

## LIKELIHOOD RATIOS FOR CONTINUOUS TEST RESULTS—MAKING THE CLINICIANS' JOB EASIER OR HARDER?

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**Abstract**—Clinicians' paradigms for considering diagnostic test results require decisions based on the actual test value. However, when the test result is reported on a continuous scale each possible outcome may not result in unique actions. To simplify decision making, clinicians often break down the continuous scale into dichotomous or ordered outcomes.

Likelihood ratios, reported with the test outcome, help summarize the impact of diagnostic tests. Although commonly applied to dichotomous outcomes, likelihood ratios can also be applied to ordinal or continuous results. This application allows investigators to consider the effect of clinically simplifying continuous data into dichotomous or ordinal categories. The parameters of a simple logistic regression equation summarize continuous likelihood ratios, evaluate covariates, generate likelihood ratio lines, and help assess the statistical significance of more complex models. Having visually inspected likelihood ratio lines and considered statistical differences, the investigator should choose the test report format that best accounts the realities driving clinical decisions.

Diagnosis      Sensitivity      Specificity      Likelihood ratios      Test

### INTRODUCTION

Diagnostic tests are used as predictors of target disorders, but their results may be reported in several different ways. In particular, the investigator often has the option of reporting diagnostic test results using several different measurement scales such as dichotomous, ordinal, or continuous. Likelihood ratios provide a method for describing these results, though they are typically calculated and reported only for dichotomous or ordinal test outcomes. We will provide a brief review of likelihood ratios, showing how the likelihood ratio can be determined for 3 general cases: (1) dichotomous tests as a special case of an ordinal scale, (2) ordinal scales with more than 2 ranks, and (3) continuous diagnostic tests.

When the underlying test result is continuous, the result can be reported in any one of these 3 formats. However, continuous likelihood ratios convey the most clinical information for assessing the impact of individual test results [1]. The parameters of a simple logistic regression equation can be used to summarize continuous likelihood ratios, and they can be evaluated to determine if they are statistically appropriate and clinically useful. Because likelihood ratios combine sensitivity and specificity, this approach carries an important advantage over other logistic models that require fitting separate regression equations for sensitivity and specificity [2].

Dichotomous diagnostic tests require classifying results as "positive" vs "negative". Elements of the physical examination are often expressed

Table 1

	Disease +	Disease -
Diagnostic Test +	A	B
Diagnostic Test -	C	D

Prevalence =  $(a + c)/(a + b + c + d)$

Sensitivity =  $a/(a + c)$

Specificity =  $d/(b + d)$

Positive predictive value =  $a/(a + b)$

Negative predictive value =  $d/(c + d)$

Likelihood ratio positive = sensitivity/(1 - specificity)

Likelihood ratio negative = (1 - sensitivity)/specificity

this way, such as the presence of an S3 gallop for heart failure or shifting dullness for ascites. The clinician's overall impression of the likelihood of a target disorder can be reported in more than two levels, such as "high likelihood", "intermediate", or "low likelihood." Laboratory results can also be reported on ordinal or continuous scales; for example, a serum ferritin can be classified by four levels:  $\leq 18$ , 19–45, 46–100, or  $> 100$  ng/ml [3]. Generally, laboratory results are reported with the raw value compared to reference ranges that allow classifying the result as "high", "normal", or "low".

Considering test results as dichotomous or ordinal is more convenient than accepting test results as continuous. Investigators and clinicians must decide whether these advantages outweigh the increased predictive ability conferred by the continuous result. For example, when the serum ferritin is dichotomized at 45 ng/ml, patients with a ferritin value of 44 ng/ml would be treated as identical to those with a value of 19 ng/ml, but differently from those with values of 46 ng/ml. Test report information might be improved with appropriate multilevel or continuous likelihood ratios by accounting for the increasing likelihood of iron deficiency with the level of serum ferritin.

