic dynamics scenarios (uncontrolled propagation). This work contributes to our knowledge of SC resistance to super-disruptions like a pandemic in various ways. First, rather than treating pandemic dynamics as a single disruptive

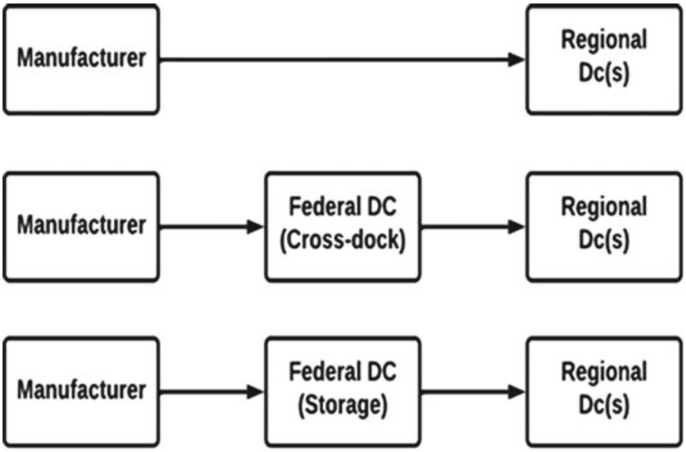


Fig. 4.6 Supply chain structures and sourcing strategies for analysis

event, we describe them as a distinct system that affects the SC system's capacity, supply, and demand (see Fig. 4.6).

Model of operational SC dynamics

Rozhkov et al. (2022) used a special production-inventory control strategy that takes into account product perishability and is similar to Nahmias' (2011). Regional distribution centers (RDC) are catered either from inventory batch I at the federal DC (FDC) or by direct shipments from the manufacturer, depending on the sourcing strategy. Figure 4.7 depicts supply chain network design for pandemic scenario.

For deteriorated items, the two and three echelon SC,

$$L_{\text{two stage}} = L_{1\text{RDC}};\, L_{\text{three stage}} = L_{1\text{RDC}} + L_{2\text{FDC}} \tag{4.9}$$

For modeling purpose, we assume that the product shelf life is denoted by η and the inventory freshness level (in days) at the SC echelon addressing customer demand is specified as C_d. They are employed to determine the

$$C_d^{\max} = \eta - L \tag{4.10}$$

Each SC echelon has a minimal freshness level f that cannot be exceeded (i.e., shelf-life threshold).

For three-echelon setup:

$$C_d > f_{\text{RDC}} \times \eta - f_{\text{FDC}} \times \eta + L \tag{4.11}$$

Based on predicted shipments and inventory dynamics, the order quantity calculation method is used. Orders x_t that have already been placed within the order fulfillment cycle α are included in the future shipments set $J = \{x_t, x_{t+1}, \ldots\ldots, x_{t+\alpha}\}$.

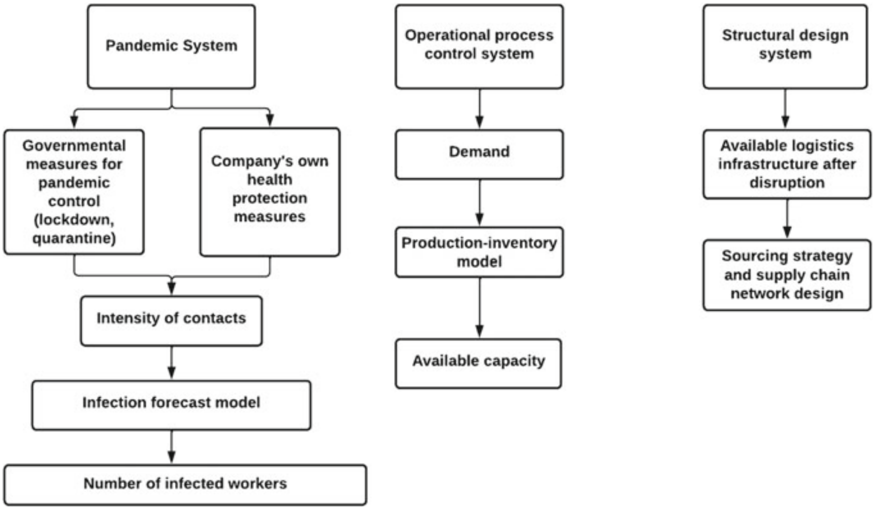


Fig. 4.7 Supply chain network design for pandemic scenario

Keep in mind that $\alpha = L$ is used for three-echelon SC, and $\alpha = L + \varepsilon$ is used for shipments from the manufacturer without inventory holding at the FDC, where ε is the production freeze time.

The projected inventory future state set I^P is determined for each planning period $b, b\epsilon(t, t+\alpha)$ by iteratively merging sets I and J under the constraint that each future shipment x from the set J complies with conditions (4.9) and (4.10). Order $x_{t+\alpha}$ is placed if the expected inventory level at period $t+\alpha$ matching constraint (4.10) is smaller than the intended inventory level (Sethi et al. 2003). The order size is divided by the minimum order size $\overline{\theta}$.

$$x_\alpha = \overline{\theta} \times \text{ceil}\left(\frac{S - I^P_{t+\alpha}}{\overline{\theta}}\right) \tag{4.12}$$

4.7 Literature Survey on AI, NIO, and Supply Chain Management (SCM)

The current study provides overview and research contributions on AI and NIO algorithms that address various issues of supply chain and logistics. A demanding new area of artificial intelligence (AI) known as "nature-inspired intelligence" (NII) is particularly adept at solving tough optimization issues. Similar methods are applied either as independent algorithms or as hybrid systems, that is, in conjunction with other AI techniques. Tables 4.2 and 4.3 provide some important contributions on SCM via AI and/or NIO.

4.8 Research Directions for the Future and Closing Remarks

In the domain of AI and NIO applications, the supply chain is fairly reviewed. We have presented a framework for general aspects about the behavioral and psychological aspects of using AI-driven systems in supply chain management. Our study shows that there are enormous applications of SCM. The use of AI and NIO methodologies can go well beyond what has been discussed here. There have been a number of meta-heuristic algorithms proposed to resolve optimization problems in AI throughout the last 30 years. The NIO techniques have made significant progress since their introduction and can be utilized in a variety of application in SCM domains. We have discussed a model to explain a two-layer supply chain. According to our survey in all directions of AI, NIO algorithms, and SCM, we can finally conclude that AI and NIO techniques are very useful in SCM for sustainable business development.

References

Agbehadji IE, Awuzie BO, Ngowi AB, Millham RC (2020) Review of big data analytics, artificial intelligence and nature-inspired computing models towards accurate detection of COVID-19 pandemic cases and contact tracing. Int J Environ Res Public Health 17(15):1–16

Ali SM, Paul SK, Ahsan K, Azeem A (2011) Forecasting of optimum raw material inventory level using artificial neural network. Int J Oper Quant Manag 17:333–348

Canhoto AI, Clear F (2020) Artificial intelligence and machine learning as business tools: a framework for diagnosing value destruction potential. Bus Horiz 63(2):183–193

Costa A, Celano G, Fichera S, Trovato E (2010) A new efficient encoding/decoding procedure for the design of a supply chain network with genetic algorithms. Comput Ind Eng 59(4):986–999

Dirican C (2015) The impacts of robotics, artificial intelligence on business and economics. Proc Soc Behav Sci 195:564–573

Elkhechafi M, Benmamoun Z, Hachimi H, Elkettani Y (2018) Firefly algorithm for supply chain optimization. Lobachevskii J Math 39(3):355–367

Farooq MU, Hussain A, Masood T, Habib MS (2021) Supply chain operations management in pandemics: a state-of-the-art review inspired by covid-19. Sustainability (Switzerland) 13(5):1–33

Giri BC, Sarker BR (2019) Coordinating a multi-echelon supply chain under production disruption and price-sensitive stochastic demand. J Ind Manag Optim 15(4):1631–1651

Ip WH, Li Y, Man KF, Tang KS (2000). Multi-product planning and scheduling using genetic algorithm approach. Comput Ind Eng 38(2): 283–296. https://doi.org/10.1016/S0360-8352(00)00044-9

Islam SMS, Hossain R, Yasmin MJ (2020) A Green integrated inventory model for a three tier supply chain of an agricultural product. Eng Int 8(2):73–86

Jaggi CK, Khanna A (2010) Supply chain models for deteriorating items with stock-dependent consumption rate and shortages under inflation and permissible delay in payment. Int J Math Oper Res 2(4):491–514

Jana R, Sharma DK, Mehta P (2022) Managing the supply of emergency relief materials through a differential evolution approach. Ann Oper Res. https://doi.org/10.1007/s10479-021-04267

Jang JS (1993) ANFIS: adaptive-network-based fuzzy inference system. IEEE Trans Syst Man Cybern 23: 665–685. https://doi.org/10.1109/21.256541

Jarrahi MH (2018) Artificial intelligence and the future of work: Human-AI symbiosis in organizational decision making. Bus Horiz 61(4):577–586

Jeong B, Jung H, Park N (2002) A computerized forecasting system using genetic algorithm in supply chain management. J Syst Softw 60(3):223–237

Kennedy J, Eberhart R (1995) Particle swarm optimization. In: Proceedings of ICNN'95-international conference on neural networks, vol 4, pp 1942–1948

Luan J, Yao Z, Zhao F, Song X (2019) A novel method to solve supplier selection problem: hybrid algorithm of genetic algorithm and ant colony optimization. Math Comput Simul 156:294–309

Maiti AK, Bhunia AK, Maiti M (2006) An application of real-coded genetic algorithm (RCGA) for mixed integer non-linear programming in two-storage multi-item inventory model with discount policy. Appl Math Comput 183(2):903–915

Martínez-López FJ, Casillas J (2013) Artificial intelligence-based systems applied in industrial marketing: an historical overview, current and future insights. Ind Mark Manage 42(4):489–495

Mentzer JT, DeWitt W, Keebler JS, Min S, Nix NW, Smith CD, Zacharia ZG (2001) Defining supply chain management. J Bus Logist 22(2):1–25

Min H (2010) Artificial intelligence in supply chain management: Theory and applications. Int J Log Res Appl 13(1):13–39. https://doi.org/10.1080/13675560902736537

Miranda V, Alves R (2013) Differential evolutionary particle swarm optimization (DEEPSO): a successful hybrid. In: BRICS Congress on Computational Intelligence and 11th Brazilian Congress on Computational Intelligence, 368–374

Mittal M, Swain PR, Rana H (2019) A nature inspired optimisation method for supply chain management problem. In: Amity International Conference on Artificial Intelligence (AICAI), 505–509

Modgil S, Gupta S, Stekelorum R, Laguir I (2021) AI technologies and their impact on supply chain resilience during COVID-19. Int J Phys Distrib Logist Manag 52(2):130-149. https://doi.org/10.1108/IJPDLM-12-2020-0434

Nahmias S (2011) Perishable inventory systems. Springer Science & Business Media, New York 160. https://doi.org/10.1007/978-1-4419-7999-5

Rozhkov M, Ivanov D, Blackhurst J, Nair A (2022) Adapting supply chain operations in anticipation of and during the COVID-19 pandemic. Omega (United Kingdom) 110:102635

Sadrnia A, Soltani HR, Zulkifli N, Ismail N, Ariffin MKA (2014) A review of nature-based algorithms applications in green supply chain problems. Int J Eng Technol 6(3):204–211

Sethi SP, Yan H, Zhang H (2003) Inventory models with fixed costs, forecast updates, and two delivery modes. Oper Res 51(2):321–328

Shadkam E (2021) Cuckoo optimization algorithm in reverse logistics: a network design for COVID-19 waste management, waste management and research. J Sustain Circ Econ 40(4):458–469. https://doi.org/10.1177/0734242X211003947

Song Z, Tang W, Zhao R, Zhang G (2021) Inventory strategy of the risk averse supplier and overconfident manufacturer with uncertain demand. Int J Prod Econ 234:108066. https://doi.org/10.1016/j.ijpe.2021.108066

Soni G, Jain V, Chan FTS, Niu B, Prakash S (2019) Swarm intelligence approaches in supply chain management: potentials, challenges and future research directions. Supply Chain Manag 24(1):107–123

Szmelter-Jarosz A, Ghahremani-Nahr J (2021) A neutrosophic fuzzy optimisation model for optimal sustainable closed-loop supply chain network during COVID-19. J Risk Financ Manag 14(11):519–531

Teodorovic D, Popovic J, Pavkovic G, Kikuchi S (2002) Intelligent airline seat inventory control system. Transp Plan Technol 25(3):155–173

Xie C, Wang L, Yang C (2021) Robust inventory management with multiple supply sources. Eur J Oper Res 295(2):463–474

Zhang X, Chan FTS, Adamatzky A, Mahadevan S, Yang H, Zhang Z, Deng Y (2017) An intelligent physarum solver for supply chain network design under profit maximization and oligopolistic competition. Int J Prod Res 55(1):244–263

Zivkovic M, Bacanin N, Djordjevic A, Antonijevic M, Strumberger I, Rashid TA (2021) Hybrid genetic algorithm and machine learning method for COVID-19 cases prediction, Proc Int Conf Sustain Expert Syst Springer, Singapore 169–184

Chapter 5
An EPQ Model for Imperfect Production System with Deteriorating Items, Price-Dependent Demand, Rework and Lead Time Under Markdown Policy

Srabani Shee and Tripti Chakrabarti

Abstract The inventory model with price-dependant demand and imperfect production has been studied in this paper under markdown policy. Herein, we are assuming that the items which can be serviced and reworked are deteriorating with time. The markdown policy is being adopted to lower the on-hand inventory and to enhance the total revenue or the rate of sale of goods. Generally, such policies are adopted at the end of the session for stock clearance or for selling off any obsolete goods at the end of the stock's life. Herein, shortages of items are accepted during lead time and it is fully backlogged. For obtaining the optimal total cost, a solution procedure has been developed in this model. For exemplification of the procedure, a numerical example with practical application has been presented. Sensitivity Analysis has been used to express the impact of system descriptors on the system metrics.

Keywords EPQ models · Deterioration · Price-dependent demand · Lead tim · Imperfect production · Rework · Shortages · Markdown policy

5.1 Introduction

EPQ is one of the most important research areas in production and inventory control. EPQ model helps in obtaining the optimal quantity of products being produced. Various assumptions have gone behind growth of the classical EPQ model. Since then, the model has been extended by relaxing one or more of its assumptions.

Certain products like fruits, vegetables and milk have a deterioration feature. Deterioration is defined as obsolescence, evaporation, decay and loss of quality marginal value of a product resulting into decreasing effectiveness from the initial condition.

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The products stored in the inventory for a long duration will result in the higher cost of the deteriorating items. Sometimes, retailers use the markdown strategy to reduce their on-hand inventory. The retailers attempt to raise their profit by assuming that demand will rise with decreasing price.

In this proposed study, we have considered a markdown policy on an EPQ model for deteriorating products with price-dependent demand rate. In this EPQ model, the items produced are imperfect and the same undergoes a rework process. Here, shortages are allowed during lead time. The main purpose of this model is to establish the optimum time to reduce the total cost and inventory. All the theoretic models are explained with the support of a numerical problem using LINGO 17.0 software.

5.2 Literature Survey

In recent decades many researchers have developed a production inventory model assuming selling price-dependent (SPD) demand. An EOQ model with SPD demand has been developed by Mondal and Bhunia (2003). Saha and chakrabarti (2016) introduced a buyer–vendor EOQ model with time adjusting holding cost where the lead time has been considered as a decision variable. Abdoli (2016) considered an inventory model with a fluctuating demand rate for deteriorating and improving items. Saha and Chakrabarti (2017) have developed a fuzzy inventory model for worsening items in a supply chain system with SPD demand. Also, Saha and Chakrabarti (2019) presented an inventory control model with shortages by including the features of time-dependent deterioration and SPD ramp type demand.

In a practical production environment, production of faulty products along with the non-faulty is very much unavoidable. The same might be due to various factors like damages, imperfect production process and natural disasters. Hence, imperfect production system or defective products will be an inevitable part of an inventory model. Many researchers have studied the effect of imperfect (non-standard) quality production on EPQ models. It has been assumed by Hayek and Salameh (2001) that all the defective products that are being produced can be repaired and derived into an optimal EPQ model. Van der Laan and Teunter (2002) made an attempt to find the explanation for the non-optimal state in an inventory model with reproduction. Chiu (2003) has reflected upon a finite EPQ model for repairable faulty products with a random defective rate and reworking of the same to obtain an optimal operating policy with a lot size that will reduce the total inventory cost. Jamal et al. (2004) assumed a specific production model including two cases of rework process to minimise the total inventory cost. Among the most recent work, Sarkar et al. (2010) established an EPQ model where the demand is stock dependent in an imperfect production process. Kundu and Chakrabarti (2015) created a multi-stage supply chain model where the production process is imperfect. Khanna et al. (2017) developed an imperfect EPQ model under a two-level trade credit considering inspection faults, return of sales and rework. Pal and Mahapatra (2017) have designed a three-layer supply chain model involving the supplier, manufacturer and retailer wherein, they have assumed an imperfect EPQ model with inspection faults and stochastic demand with shortages. Saha and Chakrabarti (2018a, b) investigated an imperfect production inventory model where they considered advertisement and stock-dependent demand for deteriorating items under trade credit. Saha and Chakrabarti (2018a, b) considered an imperfect quality inventory model with Learning Effects and Imprecise Inventory Related Costs where market demand is considered as imprecise in nature and shortages are allowed under Screening Errors.

Inventory model with price-dependent demand and markdown price is also seeking the consideration of researchers. Urban and Baker (1997) developed optimum ordering and pricing plans with multivariate demand and discounts in single period atmosphere. Widyadana and Wee (2007) have developed an inventory model for deteriorating items under the markdown policy wherein demand is price-dependent. Roy Monami Das (2018) introduced a production inventory model for deteriorating products under the markdown policy with variable production rate and stock and sales price sensitive demand. Omar and Kamaruzaman (2019) developed a production model of deferred decaying items with inventory and price-dependent demand under the markdown policy. Monika Rani (2020) has designed a production inventory model for delayed deteriorating products where demand is dependent on selling price and time under the markdown policy.

5.3 Assumptions and Notations

5.3.1 Notations

The model has been established based on the below mentioned notations.

$D(\gamma)$ Demand rate (units/unit time).
P Production rate (unit/unit time) ($P > D$).
P_r Rework rate (unit/unit time) ($P_r > D$).
δ Markdown rate.
α Ratio of good quality items ($0 < \alpha < 1$).
θ Deterioration rate (per unit time) ($0 < \theta < 1$).
γ Selling price (per unit).
T_1 Production start time.
T_2 Time at which all back-orders are emptied.
T_3 Production stop and rework start time.
T_4 Rework stoppage and markdown offer time.
T_5 Length of Cycle time.
c_p Production cost (including raw materials and inspection costs) (Per unit).
K Set Up cost (where K = Production set up cost + rework set up cost = $K_p + K_r$).
c_b Back order cost (per unit).

c_d	Deterioration cost (per unit).
c_r	Rework cost (per unit).
h_s	Holding cost for serviceable items (per unit).
h_r	Holding cost for reworkable items (per unit).

5.3.2 *Assumptions*

Following assumptions have been considered for development of the model.

i. Demand is SPD,where $D(\gamma) = \begin{cases} a\gamma^{-b},\ 0 \le t \le T_4 \\ a(\delta\gamma)^{-b},\ T_4 \le t \le T_5 \end{cases}$, $a, b > 0$, constants
ii. Single deteriorating type of product is considered over a finite planning horizon. The deterioration rate is constant.
iii. Production and rework rates are fixed.
iv. The required raw materials are supplied after time T_1 of order. Lead time is T_1.
v. Shortages are permissible during lead time and totally backlogged.
vi. Products produced will have certain imperfection due to usage of faulty machine affecting the production rate.
vii. All imperfect products undergo rework post stoppage of production.
viii. The demand rate is lesser than the rate at which ideal quality products are obtained post production and rework process
ix. Rework products are good like new ones.
x. One time markdown price for a planning horizon
xi. Markdown price is known from before.
xii. Once production is stopped, then markdown is offered.

5.4 Mathematical Model

The complete inventory cycle has been segregated into five stages for formulation of the model: Lead time (Stage 1), backorder replenishment phase (Stage 2), building phase of inventory (Stage 3), Rework phase (Stage 4), markdown phase (Stage 5). Figs. 5.1 and 5.2 illustrate the on hand inventory pattern of both serviceable and reworkable products during a complete cycle T. Inventory opens with zero stock level at $t = 0$. The suppliers always deliver the raw materials after a span T_1 when shortage starts accumulating at rate $D(\gamma)$. T_1 is time when production is started to cater the present and backlogged demand and continues till $t = T_3$. P units of the items are manufactured. Due to the imperfect manufacturing process, the manufactured quantity P includes a fraction αP of non-defective items with $(1 - \alpha)P$ of defective items. The shortage of the inventory level touches zero at time $t = T_2$; post which a certain level of inventory gets added. Till Stage 2, no serviceable items are held in inventory and thus no deterioration occurs; the imperfect products start accumulating

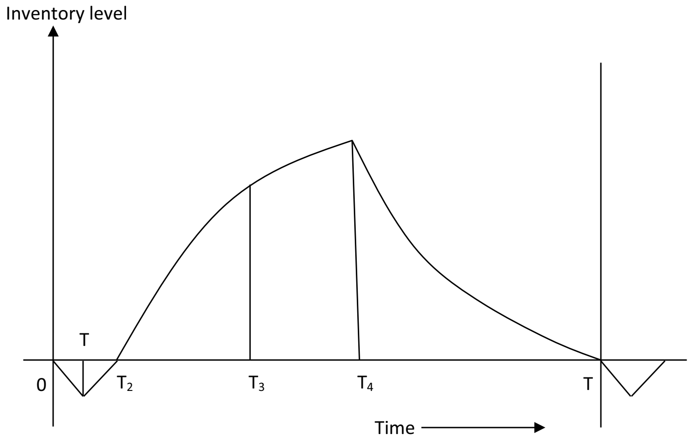

Fig. 5.1 Pattern of serviceable inventory in the course of a complete cycle

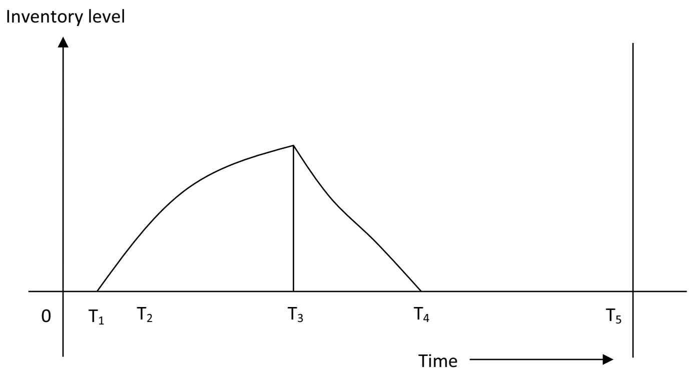

Fig. 5.2 Pattern of reworkable inventory in the course of a complete cycle

from T_1 and hence suffer deterioration with time. At $t = T_3$ all defective products accumulated in the inventory go through the rework process at a rate P_r. At $t = T_4$, the reworkable stock level touches zero and the rework stops. Also, markdown is offered at time $t = T_4$, and as an outcome, demand goes up due to decrease in the selling price. The serviceable stock then gets replenished due to demand and deterioration until $t = T_5$. The cycle stops with zero inventories at time $t = T_5$. The rate of deterioration θ is very small and hence for ease of calculation, we can ignore the second and higher powers of θ.

The governing equations of serviceable inventory level for the five phases are constructed as follows:

$$\frac{dI_1(t)}{dt} = -a\gamma^{-b}, 0 \le t \le T_1, I_1(0) = 0 \tag{5.1}$$

$$\frac{dI_2(t)}{dt} = \alpha P - a\gamma^{-b}, T_1 \le t \le T_2, I_2(T_2) = 0 \tag{5.2}$$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = \alpha P - a\gamma^{-b}, T_2 \le t \le T_3, I_3(T_2) = 0 \tag{5.3}$$

$$\frac{dI_4(t)}{dt} + \theta I_4(t) = P_r - a\gamma^{-b}, T_3 \le t \le T_4, I_4(T_3) = I_3(T_3) \tag{5.4}$$

$$\frac{dI_5(t)}{dt} + \theta I_5(t) = -a(\delta\gamma)^{-b}, T_4 \le t \le T_5, I_5(T_5) = 0 \tag{5.5}$$

The governing equations of reworkable inventory level are

$$\frac{dI_6(t)}{dt} + \theta I_6(t) = (1-\alpha)P, T_1 \le t \le T_3, I_6(T_1) = 0 \tag{5.6}$$

$$\frac{dI_7(t)}{dt} + \theta I_7(t) = -P_r, T_3 \le t \le T_4, I_7(T_4) = 0 \tag{5.7}$$

Solving the differential equations given by (5.1)–(5.7), we get

$$I_1(t) = -a\gamma^{-b}t, 0 \le t \le T_1 \tag{5.8}$$

$$I_2(t) = (\alpha P - a\gamma^{-b})(t - T_2), T_1 \le t \le T_2 \tag{5.9}$$

$$I_3(t) = \frac{(\alpha P - a\gamma^{-b})}{\theta}\left\{1 - e^{\theta(T_2 - t)}\right\}, T_2 \le t \le T_3 \tag{5.10}$$

$$I_4(t) = \frac{e^{-\theta t}}{\theta}(\alpha P - a\gamma^{-b})(e^{\theta T_3} - e^{\theta T_2}) + \frac{(P_r - a\gamma^{-b})}{\theta}\left\{1 - e^{\theta(T_3 - t)}\right\},$$
$$T_3 \le t \le T_4 \tag{5.11}$$

$$I_5(t) = \frac{a(\delta\gamma)^{-b}}{\theta}\left\{e^{\theta(T_5 - t)} - 1\right\}, T_4 \le t \le T_5 \tag{5.12}$$

$$I_6(t) = \frac{(1-\alpha)P}{\theta}\left\{1 - e^{\theta(T_1 - t)}\right\}, T_1 \le t \le T_3 \tag{5.13}$$

$$I_7(t) = \frac{P_r}{\theta}\left\{e^{\theta(T_4 - t)} - 1\right\}, T_3 \le t \le T_4 \tag{5.14}$$

Using $I_1(T_1) = I_2(T_1)$, we get the following relation

$$-a\gamma^{-b}T_1 = (\alpha P - a\gamma^{-b})(T_1 - T_2) \tag{5.15}$$

Using $I_4(T_4) = I_5(T_4)$, we get the following relation

$$\frac{e^{-\theta T_4}}{\theta}(\alpha P - a\gamma^{-b})(e^{\theta T_3} - e^{\theta T_2}) + \frac{(P_r - a\gamma^{-b})}{\theta}\left\{1 - e^{\theta(T_3 - T_4)}\right\}$$
$$= \frac{a(\delta\gamma)^{-b}}{\theta}\left\{e^{\theta(T_5 - T_4)} - 1\right\} \tag{5.16}$$

Using $I_6(T_3) = I_7(T_3)$, we get the following relation

$$\frac{(1-\alpha)P}{\theta}\left\{1 - e^{\theta(T_1 - T_3)}\right\} = \frac{P_r}{\theta}\left\{e^{\theta(T_4 - T_3)} - 1\right\} \tag{5.17}$$

Taylor's Series can be used on the relations given by (5.15)–(5.17) to find the time periods. The time spans $T_j^{'}s(j = 1, 2, \ldots, 5)$ are of the form

$$T_2 = \frac{\alpha P}{(\alpha P - a\gamma^{-b})}T_1 \tag{5.18}$$

$$T_4 = T_3 + \frac{(1-\alpha)P}{P_r}\left\{T_3 - T_1 - \frac{\theta}{2}(T_3 - T_1)^2\right\} \tag{5.19}$$

$$T_5 = T_4 + \frac{(\alpha P - a\gamma^{-b})}{a(\delta\gamma)^{-b}}(1 - \theta T_4)(T_3 - T_2 + \frac{\theta}{2}T_3^2 - \frac{\theta}{2}T_2^2)$$
$$+ \frac{(P_r - a\gamma^{-b})}{a(\delta\gamma)^{-b}}\left\{T_4 - T_3 - \frac{\theta}{2}(T_4 - T_3)^2\right\} \tag{5.20}$$

Now, the total deteriorated items per cycle.

$$= \text{Serviceable items deteriorated} + \text{Reworkable items deteriorated}$$
$$\Big[\left\{(\alpha P - a\gamma^{-b})(T_3 - T_2) - I_3(T_3)\right\} + \left\{(P_r - a\gamma^{-b})(T_4 - T_3) + I_3(T_3) - I_4(T_4)\right\}$$
$$+ \left\{I_4(T_4) - a(\delta\gamma)^{-b}(T_5 - T_4)\right\}\Big] +$$
$$[\{(1-\alpha)P(T_3 - T_1) - I_6(T_3)\} + \{I_6(T_3) - P_r(T_4 - T_3)\}]$$
$$= P(T_3 - T_1) + \left\{a(\delta\gamma)^{-b} - a\gamma^{-b}\right\}T_4 - a(\delta\gamma)^{-b}T_5 \tag{5.21}$$

Therefore, the per unit time (PUT) deterioration cost is

$$D_c = \frac{c_d}{T_5}\left[P(T_3 - T_1) + \left\{a(\delta\gamma)^{-b} - a\gamma^{-b}\right\}T_4 - a(\delta\gamma)^{-b}T_5\right] \tag{5.22}$$

The production cost PUT is

$$P_c = \frac{c_p}{T_5} P(T_3 - T_1) \tag{5.23}$$

The Set up cost PUT is

$$S_c = \frac{K}{T_5} \tag{5.24}$$

The Back order cost PUT is

$$B_c = \frac{c_b}{T_5} a\gamma^{-b} T_1 \frac{T_2}{2} \tag{5.25}$$

The rework cost PUT is

$$R_c = \frac{c_r}{T_5} P_r(T_4 - T_3) \tag{5.26}$$

The holding cost per unit time is

$$\begin{aligned}
H_c = {} & \text{holding cost for serviceable products PUT} \\
& + \text{holding cost for reworkable products PUT} \\
= {} & \frac{h_s}{T_5}\left[\int_{T_2}^{T_3} I_3(t)dt + \int_{T_3}^{T_4} I_4(t)dt + \int_{T_4}^{T_5} I_5(t)dt\right] \\
& + \frac{h_r}{T_5}\left[\int_{T_1}^{T_3} I_6(t)dt + \int_{T_3}^{T_4} I_7(t)dt\right] \\
= {} & \frac{h_s}{T_5}\Bigg[\frac{(\alpha P - a\gamma^{-b})}{2}(T_2 - T_3)^2 + (\alpha P - a\gamma^{-b})(T_3 - T_2 + \frac{\theta}{2}T_3^2 - \frac{\theta}{2}T_2^2) \\
& \times (T_4 - T_3 + \frac{\theta}{2}T_3^2 - \frac{\theta}{2}T_4^2) + \frac{(P_r - a\gamma^{-b})}{2}(T_3 - T_2)^2 + \frac{a(\delta\gamma)^{-b}}{2}(T_5 - T_4)^2\Bigg] \\
& + \frac{h_r}{T_5}\left[\frac{(1-\alpha)P}{2}(T_3 - T_1)^2 + \frac{P_r}{2}(T_4 - T_3)^2\right]
\end{aligned} \tag{5.27}$$

Adding all cost functions viz. (5.22)–(5.27) and by applying the time relations viz. (5.18)–(5.20), the total inventory cost/unit time during a cycle period, as a function of T_3, is given by

$$
\begin{aligned}
TIC(T_3) &= D_c + P_c + S_c + B_c + R_c + H_c \\
&= \frac{c_d}{T_5}\left[P(T_3 - T_1) + \left\{a(\delta\gamma)^{-b} - a\gamma^{-b}\right\}T_4 - a(\delta\gamma)^{-b}T_5\right] \\
&+ \frac{c_p}{T_5}P(T_3 - T_1) + \frac{K}{T_5} + \frac{c_b}{T_5}a\gamma^{-b}T_1\frac{T_2}{2} + \frac{c_r}{T_5}P_r(T_4 - T_3) \\
&+ \frac{h_s}{T_5}\frac{(\alpha P - a\gamma^{-b})}{2}(T_2 - T_3)^2 + \frac{h_s}{T_5}(\alpha P - a\gamma^{-b}) \\
&+ (T_3 - T_2 + \frac{\theta}{2}T_3^2 - \frac{\theta}{2}T_2^2)(T_4 - T_3 + \frac{\theta}{2}T_3^2 - \frac{\theta}{2}T_4^2) \\
&+ \frac{h_s}{T_5}\frac{(P_r - a\gamma^{-b})}{2}(T_3 - T_2)^2 + \frac{h_s}{T_5}\frac{a(\delta\gamma)^{-b}}{2}(T_5 - T_4)^2 \\
&+ \frac{h_r}{T_5}\left[\frac{(1-\alpha)P}{2}(T_3 - T_1)^2 + \frac{P_r}{2}(T_4 - T_3)^2\right]
\end{aligned} \tag{5.28}
$$

Since the total inventory cost function PUT obtained from Eq. (5.28) is highly non-linear, the partial derivatives are very complex. Thus, it is problematic to get an analytical solution for the optimal value of T_3. Likewise, the analytical check of the cost function for convexity could not be done due to the high complexity involved. To check the convexity of the cost function, a numerical example has been given.

5.5 Numerical Illustration

We consider the following numerical values of the parameters in appropriate units to analyse the model:

$P = 300,\ a = 200,\ \gamma = 150,\ b = 0.045, \delta = 0.6, P_r = 240,\ \theta = 0.06,\ \alpha = 0.95,\ c_p = 25,$
$c_d = 10,\ c_r = 3,\ c_b = 2,\ k_p = 150,\ k_r = 100,\ h_s = 4,\ h_r = 4.5,\ T_1 = 0.4$ in appropriate units

We obtain the optimal total cost $TIC = 4351.06$ and $T_3 = 1.012275$ in appropriate units.

Sensitivity Analysis

The value of each of the parameters P, P_r, θ, c_p, c_d, c_r, c_b, K, h_s, h_r, δ is changed to perform the sensitivity analysis, considering one parameter at a time and keeping the remaining parameters constant. We will now look into the sensitivity of the optimal solution with respect to the changes in the values of different parameters connected with the model (Tables 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 5.10, 5.11 and Figs. 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 5.10, 5.11, 5.12, 5.13).

Table 5.1 Sensitivity of parameter P on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
P	390	4405.42	0.8117902
	330	4373.04	0.9237332
	300	4351.06	1.012275
	270	4323.40	1.147144
	210	4247.18	1.965929

Table 5.2 Sensitivity of parameter P_r on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
P_r	312	4352.75	1.001296
	264	4351.68	1.008380
	240	4351.06	1.012275
	216	4350.37	1.016403
	168	4348.71	1.025217

Table 5.3 Sensitivity of parameter θ on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
θ	0.078	4358.56	0.9713115
	0.066	4353.89	0.9981551
	0.06	4351.06	1.012275
	0.054	4347.88	1.026899
	0.042	4340.46	1.057797

Table 5.4 Sensitivity of parameter c_p on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
c_p	32.5	5558.44	0.9739816
	27.5	4753.81	0.9992262
	25	4351.06	1.012275
	22.5	3948.01	1.025633
	17.5	3141.04	1.053369

Table 5.5 Sensitivity of parameter c_d on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
c_d	13	4353.47	1.0000383
	11	4351.89	1.008265
	10	4351.06	1.012275
	9	4350.2	1.016331
	7	4348.4	1.024589

Table 5.6 Sensitivity of parameter c_r on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
c_r	3.9	4358.18	1.012171
	3.3	4353.43	1.012240
	3	4351.06	1.012275
	2.7	4348.68	1.012309
	2.1	4343.94	1.012378

Table 5.7 Sensitivity of parameter c_b on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
c_b	2.6	4366.25	1.026555
	2.2	4356.16	1.017071
	2	4351.06	1.012275
	1.8	4345.92	1.007441
	1.4	4335.52	0.9976585

Table 5.8 Sensitivity of parameter K on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
K	325	4413.93	1.071335
	275	4372.65	1.032566
	250	4351.06	1.012275
	225	4328.78	0.9912983
	175	4281.68	0.9469790

Table 5.9 Sensitivity of parameter h_s on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
h_s	5.2	4353.01	0.9944634
	4.4	4351.78	1.005676
	4	4351.06	1.012275
	3.6	4350.26	1.019703
	2.8	4348.34	1.037767

Table 5.10 Sensitivity of parameter h_r on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
h_r	5.35	4353.28	1.010056
	4.95	4352.23	1.011097
	4.5	4351.06	1.012275
	4.05	4349.88	1.013459
	2.65	4346.2	1.017189

Table 5.11 Sensitivity of parameter δ on *TIC* and T_3

Parameter	Change values	Changes in *TIC*	Changes in T_3
δ	0.78	4346.63	1.028482
	0.66	4348.91	1.020400
	0.6	4351.06	1.012275
	0.54	4353.07	1.0004102
	0.42	4354.94	0.9958811

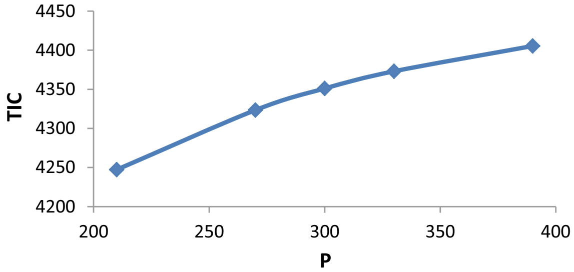

Fig. 5.3 Impact of *P* on *TIC*

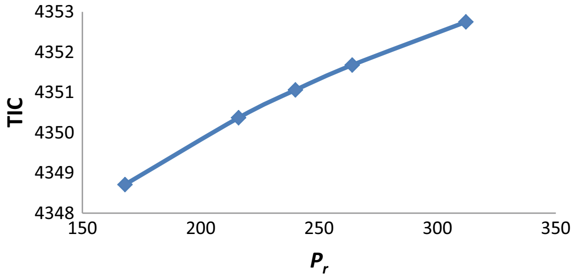

Fig. 5.4 Impact of P_r on *TIC*

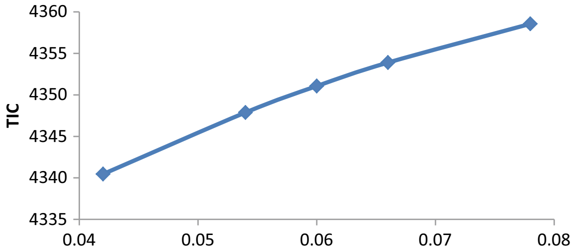

Fig. 5.5 Impact of θ on *TIC*

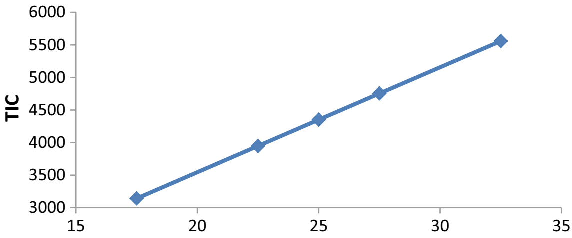

Fig. 5.6 Impact of c_p on *TIC*

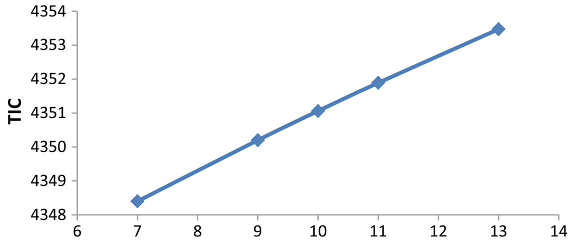

Fig. 5.7 Impact of c_d on *TIC*

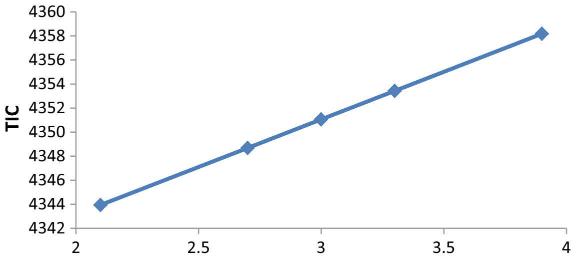

Fig. 5.8 Impact of c_r on *TIC*

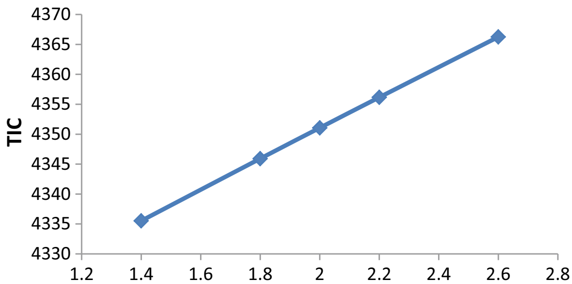

Fig. 5.9 Impact of c_b on *TIC*

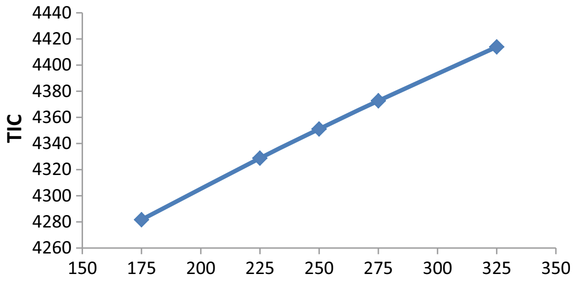

Fig. 5.10 Impact of K on TIC

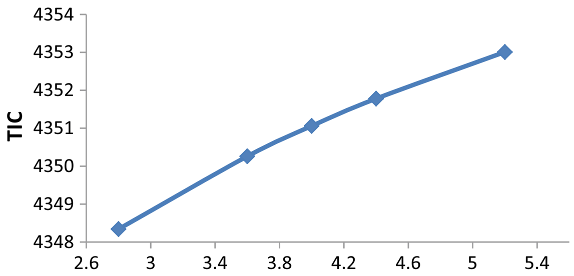

Fig. 5.11 Impact of h_s on TIC

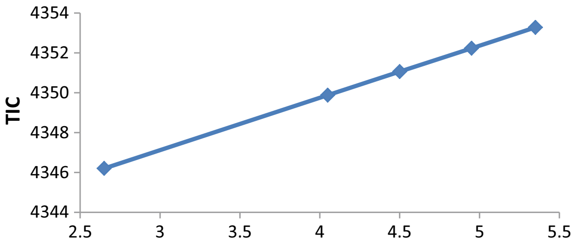

Fig. 5.12 Impact of h_r on TIC

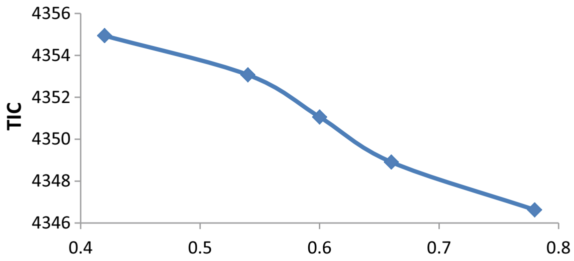

Fig. 5.13 Impact of δ on TIC

Observations

Based on numerical results summarised in the tables we notice the following:

- The total inventory cost *TIC* is highly sensitive to the production cost C_p, production rate P and set up cost K.
- Total cost *TIC* is moderately sensitive towards P_r, θ, c_d, c_r, c_b, h_s, h_r, δ.
- As the production rate P and production cost C_p, rise (or declines), the value of T_3 decreases (or increases) but increases (or decreases) the value total cost *TIC*. That is why, to produce larger lots of items, there is an increase in the total cost.
- The set up cost rises (or declines) the total inventory cost *TIC* and the time T_3 also increases (or decreases).
- The markdown rate δ surges (or diminishes) the value of total inventory cost *TIC* decreases (or increases) but the change in rate of markdown directly affects time unit T_3.

5.6 Conclusions

In this proposed system, an imperfect EPQ model under the markdown policy with a selling price-dependent demand rate has been established wherein items are deteriorating with time. This model will be relevant for certain perishable T_3 products, whose market demand is less than the other products and there is a continuous deterioration with time. Since the raw materials are supplied after some time, the lead time occurs. The results demonstrate that the markdown rate gives a substantial contribution in optimising the total cost and a policy maker must be very attentive in setting the markdown rate, because the optimum policy varies from case to case. Also, sensitivity analysis concludes that delay in the application of the markdown policy will lead to a higher profit and aside, as markdown rises, the total cost will decrease.

In this study, we have not examined the effect of other parameters of different deteriorating rates for finding the optimal cost. For further modification, different types of deteriorating functions can be taken into consideration. The present study can also be stretched by considering the stochastic demand rate in place of the price-dependent demand rate.

References

Abdoli M (2016) Inventory model with variable demand rate under stochastic inflation for deteriorating and ameliorating items with permissible delay in payment. Int J Oper Res. 27(3):375

Chiu YP (2003) Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. Eng Optim 35(4):427–437

Hayek PA, Salameh MK (2001) Production lot sizing with the reworking of imperfect quality items produced. Product Plan Control 12(6):584–590

Jamal AMM, Sarker BR, Mondal S (2004) Optimal manufacturing batch size with rework process at a single-stage production system. Comput Ind Eng 47:77–89

Kamaruzaman AN, Omar M (2019) An EPQ model of delayed deteriorating items with price and inventory level dependent demand under markdown policy. J Mech Cont Math Sci Special Issue-1, 383–389

Khanna A, Kishore A, Jaggi C (2017) Strategic production modeling for defective items with imperfect inspection process, rework, and sales return under two-level trade credit. Int J Ind Eng Comput 8(1):85–118

Kundu S, Chakrabarti T (2015) An integrated multi-stage supply chain inventory model with imperfect production process. Int J Indust Eng Comput, Growing Sci 6(4):565–580

Mondal B, Bhunia AK, Maity M (2003) An inventory system of ameliorating items for price dependent demand rate. Comput Ind Eng 45(3):443–456

Pal S, Mahapatra GS (2017) A manufacturing oriented supply chain model for imperfect quality with inspection errors, Stochastic demand under rework and shortages. Comput Ind Eng 106:299–314

Rani M (2020) An Integrated EPQ inventory model for delayed deteriorating items with time and price dependent demand with inflation under discount policy. Int J Innov Sci Res Technol 5(5):1115–1119

Roy D, Monami (2018) An EPQ model with variable production rate and markdown policy for stock and sales price sensitive demand with deterioration. Int J Eng Sci Math 7(4):260–268

Saha S, Chakrabarti T (2019) An EOQ inventory Model for time-dependent deteriorating items under price-dependent ramp type demand with shortages. Int J Adv Res Sci Eng Technol 6(8)

Saha S, Chakrabarti T (2016) A buyer-vendor EOQ model with time varying holding cost involving lead time as a decision variable under an integrated supply chain system. Int J Sci Eng Res 7(1):352–364

Saha S, Chakrabarti T (2017) Fuzzy inventory model for deteriorating items in a supply chain system with price dependent demand and without backorder. Am J Eng Res 6(6):183–187

Saha S, Chakrabarti T (2018a) Two- Echelon Supply Chain model for deteriorating items in an imperfect production system with advertisement and stock-dependent demand under trade credit. Int J Supply Operat Manage 5(3):207–217

Saha S, Chakrabarti T (2018b) Inventory model for items with imperfect quality subject to learning effects and imprecise inventory related costs and imprecise market demand having shortages under screening errors. Handbook of Research on Promoting Business Process Improvement through Inventory Control Techniques, Chapter 16:284–304

Sarkar B, Chaudhuri K, Sana SS (2010) A stock dependent inventory model in an imperfect production process. Int J Procur Manage 3(4):361–378

Teunter R, Van der Laan E (2002) On the non-optimality of the average cost approach for inventory models with remanufacturing. Int J Product Res 79(1):67–73

Urban TL, Baker RC (1997) Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns. Eur J Oper Res 103(3):573–583

Widyadana GA, Wee HM (2007) A replenishment policy for items with price-dependent demand and deteriorating under markdown policy. J. Teknik Industri 9(2):75–84

Chapter 6
Retrial Inventory-Queueing Model with Inspection Processes and Imperfect Production

Palak Mehta, Madhu Jain, and Sibasish Dhibar

Abstract This study examines the inventory-queueing system (IQS) with balking and retrial orbits. IQS may produce defective items which wait in the queue during the inspection process. The inventory formed in the production process is checked by an inspector to identify misclassifications of type-I and type-II. The arriving jobs either leave after getting served with exactly one item or retry after some time if the server is occupied. The impatience jobs may balk from the system. The joint stationary distributions have been derived in an explicit product form and further used to obtain the counts of the jobs, the available inventory, and the counts of units in the inspection system. The gradient descent method has been implemented to minimize the total cost function so as to find the optimal production rate.

Keywords Retrial queue · Inventory · Balking · Imperfect production · Imperfect inspection · Gradient descent method

6.1 Introduction

The interrelationship between the integrated supply chain and the inventory-queueing system opened up new opportunities to explore them. Several researchers have developed inventory-queue models to study the production and waiting issues. The significance of such studies is well known in providing the managerial insights with regard to the demand prediction and queue formation. In the practical scenario, the clients' demand cannot be fulfilled immediately. Thus, the queue is formed, and the service provider may take time to serve the customers. Some queueing-inventory models are

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studied by Graves (1982), Sigman and Simchi-Levi (1992), Berman et al. (1993), Schwarz et al. (2006), Albrecher et al., (2017), etc.

The retrial queueing models with impatience behaviour of the customers provide more accurate prediction of inventory-queueing system (IQS). The retrial attempts of the customers/jobs seem quite common in the real-life congestion scenario. Wang (2015) studied the retrial queue inventory model by considering the two demand classes and optimized the number of servers, stock and reorder levels using combined enumerative and quasi-Newton approaches. Manikandan and Nair (2017) presented the M/M/1 retrial queue inventory model and established the stability condition for the system. They employed an algorithmic approach to solve this problem. Shajin and Krishnamoorthy (2020) analysed retrial IQS by considering the stochastic decomposition. Recently, Jaganathan et al. (2021) studied the stochastic model for single server retrial queue by assuming the state-dependent rates.

The production of items for most of the inventory system cannot be 100% perfect in the real-time scenario. With the aim to gain the customers' loyalty and for their satisfaction, it becomes important for the manufacturers to provide the quality product. Therefore, the produced items should be screened via the inspection process to separate the defective items. Al-Salamah (2011) analysed the economic order quantity model by considering the destructive sampling and inspection errors. Yu and Chen (2018) studied a single vendor and single retailer production inventory system in which the defective items are screened by the 100% inspection process. They studied the model to establish the vendor and buyer relationship. The optimization has been done for the quantitative assessment of annual profits, optimal size of shipments, etc. Adak and Mahapatra (2021) studied the two-layer supply chain production system in the crisp and fuzzy environment by inclusion of defective and non-defective items, and screening of the items produced.

Due to the various external factors, it may happen that the inspector can make some inspection-errors i.e., he may identify a defective item as a non-defective item and vice-versa. To identify a non-defective (defective) as a defective (non-defective) item is termed as Type-I (II) error. Yu et al. (2009) investigated mixed policy for the inventory-queue model with errors of Type-I and Type-II. The net profit of the mixed inspection policy was obtained for the long term. Taheri-Tolgari et al. (2019) investigated the defective quality process in a production system in the fuzzy environment and developed the inspection policy through preventative maintenance. The partial backlogging occurred in their model due to the shortage caused by preventive maintenance. Aghsami (2021) investigated the imperfect production system with imperfect inspection in the IQS. They obtained joint stationary distribution and used a non-linear programming approach to evaluate the design parameters at a minimum cost. Asadkhani et al. (2022) developed four economic order quantity models. They included imperfection of the types of salvage, repairable, scrap and reject items. They also considered the learning in the inspection process by considering the human tendency to improve itself.

By reviewing the relevant literature, it appears that the retrial IQS was not studied in the realistic scenario of production of defective items. The practical scenarios may have various factors that makes it impossible to deliver hundred percent non-defective items. As a result, studies on the retrial inventory-queueing models with defective products seem to be more realistic and practical. The delivery of a defective item causes customer dissatisfaction, which leads to a loss of business and client loyalty. Therefore, the inspection process can be proved to be advantageous for both the customers' satisfaction and profitability. Also, different external factors can make the inspection process imperfect. In this chapter, we study a retrial queue inventory model by considering the queues for the arrivals of jobs, inventory level and inspection process. The joint stationary distribution has been obtained for this model and performance indices such as means count of jobs in the system, in the retrial orbit, mean items in the inventory and in the inspection system etc. are derived. The sensitivity analysis has been performed for the performance indices with respect to key parameters. To find the optimal production rate and minimum total cost, the gradient descent method has been used. In this article, our research aims to provide the manufacturers and system designers a deep understanding of IQS with retrials to the academicians and system designers. The rest of the study is presented in the various sections as follows: Sect. 6.2 describes the model by including the assumptions made for the mathematical formulation. In Sect. 6.3, the analysis of the model has been done. Section 6.4 consists of system performance indices, whereas Sect. 6.5 and 6.6 deal with the cost optimization and sensitivity analysis, respectively. The chapter is concluded by highlighting the silent features in Sect. 6.7.

6.2 Model Description

The imperfect production system having the single server retrial queue and provision of imperfect inspection is studied. The integrated system consists of a queueing system having the retrial orbit, production system and inspection system as depicted in Fig. 6.1. To study this inventory-queueing model, the following assumptions are made:

- The arrival of the jobs follows Poisson process with arrival rate λ. The jobs are served by the single server following exponential distribution (Exp-D) with rate μ. The service is done according to the FCFS discipline.
- If the server is occupied, the jobs wait in the retrial orbit. The jobs either gets served immediately or retry to get served from the retrial orbit. The jobs retry from the orbit according to Exp-D with rate γ. The jobs may balk on arrival from the system with probability q.
- The production process is governed by Poisson process with rate η. The probability of items to be defective is σ. The percentage of defective products is constant.
- There is unlimited room in the service system for the jobs and for the storage of items. If the available inventory reduces to zero, the existing jobs wait for the inventory to be replenished, but new jobs are unwilling to join the system, resulting in lost sales.

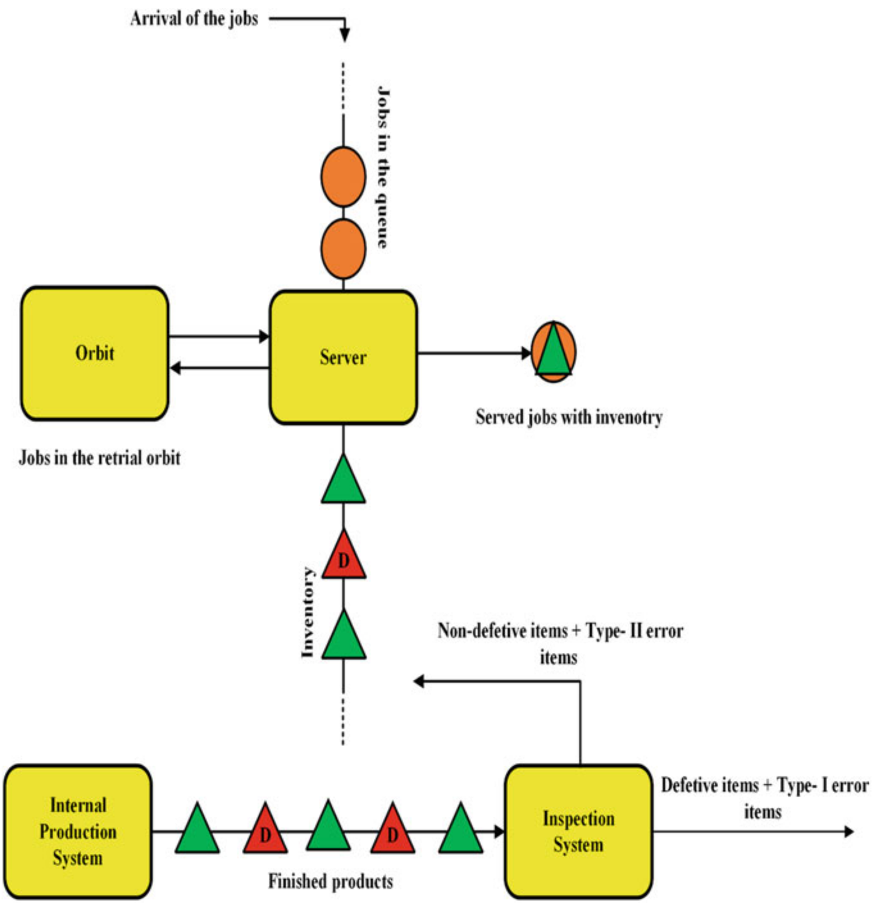

Fig. 6.1 The inventory retrial queueing system with imperfect production and inspection

- After the production process, inspection takes place for 100% of the produced items. The inspection system consists of one inspector who checks the produced items. The inspection may be imperfect i.e., defective items may be classified as non-defective and vice-versa. The rate of inspection of the items is considered as ω.
- As the inspection process is imperfect, this may result in misclassification of Type-I and Type-II with probabilities σ_1 and σ_2, respectively. In type-I misclassification, a non-defective item is identified as defective, whereas in type-II misclassification, a defective item is classified as non-defective. The probability of identification of an item as defective by the inspector is σ_e. The probability σ_e is computed by $\sigma_e = \sigma_1(1 - \sigma) + (1 - \sigma_2)\sigma$.
- To ensure the ergodicity of the queueing system, the assumption of $\lambda < \mu$ and $\eta < \omega$ is maintained.

It is considered that the service times, production times, inter-arrival times and inspection time are mutually independent. Consequently, a stochastic process can be structured given as.

$$S = \{(\aleph(t), \Re(t), \Im(t), \Upsilon(t)) : t \geq 0\} \tag{6.1}$$

where $\aleph(t)$,$\Re(t)$, $\Im(t)$ and $\Upsilon(t)$ represent the count of jobs in the system, in the retrial orbit, the counts of available inventory and the count of items in the inspection system at time t, respectively. It is evident that the waiting time in each state is distributed exponentially, and the stochastic process is a 4-D continuous time Markov chain (CTMC).

Therefore, the state space for the concerned system is defined as:

$$Q(k, r, l, m) = \{(k, r, l, m), k, r, l, m \geq 0\} \tag{6.2}$$

The joint stationary distribution can be obtained using the state space with limiting case, when

$t \to \infty$ and given by

$$\lim_{t\to\infty} Q(\aleph(t), \Re(t), \Im(t), \Upsilon(t)) = Q(k, r, l, m) \tag{6.3}$$

The transition rates for system states are given as follows:

$\delta(k, r, l, m)(k + 1, r, l, m) = \lambda q, k \geq 0;\ \ r \geq 1;\ \ l \geq 1; m \geq 0;$

$\delta(k + 1, r, l + 1, m)(k + 1, r, l, m) = \mu, k \geq 0;\ \ r \geq 1;\ \ l \geq 0; m \geq 0;$

$\delta(k, r, l, m)(k, r, l, m + 1) = \eta, k \geq 0;\ \ r \geq 1;\ \ l \geq 0; m \geq 0;$

$\delta(k, r + 1, l, m)(k, r, l, m) = \gamma, k \geq 0;\ \ r \geq 1;\ \ l \geq 0; m \geq 0;$

$\delta(k, r, l, m + 1)(k, r, l, m) = \omega\sigma_e, k \geq 0;\ \ r \geq 1;\ \ l \geq 0; m \geq 0;$

$\delta(k, r, l, m + 1)(k, r, l + 1, m) = \omega(1 - \sigma_e), k \geq 0;\ \ r \geq 1;\ \ l \geq 0; m \geq 0;$

6.3 Joint Probability Distributions

This section aims to derive the expressions for the joint probability distributions $Q(k, r, l, m)$. As the integrated inventory-queue system consists of three different independent systems dealing with (i) the retrial queue which is modelled by M/M/1/∞ retrial queue with balking, (ii) the production process which is considered as M/M/1/∞ queue and (iii) the inspection process which is also formulated as M/M/1/∞ model. Now, we will derive the expressions of the queue size distributions for the three processes.

The steady state equations for various system states are framed using inflow–outflow of the various processes involved.