Likelihood ratios reflect the probability that a patient with the target disorder has the test finding compared to the probability that a patient without the target disorder has the same result. This definition applies whether the diagnostic test is reported on a dichotomous, ordinal, or continuous scale. We will discuss LRs in the  $2 \times 2$  table case that address the clinical question "What is the probability that my patient has the target disorder given a certain test result?" when the test result is dichotomized into positive or negative outcomes (Table 1). Next, we will consider test results with more than 2 ordered levels, amplifying the proposals of others for multilevel results [4, 5]. Finally, we will quantify the loss of pre-

dictive ability when continuous predictors are reported in ordinal categories. When this loss of predictive ability is both statistically and clinically significant, the investigator should consider reporting the continuous likelihood ratio rather than the multilevel likelihood ratios. Although the final decision about test report format should be made on clinical grounds, investigators should feel reassured about multilevel formats when the loss of predictive ability is not statistically significant.

#### LIKELIHOOD RATIOS CALCULATED FROM DATA IN AN $n \times 2$ TABLE

Calculating the sensitivity and specificity of tests measured on continuous scales (e.g. serum ferritin, creatine phosphokinase, millimeters of ST-segment depression on an exercise tolerance test, or ejection fraction) requires selecting a cutpoint beyond which the result is considered positive for the target disorder. Although sensitivity and specificity are fundamental values calculated from the columns of  $2 \times 2$  tables, in actual practice the clinician infers "horizontally" across the table, rather than "vertically". Rather than being concerned with the question "What proportion of patients with the target disorder have positive test results?", the clinical thought process addresses the question "What is the probability my patient has the target disorder given a positive (or negative) test result?" While it might seem that positive predictive value is the most appropriate value to report, it is not generalizable since its value depends on the target disorder's prevalence. Like sensitivity and specificity, likelihood ratios have the advantage that they are independent of the prevalence of the target condition. This independence holds when the patients studied with the target condition are representative of all those with the condition; similarly, patients studied without the target condition must be representative of all those without the condition.

To better understand how likelihood ratios can be used to estimate the probability of a target disorder, it helps to refer to the conventional form of Bayes rule. The conditional probabilities of the presence or absence of the target condition are:

$$P(D+|T+) = \frac{P(D+)}{P(T+) \cdot P(T+|D+)} \quad (1)$$

and

$$P(D-|T+) = \frac{P(D-)}{P(T+) \cdot P(T+|D-)} \quad (2)$$

where  $D+ =$  target disorder present,  $D- =$  target disorder absent,  $T+ =$  test positive,  $T- =$  test negative [6]. In words, the left side of equation (1) reads “the probability of the target disorder given the patient has a positive test.” The numerator of the right side of equation (1) reads “the probability of any patient having the target disorder”, whereas the denominator is the “probability of any patient having a positive test multiplied by the probability of a positive test given the patient has the target disorder.” Equation (2) appears similar, but represents the probability that a patient will be free of the target disorder despite a positive test. Thus, equations (1) and (2) describe the only two situations for patients who have a given test result: they either have the target disorder or they do not. Dividing equation (1) by equation (2) gives the following:

$$\frac{P(D+|T+)}{P(D-|T+)} = \frac{P(D+)}{P(D-)} \cdot \frac{P(T+|D+)}{P(T+|D-)} \quad (3)$$

This equation can be recognized more easily in its general form:

$$\begin{aligned} &\text{posterior odds of target disorder} \\ &= \text{prior odds} \times \text{LR} + . \quad (4) \end{aligned}$$

The posterior odds of the target disorder are clinically relevant because they combine the assessment of the odds of the target disorder before a test result is obtained (the prior odds) and the likelihood of the test result given the target disorder (the likelihood ratio). By understanding the prior odds and likelihood ratio the clinician, before ordering a test, estimates the impact of any result on their assessment of the target disorder. Tests that will not have a meaningful impact on the posterior odds of the target disorder can be avoided.

While the above development is described in terms of  $2 \times 2$  tables, likelihood ratios can also be calculated from tables with more than two rows. There are two different likelihood ratios that can be calculated at each level of an  $n \times 2$  table. In the first approach the likelihood ratio is used to answer the question “what is the proportion of patients with the target disorder who have a certain test result?”. This would be the relevant likelihood ratio when clinical decisions differ for each test result level. These multilevel likelihood ratios derive from dividing the probability of a patient having a certain test result among all those with the target disorder by the probability of a patient having the same

result among those without the target disorder.

A second approach is used when the clinician is not really interested in the likelihood ratio at each level, but wants to treat patients similarly when their results are the same or more abnormal than the specified level. These dichotomized likelihood ratios are determined by sequentially considering values above each cutpoint as positive, and those below as negative for the target disorder. The confidence interval around a likelihood ratio can be calculated from a Taylor series approximation [7].

#### LIKELIHOOD RATIOS CALCULATED FROM LOGISTIC REGRESSION MODELING

There is an alternative derivation for likelihood ratios of a  $2 \times 2$  table that generalizes to continuous test outcomes. This derivation requires recognizing that test results and the target disorder status can be modeled using logistic regression. Logistic models have a dependent dichotomous variable (i.e. the presence/absence of the target disorder) and independent predictors (i.e. diagnostic tests) that yield the clinically important probability of the target disorder given any test outcome. Logistic regression is especially appropriate here since the dependent variable is the log odds of the target disorder.

To perform logistic regression on dichotomized test results we first define a variable,  $x$ , so that  $x = 1$  if the diagnostic test result is considered positive and  $x = 0$  if the result is negative. We also define a variable,  $D$ , so that  $D = 1$  when the target disorder is present and  $D = 0$  when the target disorder is absent. Next, we perform logistic regression of  $D$  on  $x$ , solving for  $\alpha$  and  $\beta$ :

$$\ln \frac{P(D = 1|x)}{P(D = 0|x)} = \alpha + \beta x. \quad (5)$$

In terms of the odds of the target disorder for a certain result, equation (5) can be rewritten:

$$\frac{P(D = 1|x)}{P(D = 0|x)} = \exp(\alpha + \beta x). \quad (6)$$

When  $x = 1$  the left sides of the equations apply to a “positive” test so that the numerator represents patients with true positive results and the denominator represents those with false positive outcomes. Similarly, when  $x = 0$  the results apply to a “negative” test result so that the numerator represents patients with false negative results while the denominator represents those with true negative outcomes.

For any  $2 \times 2$  table, and treating  $\alpha$  and  $\beta$  as known constants, equation (7) below demonstrates that we can find a value of  $x$  between 0 and 1 (we will call the value  $x'$ ) for which  $\alpha + \beta x'$  is the log odds at the target disorder's prevalence. Interestingly, this value of  $x'$  describes the proportion of patients in the source population with an abnormal test result and is best understood by working through an example (see below). To solve for  $x'$ , we estimate  $\alpha$  and  $\beta$  from logistic regression using the  $2 \times 2$  table results as predictors of the target disorder. We also directly estimate from the  $2 \times 2$  table the prevalence of the target disorder in the population studied,  $p'$ , to derive the prior odds of the target disorder. Substituting these values ( $\alpha$ ,  $\beta$ , and  $p'$ ) into equation (5) gives us the expression:  $\ln[p'/(1-p')] = \alpha + \beta x'$  and solving for  $x'$  gives:

$$x' = \frac{\ln[p'/(1-p')] - \alpha}{\beta}. \quad (7)$$

Algebraically manipulating equation (4) to solve for the likelihood ratio gives the equation  $LR = \text{posterior odds/prior odds}$ . We can take the logarithm of both side of this equation so that:

$$\begin{aligned} \ln LR &= \ln \frac{P(\text{target disorder}|x)}{P(\text{no target disorder}|x)} \\ &\quad - \ln \frac{P(\text{target disorder})}{P(\text{no target disorder})}. \end{aligned} \quad (8)$$

To obtain the second term of the right side of the equation we set the probability of the target disorder = prevalence, so that substitution gives:

$$\ln LR = (\alpha + \beta x) - (\alpha + \beta x') = \beta(x - x'). \quad (9)$$

Exponentiating this equation allows us to solve for the likelihood ratio:

$$LR = \exp[\beta(x - x')]. \quad (10)$$

The simple equation provides the solution for both the positive and negative likelihood ratio. The variable  $x$  has only two values:  $x = 1$  when the test is positive and  $x = 0$  when the test is negative. Consequently, when  $x = 1$  the likelihood ratio is the positive likelihood ratio; when  $x = 0$  the negative likelihood ratio is obtained.