6.3.1 Governing Equations

$$
\begin{aligned}
(\lambda q+\lambda)Q(k,0,0,0) &= \omega\sigma_e Q(k,0,0,1)+\mu Q(k+1,0,0,0)\\
&\quad +\gamma Q(k,r+1,0,0),\ \ k\geq 0;\quad r\geq 0
\end{aligned}
\tag{6.4}
$$

$$
(\lambda+j\gamma)Q(k,r,0,0)=\mu Q(k,r,0,0);\ \ k\geq 0;\ \ r\geq 0;\ \ j\geq 1 \tag{6.5}
$$

$$
\begin{aligned}
(\eta+\omega)Q(k,0,0,m) &= \eta Q(k,0,0,m-1)+\omega\sigma_e Q(k,0,0,m+1)\\
&\quad +\mu Q(k+1,0,1,v);\quad k\geq 1;\ \ m\geq 1
\end{aligned}
\tag{6.6}
$$

$$
\begin{aligned}
(\eta+\lambda)Q(0,r,l,0) &= \omega(1-\sigma_e)Q(0,r,l-1,1)\\
&\quad +\omega\sigma_e Q(0,r,l,1)+\mu Q(1,r,l+1,0);\quad r\geq 1;\ \ l\geq 1
\end{aligned}
\tag{6.7}
$$

$$
\begin{aligned}
(\eta+\mu+\lambda)Q(k,r,l,0) &= \omega(1-\sigma_e)Q(k,r,l-1,1)\\
&\quad +\omega\sigma_e Q(k,r,l,1)+\mu Q(n+1,r,l+1,0)\\
&\quad +\lambda Q(k-1,r,l,0)\quad k\geq 1;\ \ r\geq 1; l\geq 1
\end{aligned}
\tag{6.8}
$$

$$
\begin{aligned}
(\eta+\mu+\lambda)Q(0,r,l,m) &= \eta Q(0,r,l,m-1)+\omega(1-\sigma_e)Q(0,r,l-1,m+1)\\
&\quad +\omega\sigma_e Q(0,r,l,m+1)+\mu Q(1,r,l+1,m)\\
&\quad r\geq 1; l\geq 1;\ \ m\geq 1
\end{aligned}
\tag{6.9}
$$

$$
\begin{aligned}
(\eta+\mu+\lambda+\omega)Q(k,r,l,m) &= \eta Q(k,r,l,m-1)+\omega(1-\sigma_e)Q(k,r,l-1,m+1)\\
&\quad +\omega\sigma_e Q(k,r,l,m+1)+\mu Q(k+1,r,l+1,m)\\
&\quad +\lambda Q(k-1,r,l,m),\ k\geq 1;\ \ r\geq 1; l\geq 1; m\geq 1
\end{aligned}
\tag{6.10}
$$

$$
\sum_k\sum_r\sum_l\sum_m Q(k,r,l,m)=1 \tag{6.11}
$$

Remark 1 For events m and (k, r, l), the joint probability distribution is given by:

$$Q((k, r, l), m) = f(k, r, l).f_m(m) \tag{6.12}$$

due to the independence of both the events. Here, $f(k, r, l)$ represents the joint probability distribution of k jobs in the system, r jobs in the retrial orbit, and l items at the inventory level,$f_m(m)$ denotes the stationary distribution of m items in the inspection system.

Further Eq. (6.12) can be rewritten as:

$$Q(k, r, l, m) = f_{k,r}(k, r).f_l(l).f_m(m) \tag{6.13}$$

where,$f_{k,r}(k, r)$, $f_l(l)$ are the stationary distribution of k jobs in the system and r jobs in the retrial orbit, and the stationary distribution of l items in the inventory level.

6.3.2 *Derivation of Joint Probability Distribution Function*

The probability distributions of the system can be obtained by solving Eqs. (6.4)–(6.11). However, the joint probability distribution $Q(k, r, l, m)$ is obtained using Eq. (6.13) through the limiting process.

(i) ***Stationary probability distribution for the retrial queue of jobs*** ($f_{k,r}(k, r)$)***:***

The expression for $f_{k,r}(k, r)$ can be derived using results for the M/M/1 retrial queue with balking Gross and Harris (Gross and Harris 1998), by considering the arrival rate λ, service rate μ, and retrial rate α, and balking probability q. Therefore, we obtain

$$f_{k,r}(k, r) = \left\{ (1-\rho)^{\frac{\lambda}{\gamma}+1}(1+\rho)\left[\frac{\rho^n}{n!\gamma^n}\left\{ \prod_{j=0}^{k-1}(\lambda q + j\gamma) + \prod_{j=1}^{k}(\lambda q + j\gamma) \right\}\right]\right\} \tag{6.14}$$

(ii) ***Stationary probability distribution for the inventory*** ($f_l(l)$)***:*** The inventory of items forms the M/M/1 queue with arrival rate $\eta(1-\sigma_e)$ and service rate λ. Therefore, we have

$$f_l(l) = \left(1 - \frac{\eta(1-\sigma_e)}{\lambda}\right)\left(\frac{\eta(1-\sigma_e)}{\lambda}\right)^l \tag{6.15}$$

(iii) ***Stationary probability distribution for the inspection*** ($f_m(m)$)***:*** The items in the inspection system form the M/M/1 queue with arrival rate η and service rate ω. Therefore

$$f_m(m) = \left(1 - \frac{\eta}{\omega}\right)\left(\frac{\eta}{\omega}\right)^m \tag{6.16}$$

Therefore, the joint probability distribution Eq. (6.13) yields,

$$Q(k,r,l,m) = \left\{(1-\rho)^{\frac{\lambda}{\gamma}+1}(1+\rho)\left[\frac{\rho^n}{n!\gamma^n}\left\{\prod_{j=0}^{k-1}(\lambda q + j\gamma) + \prod_{j=1}^{k}(\lambda q + j\gamma)\right\}\right]\right\} \times \left(1-\frac{\eta(1-\sigma_e)}{\lambda}\right)\left(\frac{\eta(1-\sigma_e)}{\lambda}\right)^l \times \left(1-\frac{\eta}{\omega}\right)\left(\frac{\eta}{\omega}\right)^m \tag{6.17}$$

where, $\rho = \frac{\lambda}{\gamma}$.

6.4 System Performance Indices

This section presents the important performance indices such as mean count of jobs in the system and in the orbit, expected available inventory level and expected count of items in the inspection system, etc. The performance indices are formulated based on the classical M/M/1 retrial queue with balking strategy (Gross and Harris 1998) and M/M/1 models for inventory and inspection queues and are given as follows:

(i) The mean count of jobs in the system is

$$EL_S = \sum_k \sum_r \sum_l \sum_m kQ(k,r,l,m) = \sum_k kf_k(k) = \frac{\rho}{1-\rho} \times \frac{\lambda+\gamma}{\gamma} \tag{6.18}$$

(ii) The mean count of jobs in the orbit is

$$EL_O = \sum_k \sum_r \sum_l \sum_m rQ(k,r,l,m) = \sum_r rf_r(r) = \frac{\rho^2}{1-\rho} \times \frac{\mu+\gamma}{\gamma} \tag{6.19}$$

(iii) The expected available inventory level is

$$E_{lkrm} = \sum_k \sum_r \sum_l \sum_m lQ(k,r,l,m) = \sum_l lf_l(l) = \left(1-\frac{\eta(1-\sigma_e)}{\lambda}\right) \tag{6.20}$$

(iv) The expected counting of items in the inspection system is

$$E_{lkrs} = \sum_k \sum_r \sum_l \sum_m mQ(k,r,l,m) = \sum mf_m(m) = \frac{\eta}{\omega-\eta} \tag{6.21}$$

(v) The expected number of lost sales is

$$E_{loss} = \sum_{k}\sum_{r}\sum_{m} \lambda Q(k, r, 0, m) = \lambda f_l(0) \tag{6.22}$$

(vi) The probability of the server being in idle state

$$\begin{aligned}\sigma_{ssi} &= \sum_{r}\sum_{l}\sum_{m} Q(0, r, l, m) + \sum_{k\geq 1}\sum_{r}\sum_{m} Q(k, r, 0, m)\\ &= f_k(0) + f_l(0)(1 - \sigma_k(0))\end{aligned} \tag{6.23}$$

(vii) The probability of inspector being in idle state is

$$\sigma_{sii} = \sum_{r}\sum_{l}\sum_{m} \lambda Q(k, r, l, 0) = f_m(0) \tag{6.24}$$

(viii) The probability that the server and the inspector are in the idle state

$$\begin{aligned}\sigma_{tsi} &= \sum_{r}\sum_{l}\sum_{m} Q(0, r, l, 0) + \sum_{k\geq 1} Q(k, r, 0, 0)\\ &= f_k(0).f_m(0) + f_l(0).f_m(0).(1 - f_k(0))\end{aligned} \tag{6.25}$$

6.5 Cost Optimization

This section contains the cost optimization formulation by considering the cost function for the concerned model as a function of the production rate η. The cost optimization is done to determine the optimal production rate and corresponding minimum total cost per unit time (PUT). The cost factors associated with different activities are given in Table 6.1.

Table 6.1 Cost factors

C_H: The holding cost PUT for each inventory unit
C_W: Waiting costs PUT for each job in the stock out zone
C_σ: The cost PUT of production of each item
C_{T1}: The cost PUT incurred when Type-I misclassification happens
C_{T2}: The cost PUT incurred when Type-II misclassification happens
C_L: The cost incurred for losing a job

Table 6.2 GDM algorithm

Input: Initial value η_0, function $ETC(\eta)$

Output: η^*, a local optimal of the expected cost function $ETC(\eta)$

Begin

$j \leftarrow 0;$

while STOP-CRITERIA and $(j < j_{\max})$ do

$\quad \eta^{(j+1)} \leftarrow \eta^{(j)} - \alpha^{(j)} \nabla ETC(\eta);$

$\quad$ with $\alpha^{(j)} = \arg\min_{\alpha \in \mathbb{R}_+} ETC\left(\eta^{(j)} - \alpha \nabla ETC(\eta)\right);$

$\quad j \leftarrow j+1;$

return $\eta^{(j)}$

end

The expected total cost PUT can be expressed as

$$ETC(\eta) = C_H EL_{inv} + C_W EL_C \sigma_i(0) + C_\sigma C_{T1}(1-\sigma)\sigma_1 + \eta C_{T2}\sigma\sigma_2 + C_L EL_{loss} \tag{6.26}$$

The expected total cost (ETC) function is nonlinear, so it is difficult to find an optimal value for the production rate η by the classical optimization approach. Therefore, to find the minimum ETC, the gradient descent method (GDM) is used. Using the gradient descent method (GDM) algorithm, we obtain the optimal production rate (η^*) and the respective minimum total cost (ETC^*).

Hence, the optimization problem is

$$ETC\left(\eta^*\right) = \min ETC(\eta) \tag{6.27}$$

To implement cost optimization, the iterative steps of the gradient descent method (GDM) approach given in Table 6.2, are used.

6.6 Numerical Illustration and Sensitivity Analysis

This section presents the numerical results to examine the behaviour of system performance measures. We shall conduct the sensitivity analysis to explore how certain important parameters affect the total cost that was formulated in the previous section. The study of the impact of change that the system parameters make on the cost function can help the organizations in making decision. In Figs. 6.2(i)–(ii), 6.3(i)–(ii), 6.4(i)–(iii), and 6.5(i)–(iii), we analyse the impact of the arrival rate λ, service rate

Table 6.3 The optimal rate of production η^* and corresponding optimal total cost ETC^* w.r.t. C_{T1} and σ_1)

σ_1		C_{T1}					
		150	200	250	300	350	400
0.00	η^*	37.63	37.63	37.63	37.63	37.63	37.63
	ETC^*	3900.21	3900.21	3900.21	3900.21	3900.21	3900.21
0.02	η^*	30.86	30.75	30.45	30.32	30.14	30.04
	ETC^*	4100.56	4136.36	4163.58	4261.25	4278.21	4289.78
0.04	η^*	30.68	30.52	30.42	30.38	30.12	30.01
	ETC^*	4236.21	4287.32	4299.65	4312.66	4365.68	4392.56
0.06	η^*	30.89	30.52	30.36	30.22	30.12	30.08
	ETC^*	4412.35	4562.25	4668.21	4688.58	4763.25	4789.36
0.08	η^*	32.65	32.75	32.66	32.52	32.49	32.56
	ETC^*	4768.58	4898.54	4968.69	5169.73	5278.63	5398.59
0.10	η^*	33.56	33.45	33.44	33.33	33.25	33.12
	ETC^*	4800.52	4823.78	4936.25	5062.21	5156.14	5236.54

μ, other parameters (σ, σ_1 and σ_2) and cost elements on the expected system length EL_S, the expected orbit length EL_O and the expected total cost ETC of the system.

Now, we select the default system parameters as
$\lambda = 30,\ \mu = 40,\ \omega = 50,\ q = 0.7,\ \gamma = 7, \sigma_1 = 0.05,\ \sigma_2 = 0.05,\ \sigma = 0.1,$

The default cost factors chosen are
$C_H = 20,\ C_W = 70, C_\sigma = 90,\ C_{T1} = 200, C_{T2} = 400, C_L = 120,$

The optimal production rate and corresponding optimal minimum cost value are given in Table 6.3.

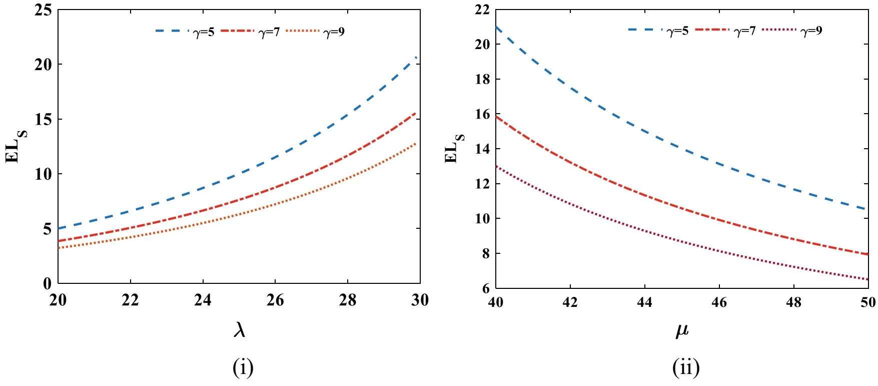

Fig. 6.2 The expected system length EL_S versus (i) λ (ii) μ for different values of γ

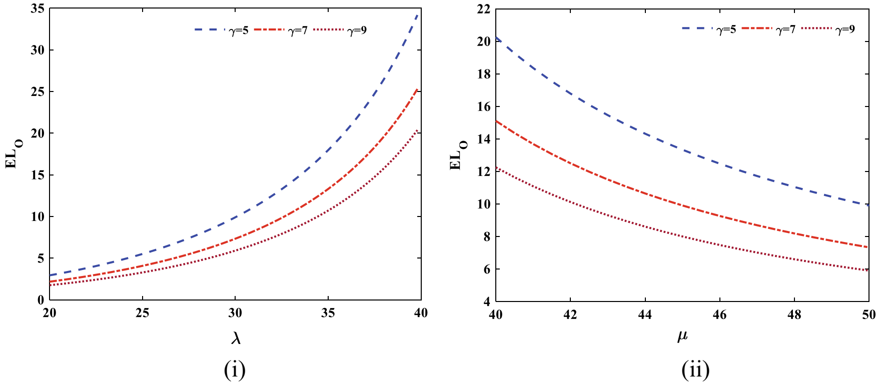

Fig. 6.3 The expected orbit length EL_O versus (i) λ (ii) μ for different values of γ

It can be seen in Fig. 6.2 (i) and Fig. 6.3 (i) that with the increasing arrival rate, the expected system size and orbit length increases. On the contrary, with the increasing retrial rate, EL_S and EL_O are decreasing. Similary, Fig. 6.2(ii) and Fig. 6.3(ii) depict that an increment in the service rate μ results in the decrease in the EL_O. We notice that by increasing the retrial rate there is a decreasing trend for EL_O. Figure 6.4 (i)-(iii) depict that on increasing the service rate μ, the total cost (ETC) firstly shows the decreasing trend and then increases. It also depicts that ETC increase with increasing probabilities σ, σ_1, σ_2.

In Fig. 6.5 (i)–(iii), we can observe that the expected total cost (ETC) increases linearly with the increase in the cost factors C_σ, C_{T1} and C_{T2}. It can also be observed that the value of ETC increases with the increase in the probabilities σ, σ_1 and σ_2.

6.7 Conclusions

The performance analysis of the queueing-inventory system in which jobs retry and the impatience job may balk has been studied. The explicit solution in the product form has been obtained for the joint stationary distribution, which is further used in computing performance measures such as mean count of jobs in the system, in the orbit, mean available inventory and so on. The performance measures established in this chapter may help the system designers in the decision making so as to provide quality service, and deliver quality items to their consumers. The cost optimization using the gradient descent method has been done and the optimal production rate has been obtained. The cost optimization can provide valuable insights to the decision makers to achieve huge profit margins from both the management and economic

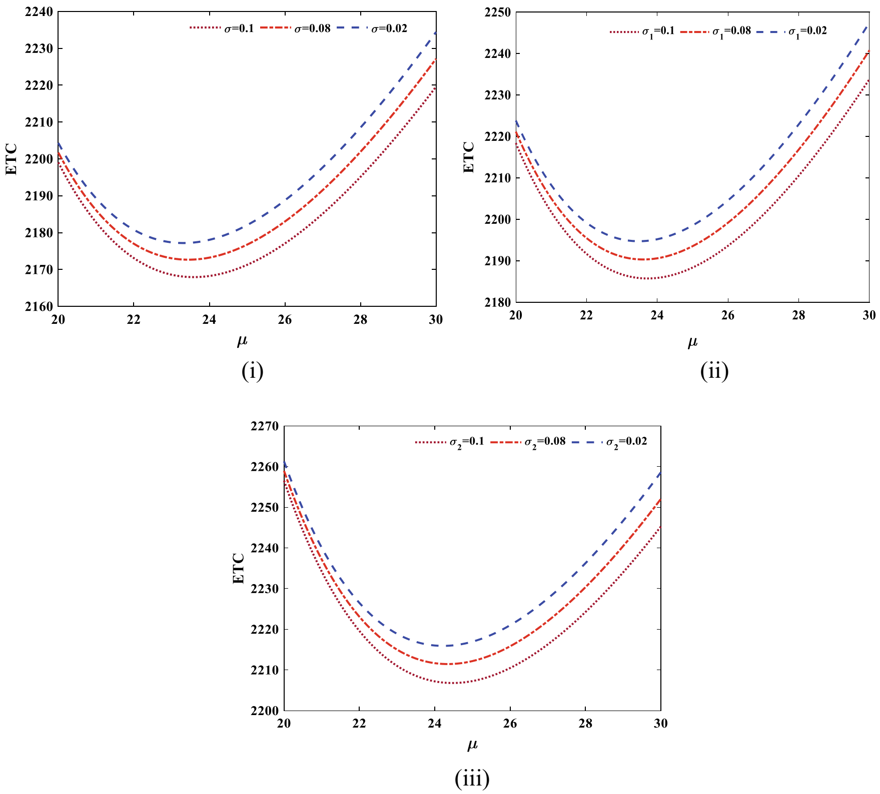

Fig. 6.4 The expected total cost (ETC) versus μ with varying parameters (i) σ (ii) σ_1 (iii) σ_2

viewpoint. Further, this study can be extended to the queueing inventory models with vacation/working vacation.

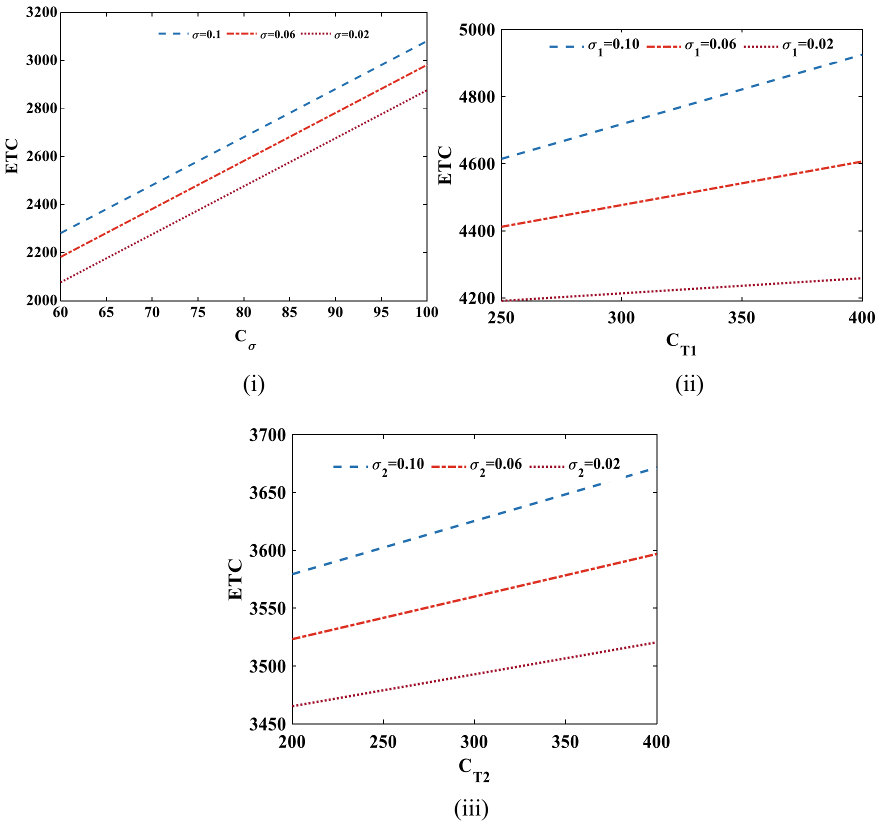

Fig. 6.5 The expected total cost (*ETC*) versus μ with varying parameters (i) C_σ (ii) C_{T1} (iii) C_{T2}

References

Adak S, Mahapatra GS (2021) Two-echelon imperfect production supply chain with probabilistic deterioration rework and reliability under fuzziness. J Manage Anal 9(2):287–311

Aghsami A, Samimi Y, Aghaei A (2021) A novel Markovian queueing-inventory model with imperfect production and inspection processes: a hospital case study. Comput Ind Eng 162:107772

Albrecher H, Boxma O, Essifi R, Kuijstermans R (2017) A queueing model with randomized depletion of inventory. Probab Eng Inf Sci 31(1):43–59

Al-Salamah M (2011) Economic order quantity with imperfect quality, destructive testing acceptance sampling, and inspection errors. Adv Manage Appl Econ 1(2):59–75

Asadkhani J, Mokhtari H, Tahmasebpoor S (2022) Optimal lot-sizing under learning effect in inspection errors with different types of imperfect quality items. Oper Res Int J 22:2631–2665

Berman O, Kaplan EH, Shevishak DG (1993) Deterministic approximations for inventory management at service facilities. IIE Trans 25(5):98–104

Graves SC (1982) The application of queueing theory to continuous perishable inventory systems. Manage Sci 28(4):400–406

Gross D, Harris CM (1998) Fundamentals of queueing theory. Wiley, New York

Jeganathan K, Selvakumar S, Anbazhagan N, Amutha S, Hammachukiattikul P (2021) Stochastic modeling on M/M/1/N inventory system with queue-dependent service rate and retrial facility. AIMS Math 6(7):7386–7420

Manikandan R, Nair SS (2017) M/M/1/1 queueing-inventory system with retrial of unsatisfied customers. Commun Appl Anal 21(2):217–236

Schwarz M, Sauer C, Daduna H, Kulik R, Szekli R (2006) M/M/1 queueing systems with inventory. Queueing Syst 54(1):55–78

Shajin D, Krishnamoorthy A (2020) Stochastic decomposition in retrial queueing inventory system. RAIRO-Operat Res 54(1):81–99

Sigman K, Simchi-Levi D (1992) Light traffic heuristic for an M/G/1 queue with limited inventory. Ann Oper Res 40(1):371–380

Taheri-Tolgari J, Mohammadi M, Naderi B, Arshadi-Khamseh A, Mirzazadeh A (2019) An inventory model with imperfect item, inspection errors, preventive maintenance and partial backlogging in uncertainty environment. J Indust Manage Optim 15(3):1317–1344

Wang FF (2015) Approximation and optimization of a multi-server impatient retrial inventory-queueing system with two demand classes. Quality Technol Quant Manage 12(3):269–292

Yu HF, Chen Y (2018) An integrated inventory model for items with acceptable defective items under warranty and quality improvement investment. Qual Technol Quant Manage 15(6):702–715

Yu HF, Yu WC, Wu WP (2009) A mixed inspection policy for CSP-1 and precise inspection under inspection errors and return cost. Comput Ind Eng 57(3):652–659

Chapter 7
Inventory Model for Growing Items and Its Waste Management

Jagannath Biswas, Nirmal Kumar Duari, and Tripti Chakrabarti

Abstract A novel mathematical model for the inventory control of growing items has been developed by considering the post-pandemic scenario. The farming of growing items like poultry can be tackled by well-managed vaccination process. The main objective of our investigation in the present chapter is to examine how a poultry farmer can manage the farm in spite of any type of viral flu scenarios. The model developed can provide an insight to save a growing item farm from any pandemic situation and can maximize the profit by finding the optimum order quantity, etc. We have calculated the profit from the slaughtering of livestock by considering all possible characteristics of a poultry firm which may be affected by viral diseases. Moreover, some of the items considered as wastes are deteriorating as well as ameliorating. The goal of the development of the mathematical model is to determine the profit from the slaughtering of livestock as well as profit from well managing the waste in another inventory. The present work is a blend of two inventory models where the first one is an inventory model of growing items considering viral disease & vaccination; whereas the second one is a sustainable waste management model. So in this, we have considered simultaneously viral disease & vaccination alongside general inventory issues. In the arena of micro, small and medium enterprises (MSMEs), these models are of great utility.

Keywords Growing item · Inventory · Viral Flu · Vaccination · Optimization · Waste management

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7.1 Introduction

The traditional EOQ model determines the optimal order quantity and optimum costs by considering the demand of items in the market. Many types of the EOQ models in different contexts have been proposed in the literature till now. EOQ/EPQ models can be applied to several products. All products are not of similar quality and similar economic features, and this is why various models incorporate different quality of products in the market. For deteriorating items such as food, medicine, vegetables, milk products, etc. which eventually decay with time, the inventory models called "EOQ/EPQ models for deteriorated items" have been studied. The non-deteriorating products can be kept in store room for any amount of time, but this is not true for the perishable products. Imperfect products, like electronic items or clothes, for which all the finished (produced) products are of perfect quality, there is another model called "EOQ/EPQ model for imperfect items" that has also received huge attention. Another instance is of repairable products, such as war materials and reusable products, use & throw away pens, soft drink bottles etc. All these inventory models have a similar property that during storage/holding time, inventory items remain as usual (like computers, cars, toothpaste etc.), or deteriorate (like medicine, vegetables, fruits, fast food etc.). But when we talk about the livestock or any other product like fish, then it is interesting to know that their weights gain over time. The inventory issues of the growing items like poultry farm, fish cultivation, cattle farming etc., are also very important as there is a major portion of micro, small and medium enterprise (MSME) sector which depends on these businesses.

Recently after the corona pandemic, every type of firm has started to think differently to cope up with new challenges. In this context, a poultry firm or cultivation of any livestock is also not out of special care. Before the corona pandemic, several researchers made so many models and predictions for inventory-related issues of industries. In future, there may be a situation of devastation by sudden attack of unknown viruses. In the pandemic situation, not only the livestock are affected, all the human resources working in those firms, are also get affected economically. So there is necessity to make inventory models considering the virus attack and vaccination also.

7.1.1 Literature Survey

Rezaei (2014) pioneered the inventory model for growing items where he only considered the classical inventory model and optimized the profit and the slaughter time for livestock. Ritha and Haripriya (2015) considered the inventory model of growing items where they investigated the past few years' data to reveal that there was a growth in the food industry with a huge change to meet the demand. Zhang et al. (2016) discussed an inventory management problem of growing items by considering the carbon emission and showed that instead of feeding time if there is change in the order quantity then retailers can mitigate carbon emission. Dhanam and Jesintha (2017) developed a fuzzy inventory model for growing items where the demand rate was random. Sebatjane (2018) proposed an inventory system of growing items, where all the items are screened before going to slaughtering and selling, after which the defected items are rejected. They also showed that the final weight at the time of slaughtering is most sensitive to the optimal order quantity. Nobil et al. (2019) studied an inventory system of poultries wherein they considered that the shortages were fully backordered and additionally they considered hygiene in place of farming. Khalilpourazari and Pasandideh (2019) developed a multi-item economic order quantity for growing items and they used several modern optimization techniques to find solutions. Malekitabar et al. (2019) proposed a model where they considered the mortality rate as well as the growing rate of the growing items. Gharaei and Almehdawe (2019) provided a new generation of inventory models, entitled Economic Growing Quantity (EGQ), which was designed for growing items but after death, they considered the holding cost and deterioration rate of the dead items. Sebatjane and Adetunji (2019) developed a model for growing items by considering the incremental quantity discount at the end of the supplier. Khalilpourazari et al. (2019) studied a multi-item economic order quantity model with imperfect items where the inspection process was imperfect and different types of errors of inspection were considered. Hidayat et al. (2020) developed an Economic Order Quantity model for growing items where incremental quantity discount was considered as taken in Sebatjane and Adetunji (2019). They included the two constraints due to limited budget and capacity of storage. Sebatjane and Adetunji (2020) considered an Economic Order Quantity model for the growing items where demand was a function of price and freshness. They solved the model for the optimal values of price, number of orders, and number of shipments. Sebatjane and Adetunji (2020) considered an Economic Order Quantity model for the imperfect growing items to get an optimal value of order quantity, cycle time & number of shipments. Gharaei and Almehdawe (2020), Mokhtari et al. (2020) etc. also did similar studies. Nishandhi (2020) also developed an Economic Order Quantity model for growing items and allowed shortages with full backordering and imperfect items to optimize order quantity, backordering quantity & cycle time. Pourmohammad-Zia and Karimi (2020) considered an Economic Order Quantity model for the growing items with imperfect quality to optimize order quantity & cycle time. Afzal and Alfares (2020) investigated an Economic Order Quantity model for growing items by allowing shortage along with full backordering and imperfect quality. They did this analysis to optimize order quantity, backordering quantity & cycle time.

Similar models were also developed by Alfares and Afzal (2021), Sebatjane and Adetunji (2021), Mittal and Sharma (2021), etc. Gharaei and Almehdawe (2021) considered carbon tax to develop an inventory model for growing items and optimized order quantity & cycle time. De-la-cruz-marquez et al. (2021) developed an inventory model for growing items with imperfect quality when demand is price-dependent under carbon emission & shortage. De-la-cruz-marquez et al. (2021) assumed price-dependent polynomial demand function with shortage and full backordering for imperfect quality items and carbon tax in consideration to optimize selling price, order quantity, backordering quantity & cycle time.

7.1.2 Motivation

In Food Processing Industries sector there are six main areas of interest, such as Fruits and Vegetables; Grain processing (including pulse and oilseed processing); Dairy processing; Meat and poultry processing; Fish processing and Consumer (ready to eat/drink) foods. According to the data provided by the Department of Animal Resource Development, Govt. of West Bengal, India, only 2% of the total production of milk (cow/ buffalo) is being marketed by the organized sector. Kolkata region alone has a demand of 14.2% of the total demand in the state daily. So, there is a major portion of the market which is met by the unorganized sector. The West Bengal state has huge amount of sheep and goat population. Also the state has a famous indigenous pig breed which is named as 'Ghungroo'. West Bengal only produces 67% of the per capita requirement of the State. In spite of having about 53 million poultry birds, the state has not enough to meet the local demand of people and commercial bakeries and confectioneries. West Bengal produces 50% of its demand of eggs per day. West Bengal imports eggs from mainly Andhra Pradesh.

India is facing power crisis. Stocks of coal in electricity power plants have fallen to unexpectedly low levels and many states are very much worried of power blackouts. Several states of India have issued warnings that coal supplies are being too low in thermal power plants. Several of the country's plants (run by coal) were in the supercritical stage, which means their stocks could run out early. Heavy monsoon rains have also affected domestic coal mines due to flooding. Also dispatch of coal from the mines is being problematic. To solve these types of problems, generally coal is imported from other countries. But global energy crisis increases the international prices. So, it has been a financial challenge to import coal. So, in order to maintain an economic balance in the case of coal supply, it is our duty to think carefully about a parallel source of energy so that we can see a better future. We can see that there is a lack of attention in the field of food processing industry and we have not done enough research yet for further development of this area and also another research gap in this literature is that nobody has considered the possibility of a sustainable model by using waste from poultry farm in another production business like fertilizers, bio-gas etc. Moreover, in the upcoming days, sustainable development is of utmost care for every nation due to crisis in fossil fuels in recent years. Speaking another way, we can say that this type of model can help farmers to estimate every necessary measure proactively to maintain their business in an eco-friendly manner and also situations when disease like bird flu happens.

7.2 Mathematical Model and Analysis

Let the livestock during the growing cycle be affected by a viral flu that hits our society. As a result, some amount of livestock gets contaminated and we are trying to treat them by vaccination. Now it's obvious that all the contaminated items will not get back to a healthy state. Moreover, we will consider different prices of vaccines as per their quality.

Assumptions

y	Number of baby items being ordered
w_t	Weight of unit item at time t
p	Purchasing price per unit weight
s	Selling price per weight unit
c_f	Feeding cost per unit item
h	Annual holding cost per unit weight
m	Set-up cost per growing cycle
Q_t	Total weight of inventory at time t
d	Annual demand rate
a	Asymptotic weight
k	Growth rate
$z_1(t)$	Diseased item after time t during growing period
$z_2(t)$	Healthy item after vaccination at time t during growing period
u	Rate of contamination
v	Rate of convalescence after vaccination
c_v	Cost of vaccination per item
n	Shape parameter of the growth function
b	Integration constant of the growth function

Let w_0 be the weight of the new-born baby chick and w_1 the final weight at the time of slaughtering, respectively. Total revenue is $sw_1\int_0^{t_1} y(1 - z_1(t) + z_2(t))dt$, where $z_1(t)$ is the diseased item and $z_2(t)$ is the healthy item after vaccination at time t during the growing period.

Total purchasing cost is pyw_0.

Total feeding cost is $c_f\int_0^{t_1} y(1 - z_1(t) + z_2(t))dt$.

t_1 is the length of the growing cycle and t_2 is the slaughtering period.

$(y(1 - z_1(t) + z_2(t))\ w_1/2)$ is the average weight of the inventory in the growing period.

Total holding cost is $ht_2\left(w_1 \int_0^{t_1} y(1 - z_1(t) + z_2(t))\ dt/2\right)$.

$$\text{Final profit} = \text{total income} - (\text{purchase cost} + \text{vaccination cost} + \text{production cost} + \text{holding cost} + \text{setup cost}).$$

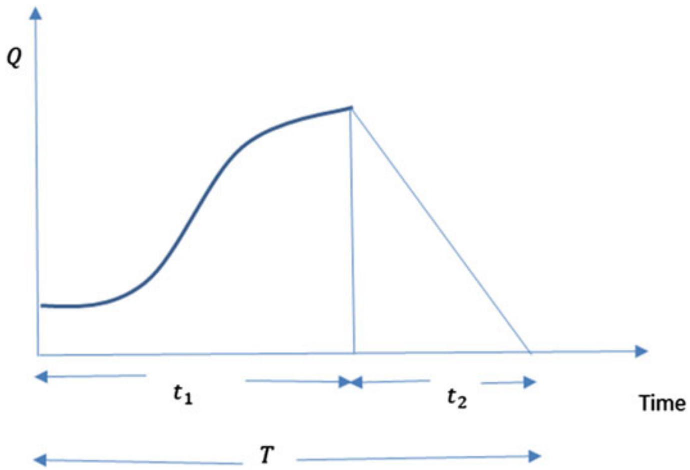

Fig. 7.1 Graphical view of growing cycle & slaughter cycle of growing items

The total profit function per cycle is:

$$TP = sw_1 \int_0^{t_1} (y - z_1(t) + z_2(t))\mathrm{dt} - pyw_0 - c_f \int_0^{t_1} y(1 - z_1(t) + z_2(t))f(t)$$
$$- \; ht_2\left(w_1 \int_0^{t_1} y(1 - z_1(t) + z_2(t))\; dt/2\right) - m - c_v \tag{7.1}$$

At the beginning of the growing cycle, the level of inventory is $Q_0 = yw_0$. At the end of the growing cycle t_1, the animal weight becomes w_1 and the total weight of the inventory $Q_1 = w_1 \int_0^{t_1} y(1 - z_1(t) + z_2(t))dt$ (Fig. 7.1).

According to Richards (1959) the weight function of the poultry can be expressed as follows:

$$w_t = a\left(1 + be^{-kt}\right)^{-1/n} \tag{7.2}$$

According to Goliomytis et al. (2003), production (feeding) function $f(t)$, is:

$$f(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 \tag{7.3}$$

where $b_0 = 532.2$, $b_1 = 67.15$, $b_2 = -0.651$, $b_3 = 0.0018$

Therefore, the total profit function

$$
\begin{aligned}
TP &= sw_1\int\limits_0^{t_1} y(1-z_1(t)+z_2(t))dt - pyw_0 \\
&\quad - c_f\int_0^{t_1} y(1-z_1(t)+z_2(t))\left(532.2+67.15t-0.651t^2+0.0018t^3\right)dt \\
&\quad - h\left(w_1\int_0^{t_1} y(1-z_1(t)+z_2(t))dt/d\right)\left(w_1\int_0^{t_1} y(1-z_1(t)+z_2(t))dt/2\right) \\
&\quad - m - c_v \\
&= sw_1\int_0^{t_1} y(1-z_1(t)+z_2(t)) - pyw_0 \\
&\quad - c_f\int_0^{t_1} y(1-z_1(t)+z_2(t))\left(532.2+67.15t-0.651t^2+0.0018t^3\right)dt \\
&\quad - \frac{hw_1{}^2}{2d}\left\{\int_0^{t_1} y(1-z_1(t)+z_2(t))dt\right\}^2 - m - c_v
\end{aligned}
\tag{7.4}
$$

Now

$$
w_1 = a\left(1+be^{-kt_1}\right)^{-1/n} \text{ implies that } t_1 = -\frac{1}{k}\ln\left[\frac{1}{b}\left\{\left(\frac{a}{w_1}\right)^n\right\}-1\right] \tag{7.5}
$$

Now, since we cannot give life to every diseased item by vaccination, $z_1(t) > z_2(t)$. So $z_1(t) - z_2(t)$ represents the number of items declared dead by the viral flu at any time t. Since animals don't get precautious like human beings, they get affected more rapidly than we can think of, and also we can make them well at a lesser rate than the rate of getting affected. So let us assume,

$$
z_1(t) - z_2(t) = \frac{u}{u+v}sin^2\left(\frac{tt_1}{1000}\right). \tag{7.6}
$$

where u, v are defined above and t_1 is the growing period given by (7.5). Here, initially the number of items is $y(1-z_1(t)+z_2(t)) = y\left(1-\frac{u}{u+v}sin^2\left(\frac{0*t_1}{1000}\right)\right) = y$ and after a time $t = \tilde{t}$, when the bird flu attacks first, some items will get killed before realization of the pandemic and after getting some vaccinations, the number of items becomes Y$= y\left(1-\frac{u}{u+v}sin^2\left(\frac{\tilde{t}t_1,}{1000}\right)\right)$ which is always less than y (as $u > 0, v > 0$).

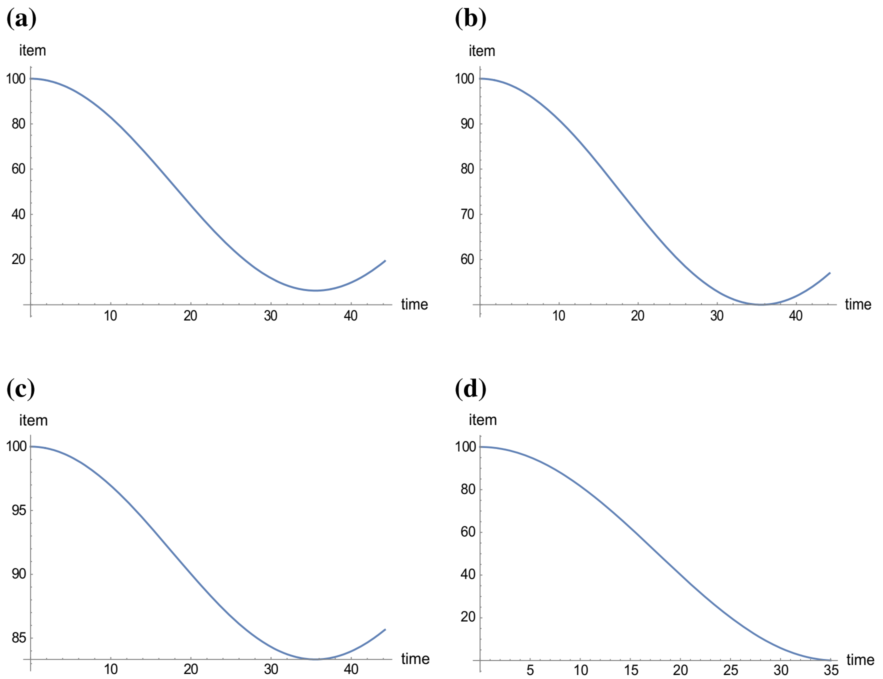

Fig. 7.2 **a** Curve representing Y when u = 15, v = 1, initial purchased item y = 100. **b** Curve representing Y when u = 15, v = 15, initial purchased item y = 100. **c** Curve representing Y when u = 15, v = 75, initial purchased item y = 100. **d** Curve representing Y when v = 0, initial purchased item y = 100

The above figures convey that a higher quality vaccine will help us to maintain a higher number of poultry in the growing cycle and most importantly, from Fig. 7.2d we can say that no use of vaccines gradually kills all items.

So *TP* will be

$$= sw_1 \int_0^{t_1} y(1 - \frac{u}{u+v} sin^2(\frac{tt_1}{1000}))dt - pyw_0 - c_f \int_0^{t_1} y\left(1 - \frac{u}{u+v} sin^2\left(\frac{tt_1}{1000}\right)\right)$$
$$\left(532.2 + 67.15t - 0.651t^2 + 0.0018t^3\right)dt$$
$$- \frac{hw_1{}^2}{2d}\left\{\int_0^{t_1} y\left(1 - \frac{u}{u+v} sin^2\left(\frac{tt_1}{1000}\right)\right)dt\right\}^2 - m - c_v \tag{7.7}$$

So without loss of generality, we can take the value of w_1 (weight of an adult chicken) $\in$ (2000gm, 3000gm), then from Eq. (7.5), $t_1 \in$ (39days, 50days) and when the weight of an adult chicken becomes 2.5 kgs, then $t_1 = 44$ days (approx.).

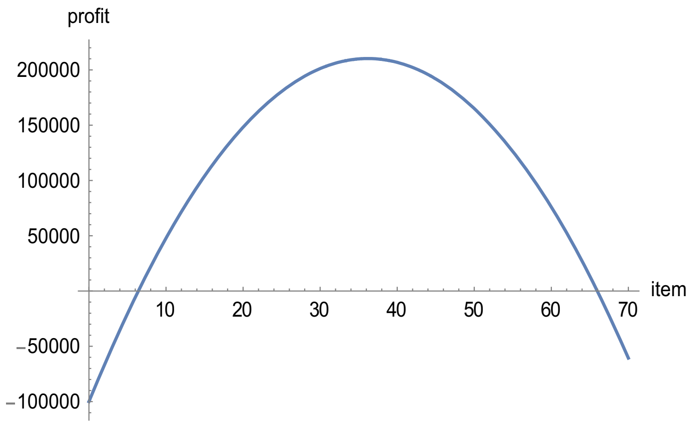

Fig. 7.3 Profit function for the values as in (7.10)

Now

$$\frac{\partial(TP)}{\partial y} = 0 \Rightarrow y = \frac{SW1\left\{\int\limits_0^{t1}\left(1 - \frac{u}{u+v}\sin 2\left(\frac{tt1}{1000}\right)\right)\right\} - pwo - cf\left\{\int\limits_0^{t1}\left(1 - \frac{u}{u+v}\sin 2\left(\frac{tt1}{1000}\right)\right)(532.2 + 67.15t - 0.651t2 + 0.0018t3)dt\right\}}{\frac{hw12}{d}\left\{\int\limits_0^{t1}\left(1 - \frac{u}{u+v}\sin 2\left(\frac{tt1}{1000}\right)\right)dt\right\}} \tag{7.8}$$

and

$$\frac{\partial^2(TP)}{\partial y^2} = -\frac{hw_1{}^2\int_0^{t_1}\left(1 - \frac{u}{u+v}sin^2\left(\frac{tt_1}{1000}\right)\right)^2}{d} < 0 \text{ since } h > 0, d > 0) \tag{7.9}$$

So TP is a concave function of y.

7.3 Profit Function

Since $0 < \dfrac{u}{u+v} < 1$, we take any positive fraction for $\dfrac{u}{u+v} = x(say)$, let's say 0.291667; d = 1,000 kg/day; h = Rs. 2.5/day, s = Rs. 200/kg; p = Re. 1/gm; $w_0 = 25$ gm/chick; $w_1 = 2.5$ kg/*chicken*; $c_f =$ Rs. 0.0048/gm chicken/day (an adult chicken of weight 2.5 kg consume approx. 500 gm feed per day); m = Rs. 100,000; $c_v = 800 + 10(v/u)^2 = 800 + 10\left(\frac{1-x}{x}\right)^2 = 859$ (7.10)

So TP is maximum at Rs. 210248(approx.) for the value $y = y^* = 36\text{(approx.)}/day$ for $t_1 = 44$ days (approx.)

Now we will analyse different market conditions and depict some figure regarding the relation between the profit function TP and its various parameters to understand the effect of different parameters on profit function as well as on order quantity.

From Table 7.1, we observe that with fixed value of u and growing value of v, profit is slipping downwards because of the higher cost of the high quality vaccine. We notice that with a fixed contamination rate, less vaccination cost makes the flock of poultry more vulnerable to death and as a consequence the profit is again falling downwards (Table 7.1 and Fig. 7.4).

Here, we can observe in a very expected way that in Table 7.2 and Fig. 7.5, with increasing purchasing cost, the total profit goes down. But this cost does not reflect much in the profit.

According to this comparison, a lower demand makes less profit and a higher demand makes a higher profit and the rate of increasing profit is too high compared to the mitigating demand. Here, we have modeled the profit function in such a way that

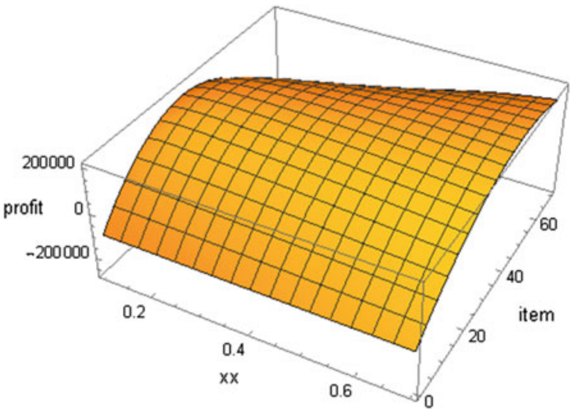

Fig. 7.4 Profit function (TP) versus purchased item(y) versus $\frac{u}{u+v}$(xx in figure)

Table 7.1 Relation between profit and vaccination cost and contamination rate

Change in **v** (%)	**u**	**v**	**u/(u + v)**	**TP**	Change in **TP** (%)	**y**	Change in **y** (%)
−75	100	61	0.62112	203,874	−3.03	47	30.56
−50	100	122	0.45045	208,298	−0.93	41	13.89
−25	100	182	0.35461	209,847	−0.19	38	5.56
0	**100**	**243**	**0.291667**	**210,248**	0.00	**36**	0
25	100	304	0.247525	209,973	−0.13	35	−2.77
50	100	365	0.215054	209,224	−0.49	34	−5.55
75	100	425	0.190476	208,124	−1.01	34	−5.55

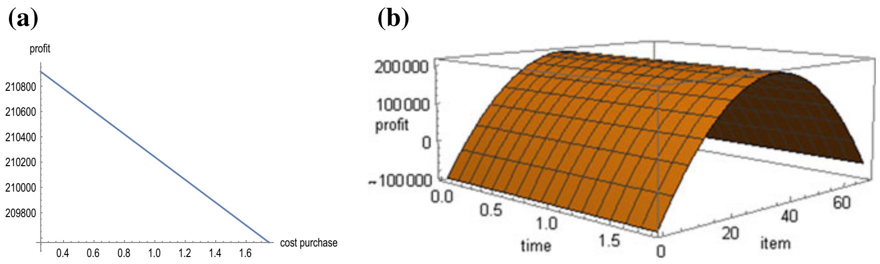

Fig. 7.5 **a** Profit function (TP) versus purchase cost (p). **b** Profit function (TP) versus purchased item(y) versus purchase cost (p)

Table 7.2 Relation between profit and purchase cost

Change in values of **p** (%)	**p**	**TP**	Change in values of **TP** (%)
−75	0.25	210,927	+0.32
−50	0.5	210,701	+0.22
−25	0.75	210,475	+0.11
0	**1**	**210,248**	**0**
+25	1.25	210,022	−0.11
+50	1.5	209,796	−0.21
+75	1.75	209,570	−0.32

Table 7.3 Relation between profit and feeding cost

Change in values of $\mathbf{c_f}$ (%)	$\mathbf{c_f}$	**TP**	Change in values of **TP** (%)
−75	0.0012	217,752	+3.57
−50	0.0024	215,241	+2.37
−25	0.0036	212,740	+1.19
0	**0.0048**	**210,248**	**0**
+25	0.006	207,767	−1.18
+50	0.0072	205,296	−2.36
+75	0.0084	202,834	−3.53

the pandemic situation can be considered with no hesitation. Moreover, we have done some sensitivity analysis to understand the effect of different parameters. Among all type of parameters, the most effective are set-up cost, holding cost, selling price and demand as can be followed from Tables 7.5, 7.6, 7.7. Apart from that we have successfully designed the model with respect to contamination rate, convalescence rate and vaccination cost.

Table 7.4 Relation between profit and set-up cost

Change in values of **m** (%)	**m**	**TP**	Change in values of **TP** (%)
−30	70,000	240,248	+14.27
−20	80,000	230,248	+9.51
−10	90,000	220,248	+4.76
0	**100,000**	**210,248**	0
+10	110,000	200,248	−4.76
+20	120,000	190,248	−9.51
+30	130,000	180,248	−14.27

Table 7.5 Relation between profit and holding cost

Change in **h** (%)	**h**	**TP**	Change in **TP** (%)	**y**	Change in **y** (%)
−75	0.625	1,140,990	+442.69	145	302.78
−50	1.25	520,497	+147.56	72	100.00
−25	1.875	313,664	+49.19	48	33.33
0	**2.5**	**210,248**	0	**36**	0
+25	3.125	148,199	−29.51	28	−22.22
+50	3.75	106,832	−49.19	24	−33.33
+75	4.375	77,284	−63.24	21	−41.67

Table 7.6 Relation between profit and selling cost

Change in value of **s** (%)	**s**	**TP**	Change in value of **TP** (%)	**y**
−15	0.17	118,840	−43.48	30
−10	0.18	147,541	−29.83	32
−5	0.19	178,011	−15.33	34
0	**0.2**	**210,248**	0	**36**
+5	0.21	244,254	16.17	38
+10	0.22	280,027	33.19	40
+15	0.23	317,569	51.04	42

Table 7.7 Relation between profit and demand

Change in **d** (%)	**d**(kg)/day	**TP**	Change in **TP** (%)	**Y**	Change in **y** (%)
−30	700	117,174	−44.27	25	−30.56
−20	800	148,199	−29.51	29	−19.44
−10	900	179,224	−14.76	33	−8.33
0	**1000**	**210,248**	0	**36**	0
+10	1100	241,273	14.76	40	11.11
+20	1200	272,298	29.51	43	19.44
+30	1300	303,323	44.27	47	30.56

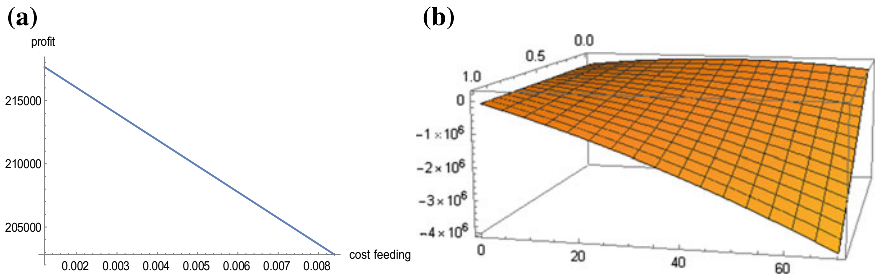

Fig. 7.6 **a** Profit function (TP) versus feeding cost (c_f). **b** Profit function (TP) versus purchased item(y) versus feeding cost (c_f)

Firstly, from Table 7.1, we can find that how a higher contamination rate can mitigate the profit, on the other hand, we also observe that a costlier vaccine can help us to grow more livestock than normal. Another important note is that vaccination cost depends on the ratio of the rate of contamination versus convalescing.

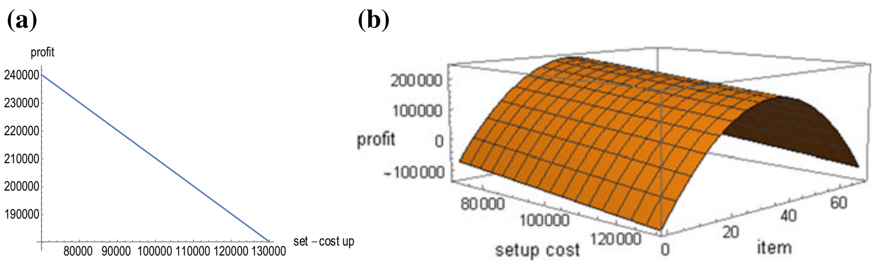

Fig. 7.7 **a** Profit function (TP) versus set-up cost (m). **b** Profit function (TP) versus purchased item(y) versus set-up cost (m)

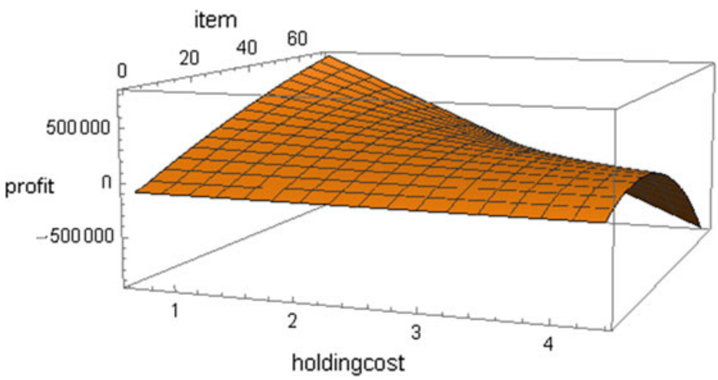

Fig. 7.8 Profit function (TP) versus purchased item(y) versus holding cost (h)

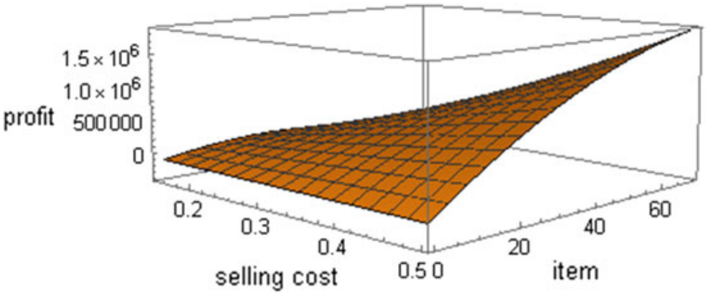

Fig. 7.9 Profit function (TP) versus purchased item(y) versus selling price (s)

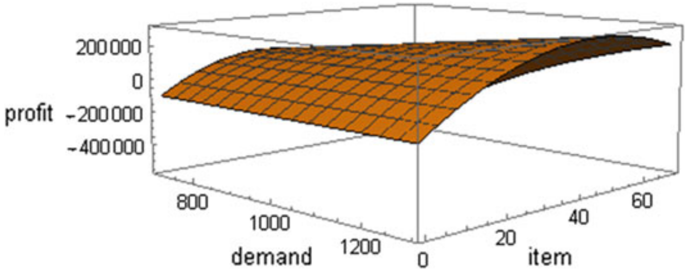

Fig. 7.10 Profit function (TP) versus purchased item(y) versus demand (d)

7.4 Profit from Waste Management

Now if we consider the waste made from the whole growing cycle as well as the slaughter cycle, then we have to consider this model as another EPQ inventory model where items like feeding material, cow dung or waste from slaughtering are the products which are to be stored at inventories in separate warehouses. According to that, we have to assume the following new parameters as well:

c_{ff}	cost of per unit weight fresh feed
w_{ff}	weight of fresh feed bought for unit weight of animal
w_d	weight of dung from unit weight of animal
w_{uf}	weight of feed (unusable) from unit weight of animal
w_s	weight of slaughter waste from each adult animal
h_{ff}	holding cost of unit weight fresh feed
h_d	holding cost of unit weight dung
h_{uf}	holding cost of unit weight unusable feed
h_s	holding cost of unit weight slaughter waste
$de_d(t)$	demand of animal waste
$de_{uf}(t)$	demand of unusable animal feed
$de_s(t)$	demand of slaughter waste
$I_{ff}(t)$	inventory of fresh feed at any time t

$I_d(t)$	inventory of animal dung at any time t
$I_{uf}(t)$	inventory of unusable feed at any time t
$I_s(t)$	inventory of slaughter waste at any time t
$\theta_{ff}(t)$	deteriorating rate of fresh feed
$\theta_d(t)$	ameliorating rate of dung
$\theta_{uf}(t)$	ameliorating rate of unusable feed
$\theta_s(t)$	ameliorating rate of slaughter waste
c_{pff}	cost for making pollution due to unit weight of fresh feed
c_{pd}	cost for making pollution due to unit weight of dung
c_{puf}	cost for making pollution due to unit weight of unusable feed
c_{ps}	cost for making pollution due to unit weight of slaughter waste
s_d	selling cost of unit weight of dung
s_{uf}	selling cost of unit weight of unusable feed
s_s	selling cost of unit weight of slaughter waste

Since $y(1-\frac{u}{u+v}sin^2(\frac{tt_1}{1000}))$ is the number of items at any point of time t,

Total feed we buy is $= w_{ff}\int_0^{t_1} y(1-\frac{u}{u+v}sin^2(\frac{tt_1}{1000}))A(1+be^{-kt})^{-1/n}dt$

So, purchase cost of fresh feed is

$$= c_{ff}w_{ff}\int_0^{t_1} y(1-\frac{u}{u+v}sin^2(\frac{tt_1}{1000}))A(1+be^{-kt})^{-1/n}dt \tag{7.11}$$

Total weight of fresh feed at the time of growing cycle is

$$I_{ff}(t) = w_{ff}\int_0^{t_1} y(1-\frac{u}{u+v}sin^2(\frac{tt_1}{1000}))A(1+be^{-kt})^{-1/n}\theta_{ff}(t)dt \tag{7.12}$$

So total weight of animal dung at the time of growing cycle is

$$I_d(t) = w_d\int_0^{t_1} y(1-\frac{u}{u+v}sin^2(\frac{tt_1}{1000}))A(1+be^{-kt})^{-1/n}\theta_d(t)dt \tag{7.13}$$

Total weight of unusable animal feed at the time of growing cycle is

$$I_{uf}(t) = w_{uf}\int_0^{t_1} y(1-\frac{u}{u+v}sin^2(\frac{tt_1}{1000}))A(1+be^{-kt})^{-1/n}\theta_{uf}(t)dt \tag{7.14}$$

Total weight of slaughter waste at the time of slaughter cycle is

$$I_s(t) = w_s \int_{t_1}^{T} \{ \int_{0}^{t_1} y(1 - \frac{u}{u+v} sin^2(\frac{tt_1}{1000}))\} A(1 + be^{-kt_1})^{-1/n} dt\theta_s(t)dt \quad (7.15)$$

Let

$$I(t) = I_{ff}(t) + I_d(t) + I_{uf}(t) + I_s(t) \quad (7.16)$$

$$\text{Now, holding cost} = h_{ff} I_{ff}(t) + h_d I_d(t) + h_{uf} I_{uf}(t) + h_s I_s(t) \quad (7.17)$$

$$\text{Pollution cost} = c_{pff} I_{ff}(t) + c_{pd} I_d(t) + c_{puf} I_{uf}(t) + c_{ps} I_s(t) \quad (7.18)$$

Then total cost regarding the second inventory model is

$$\begin{aligned} TC_2 = {} & c_{ff} w_{ff} \int_0^t y\left(1 - \frac{u}{u+v} sin^2\left(\frac{tt_1}{1000}\right)\right) A\left(1 + be^{-kt}\right)^{-1/n} dt + h_{ff} I_{ff}(t) \\ & + h_d I_d(t) + h_{uf} I_{uf}(t) + h_s I_s(t) + c_{pff} I_{ff}(t) \\ & + c_{pd} I_d(t) + c_{puf} I_{uf}(t) + c_{ps} I_s(t) \end{aligned}$$

$$\begin{aligned} = {} & c_{ff} w_{ff} \int_0^t y\left(1 - \frac{u}{u+v} sin^2\left(\frac{tt_1}{1000}\right)\right) A\left(1 + be^{-kt}\right)^{-1/n} dt + \left(h_{ff} + c_{pff}\right) I_{ff}(t) \\ & + \left(h_d + c_{pd}\right) I_d(t) + \left(h_{uf} + c_{puf}\right) I_{uf}(t) + (h_s + c_{ps}) I_s(t) \end{aligned} \quad (7.19)$$

$$\text{And total selling cost is} = s_d I_d(t) + s_{uf} I_{uf}(t) + s_s I_s(t) \quad (7.20)$$

So total profit generated from this second model is $= TP_2$

$$\begin{aligned} & = \left\{ s_d I_d(t) + s_{uf} I_{uf} + s_s I_s(t) \right\} \\ & \quad - \left\{ c_{ff} w_{ff} \int_0^t y\left(1 - \frac{u}{u+v} \sin^2\left(\frac{tt_1}{1000}\right)\right) A\left(1 + be^{-kt}\right)^{-1/n} dt \right. \\ & \quad + \left(h_{ff} + c_{pff}\right) I_{ff}(t) + \left(h_d + c_{pd}\right) I_d(t) + \left(h_{uf} + c_{puf}\right) I_{uf}(t) \\ & \quad \left. + \left(h_s + c_{ps}\right) I_s(t) \right\} \end{aligned}$$

$$= \left(s_d - h_d - c_{pd}\right) I_d(t) + \left(s_{uf} + h_{uf} - c_{puf}\right) I_{uf}(t) + \left(s_s - h_s - c_{ps}\right) I_s(t)$$

$$- \left(h_{ff} + c_{pff}\right) I_{ff}(t) - c_{ff}\, w_{ff} \int_0^t y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt1}{1000}\right)\right) A\left(1 + be^{-kt}\right)^{-1/n} dt \tag{7.21}$$

Now total profit becomes

$$\begin{aligned}
TP = TP_1 + TP_2 = &\left[sw_1 \int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right) dt - pyw_0 \right.\\
&- c_f \int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right)\left(532.2 + 67.15t - 0.6510t^2 + 0.018t^3\right) dt\\
&\left. - \frac{hw_1^2}{2d}\left\{\int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right)\right\}^2 - m - c_v \right]\\
&+ \left[\left(s_d - h_d - c_{pd}\right) w_d \int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right) A\left(1 + be^{-kt}\right)^{-1/n} \theta_d(t) dt \right.\\
&+ \left(s_{uf} - h_{uf} - c_{puf}\right) w_{uf} \int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right) A\left(1 + be^{-kt}\right)^{-1/n} \theta_{uf}(t) dt\\
&+ \left(s_s - h_s - c_{ps}\right) w_s \int_{t_1}^{T} \left\{\int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right)\right\} A\left(1 + be^{-kt_1}\right)^{-1/n} dt\ \theta_s(t) dt\\
&- \left(h_{ff} - c_{pff}\right) w_{ff} \int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right) A\left(1 + be^{-kt}\right)^{-1/n} \theta_{ff}(t) dt\\
&\left. - c_{ff} w_{ff} \int_0^{t} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right) A\left(1 + be^{-kt}\right)^{-1/n} dt \right]
\end{aligned} \tag{7.22}$$

We can easily observe that if we take

$$s_d > h_d + c_{pd}, s_{uf} > h_{uf} + c_{puf} and s_s > h_s + c_{ps.....} \tag{7.23}$$

then the profit derived from waste management is positive.