The confidence interval for equation (10) is derived from the standard asymptotic results to give an approximate 95% confidence interval:

$$\exp[(\beta \pm 1.96 \text{ se}(\beta)) \cdot (x - x')]. \quad (11)$$

### *An example for the $2 \times 2$ table*

Suppose a sample of 40 diseased and 50 non-diseased individuals are all given a diagnostic test where a prespecified cutpoint yields  $a = 30$ ,  $b = 5$ ,  $c = 10$ ,  $d = 45$  (see Table 1). The estimated sensitivity = 0.75 specificity = 0.90, and prevalence = 0.44. A logistic model where the probability of the target disorder is a function of the test result yields  $\alpha = -1.5041$  and  $\beta = 3.2958 \pm 0.596$ . From the above results,  $x' = 0.3887$  [see equation (7)] and we note that this also represents the probability of an abnormal test in the study population (from Table 1 this would be

$$\frac{a + b}{a + b + c + d} = 35/90).$$

From the  $2 \times 2$  table we use the sensitivity (0.75) and specificity (0.90) to get the  $LR+ = 7.5$  and  $LR- = 0.28$ . Since likelihood ratios are point estimates we also want to calculate their 95% confidence interval; a Taylor series approximation yields 95% confidence intervals of  $3.20 < LR+ < 17.56$  and  $0.16 < LR- < 0.48$  [7]. Alternatively, we can use equation (10) to estimate the likelihood ratios from the logistic model: for the positive likelihood ratio we use  $x = 1$  in equation (10) and for the negative likelihood ratio we use  $x = 0$ . While the point estimates are identical to the above, the confidence intervals around these results are derived from equation (11) and are narrower than those from the Taylor series expansion:  $3.67 < LR+ < 15.32$  and  $0.18 < LR- < 0.44$ .

### LIKELIHOOD RATIOS FOR CONTINUOUS PREDICTORS

The above derivation took a test result and recoded it as a dichotomous positive ( $x = 1$ ), or negative ( $x = 0$ ) finding. However, suppose we chose to retain the actual test value without reclassifying each result as positive or negative? When we retain the actual test value rather than dichotomizing results, we can estimate the likelihood ratios for each continuous outcome. From our earlier serum ferritin example we would need a likelihood ratio for a serum ferritin of exactly 44 ng/ml, rather than using a single likelihood ratio applicable to all values between 19 and 45 ng/ml.

Calculating a likelihood ratio for continuous tests requires assuming the logistic regression model specified in equation (6). As in the dichotomous case, the likelihood ratio is again defined by equation (10). However, the value  $x$

Table 2

	Non-Responder	Responder
Overweight	29	18
Not Overweight	15	32

now represents an actual test outcome rather than a dichotomized result. The continuous  $x'$  has a concrete interpretation: a group of individuals with an average test result of  $x'$  would have the same prevalence of the target disorder as the source population. Alternatively stated, an individual in the source population with a test result of  $x'$  has an odds of disease the same as the prior odds—the test result is indeterminate ( $LR \approx 1$ , [8]). As with all logistic regression models, no distributional assumptions (e.g. normality) are required for the predictors.

#### *An example for continuous predictors*

A study performed to evaluate the efficacy of hepatitis B vaccine found that the Overweight Index was a “screening” test for adequate vaccine response [9]. For illustrative purposes, we will first dichotomize the results of the Overweight Index into “overweight” (index  $\geq 31$ ) or “not overweight” (index  $< 31$ ) and compare it to the reference standard of immunogenicity status based on vaccine response (non-responder vs responder, see Table 2). Then, we will show that logistic regression on dichotomous data yields the same likelihood ratio as those obtained directly from the  $2 \times 2$  table. Finally, we will demonstrate how to apply the logistic regression model to actual test values.

We define the target disorder of impaired immunogenicity as a person who does not respond to the vaccine, and find the pretest probability of non-response is 47% (N.B. although this example may seem unusual, the principles apply exactly to those for tests more commonly used in clinical diagnosis). The positive likelihood ratio for the odds of an overweight person having impaired immunogenicity is 1.83 while the negative likelihood ratio indicating the odds of a normal or underweight persons having impaired immunogenicity is 0.53.

Alternatively, we could perform a logistic regression of the immunogenicity status on the diagnostic screening test result to find the value  $x'$  from equation (7) when the Overweight Index is dichotomized. The logistic regression yields the regression parameters  $\alpha = -0.76$  and

$\beta = 1.23 \pm 0.43$ . The prior odds of the target disorder are  $0.47/0.53 = 0.88$ , and

$$x' = \frac{\ln(0.88) + 0.76}{1.23} = 0.51$$

[see equation (7)]. Using equation (10) with  $x = 1$  we find the positive likelihood ratio =  $\exp[1.23 \cdot (1 - 0.51)] = 1.83$ , and for  $x = 0$  we find the negative likelihood ratio =  $\exp[1.23 \cdot (0 - 0.51)] = 0.53$ . These values are identical to those derived directly from the  $2 \times 2$  table (see Table 2).

We now use a logistic regression model for continuous values for the Overweight Index to find  $\alpha = -3.90$  and  $\beta = 0.11 \pm 0.034$ . Using the actual prevalence of impaired immunogenicity in our population we solve equation (7) for  $x'$  and find  $x' = 34.29$ . Thus, a patient with an Overweight Index of 34.29 will respond to the vaccine with a rate that approximates the overall population response (i.e. the prevalence).

If your patient appears overweight and has an Index of 50, then the likelihood ratio for impaired immunogenicity ( $LR_{50}$ ) is  $\exp[0.11 \cdot (50 - 34.29)] = 5.63$  [see equation (10)]. The posterior odds of impaired immunogenicity can be estimated from the overall prevalence and the  $LR_{50}$  [posterior odds =  $0.88 \cdot 5.63 = 4.95$ , see equation (4)]. The posterior odds correspond to a posterior probability of 83% vs the prior probability of 47%. For the overweight patient, the likelihood ratio for a dichotomized outcome is 1.83 vs the  $LR = 5.63$  for the continuous outcome. Clinicians who use the dichotomous ratio of 1.83 might choose to vaccinate the patient with an Overweight Index of 50 without subsequently checking the response status. However, the continuous likelihood ratio of 5.63 is the true likelihood ratio for this overweight patient and suggests more strongly that the patient has impaired immunogenicity and is unlikely to respond to the vaccine.

#### **ADJUSTING THE LIKELIHOOD RATIO FOR COVARIATES**

The previous analysis assumed that the relationship between  $x$  and  $D$  was linear in the log odds, and that covariates were not present. We can demonstrate the method to handle covariates by continuing with the example of predicting response to hepatitis B vaccine based on our Overweight Index test. Suppose that we believe age is an important covariate that, together with the Overweight Index, provides a better test for

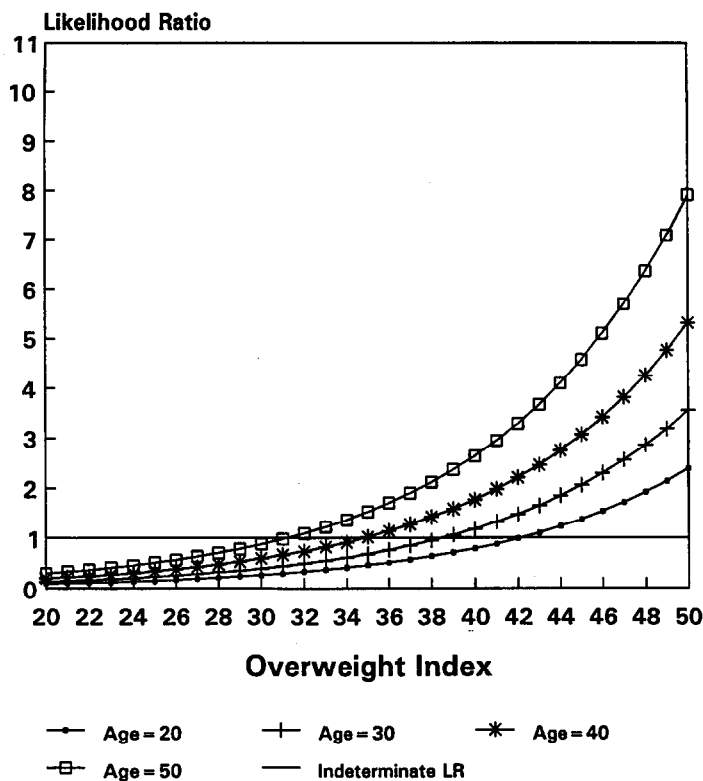


Fig. 1. The continuous likelihood level for impaired immunogenicity as a function of age. The horizontal line at a likelihood ratio of 1 indicates an indeterminate likelihood ratio.