Moreover, we have to think about different deteriorating as well as different ameliorating rates stated above. Hence,

$\theta_{ff}(t)$ = deteriorating rate of fresh feed = $\exp(\theta_1 t)$
$\theta_d(t)$ = ameliorating rate of dung = $\exp(\theta_2 t)$
$\theta_{uf}(t)$ = ameliorating rate of unusable feed = $\exp(\theta_3 t)$
$\theta_s(t)$ = ameliorating rate of slaughter waste = $\exp(\theta_4 t)$

where, $\theta_1 \neq \theta_2 \neq \theta_3 \neq \theta_4$ and all of them are $\neq 0$

So the profit from waste management is

$$TP_2 = \left(s_d - h_d - c_{pd}\right)w_d \int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right)A\left(1 + be^{-kt}\right)^{-1/n}\exp(\theta_2 t)dt$$

$$+ \left(s_{uf} - h_{uf} - c_{puf}\right)w_{uf} \int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right)A\left(1 + be^{-kt}\right)^{-1/n}\exp(\theta_3 t)dt$$

$$+ \left(s_s - h_s - c_{ps}\right)w_s \int_{t_1}^{T}\left\{\int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right)dt\right\}A\left(1 + be^{-kt_1}\right)^{-1/n}\exp(\theta_4 t)dt$$

$$- \left(h_{ff} + c_{pff}\right)w_{ff} \int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right)A\left(1 + be^{-kt_1}\right)^{-1/n}\exp(\theta_1 t)dt$$

$$- c_{ff}w_{ff} \int_0^{t_1} y\left(1 - \frac{u}{u+v}\sin^2\left(\frac{tt_1}{1000}\right)\right)A\left(1 + be^{-kt_1}\right)^{-1/n}dt \tag{7.24}$$

Hence for the value of y = 36/day, $t_1 = 44$ days (when optimising model 1), TP_2 is a function of T[where $(T - t_1)$ is the slaughter cycle].

If we take $s_d - h_d - c_{pd} = Re.1/\text{gm}$, $\left(s_{uf} - h_{uf} - c_{puf}\right) = Rs.1.1/\text{gm}$, $\left(s_s - h_s - c_{ps}\right) = Rs.0.1/\text{gm}$, $w_d = 0.01\text{gm}$, $w_{uf} = 0.05\text{gm}$, $w_s = 0.1\text{gm}$, $w_{ff} = 0.015\text{gm}$, $\theta_1 = 0.01$, $\theta_2 = 0.02$, $\theta_3 = 0.025$, $\theta_4 = -0.025$, $\left(h_{ff} + c_{pff}\right) = \text{Re}.1/\text{gm}$, $c_{ff} = \text{Rs. } 0.0048/\text{gm}$ chicken/day.

Then TP_2 is maximum with Rs. 150,891/- (approx…) for T = 48 days (approx…) (Using Wolfram Mathematica Software we have calculated). So the slaughter period for one cycle becomes 4 days (approx.). As we can see, waste management can make our business more profitable.

7.5 Conclusions

So considering a model for growing items in the post pandemic scenario in model 1, we have considered the waste coming from this business as a resource to make profit. With different values for different livestocks (provide condition (7.19) holds), we can estimate different levels of profit with different slaughter cycles. We can apply various types of growing items to illustrate this model. Along with that, we will consider different scenarios as and when the situation arises; may be the costs are fuzzy in nature and the number of diseased animals is obviously of a fuzzy number. If any natural phenomena would affect the business, we could add some other variable as well, most specifically, the terms associated with the diseased item & vaccinated item are stochastic in nature, so the model then would be a fuzzy stochastic one. So, in total, this model is quite practical as here we have considered pollution as well as the waste management specially.

References

Afzal AR, Alfares HK (2020) An inventory model for growing items with quality inspections and permissible shortages. In: Proceedings of the 5th NA international conference on industrial engineering and operations management, Detroit, MI, USA, August 2020, pp 1153–1160

Alfares HK, Afzal AR (2021) An economic order quantity model for growing items with imperfect quality and shortages. Arab J Sci Eng 46(2):1863–1875

De-la-Cruz-Márquez CG, Cárdenas-Barrón LE, Mandal B (2021) An inventory model for growing items with imperfect quality when the demand is price sensitive under carbon emissions and shortages. Math Probl Eng. https://doi.org/10.1155/2021/6649048

Dhanam K, Jesintha W (2017) Fuzzy inventory model for slow and fast growing items with deterioration constraints in a single period. Int J Manag Soc Sci 5(8):308–317

Gharaei A, Almehdawe E (2020) Economic growing quantity. Int J Prod Econ 223:107517

Gharaei A, Almehdawe E (2021) Optimal sustainable order quantities for growing items. J Clean Prod 307. https://doi.org/10.1016/j.jclepro.2021.127216

Goliomytis M, Panopoulou E, Rogdakis E (2003) Growth curves for body weight and major component parts, feed consumption and mortality of male broiler chickens raised to maturity. Poult Sci 82(7):1061–1068

Hidayat YA, Riaventin VN, Jayadi O (2020) Economic order quantity model for growing items with incremental quantity discounts, capacitated storage facility, and limited budget. Jurnal Teknik Industri, 22(1). https://doi.org/10.9744/jti.22.1.1-10

Khalilpourazari S, Pasandideh SHR (2019) Modelling and optimization of multi-item multi-constrained EOQ model for growing items. Knowl-Based Syst 164:150–162. https://doi.org/10.1016/j.knosys.2018.10.032

Khalilpourazari S, Pasandideh SHR, Niaki STA (2019) Optimizing a multi-item economic order quantity problem with imperfect items, inspection errors, and backorders. Soft Comput 23:11671–11698. https://doi.org/10.1007/s00500-018-03718-1

Malekitabar M, Yaghoubi S, Gholamian M (2019) A novel mathematical inventory model for growing-mortal items (case study: rainbow trout). Appl Math Model 71:96–117

Mittal M, Sharma M (2021) Economic ordering policies for growing items (poultry) with trade-credit financing. Int J Appl Comput Math 7(39). https://doi.org/10.1007/s40819-021-00973-z

Mokhtari H, Salmasnia A, Asadkhani J (2020) A new production-inventory planning model for joint growing and deteriorating items. Int J Supply Oper Manag 7(1):1–16. https://doi.org/10.22034/ijsom.2020.1.1

Nishandhi F (2020) An economic order quantity model for growing items with imperfect quality and budget capacity constraint to limit the purchase. Solid State Technol 63(6):7852–7858

Nobil AH, Sedigh AHA, Cárdenas-Barrón LE (2019) A generalized economic order quantity inventory model with shortage: case study of a poultry farmer. Arab J Sci Eng 44:2653–2663. https://doi.org/10.1007/s13369-018-3322-z

Pourmohammad-Zia N, Karimi B (2020) Optimal replenishment and breeding policies for growing items. Arab J Sci Eng 45(8):7005–7015

Rezaei J (2014) Economic order quantity for Growing Items. Int J Prod Econ 155(5):109–113

Richards FJ (1959) A flexible growth function for empirical use. J Exp Bot 10(2):290–301. https://doi.org/10.1093/jxb/10.2.290

Ritha W, Haripriya S (2015) Inventory model on organic poultry farming. Int J Res Eng Appl Sci 5:30–38

Ritha W, Haripriya S (2015) Inventory model on organic poultry farming. Int J Res Eng Appl Sci 5:30–38

Sebatjane M (2018) Economic order quantity model for growing items with imperfect quality. Oper Res Perspect. https://doi.org/10.1016/j.orp.2018.11.004

Sebatjane M, Adetunji O (2019) Economic order quantity model for growing items with incremental quantity discounts. J Ind Eng Int 15:545–556

Sebatjane M, Adetunji O (2020a) A three-echelon supply chain for economic growing quantity model with price- and freshness-dependent demand: pricing, ordering and shipment decisions. Oper Res Perspect 7:2214–7160. https://doi.org/10.1016/j.orp.2020.100153

Sebatjane M, Adetunji O (2020b) Optimal inventory replenishment and shipment policies in a four-echelon supply chain for growing items with imperfect quality. Prod Manuf Res 8(1):130–157. https://doi.org/10.1080/21693277.2020.1772148

Sebatjane M, Adetunji O (2020c) Three-echelon supply chain inventory model for growing items. J Model Manag 15(2):567–587. https://doi.org/10.1108/JM2-05-2019-0110

Sebatjane M, Adetunji O (2021) Optimal lot-sizing and shipment decisions in a three-echelon supply chain for growing items with inventory level-and expiration date-dependent demand. Appl Math Model 90:1204–1225

Zhang Y, Li L, Tian X, Feng C (2016) Inventory management research for growing items with carbon-constrained. In: 35th Chinese control conference (CCC), pp 9588–9593

Chapter 8
Pavement Cracks Inventory Survey with Machine Deep Learning Models

Aaron Rasheed Rababaah

Abstract Pavement maintenance poses serious economic consequences as they have been reported to cost annually billions of dollars in the US alone. Traditional approaches of handling pavement cracks surveys and inventory are tedious, ineffective, including safety risks and error prone. Investing in machine intelligent solutions should effectively address all of these problems and concerns. In this chapter, we present a proposed solution of the survey and inventory of pavement cracks based on Deep Learning (DL) models implementing Convolution Neural Networks (CNN) to detect and classify five types of road problems including longitudinal, traverse, block, alligator cracks and potholes. The development process is detailed and the experimental work, results and observations are discussed.

Keywords Pavement cracks · Automatic detection · Intelligent survey · Crack inventory · Deep learning · Machine intelligence

8.1 Introduction

Physical structures such as buildings, houses, roads, silos, chimneys, bridges etc. may suffer from distresses that may endanger their stability (Rababaah et al. 2005). Engineers are very interested in tackling this serious problem by evaluating the health of these structures repeatedly with an appropriate cycle so they can discover the early signs of risks as early as possible (Wolfer and Rababaah 2007). Our scope for this chapter is pavement road networks. According to the USA Department of Transportation (DOT 2021), there are several types of distresses that can occur in a road including: cracks, potholes, deformation, defects, etc. Samples of images that contain pavement with no cracks are shown in Fig. 8.1. Samples of images that contain pavement with cracks are depicted in Fig. 8.2.

Manual methods entail maintenance crew personnel to physically be on the site to inspect and survey the road networks. The crew has to visually scan defects, measure

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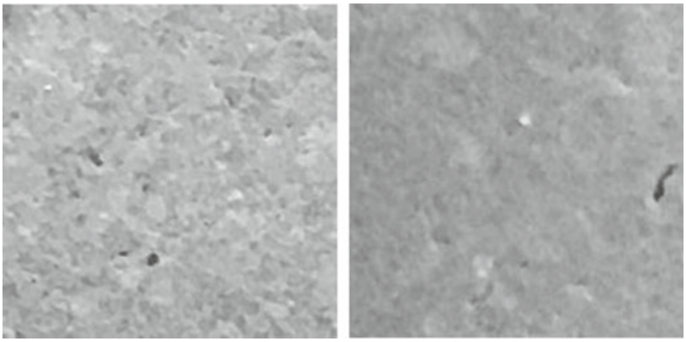

Fig. 8.1 Sample images with no crack pavement

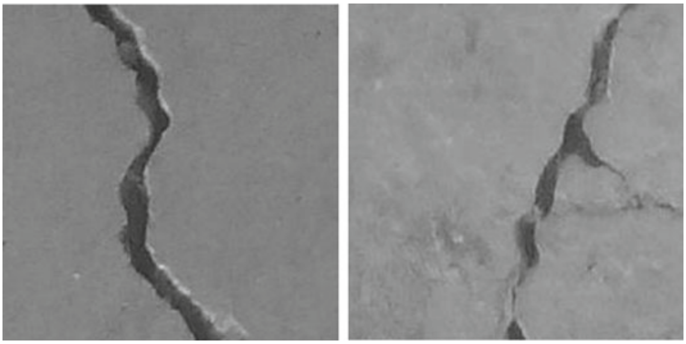

Fig. 8.2 Sample images with cracks in pavement

them and document the collected data to be used in inventory information for later analysis and decision making. This process is shown in Fig. 8.3.

The typical process of road defects inventory management and handling is illustrated in Fig. 8.4. As it can be seen, a vehicle is equipped with scanning devices such as a reliable digital camera and a computer on board. The vehicle travels over the target strip of road taking video of the road as it drives. The collected video is archived in a database for later analysis. The geospatial location information of the strip is recorded as well. This is essential because, after the analysis, the identified defects need to be localized on the map so that the maintenance schedule can be produced.

From the economical perspective, the USA government, Department of Transportations reported (USA-DOT 2021) that pavement cracks cost an average of $600/mile/year. Furthermore, it is reported by (USA-DOT 2017) as well that there are 4,183,707 miles of road in USA. Therefore, the total cost per year of pavement cracks per year can be estimated as 600 * 4,183,707 = $5.5 billion.

Fig. 8.3 Manual methods for pavement survey

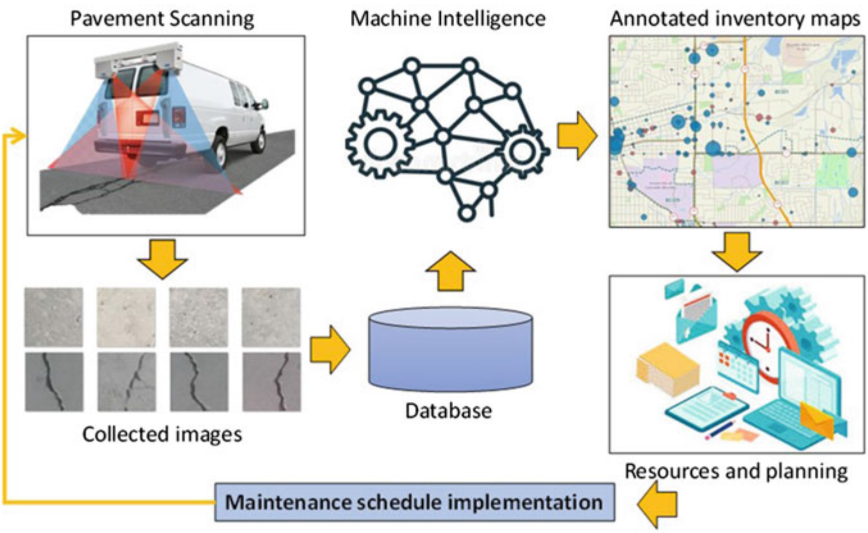

Fig. 8.4 Typical road defects inventory management

The machine intelligence role replaces human decisions onsite to visually observe and record the road defects. Automating this component in the process is a significant gain in many aspects including efficiency, accuracy and safety.

8.2 Literature Survey

There are number of techniques reported in the literature that address the problem of pavement crack detection and/or calcination: edge detection as in (Wei et al. 2010), probabilistic based proposed by Lin et al. (2019), morphological wavelets presented by Wu et al. (2016), Convolutional Neural Networks—CNN introduced in Ali et al. (2019) and (Yusof et al. 2018), Transferred knowledge of CNN proposed by (Nie and Wang 2019) and a comparative study of Artificial Neural Networks—ANN, Genetic Algorithms—GA and Self-organizing Maps—SOM by Rababaah et al. (2005). Previous work describes a number of challenges. The work of Wei et al. (2010), Yusof et al. (2018) reported that weak signals and image complexity influence the reliability of detection and classification of cracks. Blurry images, discontinuous cracks, crack degradation and crack shadows are reported as major challenges in the work of Wu et al. (2016). Lighting variation and minimal contrast in images are addressed in Yusof et al. (2018). The challenge of hand-crafted filters in producing a reliable classifier is addressed in Rababaah et al. (2005) and the real time processing challenge is reported as a challenge in Wolfer and Rababaah (2007). It is to be noted that the objective of this paper is to investigate the proposed CNN solution to classify the sub-regions of a gray-scale image into two classes—crack and pavement objects.

The work of Lin et al. (2019) presented a method for pavement crack detection based on classical image processing techniques using the hidden Markov model and expectation maximization. The authors reported that their solution was effective and produced reliable crack detection. A Beamlet transform-based method was presented by Wei et al. (2010) which is based on dimensional singularity analysis. The method requires images to be in binary form as an input. The solution is compared to traditional edge detection methods and found to be more reliable. In the study by Wu et al. (2016), a morphological wavelets-based method was proposed to effectively extract pavement cracks. The authors reported that their method was found to be effective compared to traditional techniques. The authors of Ali et al. (2019), presented the deployment of a CNN model that was trained and tested on real world images collected using a UAV. The model detection testing accuracy was 90%. The work of Yusof et al. (2018) introduced a CNN-based solution for pavement crack detection. Their solution required raw images to be in binary format as an input. The authors reported a detection accuracy of 99.2%. The work of Nie and Wang (2019) presented a method for pavement crack detection based on YOLO (Redmon et al. 2016). This method is known in CNN as transferred knowledge where, a pretrained network can be used to be partially trained to adapt to new data in a different domain. The authors reported a detection accuracy of 88%. In the work of Rababaah et al. (2005), the authors proposed a classical machine learning approach using the MLP, SOM and GA methods. The authors reported an exceptional detection and classification accuracy above 97%.

8.3 Technical Background

In this section, the fundamental architecture of CNNs and related theoretical background are discussed. Figure 8.5 illustrates the various layers in a CNN model including: input, convolution, activation, pooling, flatten, fully connected and soft max layers. The technical background of these layers is presented as follows.

8.3.1 *Convolution*

In general, convolution is defined as a mathematical operation that produces a function by integrating the product of two functions which measures the overlap between the two functions as one function is shifted over the other (Kim 2013). This definition is applied to 1D and 2D signals as in digital signals and digital images, respectively. Furthermore, the definition applies to continuous and discrete signals. Since our

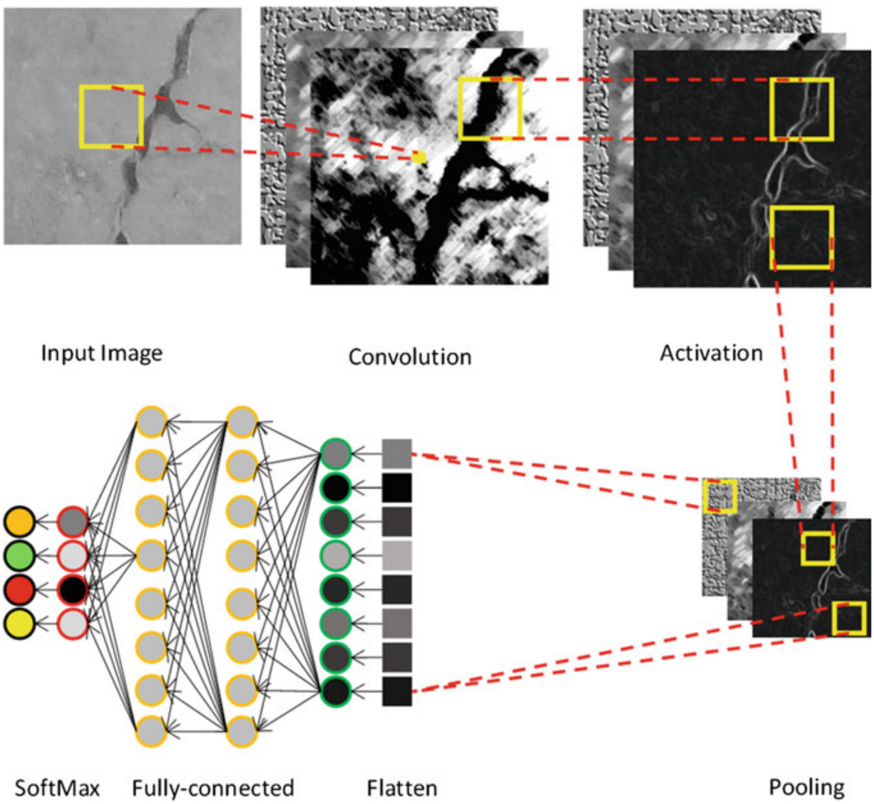

Fig. 8.5 Typical CNN architecture with fundamental layers

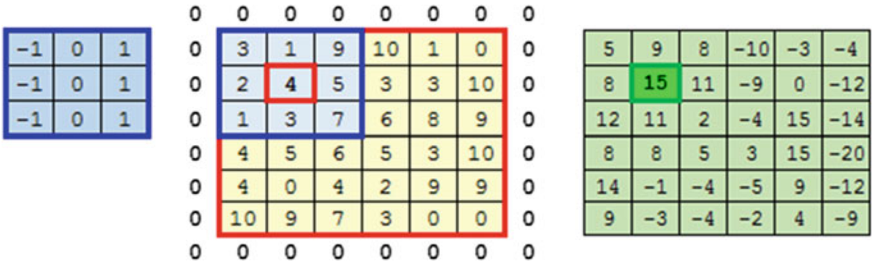

Fig. 8.6 left—3 × 3 kernel, middle—input image with current cell highlighted for convolution, right—output image after complete convolution for all cells

domain is digital images then, the formula for convolution for 2D discrete functions is given in Eq. (8.1). An illustration of the 2D convolution is given in Fig. 8.6. It should be observed that for the boarder cells of the input image zero-padding is required since scanning/sliding the kernel window crosses the edges to undefined indexes in all four directions. Furthermore, a popular kernel of Prewitt (Mutneja 2015) vertical edge detection is demonstrated in this example.

$$y[p,q] = \sum_{i=-n/2}^{n/2} \sum_{j=-m/2}^{m/2} x[p+i, q+j] \cdot h[i+n/2, j+n/2] \tag{8.1}$$

where:

y: output image
h: kernel/filter
x : input image
n : height (number of rows) of the kernel
m : width (number of columns) of the kernel
p, q : indexes of input/output images
i, j : indexes of kernel.

8.3.2 Activation

The role of activation operation at this statge is to map the produced mangnitudes by the convolution layer from a domain that includes negative values into a domain without negative values. In a sense, it is a filter for negative values. There are various activation functions that are availabe in CNNs including: linear rectified unit, binary step, sigmoid, tanh, softplus, etc. The linear rectified unit (ReLU) is the most popular activation function due to its computational efficiency and better convergence. The mathematical expression of ReLU is given in Eq. (8.2) and illustrated in Fig. 8.7.

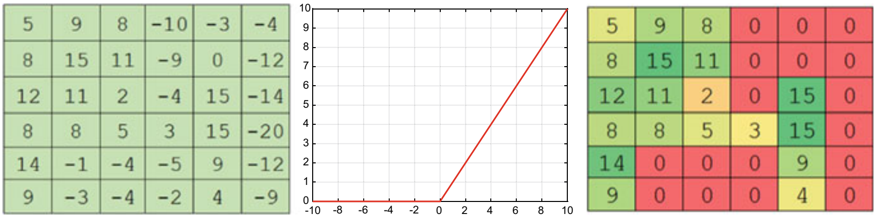

Fig. 8.7 left—input matrix, middle—Rectified linear unit function, right—output matrix after applying ReLU

$$R(x) = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{8.2}$$

8.3.3 *Max Pooling*

Max pooling is a down-sampling operation that uses the similar mechanism of sliding kernel used in convolution with predefined functions such as mean, median and maximum. The differences between convolution and pooling are listed in Table 8.1.

Max pooling can be mathematically expressed using the model in (8.3). Observe that the model is expressed based on row-wise computation in the kernel window. The unition notation is used to construct a set of returned max values of all rows after which another max function is applied to return the overall max of the row max values. An illustration of the max-pooling operation is given in Fig. 8.8. We can observe that the used kernel size is 2×2 with a stride of 2. The stride defines the amount of index shifting/sliding after each application of max-pooling.

$$y[p, q] = Max\left(\bigcup_{i=0}^{i=n-1} Max(x[p+i, q : q+m])\right) \tag{8.3}$$

Table 8.1 Comparison between convolution and pooling

Aspect	Convolution	Pooling
Main idea	Filtering	Aggregation
Objectives	Detect features	Down-sample
Weights	Learned	Not used
Input/output sizes	No change	Output < input
Functions used	Undefined	Predefined

5	9	8	0	0	0
8	15	11	0	0	0
12	11	2	0	15	0
8	8	5	3	15	0
14	0	0	0	9	0
9	0	0	0	4	0

15	11	0
12	5	15
14	0	9

Fig. 8.8 left—input image broken into 2 × 2 sub-regions, right—the output image after applying max-pooling with 2 × 2 filter and a stride of 2

where:

y : the output image
x : the input image
n : number of rows in kernel
m : number of columns in kernel
p, q : indexes of output image
i: row index in the kernel
q: $q + m$: range of columns in a row.

8.3.4 *Flatten Layer*

This layer is the interface between the concept of CNNs and classical ANNs. Since ANNs require a feature vector (1D array) and since the final output of a CNN is a volume of data, which is a stack of 2D matrices, then, a transformation of the output of a CNN to a feature vector that is compatible with an ANN is required which it does in the Flatten layer. It is to be observed that the pattern {Convolution, ReLU, MaxPool} can be repeated n times based on the complexity of the problem domain. For example, GoogleNet (Szegedy 2015) has a depth of 10 sequences of the fundamental layers, whereas AlexNet (Krizhevsky et al. 2013) has a depth of 5. This concept is depicted in Fig. 8.9.

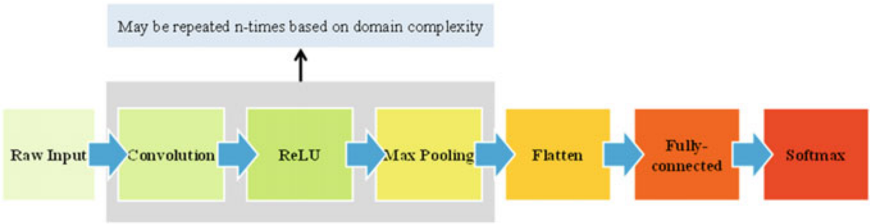

Fig. 8.9 Layers of CNN that can be repeated in sequence patterns

8.3.5 Fully Connected Layers

ANNs are mathematical models inspired by the biology of the brain and the nervous system. ANNs have various types of types and architectures including Perceptron, Feed Forward Networks (FFN), Radial Basis, Deep Feed Forward, Recurrent Networks, Belief Networks, etc. (Aggarwa 2018). Among these architectures, we focus on FFN and specifically, the Multi-Layer Perceptron (MLP) architecture, as it is typically been used for pattern recognition (Campos et al. 2004; Hasan et al. 2020; Rababaah 2020).

The computational unit of an MLP is the perceptron, shown in Fig. 8.10 (left). The perceptron models the biological nervous cell in a human body/brain. Three main operations are carried out in a perceptron namely: each stimulus "x_i" in [x_1, x_2, … x_n] is multiplied by its weight "ω_i", a weighted summation of all stimuli is computed adding a negative bias "θ" with a weight of "1" then finally the combined result is fed into an activation function, typically, the sigmoid function to map it to the interval [0, 1]. This process is mathematically expressed in Eq. (8.4).

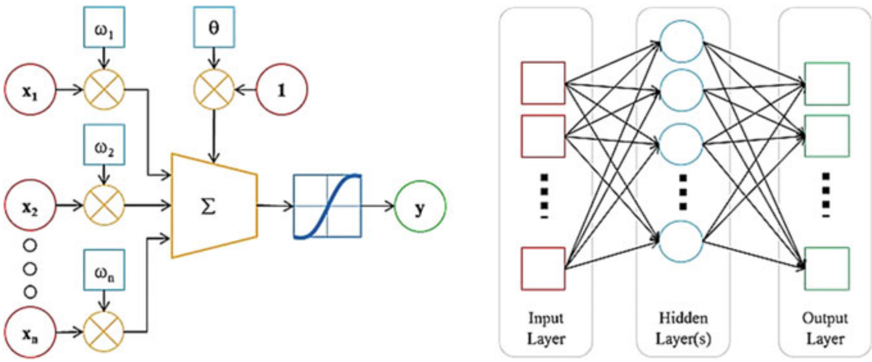

Fig. 8.10 left: the perceptron model, right: MLP model

$$y = \sigma\left(\theta + \sum_{i=1}^{n} \omega_i x_i\right) \tag{8.4}$$

where:

y: the output signal
w_i: the ith weigh associated with each stimulus x_i
n: number of stimuli
σ: the sigmoid function given in Eq. (8.5)

$$\sigma(x) = \frac{1}{1 + e^{-x}} \tag{8.5}$$

8.3.6 Classifcation

It is common that the output of the network is of a multi-class. Hence, the softmax model is used to produce a score in the range [0, 1] that represents the probability that the input vector belongs to an output class. The softmax function is a computation of the normalized exponential of all entries of the finally produced vector of the network. The softmax function is mathematically expressed in Eq. (8.6).

$$S(x[i]) = \frac{e^{x[i]}}{\sum_{k=1}^{m} e^{x[k]}} \tag{8.6}$$

where:
x: output vector right before softmax
i: the index an element in x
m: the size of the vector.

Figure 8.10 (right) illustrates a typical MLP network where components are recognized: input layer, hidden layers and output layer. The input layer is the input stimuli vector, the hidden layers are constructed of perceptrons. The first hidden layer is connected to the input and the last hidden layer is connected to the output layer. All hidden layers are interconnected with each other. The number perceptrons in the hidden layers and the depth of the hidden layers depend on the complexity of the problem.

After the MLP is constructed, a learning algorithm using Back Propagation is performed to train the network on the training dataset. The training algorithm is illustrated in Fig. 8.11. The backpropagation training algorithm computes forward responses from the input layer through the hidden layers until the output layer. For each training epoch, Mean Square Error (MSE) is computed between the predicted output vector and the class labels of the input vectors according to Eq. (8.7). In each

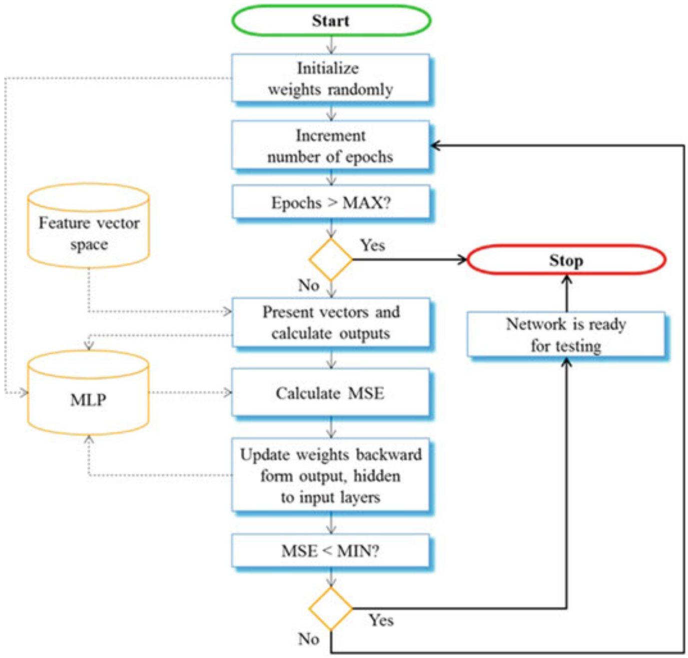

Fig. 8.11 Block diagram of MLP training algorithm

MSE computation cycle, all weights of the network are updated with an amount proportional to the MSE in the current cycle. When training is completed after reaching a satisfactory error rate, a testing set (unseen by the network) is used to validate the trained MLP.

$$MSE = \frac{1}{n}\sum_{k=1}^{n}(O_k - I_k)^2 \tag{8.7}$$

where:

MSE: mean square error

n: number of training vectors

O_k: output predicted label

I_k: input vector label

All mathematical models used in this section are based on the work of Haykin (2008).

8.4 Experimental Work

Our dataset was collected from the public-access dataset created by Özgenel (2019). The dataset contains 1000 samples of 500 pavement images and 500 crack images. Samples of these images are shown in Figs. 8.12 and 8.13, respectively.

The dataset was spilt into 80% for training and 20% for validation. The deep learning model was constructed using (MATLAB 2018) with the architecture illustrated in Fig. 8.5. We conducted 10 different experiments using the described dataset. Each of these experiments produced training curves and confusion matrix for results validation. Figures 8.14, 8.16, 8.18, 8.20 and 8.22 show the training results of the model with kernels from 1 to 10. Each figure of these contains two curves. The top one shows the training classification accuracy of the model, whereas the bottom one shows the training loss function as the mean square error. The Figs. 8.13, 8.15, 8.17, 8.19 and 8.21 depict the resulting confusion matrix from each experiment. The confusion matrix annotates the pavement class with "0" and the crack class with "1". The overall model accuracy can be seen at the bottom right corner/cell of the matrix. The summary of all the experimental results is listed and plotted in Fig. 8.24. The table to the left of the figure displays the model classification accuracies (MCA) in each experiment and it displays the mean accuracy and standard deviation of all experiments.

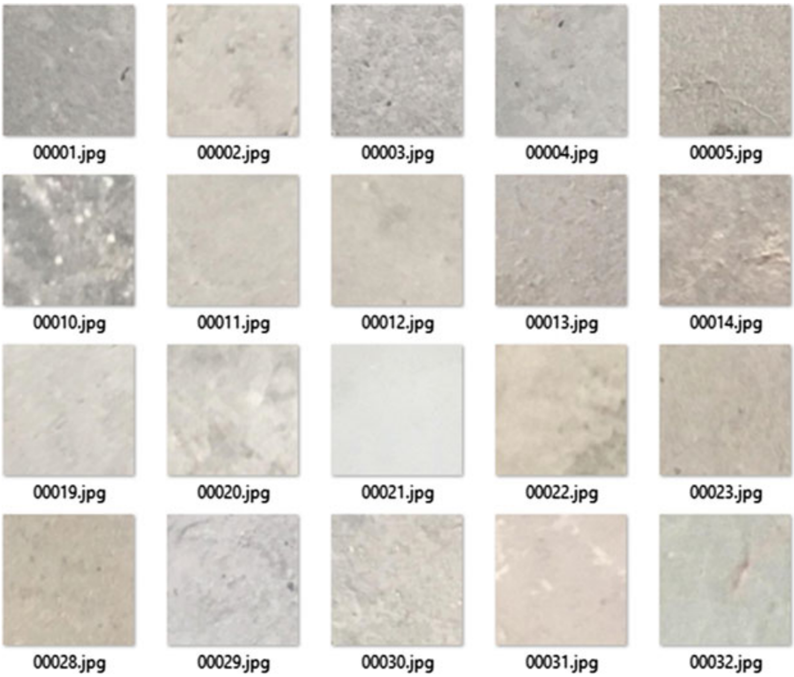

Fig. 8.12 Samples of pavement with no cracks

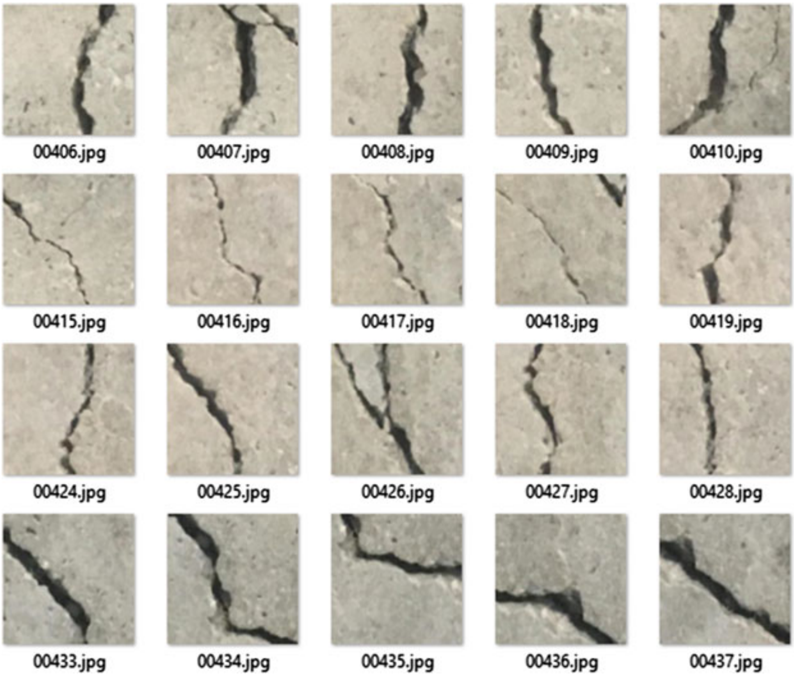

Fig. 8.13 Samples of pavement with cracks

8.5 Observations on the Results

- All tested models produced reliable classification accuracies ranging from 85 to 98.5% as it can be observed in Fig. 8.24.
- The lowest performing model was with number of kernels = 1. This is expected since number of kernels represents the number of features used to train the model.
- The highest performing model was with number of kernels = 3. This may not be expected since we tend to think of increasing the number of features that should improve the model performance. In this case, 3 features were better than the higher number of features as we tested a number of features between 1 and 10.
- The mean classification accuracy was 94.1% which is reliable. This is just to compute an average MCA among all models but if we wanted the best performing model, we should choose the best model with an MCA of 98.5 at number of kernels = 3.
- The standard deviation of the resulting MCAs was 0.045 as it can be seen in Fig. 8.24. This standard indicates the stability of the considered model as the variation in performance is insignificant.
- The overall impression of the model performance is positively supported by a reliable experimental evidence.

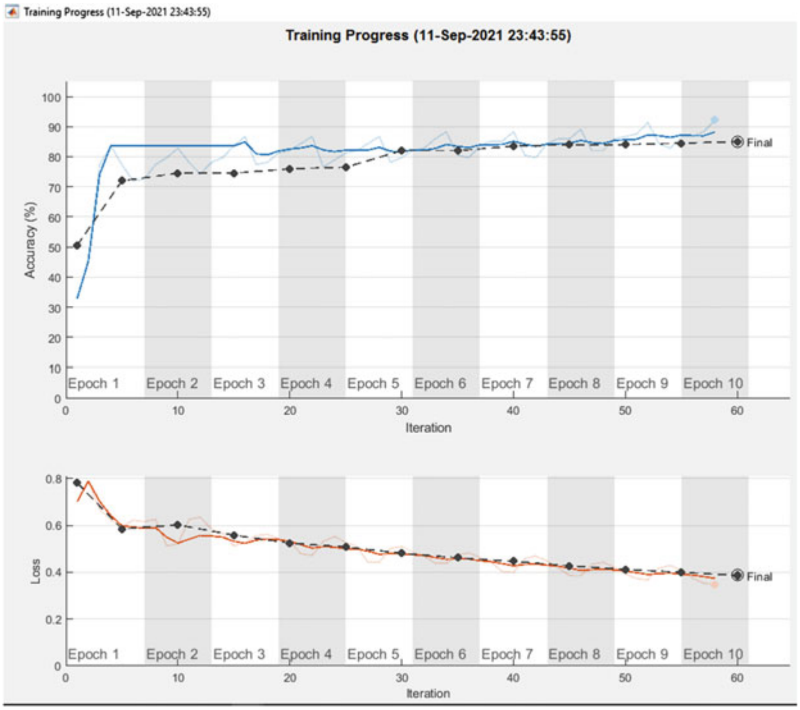

Fig. 8.14 Training results with number of kernels = 1

8.6 Conclusion

This chapter presented an investigation of the effectiveness of Deep Learning (DL) using Convolution Neural Networks (CNN) in solving the problem of manual inventory management of pavement road defects. Specifically, the chapter considered the pavement cracks as they make up the vast majority of road defects. A DL model using CNN architecture was constructed and tested using MatLab development environment. A public dataset was acquired with 1000 samples images of crack and no crack samples. Ten different experiments were conducted to evaluate the performance of the model and the results showed that CNN models are effective and reliable in classifying the road cracks automatically. This approach can be applied to automate the conventional methods of pavement defects inventory and maintenance planning. Significant gains are expected in time efficiency, accuracy and safety. As a future work, the method may be extended to cover crack types classification as well as other defects such as surface deformation and potholes.

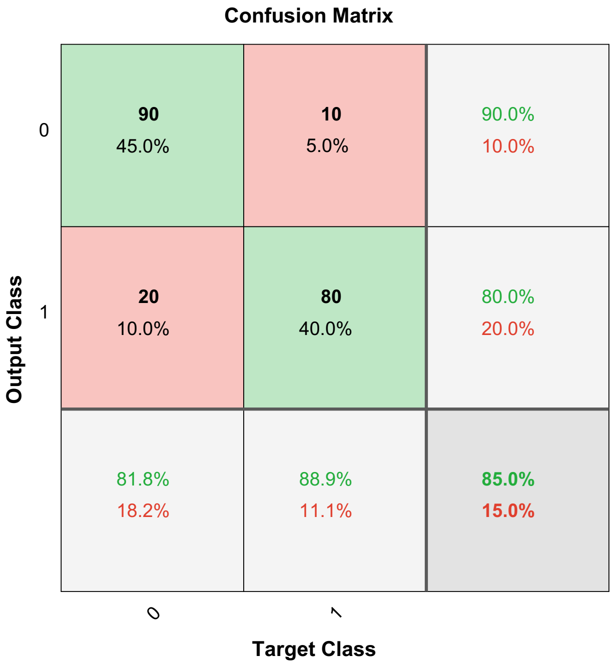

Fig. 8.15 Confusion matrix of the model with kernels = 1

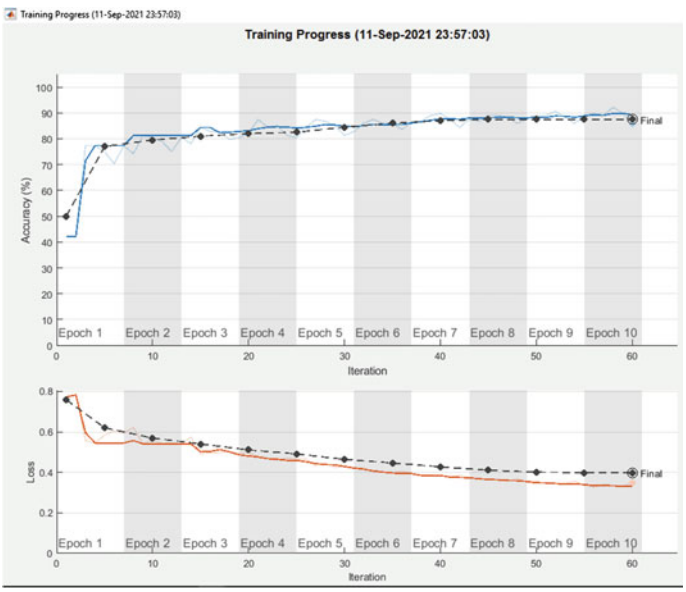

Fig. 8.16 Training results with number of kernels = 2

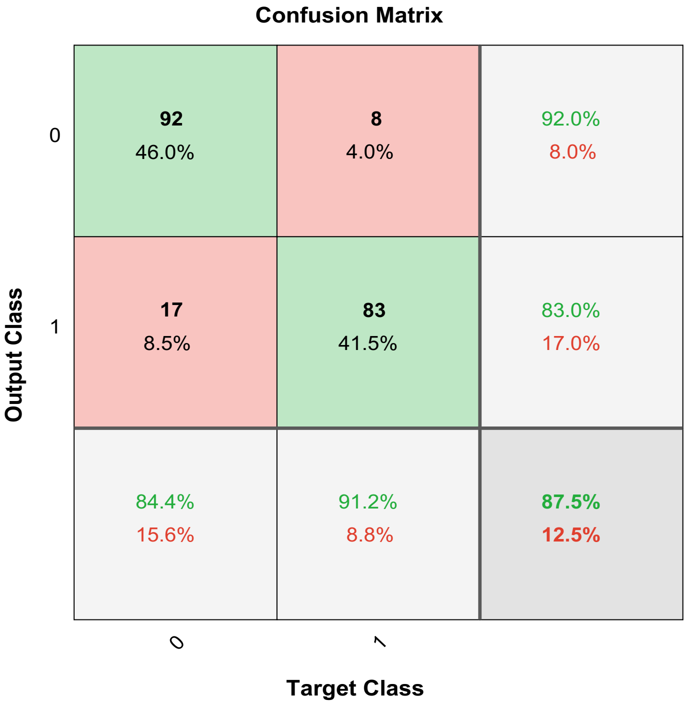

Fig. 8.17 Confusion matrix of the model with kernels = 2

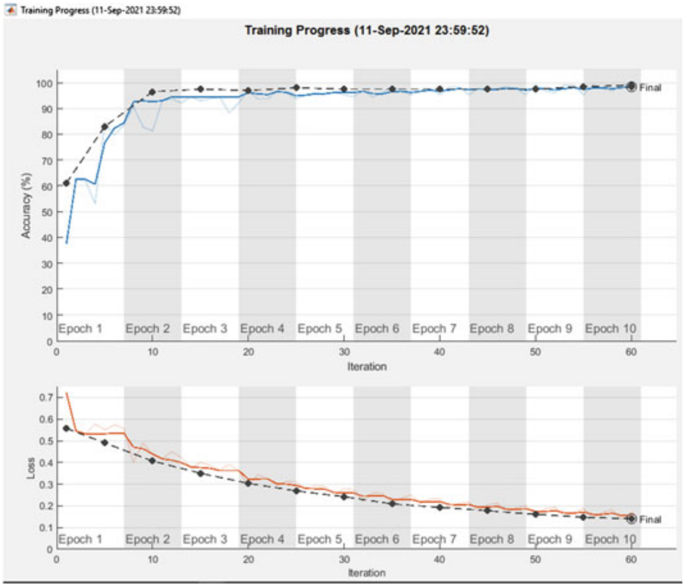

Fig. 8.18 Training results with number of kernels = 3

Fig. 8.19 Confusion matrix of the model with kernels = 3

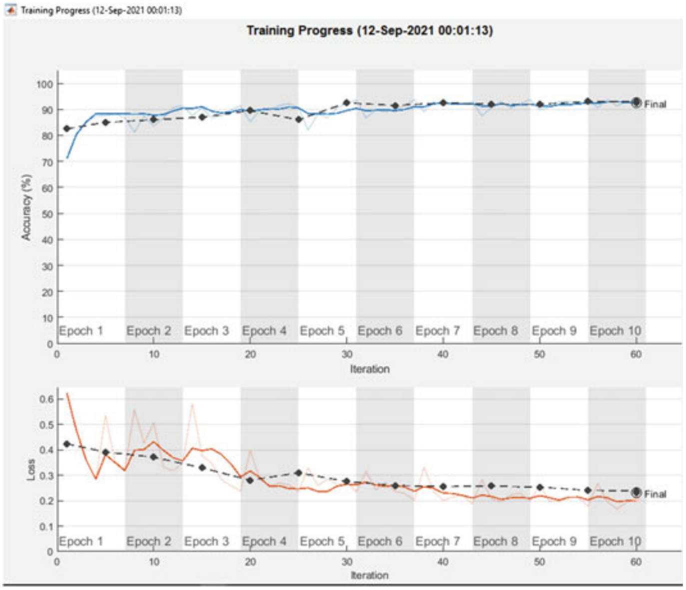

Fig. 8.20 Training results with number of kernels = 4

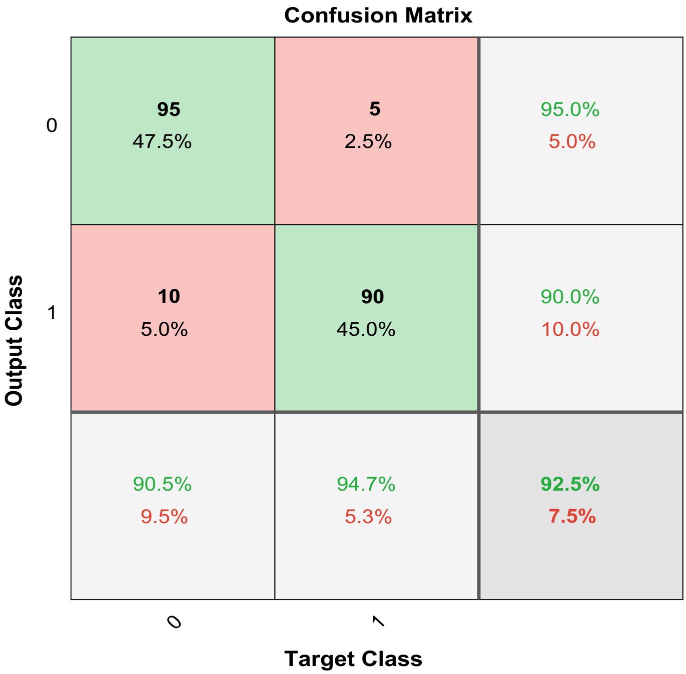

Fig. 8.21 Confusion matrix of the model with kernels = 4

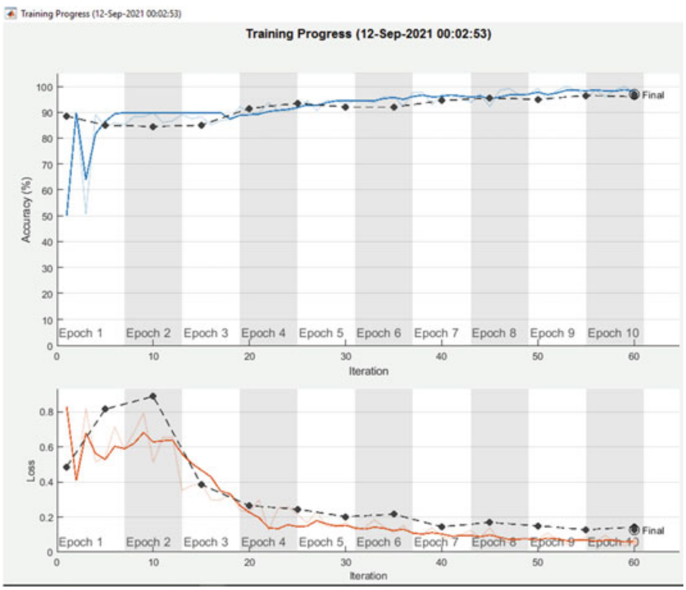

Fig. 8.22 Training results with number of kernels = 5

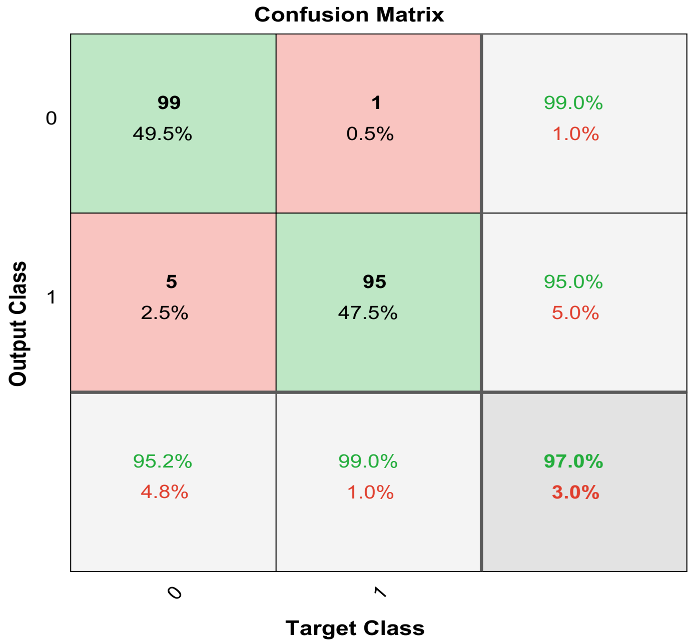

Fig. 8.23 Confusion matrix of the model with kernels = 5

Kernels	MCA
1	85.00%
2	87.50%
3	98.50%
4	92.50%
5	97.00%
6	95.50%
7	98.00%
8	97.00%
9	92.00%
10	97.50%
μ	94.05%
σ	0.044522

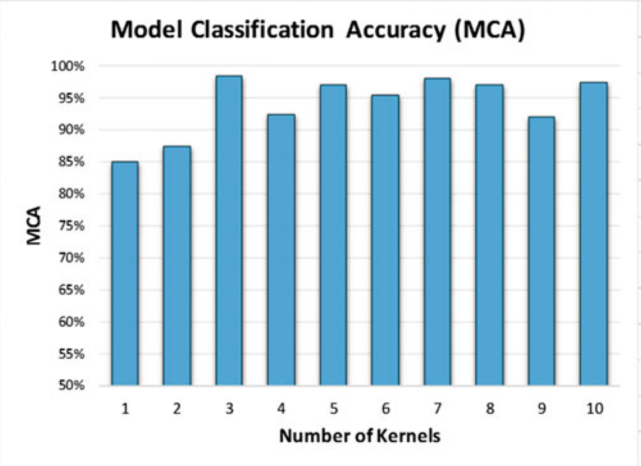

Fig. 8.24 Summary of all experiments

References

Aggarwa CC (2018) Neural networks and deep learning: a textbook, 978-3-319-94463-0. Springer

Ali L, Valappil NK, Kareem DNA, John MJ, Al Jassmi H (2019) Pavement crack detection and localization using convolutional neural networks (CNNs). In: 2019 International conference on digitization (ICD), pp 217–221

Campos PG, Oliveira EM, Ludermir TB, Araujo AF (2004) MLP networks for classification and prediction with rule extraction mechanism. IEEE international joint conference on neural networks. IEEE, Budapest, pp 1387–1392

DOT (2021) Distress identification manual for the long-term pavement performance program, 5th Revised Edition. Publication Number: FHWA-HRT-13-09. https://www.fhwa.dot.gov/publications/research/infrastructure/pavements/ltpp/13092/001.cfm. Accessed 9 Sept 2021

Hasan S, Erdal A, Taskin K (2020) Design of feedforward neural networks in the classification of hyperspectral imagery using superstructural optimization. Remote Sens. https://doi.org/10.3390/rs12060956

Haykin S (2008) Neural networks and learning machines. Pearson, New York City, New York, USA

Kim S (2013) Convolution. In: Kim S (ed) Applications of convolution inimage processing with MATLAB. University of Washington, Washington, USA, pp 4–6

Krizhevsky A, Sutskever I, Hinton GE (2013) ImageNet classification with deep convolutional neural networks. In: Neural information processing systems conference. https://papers.nips.cc/paper/4824-imagenet-classification-with-deep-convolutional-neural-networks.pdf

Lin MR, Zhou QY, Xu X (2019). Automatic pavement crack detection using HMRF-EM algorithm. In: 2019 international conference on computer, information and telecommunication systems (CITS), pp 1–5

MATLAB (2018) 9.4.0.813654 (R2018a). The MathWorks Inc., Natick, MA, USA. https://www.mathworks.com/

Mutneja V (2015) Methods of image edge detection: a review. J Electr Electron Syst 4(2). https://doi.org/10.4172/2332-0796.1000150

Nie M, Wang C (2019) Pavement crack detection based on yolo v3. In: 2019 2nd international conference on safety produce informatization (IICSPI), pp 327–330

Özgenel ÇF (2019) Concrete crack images for classification. Mendeley Data, V2. https://doi.org/10.17632/5y9wdsg2zt.2. https://data.mendeley.com/datasets/5y9wdsg2zt/2

Rababaah A (2020) CS MS thesis. www.iusb.edu: https://clas.iusb.edu/math-compsci/_prior-thesis/HRababaah_thesis.pdf

Rababaah A, Vrajitoru D, Wolfer J (2005) Asphalt pavement crack classification: a comparison of GA, MLP, and SOM. In: GECCO, genetic and evolutionary computation conference late-breaking paper, June 2005

Redmon J, Divvala S, Girshick R, Farhadi A (2016) You only look once: unified, real-time object detection. In: IEEE conference on computer vision and pattern recognition (CVPR). IEEE, Las Vegas, NV, USA, pp 779–788. https://doi.org/10.1109/CVPR.2016.91

Szegedy C (2015) Going deeper with convolutions. Computer Vision Foundation. https://static.googleusercontent.com/media/research.google.com/en//pubs/archive/43022.pdf

USA-DOT (2017) USA Department of Transportation report on mileage of road networks in USA as of 2017. https://www.fhwa.dot.gov/policyinformation/statistics/2017/hm16.cfm. Accessed 3 Sept 2021

USA-DOT (2021) Department of Transportation report on pavement cracks cost. https://www.fs.usda.gov/Internet/FSE_DOCUMENTS/fseprd528063.pdf. Accessed 3 Sept 2021

Wei N, Zhao X, Dou XY, Song H, Wang T (2010) Beamlet transform based pavement image crack detection. In: 2010 international conference on intelligent computation technology and automation, pp 881–883

Wolfer J, Rababaah A (2007) Cracking pavement: an assessment of the multi-layer perceptron and self-organizing feature map for categorizing asphalt cracks. In: Saftey, health and environmental world congress—SHEWC. Santos, Brazil, pp 1–6

Wu G, Sun X, Zhou L, Zhang H, Pu J (2016) Research on morphological wavelet operator for crack detection of asphalt pavement. In: 2016 IEEE international conference on information and automation (ICIA), pp 1573–1577

Yusof N, Osman MK, Noor MHM, Ibrahim A, Tahir NM, Yusof NM (2018) Crack detection and classification in asphalt pavement images using deep convolution neural network. In: 2018 8th IEEE international conference on control system, computing and engineering (ICCSCE), pp 227–232

Chapter 9
Decarbonisation Through Production of Rhino Bricks From the Waste Plastics: EPQ Model

Nabajyoti Bhattacharjee, Nabendu Sen, and Dinesh K. Sharma

Abstract The carbon dioxide emission due to plastic disposition is an issue of concern in the direction of global warming and environmental decarbonisation is a challenging objective in the area of sustainability and it can be attained through various strategies. The major source of plastic is deformed plastics from the industry and the plastic waste gathered from the consumers, and all these plastic wastes accumulated in a large dump yard. One of the strategies is to reuse the plastic from the dump yard so that the net quantity left for carbon emission is less as a result total carbon emission reduces in the environment from the dump yard. In this paper, a strategy is developed to decarbonise the environment through a reusing policy of waste plastic from a dump yard in a rhino brick industry where each unit of production contains 20% of plastic material per unit time. Optimal production rate of plastic and rhino brick are determined along with the length of production for rhino bricks. So that, the period can be determined after which the net carbon dioxide emission is less than the prescribed limit. The model is validated with numerical examples along with sensitivity analysis of the parameters. Managerial implications are provided to discuss the implication and impact of the proposed model in the decarbonisation process of the environment.