predicting immunogenicity status. We create another logistic regression model with the terms  $\beta_1 x_1$  for the Overweight Index and  $\beta_2 x_2$  for age:

$$\frac{P(D = 1|x)}{P(D = 0|x)} = \exp(\alpha + \beta_1 x_1 + \beta_2 x_2).$$

The equation yields  $\alpha = -5.56$ ,  $\beta_1 = 0.11$ , and  $\beta_2 = 0.04$ . We perform a similar manipulation by retaining  $x_1'$  and solving  $x_2'$  as generalized from equation (7):

$$\ln \frac{P'}{1 - P'} - \alpha = \beta_1 x_1' + \beta_2 x_2'.$$

These results suggest an infinite number of  $x_1'$  and  $x_2'$  pairs for the equation  $\ln(0.88) - (-5.56) = 0.11x_1' + 0.04x_2'$ . However, we do not have to solve for all the pairs since we simplify matters by retaining  $x_1' = 34.29$  (N.B. we made the simplifying assumption of no interaction between the Overweight Index and age). This allows us to solve the equation for only one value,  $x_2' = 41.45$ . Using these values, we can now see that the likelihood ratio for a patient with an Overweight Index = 50 who is 50 years of age would be:

$$\begin{aligned} \text{LR}_{\text{OI} = 50, \text{age} = 50} &= \exp[0.11 \cdot (50 - 34.29) \\ &\quad + 0.04 \cdot (50 - 41.45)] = 7.93. \end{aligned}$$

This relationship can be expressed graphically with different lines for any desired age (Fig. 1).

#### IS A CONTINUOUS LIKELIHOOD RATIO "BETTER" THAN AN ORDINALIZED LIKELIHOOD RATIO?

Although likelihood ratios can be calculated for any continuous diagnostic test, the question is whether the benefit is worth the trouble? When the advantage is not obvious clinically, it may be simpler to retain the multilevel format with a few important cutpoints. Nonetheless, with fewer strata more clinical information is lost; the likelihood ratio range from the lowest value to the highest value of each stratum may be all that is necessary to express the clinical impact of fewer cutpoints.

From a purely statistical perspective, an *ad hoc* statistical test can be generated to determine whether the continuous predictors provide significantly more information than the ordered outcomes [10]. First, fit a logistic regression model with the dichotomous or ordinal predictor only. Then, fit another logistic regression model with the same dichotomous or ordinal predictors *and* the continuous predictor. We then compare the models using a likelihood ratio test that yields a chi-squared test with 1 *df*