Keywords Sustainability · Production inventory · Deterioration · Defective items · Salvage cost · Carbon emission · Rhino bricks · Decarbonisation

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9.1 Introduction

In the present era of sustainable development, the emission of greenhouse gases from inventory, production, and wasteland is a challenging issue for policymakers. The net carbon emission from plastic disposal is one of the major causes for the increase in the temperature of this planet. Many steps are taken to reduce carbon emissions and one of the major steps is the reuse of waste plastic. Reuse is an amazing concept through which the emission in the open space can be reduced further and as a result, it is considered as one of the impactful processes to optimize global warming. The other effective techniques to reduce GHG or carbon emission are as follows: implementing green technologies, setting carbon caps in production, and tracing the carbon footprint in the inventory and surroundings. Many research works have been done in this direction to provide an effective method to decarbonise the inventory and production sector and as a consequence decarbonisation of the environment (Fig. 9.1).

Recently, green policies are implemented to control the carbon emission and in this direction, an inventory model had been proposed by Mashud et al. (2021a, b). Sustainable inventory models are proposed by Mashud et al. (2021a) to limit the carbon emission with greenhouse farming. For other related work on green inventory to maintain the level of carbon emission in the environment can refer to Ritha and Poongodisathiya (2018) and Mashud et al. (2020). Green policies are implemented to reduce the level of carbon emission in the environment and in this direction Dutta (2017), Dutta et al. (2019) and Dutta (2020) done an extensive study in production and consumption inventory modelling. Recently, the United Kingdom Secretary of State presented a report HM Government(2021) regarding the Industrial decarbonisation strategy in which a theoretical background is created to practically produce a net zero carbon emission in production sectors through certain policies. In a row to decarbonise the environment through inventory and operations, Dieguez et al. (2021) proposed a model to reduce emissions in the energy sector. For further study on the modelling to discuss the strategy of decarbonisation process we refer to Sluisveld

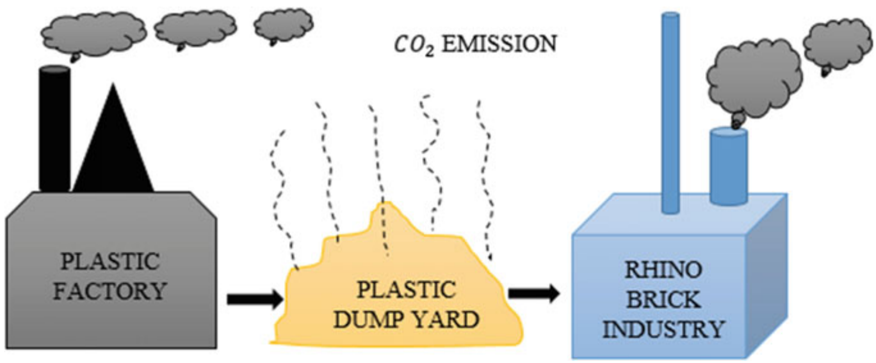

Fig. 9.1 Pictorial representation of strategy implemented for decarbonisation

et al. (2020). There are several strategies implemented by the agencies to reduce the carbon emission in the environment. In this regard, a method has been proposed in this paper to decarbonise the environment. In the next section, the motivation and description of the problem have been discussed.

9.2 Motivation and Problem Description

In the present world, global warming is a major threat to the earth and its inhabitants. There are several factors that are responsible for global warming such as the Emission of greenhouse gases like CFC, pollution particles, etc., increase in the level of CO_2, Deforestation, etc., and looking at the worst impact of global warming, various measures were taken to reduce the level of emissions and increasing the green environment and green technologies through policy making. In this direction, reusing the plastic for other production is one of the major development through which carbon emission from the dump yard can be reduced to a permissible limit. The industry of rhino bricks is one such production sector where bricks are made with plastic as one of the compositions. The plastic utilized in the production of rhino bricks is the waste plastic materials from different sources or from the waste dump yards. The rhino brick production is a sustainable green process through which the excess emission of carbon based gasses can be minimized to a considerable amount. The production process of rhino brick is cost effective which satisfies the economic aspect of sustainable development.

Thus motivated from the above discussion, an attempt has been made to develop two production inventory models one for the plastic factory and other for the rhino bricks. The objective of this paper is to estimate the optimal production rate of plastic and rhino bricks so that, the net carbon emission from the dump yard can be minimized to a permissible amount. Moreover, the optimum length of the production of rhino brick is predicted in order to determine the time required to reduce the net carbon emission in the dump yard. Considering the economic factor of sustainability, the decision variables are obtained to minimize the overall cost of production (Fig. 9.2).

9.3 Notations

The following notations we consider in this model:

Symbol	Unit	Meaning
$\boldsymbol{D}_1(\boldsymbol{D}_2)$	Units	Demand of plastic (rhino bricks)
$\boldsymbol{Q}(\boldsymbol{t})$	Unit per unit time	Inventory level of plastic at any time t

(continued)

(continued)

Symbol	Unit	Meaning
$\boldsymbol{B(t)}$	Unit per unit time	Inventory level of rhino bricks at any time t
$\boldsymbol{\theta}$	Constant	Rate of deterioration of plastic $0 < \theta < 1$
$\boldsymbol{\Phi}$	Constant	Rate of defective bricks $0 < \Phi < 1$
$\boldsymbol{\rho}$	Constant	Rate of carbon emission in dump yard $0 < \rho < 1$
$\boldsymbol{h}$	Rupee per unit per unit time	Holding cost per unit per unit time of plastic
$\boldsymbol{h_{rb}}$	Rupee per unit per unit time	Holding cost per unit per unit time of rhino bricks
$\boldsymbol{c}$	Rupee per unit per unit time	Deterioration cost per unit per unit time of plastic
$\boldsymbol{c_{rb}}$	Rupee per unit per unit time	Deterioration per unit per unit time of rhino bricks
$\boldsymbol{c_{e_1}}$	Rupee per unit per unit time	Carbon tax in production per unit per unit time of plastic
$\boldsymbol{c_{e_2}}$	Rupee per unit per unit time	Carbon tax in production per unit per unit time of rhino bricks
$\boldsymbol{c_{eh}}$	Rupee per unit per unit time	Carbon tax in holding per unit per unit time of rhino bricks
$\boldsymbol{\alpha}$	Rupee per unit per unit time	Fixed production cost of plastic
$\boldsymbol{\alpha_{rb}}$	Rupee per unit per unit time	Fixed production cost of rhino bricks
$\boldsymbol{v}$	Rupee per unit per unit time	Labour cost in the production of plastic
$\boldsymbol{v_{eb}}$	Rupee per unit per unit time	Labour cost in the production of rhino bricks
$\boldsymbol{s}$	Rupee per unit per unit time	Tool die cost in the production of plastic
$\boldsymbol{s_{rb}}$	Rupee per unit per unit time	Tool die cost in the production of rhino bricks
$\boldsymbol{c_t}$	Rupee per unit per unit time	Transportation cost per unit per unit time
$\boldsymbol{p}$	Rupee per unit per unit time	Salvage cost of defective bricks
$\boldsymbol{T}$	Unit time	Production period of plastic
$\boldsymbol{T_1}$	Unit time	Production period of rhino bricks
$\boldsymbol{Q_0}$	Unit	In hand inventory at time $t = T$
$\boldsymbol{L}$	Constant	Target limit of carbon emission

9.4 Assumptions

The following assumptions are made:

(a) Lead time is zero
(b) Production rate P_1 of plastic in the factory is constant and the production rate P_2 of rhino brick is also constant. Although, during the production of rhino brick, 20% of P_2 is the plastic drawn from the dump yard.
(c) The demand of plastic D_1 and rhino brick D_2 is constant.
(d) Carbon tax is imposed on production inventory of both plastic factory and rhino brick industry. Along with carbon tax on production inventory the government

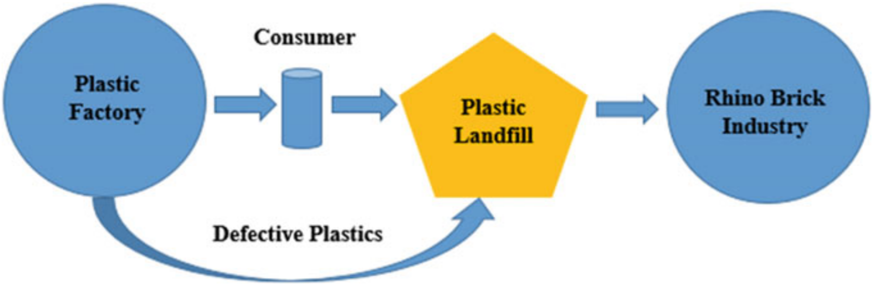

Fig. 9.2 Block-Arrow diagram of decarbonisation strategy

impose carbon tax for the carbon emission in dump yard on both the factories. However, the industry of rhino bricks follows the sustainable model of consumption and production by reusing the plastic waste therefore the carbon tax for the industry producing rhino bricks reduces with the reduction in the amount of carbon emission in the dump yard.

(e) The plastic produced in the factory is transferred to the dump yard after usage in zero time. And the rhino brick industry utilizes the waste plastic from a single source that is the dump yard.

(f) Transportation cost per unit per unit time of defective plastic from factory to dump yard and from dump yard to rhino brick industry are considered the same in the model.

(g) The production of plastic runs non-stop in the factory and reduces the model for a finite time frame. The production period is constant and the produced plastic is delivered to dump yard instantly. The delay due to shortage and usage is not considered in the study.

(h) The deterioration of plastic is considered during the production to maintain a realistic picture. But plastic is a non-degradable material so the deterioration referred to is the rate at which the finished material is wasted and not fit for use. However, the final place for all the waste product is the dump yard therefore all the deteriorated product is also transferred to dump yard and a cost of transportation is associated with it, in the total cost of production of plastic.

(i) Defective brick is considered in the model. The defective bricks cannot be used directly in the construction, however, it is utilized for other purposes therefore the defective bricks can be sold at a price less than the price of non-defective bricks. Therefore, a salvage cost is associated with the total cost of production.

(j) In the dump yard, the plastic is transported from the consumer and factory therefore, the volume of plastic in the dumb yard is given by the equation $V = D_1T + \theta\int_0^T Q(t)dt$. During the production of rhino bricks, 20% of P_2 is the plastic for the dump yard therefore the total usage of the plastic in the industry of rhino bricks is $W = \frac{1}{5}P_2T_1$. Thus the net volume of plastic in the dump yard is $V_0 = D_1T + \theta\int_0^T Q(t)dt - \frac{1}{5}P_2T_1$. Let ρ be the rate of carbon emission per unit volume of plastic in the dump. The carbon tax imposed by the government on the plastic factory is $CPF = c_e \rho V$ and on the rhino brick industry is $CRBI = c_e \rho V_0$. The objective is net carbon emission is less than the permissible limit that is, $\rho V_0 < L$.

9.5 Model Formulation

Plastic Factory

The inventory of plastic in the factory for the period $[0, T]$ is given by;

$$\frac{dQ}{dt} + \theta Q = P_1 - D_1 \;\; P_2^* \geq D_2, \tag{9.1}$$

where, $Q(0) = 0$ *and* $Q(T) = Q_0$

The inventory level of plastic in the factory during the period $[0, T]$ is given by;

$$Q(t) = \frac{P_1 - D_1}{\theta}\left(1 - e^{-\theta t}\right) \tag{9.2}$$

The cost associated with the supply chain to maintain the production factory.

Setup cost:

$$SC = O \tag{9.3}$$

Holding cost:

$$HC = h\left[\frac{(P_1 - D_1)}{\theta}\left(T + \frac{e^{-\theta T} - 1}{\theta}\right)\right]. \tag{9.4}$$

Deterioration cost:

$$DC = c\theta \int_0^T Q(t)dt = c(P_1 - D_1)\left(T + \frac{e^{-\theta T} - 1}{\theta}\right) \tag{9.5}$$

Production cost:

$$PC = \left(\alpha P_1 + v + s P_1^2\right)T \tag{9.6}$$

Carbon tax in production:

$$CTP = c_{e_1} P_1 T \tag{9.7}$$

Carbon tax in holding:

$$CTH = c_{eh}\left[\frac{(P_1 - D_1)}{\theta}\left(T + \frac{e^{-\theta T} - 1}{\theta}\right)\right] \tag{9.8}$$

Transportation cost:

$$TC = c_t(P_1 - D_1)\left(T + \frac{e^{-\theta T} - 1}{\theta}\right) \tag{9.9}$$

Overall cost of production of plastic in the period $[0, T]$ is

$$\begin{aligned} OCP =& O + c(P_1 - D_1)\left(T + \frac{e^{-\theta T} - 1}{\theta}\right) + \left(\alpha P_1 + v + sP_1^2\right)T \\ &+ c_t(P_1 - D_1)\left(T + \frac{e^{-\theta T} - 1}{\theta}\right) + h\left[\frac{(P_1 - D_1)}{\theta}\left(T + \frac{e^{-\theta T} - 1}{\theta}\right)\right] \\ &+ c_e\rho V + c_{e_1}P_1T + c_{eh}\left[\frac{(P_1 - D_1)}{\theta}\left(T + \frac{e^{-\theta T} - 1}{\theta}\right)\right] \end{aligned} \tag{9.10}$$

Neglecting the terms containing θ^2 in the expression of OCPP and on simplification we get,

$$\begin{aligned} OCP =& O + \left(\alpha P_1 + v + sP_1^2 + c_{e_1}P_1\right)T \\ &+ \frac{1}{2}(P_1 - D_1)(c\theta + c_t\theta + h + c_e\rho + c_{eh})T^2 \\ &+ \frac{1}{3}\theta(P_1 - D_1)(h + c_e\rho + c_{eh})T^3 \end{aligned} \tag{9.11}$$

Rhino Brick Industry

The differential equation to represent the production of rhino brick in the period $[0, T_1]$ is given by;

$$\frac{dB}{dt} + \Phi B = P_2 - D_2 \tag{9.12}$$

where, $B(0) = 0$

The inventory level of plastic in the factory during the period $[0, T_1]$ is given by;

$$B(t) = \frac{P_2 - D_2}{\Phi}\left(1 - e^{-\Phi t}\right) \tag{9.13}$$

Setup cost:

$$SC = O_{rb} \tag{9.14}$$

Holding cost:

$$HC = h_{rb}\left[\frac{(P_2 - D_2)}{\Phi}\left(T_1 + \frac{e^{-\Phi T_1} - 1}{\Phi}\right)\right] \tag{9.15}$$

Deterioration cost:

$$DC = c_{rb}\Phi\int_0^{T_1} B(t)dt = c_{rb}(P_2 - D_2)\left(T_1 + \frac{e^{-\Phi T_1} - 1}{\Phi}\right) \tag{9.16}$$

Production cost:

$$PC = \left(\alpha_{rb}P_2 + v_{rb} + s_{rb}P_2^2\right)T_1 \tag{9.17}$$

Carbon tax in production:

$$CT = c_{e_2}P_2T_1 \tag{9.18}$$

Transportation cost of plastic from the dump yard:

$$TC = \frac{1}{5}c_t P_2 T_1 \tag{9.19}$$

Salvage cost:

$$SVC = p(P_2 - D_2)\left(T_1 + \frac{e^{-\Phi T_1} - 1}{\Phi}\right) \tag{9.20}$$

Overall cost of production of rhino brick is given by

$$\begin{aligned} OCRBP =& O_{rb} + h_{rb}\left[\frac{(P_2 - D_2)}{\Phi}\left(T_1 + \frac{e^{-\Phi T_1} - 1}{\Phi}\right)\right] \\ &+ c_{rb}(P_2 - D_2)\left(T_1 + \frac{e^{-\Phi T_1} - 1}{\Phi}\right) \\ &+ \left(\alpha_{rb}P_2 + v_{rb} + s_{rb}P_2^2\right)T_1 + c_{e_2}P_2T_1 \\ &+ \frac{1}{5}c_t P_2 T_1 + c_e\rho V_0 - p(P_2 - D_2)\left(T_1 + \frac{e^{-\Phi T_1} - 1}{\Phi}\right) \end{aligned} \tag{9.21}$$

Neglecting the terms containing θ^2 in the expression of OCRBP and on simplification we get;

$$OCRBP = O_{rb} + \left(v_{rb} + \left(\alpha_{rb} + c_{e_2} + \frac{1}{5}c_t - \frac{1}{5}c_e\rho\right)P_2 + s_{rb}P_2^2\right)T_1 \\ + \frac{1}{2}(P_2 - D_2)(h_{rb} + c_{rb}\Phi + c_e\rho - p\Phi)T_1^2 \\ + \frac{1}{3}\Phi(P_2 - D_2)(c_e\rho - h_{rb})T_1^3 \tag{9.22}$$

9.6 Solution Procedure

The formulated model has two objective functions of minimization type;

$$\min f(P_1, P_2, T_1) = U_0 + U_1T + V_1T_1 + U_2T^2 + V_2T_1^2 + U_3T^3 + V_3T_1^3 \tag{9.23}$$

where,

$$U_0 = O + O_rb$$

$$U_1 = \alpha P_1 + v + sP_1^2 + c_{e_1}P_1$$

$$U_2 = \frac{1}{2}(P_1 - D_1)(c\theta + c_t\theta + h + c_e\rho + c_{eh})$$

$$U_3 = \frac{1}{3}\theta(P_1 - D_1)(h + c_e\rho + c_{eh})$$

$$V_1 = v_{rb} + \left(\alpha_{rb} + c_{e_2} + \frac{1}{5}c_t - \frac{1}{5}c_e\rho\right)P_2 + s_{rb}P_2^2$$

$$V_2 = \frac{1}{2}(P_2 - D_2)(h_{rb} + c_{rb}\Phi + c_e\rho - p\Phi)$$

$$V_3 = \frac{1}{3}\Phi(P_2 - D_2)(h_{rb} - c_e\rho)$$

Subject to the constraints

$$\rho\left[D_1T + \frac{1}{2}\theta(P_1 - D_1)T^2 - \frac{1}{5}P_2T_1\right] < L$$

$$P_1^*, P_2^*, T_1^* > 0$$

$$P_1^* \geq D_1$$

$$P_2^* \geq D_2$$

The decision variable is;
Production rate of plastic factory P_1 units/unit time
Production rate of rhino brick P_2 units/unit time
Production period for rhino bricks T_1 unit time.

The objective function is a nonlinear constraint minimization problem with three independent decision variables. In order to simplify the computational process, the model is solved with LINGO 19.0 to get the minimum overall cost of production and the optimal values of the decision parameters. The solver selected for solving the objective function is Nonlinear Solver and a graphical presentation is provided to depict the convexity in SciLab 6.1.0. The system in which the model is coded for optimisation and graphical representation has 12 GB RAM, i3-intel core processor in Windows 10 environment.

9.7 Numerical Illustration

In the absence of actual data hypothetical values of the parameters are considered in the numerical examples.

Example 9.1 ***When demand of rhino brick is less than demand of plastic***-The rate of carbon emission from the dump yard is $\rho = 0.65$. The government tax for each unit of carbon emission is $Rs.250$ per unit of emission and the permission limit of carbon emission is 50 units. There are two production sectors: Plastic factory and Rhino Brick Industry so the following values are considered for the parameters. Setup cost: $O = Rs.1000$ and $O_{rb} = Rs.1200$. Fixed production cost: $\alpha = Rs.100$ and $\alpha_{rb} = Rs.75$. Labour cost associated with the production: $v = Rs.1300$ and $v_{rb} = Rs.1000$. Tool die cost: $s = Rs.0.008$ and $s_{rb} = Rs.0.001$. Carbon tax in the production: $c_{e_1} = Rs.1.2$ and $c_{e_2} = Rs.0.7$. Carbon tax in holding in plastic factory: $c_{eh} = Rs.0.65$. Holding cost: $h = Rs.0.4$ and $h_{rb} = Rs.0.5$. Deterioration cost: $c = Rs.1$ and $c_{rb} = Rs.1.5$. Transportation cost: $c_t = Rs.2.5$. Salvage cost of defective bricks: $p = Rs.5$. Demand: $D_1 = 300$ units and $D_2 = 150$ units. Rate of deterioration: $\theta = 0.005$ and $\Phi = 0.001$. Production period of plastic: $T = 30$ days.

Solution: The optimal production rate of plastic is 300 units per unit time and the optimal production rate of rhino brick is 150 units per unit time. The minimum overall cost of production of plastics is $OCP = Rs.972400$ and rhino brick is $OCRBP = Rs.2232726$. And the optimal production period in order to reduce the net carbon emission to a permissible limit is $T_1 = 297.43359$ days approximately 9.914 months.

Example 9.2 ***When demand of rhino brick is more than demand of plastic***-The rate of carbon emission from the dump yard is $\rho = 0.65$. The government tax for each unit of carbon emission is $Rs.250$ per unit of emission and the permission limit of carbon emission is 50 units. There are two production sectors: Plastic factory and Rhino Brick Industry so the following values are considered for the parameters. Setup cost: $O = Rs.1000$ and $O_{rb} = Rs.1200$. Fixed production cost: $\alpha = Rs.100$ and $\alpha_{rb} = Rs.75$. Labour cost associated with the production: $v = Rs.1300$ and $v_{rb} = Rs.1000$. Tool die cost: $s = Rs.0.008$ and $s_{rb} = Rs.0.001$. Carbon tax in the production: $c_{e_1} = Rs.1.2$ and $c_{e_2} = Rs.0.7$. Carbon tax in holding in plastic factory: $c_{eh} = Rs.0.65$. Holding cost: $h = Rs.0.4$ and $h_{rb} = Rs.0.5$. Deterioration cost: $c = Rs.1$ and $c_{rb} = Rs.1.5$. Transportation cost: $c_t = Rs.2.5$. Salvage cost of defective bricks: $p = Rs.5$. Demand: $D_1 = 250$ units and $D_2 = 450$ units. Rate of deterioration: $\theta = 0.005$ and $\Phi = 0.001$. Production period of plastic: $T = 30$ days.

Solution: The optimal production rate of plastic is 2 50 units per unit time and the optimal production rate of rhino brick is 4 50 units per unit time. The minimum overall cost of production of plastics is $OCP = Rs.814000$ and rhino brick is $OCRBP = Rs.1703776$. And the optimal production period in order to reduce the net carbon emission to a permissible limit is $T_1 = 82.4783$ days approximately 2.749 months.

Example 9.3 Rhino brick production is a sustainable production sector therefore in this example we adopt the value of the cost parameters of Gautam et al. (2020) for the cost parameters of Rhino brick industry $(\boldsymbol{D}_1 < \boldsymbol{D}_2)$. The rate of carbon emission from the dump yard is $\rho = 0.65$. The government tax for each unit of carbon emission is $Rs.250$ per unit of emission and the permission limit of carbon emission is 50 units. There are two production sectors: Plastic factory and Rhino Brick Industry so the following values are considered for the parameters. Setup cost: $O = Rs.1000$ and $O_{rb} = Rs.600$. Fixed production cost: $\alpha = Rs.100$ and $\alpha_{rb} = Rs.25$. Labour cost associated with the production: $v = Rs.1300$ and $v_{rb} = Rs.1300$. Tool die cost: $s = Rs.0.008$ and $s_{rb} = Rs.0.008$. Carbon tax in the production: $c_{e_1} = Rs.1.2$ and $c_{e_2} = Rs.3$. Carbon tax in holding in plastic factory: $c_{eh} = Rs.0.65$. Holding cost: $h = Rs.0.4$ and $h_{rb} = Rs.1.5$. Deterioration cost: $c = Rs.2$ and $c_{rb} = Rs.250$. Transportation cost: $c_t = Rs.2.5$. Salvage cost of defective bricks: $p = Rs.5$. Demand: $D_1 = 140$ units and $D_2 = 250$ units. Rate of deterioration: $\theta = 0.005$ and $\Phi = 0.001$. Production period of plastic: $T = 30$ days.

Solution: The optimal production rate of plastic is 140 units per unit time and the optimal production rate of rhino brick is 250 units per unit time. The minimum overall cost of production of plastics is $OCP = Rs.468744$ and rhino brick is $OCRBP = Rs.56692.31$. And the optimal production period in order to reduce the net carbon emission to a permissible limit is $T_1 = 82.461$ days approximately 8.837 months.

Example 9.4 Rhino brick production is a sustainable production sector therefore in this example we adopt the value of the cost parameters of Gautam et al. (2020) for the cost parameters of Rhino brick industry ($\boldsymbol{D}_1 > \boldsymbol{D}_2$). The rate of carbon emission from the dump yard is $\rho = 0.65$. The government tax for each unit of carbon emission is $Rs.250$ per unit of emission and the permission limit of carbon emission is 50 units. There are two production sectors: Plastic factory and Rhino Brick Industry so the following values are considered for the parameters. Setup cost: $O = Rs.1000$ and $O_{rb} = Rs.600$. Fixed production cost: $\alpha = Rs.100$ and $\alpha_{rb} = Rs.25$. Labour cost associated with the production: $v = Rs.1300$ and $v_{rb} = Rs.1300$. Tool die cost: $s = Rs.0.008$ and $s_{rb} = Rs.0.008$. Carbon tax in the production: $c_{e_1} = Rs.1.2$ and $c_{e_2} = Rs.3$. Carbon tax in holding in plastic factory: $c_{eh} = Rs.0.65$. Holding cost: $h = Rs.0.4$ and $h_{rb} = Rs.1.5$. Deterioration cost: $c = Rs.2$ and $c_{rb} = Rs.250$. Transportation cost: $c_t = Rs.2.5$. Salvage cost of defective bricks: $p = Rs.5$. Demand: $D_1 = 250$ units and $D_2 = 140$ units. Rate of deterioration: $\theta = 0.005$ and $\Phi = 0.001$. Production period of plastic: $T = 30$ days.

Solution: The optimal production rate of plastic is 250 units per unit time and the optimal production rate of rhino brick is 140 units per unit time. The minimum overall cost of production of plastics is $OCP = Rs.813000$ and rhino brick is $OCRBP = Rs.221048.6$. And the optimal production period in order to reduce the net carbon emission to a permissible limit is $T_1 = 265.1099$ days approximately 2.7949 months.

Example 9.5 The emission rate of $\boldsymbol{CO_2}$ from plastic waste provided in the report of Greenhouse gas emission and natural capital implications of plastics (2021) ($\boldsymbol{D}_1 > \boldsymbol{D}_2$)-The rate of carbon emission from the dump yard is $\rho = 0.15$ [15% emission from waste plastic, Eionet Report—ETC/WMGE 2021/3]. The government tax for each unit of carbon emission is $Rs.250$ per unit of emission and the permission limit of carbon emission is 50 units. There are two production sectors: Plastic factory and Rhino Brick Industry so the following values are considered for the parameters. Setup cost: $O = Rs.1000$ and $O_{rb} = Rs.600$. Fixed production cost: $\alpha = Rs.100$ and $\alpha_{rb} = Rs.25$. Labour cost associated with the production: $v = Rs.1300$ and $v_{rb} = Rs.1300$. Tool die cost: $s = Rs.0.008$ and $s_{rb} = Rs.0.008$. Carbon tax in the production: $c_{e_1} = Rs.1.2$ and $c_{e_2} = Rs.3$. Carbon tax in holding in plastic factory: $c_{eh} = Rs.0.65$. Holding cost: $h = Rs.0.4$ and $h_{rb} = Rs.1.5$. Deterioration cost: $c = Rs.2$ and $c_{rb} = Rs.250$. Transportation cost: $c_t = Rs.2.5$. Salvage cost of defective bricks: $p = Rs.5$. Demand: $D_1 = 250$ units and $D_2 = 140$ units. Rate of deterioration: $\theta = 0.005$ and $\Phi = 0.001$. Production period of plastic: $T = 30$ days.

Solution: The optimal production rate of plastic is 250 units per unit time and the optimal production rate of rhino brick is 140 units per unit time. The minimum overall cost of production of plastics is $OCP = Rs.813000$ and rhino brick is $OCRBP = Rs.1109246$. And the optimal production period in order to reduce the net carbon emission to a permissible limit is $T_1 = 255.9524$ days approximately 2.7949 months.

Example 9.6 The emission rate of CO_2 from plastic waste provided in the report of Greenhouse gas emission and natural capital implications of plastics (2021) ($\boldsymbol{D}_1 < \boldsymbol{D}_2$)-The rate of carbon emission from the dump yard is $\rho = 0.15$ [15% emission from waste plastic, Eionet Report—ETC/WMGE 2021/3]. The government tax for each unit of carbon emission is $Rs.250$ per unit of emission and the permission limit of carbon emission is 50 units. There are two production sectors: Plastic factory and Rhino Brick Industry so the following values are considered for the parameters. Setup cost: $O = Rs.1000$ and $O_{rb} = Rs.600$. Fixed production cost: $\alpha = Rs.100$ and $\alpha_{rb} = Rs.25$. Labour cost associated with the production: $v = Rs.1300$ and $v_{rb} = Rs.1300$. Tool die cost: $s = Rs.0.008$ and $s_{rb} = Rs.0.008$. Carbon tax in the production: $c_{e_1} = Rs.1.2$ and $c_{e_2} = Rs.3$. Carbon tax in holding in plastic factory: $c_{eh} = Rs.0.65$. Holding cost: $h = Rs.0.4$ and $h_{rb} = Rs.1.5$. Deterioration cost: $c = Rs.2$ and $c_{rb} = Rs.250$. Transportation cost: $c_t = Rs.2.5$. Salvage cost of defective bricks: $p = Rs.5$. Demand: $D_1 = 140$ units and $D_2 = 250$ units. Rate of deterioration: $\theta = 0.005$ and $\Phi = 0.001$. Production period of plastic: $T = 30$ days.

Solution: The optimal production rate of plastic is 140 units per unit time and the optimal production rate of rhino brick is 250 units per unit time. The minimum overall cost of production of plastics is $OCP = Rs.468744$ and rhino brick is $OCRBP = Rs.536500$. And the optimal production period in order to reduce the net carbon emission to a permissible limit is $T_1 = 77.333$ days approximately 8.837 months.

9.8 Sensitivity Analysis

In this section, the flexibility and robustness of the model is tested through sensitivity analysis of the parameters. The changes in the parameters can be made through the equation $N \to (1 + r) * N$ where, $r \in \{-0.5, -0.25, 0.25, 0.5\}$. The comparison of changed values with the change in parameters is made with Example 9.3 (Table 9.1).

9.9 Managerial Implications

The purpose of this paper is to understand the impact of rhino brick production on decarbonising the environment by reducing the level of carbon emission in plastic dump yard. There are several highlights that, can be made from the above numerical analysis and it can be articulated as follows:

I. From Example 9.1, it can be noticed that when the production rate of plastic is double the production rate of rhino bricks then the time to reduce the carbon emission from the dump yard created in 30 days of plastic production and consumption can be controlled to a permissible limit in more than 9 months.

Table 9.1 Sensitivity analysis

Parameter	% Changed (%)	T_1	f	OCP	OCRBP
$\rho = 0.65$	4	82.52	498,657.8	468,744	29,913.76
	2	82.49	512,052	468,744	43,303.14
	−2	82.43	538,810	468,744	70,065.62
	−4	82.39	552,171	468,744	83,427.4
$c_e = 250$	4	82.46	525,477.5	468,744	56,733.54
	2	82.46	525,457.9	468,744	56,712.92
	−2	82.46	525,416.7	468,744	56,671.69
	−4	82.46	525,395	468,744	56,651.08
$L = 50$	4	82.4	525,353	468,744	56,608.80
	2	82.43	525,374	468,744	56,629.94
	−2	82.49	525,416	468,744	56,672.22
	−4	82.52	525,437	468,744	56,693.35
$c_t = 2.5$	4	82.46	525,395	468,744	56,651.08
	2	82.46	525,395	468,744	56,651.08
	−2	82.46	525,395	468,744	56,651.08
	−4	82.46	525,395	468,744	56,651.08
$p = 5$	4	82.46	525,395	468,744	56,651.08
	2	82.46	525,395	468,744	56,651.08
	−2	82.46	525,395	468,744	56,651.08
	−4	82.46	525,395	468,744	56,651.08
T	4	85.82	546,453	487,493	58,959.4
	2	84.14	535,924	478,118.9	57,805.24
	−2	80.78	514,866	459,369.1	55,496.92
	−4	79.1	504,337	449,994.2	54,342.76
α, α_{rb}	4	82.46	5,628,851.7	485,544	77,307.69
	2	82.46	544,144	477,144	67,000
	−2	82.46	506,729	460,344	46,384.62
	−4	82.46	488,020.9	451,944.0	36,076.92
v, v_{rb}	4	82.46	531,284	470,304	60,980.31
	2	82.46	528,360	469,524	58,836.31
	−2	82.46	522,512	467,964	54,548.31
	−4	82.46	519,588	467,184	52,404.31
s, s_{rb}	4	82.46	521,426	467,372.2	54,053.54
	2	82.46	520,507	467,278.1	53,228.92
	−2	82.46	518,670	467,089.9	51,579.69
	−4	82.46	517,751	466,995.8	50,755.08

II. While in Example 9.2, where the rate of plastic production is less than the production rate of rhino bricks then the production period to reduce the carbon emission to a permissible limit is significantly less as compared to the production period obtained in Example 9.1. Furthermore, the overall cost of production reduces for both the plastic factory and rhino bricks industry. The reason is that, the consequence of less plastic production is that, the overall cost of production and the quantity produced gets reduced. As a result, the amount of plastic disposal is less and therefore the total time to reuse the necessary amount of waste plastics is considerably low as a consequence of it the length of production considered in the model is small and simultaneously the overall cost of production cost shrinks.

III. In Examples 9.3 and 9.4, the values of the cost parameters of rhino bricks production inventory are adopted from Gautam et al. (2020). The overall production cost reduces in Example 9.3 as compared to Example 9.4, and the length of the production period also gets reduced which suggests that changing the cost parameters and taking the demand of rhino bricks more than the demand of plastic improves the decarbonisation speed.

IV. Taking $\rho = 15\%$ and keeping rest of the parameters as Examples 9.3 and 9.4 in the Examples 9.5 and 9.6 it can be observed that, a low rate of carbon emission and high demand for rhino bricks can reduce the production cost significantly and increases the speed of decarbonisation.

V. The total cost of production increases with the decrease in the rate of carbon emission. Moreover, production length of rhino brick tends to increase with the increase in the rate of carbon emission.

VI. Production cost of rhino brick increases with the increase in the carbon tax imposed by the government. Whereas, both the cost of production and length of production decreases with the increase in carbon cap of the landfill.

VII. Planning horizon is a significant parameter because it has an impact on the production cost of both plastic factory and rhino brick industry. Overall cost of plastic production decreases and cost of rhino brick production increases. Further, by increasing the planning horizon it can be observed that, the length of production of rhino brick increases.

VIII. Overall cost of plastic production and rhino brick production increases with the increase in fixed production cost, labour cost and tool die cost.

9.10 Conclusions

This is a interconnected production inventory model to determine the production rate so that, through reuse of waste plastics from the dumb yard environment can be decarbonised. A single flow of plastic is assumed in the model:

$$\boldsymbol{Plastic\ Production} \rightarrow \boldsymbol{Consumer} \rightarrow (\boldsymbol{Disposition} + \boldsymbol{Defective\ Plastic}) \\ \rightarrow \boldsymbol{Rhino\ Brick\ production} \rightarrow \boldsymbol{Consumer}$$

The strategy proposed in the model suggests that, if the demand of rhino bricks increases in the society then the duration of decarbonisation reduces. Furthermore, the production rate can impact the process of decarbonisation to a larger extent and it takes a lot of time to reduce the level of carbon dioxide in the environment. The model assumes a single plastic production factory, single consumer and a single rhino brick production industry, it can be observed that single production unit of rhino brick has a little impact on decarbonising the environment. Therefore, it is adviced to increase the number of rhino brick production units in order to accelerate the process of decarbonisation. There are two types of carbon tax imposed on both the production sectors: Carbon emission tax in production and carbon tax for emission caused due to waste plastic in dump yard. The carbon tax caused due to waste plastic is taken from both the plastic factory and rhino brick industry, however, the tax collected from the rhino brick industry reduces with time but it will increase for plastic factory.

The model is solved to obtain the minimum overall production cost for both the production units subject to the condition that, the total carbon emission reduces to a permissible limit. Further, the model is analysed with multiple numerical examples constructed from hypothetical values of parameters chosen in such as a manner in order to study certain situations such as a comparison between production rates and the corresponding time to reduce the amount of emission from the atmosphere through the production of rhino bricks.

The proposed model is an integrated design of two production inventory models both are connected to a dump yard of waste plastic. In this paper, a flow of plastic from plastic factory to rhino brick industry is developed and studied to determine the time period to reduce the level of emission from the atmosphere. The strategy to decarbonise the environment through the production of rhino brick is developed for the first time in the literature on production inventory and sustainability. Price factor is not considered in this model as the major emphasis is given on production rate and optimal time period to reduce the carbon emission in the environment. However, the price factor is important in order to increase the demand and simultaneously increase the production rate. Furthermore, considering the finite production period is also a significant variable which provide a space to understand the length of production that can limit the carbon emission both in the production and atmosphere independent from production unit. From the above discussion an insight can be made on the extension of the present research work. The extension of the model can be made by including the parameters of price and finite production period.

References

Dutta TK (2017) Effect of green technology on a production-inventory system with carbon tax. Adv Operat Res Hindawi 4834839:1–13

Dutta TK (2020) A new green efficiency-based carbon taxing policy and its effects on a production-inventory system with random carbon emission and green investment. Adv Oper Res Hindawi Article ID 3451981:1–13

Dutta TK, Nath P, Choudhury KD (2019) A hybrid carbon policy inventory model with emission source-based green investments. Opsearch 57:202–220

Dieguez MS, Fattahi A, Espana GM, Faaij A (2021) Modelling of decarbonisation transition in national integrated energy system with hourly operational resolution. Adv Appl Energy 3:100043

HM Government (2021) Industrial Decarbonisation Strategy. [Online] Available: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/970229/Industrial_Decarbonisation_Strategy_March_2021.pdf

Mashud AHM, Roy D, Daryanto Y, Chakraborty R, Tseng M-L (2021a) A sustainable inventory model with controllable carbon emissions and deterioration and advance payments. J Clean Prod 296(12):126608

Mashud AH, Pervin M, Mishra U, Daryanto Y, Tseng M-L, Ming K, Lim (2021b) A sustainable inventory model with controllable carbon emissions in green-warehouse farms. J Clean Prod 298(5):126777

Mashud AH, Md R, Dipa D, Ali MH (2020) A sustainable inventory model with imperfect products, deterioration, and controllable emissions. Mathematics 8(2049):1–21

Ritha W, Poongodisathiya S (2018) Optimal green inventory model to regulate Eco-friendly Environment. Int J Latest Res Eng Technol 4(5):13–16

Sluisveld MV, Schneider C, Lechtenbohmer S (2020) Implications for modelling decarbonisation. Project NR 730053:1–14

Chapter 10
Cost Analysis of Supply Chain Model for Deteriorating Inventory Items with Shortages in Fuzzy Environment

S. S. Mishra, S. Rawat, S. K. Yadav, and P. P. Mishra

Abstract The paper mainly focuses on cost analysis of supply chain for deteriorating inventory items with shortages in a fuzzy environment by making use of the fuzzy paradigm. The deteriorating items have their limited lives and decrease with respect to the terminating time of the season. The total cost function of the model has been constructed and is further subjected to fuzzy optimization. Optimization of total cost function leads to the development of a system of non-linear equations which is solved and computed by Mathematica software. Thus, finally the optimal total cost of the model has been computed in crisp and fuzzy environments as the most important performance measures of the model. The sensitivity analysis of the model has been discussed to gain a deeper insight into the investigation as well as application of the model.

Keywords Supply chain · Inventory control · Fuzzy optimization · Fuzzy paradigm

10.1 Introduction

Deteriorating items, in general, are such items which are subject to deterioration and devaluation with the increase of time. It may be in different forms such as loss of utility, marginal value because of the introduction of new technology and obsolete fashion etc. Category of items includes edible, non-edible and any kind of usable

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items. Ghare and Schrader (1963) comprehensively considered these relevant issues in their study.

Zimmerman (2001) conceptualized and developed the fuzzy set and suggested their applications. The nitty–gritty of fuzzy arithmetic for theoretical development along with their applications was given by Kaufmann and Gupta (1991). Pathak and Sarkar (2012), Das et al. discussed (2021), suggested in their study the following points (i) deterioration attempted by random Weibull distribution (ii) partial backlogging (iii) ramp-type demand (iv) trade credit financing and inflation under. Model with infinite time-horizon without shortage was investigated by Datta and Pal (1990) and Balkhi and Benkherouf (2004). Dye et al. (2007) discussed a deterministic model with deteriorating inventory which is endowed with replenishment in finite nature, back logging with time and capacity constraints. Sekar and Uthayakumar (2021) discussed a deteriorating inventory model for determining optimal replenishment run time with failure rework and shortage.

Mishra et al. (2004) discussed the computational approach to the supply chain in the fuzzy case with spoilage of items. Patrro et al. (2019) analyzed the EOQ model for fuzzy defective rate with allowable proportional discount and Rani et al. (2019) discussed the fuzzy inventory model for a green supply chain with deteriorating products. Jeyanthi et al. (2022) developed a nonlinear inventory model for deteriorating items solving by using the fractional differential method.

Deterioration is a quintessential property of any production system. The following researchers have reported their significant contributions in this area which include Sharma (1987), Mishra and Mishra (2011) and Pathak and Sarkar (2013), Zadeh and Fax (1965) redefined the concept of fuzzy sets and its application and Kruse et al. (1994) discussed the basic foundations of a fuzzy system having useful applications in the supply chain model. Bellman and Zadeh (1970), Prasath and Seshaiah (2013), Pattnaik (2013) and John Yen Reza Lengari (2005) attempted to investigate novel aspects of the inventory model in supply chain in deterministic as well as fuzzy cases. An economic production lot size model was analyzed by Wee (1993) including deterioration and backordering in a partial manner. Mishra (2017) has attempted to develop an intelligent index applied in a complex situation of supply chain by making use of neural computing. Some more advanced approaches have been used to study inventory flow and control through supply chain by Whitin (1953), Ackoff and Sasieni (1968), Sharma (2012), Crozevic and Puska (2018), Miguel et al. (2020), Nima and Dizbin (2020) and Mishra et al. (2020). An inventory model with deteriorating items and stock-dependent demand in power form was attempted by Mishra and Singh (2012a, b), Singh and Singh (2013), Deb and Chaudhury (1986), and Liu and Liu (2002). Mishra et al. (2017) reported their useful and fundamental results of investigation related to the supply chain model with important characteristics including ramp-type demand and deterioration.

In the above series of works, no cost analysis has been used in a fuzzy environment to investigate such models of supply chain to provide optimal performance measures of the model in fuzzy uncertainty. Keeping this in mind, a supply chain model with special characteristics of deterioration, ramp-type demand and occurrence of shortage has been further subjected to investigation by using a fuzzy paradigm (FP) as a different approach to deal with an environment of uncertainty. FP is defined here by the process of a series of tasks that include modeling, fuzzification, operations, defuzzification, optimization and computing; expert and control systems in order to accomplice a goal of realizing more realistic results and applications with enhanced computational intelligence in terms of the performance measures of the model under consideration.

The model has been solved by Mathematica version 8.0. Total optimum cost of the model in fuzzy and crisp systems have also been computed and a comprehensive comparison has been made in order to reveal the significant effect on performance measure because of a new paradigm of computing to gain an insightful and broader spectrum of the model for its efficient application.

Presentation of a sensitivity analysis in the form of a variational study of parameters of the model has formed the basis to critically examine the variational nature of the model in order to provide a better perspective of the problem. The sections introduction, notations and assumptions; developing crisp model; developing the model and computing its solution by using FP; numerical computing and sensitivity analysis; and conclusion form the paper in completeness.

10.2 Assumptions and Notations

We use the following set of assumptions and notations while developing the present model.

Assumptions

The assumptions are followed in the paper:

i. Inventory model is developed for a single perishable product.
ii. Rate of replenishment is instantaneous, it is uniform with finite size.
iii. Lead time is zero.
iv. Ordered units are fresh.
v. Shortages are allowed, it is fully backlogged having lost sale cost zero.
vi. D(t), demand rate as a function of time t and a ramp-type function having following functional form:

D(t) = D[t − (t − μ)$H(t-\mu)$], , where D (>0) is a constant and $H(t-\mu)$ is the well known Heaviside's function. Figure 10.1 depicts the demand function.

Notations

We have developed an inventory model under the following notations and assumptions.

i. c_h: Holding cost per unit/unit time.
ii. c_d: Cost per deteriorated item.
iii. c_s: Shortage cost per unit/ unit time.

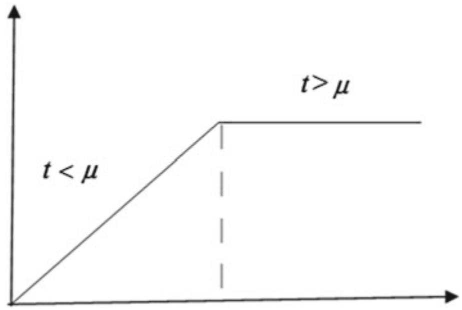


Fig. 10.1 Relationship between ramp-type demand rate and time

iv. C_p: Cost of purchasing per unit/ unit time.
v. μ: Time when demand becomes constant.
vi. t_1: Time when demand becomes zero.
vii. T: Length of per replenishment cycle.
viii. Q: Total amount of inventory purchased at the beginning of per cycle.
ix. S: Initial level of inventory after backlogging demand is fulfilled.
x. W: Total quantity of deteriorated units.
xi. $TAC\,(t_1)$: Total average cost per replenishment cycle.
xii. $T\tilde{A}C\,(t_1)$: Fuzzified Total cost per unit time.

10.3 Development and Analysis of the Model in Crisp Form

(0, μ), (μ, t_1) and (t_1, T) are inventory cycles. In (0, μ), instantaneous refilling takes place at t = 0. At each beginning period, the total inventory purchased or produced is Q and let us S (>0) be as initial stock. At time t = μ, demand becomes constant. The inventory level slowly decreases in the period (0, t_1) and finally drops to zero at t = t_1 because of market demand and deterioration. Shortages occur during the period (t_1, T) which is fully backlogged. The stock gradually goes to max. shortage stock (Q − S) at time t = T. The inventory system behaves through the following Fig. 10.2.

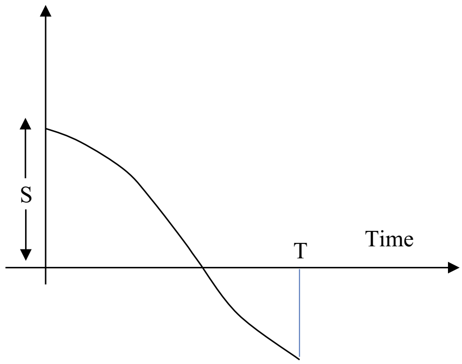
Mishra and Singh (2012) obtained the following as

$$TAC\,(t_1, T) = \frac{D\mu}{T}\left[c_h\left(\frac{t_1^2}{2} - \frac{\mu^2}{6} - \frac{\delta\,\mu^3}{120} + \frac{\delta\,t_1^3}{20}\right) + \frac{c_d\,\delta}{12}\left(t_1^4 - \frac{\mu^4}{5}\right) + \frac{c_s}{2}\,(T - t_1)^2\right] + \frac{D\mu}{T}\left[c_p\left\{t_1\left(1 + \frac{\delta\,t_1^3}{12}\right) - \frac{\mu}{2}\left(1 + \frac{\delta\,\mu^3}{30}\right)\right\}\right],\quad 0 < \delta << 1 \tag{10.1}$$

Optimal t_1 and T are obtained for minimum $TAC\ (T,\ t_1)$ by solving the following equations with having satisfied with embedded sufficient conditions:

$$\frac{\partial\,TAC\,(t, T)}{\partial t_1} = 0 \ \ \&\ \ \frac{\partial\,TAC\,(t, T)}{\partial T} = 0 \tag{10.2}$$

Fig. 10.2 Complete backlogging inventory



$$\frac{\partial TAC\,(t, T)}{\partial t_1} = 0$$

$$\Rightarrow \frac{D\mu}{T}\left[c_h\left(t_1 + \frac{3}{20}\delta t_1^2\right) + \frac{1}{3}c_d\,\delta\,t_1^3 + c_s\,(T - t_1) + c_p\left(1 + \frac{1}{3}\delta\,t_3^1\right)\right] = 0 = 0$$

$$\Rightarrow (c_h + c_s)\,t_1 + \frac{3}{20}c_h\,\delta\,t_1^2 + \frac{1}{3}\left(c_d\,\delta + c_p\,\delta\right)t_1^3 + c_p - c_s\,T = 0 \tag{10.3}$$

$$\frac{\partial TAC\,(t, T)}{\partial T} = 0$$

$$\Rightarrow -\frac{D\mu}{T^2}\left[c_h\left(\frac{t_1^2}{2} - \frac{\tilde{\mu}^2}{6} - \frac{\delta\,\tilde{\mu}^3}{120} + \frac{\delta\,t_1^3}{20}\right) + \frac{\tilde{c}_d\,\delta}{12}\left(t_1^4 - \frac{\tilde{\mu}^4}{5}\right) + \frac{\tilde{c}_s}{2}(T - t_1)^2\right.$$
$$+\,\tilde{c}_p\left\{t_1\left(1 + \frac{\delta\,t_1^3}{12}\right) - \frac{\tilde{\mu}}{2}\left(1 + \frac{\delta\,\tilde{\mu}^3}{30}\right)\right\} + \frac{\partial}{\partial T}\,\frac{D\mu\,c_s}{2T}\,(T - t_1)^2 = 0 \quad .$$

$$\Rightarrow c_h\left(\frac{t_1^2}{2} - \frac{\mu^2}{6} - \frac{\delta\,\mu^3}{120} + \frac{\delta\,t_1^3}{20}\right) + \frac{c_d\,\delta}{12}\left(t_1^4 - \frac{\mu^4}{5}\right) + \frac{c_s}{2}(T - t_1)^2$$
$$+\,c_p\left\{t_1\left(1 + \frac{\delta\,t_1^3}{12}\right) - \frac{\mu}{2}\left(1 + \frac{\delta\,\mu^3}{30}\right)\right\}$$
$$+\,(T - t_1) = 0 \tag{10.4}$$

10.4 Developing Model and Computing Its Solution by Using FP

The following steps are undertaken one by one to accomplice the goal of the paper mainly as the total cost of the model which is to be optimal.

Fuzzification

We consider following fuzzy numbers to fuzzify the model.

Let $\tilde{c}_h = (c_{h_1}, c_{h2}, c_{h3}, c_{h4})$, $\tilde{c}_p = (c_{p_1}, c_{p_2}, c_{p_3}, c_{p_4})$, $\tilde{c}_s = (c_{s_1}, c_{s_2}, c_{s_3}, c_{s_4})$.

$\tilde{\mu} = (\mu_1, \mu_2, \mu_3, \mu_4)$ be trapezoidal fuzzy numbers. These are used to fuzzify the total average cost function as

$$\begin{aligned}
\tilde{T}AC(t_1) =& \frac{D\tilde{\mu}}{T}\left[\tilde{c}_h\left(\frac{t_1^2}{2} - \frac{\tilde{\mu}^2}{6} - \frac{\delta\,\tilde{\mu}^3}{120} + \frac{\delta\, t_1^3}{20}\right) + \frac{\tilde{c}_d\,\delta}{12}\left(t_1^4 - \frac{\tilde{\mu}^4}{5}\right) + \frac{\tilde{c}_s}{2}(T-t_1)^2\right] \\
&+ \frac{D\tilde{\mu}}{T}\left[\tilde{c}_p\left\{t_1\left(1 + \frac{\delta\, t_1^3}{12}\right) - \frac{\tilde{\mu}}{2}\left(1 + \frac{\delta\tilde{\mu}^3}{30}\right)\right\}\right] \\
&= \left(\frac{D\mu_1}{T}, \frac{D\mu_2}{T}, \frac{D\mu_3}{T}, \frac{D\mu_4}{T}\right) \\
&\left[(c_{h1}, c_{h2}, c_{h3}, c_{h4})\left(\frac{t_1^2}{2} - \frac{\mu_1^2}{6} - \frac{\delta\,\mu_1^3}{120} + \frac{\delta\, t_1^3}{20}, \frac{t_1^2}{2} - \frac{\mu_2^2}{6} - \frac{\delta\,\mu_2^3}{120} + \frac{\delta\, t_1^3}{20},\right.\right. \\
&\left.\frac{t_1^2}{2} - \frac{\mu_3^2}{6} - \frac{\delta\,\mu_3^3}{120} + \frac{\delta\, t_1^3}{20}, \frac{t_1^2}{2} - \frac{\mu_4^2}{6} - \frac{\delta\,\mu_4^3}{120} + \frac{\delta\, t_1^3}{20}\right) \\
&+ \left(\frac{c_{d1}\,\delta}{12}, \frac{c_{d2}\,\delta}{12}, \frac{c_{d3}\,\delta}{12}, \frac{c_{d4}\,\delta}{12}\right)\left(t_1^4 - \frac{\mu_1^4}{5}, t_1^4 - \frac{\mu_2^4}{5},\right. \\
&\left. t_1^4 - \frac{\mu_3^4}{5}, t_1^4 - \frac{\mu_4^4}{5}\right) + \left(\frac{c_{s1}}{2}, \frac{c_{s2}}{2}, \frac{c_{s3}}{2}, \frac{c_{s4}}{2}\right)(T-t_1)^2 \\
&+ (c_{p1}, c_{p2}, c_{p3}, c_{p4})\left\{t_1\left(1 + \frac{\delta\, t_1^3}{12}\right) - \right. \\
&\left(\frac{\mu_1}{2}, \frac{\mu_2}{2}, \frac{\mu_3}{2}, \frac{\mu_4}{2}\right) \\
&\left.\left.\left(1 + \frac{\delta\,\mu_1^3}{30}, 1 + \frac{\delta\,\mu_2^3}{30}, 1 + \frac{\delta\,\mu_3^3}{30}, 1 + \frac{\delta\,\mu_4^3}{30}\right)\right\}\right]
\end{aligned}$$

$$\begin{aligned}
&\frac{D}{T}\left[\left(\mu_1 c_{h1}\left(\frac{t_1^2}{2} - \frac{\mu_1^2}{6} - \frac{\delta\,\mu_1^3}{120} + \frac{\delta\, t_1^3}{20}\right),\right.\right. \\
&\mu_2 c_h\left(\frac{t_1^2}{2} - \frac{\mu_2^2}{6} - \frac{\delta\,\mu_2^3}{120} + \frac{\delta\, t_1^3}{20}\right), \\
&\mu_3 c_{h3}\left(\frac{t_1^2}{2} - \frac{\mu_3^2}{6} - \frac{\delta\,\mu_3^3}{120} + \frac{\delta\, t_1^3}{20}\right), \\
&\left.\mu_4 c_4\left(\frac{t_1^2}{2} - \frac{\mu_4^2}{6} - \frac{\delta\,\mu_4^3}{120} + \frac{\delta\, t_1^3}{20}\right)\right) \\
&+ \left(\frac{\mu_1 c_{d1}\delta}{12}\left(t_1^4 - \frac{\mu_1^4}{5}\right), \frac{\mu_2 c_{d2}\delta}{12}\left(t_1^4 - \frac{\mu_2^4}{5}\right), \frac{\mu_3 c_{d3}\delta}{12}\left(t_1^4 - \frac{\mu_3^4}{5}\right), \frac{\mu_4 c_{d4}\delta}{12}\right. \\
&\left.\left(t_1^4 - \frac{\mu_4^4}{5}\right)\right) + \left(\frac{\mu_1\, c_{s1}}{2}(T-t_1)^2, \frac{\mu_2\, c_{s2}}{2}(T-t_1)^2, \frac{\mu_3\, c_{s3}}{2}(T-t_1)^2, \frac{\mu_4\, c_{s4}}{2}(T-t_1)^2\right)
\end{aligned}$$

$$
\begin{aligned}
&+ \big(c_{p1}\mu_1 t_1\left(1+\frac{\delta t_1^3}{12}\right), c_{p1}\mu_1 t_1\left(1+\frac{\delta t_1^3}{12}\right), c_{p1}\mu_1 t_1\left(1+\frac{\delta t_1^3}{12}\right),\\
&c_{p1}\mu_1 t_1\left(1+\frac{\delta t_1^3}{12}\right), c_{p4}\mu_4 t_1\left(1+\frac{\delta t_1^3}{12}\right)\Big)\\
&+\left(c_{p1}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_1}{2}\left(1+\frac{\delta \mu_1^3}{30}\right)\right)\right.\\
&c_{p1}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_2}{2}\left(1+\frac{\delta \mu_2^3}{30}\right)\right),\\
&c_{p2}(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_3}{2}\left(1+\frac{\delta \mu_3^3}{30}\right), c_{p1}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_4}{2}\left(1+\frac{\delta \mu_4^3}{30}\right)\right)\Big)\Big]\\
&= (W, X, Y, Z);
\end{aligned}
$$

where

$$
\begin{aligned}
W =&\frac{D}{T}\Bigg[\Bigg(\mu_1 c_{h1}\left(\frac{t_1^2}{2}-\frac{\mu_1^2}{6}-\frac{\delta \mu_1^3}{120}+\frac{\delta t_1^3}{20}\right)\\
&+\frac{\mu_1 c_{di}\delta}{12}\left(t_1^4-\frac{\mu_1^4}{5}\right)+\frac{\mu_1 c_{s1}}{2}(T-t_1)^2\\
&+c_{p1}\mu_1 t_1\left(1+\frac{\delta t_1^3}{12}\right)+c_{p1}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_1}{2}\left(1+\frac{\delta \mu_1^3}{30}\right)\right)\Bigg]\\
X =&\frac{D}{T}\Bigg[\Bigg(\mu_2 c_{h1}\left(\frac{t_1^2}{2}-\frac{\mu_2^2}{6}-\frac{\delta \mu_2^3}{120}+\frac{\delta t_1^3}{20}\right)+\frac{\mu_2 c_{di}\delta}{12}\left(t_1^4-\frac{\mu_1^4}{5}\right)\\
&+\frac{\mu_2 c_{s1}}{2}(T-t_1)^2+c_{p1}\mu_2 t_1\left(1+\frac{\delta t_1^3}{12}\right)\\
&+c_{p1}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_2}{2}\left(1+\frac{\delta \mu_2^3}{30}\right)\right)\Bigg]\\
Y =&\frac{D}{T}\Bigg[\Bigg(\mu_3 c_{h1}\left(\frac{t_1^2}{2}-\frac{\mu_3^2}{6}-\frac{\delta \mu_3^3}{120}+\frac{\delta t_1^3}{20}\right)+\frac{\mu_3 c_{di}\delta}{12}\left(t_1^4-\frac{\mu_3^4}{5}\right)\\
&+\frac{\mu_3 c_{s1}}{2}(T-t_1)^2+c_{p1}\mu_3 t_1\left(1+\frac{\delta t_1^3}{12}\right)\\
&+c_{p3}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_3}{2}\left(1+\frac{\delta \mu_3^3}{30}\right)\right)\Bigg]\\
Z =&\frac{D}{T}\Bigg[\Bigg(\mu_4 c_{h1}\left(\frac{t_1^2}{2}-\frac{\mu_4^2}{6}-\frac{\delta \mu_4^3}{120}+\frac{\delta t_1^3}{20}\right)\\
&+\frac{\mu_4 c_{di}\delta}{12}\left(t_1^4-\frac{\mu_4^4}{5}\right)+\frac{\mu_4 c_{s1}}{2}(T-t_1)^2+c_{p1}\mu_4 t_1\left(1+\frac{\delta t_1^3}{12}\right)
\end{aligned}
$$

$$+ c_{p4}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_4}{2}\left(1+\frac{\delta \mu_4^3}{30}\right)\right]$$

Operations

Operations consist of various rules of fuzzy logic including sum, product, alpha cuts, left and right alpha cuts etc. which are applied here. The α-cut, $C_L(\alpha)$ & $C_R(\alpha)$ of trapezoidal fuzzy number TAC are given as

$$C_L(\alpha) = W + (X - W)\alpha$$

$$\begin{aligned}
C_L(\alpha) =& \frac{D}{T}\left[\left(\mu_1 c_{h1}\left(\frac{t_1^2}{2}-\frac{\mu_1^2}{6}-\frac{\delta\mu_1^3}{120}+\frac{\delta t_1^3}{20}\right)+\frac{\mu_1 c_{di}\delta}{12}\left(t_1^4-\frac{\mu_1^4}{5}\right)\right.\right.\\
&+\frac{\mu_1 c_{s1}}{2}(T-t_1)^2 c_{p1}\mu_1 t_1\left(1+\frac{\delta t_1^3}{12}\right)+c_{p1}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_1}{2}\left(1+\frac{\delta\mu_1^3}{30}\right)\right]\\
&+\frac{D}{T}\left[\left(\mu_2 c_{h1}\left(\frac{t_1^2}{2}-\frac{\mu_2^2}{6}-\frac{\delta\mu_2^3}{120}+\frac{\delta t_1^3}{20}\right)\mu_1 c_{h1}\left(\frac{t_1}{2}-\frac{\mu_1}{6}-\frac{\delta\mu_1^3}{120}+\frac{\delta t_1^3}{20}\right)\right.\right.\\
&+\frac{\mu_2 c_{d2}\delta}{12}\left(t_1^4-\frac{\mu_2^4}{5}\right)-\frac{\mu_1 c_{di}\delta}{12}\left(t_1^4-\frac{\mu_1^4}{5}\right)+c_{p2}\mu_2 t_1\left(1+\frac{\delta t_1^3}{12}\right)\\
&-c_{p1}\mu_1 t_1\left(1+\frac{\delta t_1^3}{12}\right)+c_{p2}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_2}{2}\left(1+\frac{\delta\mu_2^3}{30}\right)\right.\\
&\left.-c_{p2}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_2}{2}\left(1+\frac{\delta\mu_2^3}{30}\right)\right)\right]\alpha
\end{aligned}$$

and

$$C_L(\alpha) = Z + (Z - Y)\alpha$$

$$\begin{aligned}
C_R(\alpha) =& \frac{D}{T}\left[\left(\mu_4 c_{h4}\left(\frac{t_1^2}{2}-\frac{\mu_4^2}{6}-\frac{\delta\mu_4^3}{120}+\frac{\delta t_1^3}{20}\right)+\frac{\mu_4 c_{d4}\delta}{12}\left(t_1^4-\frac{\mu_4^4}{5}\right)\right.\right.\\
&+\frac{\mu_4 c_{s4}}{2}(T-t_1)^2+c_{p4}\mu_4 t_1\left(1+\frac{\delta t_1^3}{12}\right)\\
&+c_{p4}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_4}{2}\left(1+\frac{\delta\mu_4^3}{30}\right)\right]\\
&+\frac{D}{T}\left[\left(\mu_4 c_{h4}\left(\frac{t_1^2}{2}-\frac{\mu_4^2}{6}-\frac{\delta\mu_4^3}{120}+\frac{\delta t_1^3}{20}\right)\right.\right.\\
&-\mu_3 c_{h3}\left(\frac{t_1^2}{2}-\frac{\mu_3^2}{6}-\frac{\delta\mu_3^3}{120}+\frac{\delta t_1^3}{20}\right)\\
&+\frac{\mu_4 c_{d4}\delta}{12}\left(t_1^4-\frac{\mu_4^4}{5}\right)-\frac{\mu_3 c_{d3}\delta}{12}\left(t_1^4-\frac{\mu_3^4}{5}\right)
\end{aligned}$$

$$+ c_{p4}\mu_4 t_1\left(1+\frac{\delta\, t_1^3}{12}\right) - c_{p3}\mu_3 t_1\left(1+\frac{\delta\, t_1^3}{12}\right)$$
$$+ c_{p4}\left(t_1\left(1+\frac{\delta\, t_1^3}{12}\right) - \frac{\mu_3}{2}\left(1+\frac{\delta\,\mu_3^3}{30}\right)\right.$$
$$\left.- c_{p3}\left(t_1\left(1+\frac{\delta\, t_1^3}{12}\right) - \frac{\mu_3}{2}\left(1+\frac{\delta\,\mu_3^3}{30}\right)\right)\right]\alpha$$

Defuzzification

There are several methods of defuzzification laid down in the literature. Here, we use the "signed distance method" to the fuzzy number $TAC\,(t_1, T)$ which is given by

$$\begin{aligned}
TAC_{sd}\,(t_1, T) =& \frac{1}{2}\int_0^1 [C_L(\alpha) + C_R(\alpha)]\, d\alpha \\
&= \frac{D}{2T}\left[\left(\mu_1 c_{h1}\left(\frac{t_1^2}{2} - \frac{\mu_1^2}{6} - \frac{\delta\,\mu_1^3}{120} + \frac{\delta\, t_1^3}{20}\right)\right.\right. \\
&+ \frac{\mu_1 c_{di}\delta}{12}\left(t_1^4 - \frac{\mu_1^4}{5}\right) + \frac{\mu_1\, c_{s1}}{2}(T - t_1)^2 + c_{p1}\mu_1 t_1\left(1+\frac{\delta\, t_1^3}{12}\right) \\
&\left.+ c_{p1}\left(t_1\left(1+\frac{\delta\, t_1^3}{12}\right) - \frac{\mu_1}{2}\left(1+\frac{\delta\,\mu_1^3}{30}\right)\right)\right] \\
&+ \frac{D}{4T}\left[\left(\mu_2 c_{h1}\left(\frac{t_1^2}{2} - \frac{\mu_2^2}{6} - \frac{\delta\,\mu_2^3}{120} + \frac{\delta\, t_1^3}{20}\right)\right.\right. \\
&- \mu_1 c_{h1}\left(\frac{t_1}{2} - \frac{\mu_1}{6} - \frac{\delta\,\mu_1^3}{120} + \frac{\delta\, t_1^3}{20}\right) \\
&+ \frac{\mu_2 c_{d2}\delta}{12}\left(t_1^4 - \frac{\mu_2^4}{5}\right) - \frac{\mu_1 c_{di}\delta}{12}\left(t_1^4 - \frac{\mu_1^4}{5}\right) \\
&+ c_{p2}\mu_2 t_1\left(1+\frac{\delta\, t_1^3}{12}\right) - c_{p1}\mu_1 t_1\left(1+\frac{\delta\, t_1^3}{12}\right) \\
&+ c_{p2}\left(t_1\left(1+\frac{\delta\, t_1^3}{12}\right) - \frac{\mu_2}{2}\left(1+\frac{\delta\,\mu_2^3}{30}\right)\right) \\
&\left.- c_{p2}\left(t_1\left(1+\frac{\delta\, t_1^3}{12}\right) - \frac{\mu_2}{2}\left(1+\frac{\delta\,\mu_2^3}{30}\right)\right)\right] \\
&+ \frac{D}{T}\left[\left(\mu_4 c_{h4}\left(\frac{t_1^2}{2} - \frac{\mu_4^2}{6} - \frac{\delta\,\mu_4^3}{120} + \frac{\delta\, t_1^3}{20}\right)\right.\right.
\end{aligned}$$

$$+\frac{\mu_4 c_{d4}\delta}{12}\left(t_1^4-\frac{\mu_4^4}{5}\right)+\frac{\mu_4\, c_{s4}}{2}(T-t_1)^2$$
$$+c_{p4}\mu_4 t_1\left(1+\frac{\delta\, t_1^3}{12}\right)+c_{p4}\left(t_1\left(1+\frac{\delta\, t_1^3}{12}\right)\right.$$
$$\left.\left.-\frac{\mu_4}{2}\left(1+\frac{\delta\,\mu_4^3}{30}\right)\right]-\frac{D}{T}\left[\left(\mu_4 c_{h4}\left(\frac{t_1^2}{2}-\frac{\mu_4^2}{6}-\frac{\delta\,\mu_4^3}{120}+\frac{\delta\, t_1^3}{20}\right)\right.\right.$$
$$-\mu_3 c_{h3}\left(\frac{t_1^2}{2}-\frac{\mu_3^2}{6}-\frac{\delta\,\mu_3^3}{120}+\frac{\delta\, t_1^3}{20}\right)-\frac{\mu_4 c_{d4}\delta}{12}\left(t_1^4-\frac{\mu_4^4}{5}\right)$$
$$-\frac{\mu_3 c_{d3}\delta}{12}\left(t_1^4-\frac{\mu_3^4}{5}\right)+c_{p4}\mu_4 t_1\left(1+\frac{\delta\, t_1^3}{12}\right)-c_{p3}\mu_3 t_1\left(1+\frac{\delta\, t_1^3}{12}\right)$$
$$+c_{p4}\left(t_1\left(1+\frac{\delta\, t_1^3}{12}\right)-\frac{\mu_3}{2}\left(1+\frac{\delta\,\mu_3^3}{30}\right)\right)$$
$$\left.-c_{p3}\left(t_1\left(1+\frac{\delta\, t_1^3}{12}\right)-\frac{\mu_3}{2}\left(1+\frac{\delta\,\mu_3^3}{30}\right)\right)\right]$$

10.5 System of Non-Linear Equations and Its Solution

Optimal t_1 and T are obtained for minimum $TAC_{ds}\,(t_1, T)$ by solving the system of non-linear equations which is obtained by the following necessary conditions as. $\frac{\partial TAC_{sd}\,(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TAC_{sd}\,(t_1,T)}{\partial T}=0$, along with sufficient conditions as. $\left(\frac{\partial^2 T\tilde{A}C(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 T\tilde{A}C(t_1,T)}{\partial T^2}\right)-\left\{\frac{\partial^2 T\tilde{A}C(t_1,T)}{\partial t\,\partial T}\right\}^2>0$ and $\left(\frac{\partial^2 T\tilde{A}C(t_1,T)}{\partial t_1^2}\right)<0$

By the first condition, we get as

$$\frac{D}{2T}\left[\left(\mu_1 c_{h1}\left(\frac{t_1^2}{2}-\frac{\mu_1^2}{6}-\frac{\delta\,\mu_1^3}{120}+\frac{\delta\, t_1^3}{20}\right)+\frac{\mu_1 c_{di}\delta}{12}\left(t_1^4-\frac{\mu_1^4}{5}\right)\right.\right.$$
$$+\frac{\mu_1\, c_{s1}}{2}(T-t_1)^2+c_{p1}\mu_1 t_1\left(1+\frac{\delta\, t_1^3}{12}\right)$$
$$\left.+c_{p1}\left(t_1\left(1+\frac{\delta\, t_1^3}{12}\right)-\frac{\mu_1}{2}\left(1+\frac{\delta\,\mu_1^3}{30}\right)\right)\right]$$
$$+\frac{D}{4T}\left[\left(\mu_2 c_{h2}\left(\frac{t_1^2}{2}-\frac{\mu_2^2}{6}-\frac{\delta\,\mu_2^3}{120}+\frac{\delta\, t_1^3}{20}\right)\right.\right.$$
$$-\mu_1 c_{h1}\left(\frac{t_1}{2}-\frac{\mu_1}{6}-\frac{\delta\,\mu_1^3}{120}+\frac{\delta\, t_1^3}{20}\right)$$

$$+\frac{\mu_2 c_{d2}\delta}{12}\left(t_1^4-\frac{\mu_2^4}{5}\right)-\frac{\mu_1 c_{di}\delta}{12}\left(t_1^4-\frac{\mu_1^4}{5}\right)$$
$$+c_{p2}\mu_2 t_1\left(1+\frac{\delta t_1^3}{12}\right)-c_{p1}\mu_1 t_1\left(1+\frac{\delta t_1^3}{12}\right)$$
$$+c_{p2}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_2}{2}\left(1+\frac{\delta \mu_2^3}{30}\right)\right.$$
$$\left.-c_{p1}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_1}{2}\left(1+\frac{\delta \mu_1^3}{30}\right)\right)\right]$$
$$+\frac{D}{T}\left[\left(\mu_4 c_{h4}\left(\frac{t_1^2}{2}-\frac{\mu_4^2}{6}-\frac{\delta \mu_4^3}{120}+\frac{\delta t_1^3}{20}\right)\right.\right.$$
$$+\frac{\mu_4 c_{d4}\delta}{12}\left(t_1^4-\frac{\mu_4^4}{5}\right)+\frac{\mu_4 c_{s4}}{2}(T-t_1)^2$$
$$+c_{p4}\mu_4 t_1\left(1+\frac{\delta t_1^3}{12}\right)+t_1\left(1+\frac{\delta t_1^3}{12}\right)+c_{p4}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)\right.$$
$$\left.\left.-\frac{\mu_4}{2}\left(1+\frac{\delta \mu_4^3}{30}\right)\right]-\frac{D}{T}\left[\left(\mu_4 c_{h4}\left(\frac{t_1^2}{2}-\frac{\mu_4^2}{6}-\frac{\delta \mu_4^3}{120}+\frac{\delta t_1^3}{20}\right)\right.\right.$$
$$-\mu_3 c_{h3}\left(\frac{t_1^2}{2}-\frac{\mu_3^2}{6}-\frac{\delta \mu_3^3}{120}+\frac{\delta t_1^3}{20}\right)$$
$$-\frac{\mu_4 c_{d4}\delta}{12}\left(t_1^4-\frac{\mu_4^4}{5}\right)-\frac{\mu_3 c_{d3}\delta}{12}\left(t_1^4-\frac{\mu_3^4}{5}\right)$$
$$+c_{p4}\mu_4 t_1\left(1+\frac{\delta t_1^3}{12}\right)-c_{p3}\mu_3 t_1\left(1+\frac{\delta t_1^3}{12}\right)$$
$$-c_{p4}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_3}{2}\left(1+\frac{\delta \mu_3^3}{30}\right)\right)$$
$$\left.-c_{p3}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_3}{2}\left(1+\frac{\delta \mu_3^3}{30}\right)\right)\right]=0$$

$$2\left[\mu_1 c_{h1}\left(t_1+\delta t_1^2\right)+\frac{c_{d1}\delta t_1^3}{3}+c_{s1}(T-t_1)\right.$$
$$\left.+c_{p1}\mu_1\left(1+\frac{\delta t_1^3}{3}\right)\right]+\mu_2 c_{h2}\left(t_1+\delta t_1^2\right)-\mu_1 c_{h1}\left(t_1+\delta t_1^2\right)$$
$$+\frac{c_{d2}\delta t_1^3}{3}-\frac{c_{d1}\delta t_1^3}{3}+c_{s2}(T-t_1)-c_{s1}(T-t_1)$$
$$+c_{p2}\left(1+\frac{\delta t_1^3}{3}\right)-c_{p1}\left(1+\frac{\delta t_1^3}{3}\right)+$$

$$2\,[\mu_4\,c_{h4} + (t_1 +\ \delta\,t_1^2)\frac{c_{d4}\,\delta\,t_1^3}{3}$$
$$+\ c_{s4}(T - t_1) +\ c_{p4}\mu_1\left(1 +\ \frac{\delta\,t_1^3}{3}\right)\Bigg]$$

$$-\mu_4\,c_{h4}\left(t_1 + \delta\,t_1^2\right) - \mu_3\,c_{h3}\left(t_1 + \delta\,t_1^2\right) - \frac{c_{d2}\,\delta\,t_1^3}{3} - \frac{c_{d1}\,\delta\,t_1^3}{3}$$

$$-c_{s4}\,(T - t_1) + c_{s3}\,(T - t_1)\ - c_{p4}\left(1 + \frac{\delta t_1^3}{3}\right) + c_{p3}\left(1 + \frac{\delta t_1^3}{3}\right) =\ 0$$

$$\mu_1\,c_{h1}\big(t_1 +\ \delta\,t_1^2\big)\ +\ \mu_2\,c_{h2}\big(t_1 +\ \delta\,t_1^2\big)\ +\ \frac{c_{d1}\,\delta\,t_1^3}{3} + \frac{c_{d2}\,\delta\,t_1^3}{3}$$
$$+\ c_{s1}(T - t_1)\ +\ c_{s2}(T - t_1)\ + c_{p2}\left(1 + \frac{\delta t_1^3}{3}\right)$$

$$+\,c_{p1}\left(1 + \frac{\delta t_1^3}{3}\right) + \frac{c_{d4}\,\delta\,t_1^3}{3} + \frac{c_{d3}\,\delta\,t_1^3}{3} + c_{s4}(T - t_1)\ +\ c_{s3}(T - t_1)$$
$$+\,c_{s4}(T - t_1) + c_{s3}(T - t_1)\ + c_{p4}\left(1 + \frac{\delta t_1^3}{3}\right) + c_{p3}\left(1 + \frac{\delta t_1^3}{3}\right) +$$

$$\mu_1\,c_{h1}\big(t_1 +\ \delta\,t_1^2\big)\ +\ \mu_2\,c_{h2}\big(t_1 +\ \delta\,t_1^2\big) = 0.$$

Second condition gives us

$$-\frac{D}{2T^2}\Bigg[\left(\mu_1 c_{h1}\left(\frac{t_1^2}{2} - \frac{\mu_1^2}{6} - \frac{\delta\,\mu_1^3}{120} + \frac{\delta\,t_1^3}{20}\right) + \frac{\mu_1 c_{di}\,\delta}{12}\left(t_1^4 - \frac{\mu_1^4}{5}\right)\right.$$
$$+\ \frac{\mu_1\,c_{s1}}{2}(T - t_1)^2 + c_{p1}\mu_1 t_1\left(1 +\ \frac{\delta\,t_1^3}{12}\right)$$

$$+\,c_{p1}\left(t_1\left(1 +\ \frac{\delta\,t_1^3}{12}\right) - \frac{\mu_1}{2}\left(1 +\ \frac{\delta\,\mu_1^3}{30}\right)\right]\Bigg] + \frac{D}{2T}\,[c_{s1}\,(T - t_1)]$$
$$-\frac{D}{4T^2}\Bigg[\left(\mu_2 c_{h2}\left(\frac{t_1^2}{2} - \frac{\mu_2^2}{6} - \frac{\delta\,\mu_2^3}{120} + \frac{\delta\,t_1^3}{20}\right)\right.$$

$$-\ \mu_1 c_{h1}\left(\frac{t_1}{2} - \frac{\mu_1}{6} - \frac{\delta\,\mu_1^3}{120} + \frac{\delta\,t_1^3}{20}\right) + \frac{\mu_2 c_{d2}\delta}{12}\left(t_1^4 - \frac{\mu_2^4}{5}\right)$$
$$-\ \frac{\mu_1 c_{di}\,\delta}{12}\left(t_1^4 - \frac{\mu_1^4}{5}\right) + \frac{\mu_2\,c_{s2}}{2}(T - t_1)^2 - \frac{c_{s1}\mu_1}{2}$$

$$+\ (T - t_1)^2 +\ c_{p2}\left(t_1\left(1 +\ \frac{\delta\,t_1^3}{12}\right) - \frac{\mu_2}{2}\left(1 +\ \frac{\delta\,\mu_2^3}{30}\right)\right)$$

$$- c_{p1}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_1}{2}\left(1+\frac{\delta \mu_1^3}{30}\right)\right]$$

$$\frac{D}{4T}[(c_{s2}(T-t_1)\ - c_{s1}(T-t_1)]$$

$$-\frac{D}{2T^2}\left[\left(\mu_4 c_{h4}\left(\frac{t_1^2}{2}-\frac{\mu_4^2}{6}-\frac{\delta \mu_4^3}{120}+\frac{\delta t_1^3}{20}\right)\right.\right.$$

$$+\frac{\mu_4 c_{d4}\delta}{12}\left(t_1^4-\frac{\mu_4^4}{5}\right)+$$

$$\frac{\mu_4\, c_{s4}}{2}(T-t_1)^2 + + c_{p2}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_2}{2}\left(1+\frac{\delta \mu_2^3}{30}\right)\right]$$

$$+\frac{D}{2T}\left[c_{s1}\left(T-t_1\right)\right]+\frac{D}{4T^2}$$

$$\left[\left(\mu_4 c_{h4}\left(\frac{t_1^2}{2}-\frac{\mu_4^2}{6}-\frac{\delta \mu_4^3}{120}+\frac{\delta t_1^3}{20}\right)-\mu_3 c_{h3}\left(\frac{t_1}{2}-\frac{\mu_3}{6}-\frac{\delta \mu_3^3}{120}+\frac{\delta t_1^3}{20}\right)\right.\right.$$

$$+\frac{\mu_4 c_{d4}\delta}{12}\left(t_1^4-\frac{\mu_4^4}{5}\right)$$

$$-\frac{\mu_3 c_{d3}\delta}{12}\left(t_1^4-\frac{\mu_3^4}{5}\right)+\frac{\mu_4\, c_{s4}}{2}(T-t_1)^2-\frac{c_{s3}\mu_3}{2}$$

$$+\ c_{p4}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_4}{2}\left(1+\frac{\delta \mu_4^3}{30}\right)\right.$$

$$-c_{p3}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_3}{2}\left(1+\frac{\delta \mu_3^3}{30}\right)\right]$$

$$-\frac{D}{4T}[(c_{s4}(T-t_1)\ - c_{s3}(T-t_1)] = 0$$

$$\mu_1 c_{h1}\left(\frac{t_1^2}{2}-\frac{\mu_1^2}{6}-\frac{\delta \mu_1^3}{120}+\frac{\delta t_1^3}{20}\right)+\frac{\mu_1 c_{di}\delta}{12}\left(t_1^4-\frac{\mu_1^4}{5}\right)$$

$$+\mu_1\, c_{s1}\left(T-t_1\right)^2 + c_{p1}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)\right.$$

$$-\frac{\mu_1}{2}\left(1+\frac{\delta \mu_1^3}{30}\right)-\ T\, c_{s1}(T-t_1)$$

$$+\mu_2 c_{h2}\left(\frac{t_1^2}{2}-\frac{\mu_2^2}{6}-\frac{\delta \mu_2^3}{120}+\frac{\delta t_1^3}{20}\right)+\frac{\mu_2\, c_{d2}\delta}{12}\left(t_1^4-\frac{\mu_2^4}{5}\right)$$

$$+\mu_2\, c_{s2}\left(T-t_1\right)^2 + c_{p2}\left(t_1\left(1+\frac{\delta t_1^3}{12}\right)-\frac{\mu_2}{2}\left(1+\frac{\delta \mu_2^3}{30}\right)\right.$$

$$- T\, c_{s2}(T - t_1) + \mu_4 c_{h4} \left(\frac{t_1^2}{2} - \frac{\mu_4^2}{16} - \frac{\delta\, \mu_4^3}{120} + \frac{\delta\, t_1^3}{20} \right)$$

$$+ \frac{\mu_4\, c_{d4} \delta}{12} \left(t_1^4 - \frac{\mu_4^4}{5} \right) + \mu_4\, c_{s4}\, (T - t_1)^2$$

$$+ c_{p4} \left(t_1 \left(1 + \frac{\delta\, t_1^3}{12} \right) - \frac{\mu_4}{2} \left(1 + \frac{\delta\, \mu_4^3}{30} \right) \right.$$

$$- T\, c_{s4}(T - t_1) + \mu_3 c_{h3} \left(\frac{t_1^2}{2} - \frac{\mu_3^2}{6} - \frac{\delta\, \mu_3^3}{120} + \frac{\delta\, t_1^3}{20} \right)$$

$$+ \frac{\mu_3 c_{d3} \delta}{12} \left(t_1^4 - \frac{\mu_3^4}{5} \right) + \mu_3\, c_{s3}\, (T - t_1)^2 +$$

$$c_{p3} \left(t_1 \left(1 + \frac{\delta\, t_1^3}{12} \right) - \frac{\mu_3}{2} \left(1 + \frac{\delta\, \mu_3^3}{30} \right) - T\, c_{s3}(T - t_1) \right. = 0.$$

10.6 Numerical Computing and Sensitivity Analysis

Here, a numerical computing of the results will form a basis of sensitivity of a model. The sensitivity analysis is an important aspect of supply chain model and its flow of items which primarily seeks to study variations of various parameters involved therein with respect to the objective performance measure of the model. This analysis envisions in itself the judgment of validity and feasibility of the model by using variational data for the validation approach to the model. Tables 10.2, 10.3, 10.4, 10.5 and 10.6 display various numerically computed parameters and corresponding objective performance measures of the model. If we look at Tables 10.2, 10.3, 10.4, 10.5 and 10.6, both kinds of correlations between the first half cycle time (FHCT) and total optimum cost (TOC); total cycle time (TCT) and total optimum cost (TOC) exist. FHCT and TOC have a negative correlation whereas TCT and TOC have a positive correlation. Interesting aspect in this case is the negative trend when TOC increases as FHCT diminishes and tends to be absent; this is because deterioration takes place in this cycle which happens to add to the total cost of the system. From Tables 10.4, 10.5 and 10.6, the correlations between shortage cost and total optimal cost; purchasing cost and total optimal cost; holding cost and total optimal cost are positive which are important to analyze the validity of the model (Fig. 10.3).

Crisp Model

$c_h = 5/unit/year, c_s = 15/unit/year, c_p = 100/unit/year, c_d = 3/unit/year, \delta = 0.01, \mu = 0.15, D = 1000$ The solution of crisp model is TAC = Rs 404.3642, t = 0.4532, $T = 1.6436$.

Fuzzy Model: The convexity is given in fuzzy environment (Fig. 10.4).

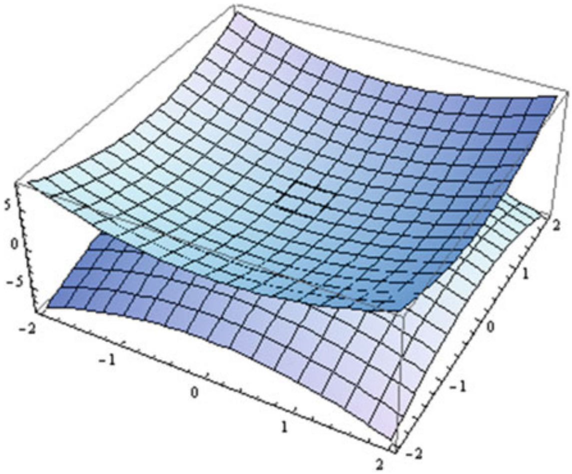


Fig. 10.3 Convexity of TAC (t1, T) for Crisp Model

Table 10.1 Comparison of Results is given in two environments

Model	Optimal t_1	Optimal T	Optimal c_s	Optimal TAC
CRISP	0.4532	1.6436	59.4336	404.3642
FUZZY	0.6605	1.1687	68.2235	415.3136

Table 10.2 Sensitivity Analysis on parameter μ

Defuzzified value of μ (per unit per year)	Fuzzified value of parameter	t_1 (year)	T (year)	$T\tilde{A}C$ (t, T)
90	(60, 80, 100, 120)	0.9236	1.003	402.035
100	(70, 90, 110, 130)	0.6790	1.5479	419.824
110	(80, 100, 120, 140)	0.6605	1.8067	425.133
120	(90, 110, 130, 150)	0.6352	1.9303	433.392
130	(100, 120, 140, 160)	0.6126	1.9484	456.634

Table 10.3 Sensitivity Analysis on parameter θ

Defuzzified value of θ *(per unit per year)*	Fuzzified value of parameter	t_1(year)	T (year)	$T\tilde{A}C$ (t, T) (Rs.)
0.006	(0.001, 0.004, 0.008, 0.012)	0.5645	1.6832	412.411
0.008	(0.002, 0.006, 0.010, 0.014)	0.6387	1.6530	416.572
0.010	(0.004, 0.008, 0.012, 0.016)	0.6152	1.7767	421.553
0.012	(0.006, 0.010, 0.014, 0.018)	0.6625	1.7451	427.717
0.014	(0.008, 0.012, 0.016, 0.020)	0.6791	1.9660	433.809

Table 10.4 Sensitivity Analysis on parameter C_s

Defuzzified value of C_s (per unit per year)	Fuzzified value of parameter	t_1(year)	T (year)	$T\tilde{A}C$ (t, T)
11	(8, 10, 12, 14)	0.6908	1.7092	410.162
13	(10, 12, 14, 16)	0.6691	1.7287	414.331
15	(12, 14, 16, 18)	0.6266	1.7567	417.553
17	(14, 16, 18, 20)	0.6093	1.7732	421.504
19	(16, 18, 20, 22)	0.7121	1.8737	424.745

Table 10.5 Sensitivity Analysis on parameter Cp

Defuzzified value of Cp (per unit per year)	Fuzzified value of parameter	t_1(year)	T (year)	$T\tilde{A}C$ (t, T)
16	(10, 14, 18, 22)	0.6131	1.8951	415.009
18	(12, 16, 20, 24)	0.6169	1.8956	418.165
20	(14, 18, 22, 26)	0.6589	1.8923	422.313
22	(16, 20, 24, 28)	0.6767	1.9389	426.455
24	(18, 22, 26, 30)	0.6776	1.9434	429.588

Table 10.6 Sensitivity Analysis on parameter C_h

Defuzzified value of C_h (per unit per year)	Fuzzified value of parameter	t_1(year)	T (year)	$T\tilde{A}C$ (t, T)
3	(0, 2, 4, 6)	0.7113	1.0552	414.609
4	(1, 3, 5, 7)	0.7444	1.2596	425.357
5	(2, 4, 6, 8)	0.7578	1.2998	437.658
6	(3, 5, 7, 9)	0.8651	1.4091	444.817
7	(4, 6, 8, 10)	0.8945	1.4601	451.063

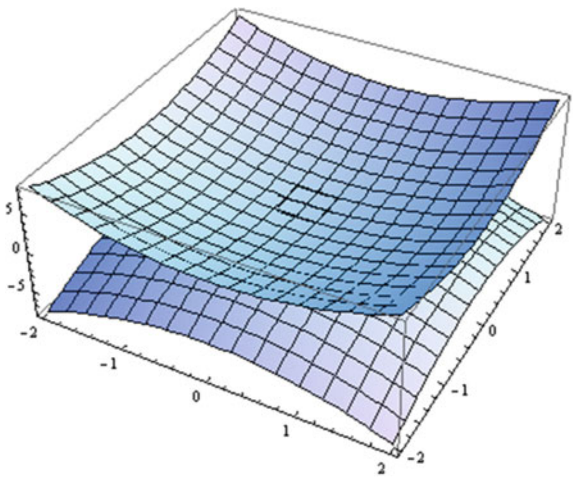


Fig. 10.4 Convexity of TAC (t_1, T) for Fuzzy Model

10.7 Conclusions

Optimization of the supply chain model nowadays profusely attracts the attention of researchers engaged in the field. The optimization analysts seek to know an advanced technique to optimize the inventory flow in supply chain. Upon revisiting the sequence of techniques involving the elements of certainty and uncertainty such as dynamic programming, heuristic and statistical, classical and fuzzy paradigm etc., domain experts' discriminating intellects make a final decision to prefer the fuzzy paradigm to sensibly handle the deterministic uncertainty of various system data connected with the model under reference. After applying fuzzy and crisp approach paradigms and subjecting the model with differential properties to the computational approach of optimization, Table 10.1 presents the comparative optimal values of total optimal cost as the main performance measure of the model. Tables 10.2, 10.3, 10.4, 10.5 and 10.6 intend to present various optimal values of the model in a fuzzy environment. Efficiency measures in terms of optimal performance measures by applying different methods of defuzzification will be intended to form the future plan of action for investigating the problem under discussion. Intuitionistic fuzzy paradigm as an improved approach will also be suggested to compute the optimal performance measure in a complex situation of supply chain model in which queued customers are classified to gain insight into a much closer realistic result as a decision indicator to the problem under investigation.

References

Ackoff RL, Sasieni MW (1968) Fundamentals of operations research. Wiley Press

Balkhi TZ, Benkherouf L (2004) On an inventory model for deteriorating items with stock dependent time varying demand rates. Comput Oper Res 31:223–240

Belloman RE, Zadeh LA (1970) Decision-making in a fuzzy environment. Manag Sci 17(4):B-141-B-164

Das SC, Manna AK, Rahman S, Shaikh AA, Bhunia AK (2021) An inventory model for non-instantaneous deteriorating items with preservation technology and multiple credit periods-based trade credit financing via particle swarm optimization. Soft Comput 25:5365–5384

Datta TK, Pal AK (1990) A note on an inventory model with inventory-level-dependent demand rate. J Oper Res Soc 41(10):971–975

Deb M, Chaudhuri KS (1986) An EOQ model for items with finite rate of production and variable rate of deterioration. Opsearch 23:175–181

Dye CY, Ouyang LY, Hsieh TP (2007) Deterministic inventory model for deteriorating items with capacity constraint and time proportional backlogging rate. Eur J Oper Res 178(3):789–807

Ghare PM, Schrader GH (1963) A model for exponential decaying inventory systems. Int J Prod Econ 21:449–460

Jeyanthi MV, Uduman PS, Dowlath F (2022) Solving nonlinear inventory model for deteriorating items using fractional differential method. J Environ Account Manag 10(1):31–38

John Yen Reza Lengari (2005) Fuzzy logic intelligence. Pearson Prentice Hall, Pearson education Inc., Control an Information
Kaufmann A, Gupta MM (1991) Introduction to fuzzy arithmetic theory and applications. Van Nostrand Reinhold, New York
Kozarević S, Puška A (2018) Use of fuzzy logic for measuring practices and performances of supply chain. Opera Res Perspect 5:150–160
Kruse K, Gebhardt JE, Klowon F (1994) Foundations of fuzzy systems, 1st edn. Wiley, New York. ISBN: 047194243X
Liu B, Liu YK (2002) Expected value of fuzzy variable and fuzzy expected value model. IEEE Trans Fuzzy Syst 10(4):445–450
Miguel J, Carlos O-P, Ruben Y-P (2020) Inventory and fleet purchase decisions under a sustainable regulatory environment. J Supply Chain Forum 21(1):53–65
Mishra SS (2017) Developing intelligent index for fuzzified inventory of supply chain by using neural computing. Int J Oper Res Inderscience 29(4):460–477
Mishra SS, Mishra PP (2011) A (Q, R) model for fuzzified deterioration under cobweb phenomenon and permissible delay in payment. Comput Math Appl 61(4):921–932
Mishra SS, Singh PK (2012) Computational approach to an inventory model with ramp- type demand and linear deterioration. Int J Oper Res Inderscience 15(3):337–357
Mishra SS, Pandey H, Singh RS (2004) A fuzzified deteriorating inventory model with breakdown of machine and shortage cost. Int J Math Sci 3:241–255
Mishra SS, Sharma DK, Yadav SK, Rawat S (2017) Computational approach to fuzzified profit optimization of inventory flow in supply chain with deteriorating items. Int J Oper Res Inderscience 30(1):83–98
Mishra PP, Poongodi T, Yadav SK, Mishra SS (2020) Algorithmic approach to time-cost analysis of queued commodity flowing through critical path. Int J Math Oper Res 18(2):169–185
Mishra SS, Singh PK (2012a) Computing of total optimal cost of an EOQ model with quadratic deterioration and occurrence of shortages. Int J Manag Sci Eng Manag, World Academic Press (Taylor and Francis), ISSN 1750–9653, 7(4): 243–252
Pathak S, Sarkar (Mondol) S (2012b) A fuzzy EOQ inventory model for random Weibull deterioration with Ramp-Type demand, parial backlogging and inflation under trade credit financing. Int J Res Commer IT Manag 2(2):8–18
Pathak S, Sarkar (Mondol) S (2013) Fuzzy production inventory model for deteriorating items with shortage under the effect of the time dependent learning and forgetting: a possibility/necessity approach. Opsearch 50(2):149–181
Patrro R, Nayak MM, Acharya M (2019) An EOQ model for fuzzy defective rate with allowable proportional discount. Opsearch 56(1):191–215
Pattnaik M (2013) Optimization in an instantaneous economic order quantity (EOQ) model incorporated with promotional effort cost, variable ordering cost and units lost due to deterioration. Uncertain Supply Chain Manag 1:57–66
Prasath GM, Arun, Seshaiah CV (2013) A fuzzy production-distribution Inventory Model with Shortage. Int J Innov Appl Stud 2:131–137
Rani S, Ali R, Agarwal A (2019) Fuzzy inventory model for deteriorating items in a green supply chain carbon concerned demand. Opsearch 56(1):191–215
Sekar T, Uthayakumar R (2021) A deteriorating inventory model for determining optimal replenishment run time with failure rework and shortage. Int J Syst Sci Oper Logisti 8(1):69–86
Sharma KVS (1987) A deterministic order-level inventory model for deteriorating items with two facilities. Eur J Oper Res 29:70–72
Sharma S (2012) Supply chain management. Oxford University Press, USA
Singh C, Singh SR (2013) Optimal ordering policy for deteriorating items with power-from stock dependent demand under two-warehouse storage facility. Opsearch 50(2):182–196
Wee HM (1993) Economic Production lot size model for deteriorating items with partial backordering. Comput Ind Eng 24:449–458
Whitin TM (1953) Theory of inventory management. Princeton University Press, Princeton, N.J.

Zadeh LA (1965) Fuzzy sets and systems. In: Fox J (ed) System theory. Polytechnic Press, Brooklyn, NY, pp 29–39
Zimmermann H-J (2001) Fuzzy set theory and its applications. Springer, New York

Chapter 11
Multi-echelon Inventory Planning in Supply Chain

Shalini Rathore, Madhu Jain, and Rakesh Kumar Meena

Abstract This chapter deals with an inventory problem in a multi-echelon set up for scrutinizing the execution of a single manufacturer and multiple retailer's supply chain. I identical products are supplied to the K identical retailer through a single manufacturer. The demand for the individual product for different retailers is independent, identical, and generally distributed. The manufacturer waits for q orders to be accumulated and then produces multiple products in lots of q units. The impact of the product type on the inventory cost and the impact of the lot size on the manufacturer's lead time are studied. This multi-echelon inventory model can explore the impact of various factors like demand rate, product variety, numbers of retailers, setup time, and processing time on total inventory cost.

Keywords Multi-echelon · Supply chain · Batch order · Inventory model · Setup time

11.1 Introduction

Nowadays, the supply chain is considered to be a web of different activities and planning in different areas. A series of different steps are found between a company and its supplier that performs the manufacture of raw material into a specific product or service and finally distributes these products or services to customers. Broad areas for supply chain planning include the supply of raw materials, vendors, retailers, logistic

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companies, warehouses, distribution channel partners, and many others. Lots of planning is required for the smooth running of the supply chain that focuses on planning supply, production, capacity, distribution, and inventory. Supply chain management covers the overall management and planning of entire important activities from manufacturing to distribution, including coordination with suppliers, customers, intermediaries, logistics management activities, third-party service providers, manufacturing operations, channel partners, sourcing and procurement services, conversion, etc. Supply chain management (SCM) ensures the coordination between customers' demand and supply management within and across organizations. It is an integrated function that moves the coordination of operations with and across product design, sales, marketing, finance, and information technology.

Planning of inventory is the most essential part of SCM; as such different kinds of inventories need further attention. A deteriorating item like eatable products, cosmetics, medicines, seasonal items etc. need to be stored in a specific environment, whereas large expensive item needs more space. So, the inventory optimization provides the idea for the safety stock as per the customers' demand. The focus of the supply chain tends to be on inventory optimization with consideration of all important aspects. Management decisions arise out of the productive nature of the organizations. In a general sense, it could be suggested that improving productivity is the basic motivation behind all management decision-making. Therefore, productivity can be considered an inclusive concept of the relation between the outputs and the inputs of a productive system.

11.2 Literature Survey

Past research work on inventory management systems consists of deteriorating items, mainly consumable items that need repetitive review on stock level because money is concerned with it. Some of the previous studies include the works of Sherbrooke (1968) and Moinzadeh and Lee (1986). Van Beek (1981) developed an inventory model by assuming the order size fixed to Q and inventory position level b; whenever the inventory level is down below b, they reorder amount Q. They have used the replenishment cycle approach in their study. It is generally expected that an incoming lot of defective goods may be repairable or non-repairable in a lot, which means some parts of the lot are recoverable, and the remaining is not recovered.

Many authors have done substantial works for multi-echelon inventory models with recoverable items. Simon (1971) investigated the model containing the Poisson distributed demand and deterministic lead time for low demand items. This inventory model was again extended by Shanker (1981) with the compound Poisson case. Many standard textbooks (cf. Hadley and Whiten 1963; Silver and Peterson 1985) cover the elementary concepts of the (R, S) inventory problems; these studies mainly emphasize the analysis of optimal values for R and the computation of specific performance measures.

Many organizations use the management of multiple distribution channels to find ways to attract maximum customers from various channels. Many organizations use multiple channels to allocate their goods in different stores through online orders and social media to get maximum customers from different market segments. Alptekinoğlu and Tang (2005) presented an inventory model for analyzing the supply chain with stochastic demand.

Bhattacharya (2005) studied a two-item inventory model for deteriorating items where the demand rate is considered to be linear stock dependent that gave a new direction to retail marketing due the fact that there may be increase in the demand of goods by attractively displaying them. Lin and Lin (2005) proposed a model to study the strategy which dealt with the production size inventory policies and order policies. The two-echelon inventory model combines a periodical commodity's manufacturing channels and market. McTavish and Goyal (1989) considered an integrated marketing and production decision planning. Rao (1990) suggested the joint determination of optimal inventory and marketing policy. An approach toward developing an acquisition policy for a single-item and multi-supplier was proposed by Rosenblatt et al. (1998).

Retail inventory planning has been a complex task, even under the best condition. This process is incredibly challenging due to its vary nature. Since the firm must often engage with an extreme number of unique products, their sales level may be prolonged (Aggarwal 1983). The inventory planning circumstances of low usage goods are present in many sectors including garment outlets, gift and handicraft handlooms, stationeries, motor parts, hospitals, etc. The inventory control of such items was considered by Cohen et al. (1986).

The conventional inventory control judgments regarding the timing and sizing of the order have been studied by Feeney and Sherbrooke (1966) to propose the (S-1, S) policy with a compound Poisson demand rate. Archibald and Silver (1978) analyzed the (s, S) model by including the compound Poisson demand. Chatfield and Goodhardt (1973) studied a inventory model having the inter-purchase times governed by Erlang distribution. Graves (1985) used one-for-one replenishment policy for a repairable item in his multi-echelon inventory model. Thonemann and Bradley (2002) analyzed the supply chain performance by using different products of different retailers. They investigated the SCM having many retailers and a single manufacturer. An organized literature review on the significance of the SCM coordination has been presented by Arshinder et al. (2008).

Three different types of channel structures viz. decentralized, semi-integrated and integrated have been studied by Huang and Huang (2010). Many authors study the uncertainty of demand in a multi-echelon supply chain model. (He and Zhao 2012; Hu et al. 2013; Wang 2009). Again, for the stochastic demand, a single supplier and manufacturer supply chain management problem of production and procurement was considered by Xu (2010). The supply chain management from the vendor side for a multi-retailer was studied by Mateen et al. (2015) under stochastic demand. Almeder et al. (2015) considered lead time in multi-level business problems. A decentralized approach for the SCM having a single manufacturer which has a random yield and one retailer facing uncertain demand, has been studied by Yin and Ma (2015). Zhao et al. (2016) formulated the integrated multi-stage supply chain with optimal production-inventory policy with time-varing demand.

Dai et al. (2017) considered a multi-echelon inventory system for a supply chain having three types of demand rates. The impact of centralization on the supply chain for the deteriorating item was explored by Hosseinifard and Abbasi (2018). Qu et al. (2018) studied the incentive problems associated with inventory investment in a supply chain. A three-echelon model for a supply chain with multiple items and multiple clients was presented by Salas Navarro et al. (2020). The supply chain and transportation problems have been studied by Nasseri and Bavandi (2020) by using a fuzzy programming approach for the decision-making support. Halat et al. (2021) presented a multi-echelon supply chain in which they considered four structures concerning the decision-making of supply chain members. Two inventory models with starting shortages and without shortages for perishable products in a supply chain are proposed by Dai et al. (2022).

The contents of the chapter are organized as follows: Sect. 11.3 presents a multi-echelon supply chain inventory model by defining the requisite assumptions and notations used. The expected lead time is outlined in Sect. 11.4. In Sect. 11.5, we find the optimal batch size. In Sect. 11.6, some special cases are taken into consideration. Section 11.7 is devoted to find the cost minimization problem. To investigate the impact of various parameters, numerical results are displayed graphically in Sect. 11.8. We review the findings and explore how the model might be improved in Sect. 11.9. Some promising topics for future study are also suggested.

11.3 Model Description

We consider a multi-echelon model for a supply chain having a single manufacturer and multiple retailers. The manufacturer provides I identical items to K retailers. The demand for the individual product for different retailers is independent, identical, and general distributed. The total cost includes the fixed unit cost, holding cost and penalty cost associated with a backorder. The manufacturer waits for q orders to be accumulated and then produces multiple products in lots of q units.

The symbols used to describe the inventory model for SCM are as given below:

I	Total number of products
K	Numbers of retailers
Λ	Combined demand rate of retailers
λ_{ik}	Demand rate of product i from retailer k
T	Changeover time
T	Per item manufacturer time
Q	Batch size

U	Transportation time from manufacturer to retailers
LT	Manufacturing lead time
τ	Total lead time to retailer, E[LT] + u
ρ	Manufacturing utilization
C_A^2 (C_s^2)	Square coefficient of variation of demand (supply)
W_B (W_p)	Waiting time during the batch (process) buffer
S	Batch service time

11.4 Expected Lead Time

Suppose that the manufacturer produces I products for K identical retailers. All the I products are of same cost, and the demand for every ith ($i = 1,2,\ldots,I$) product by kth ($k = 1,2,\ldots,K$) retailers is independent, identical, and general distributed. The overall supply chain demand rate is given by

$$\Lambda = \sum_{i=1}^{I}\sum_{k=1}^{K}\lambda_{ik} = IK\lambda$$

The manufacturer waits for q orders to be accumulated and then produces multiple products in lots of q units. First in first out (FIFO) rule is applied to process the batches of q units. The time to produce a lot of q units is $T + qt$, where T is the setup time and t is the unit manufacturer time; both manufacturing and setup times are predictable and are independent of the product produced. Let the time consumed in transportation from the manufacturer to the retailer be u.

(i) The average lead time, of an order is, given by

$$\text{E [LT]} = \text{E}\left[\text{W}_\text{B} + \text{W}_\text{p} + \text{S}\right] \tag{11.1}$$

The manufacturer, which is not always optimal, assumes that it uses constant batch sizes. It is taken into consideration due to mathematical tractability.

(ii) The average time spent in the batch buffer is

$$\text{E [W}_\text{B}] = \frac{(q-1)I}{2\Lambda} \tag{11.2}$$

(iii) The average service time of a batch is given by

$$E\,[S] = T + qt \tag{11.3}$$

Here, the G/G/1 model is used to derive the average time an order spends in the process buffer.

(iv) In the process buffer, the average time spent by an order is given by

$$\mathrm{E}\left[\mathrm{W_p}\right] = \frac{\Lambda(T+qt)^2}{Q(t)}A,$$
$$\text{where} \quad \mathrm{A} = \left(C_A^2 + C_S^2\right); \quad Q(t) = [q(1-\Lambda t) - \Lambda t]. \tag{11.4}$$

Equation (11.4) is a closed-form formula for the amount of time an order is expected to spend in the system. The average time an order spends in the queueing system and the transportation time from the manufacturer to the store add up to the expected replenishment lead time of our multi-echelon supply chain inventory model. By using Eqs. (11.1)–(11.4), we obtain the averaage lead time as

$$E[LT] = \frac{q-1}{2\Lambda}I + \frac{\Lambda(T+qt)^2}{Q(t)}A + T + qt \tag{11.5}$$

(v) The minimal expected lead time is

$$\tau = \min \frac{q-1}{2\Lambda}I + \frac{\Lambda(T+qt)^2}{Q(t)}A + T + qt + u \tag{11.6}$$

(vi) The utilization rate is given by

$$\rho = \Lambda\left(\frac{T}{q} + t\right). \tag{11.7}$$

11.5 Optimal Policy

The objective of our investigation in this section is to minimize the expected lead time to determine the optimal batch size q. For this purpose, we use the classical approach as follows.

Consider $\frac{\partial \tau}{\partial q} = 0$ which gives

$$q_1^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1 + \sqrt{\frac{A}{R}}\right], \tag{11.8}$$

$$q_2^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1 - \sqrt{\frac{A}{R}}\right] \tag{11.9}$$

$$\text{where} \quad R = A\Lambda^2 t^2 + 2\Lambda t(1-\Lambda t) + I(1-\Lambda t) \tag{11.10}$$

It is evident that q_2^* is not feasible because it suggests a utilization rate greater than one. So, we use q_1^* to determine the optimal utilization rate ρ^* and lead time τ^*.

Thus, the optimal utilization rate is given by

$$\rho^* = \frac{\sqrt{R} + \Lambda t\sqrt{A}}{\sqrt{R} + \sqrt{A}} \tag{11.11}$$

Also, the optimal lead time is

$$\tau = \frac{\left[\Lambda T(1-\Lambda t) - (1-\Lambda t)^2\right] + 2\Lambda T\left[A\Lambda t - \Lambda t + \sqrt{AR} + 1\right]}{2\Lambda(1-\Lambda t)^2} + u. \tag{11.12}$$

11.6 Some Special Cases

Case 1: If we take $c_A^2 = 1$ and $c_s^2 = 1$, then the process buffer is governed by the M/M/1 model. In this case, the optimal values of different system parameters reduce to:

$$q_1^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1 + \sqrt{\frac{2}{R}}\right] \tag{11.13}$$

$$\rho^* = \frac{\sqrt{R} + \Lambda t\sqrt{2}}{\sqrt{R} + \sqrt{2}} \tag{11.14}$$

$$\tau^* = \frac{\left[\Lambda T(1-\Lambda t) - (1-\Lambda t)^2\right] + 2\Lambda T\left[\Lambda t + \sqrt{2R} + 1\right]}{2\Lambda(1-\Lambda t)^2} + u \tag{11.15}$$

Case 2: For the M/D/1 model for process buffer, we set $c_A^2 = 1$ and $c_s^2 = 0$. Also, for the $E_2/E_2/1$ model we have $c_A^2 = 0.5$ and $c_s^2 = 0.5$. For both models, the optimal batch size, utilization rate, and the optimal expected lead time are given by

$$q_1^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1 + \frac{1}{\sqrt{R}}\right] \tag{11.16}$$

$$\rho^* = \frac{\sqrt{R} + \Lambda t}{\sqrt{R} + 1} \tag{11.17}$$

and $$\tau^* = \frac{\left[\Lambda T(1-\Lambda t) - (1-\Lambda t)^2\right] + 2\Lambda T\left[\sqrt{R} + 1\right]}{2\Lambda(1-\Lambda t)^2} + u \tag{11.18}$$

In this case, our results coincide with Thonemann and Bradley (2002).

Case 3: Setting $c_A^2 = 0.5$ and $c_s^2 = 1$, we propose the E_2/M/1 model for the buffer process. In this case, the q^*, ρ^* and τ^* are given below:

$$q_1^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1 + \frac{\sqrt{1.5}}{\sqrt{R}}\right] \tag{11.19}$$

$$\rho^* = \frac{\sqrt{R} + \Lambda t\sqrt{1.5}}{\sqrt{R} + \sqrt{1.5}} \tag{11.20}$$

$$\text{and} \quad \tau^* = \frac{\left[\Lambda T(1-\Lambda t) - (1-\Lambda t)^2\right] + 2\Lambda T\left[0.5\Lambda t + \sqrt{1.5R} + 1\right]}{2\Lambda(1-\Lambda t)^2} + u. \tag{11.21}$$

11.7 Cost Minimization Analysis

In this section, we discuss the cost minimization problem. For this purpose, we obtain the retailor's cost for a given replenishment lead time. Now we define some notations as shown below:

C_{ik}	Unit cost of ith ($i = 1,2,3,\ldots,J$) item at kth ($k = 1,2,3,\ldots,K$) retailer
C_h	Holding cost
C_p	Backorder penalty cost
z	Optimal order up to level
TC	Total cost
Y	Limiting distribution of the number of order in resupply

The cost minimization problem for the retailer is

$$\text{Min } TC = \left\{C_{ik}\lambda + C_h E[s-y]^+ + C_p E[y-s]^+\right\} I \tag{11.22}$$

where

$$[x]^+ = \begin{cases} x, & x > 0 \\ 0, & x < 0 \end{cases}$$

The cost minimization problem in case when replenishment times are i.i.d., is given by

$$z^* = F_{\lambda\tau}^{-1}\left(\frac{C_p}{C_h + C_p}\right) \tag{11.23}$$

where $F_{\lambda\tau}$ is used for distribution function of a Poisson distribution with mean $\lambda\tau$. Also

$$z^* = \frac{\tau\Lambda}{IN} + v\sqrt{\frac{\tau\Lambda C_a^2}{IN}} \tag{11.24}$$

where demand is a normally distributed variate having mean $\lambda\tau$ and variance $\lambda\,\tau c_a^2$.

$$\text{Also} \quad v = \Phi^{-1}\left(\frac{C_p}{C_h + C_p}\right). \tag{11.25}$$

Here, Φ represents the CDF of the standard normal distribution. Let $G(y)$ be the CDF of normal distribution with mean $\lambda\tau$ and variance $\tau\lambda C_a^2$, then

$$\begin{aligned} E\left\{C_h\left(z^* - y\right) + C_p\left(y - z^*\right)\right\} &= C_h \int_{y=0}^{S^*} \left(z^* - y\right) dG(y) + C_p \int_{y=S^*}^{\infty} \left(y - z^*\right) dG(y) \\ &= \left[C_h Z + (C_p + C_h)L(v)\right]\sqrt{\frac{\tau\Lambda C_a^2}{IN}} \end{aligned} \tag{11.26}$$

where

$$L(v) = \int_{\xi=v}^{\infty} (\xi - TC)\,d\Phi(\xi). \tag{11.27}$$

Using Eq. (11.26) in (11.22), we get the total cost (TC) as

$$TC = \frac{C_{jk}\Lambda}{N} + \left[C_h Z + (C_p + C_h)L(v)\right]\sqrt{\frac{\tau\Lambda C_a^2}{IN}} \tag{11.28}$$

We now look at the impact of the item type on a retailer's cost. We suppose that the supply chain contains a large number of products I and retailers K. Using (11.21) and (11.28) as inputs, the minimum cost is derived at the retailer's end. Thus,

$$\begin{aligned} TC^* = r(v)&\sqrt{\left[\frac{\left(\Lambda T(1 - \Lambda t) - (1 - \Lambda t)^2\right)I + 2\Lambda T\left(\Lambda t A + \sqrt{AR} - \Lambda t + 1\right)}{2N(1 - \Lambda t)^2} + \frac{u}{N}\right] I C_a^2} \\ &+ \frac{C\Lambda}{N} \end{aligned} \tag{11.29}$$

where $r(v) = C_h v + (C_p + C_h)L(v)$.

11.8 Numerical Results

Using the MATLAB software, the following section calculates numerical results for the average lead time and total optimal cost. The graphical presentation has also been done in Figs. 11.1, 11.2, 11.3 and 11.4. In Fig. 11.1, we depict the effect of aggregate demand on the total optimal cost. It is observed that total optimal cost aggregate demand is increasing as retailers increase their demand rate so that the profit will be automatically maximized. The effect of the counts of retailers over the total optimal cost for different levels of product type is shown in Fig. 11.2. It is noted that optimal cost decreases with the increase in the number of retailers. From Fig. 11.3, we notice that as changeover time increases, the total optimal cost for different levels of product type (I) also increases. In Fig. 11.4, we notice that the mean lead time is a linearly increasing function of product variety (I).

Fig. 11.1 Total optimal cost versus aggregate demand

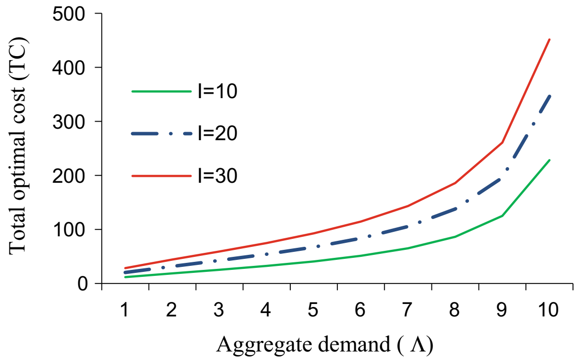
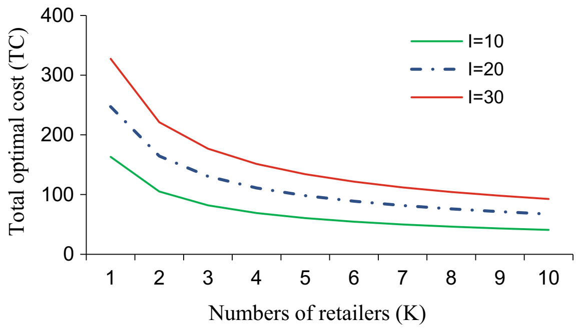


Fig. 11.2 Total optimal cost versus numbers of retailers (K)



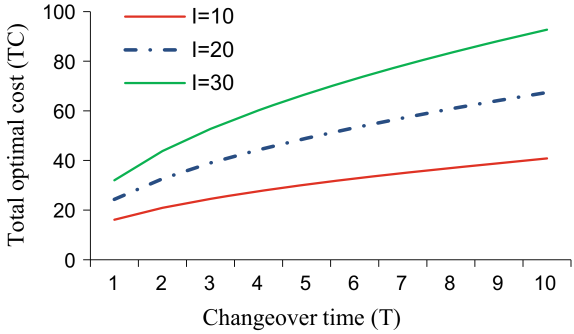


Fig. 11.3 Total optimal cost versus changeover time

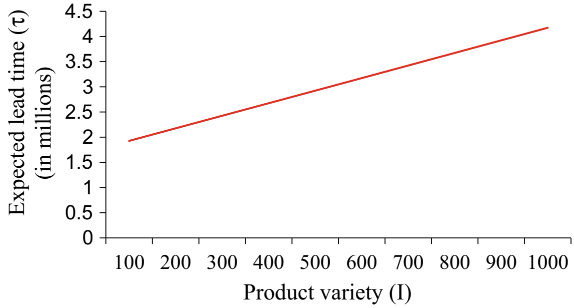


Fig. 11.4 Expected lead time versus product variety

11.9 Conclusions

We have explored the effect of different parameters in a multi-echelon supply chain dealing with a single manufacturer and multiple retailers by developing the generic G/G/1 model. The manufacturer supplies multi-products to several identical retailers. We have incorporated the realistic feature according to which the manufacturer waits for a fixed number of orders to be accumulated and then produces multiple products in batches. We have studied the impact of batch size and product type on the mean lead time and discovered that as the batch size and product diversity grow, so does the expected lead time. We also notice that the total optimal cost seems to increase as the aggregate demand and changeover time T grow up. The investigation can be extended by considering the impact of imperfect items.