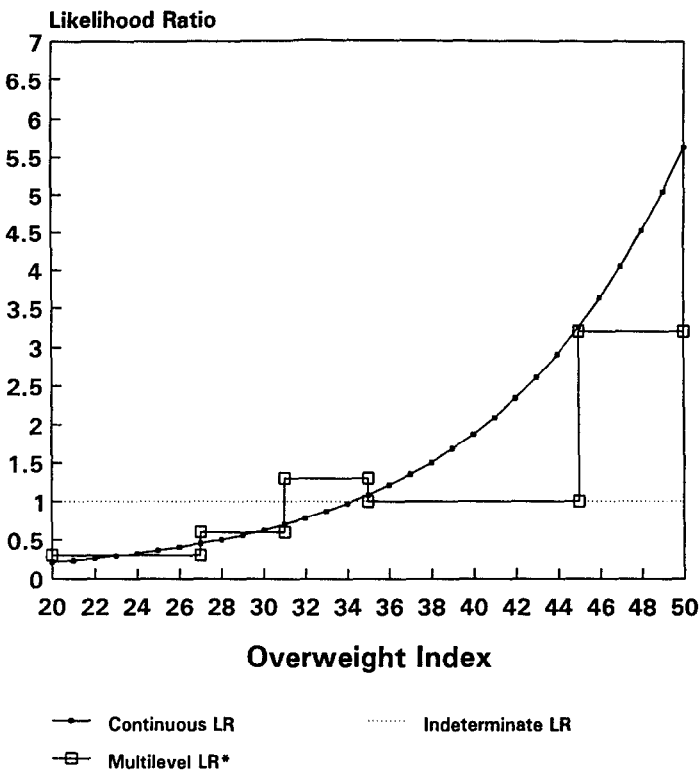


Fig. 2(A). The continuous likelihood ratio line (solid) versus the multilevel likelihood ratio (box). The multilevel likelihood ratio is calculated for the midpoint of each overweight index quintile. \*Multilevel likelihood ratio at each quintile midpoint.

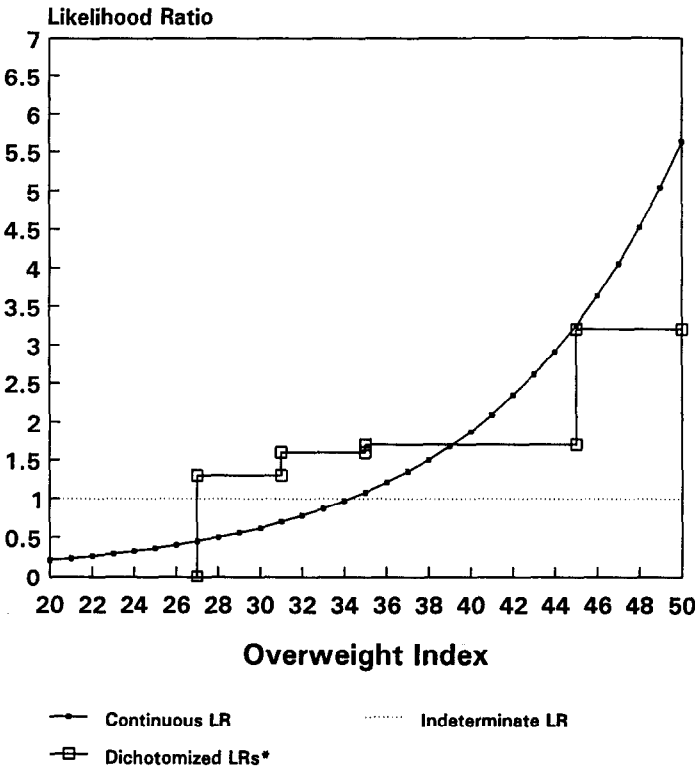


Fig. 2(B). The continuous likelihood ratio line (solid) versus the dichotomous likelihood ratio (box). The dichotomized likelihood ratios are the likelihood ratios for a patient with an Overweight Index equal to or greater than the specified points. \*Likelihood ratio for index equal to or greater than the quintile midpoint.

(N.B. the likelihood ratio here is not that of a diagnostic test, but refers to the statistically derived likelihood ratio for the model's fit). The difference between  $-2 \cdot \log LR$  for the two regression models produces the chi-squared statistic, indicating the additional predictive ability provided by the continuous formulation. If this test is non-significant, then the continuous predictor does not add useful information to that conferred by the ordinal outcome.

For example, in the hepatitis B study the  $-2 \cdot \log LR$  for the model with Overweight Index expressed as an ordinal outcome in quintiles was 120.04. The  $-2 \cdot \log LR$  for the model with the continuous Overweight Index was 113.53. The difference between these two values is 6.51 (i.e.  $120.04 - 113.53$ ) and is a chi-square statistic with 1 df under the null hypothesis that the continuous predictor does not add more predictive ability than the multilevel predictor [10]. Although the continuous predictor added information ( $p = 0.01$ ), that may not always occur. Failure to find significance may have alternative explanations other than absence of a true difference. Examples of causes that might lead to an incorrect assumption of no difference between continuous and multilevel predictor models include the cutpoint and number of levels, an inadequate sample size, or a need for more complex logistic models.