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References

Aggarwal V (1983) A closed-from approach for multi-item inventory grouping. Naval Res Logist Quar 30(3):471–485

Almeder C, Klabjan D, Traxler R, Almada-Lobo B (2015) Lead time considerations for the multi-level capacitated lot-sizing problem. Eur J Oper Res 241(3):727–738

Alptekinoğlu A, Tang CS (2005) A model for analyzing multi-channel distribution systems. Eur J Oper Res 163(3):802–824

Archibald BC, Silver EA (1978) (s, S) Policies under continuous review and discrete compound poisson demand. Manage Sci 24(9):899–909

Arshinder KA, Deshmukh SG (2008) Supply chain coordination: perspectives, empirical studies and research directions. Int J Prod Econ 115(2):316–335

Bhattacharya DK (2005) On multi-item inventory. Eur J Oper Res 162(3):786–791

Chatfield C, Goodhardt GJ (1973) A consumer purchasing model with erlang inter-purchase times. J Am Stat Assoc 68(344):828–835

Cohen MA, Kleindorfer PR, Lee HL (1986) Optimal stocking policies for low usage items in multi-echelon inventory systems. Naval Res Logist Quar 33(1):17–38

Dai Z, Aqlan F, Gao K (2017) Optimizing multi-echelon inventory with three types of demand in supply chain. Transp Res Part E: Logist Transp Rev 107:141–177

Dai Z, Gao K, Zheng X (2022) Optimizing two multi-echelon inventory systems for perishable products with price and stock dependent demand in supply chain. Scientia Iranica 29(1):320–342

Feeney GJ, Sherbrooke CC (1966) The (S− 1, S) inventory policy under compound poisson demand. Manage Sci 12(5):391–411

Graves SC (1985) A multi-echelon inventory model for a repairable item with one-for-one replenishment. Manage Sci 31(10):1247–1256

Hadley G, Whiten TM (1963) Analysis of inventory system. Prentice Hall Inc., Englewood Cliffs

Halat K, Hafezalkotob A, Sayadi MK (2021) Cooperative inventory games in multi-echelon supply chains under carbon tax policy: vertical or horizontal? Appl Math Model 99:166–203

He Y, Zhao X (2012) Coordination in multi-echelon supply chain under supply and demand uncertainty. Int J Prod Econ 139(1):106–115

Hosseinifard Z, Abbasi B (2018) The inventory centralization impacts on sustainability of the blood supply chain. Comput Oper Res 89:206–212

Hu F, Lim C-C, Lu Z (2013) Coordination of supply chains with a flexible ordering policy under yield and demand uncertainty. Int J Prod Econ 146(2):686–693

Huang Y, Huang GQ (2010) Price competition and coordination in a multi-echelon supply chain. Eng Lett 18(4):399

Lin C, Lin Y (2005) The production size and inventory policy for a manufacturer in a two-echelon inventory model. Comput Oper Res 32(5):1181–1196

Mateen A, Chatterjee AK, Mitra S (2015) VMI for single-vendor multi-retailer supply chains under stochastic demand. Comput Ind Eng 79:95–102

McTavish R, Goyal SK (1989) Integrating marketing and production decisions. Engi Costs Prod Econ 15:387–390

Moinzadeh K, Lee HL (1986) Batch size and stocking levels in multi-echelon repairable systems. Manage Sci 32(12):1567–1581

Nasseri SH, Bavandi S (2020) Solving multi-objective multi-choice stochastic transportation problem with fuzzy programming approach. In: 2020 8th Iranian joint congress on fuzzy and intelligent systems (CFIS), pp 207–210

Qu Z, Raff H, Schmitt N (2018) Incentives through inventory control in supply chains. Int J Ind Organ 59:486–513

Rao PP (1990) Joint determination of optimal inventory and marketing policies. Int J Manag Syst 6:189–194

Rosenblatt MJ, Herer YT, Hefter I (1998) An acquisition policy for a single item multi-supplier system. Manag Sci 44(11-part-2):S96–S100

Salas Navarro K, Acevedo Chedid J, Florez WF, Ospina Mateus H, Eduardo Cárdenas-Barrón L, Sankar Sana S (2020) A collaborative EPQ inventory model for a three-echelon supply chain with multiple products considering the effect of marketing effort on demand. J Indus Manag Optim 16(4):1613–1633

Shanker K (1981) Exact analysis of a two-echelon inventory system for recoverable items under batch inspection policy. Naval Res Logist Quar 28(4):579–601

Sherbrooke CC (1968) Metric: a multi-echelon technique for recoverable item control. Oper Res 16(1):122–141

Silver EA, Peterson R (1985) Decision system for inventory management production planning, 2nd edn. Wiley, Newyork

Simon RM (1971) Stationary properties of a two-echelon inventory model for low demand items. Oper Res 19(3):761–773

Thonemann UW, Bradley JR (2002) The effect of product variety on supply-chain performance. Eur J Oper Res 143(3):548–569

van Beek P (1981) Modelling and analysis of a multi-echelon inventory system: a case study. Eur J Oper Res 6(4):380–385

Wang CX (2009) Random yield and uncertain demand in decentralised supply chains under the traditional and VMI arrangements. Int J Prod Res 47(7):1955–1968

Xu H (2010) Managing production and procurement through option contracts in supply chains with random yield. Int J Prod Econ 126(2):306–313

Yin Z, Ma S (2015) Incentives to improve the service level in a random yield supply chain: the role of bonus contracts. Eur J Oper Res 244(3):778–791

Zhao ST, Wu K, Yuan X-M (2016) Optimal production-inventory policy for an integrated multi-stage supply chain with time-varying demand. Eur J Oper Res 255(2):364–379

Chapter 12
Impact of Renewable Energy on a Flexible Production System Under Preorder and Online Payment Discount Facility

S. R. Singh and Dipti Singh

Abstract Energy plays a vital role in reducing GHG emission in the world and making the environment eco-friendly. Online payment and preordered discount facility become a very important factor in inventory management. Most of the researchers used different forms of energy to reduce carbon emission. Some researchers used electrical energy, solar energy, hydro power energy etc. to reduce carbon emission. In this paper, we developed a production inventory model with a flexible production system under online payment and preorder discount facility. Numerical illustration is carried out to find the original value and optimality. Sensitivity is carried out to see the behavior of different parameters.

Keywords Renewable energy · Preorder discount and online payment facility · Flexible production system · Lead time · Carbon emission · Price-dependent demand

12.1 Introduction

In today's highly competitive market online payment and preordered discount play an important role which was also useful during the pandemic situation. Mostly online business provides this type of facility which gives better provision to their customer and themselves. Carbon emission becomes a great challenge in today's highly business scenario. Usually, two types of energy are used in production systems—renewable and non-renewable, but renewable energy is less costly and emitted low carbon than non-renewable energy. In this study, we use renewable energy in the production process which is much profitable than the traditional production process. Also,

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it is useful in the reduction of carbon emission. Most of the industries have preference to produce a product with perfect quality. Perfect products depend upon many factors such as the raw material, skilled labor inefficiency, machine breakdown and long-run machine running. For this purpose, the concept of flexibility is used. Using a model developed by Tang et al. (2004) we can quantify two key benefits of a discount program for advance bookings, which include increased sales and a better matching of supply with demand through more accurate forecasts and supply planning. Chao (2011) used the latest economic model to analyze the restructured electricity market and explored renewable energy investment under volatile pricing methods. It was found that volatile prices can lead to a higher level of investment. Lee and Moon (2014) improved energy efficiency through a traditional non-renewable energy source in a wireless sensor network. According to Kök et al. (2018), high-value investments produce better investment outcomes than flat-priced investments. Their paper discussed investing in renewable energy. Datta (2017) investigated the impact of green technology investment under a focused tax system concentrated on improving investment in transport activities with renewable energy. Hasan et al. (2020) introduced the concept of multiple discounts based on online purchase and payment, along with preordering a product into the economic order quantity model (EOQ) of a retailer. Anyaoha and Zhang (2021) developed a model with renewable energy for environmental protection: life cycle inventory of Nigeria's palm oil production. Sarkar and Chung (2021) developed a production inventory model with a flexible production system. They used renewable energy for flexible production and carbon emission reduction.

Online payment facilities also open many opportunities for any organization. It is also helpful in covid-19 situations. In today's pandemic situation, the virus spreads rapidly from one person to another person. So, in this situation online payment facility is the best way to pay the amount for the order. They concluded that the advance booking discount program can act as an effective tool to match the supply with the demand for these product lines. Hasan et al. (2021) studied the optimum price and replenishment cycle when multiple discounts were applied to customers who made purchases during the preorder period and paid online. This study works for non-instantaneous deteriorating items. Moreover, in this the effect of selling price and advertisement on customer demand is also considered. They find that the online payment system is more beneficial than the traditional one. Singh et al. (2021) developed a supply chain inventory model in which the effect of energy and carbon emission is taken into account with the trade credit policy.

In this chapter, we shall study the impact of renewable energy on the flexible production system under preorder and online payment discount facilities.

12.2 Review of Literature

See Table 12.1.

Table 12.1 Important contributions

Author's name	Type of model	Rate of production	Rate of defective	Preorder and online payment	Effect of carbon emission
Hasan et al. (2020)	EOQ	×	×	✓	×
Singh et al. (2021)	SCM	Constant	×	×	✓
Sarkar and Chung (2021)	SSPS	Flexible	Random	×	✓
Present study	SSPS	Flexible	Random	✓	✓

SSPS indicate: Sustainable smart production system; SCM: Supply chain Management

12.3 Notations and Assumptions

Notations and Assumptions are as follows

12.3.1 Notations

L	Lead time
D_1	A discount is offered for preorders and online payments
D_2	Payment online leads to a discount on price
S	Unit selling price ($/unit)
T	The length of the production cycle
R	Sustainable smart production with flexible production rates
K_b	Interest earned by the mobile banking company
S_t	SSPS Setup Cost
S_{te}	Costs associated with setting up the SSPS
S_h	The cost of holding the product
S_{he}	Costs associated with holding the energy
S_d	Obsolete inventory scrap cost
$D(S)$	Product demand (unit)
B_C	Carbon limit for SSPS (kg/year)
B_1	The amount of carbon emitted from holding the product (kg/year)
B_2	The amount of carbon emitted for obsolete items (Kg/year)
B_3	The carbon dioxide that is emitted during setup (Kg/Year)
B_4	Carbon dioxide (Kg/year) emitted during the production process
B_5	Environmental effect of carbon emission (Kg/year)

β_1	A shape parameter that is associated with carbon emission reductions
β_2	Fraction of carbon emission reduction due to investment
γ	Deterioration rate of inventory
a_1	Tool/die cost of sustainable smart production rate
a_{1e}	Cost of Energy during production process ($/Unit)
a_2	Costs associated with the development of the SSPS
a_{2e}	Energy costs for developing the SSPS
a_3	Material cost ($) for the lot of the product
$E[\mu]$	The expected value of imperfect products
S_μ	Item cost for imperfect goods ($/Item)
$S_{\mu e}$	The cost of energy associated with an imperfect product
α	The task paid for the carbon dioxide emissions ($/unit emission)
Q	Maximum amount of produced inventory
S_1	Amount of preordered and online payment inventory
S_2	Amount of inventory after fulfilled preordered inventory

Decision variables	
T	Cycle length
C	Investment in carbon emission reduction
R	Flexible production rate for sustainable production rate
S	Selling price
L	Lead time

12.3.2 Assumptions

(i) Lead time is considered in this model.
(ii) Preordered and online payment facility is undertaken in this model.
(iii) Renewable energy is used to reduce the carbon emission.
(iv) Investment in carbon emission is taken into account.
(v) Demand is dependent on selling price and discount rate

a. $$D(S) = \begin{cases} b_1 - b_2(S - D_1) - b_3(S - D_1)^2 \; if \frac{-L}{2} \le t \le 0 \\ b_1 - b_2(S - D_2) - b_3(S - D_2)^2 \quad Otherwise \end{cases}$$

(vi) Shortages are not allowed.
(vii) Deterioration is taken into account.
(viii) At the end of the preorder period, the retailer offers a discount of D_2 to the customer during t ϵ [0, T]. In order to maintain the retailer's profit from

online payments, $D_2 = \frac{SK_b}{n}$, where $1 \leq n \leq 2$, the retailer establishes n as a discount factor. Under the agreement, the retailer must offer a discount D_2 at least half of the time L. As a result, the upper limit of n is 2. On the other hand, if n is less than 1, $\frac{K_b}{n}$ value is higher than the interest paid by the mobile bank. As such, n must be less than 1. In order to maintain the profit. If n is 1, the retailer offers a full discount of E_b percentage of S that they obtain from a bank to stimulate the customer's demand without breaking the bank. To give higher discounts to customers who purchase during the preorder period with online payments $D_1 = D_2 + S_h$.

12.4 Mathematical Modeling

The mathematical model is developed for the manufacturer production system. In this model, the manufacturer orders raw material from the supplier. Supplier takes L lead time to fulfill the order. In the middle of the lead time the manufacturer announces a preorder and online payment discount policy for the retailer to increase their sales and to reduce their inventory cost. At time $t = 0$ the preordered demand S_1 comes from the retailer and at this time period the production starts. At first the manufacturer fulfills the preordered demand. After that the manufacturer fulfills the demand which arose when the preordered discount period ended. At time t_2 the production is starts. At this time the inventory level becomes S_2. After time t_2 the inventory depletes due to demand only (Fig. 12.1).

The governing differential equation describing the inventory level is given by

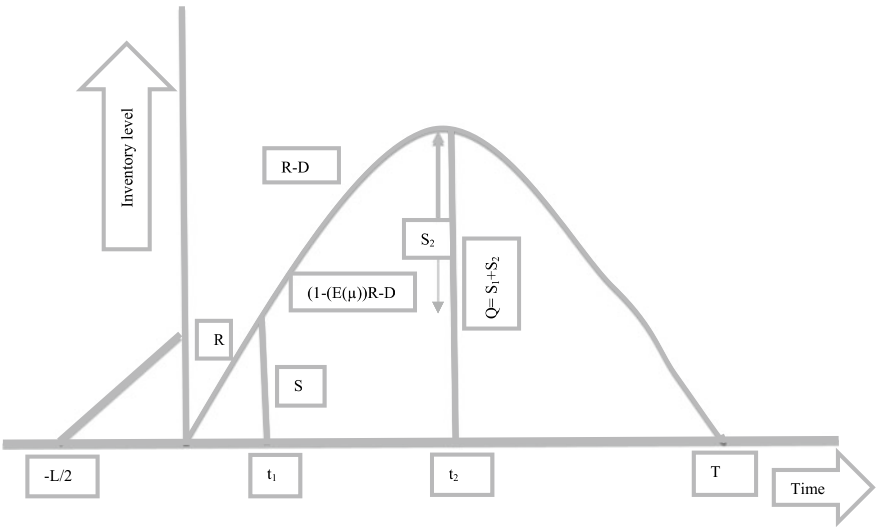

Fig. 12.1 Graphical representation of inventory level

$$\frac{dq_1(t)}{dt} = D(S), \quad \frac{-L}{2} \le t \le 0$$

$$\frac{dq_1(t)}{dt} = b_1 - b_2(S - D_1) - b_3(S - D_1)^3, \quad \frac{-L}{2} \le t \le 0 \tag{12.1}$$

With boundary condition.
At $t = \frac{-L}{2}, q_1\left(\frac{-L}{2}\right) = 0$

$$\frac{dq_2(t)}{dt} = (1 - E[\mu])R \quad, 0 \le t \le t_1 \tag{12.2}$$

With boundary condition.
At t $= 0$, $q_2(0) = 0$, at $t = t_1, q_2(t_1) = S_1$

$$\frac{dq_3(t)}{dt} = (1 - E[\mu])R - D(S)$$

$$\frac{dq_3(t)}{dt} = (1 - E[\mu])R - \left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right], t_1 \le t \le t_2 \tag{12.3}$$

With boundary condition.
At $t = t_1, q_3(t_1) = S_1, t = t_2, q_3(t_2) = S_2$

$$\frac{dq_4(t)}{dt} = -[b_1 - b_2(S - D_1) - b_3(S - D_1)]^3, \quad t_2 \le t \le T \tag{12.4}$$

With boundary condition.
At $t = t_2, q_4(t_2) = S_2, att = T, q_4(T) = 0$
After solving Eq. (12.1) we get

$$q_1(t) = D(S)\left(t + \frac{L}{2}\right), \quad \frac{L}{2} \le t \le 0 \tag{12.5}$$

After solving Eq. (12.2) we get

$$q_2(t) = (1 - E(\mu))(t - t_1) + S_1, \quad 0 \le t \le t_1 \tag{12.6}$$

And the value of S_1 is given by

$$S_1 = \frac{D(S)L}{2}$$

After solving Eq. (12.3) we get

$$q_3(t) = S_1 + ((1 - E[\mu])R - D(S))(t - t_1), \quad t_1 \le t \le t_2 \tag{12.7}$$

And the value of S_2 is given by

$$S_2 = S_1 + ((1 - E[\mu])R - D(S))(t_2 - t_1)$$

After solving Eq. (12.5) we get

$$q_4(t) = \left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right][T - t], \;\; t_2 \le t \le T \tag{12.8}$$

Therefore, the total inventory is

$$\begin{aligned}
&= \left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right]\left[\frac{-L^2}{8} + \frac{L^2}{4}\right] + \frac{[1 - E[\mu])Rt_1^2}{2} + S_1 t_1 \\
&\quad + [S_1(t_2 - t_1) + (1 - E[\mu])R - D(S)]\left(\frac{t_2^2}{2} - t_1 t_2 + \frac{t_1^2}{2}\right) \\
&\quad + \left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right] \\
&\quad \left(\frac{T^2}{2} - Tt_2 + \frac{t_2^2}{2}\right)
\end{aligned} \tag{12.9}$$

The average inventory is given by

$$\begin{aligned}
&= \frac{1}{T}\left[\left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right]\left[\frac{-L^2}{8} + \frac{L^2}{4}\right] + \frac{[1 - E[\mu]]Rt_1^2}{2} + S_1 t_1\right. \\
&\quad + \left[S_1(t_2 - t_1) + (1 - E[\mu])R - D(S)\left(\frac{t_2^2}{2} - t_1 t_2 + \frac{t_1^2}{2}\right)\right] \\
&\quad \left. + \left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right]\left(\frac{T^2}{2} - Tt_2 + \frac{t_2^2}{2}\right)\right]
\end{aligned} \tag{12.10}$$

Thus, the holding and energy cost of inventory is

$$\begin{aligned}
&= \frac{(S_h + S_{he})}{T}\left[\left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right]\left[\frac{L^2}{8}\right] + \frac{[1 - E[\mu]]Rt_1^2}{2}\right. \\
&\quad + S_1 t_1 + \left[S_1(t_2 - t_1) + (1 - E[\mu])R - D(S)\left(\frac{t_2^2}{2} - t_1 t_2 + \frac{t_1^2}{2}\right)\right] \\
&\quad \left. + \left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right]\left(\frac{T^2}{2} - Tt_2 + \frac{t_2^2}{2}\right)\right]
\end{aligned} \tag{12.11}$$

In storage, $\gamma\%$ of products become obsolete or waste. Thus, the scrap cost of the outdated product is

$$\begin{aligned}
&= \frac{\gamma(S - S_d)}{T}\left[\left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right]\left[\frac{L^2}{8}\right] + \frac{[1 - E[\mu]]Rt_1^2}{2} + S_1 t_1\right. \\
&\quad + \left[S_1(t_2 - t_1) + (1 - E[\mu])R - D(S)\left(\frac{t_2^2}{2} - t_1 t_2 + \frac{t_1^2}{2}\right)\right] \\
&\quad \left. + \left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right]\left(\frac{T^2}{2} - Tt_2 + \frac{t_2^2}{2}\right)\right]
\end{aligned} \tag{12.12}$$

We calculate the flexible production cost as well as energy cost due to production as follows:

$$\left((a_1 + a_{1e})R + \frac{(a_2 + a_{2e})}{R} + a_3\right)\left[b_1 - b_2(S - D_2) - b_3(S - D_2)^2\right] \quad (12.13)$$

The setup cost of the SSPS and energy cost for every cycle is

$$= \frac{S_t + \dot{S}_t}{T} \quad (12.14)$$

A defective item's energy cost and cost of defective items in a production system are.

$$= \frac{\left(S_\mu + \dot{S}_\mu\right)E[\mu]R}{T} \quad (12.15)$$

In order to reduce carbon emissions, the firm uses renewable or non-renewable energy instead of only non-renewable energy, which reduces the system's carbon emissions

$$\text{The investment for carbon emission reduction} = \text{C} \quad (12.16)$$

Investing in renewable energy and technological development has reduced β_2 fraction of carbon emissions only non-renewable energy, which reduces the system's carbon emissions

$$\text{The depletion of carbon emission is CR} = \beta_2\left(1 - e^{\beta_1 C}\right) \quad (12.17)$$

$$\text{The allocated carbon emission cost is } = \alpha\beta_C \quad (12.18)$$

The carbon emission costs are required in a different part of the sustainable smart production system.

The carbon emission cost per cycle related to the setup of the sustainable smart production system is

$$= \frac{\alpha B_3}{T} \quad (12.19)$$

The carbon emission cost for the holding product is given below

$$
\begin{aligned}
&= \frac{\alpha B_1}{T}\Bigg[\frac{L^2\big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]}{8} + \frac{(1-E[\mu])Rt_1^2}{2} + S_1 t_2 \\
&+ (1-E[\mu])R - \big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]\left(\frac{t_2^2}{2} - t_1 t_2 + \frac{t_1^2}{2}\right) \\
&+ \big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]\left(\frac{T^2}{2} - Tt_2 + \frac{t_2^2}{2}\right)\Bigg]
\end{aligned}
\tag{12.20}
$$

Carbon emission cost for obsolete inventory is given below

$$
\begin{aligned}
&= \frac{\alpha B_2 \gamma}{T}\Bigg[\frac{L^2\big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]}{8} + \frac{(1-E[\mu])Rt_1^2}{2} + S_1 t_2 \\
&+ (1-E[\mu])R - \big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]\left(\frac{t_2^2}{2} - t_1 t_2 + \frac{t_1^2}{2}\right) \\
&+ \big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]\left(\frac{T^2}{2} - Tt_2 + \frac{t_2^2}{2}\right)\Bigg]
\end{aligned}
\tag{12.21}
$$

The cost of carbon emission per cycle for production is given below.

$$
= \frac{T\alpha B_4\big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]}{T} \tag{12.22}
$$

The environmental impact of carbon emission for the inventory is given below

$$
= \frac{\alpha B_5 T\big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]}{T} \tag{12.23}
$$

Therefore, the total carbon emission cost is given by

$$
\begin{aligned}
&= \alpha\Bigg(\frac{B_3}{T} + \frac{B_1}{T}\Bigg[\frac{L^2\big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]}{8} + \frac{(1-E[\mu])Rt_1^2}{2} + S_1 t_2 \\
&+ (1-E[\mu])R - \big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]\left(\frac{t_2^2}{2} - t_1 t_2 + \frac{t_1^2}{2}\right) \\
&+ \big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]\left(\frac{T^2}{2} - Tt_2 + \frac{t_2^2}{2}\right)\Bigg] + \frac{B_2\gamma}{T} \\
&\Bigg[\frac{L^2\big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]}{8} + \frac{(1-E[\mu])Rt_1^2}{2} + S_1 t_2 \\
&+ (1-E[\mu])R - \big[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\big]\left(\frac{t_2^2}{2} - t_1 t_2 + \frac{t_1^2}{2}\right)
\end{aligned}
$$

$$+\Big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\Big]\left(\frac{T^2}{2}-Tt_2+\frac{t_2^2}{2}\right)\Bigg]$$
$$+\frac{TB_4\Big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\Big]}{T}+\frac{B_5T\Big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\Big]}{T}\Bigg) \tag{12.24}$$

The carbon emission after the investment is as follows

$$=\left(\frac{B_3}{T}+\frac{B_1}{T}\left[\frac{L^2\big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\big]}{8}+\frac{(1-E[\mu])Rt_1^2}{2}+S_1t_2\right.\right.$$
$$+(1-E[\mu])R-\big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\big]\left(\frac{t_2^2}{2}-t_1t_2+\frac{t_1^2}{2}\right)$$
$$\left.+\big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\big]\left(\frac{T^2}{2}-Tt_2+\frac{t_2^2}{2}\right)\right]+\frac{B_2\gamma}{T}$$
$$\left[\frac{L^2\big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\big]}{8}+\frac{(1-E[\mu])Rt_1^2}{2}+S_1t_2\right.$$
$$+(1-E[\mu])R-\big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\big]\left(\frac{t_2^2}{2}-t_1t_2+\frac{t_1^2}{2}\right)$$
$$\left.+\big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\big]\left(\frac{T^2}{2}-Tt_2+\frac{t_2^2}{2}\right)\right]$$
$$\left.+\frac{T(B_4+B_5)\big[b_1-b_2(S-D_2)-b_3(S-D_2)^2\big]}{T}\right)+\big(1-\big(1-e^{\beta_1C}\big)\big) \tag{12.25}$$

Then

$$\alpha\left[B_C-\left(\frac{B_3}{T}+\frac{B_1}{T}\left[\frac{L^2(b_1-b_2(S-D_1)-b_3(S-D_1)^2}{8}+\frac{(1-E[\mu])Rt_1^2}{2}\right.\right.\right.$$
$$S_1t_2+(1-E[\mu])R-\big(b_1-b_2(S-D_2)-b_3(S-D_2)^2\big)\left[\frac{t_2^2}{2}-t_1t_2+\frac{t_1^2}{2}\right]$$
$$\left.+\big(b_1-b_2(S-D_2)-b_3(S-D_2)^2\big)\left[\frac{T^2}{2}-Tt_2+\frac{t_2^2}{2}\right]\right]+\frac{B_2\gamma}{T}$$
$$\left[\frac{L^2(b_1-b_2(S-D_1)-b_3(S-D_1)^2)}{8}+\frac{(1-E[\mu])Rt_1^2}{2}+S_1t_2+[(1-E[\mu])R\right.$$
$$-\big[\big(b_1-b_2(S-D_2)-b_3(S-D_2)^2\big)\big]\left[\frac{t_2^2}{2}-t_1t_2+\frac{t_1^2}{2}\right]+[(b_1-b_2(S-D_2)-$$
$$\left.\left.b_3(S-D_2)^2)\big]\left[\frac{T^2}{2}-Tt_2+\frac{t_2^2}{2}\right]\right]\right]+(B_4+B_5)(b_1-b_2(S-D_2)$$

$$-b_3(S-D_2)^2)\big]\left(1-\beta_2\left(1-e^{\beta_1 C}\right)\right)\big] \tag{12.26}$$

can be saved for the reduction of carbon emission for the sustainable smart production system.

Sales Revenue: Manufacturers earn revenue from two types of stocks they sell, the first being the manufacturer's stock from preordered items that are sold in intervals $[t_1, T]$ from the warehouse.

The revenue from preordered products

$$S_{R1} = (S-D_1)\int_{\frac{-L}{2}}^{0} D dt$$

$$S_{R1} = \frac{(S-D_1)\left[b_1 - b_2(S-D_1) - b_3(S-D_1)^2\right]L}{2} \tag{12.27}$$

Revenue from Regular products

$$S_{R2} = (S-D_2)\int_0^T D dt$$

$$S_{R2} = (S-D_2)T\left[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\right] \tag{12.28}$$

Since the manufacturer has an agreement with a mobile banking company that will pay to the manufacturer K_b percentage of the total revenue from the retailer's payment through the banking system, the total revenue becomes.

$S.R = (SR_1 + SR_2)(1 + K_b)$

$$S.R = \left[\frac{(S-D_1)\left[b_1 - b_2(S-D_1) - b_3(S-D_1)^2\right]L}{2} + (S-D_2)T\right.$$
$$\left.\left[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\right](1+K_b)\right. \tag{12.29}$$

Therefore, the profit per cycle of the sustainable smart production system can be written as follows.

$$\pi(S,R,T,C,L) = \left[\left[\frac{(S-D_1)\left[b_1 - b_2(S-D_1) - b_3(S-D_1)^2\right]L}{2}\right.\right.$$
$$\left.\left.+(S-D_2)T\left[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\right]\right]\right](1+K_b)$$
$$\frac{S_t + \dot{S}_t}{T} - \left((a_1 + a_{1e})R + \frac{(a_2 + a_{2e})}{R} + a_3\right)[b_1 - b_2(S-D_2)$$

$$
-b_3(S-D_2)^2\Big] - \frac{\left(S_\mu + \tilde{S}_\mu\right)E[\mu]R}{T} - C
$$
$$
- \frac{\left[(S_h + S_{he}) + \gamma(S - S_d)\right]}{T}\left[\left[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\right]\left[\frac{L^2}{8}\right]\right.
$$
$$
+ \frac{[1-E[\mu])Rt_1^2}{2} + S_1t_1 + [S_1(t_2 - t_1) + (1-E[\mu])R -
$$
$$
D(S))\left(\frac{t_2^2}{2} - t_1t_2 + \frac{t_1^2}{2}\right)\Big] + \left[b_1 - b_2(S-D_2) - b_3(S-D_2)^2\right]\left(\frac{T^2}{2} - Tt_2\right.
$$
$$
\left.+\frac{t_2^2}{2}\right]\Big] + \alpha\Big[B_C - \left(\frac{B_3}{T} + \frac{B_1}{T}\left(\frac{L^2\left(b_1 - b_2(S-D_1) - b_3(S-D_1)^2\right.}{8}\right.\right.
$$
$$
+ \frac{(1-E[\mu])Rt_1^2}{2} + S_1t_2 + [(1-E[\mu])R - (b_1 - b_2(S-D_2)
$$
$$
-b_3(S-D_2)^2\Big)\Big]\left[\frac{t_2^2}{2} - t_1t_2 + \frac{t_1^2}{2}\right] + (b_1 - b_2(S-D_2)
$$
$$
- b_3(S-D_2)^2\Big)\left[\frac{T^2}{2} - Tt_2 + \frac{t_2^2}{2}\right]\Big] + \frac{B_2\gamma}{T}\left(\frac{L^2\left(b_1 - b_2(S-D_1) - b_3(S-D_1)^2\right)}{8}\right.
$$
$$
+ \frac{(1-E[\mu])Rt_1^2}{2} + S_1t_2 + [(1-E[\mu])R - [(b_1 - b_2(S-D_2)
$$
$$
-b_3(S-D_2)^2\Big)\Big]\left[\frac{t_2^2}{2} - t_1t_2 + \frac{t_1^2}{2}\right] + [(b_1 - b_2(S-D_2) - \; b_3(S-D_2)^2\Big)\Big]
$$
$$
\left[\frac{T^2}{2} - Tt_2 + \frac{t_2^2}{2}\right]\Big]\Big] + (B_4 + B_5)\left(b_1 - b_2(S-D_2) - b_3(S-D_2)^2\right)\Big]
$$
$$
\left(1 - \beta_2\left(1 - e^{\beta_1 C}\right)\right]\Big] \tag{12.30}
$$

Remark 1: In the absence of consideration of preordered discount and online payment facility, the study reduces to (Sarkar and Chung 2021).

Remark 2: In the absence of preordering and online billing, the impact of renewable energy, and the concept of flexible production rate with no defective goods, the result is comparable to that of the study by Mishra et al. (2020).

12.5 Solution Methodology

The proposed model specifies five independent variables in the total profit equation: L, R, C, S, and T. To find the value of each independent parameter, the following steps are taken.

Step 1. Calculate the first-order partial derivative w. r. t. all the independent variables $\frac{\partial \pi}{\partial S}, \frac{\partial \pi}{\partial T}, \frac{\partial \pi}{\partial L}, \frac{\partial \pi}{\partial C}, \frac{\partial \pi}{\partial R}$

Step 2. Equate the first-order partial derivatives to zero and solve the value of L, S,C,R, and T

Step 3. Now calculate the second-order partial derivatives w.r.t. all the independent variables like $\frac{\partial^2 \pi}{\partial L^2}, \frac{\partial^2 \pi}{\partial T^2}, \frac{\partial^2 \pi}{\partial S^2}, \frac{\partial^2 \pi}{\partial R^2}, \frac{\partial^2 \pi}{\partial C^2}$

Step 4. Now, form a Hessian matrix as follows

$$\begin{pmatrix} \frac{\partial^2 \pi}{\partial L^2} & \frac{\partial^2 \pi}{\partial L \partial T} & \frac{\partial^2 \pi}{\partial L \partial S} & \frac{\partial^2 \pi}{\partial L \partial C} & \frac{\partial^2 \pi}{\partial L \partial R} \\ \frac{\partial^2 \pi}{\partial T \partial L} & \frac{\partial^2 \pi}{\partial T^2} & \frac{\partial^2 \pi}{\partial T \partial S} & \frac{\partial^2 \pi}{\partial T \partial C} & \frac{\partial^2 \pi}{\partial T \partial R} \\ \frac{\partial^2 \pi}{\partial S \partial L} & \frac{\partial^2 \pi}{\partial S \partial T} & \frac{\partial^2 \pi}{\partial S^2} & \frac{\partial^2 \pi}{\partial S \partial C} & \frac{\partial^2 \pi}{\partial S \partial R} \\ \frac{\partial^2 \pi}{\partial C \partial L} & \frac{\partial^2 \pi}{\partial C \partial T} & \frac{\partial^2 \pi}{\partial C \partial S} & \frac{\partial^2 \pi}{\partial C^2} & \frac{\partial^2 \pi}{\partial C \partial R} \\ \frac{\partial^2 \pi}{\partial R \partial L} & \frac{\partial^2 \pi}{\partial R \partial T} & \frac{\partial^2 \pi}{\partial R \partial S} & \frac{\partial^2 \pi}{\partial R \partial C} & \frac{\partial^2 \pi}{\partial R^2} \end{pmatrix}$$

Step 5. Find H_1, H_2, H_3, H_4, and H_5, where H_1, H_2, H_3, H_4, and H_5 denote the first principal minor, second principal minor, third principal minor, fourth principal minor, and fifth principal minor, respectively. If det (H_1) < , 0, det (H_2) < 0, det (H_3) < 0, det (H_4) > 0 and det (H_5) > 0 then the matrix is negative definite matrix, and $\pi(S, R, T, C, L)$ is called the convex function.

12.6 Numerical Illustration

All the parameters are taken into appropriate units.

Input parameters $a_1 = 0.9$, $a_{1e} = 0.1$, $a_2 = 24$, $a_{2e} = 1$, $a_3 = 9$, n = 2, $B_1 = 1.6$, $B_2 = 0.6$, $B_3 = 6$, $B_4 = 0.12$, $B_5 = 0.2$, $S_h = 0.05$, $S_{he} = 0.05$, $\beta_1 = 0.3$, $\beta_1 = 0.01$, $b_1 = 9.6$, $b_2 = 0.2$, $b_3 = 0.2$, $\alpha = 0.5$, $B_C = 0.3$, $L = 0.5$, $S_d = 0.02$, $E[\mu] = 0.9$, $K_b = 0.3$

We obtain $\pi(S, R, T, C, L) = 19.3728 Lac$, $C = 0.861676$, $R = 5.17388$, $T = 6.08197$, $S = 4.27704$, $t_1 = 0.442163$, $t_2 = 5.30609$.

12.7 Concavity

See Figs. 12.2, 12.3 and 12.4.

12.8 Sensitivity Analysis

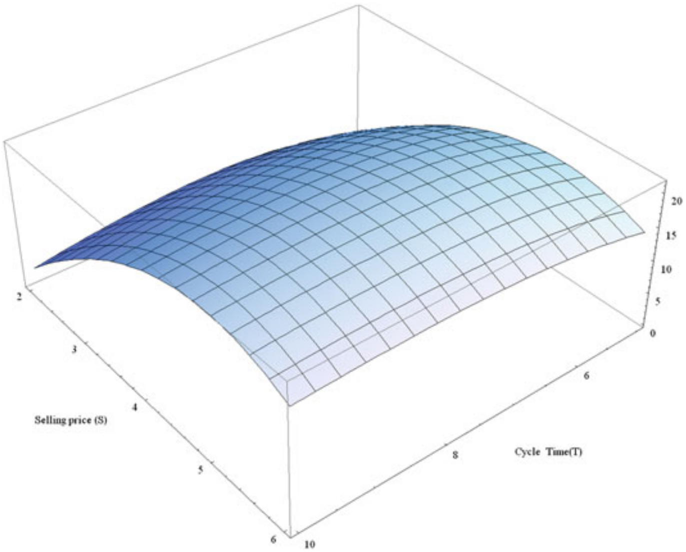

Fig. 12.2 Concavity between selling price and cycle length

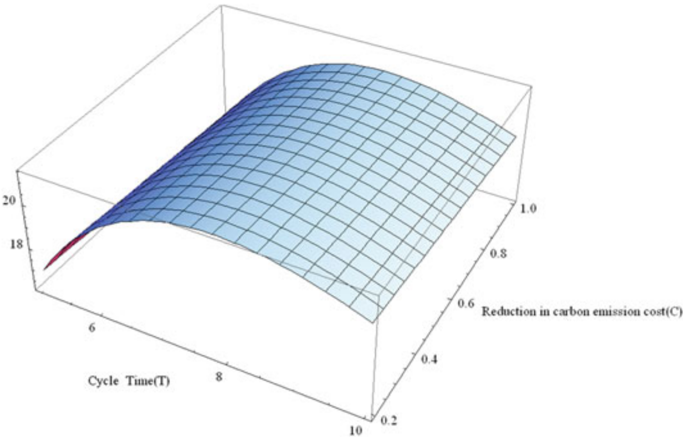

Fig. 12.3 Concavity between cycle length and reduction in carbon emission

Parameters	% Value	t_1	t_2	T	C	R	S	TP
S_h	+ 20%	1.15404	7.06703	8.03154	0.82667	10.66	4.30127	23.1861
	+ 10%	1.15404	7.06703	8.03154	0.86167	5.17388	4.30127	23.1594
	0%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3728

(continued)

(continued)

Parameters	% Value	t_1	t_2	T	C	R	S	TP
	−10%	1.7×10^{-4}	90.7272	90.7271	0.000020	22.714	0.5557	383.814
	−20%	6.1×10^{-12}	822.443	822.443	3.07×10^{-8}	79.3435	6.1×10^{-12}	3485.88
S_d	+ 20%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3737
	+ 10%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3728
	0%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3729
	−10%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.373
	−20%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	23.1706
γ	+ 20%	1.15404	7.06703	8.03154	0.82667	10.66	4.30127	19.3737
	+ 10%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3728
	0%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3728
	−10%	0.442163	5.30609	6.08197	0.86167	5.17388	4.27704	19.3729
	−20%	0.651242	4.57108	6.64889	0.942621	7.89702	4.72394	19.373
b_1	+ 20%	5.8×10^{-12}	1177.7	1177.7	14.9605	1697.94	5.8×10^{-13}	6048.56
	+ 10%	0	1.05×10^{11}	1.19×10^{11}	5.0×10^{9}	3.13×10^{9}	0	4.29×10^{11}
	0%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3728
	−10%	2.34581	4.48572	7.79814	0.124946	8.05451	4.95194	10.1857
	−20%	0.44216	5.30609	7.79001	0.861676	5.17388	4.27704	9.33119
β_1	+ 20%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.373
	+ 10%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3729
	0%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3728
	−10%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3728
	−20%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3728
β_2	+ 20%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3667
	+ 10%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3698
	0%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3728
	−10%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.3759
	−20%	0.442163	5.30609	6.08197	0.861676	5.17388	4.27704	19.379

12.9 Observation

i. On increasing the holding cost S_h the total profit increases, selling price increases, cycle time increases, and flexible production rate increases.
ii. On decreasing the holding cost S_h the total profit, flexible production rate, and cycle length highly increase but the selling price highly decreases.

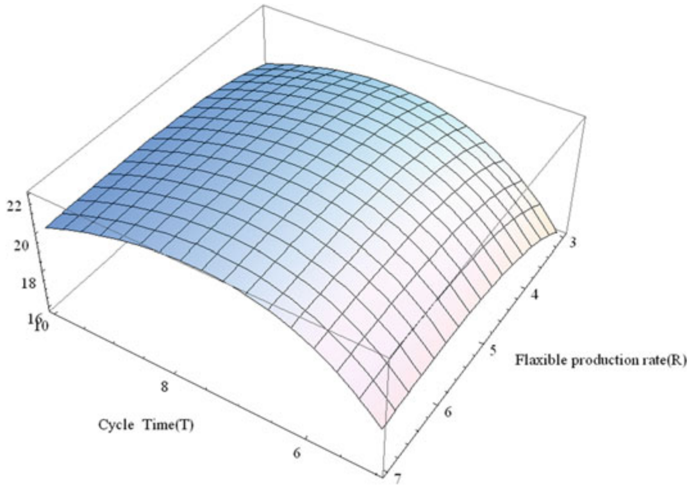

Fig. 12.4 Concavity between cycle length and flexible production rate

iii. On increasing the deterioration cost S_d the total profit decreases.
iv. On increasing the obsolete inventory, the total profit decreases and the selling price increases.
v. On increasing the demand parameter b_1 the total profit highly increases.
vi. On increasing the shape parameter of carbon emission the total profit slightly increases.
vii. On increasing the fraction of carbon emission there is reduction due to investment.

12.10 Conclusion

In this study we considered a sustainable smart production system with flexible production rate and preordered and online payment facility which is more profitable than the traditional payment method. This online payment and preordered discount facility was useful in the covid-19 pandemic situation. An investment in the reduction of carbon emission is also applied. Renewable energy saves the total cost and also reduce carbon emission. The flexible production system decreased the number of defective items and increased the total profit. This study can be further extended with a stochastic situation.

References

Anyaoha KE, Zhang L (2021) Renewable energy for environmental protection: life cycle inventory of Nigeria's palm oil production. Resour Conserv Recycl 174:105797

Chao HP (2011) Efficient pricing and investment in electricity markets with intermittent resources. Energy Policy 39(7):3945–3953

Datta TK (2017) Effect of green technology investment on a production-inventory system with the carbon tax. Adv Oper Res 2017:1–13. https://doi.org/10.1155/2017/4834839

Hasan MR, Daryanto Y, Roy TC, Feng Y (2020) Inventory management with online payment and preorder discounts. Ind Manag Data Syst 120(11):2001–2023

Hasan MR, Roy TC, Daryanto Y, Wee HM (2021) Optimizing inventory level and technology investment under a carbon tax, cap-and-trade and strict carbon limit regulations. Sustain Prod Consumpt 25:604–621

Kök AG, Shang K, Yücel Ş (2018) Impact of electricity pricing policies on renewable energy investments and carbon emissions. Manage Sci 64(1):131–148

Lee JH, Moon I (2014) Modeling and optimization of energy efficient routing in wireless sensor networks. Appl Math Model 38:2280–2289

Mishra U, Wu JZ, Sarkar B (2020) A sustainable production-inventory model for a controllable carbon emissions rate under shortages. J Clean Prod 256:120268

Sarkar M, Chung BD (2021) Effect of renewable energy to reduce carbon emissions under a flexible production system: a step toward sustainability. Energies 14(1):215

Singh SR, Yadav D, Sarkar B, Sarkar M (2021) Impact of energy and carbon emission of a supply chain management with two-level trade-credit policy. Energies 14(6):1569

Tang CS, Rajaram K, Alptekinoğlu A, Ou J (2004) The benefits of advance booking discount programs: model and analysis. Manage Sci 50(4):465–478

Chapter 13
Impact of Preservation Technology Investment and Order Cost Reduction on an Inventory Model Under Different Carbon Emission Policies

Dipti Singh, S. R. Singh, and Monika Rani

Abstract A product's preservation is a key issue in the inventory control system because it prevents deterioration of the product while it is stored in a warehouse or showroom. In today's highly competitive business all organizations try to reduce carbon emission and deterioration to make their business sustainable. This study developed a joint effect of preservation technology investment and order cost reduction under different carbon emission regulation policies in an inflationary environment. Because inflation becomes the most realistic approach in inventory modeling, we can't ignore it. So, in this paper, preservation technology is applied and taken as a decision variable. The main aim of this study is to make a sustainable inventory model and provide the best technique to reduce carbon emissions. To validate the model three different numerical examples are carried out. From sensitivity analysis, we obtain that under carbon taxation, reducing carbon emissions should have better benefits. A numerical illustration is carried out by using Software Mathematica 12.0. Sensitivity is carried out to see the behavior of different parameters on the total profit.

Keywords Preservation technology investment · Order cost reduction · Inflation · Shortages · Selling price · Time-dependent demand

13.1 Introduction

Deterioration is becoming a crucial problem all over the world. Products like food items, medicines, fruits, and fashion goods deteriorate with time and loss their original utility over their life time. Deterioration of products can be control by the investment in adequate preservation technology. So, in this paper, preservation technology is applied and taken as a decision variable. Ghare (1963) developed an inventory model with deterioration. Inflation is the most realistic concept everywhere either in business management or inventory modeling etc. So, in this paper, we take into

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account the effect of inflation. In today's scenario inflation is becoming common in every developing country like India, therefore, to make our model more realistic we have considered the factor of inflation. In the mature stage of a product's lifespan, a constant demand rate is usually valid. Buzacott (1975) was the first who used the concept of inflation in inventory modeling. Demand rates can be approximated by time-dependent functions in the growth and/or end stages of a life cycle. When analyzing real life situations, the selling price and the amount of stock available influence the demand of the product. Demand is taken as the combination of selling price-dependent and time-dependent demand. In general life, the practical experience tells that some but not every or all clients will wait in favor of backlogged merchandise for the shortage period for illustration. If we increase the waiting time then the backlogging price will be decreased. In this paper, shortage is taken into account. In today's supply chain inventory management system, order cost plays a significant role in the timely delivery of items. Goyal and Giri (2003) determined the optimal replenishment policies for the production-inventory model with deteriorating items, stock-dependent demand, and partial backlogging under the effect of inflation. An ordering procedure should not be considered as a fixed/known constraint, but rather as part of minimizing waste, meeting deadlines, improving productivity, and maximizing resource utilization. The manufacturer must reduce the order costs to minimize the total cost investment.

13.2 Literature Review

The decline of products adversely impacts the finances and brand image of a company. With such high deterioration rates, there will be higher annual costs, shortages, and sales lost. In this context, firms are interested in identifying the source of deterioration and developing ways to preserve the products they produce and increase profits. Manufacturers are under economic and environmental pressure to keep their products useful for longer and reduce waste to conserve natural resources. Deterioration was first accounted for by Whitin (1957), who considered fashion items deteriorating after a prescribed storage period. Later, Ghare (1963) developed a revised form of the EOQ model by considering exponential decay. Covert and Philip (1973) proposed an inventory model for economic ordering quantity with Weibull rate of deterioration. Murr and Marris (1975) showed that a lower temperature extends the storage life and decreases decay, demonstrating that it is an effective preservation technique. Hsu et al. (2010) developed a model on preservation technology investment for deteriorating products. The product was a seasonal product with an expiration date. Singh and Sharma (2013) presented an optimizing policy for perishable items with a ramp-type demand rate taking into account the preservation technology under two levels of trade credit financing. Ullah et al. (2019) "This suggests that preservation investment is even more beneficial in supply chains where product ordering costs are high. Also, the optimal size of shipment increases with an increase in the deterioration prevention investment. Hence, for a supply chain with high transportation costs, preservation investment is necessary to increase the shipment size instead of shipment frequency". Diaby (2000) demonstrated "a complete model to reduce setup cost and time with determining cut setup time and minimizing the total cost". Nyea et al. (2001) studied "several inventory models for optimal investment of setup cost reduction or optimal setup times; and also discussed the queuing model to estimate work in the process level in their models". Later, Freimer et al. (2006) "demonstrated improvement in quality and setup cost reduction of the process in their model". Huang et al. (2011) assumed setup cost policy reduction by an added investment cost. Sarkar et al. (2016) developed "an integrated inventory model for a setup cost-reduction policy, carbon-emission policy and used the technique of the Stackelberg game approach to find the total cost".

In an inventory control system inflation plays a very important role. To develop the models without considering the inflation always mislead the results. Buzacott (1975) presented an inventory model for economic order quantities with the time value for money. Misra (1979) discussed on optimal inventory management under inflation. Kumar and Kumar (2016) discussed the impact of system parameters in inflationary environment. Hasan et al. (2021) developed a sustainable inventory model with controllable carbon emissions, deterioration and payments. Khurana and Chaudhary (2016) proposed an inventory model using stock and price-dependent demand for deteriorating items under shortage backordering. The recent literature review to fill the gap is given below in Table 13.1.

In this paper, we developed a joint effect of preservation technology investment and order cost reduction on an inventory model under different carbon emission policies. In this paper, our main contribution is investment in order cost reduction, effect of inflation, time-dependent and selling price-dependent demand with shortage. The

Table 13.1 Recent literature review

Author's name	Demand	Preservation technology investment	Carbon emission policy	Order cost reduction
Singh et al. (2011)	Selling price dependent	No	No	No
Lou and Wang (2013)	Constant	No	No	Yes
Sepehri et al. (2021)	Price dependent	Yes	Yes	No
Xu et al. (2020)	Time dependent	No	Yes	No
Sepehri et al. (2021)	Selling price	No	Yes	No
Mashud et al. (2021)	Selling price	Yes	Yes	No
Present study	Selling price + Time-dependent demand	Yes	Yes	Yes

whole study is divided into 6 sections. In Sect. 13.3 model notations and assumption are given. In Sect. 13.4 mathematical modeling is given. In Sect. 13.5 numerical illustration is carried out. In Sect. 13.6 concavity is carried out. In Sect. 13.7 sensitivity analysis is carried out for all three cases. In Sect. 13.8, observation is made. In the last session conclusion is given.

13.3 Notations and Assumptions

13.3.1 Notations

$$\mathrm{D(P,\,t)} = \begin{cases} a - bp + e^{\eta t} & 0 \le t \le t_1 \\ D & t_1 \le t \le T \end{cases}$$

where a > b, a, b, $\eta \ge 0$

Q = Order quantity per cycle
r = Rate of Inflation
β = Backlogging rate
c = Purchasing price
h = Holding cost per unit per unit time
C_1 = Lost sale cost per unit per unit time
C_2 = Shortage cost per unit per unit time
T = Fixed inventory cycle; namely the length of the inventory replenishment
S = The amount of Carbon Emission per unit time
$\widehat{K}$ = The amount of Carbon Emission per unit time
$\hat{h}$ = The amount of Carbon Emission increased by per unit holding cost
$\hat{e}$ = The amount of Carbon Emission increased by per unit of delivery
C_p = The Carbon price per unit Carbon Emission
$\widehat{E}$ = The Capacity of Carbon Emission mandated by Government
$I_1(t)$ = Inventory level at time t in the time interval $0 \le t \le t_1$
$I_2(t)$ = Inventory level at time t in the time interval $t_1 \le t \le T$
θ = Rate of deterioration
k_0 = Fixed ordering cost
L(k) = Capital Investment
ψ = Percentage decrease in order k
ω = Opportunity cost of capital investment ($/unit/time)
C_T = Carbon tax per unit carbon emissions
C_P = Carbon trading price
U = Carbon emissions cap/limit in the cap and trade policy
w = Carbon emissions limit in the limited emission policy
T = Cycle length

P = Selling price per unit

Decision Variable

t_1 =The time at which inventories deplete to zero
k = Ordering cost per order
ξ = Preservation Technology

13.3.2 Assumptions

(1) Demand is taken as selling price and time dependent.
(2) $P(\xi) = 1 - e^{-\lambda\xi}$, where ξ is the investment in preservation technology and λ is the efficiency parameter preservation technology investment, which reduces the deterioration rate. $P(\xi) = 1 - e^{-\lambda\xi}$ is a function that is continuous concave and twice differentiable with respect to preservation investment to ξ at P (0) = 0 and $\lim_{\xi \to 0} P(\xi) = 1$ Here $P'(\xi) = -\lambda^2 e^{-\lambda\xi} < 0$.
(3) L(k) = Reorder point contributions are made by the order k, which decides on capital investments. Therefore, L (k) $= \frac{T}{\psi} \log \frac{k_0}{k}$ and where ψ is a percentage decrease in order cost k. The capital investment cost is $\frac{\omega T}{\psi} \log \frac{k_0}{k}$ to control k, where ω is the annual opportunity cost.
(4) Inflation is considered in this model.
(5) Shortage is permitted.

13.4 Mathematical Modeling

This study is developed for any retailing business. In this initially the retailer orders Q quantity of items to the supplier/manufacturer. After receiving the order from the manufacturer/supplier, the retailer inventory level depletes due to the joint effect of demand and deterioration in the time interval (0, t_1). In the time interval (t_1, T) shortage occurs. The governing differential equations describing the inventory level is given as below (Fig. 13.1).

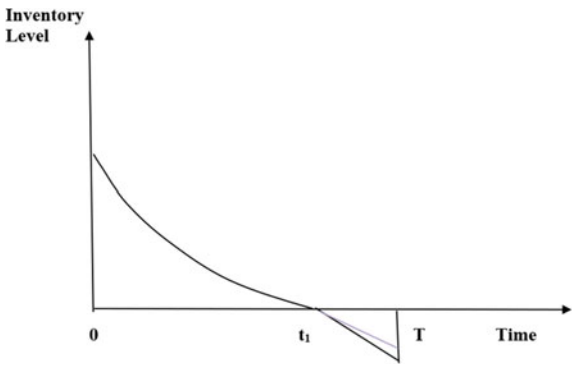

Fig. 13.1 Graphical Representation of inventory level

The governing differential equation describing the inventory level is given by

$$\frac{dI_1 t}{dt} = -D(P,t) - \theta v(\xi) I_1(t) \quad 0 \le t \le t_1 \tag{13.1}$$

$$\frac{dI_2 t}{dt} = -D(P,t)\beta \quad t_1 \le t \le T \tag{13.2}$$

With boundary conditions at t = t_1, $I_1(t) = 0$ & t = t_1, $I_2(t) = 0$ at t = T, $I_2(T) = -S$.

From Eq. (13.1)

$$\frac{dI_1(t)}{dt} = -\left(a - bP + e^{\eta t}\right) - \theta v(\xi)) I_1(t) 0 \le t \le t_1$$

$$\frac{dI_1(t)}{dt} + \theta v(\xi)) I_1(t) = -\left(a - bP + e^{\eta t}\right)$$

Solving Eqs. (13.1) and (13.2)

$$I_1(t) = \frac{(a-bP)}{\theta v(\xi)}\left[e^{\theta v(\xi)(t_1 - t)} - 1\right] + \frac{1}{(\eta + \theta v(\xi))}\left[e^{\eta t_1 + \theta v(\xi)(t_1 - t)} - e^{\eta t}\right] 0 \le t \le t_1 \tag{13.3}$$

$$I_2(t) = (a - bP)\beta(t_1 - t) \; t_1 \le t \le T \tag{13.4}$$

The backorder inventory is given as below:

$$S = (a - bP)\beta(T - t_1) \tag{13.5}$$

The order quantity is given as below:

$$Q = \frac{(a - bP)}{\theta v(\xi)}\left[e^{\theta v(\xi)(t_1)} - 1\right] + \frac{1}{(\eta + \theta v(\xi))}\left[e^{\eta t_1 + \theta v(\xi)(t_1)} - 1\right] + (a - bP)\beta(T - t_1) \tag{13.6}$$

Inventory costs:

(1)

$$\textbf{Sales revenue cost} = \mathrm{P}\left[(\mathrm{a} - \mathrm{bP})e^{-rt_1}\frac{e^{(\eta - r)t_1}}{\eta} - \frac{1}{\eta}\right] + P[(a - bP)\beta(T - t_1)] \tag{13.7}$$

(2)

$$\textbf{Purchasing cost} = \mathrm{c}\left[\frac{(a-bp)}{\theta v(\xi)}\left[e^{\theta v(\xi)(t_1)}-1\right] + \frac{1}{(\eta+\theta v(\xi))}\left[e^{\eta t_1+\theta v(\xi)(t_1)}-1\right] + (a-bp)\beta(T-t_1)\right] \tag{13.8}$$

(3)

$$\textbf{Holding cost} = \mathrm{h}\left[\frac{(a-bp)}{\theta v(\xi)}\left[\frac{-e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)} + \frac{e^{-rt_1}}{r} - \frac{1}{r}\right] + \frac{1}{(\eta+\theta v(\xi))}\left[\frac{e^{(\theta v(\xi)+\eta)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\eta-r)} + \frac{1}{(\eta+r)}\right]\right] \tag{13.9}$$

(4)

$$\textbf{Preservation Technology investment} = \xi T \tag{13.10}$$

(5)

$$\textbf{Shortage cost} = -C_2(a-bP)\beta t_1\left[e^{-rT}-e^{-rt_1}\right] + C_2(a-bP)\beta\left[\frac{-Te^{-rT}}{r} + \frac{t_1 e^{-rt_1}}{r} - \frac{e^{-rT}}{r^2} + \frac{e^{-rt_1}}{r^2}\right] \tag{13.11}$$

(6)

$$\textbf{Lost sale cost} = C_1(1-\beta)(a-bP)(T-t_1) \tag{13.12}$$

(7)

$$\textbf{Capital investment in ordering cost} = \frac{\omega T}{\psi}\log\left(\frac{k_0}{k}\right) \tag{13.13}$$

Then the total profit is given by

$$\textbf{TP} = [\textbf{Sales revenue cost} - \textbf{Purchasing cost} - \textbf{Holding cost} - \textbf{Preservation Technology investment} - \textbf{Lost sale cost} -$$

$$\textbf{Shortage cost} - \textbf{Capital investment in ordering cost}]$$

$$\begin{aligned}
\mathrm{TP} = \mathrm{P}&\left[(\mathrm{a}-\mathrm{bP})e^{-rt_1} + \frac{e^{(\eta-r)t_1}}{\eta} - \frac{1}{\eta}\right] + P(a-bP)\beta(T-t_1) \\
&-\mathrm{c}\left[\frac{(a-bp)}{\theta v(\xi)}\left[e^{\theta v(\xi)(t_1)} - 1\right] + \frac{1}{(\eta+\theta v(\xi))}\left[e^{\eta t_1+\theta v(\xi)(t_1)} - 1\right]\right. \\
&\left.+(a-bp)\beta(T-t_1)\right] - \mathrm{h}\left[\frac{(a-bp)}{\theta v(\xi)}\left[\frac{-e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)}\right.\right. \\
&\left.+\frac{e^{-rt_1}}{r} - \frac{1}{r}\right] + \frac{1}{(\eta+\theta v(\xi))}\left[\frac{e^{(\theta v(\xi)+\eta)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}\right. \\
&\left.\left.-\frac{e^{(\eta-r)t_1}}{(\eta-r)} + \frac{1}{(\eta+r)}\right] - \xi T - C_1(1-\beta)(a-bP)(T-t_1)\right. \\
&+C_2(a-bP)\beta t_1\left[e^{-rT} - e^{-rt_1}\right] + C_2(a-bP)\beta\left[\frac{-Te^{-rT}}{r} + \frac{t_1 e^{-rt_1}}{r}\right. \\
&\left.\left.-\frac{e^{-rT}}{r^2} + \frac{e^{-rt_1}}{r^2}\right] - \frac{\omega T}{\psi}\log\left(\frac{k_0}{k}\right)\right]
\end{aligned} \tag{13.14}$$

In addition, considering the inventory system's carbon footprint including the carbon emissions incurred by delivering, warehousing and fixed carbon emissions, per order, the amount of carbon emissions per inventory cycle can be calculated as

$$E(t_1) = \hat{c}Q + \hat{h}\int_0^{t_1} I_1(t)dt + \hat{k}$$

13.4.1 Profit Function Under Different Carbon Tax Regulations

The Carbon tax is charged proportionally with the total carbon emissions. Hence, the total carbon emissions cost under carbon tax policy is:

$$TC_1 = C_T(E(t_1))$$

$$\begin{aligned}
= C_T&\left[\hat{c}\left[\frac{(a-bP)}{\theta v(\xi)}\left[e^{\theta v(\xi)t_1} - 1\right] + \frac{1}{(\eta+\theta v(\xi))}\left[e^{\eta t_1+\theta v(\xi)t_1} - 1\right]\right.\right. \\
&\left.+\ (a-bP)\beta(T-t_1)\right]\Big] + \hat{h}\left[\frac{(a-bP)}{\theta v(\xi)}\left[\frac{-e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)}\right.\right.
\end{aligned}$$

$$+\frac{e^{-rt_1}}{r}-\frac{1}{r}\Bigg]+\frac{1}{(\eta+\theta v(\xi))}\Bigg[\frac{e^{(\theta v(\xi)+\eta)t_1}}{(\theta v(\xi)+r)}-\frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}-\frac{e^{(\eta-r)t_1}}{(\eta-r)}$$
$$+\frac{1}{(\eta-r)}\Bigg]+\hat{k}\Bigg] \tag{13.15}$$

The profit function considering mentioned inventory investments for this case is given by

$$X_1 = TP_1 - TC_1$$
$$=\Bigg[P\Bigg[(a-bP)e^{-rt_1}+\frac{e^{(\eta-r)t_1}}{\eta}-\frac{1}{\eta}\Bigg]+P[(a-bP)\beta(T-t_1)]$$
$$-c\Bigg[\frac{(a-bP)}{\theta v(\xi)}\Big[e^{\theta v(\xi)t_1}-1\Big]+\frac{1}{(\eta+\theta v(\xi))}\Big[e^{(\theta v(\xi)+\eta)t_1}-1\Big]$$
$$+(a-bP)\beta(T-t_1)]-h\Bigg[\frac{(a-bP)}{\theta v(\xi)}\Bigg[\frac{-e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}+\frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)}$$
$$+\frac{e^{-rt_1}}{r}-\frac{1}{r}\Bigg]+\frac{1}{(\eta+\theta v(\xi))}\Bigg[\frac{e^{(\theta v(\xi)+\eta)t_1}}{(\theta v(\xi)+r)}-\frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}$$
$$-\frac{e^{(\eta-r)t_1}}{(\eta-r)}+\frac{1}{(\eta-r)}\Bigg]-\xi T+C_2(a-bP)\beta t_1\Big[e^{-rT}-e^{-rt_1}\Big]\Big]$$
$$+C_2(a-bP)\beta\Bigg[\frac{-Te^{-rT}}{r}+\frac{t_1e^{-rt_1}}{r}+\frac{e^{-rT}}{r^2}-\frac{e^{-rt_1}}{r^2}\Bigg]-C_1(1-\beta)(a-bP)(T-t_1)$$
$$-\frac{\omega T}{\psi}\log\frac{K_0}{K}-C_T\Bigg[\hat{C}\Bigg[\frac{(a-bP)}{\theta v(\xi)}\Big[e^{\theta v(\xi)t_1}-1\Big]+\frac{1}{(\eta+\theta v(\xi))}\Big[e^{(\theta v(\xi)+\eta)t_1}-1\Big]$$
$$+(a-bP)\beta(T-t_1)]]+\hat{h}\Bigg[\frac{(a-bP)}{\theta v(\xi)}\Bigg[\frac{-e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}+\frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)}+\frac{e^{-rt_1}}{r}-\frac{1}{r}\Bigg]$$
$$+\frac{1}{(\eta+\theta v(\xi))}\Bigg[\frac{e^{(\theta v(\xi)+\eta)t_1}}{(\theta v(\xi)+r)}-\frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}-\frac{e^{-(\eta-r)t_1}}{(\eta-r)}+\frac{1}{(\eta-r)}\Bigg]\Bigg]+\hat{k}\Bigg] \tag{13.16}$$