Our next step should be to determine whether a statistically significant difference between an ordinal and continuous predictor is clinically significant. This is a difficult process, but should involve visual inspection of the likelihood ratio curves to assess the potential impact on clinical decisions. To demonstrate the graphical approach for assessing clinical significance we constructed Fig. 2(A) that shows the likelihood ratio at the midpoint of each Overweight Index quintile. It is appropriate to construct this graph when the clinician would treat patients differently based on the quintile of their test result. Each horizontal segment of the multilevel likelihood ratio line represents the midpoint of a quintile of the Overweight Index for impaired immunogenicity. This line shows the likelihood ratio for impaired immunogenicity for every possible Overweight Index value. The multilevel likelihood ratio for the Overweight Index quintile from 35–45 underestimates the risk of impaired immunogenicity compared to the continuous likelihood ratio line. If the clinician states *a priori* that the patient would be treated differently at each level, the investigator might

choose to report continuous likelihood ratios. This decision is justified by the figure since it shows that the patients in this quintile have progressively increased risk of impaired immunogenicity as the Overweight Index increases from 35 to 45.

Figure 2(B) shows the graphical result when the clinician has decided that patients with values above a given result will be treated one way and patients with lower values will receive different treatment. These clinicians have a pragmatic advantage over clinicians using continuous likelihood ratios since they make only a single treatment decision based on one Overweight Index cutpoint. The likelihood ratios in Fig. 2(B) are the likelihood ratios representing the target disorder of impaired immunogenicity when patients have an Overweight Index greater than or equal to the quintiles' midpoints. The clinicians might infer from Fig. 2(B) that using the continuous likelihood ratio does not facilitate their clinical decisions: above an Overweight Index of 47 the patient's odds of non-response increase 3-fold. Thus, clinicians might choose to vaccinate all patients and check the immune response to vaccination only for those with an Overweight Index greater than 47.

## CONCLUSIONS

We have demonstrated that likelihood ratios provide a general methodology for analyzing diagnostic tests: likelihood ratios can be determined not only for dichotomous and multilevel results, but also for continuous test results. In addition, information about covariates can be incorporated in the analysis and allow insights into important clinical variables. When clinicians use contingent probability during differential diagnosis their natural inclination is to think in probability terms such as the predictive value. However, the predictive values we obtain from literature are highly prevalent dependent and may not generalize to other clinical settings. Likelihood ratios, coupled to the prior odds, represent an alternative approach that adapts to populations with a different prevalence of the target disorder. The posterior odds generated from multiplying the likelihood ratio and prior odds can be transformed to predictive values. We prefer, along with others, to mentally derive the posterior odds from the likelihood ratios and prior odds as opposed to calculating the predictive values directly from Bayes theorem [6, 11]. Calculating predictive values directly from the sensitivity, specificity, and prevalence



only allows evaluation of dichotomous results; the sensitivity and specificity cannot be used when there are more than three ordinal levels or continuous outcomes [8, 12].

It is not necessary that clinicians themselves work through our mathematical approach to decide on the appropriate test report format. However, clinicians should understand the trade-offs investigators make when deciding to report tests on dichotomous vs continuous scales. The clinical investigator must decide the best way to report the results.

The final decision to use multilevel vs dichotomous likelihood ratios should be for clinical rather than statistical reasons. First, we recognize that no statistical test tells clinicians the appropriate response at different ranked levels of test outcomes. That is a decision based not only on the likelihood of outcomes but also the value placed on those outcomes. Secondly, statistical tests comparing ordinal to continuous results will depend strongly on source sample size and the number of levels selected. Obviously, the decision to use multilevel or dichotomous likelihood ratios becomes meaningless when the investigator fails to design a high quality study of the diagnostic test.

Although continuous test results contain the most information, clinicians traditionally compare such results to a reference range describing "high", "normal" or "low" outcomes. Coupling the raw result with the continuous likelihood ratio may facilitate clinical test interpretation. When specified test results would lead to differing patient management strategies, ordinal outcomes with the multilevel likelihood ratios

would simplify test reporting and clinical decision making. Pragmatic reasons that incorporate clinical realities should always drive the choice between continuous likelihood ratios versus simplifying to multilevel or dichotomous outcomes.

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