So Total Average profit is given by

$$
\begin{aligned}
&\text{TA } P_1(t_1, T, K, P) = \frac{1}{T}\Bigg[P\Bigg[(a-bP)e^{-rt_1} + \frac{e^{(\eta-r)t_1}}{\eta} - \frac{1}{\eta}\Bigg]\\
&+P[(a-bP)\beta(T-t_1)] - \Big[c + \hat{C}C_T\Big]\Bigg[\frac{(a-bP)}{\theta v(\xi)}\big[e^{\theta v(\xi)t_1} - 1\big]\\
&+\frac{1}{(\eta+\theta v(\xi))}\big[e^{(\theta v(\xi)+\eta)t_1} - 1\big] + (a-bP)\beta(T-t_1)\Big]\\
&-\Big(h + \hat{h}C_T\Big)\Bigg[\frac{(a-bP)}{\theta v(\xi)}\Bigg[\frac{-e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)}\\
&+\frac{e^{-rt_1}}{r} - \frac{1}{r}\Bigg] + \frac{1}{(\eta+\theta v(\xi))}\Bigg[\frac{e^{(\theta v(\xi)+\eta)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}\\
&-\frac{e^{(\eta-r)t_1}}{(\eta-r)} + \frac{1}{(\eta-r)}\Bigg] - \xi T + C_2(a-bP)\beta t_1\big[e^{-rT} - e^{-rt_1}\big]\\
&+C_2(a-bP)\beta\Bigg[\frac{-Te^{-rT}}{r} + \frac{t_1e^{-rt_1}}{r} + \frac{e^{-rT}}{r^2} - \frac{e^{-rt_1}}{r^2}\Bigg] - \hat{k}C_T\\
&-C_1(1-\beta)(a-bP)(T-t_1) - \frac{\omega T}{\psi}\log\Big(\frac{k_0}{k}\Big)\Bigg]
\end{aligned}
\tag{13.17}
$$

Profit function under cap-and-trade regulations

To determine the total carbon-emissions allowable, the government may set a limit U based on the total emissions from related sources. When the retailer exceeds the limit, it must purchase allowances from other institutions:

The total carbon emissions cost under the cap-and-trade regulations.

$$
\begin{aligned}
TC_2 = C_p\Big[\hat{C}&\Bigg[\frac{(a-bP)}{\theta v(\xi)}\big[e^{\theta v(\xi)t_1} - 1\big] + \frac{1}{(\eta+\theta v(\xi))}\Big[\Big(e^{\eta t_1+\theta v(\xi)t_1} - 1\Big)\Big]\\
&+(a-bP)\beta(T-t_1)\Big] + \hat{h}\Bigg[\frac{(a-bP)}{\theta v(\xi)}\Bigg[\frac{-e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)}\\
&+\frac{e^{-rt_1}}{r} - \frac{1}{r}\Bigg] + \frac{1}{(\eta+\theta v(\xi))}\Bigg[\frac{e^{\eta t_1+\theta v(\xi)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\eta-1)}\\
&+\frac{1}{(\eta-r)}\Bigg] + \hat{k}\Bigg] - C_P U\Big]
\end{aligned}
\tag{13.18}
$$

The profit function considering the mentioned inventory investment for this investment for this case in given by

$$
\begin{aligned}
X_2 = TP_2 - TC_p = &\Bigg[P\Bigg[(a-bP)e^{-rt_1} + \frac{e^{(\eta-r)t_1}}{\eta} - \frac{1}{\eta}\Bigg] + P[(a-bP)\beta(T-t_1)]\\
&-c\Bigg[\frac{(a-bP)}{\theta v(\xi)}\Big[\Big(e^{\theta v(\xi)t_1} - 1\Big)\Big] + \frac{1}{(\eta+\theta v(\xi))}\Big[\Big(e^{(\eta+\theta v(\xi))t_1} - 1\Big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+ (a - bP)\beta(T - t_1)] - h\Bigg[\frac{(a - bP)}{\theta v(\xi)}\Bigg[\frac{-e^{(\eta - r)t_1}}{(\theta v(\xi) + r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi) + r)} \\
&+ \frac{e^{-rt_1}}{r} - \frac{1}{r}\Bigg] + \frac{1}{(\eta + \theta v(\xi))}\Bigg[\frac{e^{(\eta t_1 + \theta v(\xi))t_1}}{(\theta v(\xi) + r)} - \frac{e^{(\eta - r)t_1}}{(\theta v(\xi) + r)} \\
&- \frac{e^{(\eta - r)t_1}}{(\eta - r)} + \frac{1}{(\eta - r)}\Bigg] - \xi T + C_2(a - bP)\beta t_1\Big[e^{-rT} - e^{-rt_1}\Big]\Big] \\
&+ C_2(a - bP)\beta\Bigg[\frac{-Te^{-rT}}{r} + \frac{t_1 e^{-rt_1}}{r} + \frac{e^{-rT}}{r^2} - \frac{e^{-rt_1}}{r^2}\Bigg] - C_1(1 - \beta)(a - bP)(T - t_1) \\
&- \frac{\omega T}{\psi}\log\frac{K_0}{K} - C_T\Big[\hat{C}\Bigg[\frac{(a - bP)}{\theta v(\xi)}\Big[e^{\theta v(\xi)t_1} - 1\Big] + \frac{1}{(\eta + \theta v(\xi))}\Big[\Big(e^{(\eta t_1 + \theta v(\xi))t_1} - 1\Big)\Big] \\
&+ (a - bP)\beta(T - t_1)]] + \hat{h}\Bigg[\frac{(a - bP)}{\theta v(\xi)}\Bigg[\frac{-e^{(\eta - r)t_1}}{(\theta v(\xi) + r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi) + r)} + \frac{e^{-rt_1}}{r} - \frac{1}{r}\Bigg] \\
&+ \frac{1}{(\eta + \theta v(\xi))}\Bigg[\frac{e^{(\eta t_1 + \theta v(\xi))t_1}}{(\theta v(\xi) + r)} - \frac{e^{(\eta - r)t_1}}{(\theta v(\xi) + r)} - \frac{e^{-(\eta - r)t_1}}{(\eta - r)} + \frac{1}{(\eta - r)}\Bigg]\Bigg] + \hat{k}\Big] + C_p U\Big]
\end{aligned} \tag{13.19}
$$

So total average profit is given by

$$
\begin{aligned}
\text{TA } P_2(t_1, T, K, P) &= \frac{1}{T}\Bigg[P\Bigg[(a - bP)e^{-rt_1} + \frac{e^{(\eta - r)t_1}}{\eta} - \frac{1}{\eta}\Bigg] \\
&+ P[(a - bP)\beta(T - t_1)] - \big[c + \hat{c}C_p\big]\Bigg[\frac{(a - bP)}{\theta v(\xi)}\big[e^{\theta v(\xi)t_1} - 1\big] \\
&+ \frac{1}{(\eta + \theta v(\xi))}\big[e^{(\theta v(\xi) + \eta)t_1} - 1\big] + (a - bP)\beta(T - t_1)] \\
&- \Big(h + \hat{h}C_p\Big)\Bigg[\frac{(a - bP)}{\theta v(\xi)}\Bigg[\frac{-e^{(\eta - r)t_1}}{(\theta v(\xi) + r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi) + r)} \\
&+ \frac{e^{-rt_1}}{r} - \frac{1}{r}\Bigg] + \frac{1}{(\eta + \theta v(\xi))}\Bigg[\frac{e^{(\theta v(\xi) + \eta)t_1}}{(\theta v(\xi) + r)} - \frac{e^{(\eta - r)t_1}}{(\theta v(\xi) + r)} \\
&- \frac{e^{(\eta - r)t_1}}{(\eta - r)} + \frac{1}{(\eta - r)}\Bigg] - \xi T + C_2(a - bP)\beta t_1\big[e^{-rT} - e^{-rt_1}\big] \\
&+ C_2(a - bP)\beta\Bigg[\frac{-Te^{-rT}}{r} + \frac{t_1 e^{-rt_1}}{r} + \frac{e^{-rT}}{r^2} - \frac{e^{-rt_1}}{r^2}\Bigg] - \hat{k}C_T \\
&- C_1(1 - \beta)(a - bP)(T - t_1) - \frac{\omega T}{\psi}\log\left(\frac{k_0}{k}\right) + \hat{k}C_p + C_p U\Bigg]
\end{aligned} \tag{13.20}
$$

Profit function under limit carbon emission regulations

As a result of this policy, the retailer must comply with a strict carbon emission limit w. So the carbon emission cost in the case is

$$
TC_3 = \phi[E(t)] - \phi w
$$

$$TC_3 = \phi\left[\frac{(a-bP)}{\theta v(\xi)}\left[e^{\theta v(\xi)t_1} - 1\right] + \frac{1}{(\eta + \theta v(\xi))}\left[e^{(\theta v(\xi)+\eta)t_1} - 1\right]\right.$$
$$+(a-bP)\beta(T-t_1)] + \hat{h}\left[\frac{(a-bP)}{\theta v(\xi)}\left[\frac{-e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)}\right.\right.$$
$$\left.+\frac{e^{-rt_1}}{r} - \frac{1}{r}\right] + \frac{1}{(\eta+\theta v(\xi))}\left[\frac{e^{(\theta v(\xi)+\eta)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\eta-r)}\right.$$
$$\left.\left.+\frac{1}{(\eta-r)}\right] + \hat{k}\right] - \Phi\mathrm{W} \tag{13.21}$$

$$X_3 = \left[P\left[(a-bP)e^{-rt_1} + \frac{e^{(\eta-r)t_1}}{\eta} - \frac{1}{\eta}\right] + P[(a-bP)\beta(T-t_1)]\right.$$
$$-c\left[\frac{(a-bP)}{\theta v(\xi)}\left[\left(e^{\theta v(\xi)t_1} - 1\right)\right] + \frac{1}{(\eta+\theta v(\xi))}\left[e^{(\theta v(\xi)+\eta)t_1} - 1\right]\right.$$
$$+ (a-bP)\beta(T-t_1)] - h\left[\frac{(a-bP)}{\theta v(\xi)}\left[\frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)}\right.\right.$$
$$\left.+ \frac{e^{-rt_1}}{r} - \frac{1}{r}\right] + \frac{1}{(\eta+\theta v(\xi))}\left[\frac{e^{(\theta v(\xi)+\eta t_1)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}\right.$$
$$\left.-\frac{e^{(\eta-r)t_1}}{(\eta-r)} + \frac{1}{(\eta-r)}\right] - \xi T + C_2(a-bP)\beta t_1\left[e^{-rT} - e^{-rt_1}\right]]$$
$$+ C_2(a-bP)\beta\left[\frac{-Te^{-rT}}{r} + \frac{t_1 e^{-rt_1}}{r} + \frac{e^{-rT}}{r^2} - \frac{e^{-rt_1}}{r^2}\right] - C_1(1-\beta)(a-bP)(T-t_1)$$
$$- \frac{\omega T}{\psi}\log\left(\frac{K_0}{K}\right) - \psi\left[\hat{C}\frac{(a-bP)}{\theta v(\xi)}\left[e^{\theta v(\xi)t_1} - 1\right] + \frac{1}{(\eta+\theta v(\xi))}\left[\left(e^{(\theta v(\xi)+\eta t_1)t_1} - 1\right)\right]\right.$$
$$+(a-bP)\beta(T-t_1)]] + \hat{h}\frac{1}{(\eta+\theta v(\xi))}\left[\frac{e^{(\theta v(\xi)+\eta t_1)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}\right.$$
$$\left.-\frac{e^{-(\eta-r)t_1}}{(\eta-r)} + \frac{1}{(\eta-r)}\right] + \hat{k}\Big] - \Phi\mathrm{w}] \tag{13.22}$$

So, the total average profit is given by

$$\mathrm{TA}\ P_3(t_1, T, K, P) = \frac{1}{T}\left[P\left[(a-bP)e^{-rt_1} + \frac{e^{(\eta-r)t_1}}{\eta} - \frac{1}{\eta}\right]\right.$$
$$+ P[(a-bP)\beta(T-t_1)] - \left[c + \hat{c}\phi\right]\left[\frac{(a-bP)}{\theta v(\xi)}\left[e^{\theta v(\xi)t_1} - 1\right]\right.$$
$$+ \frac{1}{(\eta+\theta v(\xi))}\left[e^{(\theta v(\xi)+\eta)t_1} - 1\right] + (a-bP)\beta(T-t_1)]$$
$$-\left(h + \hat{h}\phi\right)\left[\frac{(a-bP)}{\theta v(\xi)}\left[\frac{-e^{(\eta-r)t_1}}{(\theta v(\xi)+r)} + \frac{e^{\theta v(\xi)t_1}}{(\theta v(\xi)+r)}\right.\right.$$
$$\left.+\frac{e^{-rt_1}}{r} - \frac{1}{r}\right] + \frac{1}{(\eta+\theta v(\xi))}\left[\frac{e^{(\theta v(\xi)+\eta)t_1}}{(\theta v(\xi)+r)} - \frac{e^{(\eta-r)t_1}}{(\theta v(\xi)+r)}\right.$$

$$
-\frac{e^{(\eta-r)t_1}}{(\eta-r)}+\frac{1}{(\eta-r)}\Bigg]-\xi T+C_2(a-bP)\beta t_1\left[e^{-rT}-e^{-rt_1}\right]
+C_2(a-bP)\beta\left[\frac{-Te^{-rT}}{r}+\frac{t_1e^{-rt_1}}{r}+\frac{e^{-rT}}{r^2}-\frac{e^{-rt_1}}{r^2}\right]
-C_1(1-\beta)(a-bP)(T-t_1)-\frac{\omega T}{\psi}\log\left(\frac{k_0}{k}\right)+\hat{k}\phi+\phi w\Bigg] \tag{13.23}
$$

13.5 Numerical Illustration

All the units are taken into appropriate units. So, input parameters are given by:

Example 13.1 Consider a situation in which the carbon emission is reduced by the carbon tax policy. The input parameters are as follows:

$a = 1000, b = 1, P = 500, n = 3, C = 25, q = 0.5, \omega = 0.7, r = 0.5, \beta = 0.01,$
$\psi = 5,$

$$
\theta = 0.02, T = 300, \hat{k} = 0.15, C_1 = 0.2, C_P = 0.15, h = 6,
\phi = 4, C_2 = 0.1, \eta = 0.02, k_0 = 10, \hat{c} = 0.5, C_T = 0.15,
h = 6, \hat{h} = 3.
$$

We obtained TA P_1 $(t_1, T, K, P) = 5.33917 \times 10^6$, $t_1 = 271.468$, $k = 3.06604$, $\xi = 482.088\xi = 482.088$

Example 13.2 Consider a situation in which the carbon emission is reduced by the carbon tax policy. The input parameters are as follows:

$a = 1000, b = 1, P = 500, n = 3, C = 25, q = 0.5, \omega = 0.7, r = 0.5, \beta = 0.01,$
$\psi = 5,$

$$
\theta = 0.02, T = 300, \hat{k} = 0.15, C_1 = 0.2, C_P = 0.15,
h = 6, \phi = 4, C_2 = 0.1, \eta = 0.02, k_0 = 10, \hat{c} = 0.5,
C_P = 0.12, h = 6, \hat{h} = 3, U = 0.25.
$$

We obtained TA P_1 $(t_1, T, K, P) = 5.33952 \times 10^6$, $t_1 = 271.471$, $k = 3.0725577$, $\xi = 482.215\xi = 482.215$

Example 13.3 Consider a situation in which the carbon emission is reduced by the carbon tax policy. The input parameters are as follows:

$a = 1000, b = 1, P = 500, n = 3, C = 25, q = 0.5, \omega = 0.7, r = 0.5, \beta = 0.01,$
$\psi = 5,$

$$\theta = 0.02, T = 300, \hat{k} = 0.15, C_1 = 0.2, \phi = 0.08,$$
$$h = 6, \phi = 4, , C_2 = 0.1, \eta = 0.02, k_0 = 10, \hat{c} = 0.5,$$
$$C_T = 0.15, h = 6, \hat{h} = 3, W = 0.35$$

We obtain TA P_1 $(t_1, T, K, P) = 5.33999 \times 10^6$, $t_1 = 271.485$, $k = 3.0708$, $\xi = 482.2\xi = 482.2$

13.6 Concavity

For case 1 (Figs. 13.2 and 13.3).

For case 2 (Figs. 13.4 and 13.5).

For case 3 (Figs. 13.6 and 13.7).

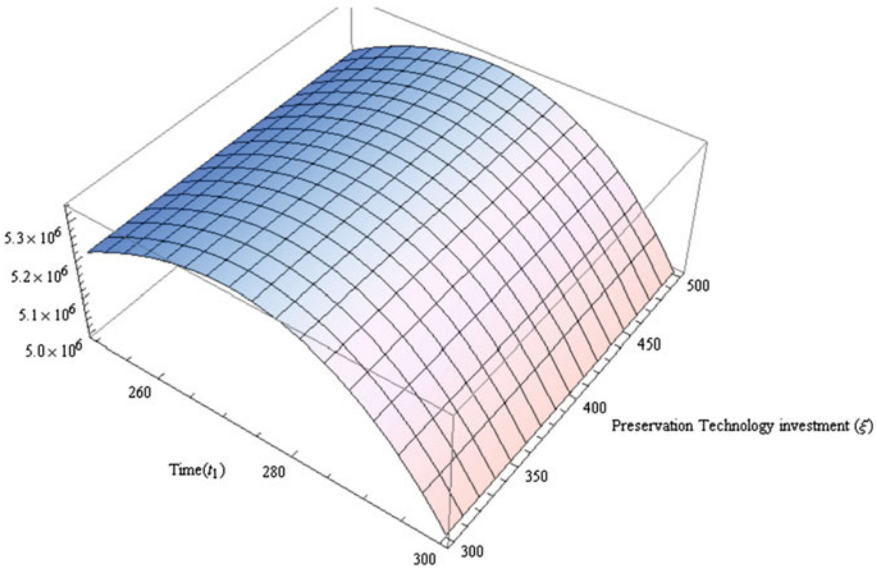

Fig. 13.2 Concavity between time, preservation technology investment, and total profit

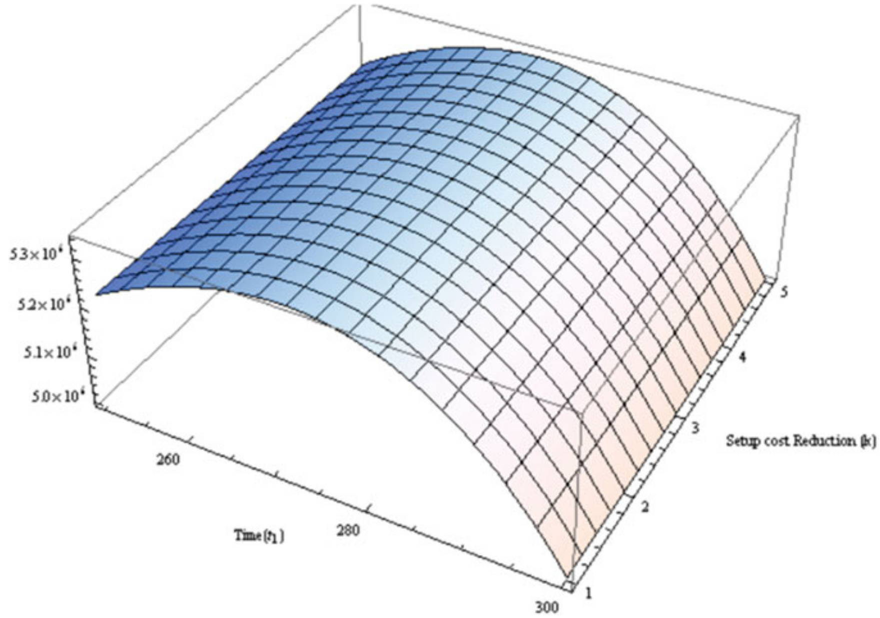

Fig. 13.3 Concavity between time, setup cost reduction, and total profit

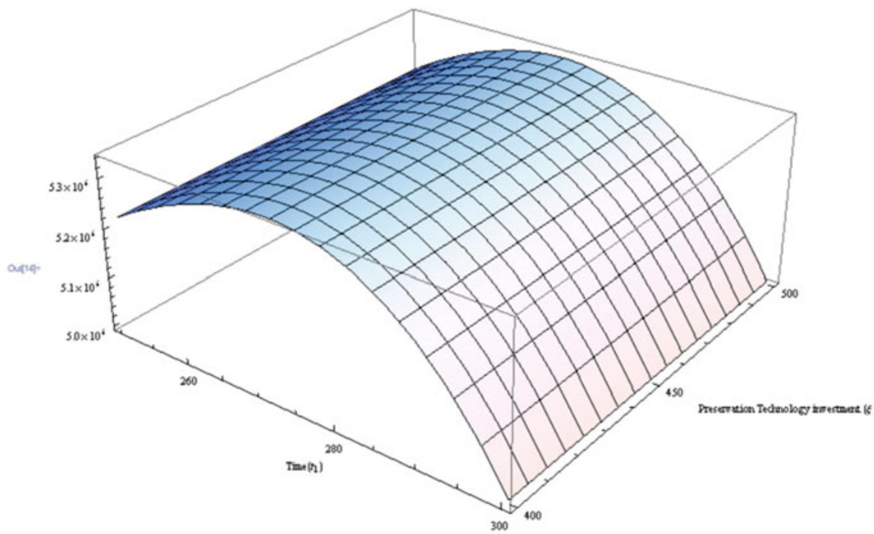

Fig. 13.4 Concavity between time, preservation technology investment, and total profit

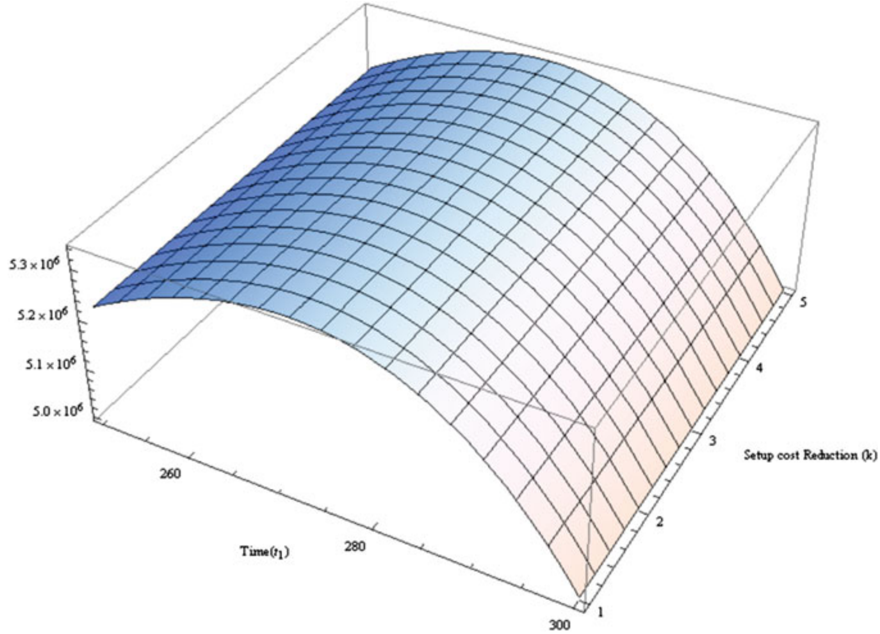

Fig. 13.5 Concavity between time, setup cost reduction, and total profit

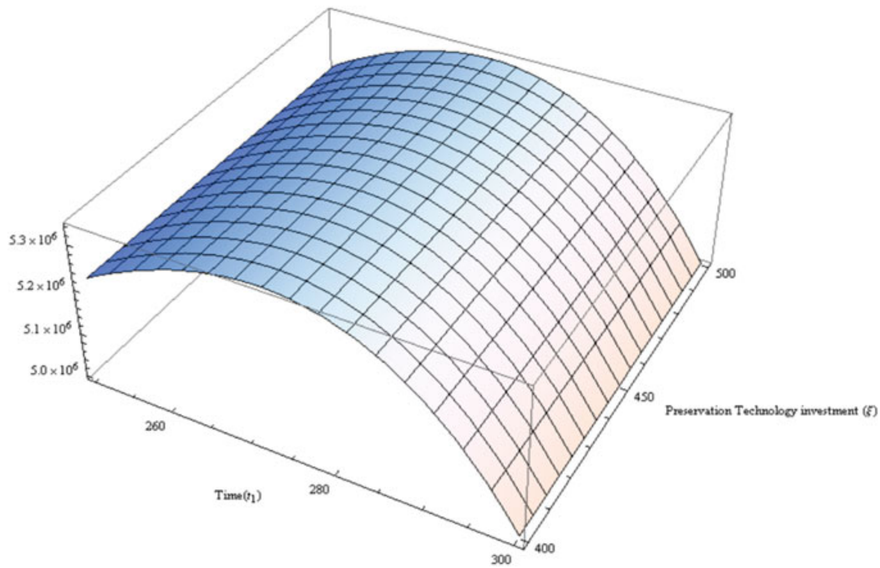

Fig. 13.6 Concavity between time, preservation technology investment, and total profit

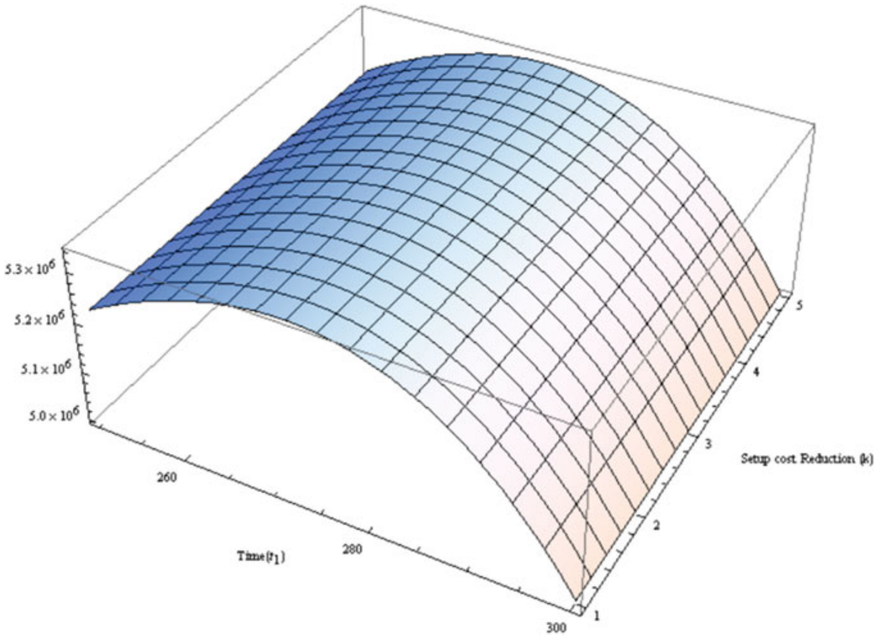

Fig. 13.7 Concavity between time, setup cost reduction, and total profit

13.7 Sensitivity Analysis

For cases 1-3, numerical results are given in Tables 13.2, 13.3, and 13.4.

13.8 Observations

(i) With the increase in the demand parameter ‘a’, the total profit increases. The investment in preservation technology and order cost reduction increase or decrease.

(ii) With the increase in the carbon tax, the total profit decreases which results in the higher carbon tax which lowers the profit.

(iii) With the increase in the demand parameter ‘b’, the total profit decreases and investment in preservation technology and order cost reduction increase or decrease.

(iv) With the increase in the inflation rate, the total profit decreases for all the three cases.

(v) With the increase in the selling price, the total profit increases and investment in preservation technology investment slightly increases.

Table 13.2 Variation of different parameters is given as below for case 1

Parameters	Value (%)	Total Profit TA P_1	t_1	ξ	K
a	5	5.8919×10^6	272.575	484.317	3.08091
	10	6.44536×10^6	273.552	486.309	3.09804
	15	6.99945×10^6	274.295	471.916	2.4478
	−5	4.78735×10^6	270.335	446.802	1.24787
	−10	4.23659×10^6	268.430	423.227	0.12627
	−15	3.68712×10^6	266.905	474.274	3.03236
b	5	5.06313×10^6	270.840	481.121	3.06710
	10	4.78735×10^6	270.335	446.802	1.24787
	15	4.51179×10^6	269.457	478.773	3.05777
	−5	5.61556×10^6	271.889	358.305	9.8769×10^{-8}
	−10	5.8919×10^6	272.575	484.317	3.08091
	−15	6.16855×10^6	273.116	484.094	3.0230
r	5	5.19863×10^6	271.196	432.883	0.475852
	10	5.05911×10^6	270.910	481.235	3.06728
	15	4.92112×10^6	270.535	439.328	0.801674
	−5	5.48053×10^6	271.432	443.363	1.00748
	−10	5.62213×10^6	271.560	430.962	0.239928
	−15	5.76336×10^6	271.841	432.274	0.298364
P	5	5.99549×10^6	304.679	542.673	3.49730
	10	6.01595×10^6	305.602	544.040	3.49213
	15	5.99765×10^6	306.351	545.601	3.50817
	−5	5.8398×10^6	302.181	538.270	3.46783
	−10	5.70550×10^6	300.593	533.476	3.45295
	−15	5.53436×10^6	298.873	523.206	3.07775
β	5	5.33919×10^6	271.458	482.190	3.07240
	10	5.33923×10^6	271.378	443.798	0.922895
	15	5.33921×10^6	271.455	482.190	3.07265
	−5	5.33916×10^6	271.461	482.175	3.07135
	−10	5.33919×10^6	271.344	437.520	0.65229
	−15	5.33914×10^6	271.455	482.166	3.07139
θ	5	5.15286×10^6	261.411	451.203	2.25010
	10	4.97455×10^6	252.003	437.385	2.85222
	15	4.80431×10^6	242.977	418.362	2.45528
	−5	5.53272×10^6	281.599	478.488	2.0181
	−10	5.73236×10^6	292.472	520.450	3.5044
	−15	5.93663×10^6	303.538	540.656	3.47876

(continued)

Table 13.2 (continued)

Parameters	Value (%)	Total Profit TA P_1	t_1	ξ	K
η	5	4.97279×10^6	266.171	472.990	3.024710
	10	4.64115×10^6	260.860	422.607	0.292175
	15	4.33973×10^6	255.679	452.244	2.82629
	−5	5.7457×10^6	276.735	492.093	3.15782
	−10	6.19909×10^6	281.798	466.454	1.37960
	−15	6.70777×10^6	287.146	466.449	0.816356
C	5	5.33997×10^6	271.647	441.936	0.83938
	10	5.33992×10^6	271.317	447.783	1.19009
	15	5.3399×10^6	271.37	431.132	0.2900821
	−5	5.3406×10^6	271.204	426.898	0.072562
	−10	5.34011×10^6	271.678	444.726	0.986017
	−15	5.34017×10^6	271.538	439.655	0.723362

Table 13.3 Variation of different parameters is given as below for case 2

Parameters	% Value	Total Profit TA P_2	t_1	ξ	K
a	5	5.89229×10^6	272.589	484.347	3.07257
	10	6.44579×10^6	273.563	486.331	3.09830
	15	6.9999×10^6	274.433	487.952	3.12748
	−5	4.78767×10^6	270.112	443.016	1.05008
	−10	4.23686×10^6	268.572	436.421	0.808278
	−15	3.68735×10^6	266.917	474.294	3.03243
b	5	5.06346×10^6	270.853	481.141	3.06703
	10	4.78767×10^6	270.112	443.016	1.05008
	15	4.51208×10^6	269.470	478.795	3.05787
	−5	5.78141×10^6	271.674	6.07913	2.8223×10^{-7}
	−10	5.89229×10^6	272.589	484.347	3.08121
	−15	6.16896×10^6	273.062	480.806	2.84571
r	5	5.19893×10^6	271.231	481.414	3.04979
	10	5.05945×10^6	270.924	481.255	3.06714
	15	4.92146×10^6	270.718	434.824	0.80028
	−5	5.4809×10^6	271.554	432.318	0.316611
	−10	5.62247×10^6	271.725	445.788	1.08011
	−15	5.7637×10^6	271.786	443.359	0.88860

(continued)

Table 13.3 (continued)

Parameters	% Value	Total Profit TA P_2	t_1	ξ	K
η	5	4.97313×10^6	265.892	422.764	0.276523
	10	4.64141×10^6	260.923	463.396	2.94307
	15	4.34001×10^6	255.691	452.266	2.82639
	−5	5.74608×10^6	276.921	473.618	2.08771
	−10	6.19951×10^6	282.011	468.140	1.41563
	−15	6.70823×10^6	287.056	449.818	0.0560086
β	5	5.33957×10^6	271.50	443.071	0.930097
	10	5.33955×10^6	271.470	482.212	3.07256
	15	5.33956×10^6	271.466	482.205	3.07245
	−5	5.33951×10^6	271.472	482.216	3.07252
	−10	5.3395×10^6	271.472	482.218	3.07263
	−15	5.33949×10^6	271.443	482.218	3.07262
θ	5	5.1532×10^6	261.572	464.779	2.97205
	10	4.97478×10^6	252.105	445.929	2.78839
	15	4.80463×10^6	243.082	430.263	2.70416
	−5	5.53308×10^6	281.602	467.243	1.45398
	−10	5.73275×10^6	292.315	475.753	0.86108
	−15	5.34011×10^6	303.553	540.683	3.4790
Cp	5	5.33945×10^6	271.465	482.203	3.07246
	10	5.33941×10^6	271.697	439.464	3.07246
	15	5.33931×10^6	271.468	482.108	0.691512
	−5	5.33959×10^6	271.47	482.213	3.06706
	−10	5.33969×10^6	271.234	437.154	0.619494
	−15	5.33973×10^6	271.479	482.229	3.07625

(vi) With the increase in the parameter 'β', the total profit slightly increases or decreases.
(vii) With the increase in the parameter 'θ', the total profit decreases for all the three cases.
(viii) With the increase in the parameter 'η', the total profit decreases for all the three cases.
(ix) With the increase in the carbon trading price, the total profit decreases for all the three cases.
(x) With the increase in the parameter ϕ the total profit decreases.

Table 13.4 Variation of different parameters is given as below for case 3

Parameter	% Value	Total Profit TA $P_3(t_1, T, K, P)$	t_1	ξ	K
a	5	5.89291×10^6	272.707	377.413	9.8137×10^{-9}
	10	6.44636×10^6	273.582	486.363	3.09851
	15	7.00052×10^6	274.452	487.962	3.12637
	−5	4.78805×10^6	270.205	480.074	3.06443
	−10	4.23722×10^6	268.867	439.150	0.938762
	−15	3.68767×10^6	266.933	474.321	3.03256
b	5	5.06393×10^6	270.979	443.186	0.952478
	10	4.78805×10^6	270.205	480.074	3.06443
	15	4.51251×10^6	269.345	432.573	0.52566
	−5	5.62059×10^6	271.523	13.8233	6.7818×10^{-8}
	−10	5.89291×10^6	272.707	377.413	9.8137×10^{-9}
	−15	6.1695×10^6	273.110	485.592	3.11237
r	5	5.19939×10^6	271.247	484.445	3.05003
	10	5.05995×10^6	271.039	445.571	1.16635
	15	4.92187×10^6	270.588	480.649	3.06343
	−5	5.48136×10^6	271.793	439.746	0.756594
	−10	5.62293×10^6	271.599	451.460	1.33571
	−15	5.76416×10^6	272.016	434.091	0.534254
η	5	4.97354×10^6	266.199	473.015	3.02366
	10	4.64181×10^6	260.932	463.349	2.93925
	15	4.34038×10^6	255.680	44,740	2.87141
	−5	5.74658×10^6	276.799	489.149	2.9878
	−10	6.2007×10^6	281.959	450.659	0.42057
	−15	6.70881×10^6	287.379	470.256	1.07376
β	5	5.34005×10^6	271.502	435.395	0.532417
	10	5.34001×10^6	271.487	482.238	3.07248
	15	5.34005×10^6	271.647	441.936	0.83938
	−5	5.34001×10^6	271.276	438.896	0.695483
	−10	5.3399×10^6	271.20	431.339	0.29006
	−15	5.33998×10^6	271.204	426.898	0.72562
θ	5	5.15366×10^6	261.590	464.830	2.97223
	10	4.97527×10^6	252.215	400.083	2.33422
	15	4.80509×10^6	242.874	400.555	1.97732
	−5	5.53356×10^6	282.017	451.10	0.457119
	−10	5.73324×10^6	292.447	469.15	0.658689
	−15	5.93746×10^6	303.570	540.714	3.47920

(continued)

Table 13.4 (continued)

Parameter	% Value	Total Profit TA $P_3(t_1, T, K, P)$	t_1	ξ	K
ϕ	5	5.33997×10^6	271.647	441.936	0.83938
	10	5.33992×10^6	271.317	447.783	1.19009
	15	5.3399×10^6	271.37	431.132	0.2900821
	−5	5.34006×10^6	271.204	426.898	0.072562
	−10	5.34011×10^6	271.678	444.726	0.986017
	−15	5.34017×10^6	271.538	439.655	0.723362

13.9 Conclusion

In this paper, we developed a joint effect of preservation technology investment and order cost reduction on an inventory model under different carbon emission policies. In this study demand is taken as selling price dependent. Preservation technology investment is applied to reduce the deterioration rate. This study provides an optimal decision method on the cycle length and investment in green technology and provides a new carbon insight for low carbon management. Hence, under carbon taxation, reducing carbon emissions should have better benefits. Sensitivity analysis shows that cap and trade and strict carbon limit policies have less impact on the total costs. For future research this study can be extended in an uncertain environment.

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References

Buzacott JA (1975) Economic order quantities with inflation. J Operat Res Soc 26(3):553–558

Covert RP, Philip GC (1973) An EOQ model for items with Weibull distribution deterioration. AIIE Trans 5(4):323–326

Diaby M (2000) Integrated batch size and setup reduction decisions in multi-product, dynamic manufacturing environments. Int J Prod Econ 67(3):219–233

Freimer M, Thomas D, Tyworth J (2006) The value of setup cost reduction and process improvement for the economic production quantity model with defects. Eur J Oper Res 173(1):241–251

Ghare PM (1963) A model for an exponentially decaying inventory. J Ind Engng 14:238–243

Goyal SK, Giri BC (2003) The production–inventory problem of a product with time varying demand, production and deterioration rates. Eur J Oper Res 147(3):549–557

Hasan MR, Roy TC, Daryanto Y, Wee HM (2021) Optimizing inventory level and technology investment under a carbon tax, cap-and-trade and strict carbon limit regulations. Sustain Prod Consum 25:604–621.

Hsu PH, Wee HM, Teng HM (2010) Preservation technology investment for deteriorating inventory. Int J Prod Econ 124(2):388–394

Huang CK, Cheng TL, Kao TC, Goyal SK (2011) An integrated inventory model involving manufacturing setup cost reduction in compound Poisson process. Int J Prod Res 49(4):1219–1228

Khurana D, Chaudhary R (2016) Optimal pricing and ordering policy for deteriorating items with price and stock dependent demand and partial backlogging. Uncertain Supply Chain Manag 4(4):307–318

Kumar N, Kumar S (2016) Inventory Model for non–instantaneous deteriorating items, stock dependent demand, partial backlogging, and inflation over a finite time horizon. Int J Supply Operat Manag 3(1):1168–1191

Lou KR, Wang WC (2013) A comprehensive extension of an integrated inventory model with ordering cost reduction and permissible delay in payments. Appl Math Model 37(7):4709–4716

Misra RB (1979) A note on optimal inventory management under inflation. Naval Res Logist Q 26(1):161–165

Murr DP, Morris LL (1975) Effect of storage temperature on post change in mushrooms. J Am Soc Hortic Sci 100:16–19

Nyea TJ, Jewkes EM, Dilsc DM (2001) Theory and methodology optimal investment in setup reduction in manufacturing systems with WIP inventories. Eur J Oper Res 135:128–141

Sarkar B, Saren S, Sarkar M, Seo YW (2016) A Stackelberg game approach in an integrated inventory model with carbon-emission and setup cost reduction. Sustainability 8(12):1244

Sepehri A, Mishra U, Sarkar B (2021) A sustainable production-inventory model with imperfect quality under preservation technology and quality improvement investment. J Cleaner Prod 310:127332

Singh S, Dube R, Singh SR (2011) Production model with Selling Price dependent demand and Partial Backlogging under inflation

Singh SR, Sharma S (2013) A global optimizing policy for decaying items with ramp-type demand rate under two-level trade credit financing taking account of preservation technology. Adv Decis Sci 2013

Ullah M, Sarkar B, Asghar I (2019) Effects of preservation technology investment on waste generation in a two-echelon supply chain model. Mathematics 7(2)

Whitin TM (1957) Theory of inventory management. Princeton University Press

Xu C, Liu X, Wu C, Yuan B (2020) Optimal inventory control strategies for deteriorating items with a general time-varying demand under carbon emission regulations. Energies 13(4):999

Chapter 14
The Impact of Corporate Credibility on Inventory Management Decisions

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Abstract With the advancement and growth of informatics, the global feud has become extra intense. A firm's supply chain management (SCM) might vary from inventory planning, scheduling, and optimization standards. Nonetheless, Artificial Intelligence (AI) has gently enhanced the decision tree in supply chains by leveraging data and information inside automated processes. The present research seeks to incorporate the concepts of AI with inventory management to facilitate a multi-objective strategy study undertaking a hybrid simulation approach. There would be two stages to it. During the first step, a non-dominated sorting genetic algorithm is applied to probe bank clients' creditworthiness, i.e. the firm. Again, the second phase employs an artificial neural network-based simulation to know the time gap during loan disbursement and restocking point. As a corollary, supply chain costs can be controlled, and overall performance metrics can be improved.

Keywords Artificial intelligence · Supply chain management · Inventory management · Hybrid model

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14.1 Introduction

Inventory management is the navigation of merchandise from the manufacturing point to storage locations and out of these venues to a point-of-sale, thus constituting an efficient and effective supply chain. Inventory management is all about having the exact stuff at the exact destination and within an exact timeframe. This prompts stock control—one that entails understanding when to order, how much to order, and where to hold supplies. Moreover, AI tends to refer to the emulation of core cognitive mechanisms via computerized infrastructure. Expert systems, language processing, voice recognition, etc. are instances of AI functionalities (Burns et al. n.d.).

AI becomes impactful as it can present organizations with details about their practices that the management usually remains uninformed. So on certain occasions, AI can function smarter than human beings; especially with regards to tedious and repetitive engagement like evaluating voluminous data. Therefore, automation has facilitated efficiency build out and also equipped businesses with additional revenue routes. Until the present boom of AI hit, it seemed hard to envision utilizing computer codes that can link banking institutions to corporate clients, but somehow it is becoming one of its extensively used approaches by corporations worldwide. It features advanced machine learning algorithms to forecast when potential buyers are expected to demand things and in which area, accordingly, thus letting them secure bank loans right on time before they run out of stock.

AI has progressed in the past few years into a formidable technology that lets devices make decisions like living beings. So far it has captivated the corporate inquisition everywhere in the world. It is seen with curiosity as the future of sophisticated technological breakthroughs after mobile and cloud platforms; it has also been termed as the "fourth industrial revolution" by experts. Corporations that embrace AI and associated technologies like machine learning and deep learning to unearth unique business insights will seize $1.2 trillion against peers who choose not to, as per Forbes (Manandhar 2019).

Lately, AI has been revolutionizing the supply chain business—delivering metadata and scientific methodology that humans only can try to match. Further, AI helps in the implementation of advanced optimization skill sets for meticulous scalability, enhanced productivity, improved quality, and reduced costs, but without obstructing workplace conditions. An attempt has been made to deploy artificial intellect towards the stock management of bank clients—firms that apply for bank loans.

14.2 Literature Review

Bank credit choices are indeed strategic policy choices centered mostly on the credibility and repaying aptitude of the applicant (Win 2018). As credit risk intendancy and bank profitability are positively related, in the same way, the efficient bank's credit risk management nurtures superior profit (Li and Zou 2014). In this process, customer records like withdrawals or deposits, might improve the quality of bank default projections. The result suggests that credit risk may be effortlessly assessed without prior financial information. Bankers may cut the lending costs and loan approval durations if the bank account information approach is used. Also, when a firm is relatively small, the likelihood of making headway rises. With small businesses, financial reporting fidelity is often considered to be poor. Nevertheless, the bank account information model can connect the dots in the data (Nemoto et al. 2018). A solid banking system has been imperative for any country to have a highly competitive economic outlook. Accountability has strengthened ever since the bank analyzed the customer's creditworthiness before granting loans.

Banks and financial organizations are frequently confronted with the dilemma—risk indicators to account for extending loan consumers. The performance of machines in learning and forecasting is critical (Turkson et al. 2016). Banks exploit automation to locate the prime causes, providing a tailored service to their clients and minimizing the potential NPAs. Resting on Advanced analytics has provided a list of resources to keep the recovery seamless and the lending process transparent enough to ensure debtors don't get over (Nirmal and Derashri 2018).

Moreover, regulatory prosecution for corporate malfeasance does have a real impact on service contracts between a company and a bank. Looking at the number of loans to businesses; the sanction that follows is often lower and even falls short of non-fraudulent entities. Following a penalty, interest rates are mostly harsher and higher than non-fraudulent peers. Consequently, malfeasance impairs the "performance-bank loan" connection, which implies that it has a serious effect on a corporation's chance to get bank loans (Chen et al. 2011).Furthermore, a rise in economic unpredictability is closely linked with banks' credit relocation plans aimed at households throughout portfolios. The swing toward personal loans reflects banks' efforts to bring down the overall risk levels (Mohapatra and Purohit 2021). Here, the proposed models helps in making decisions concerning loan amount, interest rates, and credit conditions for homogeneous risk groups. A borrower clustering model was built on hierarchical clustering and the k-means approach to cluster actual borrowers with identical creditworthiness and default risk levels. Another borrower classification model premised on the stochastic gradient boosting (SGB) prognosis of the number of clusters and hence the risk involved with a potential borrower (Orlova 2021).

Owing to pecuniary repercussions, businesses need to make sure they have a thorough hold of AI or similar technologies to guarantee an effective control. This is especially crucial with the concerns influencing the strategic value. To achieve commercial gains, AI must form part of the risk management, consisting of four major roles: risk identification, risk assessment, risk prevention, and risk control (Al-Blooshi and Nobanee 2020). Decision-makers probably gain by using the Monte Carlo tree search (MCTS) technique, which not only reduces overheads but also averts the adverse bullwhip impact. Additionally, it assists the management in reworking choices and upgrading information in real-time. Furthermore, adaptive order principles based on AI are better in proposing suitable alternatives than one-shot order practices (Preil and Krapp 2022).

According to Modgil et al. (2021), supply chain professionals should use relevant data to implant the mandatory endurance by contemplating aspects and iterations as per the planned structure. As AI has a strong bearing on supply chain success within a quick time, it is therefore proposed to be used with metadata processing considering long-term performance (Belhadi et al. 2021). The Genetic Algorithm lends more confidence to address critical challenges with precision. It applies to a variety of issues inside inventory levels that need to be maintained (Gishnu et al. 2018). Expanding consumer demands, the necessity, and also the prospect propels the networked inventory management. Intense competitive forces as an outcome of market deregulation lead to consumer needs. The networked organization is viewed as a feasible option to cope with varying business circumstances. Integrative inventory management throughout supply chains avenues are more presumptive for collective gain for these associating entities (Verwijmeren et al. 1996). The applicability of data mining tools inside the credit scoring models has garnered a lot of interest in the business community (Koh et al. 2006). By manipulating the Recency, Frequency, and Monetary model, the company can get a clear picture of its consumers and practice customer-centric marketing (Chen et al. 2012). Data mining techniques are indeed critical for the banking industry to improve strategic planning and customer relish (Hassani et al. 2018).

14.3 Objective of Study

i. To ascertain stock replenishment lag time before actually applying for a bank loan.
ii. To investigate how corporate credibility influences a bank's loan sanction decision.

14.4 Research Methodology

As the merging of two domains, AI and SCM, will present the best optimization conclusion, a hybrid technique that has been designed to identify such merging, is characterized as the integration of multiple strategies. The purpose of the hybrid approach is to select the optimal course of action by merging alternative strategies. Further simulated optimization is being used in two stages.

During the first stage, a Non-dominated Sorting Genetic Algorithm (NSGA) is introduced (it can interchangeably be referred to as genetic). This was originally used in 1995 and is one of the vital pathways that guides the favorable solution. The Genetic algorithm can be employed to categorize customer details to retrieve the particulars about the credibility of bank clients. Furthermore, focuses on prioritizing objectives also minimize worst-case scenarios that post the perfect impression as to acceptance and rejection judgment. This is derived from applicant information such as name, age, number of prior loans, and so forth. This enables the differentiation among "good" and "bad" loans, and even a forecast of default likelihood.

In the second stage, an Artificial Neural Network (ANN)-based simulation would be applied to detect the restocking point and loan disbursing mechanisms at every replenishment period (it can interchangeably be referred to as neural). A Neural is a computational model of impulses from an organism's central nervous system, or rather, proficient in machine learning and pattern recognition. The same is illustrated as interconnected networks containing "neurons" that can estimate results premised upon the information. They're meaningful in a wide variety of domains, be it economics or forensic. However, it may be used to classify large datasets only after adequate instruction (Sharma 2017).

14.5 Discussion

To undertake a hybrid approach, the integration of multiple strategies, two phases are designed as follows:

The initial step, including the genetic, will pose the subject of creditworthiness inspection of bank clients. During this phase, all sorts of details would have been collected to screen and accordingly identify prospective borrowers who satisfy specified parameters and become eligible for a loan. While in the next step, loan disbursement choice has been made; the replenishment timeline has been selected and associated with a particular demand pattern. Consequently, by devising a suitable algorithm in both stages, not only can the overall cost be minimized, but also the best decisions on loan approval, loan pay-out, and replenishment of corporate stock can be made.

14.5.1 Genetic Algorithm (NSGA-II)

To begin with the genetic algorithm, three main operations are being performed: non-dominated sorting, density estimation, and the crowded comparison operator. Here, the non-domination technique is the easiest and quickest computational procedure available to date to achieve substantial goals. Perhaps genetic may further accomplish the twin objectives by introducing the crowded comparison operator heading towards the Pareto-optimal front and maintaining a wide solution suite. The crowded comparison operator itself needs minor adjustments to an existing pair-wise tournament selection operator.

Loop—First, a major S0 is generated arbitrarily. Afterwards S0 is sorted using non-domination. A fitness grade is assigned to each solution depending on its degree of non-dominance (1 signify finest level). As for fitness, minimization is anticipated. Then binary tournament selection, recombination, and mutation operators are applied to generate a minor Z0 of size N. The approach is adjusted once the first iteration begins.

Here, Elitism is a useful concept for speeding up the process of discovering the eventual optimal assortment of responses by storing pre-identified viable solutions.

The Elitism Approach for t1 and a certain generation is as follows:

Rt = St [Zt	Major and Minor population together
F = fast-nondominated-sort(Rt) ***F = (F1; F2; : : :), all non-dominated fronts of Rt*** ***St+1 = ;*** ***until jSt+1j < N***	Repeat till you reach a major satiation point
St+1 = St+1 [Fi	Crowding-distance-assignment (Fi) calculate distance consist of ith non-dominated front in the major
Sort(St+1; n)	Sort in descending order with n
St+1 = St+1[0 : N⊥	Pick the first N elements of St + 1
Zt+1 = make-new-pop(St+1) *t = t + 1*	Use selection, crossover, and mutation to sculpt an advance population Zt + 1

The whole process begins with the generation of a composite population Rt = St [Zt]. Here, the population of Rt would be 2 N; non-domination has been used to sort the population Rt. Thereafter, the major population St + 1 is derived by inserting solutions from the first front till a size exceeds N. So, to generate St + 1 population, the solutions of the most recently approved front are sorted by n, and an aggregate of N solution is extracted. This N population will be used for selection, crossover, and mutation to produce a fresh N i.e. Zt + 1. It is significant to mention that we are using a binary tournament selection operator, yet this qualifier has been replaced with the niche comparison operator n.

Procedure—In this operation, each workaround in a GA population gets verified for dominance with a partially filled populace. To proceed, the population's initial response is kept in the set S^0. From the second solution onwards, each response s is placed in juxtaposition to all members of the set S^0. If the solution s dominates any member q of S^0, that would be excluded from S^0. In that same manner, non-members of the non-dominated get purged from S^0. Otherwise, if no member of S^0 dominates solution s, it will be assigned a place into S^0. Through this approach, the set S^0 expands with non-dominated solutions.

S^0=f1g	fast-non-dominated-sort (S) take in first member in S^0
s2S^s62S^0	For each, put in a single solution at a time
S^0 = S^0 [fsg	consist of s in S^0 transiently
zS^0 ^z6=s	for each compare s with other members of S^0
if *s z*, then *S^0 = S^0 nfqg*	if p dominates a member of S^0, delete it else if q s, then S^0 = S nfsg if s is dominated by other members of S^0

(continued)

(continued)

	do not include s in S^0 The members of S^0 will be discarded, and the method will be repeated to find new fronts

Density estimation—The density of a population denotes the extent of crowding in the region in which an actor rests. The density estimate is crucial in ascertaining the Pareto solution set. In general, it assesses the level of resemblance among individuals in a population. To measure the density of solutions encircling a specific spot in the population, consider the average distance between two places on either side of this point (along each of the objectives). This quantity in i_{distance} is a measure of the dimension of the largest cuboid enclosing point without taking into account any other points in the population; it is known as the crowding distance (Mukerjee et al. 2002). In Fig. 14.1, the crowding distance of the *i-th* solution at its front (dark ring) equals the average side-length of the cuboids.

Crowding Distance of Each Point in the Set *I*:

l = *jIj*	Crowding-distance-assignment (I) Number of solutions in I
foreach *i*, set*I[iŁdistan e* = *0*	Initialize
j 2 [1; mŁ	distance for each objective
I = sort(*I; j)*	Sort using each objective value
I[1Łdistan e = *I[lŁdistan e* = *1*	Boundary points are always selected
for *i* = *2* to *(l 1)*	For all other points
I[iŁdistan e = I[iŁdistan e + (I[i + 1Ł:j I[i 1Ł:j)	

I[i:j] denotes the j-th objective function value of the i-th person in the set I. The difficulty of the method is determined by the sorting algorithm. In the worst-case

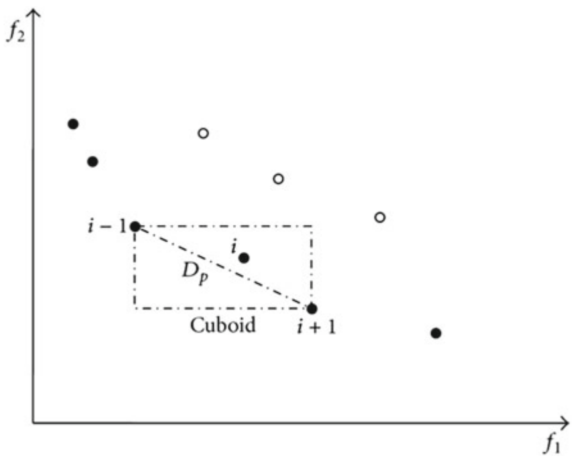

Fig. 14.1 The crowding distance computation. *Source* Reprinted from "A Location Selection Policy of Live Virtual Machine Migration for Power Saving and Load Balancing" by Zhao et al. (2013)

scenario, sorting necessitates O(*mN log N*) computations (where all solutions are in one front).

Crowded Comparison Operator

The crowded comparison operator (n) commands the algorithm's screening mechanism more toward a uniformly spread-out Pareto-optimal front at varying stages. Assuming that each customer *i* in the group holds two main aspects:

1. Non-domination rank (i_{rank})
2. Local crowding distance ($i_{distane}$) We now define a partial order n as:

$$in\,j \text{ if } (irank < jrank) \text{ or } ((irank = jrank) \text{ and } (idistan\ e > jdistan\ e))$$

In other words, simply cull the juncture featuring the least non-domination rank among two solutions (in case of unequal non-domination scores). Conversely, if both points stand on the same front, single out the point which is in a minimal points territory (The cuboid that surrounds it is bigger).

Now let us glance at the overall complexity within one algorithm iteration. As a result, the mechanism followed, as well as the worst-case scenario, is as follows:

1. Non-dominated sorting or ranking O(mN^2),)
2. Crowding distance assignment is O(mNlogN), and
3. Sort on n is O (2N log (2N)).

The worst-case complexity is $O\ (mN^2)$. The population size is given by N, while the number of objectives is given by m. Since the method can just detect N points anywhere along the Pareto-optimal front, its intricacy is proportionate to the precision with which the Pareto-optimal front has been determined (N).

Algorithm Run: GA parameters

The substantial degree of concordance between the two methodologies lends profound justification for the Pareto-frontier paradigm. The binary version of NSGA-II has been used throughout all the trials, with 5 bits assigned to each variable, bringing forth a 25-bit string for each solution. Its mutation rate is at 0.04 (1/25). The explanatory variables are Earning per share, Bank to default ratio, Price to book value, Default to equity ratio and Operating profit margin. The simulations are being executed over a total of 5 generations. To accommodate the iXi = 1 criterion, the x-values of each solution have all been normalized before computing the fitness. Table 14.1 illustrate the mean, standard deviations of returns, and Variance to judge the impact of the clients' creditworthiness. The PBV, DER, and OPM demonstrate a plausible outcome since they diverge very little compared to EPS and BDR. As a result, PBV, DER, and OPM are far more acceptable variables for gauging the corporate clients' creditworthiness.

Figure 14.2 depicts the first phase. The SO technique is being used for the first half of the year. Data has been processed with the genetic algorithm and the relevant information has been retrieved, thus vindicating the creditworthiness targets. On securing St + 1, the ideal statistics moved to execute the neural algorithm again for

Table 14.1 Descriptive statistics for clients' creditworthiness on different variables

Variable	EPS	BDR	PBV	DER	OPM
1	58	22	82.4	22.67	110.2
2	64	45	34.8	45.7	10.01
3	55.79	72.5	24.8	33.52	56.23
4	23.89	56.4	22.4	99.48	23.66
5	99.55	64.3	34.9	33.67	33.62
Sum	301.23	260.2	199.3	235.04	233.72
Mean	60.24	52.04	39.86	47.01	46.74
SD	26.95	19.62	24.45	30.44	39.27
Variance	726.38	384.97	597.87	926.79	1542.78

Source Author's own computations based on imaginary data
Notes EPS stands for earning per share, BDR stands for bank to default ratio, PBV stands for price to book value, DER stands for default to equity ratio and OPM stands for operating profit margin

the next half of the year and forecast the replenishment schedule for corporate clients who seek bankroll to manage the stock level.

The allocation X has been determined from a demand function, Z = 26 V + 10, and indeed the corresponding allocation has been derived by normalizing Xi = Zi. Considering this specific case, researchers selected 10 epsilon vectors from a range of Pareto points detected by the genetic algorithm, leading to a more uniform distribution.

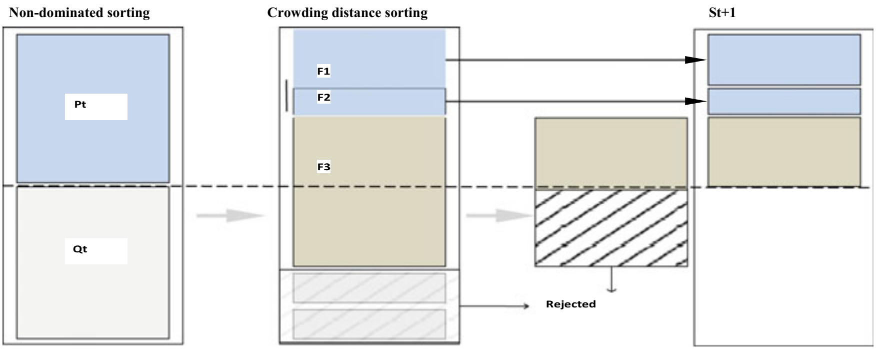

Fig. 14.2 The sorting method in NSGA-II. *Source* Reprinted from "A fast and elitist multiobjective genetic algorithm: NSGA-II." by Deb et al. (2002)

14.5.2 Neural Algorithm

There are mega options available regarding merging approaches in the supply chain. In that respect, neural is an effective AI approach. In this, projections are derived through multi-layered feed-forward networks. Here, the first is the input tier and the second and third tiers serve as hidden layers, that is, the output layers. This output layer, which primarily links up to the target values, is indeed the final layer. As outlined in Fig. 14.3, data can be shared throughout the layers via links.

Besides, input factors, such as the number of neurons, the number of layers, the training technique, the kind of activation function, the number of epochs, and the weights, do have a role in Neural efficacy. Therefore, it is crucial to comprehend the AI and simulation tools that could be implemented on the supply network. Accordingly, bankers might very well leverage this same model's output to optimize their assets based on the replenish clock and loan dissemination plan. This helps to determine the most effective solution for inventory management decisions, to uphold the corporate reputation, notably in the banking industry.

All these activities are carried out in the simulation environment described below:

Phase two commences with de-noising via Maximal Overlap Discrete Wavelet Transform (MODWT) to achieve better information quality (Dghais and Ismail 2013). Moreover, AI and data-driven simulations can be incorporated into the supply chain process if the dataset is pre-treated. AI gets exact demand estimates during each

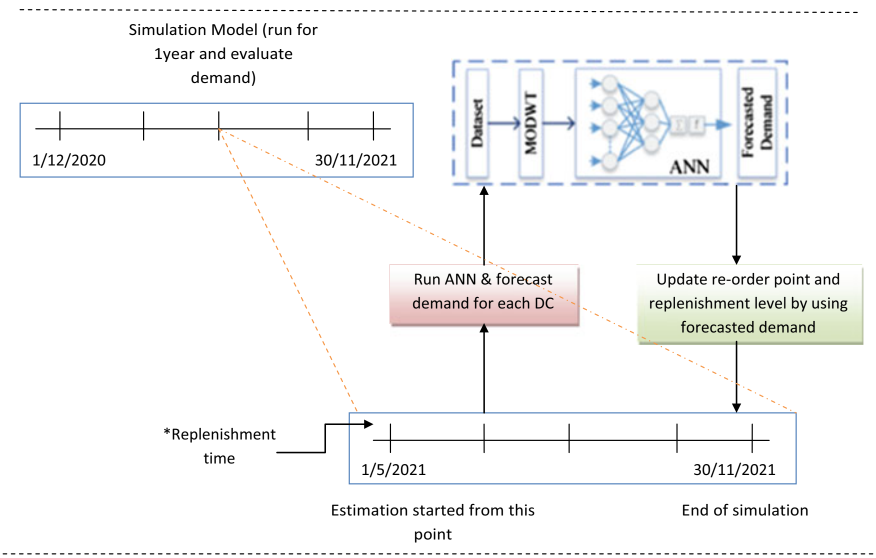

Fig. 14.3 The structure of artificial neural network and maximal overlap discrete wavelet transform. *Source* Researcher's creation; Boru et al. (2019). A Novel Hybrid Artificial Intelligence Based Methodology for the Inventory Routing Problem

replenishment cycle. The projected demand can be evaluated to ascertain the reorder point and order-up-to-level at each replenishment cycle (R).

In terms of enhancing the demand data, outliers and systematic noise can be uncovered. For each replenishing session, the GA-neural estimates supply the chain members' demand. Figure 14.3 depicts the operation of phase 2 and aids in identifying the reorder point and order level according to the forecasted demand, which has been discussed (Boru et al. 2019).

14.6 Conclusion

While living in a generation of high-performance computing, wherein neural networks of machines administrate and supervise, thereby resembling individuals across a wide range of areas. Inventory management is one such domain. The practice of ordering, processing, holding, and distributing stock is referred to as inventory management. Therefore, within the study, AI is being applied to inventory management, specifically for bank clients requesting loans. However, if a client defaults, it puts the bank in danger. To overcome this challenge, bankers can devise a two-stage hybrid technique that comprises simulated optimization. NSGA, in the opening step, filters consumer data to have details about the bank's corporate clients' creditworthiness. In the final stage, an ANN-based simulation assesses the reorder point contingent on loan disbursements timing. The purpose of using the hybrid approach is to discover the optimum solution to a particular situation.

To begin with, a multi-objective search is conducted to model the supply chain system using the NSGA-II simulation optimization. The efficiency of the supply chain system would then be enhanced by applying ANN-based demand forecasting and simulation. So, based on the executed algorithms of NSGA II and ANN, the choice regarding portfolio development and diversification is made. In the hypothetical scenario discussed here, loan allocation to Corporate client allocation is optimized across all categories. As a result, PBV, DER, and OPM are far more acceptable variables for gauging corporate clients' creditworthiness. So concludes the study from both the bank and corporate sides. According to our analysis, corporate entities can reduce their chances of running out of stock by putting more emphasis on the above positive factors. In other words, they can ensure that their loan application is not rejected due to low credibility. Also, a business should apply for a loan with a 6 months margin; otherwise, they might well run out of stock before loan is sanctioned. Furthermore, the bank will be able to improve its profitability. Whenever the actual loan pay-out is less than the demand may look for mentioned feasible variables, so that returns can indeed be augmented. Besides, this paper demonstrated that AI-based simulation may provide satisfying results in the supply chain. Data analysts, researchers, and computer engineers can use this paper and achieve fitting outcomes in the supply chain.

References

Al-Blooshi L, Nobanee H (2020) Applications of artificial intelligence in financial management decisions: a mini-review (SSRN Scholarly Paper ID 3540140). Soc Sci Res Netw. https://doi.org/10.2139/ssrn.3540140

Burns E, Laskowski N, Tucci L (n.d.) What is Artificial Intelligence (AI) and How Does it Work? *TechTarget*. *SearchEnterpriseAI*. https://www.techtarget.com/searchenterpriseai/definition/AI-Artificial-Intelligence

Boru A et al (2019) A novel hybrid artificial intelligence based methodology for the inventory routing problem. Symmetry 11(5):717. https://doi.org/10.3390/sym11050717

Belhadi A et al (2021) Artificial intelligence-driven innovation for enhancing supply chain resilience and performance under the effect of supply chain dynamism: an empirical investigation. Ann Oper Res. https://doi.org/10.1007/s10479-021-03956-x

Chen Y, Zhu S, Wang Y (2011) Corporate fraud and bank loans: evidence from china. China J Account Res 4(3):155–165. https://doi.org/10.1016/j.cjar.2011.07.001

Chen D, Sain SL, Guo K (2012) Data mining for the online retail industry: a case study of RFM model-based customer segmentation using data mining. J Database Mark Cust Strategy Manag 19(3):197–208. https://doi.org/10.1057/dbm.2012.17

Dghais A, Ismail MT (2013) A comparative study between discrete wavelet transform and maximal overlap discrete wavelet transform for testing stationarity. Int J Math Sci Eng 7:471–475

Deb K et al (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans Evolut Comput, 182–197

Hassani H, Huang X, Silva E (2018) Digitalisation and big data mining in banking. Big Data Cognit Comput 2(3). https://doi.org/10.3390/bdcc2030018

Koh H, Tan WC, Goh CP (2006) A two-step method to construct credit scoring models with data mining techniques. Undefined 1(1):96–118. https://www.semanticscholar.org/paper/A-Two-step-Method-to-Construct-Credit-Scoring-with-Koh-Tan/d349b3dd9d4112cb73a3f9575e086c55b475d085

Li F, Zou Y (2014) The impact of credit risk management on profitability of commercial banks: a study of Europe

Modgil S et al (2021) AI technologies and their impact on supply chain resilience during COVID-19. Int J Phys Distrib Logist Manag. https://doi.org/10.1108/IJPDLM-12-2020-0434

Mohapatra S, Purohit SM (2021) The implications of economic uncertainty for bank loan portfolios. Appl Econ 53(45):5242–5266. https://doi.org/10.1080/00036846.2021.1922589

Gishnu M, Kumar UV, G D, Baby S, John K (2018) Modified genetic algorithm for inventory optimization in a serial supply chain. Int Res J Eng Technol, 5(5)

Mukerjee A et al (2002) Multi–objective Evolutionary algorithms for the risk–return trade–off in bank loan management. https://doi.org/10.1111/1475-3995.00375

Manandhar S (2019) Evolution of AI. Le apfrog technology. https://www.lftechnology.com/blog/ai/ai-evolution/

Nemoto N et al (2018) Credit risk reduction effect on small and medium-sized enterprise finance through the use of bank account information. Asian Development Bank Institute, 30

Nirmal D, Derashri P (2018) Banks NPAs and use of artificial intelligence to manage

Orlova EV (2021) Methodology and models for individuals' creditworthiness management using digital footprint data and machine learning methods. Mathematics 9(15). https://doi.org/10.3390/math9151820

Preil D, Krapp M (2022) Artificial intelligence-based inventory management: a Monte Carlo tree search approach. Ann Oper Res 308(1–2):415–439. https://doi.org/10.1007/s10479-021-03935-2

Sharma S (2017) Artificial Neural Network (ANN) in machine learning. Data Science Central. https://www.datasciencecentral.com/artificial-neural-network-ann-in-machine-learning/

Turkson RE, Baagyere EY, Wenya GE (2016) A machine learning approach for predicting bank credit worthiness. Third Int Conf Artif Intell Pattern Recognit (AIPR) 2016:1–7. https://doi.org/10.1109/ICAIPR.2016.7585216

Verwijmeren M, van der Vlist P, van Donselaar K (1996) Networked inventory management information systems: materializing supply chain management. Int J Phys Distrib Logist Manag 26(6):16–31. https://doi.org/10.1108/09600039610125170

Win S (2018) What are the possible future research directions for bank's credit risk assessment research? A systematic review of literature. IEEP 15(4):743–759. https://doi.org/10.1007/s10368-018-0412-z

Zhao J et al (2013) A location selection policy of live virtual machine migration for power saving and load balancing. Sci World J 2013. https://doi.org/10.1155/2013/492615

Chapter 15
A Bidirectional Neural Network Dynamic Inventory Control Model for Reservoir Operation

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Abstract Determining irrigation reservoir operating policies is a complicated challenge because of the uncertainties associated with inflow hydrology, variable upstream, downstream consumption rates, meteorological, environmental flow requirements, crop water intake and soil moisture content among other variables. Artificial Intelligence (AI) method of data mining (DM), dimension reduction (DR) and principal component analysis (PCA) extract the relevant data under uncertainty and formulate the problem like human beings, and make decisions with supervised learning. Soft computing methods portray the system as an artificial neural network (ANN) using fuzzy numbers, fuzzy arithmetic and fuzzy logic (FL) for computations as input–output layers. This soft computing reservoir operation model uses a predictive bidirectional recurrent neural network (BRNN) based on Dynamic Inventory Control (DIC) Technique. The input layer consists of water demand, inflow, evaporation loss, and precipitation data to an irrigation area together with reservoir characteristics. The BRNN has one feed forward neural network (FFNN) and another back propagation neural network (BPNN) which computes the reservoir storage level with the priorities of water conservation and flood controlling goals, respectively. The forward and backward operations are communicated through weight factors assigned to them in order to obtain a better compromised solution. The output layer consists of the diverted water, the released water, the overflow, the irrigation deficit and the flood risk in monthly time scale. Both forward and backward dynamic inventory control operations work based on the previous states of the reservoir and the current states of the input.

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15.1 Introduction

Dynamic inventory control (DIC) and dynamic reservoir operation (DRO) models are two interrelated application areas of artificial intelligence sharing similar characteristics. Both models are aimed at balancing the fluctuating demand with available supply of resources or finished goods. Dynamic inventory control is a more general term with many different application areas. It is evident that the water resource management is a supply chain of special inventory problems under conditions of uncertain demand and controlled supply (Liu and Odanaka 1999). So, DRO can be regarded as a development of the fuzzy dynamic inventory control process in the area of reservoir management. To treat DRO as DIC, it is necessary to understand the complex links among hydrology, weather, water demand prediction, and their management and adaptation strategies to extreme conditions.

The supply chain in DRO is the natural and man-made water supply network. Each source of water is represented by a supply node and each water user sector is represented by a demand node. In the most used senses, reservoirs are used as storage facilities for fluids like water, oil, petrol, etc. in natural or artificial storage structures while; stocks apply to storage of raw materials, intermediates and finished goods in production and marketing settings. The underlying principle in DIC and DRO remains the same: to augment the demand side fluctuations with that of supply side constraints. This is possible by storing the excess supply of resources at times of abundances to meet future anticipated high demand at times of shortage. Due to these high similarities between the underlying philosophies of DRO and DIC, artificial intelligent methods of ANN utilize very similar architectures in their modeling frameworks; however, the data collected to train the two models are different leading to significantly different interpretations.

The DIC helps to meet the imbalances between the customers' demand and production levels by storing resources and working force during excess supply for later deficit periods from previous inventories, overtime production or change of contractual levels. In DRO also, more water is stored in the reservoir during the wet season to meet full irrigation demand during the dry season. Thus, a physical water reservoir dam is an engineering hydraulic facility designed to divert surplus out-of-control flows, such as water flowing from a barrage or river, into controlled releases that meet seasonal agriculture water demands wholly or partially. As a result, a reservoir's purpose is to transform the unpredictable and cyclical character of flows into a succession of outflows in accordance with the internal seasonal water demands. This is achieved through handling the quantity of water held and passing flows to farm units via the reservoir outflow.

A steady and suitable irrigation water delivery can enhance agricultural yield and manufacturing. Irrigation permits smallholder farmers to diversify their planting styles and transition from generating subsistence ingredients to high-price marketplace commodities. It has the capability to offer farmers with long-time period livelihoods and growth for their well-being. This is especially authentic in places wherein there are common droughts, higher evaporative losses, and growing populace pressure. It can also increase resources by means of enhancing productivity, lowering crop failure risks, and making sure of higher production in manufacturing industries. However, different improvement sectors inclusive of municipal, industrial, recreational, and agricultural are competing for fresh water, placing irrigation agriculture in jeopardy.

Expertise of reservoir regulation, including the in-field technique at the actual water consumption location, is vital for getting the most out of a reservoir's irrigation supply (Kumar et al. 2006). According to the reservoir's conditions, requirements and predicted inflow, the reservoir functioning policy specifies the volume of water to be released. In order to integrate this information and make educated decisions on reservoir management and cropping patterns, mathematical models are typically used. These models are representations of the system with a set of optimization constraints.

General purpose reservoirs are used for flood mitigation, along with water storage and discharge. The goal of the flood control operation is to lower the risk of flooding downstream of the dam. Flood water that has been temporarily kept in the reservoir will be progressively released without causing substantial downstream harm. Operating a reservoir that is intended to fulfill both functions—irrigation demand and flood control—is even more difficult. Flood wave mitigation necessitates draining the reservoir as much as possible in order to absorb any incoming flood peak. However, this decision will result in a reduction in the amount of water conserved for irrigation. Typically, such a reservoir would be gradually emptied just before monsoon rains arrived, anticipating a certain flood and hoping that the reservoir would be completely filled by the conclusion of the flood season. However, this expectation may not be realized in all years, and the reservoir may not be filled to its full capacity.

In large-scale irrigation, there are productions of varieties of crops based on crop development until harvest. Thus, supply of each unit of water is associated with yield/production, costs of production, water supply cost, reservoir operation cost and penalty costs for water loss and unmet water demand. Associated with each unit of storage and release, there are costs and benefits associated with production, storage and penalty terms for unmet demands. The optimization problem is to minimize all those costs. To overcome this problem this chapter deals with a Bidirectional Recurrent Neural Network (BRNN) approach for DRO.

The outline of the remaining sections is as follows: Sect. 15.2 explains the basics of Dynamic Inventory Control and Dynamic Reservoir Operations as well as the review of related literature. Section 15.3 outlines the common structure of the bidirectional recurrent neural network (BRNN) as it would be used in the reservoir operation model and Sect. 15.4 is application of the BRNN for the reservoir operation model.

Section 15.5 deals with the theoretical background of training and validation of the model to evaluate the measure best of fit of the model for the intended application using the input data. Section 15.6 serves as the chapter's summary and conclusion.

15.2 Basics of Dynamic Inventory Control and Dynamic Reservoir Operations

Since ancient times, mankind started to collect, transport, consume, store and stockpile resources of different kinds, at least in some primitive form, which may be the birth of inventory control practices, which is one of the richest sciences in management science and operations research. The purpose of inventory management models is to provide an appropriate method for inventory regulation, mostly with regard to costs associated with production, storage and transportation of raw materials, intermediates and finished products.

Before farmers learned irrigation and surplus production they learned to grow more crops during rainy periods and store excess production in dry forms later for dry seasons when the production of fresh crops seems impractical. The emergence of irrigation practices enhanced production, storage and trade during both dry and rainy seasons. The issue now is about storing and release of water to grow demand-based crops. Such storage is possible at the costs of construction, operation, and maintenance of water supply and costs of investment that have given birth to reservoir operation studies. According to Šustrová (2016), there is no universal model for inventory management in all applications but multiple problem-specific models have been developed in literature in search of solutions to each individual production situation on the basis of the existing models. Lest, most researchers acknowledged the first mathematical model for delivery size specification was the model developed by Ford W. Harris called Economic Order Quantity, EOQ for short (Harris 1913). This model is the simplest type of the single-product deterministic dynamic models and—a model of periodically replenished inventory with constant delivery size. The assumption for the EOQ model is a single-level production of a single-item with no capacity constraints. The demand for that item is assumed to be stationary, i.e. demand occurs continuously with a constant rate. The EOQ model is a continuous time model with an infinite planning horizon. The purpose of the model is to find the most economical optimum size of a manufacturing supply which is best for a manufacturer. This means balancing of acquisition and storage costs.

Because firms produce multiple items that are competing for available space and budget, the economic lot scheduling problem (ELSP) with capacity restrictions came into effect (Rogers 1958 and Elmaghraby 1978). Still, the ELSP is a continuous time model with infinite planning horizon that also assumes stationary demand. The optimal solution for the EOQ model is easy to derive but solving the ELSP optimally is NP-hard. Hence, heuristics dominate the area (Šustrová 2016).

Optimization of reservoir operation management is a set of rules that entails optimizing the costs and benefits obtained with storing and/or releasing water to/from the reservoir without compromising the reservoir's water conservation and environment protection objective functions and constraints. The decision of determining the quantity and timing of water to the intended purposes is often referred to as the reservoir operating policy. For instance, the key and primary task of the reservoir in irrigation is to satisfy the critical timely needs of efficient water application, whilst conserving water during the rainy season to mitigate floods at the downstream.

Lai et al. (2021) reviewed rigorously the reservoir optimization methods that were in use over the last several decades, that utilized a variety optimization tools and techniques, including single and multi-objective (MO) programming, beginning with Traditional models, Linear Programming (LP), Non-Linear Programming (NLP), Dynamic Programming (DP), deterministic DP, and stochastic DP, to recent advances in Evolutionary Algorithms (EAs) and ANN. Over the last few decades the use of modeling of reservoir operation systems simulation using ANNs is increasing. These supervised machine learning approaches are preferred to other hard computing methods because they were easily parameterized to find the regression equations between the input and output decisions. ANNs typically do not require a detailed description of the model process by which they obtain a solution but rather generate computerized programs that are used to predict models without assuming the form of the existing relationship. However, in data scarce applications of ANN, like the new reservoir operation study we are looking for, some kind of synthesis and analysis of the raw data and reservoir dynamics have to be learned. We construct a deterministic model from raw data of the proposed crops, irrigation size for demand prediction and use historical data of inflow, precipitation, evaporation loss, etc. in generating input–output data with different operating rules.

15.3 Structure of the Bidirectional Recurrent Neural Network-Based Dynamic Inventory Control Model

Optimal control theory is a well-established branch of applied mathematics that uses differential equations to transform a physical system in the shortest time. Generally, differential equations are a classical way to model dynamics in the physical world. In the context of optimal control theory, a basic inventory dynamic for continuous time units t can be expressed by the following differential equation:

$$\frac{dI(t)}{dt} = P(t-\tau) - D(t) \tag{15.1}$$

where $I(t)$ stands for the inventory level at time t, $P(t-\tau)$ stands for the production level at time $t-\tau$ where τ is the lead time. If time t is discrete, aggregate parameters are taken and the differential equation turns to the difference Eqs. (15.2a) and (15.2b):

$$I(t) - I(t-1) = P(t-\tau) - D(t) \tag{15.2a}$$

$$I(t) = I(t-1) + P(t-\tau) - D(t) \tag{15.2b}$$

The same models (15.1), (15.2a) and (15.2b) apply to DRO if the net water supply S replaces production P used in DIC.

$$V(t) - V(t-1) = S(t-\tau) - D(t) \tag{15.3a}$$

$$V(t) = V(t-1) + S(t-\tau) - D(t) \tag{15.3b}$$

V is used in DRO models to designate the volume of water in the reservoir to differentiate it from the inventory level I.

Artificial neural networks (ANNs) are sophisticated artificial intelligence (AI) systems that can replicate the functioning of vast, complex, and dynamic processes such as reservoir management. They entail data mining (DM), dimension reduction (DR), principal component analysis (PCA), cluster analysis (CA) and multiple regression (MR) to pre-process raw data. The goal is to find major trends in random processes so that future trends can be predicted from observed occurrences or current events can be corrected with certain anticipations of future expected incidents. Programming in a time series, information goes in forward (positive) and backward (negative) directions. It moves ahead from the past to the present and then to the future, and from the future to the present and then to the past in the backward direction. As a result, they can remember the past and predict the future, whereas a traditional RNN can only do so to a limited extent by postponing the output by certain time steps (Tanisaro and Heidemann 2018).

Input nodes, processing elements (neurons), and output nodes are frequently connected in ANNs. In a feed-forward or multilayer perceptron (MLP), they are structured in this order in a unidirectional manner (Dalcin et al. 2021). The input unit consists of only input nodes, whose role is to distribute the input signals to the processing units of the next hidden layer via connection weights and bias terms. Each connection has a weight w and independent terms known as biases b that are adjusted via training the sum of signals from former layers with the activation function f. Every input layer neuron is connected to each and every intermediate layer neuron, allowing the information processing work to be completed in parallel and at the same time. The link between the middle and output layers is the same (Fig. 15.1).

The processing unit implements the mapping of the data provided by the ANN to the output layer. The pattern's set of features is received from the environment's or surrounds' input layer and communicated to the middle Layer. This layer recognizes patterns in all incoming data by recoding it into an adequate internal representation which maintains the patterns' essential features. This analysis is received by the output layer, which turns it into a meaningful interpretation that can be communicated back to the environment (Fig. 15.2).

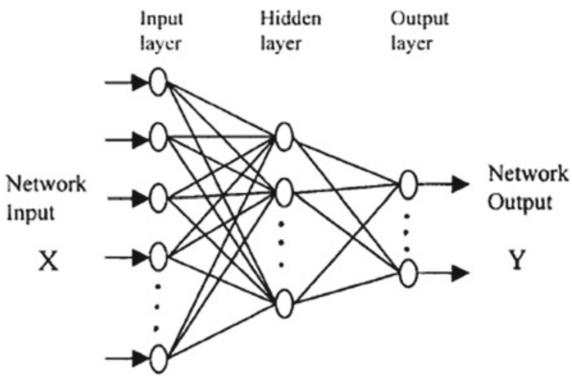

Fig. 15.1 Structure of a simple ANN with three layers

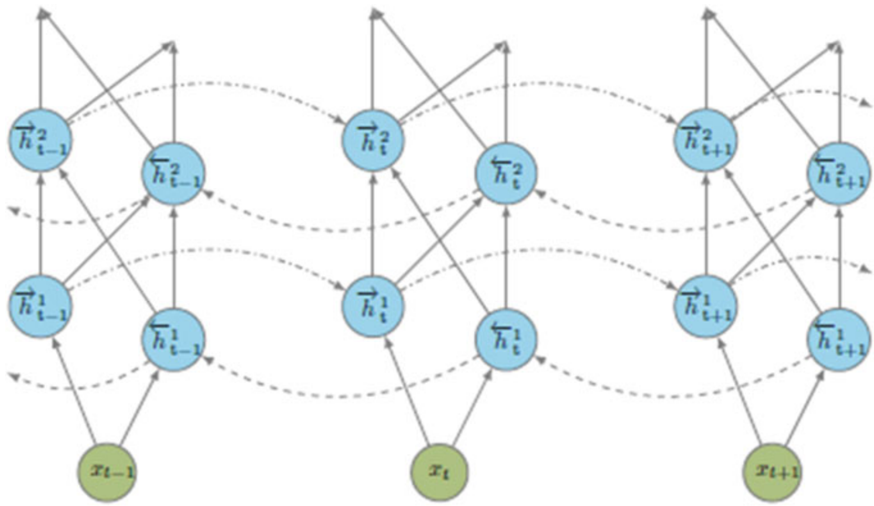

Fig. 15.2 A BRNN architecture with two hidden layers computing in forward direction FFNN and backward direction BPNN (Tanisaro and Heidemann 2018)

The model's core execution incorporates three neural network operating logics: Feed Forward Neural Network (FFNN) which is a First Come First Served (FCFS) decision rule; back-propagation Neural Network (BPNN) which is a First Come Last Served (FCLS) rule when the algorithm communicates data; the Neural Network Correction Operations (NNCO) provide progressive updates. Both the FFNN and BPNN are also recurrent neural network operations because they not only depend on current input data but also on the states of the reservoir before or after the data points are known. This gives the network structure of a bidirectional recurrent neural network (BRNN).

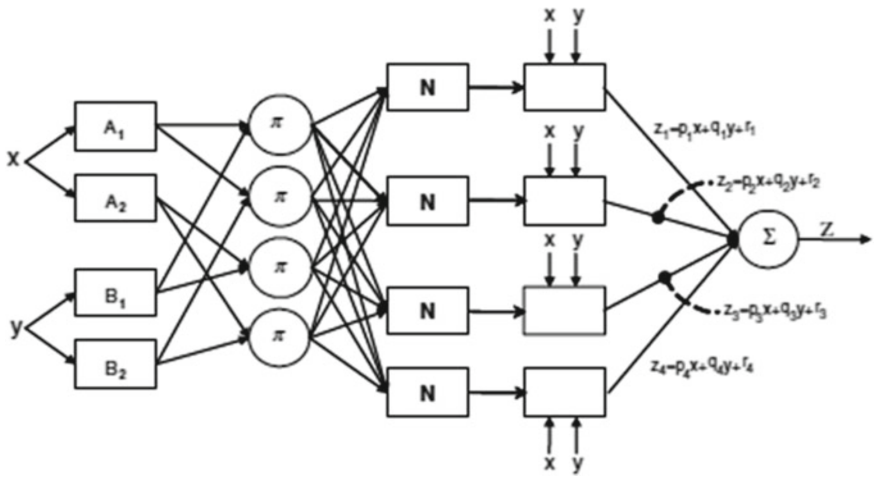

Fig. 15.3 Architecture of five layered NFS as fuzzy user interface. Square nodes have trainable parameters, but circular nodes have fixed parameters: (Chang and Chang 2006)

The structure of the BRNN comprises of two RNNs, one FFNN to compute the forward hidden sequences $\overrightarrow{h}^t$ and the second BPNN to compute the backward hidden sequences $\overleftarrow{h}^t$ for given input layer $x = (x_1, x_2, \ldots, x_T)$ of sequence and desired output layer $y = (y_1, y, \ldots, y_T)$ for T time steps. Given time sequences and training data, the objective is to discover rules that enable the output data to be predicted based on the given input data. Continuous and/or categorical variables can be used as inputs and outputs, respectively. When the outputs are continuous, they are called a regression problem when they are numerical (class labels), and a classification challenge when they are categorical (Fig. 15.3).

$\overrightarrow{h}^t$ is the state of the future from $t = 1$ to T, whereas $\overleftarrow{h}^t$ can be understood as a timeline of events from the past $t = T$ to 1. The activities in the forward and backward direction can be written by Eqs. (15.4a) and (15.4b), respectively (Tanisaro and Heidemann 2018):

$$\overrightarrow{h}^t = f\left(w_{\overrightarrow{h}^{t-1}x}\overrightarrow{h}^{t-1} + w_{\overrightarrow{h}^t x}x^t + b_{\overrightarrow{h}}\right) \tag{15.4a}$$

$$\overleftarrow{h}^t = f\left(w_{\overleftarrow{h}^{t-1}x}\overleftarrow{h}^{t-1} + w_{\overleftarrow{h}^t x}x^t + b_{\overleftarrow{h}}\right) \tag{15.4b}$$

$f(.)$ are weight factors dependent on the states of the reservoir and input layer associated with FFNN while $w_{\overrightarrow{h}^{t-1}x}$ and $w_{\overrightarrow{h}^t x}$ are weight factors dependent on the states of the reservoir and input vector associated with BPNN while $b_{\overrightarrow{h}}$ and $b_{\overleftarrow{h}}$ are the bias terms associated with FFNN and BPNN, respectively, which are independent of the states of the reservoir.

The final state h^t and output $\mathrm{y^t}_\mathrm{o}$ are computed in T frames at the last layer. In the reservoir operation model h^t is the water storage level in the reservoir at the end of the time period t given by Eqs. (15.5a) while $\mathrm{y^t}_\mathrm{o}$ represent the different output measures for reservoir release, diversion, spillover, irrigation deficit, flood risk and other variables at each of the outlets given by Eqs. (15.5b).

$$h^t = f\left(w_{\overrightarrow{h}^t}\overrightarrow{h}^t + w_{\overleftarrow{h}^t}\overleftarrow{h}^t + b_h\right) \tag{15.5a}$$

$$\mathrm{y^t}_\mathrm{o} = \mathrm{f}\left(\mathrm{w_{h^{t-1}o}h^{t-1} + w_{h^to}h^t + w_{x^to}x^t + b_o}\right) \tag{15.5b}$$

in Eqs. (15.5a) $w_{\overrightarrow{h}^t}$ and $w_{\overleftarrow{h}^t}$ are weight factors assigned to the FFNN and BPNN, respectively, b_h is the bias associated with the state of the reservoir. In Eqs. (15.5b) $\mathrm{w_{h^{t-1}o}}$ and $\mathrm{w_{h^to}}$ are weight factors assigned to an output layer that also depend on the beginning and end states of the reservoir while $\mathrm{b_o}$ is the bias associated with the output vector. The output weights are calculated at the readout connection using a linear regression of y^t.

15.4 Design of Neuro-Fuzzy Irrigation Reservoir Operation Using Bidirectional Recurrent Neural Network (BRNN)

Recently a great part of research attention focused on the so called Neuro-fuzzy systems that combine the techniques of fuzzy modeling and neural networks. They combine the advantages of fuzzy logic and neural networks, namely adaptability, quick convergence and high accuracy Šustrová (2016). Redi et al. (2021) developed a Bi-Level Neuro-Fuzzy System (BL-NFS) soft computing methodology for a new reservoir operation in the case study area in Gidabo Irrigation Dam. This chapter presents the methodology in a more elaborative way and explains the links between BL-NFS and the Bidirectional Recurrent Neural Network (BRNN).

15.4.1 Input Layer: Water Demand and Supply Analysis

Farmers lack a good understanding of on-farm water governance, notably how far to irrigate and when to irrigate, enabling farmers to over-irrigate during times of surplus and over-stress crops needlessly during times of shortage, culminating in conflicts in other aspects of the plan. Excessive and inadequate water resource distributions result from glaring holes between farmers' expertise with irrigation water and the actual water consumed in the root zone that do not even promote efficient and prompt agricultural crop absorption of water.

Predicting water demand for multiple crop irrigation is based on an optimization function with the ultimate goal of improving crop productivity, which is dependent on various controlled and uncontrolled variables, making prediction equations more complicated (Cancellirre et al. 2002). Understanding water demand and inflow estimates is the first step in operating an irrigation reservoir. The purpose of the optimization task is to identify the best releases for diverse uses and users while keeping practical constraints in mind. The objective function could be a function of water allocation costs and benefits, or of water allocation shortfalls. Minimum and maximum storage and release are common constraints in reservoir optimization models, and so are the mass conservation and other hydrological and hydraulic limitations.

Models were created with a comprehensive physical characterization of climate–soil–plant interaction offered by FAO (2006) in the early attempts to improve irrigated agriculture. It is based on the water balance between irrigation and water supply to compensate water losses from soil and plant surfaces known as evapotranspiration. Evapotranspiration refers to the loss of water from land and crop surfaces to the atmosphere through evaporation and transpiration. The transition of liquid water to water vapor and the removal of that vapor from surfaces is known as evaporation, such as the soil surface, moist vegetation, pavements, and aquatic bodies. Transpiration is defined as the evaporation of liquid within a plant and the subsequent release of water as vapor through leaf stomata.

The Blaneny Criddle, Radiation, Modified Penman Monteith, and Pan evaporation methods are all available for determining irrigation water requirements. Modified Penman Monteith is thought to provide the greatest outcomes with the fewest possible errors when compared to a living grass reference crop. As a result, it is suggested as the only approach for calculating evapotranspiration (ET0) and crop water requirements (CWR). The amount of water needed to compensate for evapotranspiration losses ET0 in a cultivated field over a given time period was denoted by ${D_i}^t$. To estimate ${D_i}^t$ irrigation scheduling and irrigation water requirement, $ET0^t$ plays the central role. ${D_i}^t$ for different irrigation schemes are calculated by using $ET0^t$, crop K^t and effective precipitation ER^t.

$$ {D_i}^t = ET0^t * K^t - P^t + b_D \tag{15.6a} $$

The irrigation water requirement (IWR) is calculated using the USDA soil conservation service approach, which estimates effective rainfall (Kassam and Smith 2001).

For determining the total irrigation water demand D^t from individual irrigation unit demand ${D_i}^t$ we can use the prediction Eq. (15.6b) below.

$$ D^t = f\left(\sum_{i=1}^{n} w_{ti}{D_i}^t + b_D\right) \tag{15.6b} $$

Equation (15.6b) gives the net irrigation water demand D^t for all cropped areas i with weight factor w_{ti} that depends on the total area of land allocated to each crop throughout the plant growth stages, evapotranspiration, irrigation efficiency, etc.

Similarly, the water supply S from an upstream area u whose demand is ${D_u}^t$ and inflow ${I_u}^t$ with the restriction of compulsory environment flow $({w_e}^t){I_u}^t$ is given by Eq. (15.7).

$$S^t = (1 - {w_e}^t){I_u}^t - {w_u}^t{D_u}^t + {w_V}^t\left(ER^t - E^t\right) + b_s \tag{15.7}$$

where ER^t and E^t are precipitation and evaporation loss with weight factors ${w_V}^t$ that is also dependent on the water volume in the reservoir and b_s is an independent bias term. These input–output forms for irrigation water prediction equations take the form of a simple ANN $y^t = f(\sum_{i=1}^n {w_i}^t x^t + b)$ that calculates the weighted sum of inputs and bias in order to directly apply the activation function.

15.4.2 Output Layer

Three important output units that determine the performance of a reservoir operation include the water diversion to the reservoir intake ${y^t}_V$ defined in terms of the volume rise from the previous state of the reservoir, the water released to each demand node ${y^t}_A$ defined in terms of the volume drop from the previous state of the reservoir and a spillover to the downstream areas ${y^t}_d$ defined in terms of the difference between the excess water supply and the volume rise that are given by Eqs. (15.8), (15.9) and (15.10), respectively

$${y^t}_V = f\left(w_{V^{t-1}V} V^t - w_{V^tV} V^{t-1} + b_V\right) \tag{15.8}$$

$${y^t}_A = f\left(w_{V^{t-1}A} V^{t-1} - w_{V^tA} V^t + b_A\right) \tag{15.9}$$

$${y^t}_d = f\left(w_{V^{t-1}d} D^{t^+} + w_{V^{t-1}d} V^{t-1} - w_{V^td} V^t + b_d\right) \tag{15.10}$$

where f is a mapping rule for transferring neurons from input to output also known as the activation function: a mathematical function that maps a neuron's net input into its output value.

15.4.3 Fuzzy Interface

According to this formulation the annual agricultural production cycle is classified into two growth seasons: a wet season, during which rain falls on the farmland and flows into the reservoir, and a dry season, during which the rainfall is at a minimum and the reservoir's base inflow is insufficient to meet the project's irrigation water requirements. As a result, the reservoir is designed to include ancillary structures to supply more water to farm units to partially or fully meet the irrigation water demand. Without a reservoir, agriculture may experience crop failure and drought.

If there is more water than is needed for irrigation, $D^t \leq S^t$ it may indicate that it is a "wet season," in which case it may be able to meet the entire irrigation water demand through the canal network without having to withdraw additional water from the reservoir. Furthermore, the excess water $D^{t+} = S^t - D^t$ may be kept in the reservoir for future shortfall periods for each time period t; $t = 1, 2, \ldots, T_1$ where T_1 is the end of the wet season. In the terminology of inventory control, this period is a "Stock in" period. On the other hand, if $D^t > S^t$ it means the water supply is less than the irrigation water demand and it may mean that t is a "dry season" in which case it may be possible to supply all the incoming inflow to farm units through the canal network without a need to store it in the reservoir. Furthermore, to satisfy the irrigation deficit $D^{t-} = D^t - S^t$, the reservoir may be used to release all or part of the irrigation water shortage. In the terminology of inventory control, this period is a "Stock out" period. This classification is a little hazy, and it can alter based on environmental, meteorological, hydrological, socioeconomic, catchment region, other knowledgeable users, political and man-made decisions, and so on.

The composition of the linear function that calculates the excess/deficit and the step function assigning the season as dry/wet leads to fuzzy classification is a nonlinear classifier which is not continuous. One way to overcome the difficulty of using it in derivative-based methods is to define a single Sigmoidal function $(x) = \frac{1}{1+e^{-x}}$.

A typical multipurpose reservoir consists of three storage levels with different operation logics: An inactive pool also known as a dead storage level (DSL), a conservation pool also known as an active storage level (ASL) and a flood control level (FCL). An operation rule is said to be linear if fixed percentage of demanded water is released to each reservoir system. This default balancing rule is referred to as the implicit storage balance (Klipsch and Evans 2007). If the implicit balance is not appropriate, user-defined rules could also be used as an explicit formula describing the inflow, storage, and release balance between the reservoirs. Reservoir operational modeling confines the permitted release range to the dam or the outlet's physical restrictions at the start of the release choice phase. If the reservoir is being built for the very first time, all water from the upstream area would be held in it until the level of water exceeds the dead storage level (DSL) for interim reservoir operation. DSL is the minimum storage level in the range across the planning horizon. It is not practicable to extract water for downstream beneficial purposes below this point. However, the water kept below the DSL level is available for in-stream applications such as fishing, navigation and water sport, and so on. On the other hand, the flood control level (FCL) is the highest water level beyond which there is no further storage space and the water overflows above the dam surface.

The conservation pool is used for a variety of purposes, including hydropower generating, navigation, water supply, and agriculture. In most cases, the inactive pool is not available to supply downstream water needs. This storage is typically set aside for hydroelectric head, recreation, ensuring a minimum level for pump diversion, and/or storing silt that will accumulate over the project's lifetime. Reservoirs are designed to release more water into a pool than that entering it at elevations above their FCL, while those at elevations below their ASL seek to discharge less water. In fuzzy terminology, DSL has a membership degree of 0 at the storage level, but FCL has the highest membership degree of 1. Storage level "small" is assigned a fuzzy membership degree of 0, whereas storage level "high" is assigned a fuzzy membership degree of 1. Thus, the storage level falls in between the minimum value DSL and maximum value FCL, $V^{\min} \leq V^t \leq V^{max}$. The other constraint is that the decision to release water at any point in time should not exceed the irrigation water demand given by $y^t{}_A \leq D^t{}_A$ or the difference between both the release and demand should be "modest." Yet another constraint is the release of water at any point in time should not exceed the canal capacity limit (CCL) $y^t{}_A \leq CCL$.

Thus, we assign fuzzy values to the excess water during the wet season and the water deficit during the dry season and define their triangular membership degrees $\mu_{D^+}\left((D^t)^+\right)$ and $\mu_{D^-}\left((D^t)^-\right)$, respectively.

This fuzzification interface transforms the crisp input data such as the reservoir storage, inflow, and release into degrees of match with linguistic values and transforms them into fuzzy variables according to the following steps (Sonawane et al. 2014).

Step 1: Based on expert knowledge, a knowledge foundation that comprises the development of the fuzzy rule set.

- Step 1a: A set of fuzzy 'If–Then' constraints in a schema
- Step 1b: A database that expresses the membership functions of the fuzzy sets used in the fuzzy rules

Step 2: A decision-making unit that executes inference operations on the rules, as well as the use of a fuzzy operator to obtain a single integer that represents each rule's premise.

Step 3: A defuzzification interface that converts the inference's fuzzy findings into a crisp output.

- Step 3a: By inference, the rule's outcome is constructed
- Step 3b: Defuzzification

15.4.4 Hidden Layer

In the reservoir operation model, there is more than one function to be approximately optimized and there are several decision-makers involved to determine the target values to water allocation and environment protection. It would be more appropriate to model the problem as a bi-level programming (BLPP) model with two hierarchical level decision-making units. Conventional Bi-level programming model is a special case of multilevel decision making process with two decision makers (DMs) in two levels or hierarchical organization. BLPP can be used to solve a variety of real-world decision-making challenges in which two decision-makers with different goals and limitations collaborate to reach a single organizational goal. The higher-level decision maker (Leader) indirectly regulates the activities of the lower-level decision maker (Follower), but he cannot dominate their decision. The aims of the two decision makers are partially at odds because one's optimizer has a detrimental impact on the other's goals, yet they want to get one mutually agreed upon the best preferred solution that is also stable. A bi-level solution is said to be stable if the follower cannot make a unilateral decision to alter the decision variable values in his own control in order to improve his own objective function value without changing the resource constraints imposed by the leader's decision variable.

For the proposed BRNN, the water conservation goal measures the unmet irrigation water demand for each time period and the objective function is minimizing the sum of the squares of deviations from the desired target water release output given by Eq. (15.11) while the environment protection goal is to minimize the maximum of all monthly spillovers for all months given by Eq. (15.12). In the two equations the minimizations are taken from all neural network operations.

$$\min_{A} f\left(\sum_{t=1}^{T} \left(y^{t}{}_{A} - D^{t}{}_{A}\right)^{2}\right) \tag{15.11}$$

$$\min max f\left(y^{t}{}_{d} - y^{max}\right) \tag{15.12}$$

In Eq. (15.11), $D^{t}{}_{A}$ is the target value for monthly water release while $y^{t}{}_{A}$ is its output value,

in Eq. (15.12), y^{Max} is the maximum allotted monthly spillover while $y^{t}{}_{d}$ is the output value for monthly spillover f which is the transfer function.

15.4.4.1 Feed Forward Neural Network Operation

In the first stage the neural networks act as an inexperienced man, learning to set its parameters to correspond to the required network topology. In the second stage, however, the network already independently transforms inputs to outputs on the basis of knowledge obtained in the first stage learning process. In the Neuro-fuzzy irrigation model, water conservation operation (WCO) for supplemental irrigation is given precedence in the first operation regulation FFNN, and to guarantee irrigation agriculture, beginning from the first rainy period, enough water should be retained in the reservoir during the wet season so as that the reservoir is projected to be completely filled by the end of the wet season in order to meet the full irrigation demand for the upcoming dry season. The operation logic begins the learning algorithm using a greedy search heuristic (GSH) in "First In, First Out" (FIFO), "First Comes First Served" (FCFS), and "A-Wait-To-See" (WTS) operation which tries to store all the excess water available beginning with the first wet period during the filling phase. This is the behavior of the irrigation manager who wants to guarantee filling of the reservoir for the coming dry season. If the same operation logic FIFO is used during the second phase of releasing water from the reservoir during the dry season, FIFO releases the full irrigation demand during the first dry periods and irrigation deficits may occur during the latter periods. However, once FIFO guarantees filling of the reservoir earlier than at the end of the wet season and there is high flood water, the training of data is made with lower connection weights in the transfer function described by Eq. (15.13).

$$\overrightarrow{V}^{t} = f\left(w_{\overrightarrow{V}^{t-1}x}\overrightarrow{V}^{t-1} + w_{\overrightarrow{V}^{t-1}x}D^{t+} - w_{\overrightarrow{V}^{t-1}x}D^{t-} + b_{\overrightarrow{V}}\right) \tag{15.13}$$

Thus, for each period t; $t = 1, 2, \ldots, T_1$ the above schedule given by the FFNN gives the end of the period state of the reservoir storage level $\overrightarrow{V}^{t}$ as a function of the beginning period state of the reservoir storage level $\overrightarrow{V}^{t-1}$ and the weighted factor of the excess D^{t+} and the deficit D^{t-}. Training algorithms based on the descending gradient depend highly on the initial conditions. An approach to reach a nearer global solution is to repetitively train the ANN with different initial conditions. At $t = 0$ we assume that the storage level is at DSL, $\overrightarrow{V}^{0} = \overrightarrow{V}^{min}$. By the default operation all the weight factors in Eq. (15.13) are equal to 1. Thus, the bias and weight factors associated with FFNN are adjusted with lower values during training. f is a mapping rule for transferring neurons from input to output also known as the activation function: a mathematical function that maps a neuron's net input into its output value. Different activation functions could be used as transfer functions of the linearity into the inputs to a non-linear output function determination. These include Sigmoid (logistic regression) function, the linear function, the hyperbolic tangent (tan h) function and the sine or cosine functions.

To activate Eq. (15.13), we need three important input data about the initial storage level $\overrightarrow{\mathrm{V}}^{\mathrm{t}-1}$ monthly excess supply $\mathrm{D}^{\mathrm{t}+}$ and monthly deficit supply $\mathrm{D}^{\mathrm{t}-}$ should be known in advance of time. The connection weight $\mathrm{w}_{\overrightarrow{\mathrm{V}}^{\mathrm{t}-1}\mathrm{x}}$ is due to the uncertainty of evaporation loss or precipitation to the reservoir surface. The connection weight $\mathrm{w}_{\overrightarrow{\mathrm{V}}^{\mathrm{t}-1}\mathrm{x}}$ depends on the initial storage $\overrightarrow{\mathrm{V}}^{\mathrm{t}-1}$ and the input data x due to uncertainty of inflow, irrigation water demand, and monthly spillover. The time independent bias term $\mathrm{b}_{\overrightarrow{\mathrm{V}}}$ is added due to uncertainty of the dead storage level as the minimum storage level as it may increase due to sedimentation deposit.

15.4.4.2 Back Propagation Operation

The default operation logic of water conservation operation (WCO) with large weight factors and $b_{\overrightarrow{V}}$ term to D^{t+} for some extent, contradicts the flood control objective because, aside from the overflow beyond the reservoir maximum capacity, "little" water is depleted for flood control operation (FCO), and once the water level reaches the highest fuzzy membership degree of 1, all incoming excess water beyond the canal capacity limit overflows.

If, on the other hand, the reservoir is not sufficiently depleted and a large-scale flood occurs, flood inundations may occur downstream. During times of strong inflow, the flood control pool is generally left empty to allow for the storage to runoff. The reservoir's ability to return the pool to its guide curve elevation is hampered by all operational constraints and physical constraints.

Unlike the operation logic in FFNN that begins the learning algorithm using a greedy search heuristic (GSH) in "First In, First Out" (FIFO), "First Comes First Served" (FCFS), "A-Wait-To-See" (WTS) operation, BPNN begins the learning algorithm using a pragmatic logic using "Last In, First Out" (LIFO), "First Comes Last Served" (FCLS), operation, which tries to store in previous periods an amount enough for the remaining dry seasons beginning with the last dry period. This is the behavior of the environment protection authority who wants to guarantee storage as minimum as possible while keeping more room in the reservoir empty so as to trap any incoming flood water.

If the reservoir reaches its maximum capacity before the end of the rainy season but still experiences high flood levels during some wet periods, it is necessary to change the default forward operation logic. As a result, it is required to release water into the environment ahead of schedule during the beginning wet periods in the hopes of storing more water before the end of the wet periods. The releases should not, however, exceed the capacity of the downstream canals. This restriction can be found in the database of the environment protection authorities.

In this case the FFNN is trained with smaller weight factors to D^{t+} and $b_{\overrightarrow{V}}$ term. On the other hand the BPNN learning algorithm computes the minimum storage level to meet full irrigation demand given by Eq. (15.14).

$$\overleftarrow{V}^{t} = f\left(w_{\overleftarrow{V}^{t+1}x} \overleftarrow{V}^{t+1} + w_{\overleftarrow{V}^{t+1}x} D^{(t+1)-} - w_{\overleftarrow{V}^{t}x} D^{t+} + b_{\overleftarrow{V}} \right) \tag{15.14}$$

Thus, for each period, the above BPNN schedule gives the previous state of the reservoir storage level $\overleftarrow{V}^{t}$ as a function of the end of the period state of the reservoir storage level $\overleftarrow{V}^{t+1}$ and the weighted factor of the excess D^{t+} and the deficit $D^{(t+1)-}$. At $t = T$ we assume that $\overrightarrow{V}^{T} = \overrightarrow{V}^{min}$. The bias term and the weight factors associated with BPNN are adjusted during training.

15.4.4.3 Neural Network Correction Operation

In both FFNN and BPNN operations, the water allocated to the periods may be less than the normal irrigation demand, or the overflow may be higher than the permitted monthly overflow beyond the canal capacity limit CCL in which case remedial steps are taken by the Neural Network Correction Operation (NNCO) given by Eqs. (15.15).

$$V^t = f\left(w_{\overrightarrow{V}^t}\overrightarrow{V}^t + w_{\overleftarrow{V}^t}\overleftarrow{V}^t + b_V\right) \tag{15.15}$$

where the weight factors $w_{\overrightarrow{V}^t}$ and $w_{\overleftarrow{V}^t}$ are associated with the states of FFNN $\overrightarrow{V}^t$ BPNN $\overleftarrow{V}^t$, respectively, while b_V is an independent bias term that would be adjusted during training data. Giving equal priorities to WCO and FCO, we get the weight factors $w_{\overrightarrow{V}^t} = w_{\overleftarrow{V}^t} = 0.5$ and the bias term $b_V = V^{min}$.

15.5 Training and Validation of the Irrigation Model Using Data

15.5.1 Training

Five parameters have biggest influence on dam reservoir functioning at the input level in dam and reservoir operation ANN models, which include inflow rate (m/s), water demand, evaporation loss from reservoir water surface (mm), precipitation (mm), and reservoir characterization such as canal capacity limit, maximum and minimum volume. The water volume of storage, evaporation, and rainfall can be computed using area-elevation and storage-elevation curves by multiplying the area by the water depth, evaporation depth, and rainfall depth in each month. Long-term river flow data can be obtained from the study area, and further processed using artificial intelligent methods of data mining (DM), dimension reduction (DR), principal component analysis (PCA), and multiple regression (MR) in pre-processing of the data for the applications of the reservoir operation.

In machine learning of the process of determining the weight factors and bias components is known as training the neural network. Training a network is conceptually similar to fitting the parameters of a regression model. It involves minimizing a loss function: a function that measures how well the output matches the desired outcome. These functions are given for BRNN by Eqs. 15.11–15.12 given in Sect. 4.4. During training, the ANN connection weights are adjusted on a regular basis through incremental simulations. Similar to supervised and unsupervised training, there are supervised and unsupervised learning methods. The use of an external guide is required for supervised learning, and the size of the neural network affects the network generalization performance. These methods are used to determine a network structure using training and generalization data in order to achieve satisfactory performance. The goal is to figure out these parameters for a model that has a certain structure based on information about the problem and a set of parameters using a limited amount of training data. The model's parameters are normally chosen via a supervised training method, but the model's structure is already set.

At random, the data is divided into three groups namely training, testing, m and validation. In all, 80% of the data is used for training, with the remaining 20% used for validation. Afterwards, the training data is separated into two parts: 70% for the training set and 30% for the testing set. These subsets are also separated in a way that ensures statistical consistency and, as a result, they reflect the same statistical population. To establish how representative the training, testing, and validation sets are of one another, the t-test and F-test are utilized.

Batch training, in which weights and biases are only updated after all of the inputs and targets are presented, can be applied to the dynamic reservoir operation network. Based on the annual totals, three water demand and three water supply fuzzy categories would naturally occur for a reservoir management model that includes both WCO and FCO. Different combinations of these three categories result in different reservoir operation model outcomes. Water availability conditions are stochastically reliant on hydrology or rainfall occurrences, and they can be classified as "low," "medium," or "high" in a fuzzy interval representation. The following nine water demand and supply conditions described in the table below provide training, testing, and validation sets. In batch training, the weight and bias terms for optimized operation depend on which of these nine combinations the observed data falls in. Because we are dealing with both wet and dry seasons in considering both WCO and FCO, it would be more appropriate to determine the batch size to annual irrigation plan and water availability condition. Generally, in BRNN training when the water demand is high and the water supply is low, the WCO is most affected and attaining a high irrigation efficiency is difficult, while the overflow in this case is low, such as the negligible flood risk. On the other hand, when the water demand is low and the water supply is high, the FCO is the most affected and the overflow in this case is high, as such the flood risk is difficult to manage while it is possible to work with a low irrigation efficiency (Table 15.1).

FFNN transmits each training data set input–output pattern from the input layer to the output layer in the forward pass. As a result of comparing the network's output to the desired goal output, errors are calculated. assuming that the distribution of

Table 15.1 Nine possible combinations of water demand and supply

	Demand			
Supply		Low	Average	High
	Low	Scenario 1	Scenario 2	Scenario 3
	Average	Scenario 4	Scenario 5	Scenario 6
	High	Scenario 7	Scenario 8	Scenario 9

errors between the estimated and desired output vectors has a Gaussian distribution, with zero mean and a fixed global data-dependent variance, the likelihood criterion is reduced to the more convenient Euclidean distance measure between the estimated and desired output vectors, or the mean-squared-error criterion, which must be kept as minimal as possible during training (Schuster and Paliwal 1997). Continuous variables make up the components of the output vectors in the reservoir operation with unimodal regression function approximation.

The BPNN on the other hand is a supervisory algorithm that regulates the weighted values in the network to identify optimal factors with backward pass. There are supervised and unsupervised backward pass. The error terms are computed by comparing the network output values for each output variable with the desired goal levels. If the error exceeds a certain level, it is projected back through the network to each preceding node, starting at the end of the dry season and ending at the start of the rainy season, and the connection weights are modified accordingly.

Back propagation, which is essentially a gradient descent technique that minimizes the flood danger and unmet irrigation water demand, is considerably the most preferred strategy for training BRNN. The network output values for each output variable are compared to the desired goal levels, and the error terms are computed. If this error exceeds some threshold value it is projected back through the network to each preceding node beginning with the end of the dry season through back to the beginning of the wet season, and as a result the connection weights are adjusted.

The BRNN may be trained using the same algorithm as a normal unidirectional RNN such that the two types of state neurons do not interact. When any type of BPNN is used, the forward and backward pass techniques become slightly more complicated because state and output neurons cannot be changed simultaneously. When using BRNN, the forward and backward journeys over time are virtually comparable to those for a normal MLP. Only at the beginning and conclusion of the training, further treatment of data is necessary. Furthermore, the forward and backward local state derivatives are unknown and set to zero here, assuming that knowledge beyond that point is unimportant for the current update. At each intermediate step, the forward and reverse state inputs have yet to be described. These can be tweaked during the learning process, although they're usually set to a fixed value of 0.5.

The extended bidirectional network's training approach over time can be characterized as follows (Schuster and Paliwal 1997):

- Step 1: Forward Pass

  Calculate all projected outputs by running all input data through the BRNN for one time slice for $1 \leq t \leq T$.

  – Step 1a: Only use the forward pass for states that are moving forward (from $t = 1$ to T) and backward states (from $t = T$ to 1)
  – Step 1b: Execute a forward pass on the output neurons

- Step 2: Backward Pass

  Compute the component of the objective function derivative for forward pass time slice

  - Step 2a: Do backward pass for output neurons
  - Step 2b: Do backward pass just for forward states (from $t = 1$ to T) and backward states (from $t = T$ to 1)
- Step 3: Update Weights.

15.5.2 Evaluation of Model Performance

The final stage in ANN modeling is the verification of the trained model by making comparisons between the observed output and the simulated one. As suggested by Fayaed et al. (2019), Root Mean Square Error (RMSE), Standard Deviation Ratio (RSR), and the Nash–Sutclieffe Efficiency Coefficient (NSE) are the evaluation indicators used to verify model accuracy. They are used to compare the simulation performance of the three neural network operation models (FFNN, BPNN, BRNN) discussed earlier under various parameter settings. In order to evaluate the effect of parameter setting on model precision and calculation speed, the time spent executing the algorithm is utilized as the basis for evaluating the model's speed. Because they reflect the mistake in the units of the constituent of interest, the indices RMSE and RSR are relevant (or squared units). The NSE is a normalized statistic that determines the magnitude of residual variance in contrast to observed data variance. As suggested by previous studies, an RMSE value less than half the standard deviation of the observed data (NSE < 0.5) may be considered low, and if 0.5 < RSR < 0.7, the model performance can be considered satisfactory, (NSE > 0.7) is considered as best fit. An NSE value < 0.0, on the other hand, implies that the mean observed value is a reliable measure than the simulated value, implying poor performance.

The calculation method for these performance measures are given by Eqs. (15.16)-(15.18):

$$RMSE = \sqrt{\sum_{t=1}^{T} \frac{\left(y^t - y_o^t\right)^2}{T}} \tag{15.16}$$

$$RSR = \frac{RMSE}{STDEV_o} = \sqrt{\frac{\sum_{t=1}^{T} \left(y^t - y_o^t\right)^2}{\sum_{t=1}^{T} \left(\overline{y}^t - y_o^t\right)^2}} \tag{15.17}$$

$$NSE = 1 - \frac{\sum_{t=1}^{T} \left(y^t - y_o^t\right)^2}{\sum_{t=1}^{T} \left(\overline{y}^t - y_o^t\right)^2} \tag{15.18}$$

15.6 Conclusion

The water resource management problem in general and the reservoir operation problem in particular are special inventory problems under conditions of uncertain demand and controlled supply. The methodology of bidirectional recurrent neural network (BRNN) is used in this study for an irrigation reservoir operation with multiple objectives at two hierarchical levels known as BLPP for short. This is to reflect that reservoirs typically play two purposes of water conservation as well as flood control. The first level decision-making unit (leader) is concerned with the objective of water conservation that was expressed by the feed forward neural network operation (FFNN), while the second level decision-making unit (follower) is concerned with the objective of environment protection that was expressed by the back propagation neural network operation (BPNN). The two decision-making units communicate their decision using the information communication system established for the purpose. The analogy of this model in inventory control models is applicable when there are two decision-making units like the production manager and the environment protection authority that determines the waste disposal control the limits for production. However, these kinds of supply chain problems are not sufficiently studied in dynamic inventory control models, hence this study suggests further research.

References

Cancellirre A, Giuliano G, Rossi G (2002) A neural network approach for deriving irrigation reservoir operating rules. Water Resour Manag 16:71–88

Chang FJ, Chang YT (2006) Adaptive Neuro-fuzzy inference system for prediction of water level in reservoir. Adv Water Resour 29:1–10

Dalcin AP, Pedrollo OC, Finck JS, Marques GF (2021) Deriving reservoir operational behavior with artificial neural networks: the case of Luiz Gonzaga dam, Brazil. Revista De Gestão De Água Da América Latina 18:e4

Elmaghraby S (1978) The economic lot scheduling problem (ELSP): review and extensions. Manage Sci 24(6):587–598

FAO (2006) Crop Evapotranspiration: FAO Irrigation and Drainage Paper 56. FAO, Rome, Italy

Fayaed SS, Fiyadh SS, Khai WJ, Ahmed AN, Afan HA, Ibrahim RK, El-Shafie A (2019) Improving dam and reservoir operation rules using stochastic dynamic programming and artificial neural network integration model. Sustainability 11:5367

Harris FW (1913) How many parts to make at once in factory. Magazine Manag 10(2):135–152

Jackson I, Tolujevs J, Kegenbekov Z (2020) Review of inventory control models: a classification based on methods of obtaining optimal control parameters. Transp Telecommun 21(3):191–202

Kassam A, Smith M (2001) FAO methodologies on crop water use and crop water productivity. Paper No.CWP-M07, Presented in expert meeting on crop water productivity held at Rome, 3 to 5 December

Kumar DN, Raju KS, Ashok B (2006) Optimal reservoir operation for irrigation of multiple crops using genetic algorithms. J Irrig Drain Eng 132:2

Lai V, Huang YF, Koo CH, Ahmed AN, El-Shafie A (2021) A review of reservoir operation optimizations: from traditional models to metaheuristic algorithms archives of computational methods in engineering

Liu B, Odanaka T (1999) Dynamic fuzzy criterion model for reservoir operations and a case study. Comput Math Appl 37:65–75

Redi M, Dananto M, Thillaigovindan N (2021) A bi-level neuro-fuzzy system soft computing for reservoir operation. Int J Adv Soft Compu Appl 13(3):223–248

Rogers J (1958) A computational approach to the economic lot scheduling problem. Manage Sci 4(3):264–291

Schuster M, Paliwal KK (1997) Bidirectional recurrent neural networks. IEEE Trans Signal Process 45(11):2673–2681

Sonawane A, Desai S, Rajurkar G, Singh D (2014) Soft computing approach for optimal reservoir operation. J Soil Water Conserv 13(1):83–88

Šustrová T (2016) A suitable artificial intelligence model for inventory level optimization. Trends Econ Manag 25(1):48–55

Tanisaro P, Heidemann G (2018) An empirical study on bidirectional recurrent neural networks for human motion recognition. Leibniz International Proceedings in Informatics. 25th International Symposium on Temporal Representation and Reasoning. Editors: Natasha Alechina, Kjetil Nørvåg, and Wojciech Penczek*; Article* 21:1–21

Thair SK, Ayad SM, Hasan HM (2015) Reservoir operation by artificial neural network model: Mosul Dam –Iraq, as a case study. Eng Tech J Part (a) 7:1697–1704

Zhang D, Peng Q, Lin J, Wang D, Liu X, Zhuang J (2019) Simulating reservoir operation using a recurrent neural network algorithm. Water 11:865