One formulation of the convex regression problem is

$$\min_{f,\beta} \quad \sum_{i=1}^{n}(Y_i - f_i)^2 \tag{1.1}$$

$$\text{such that} \quad f_{i'} \geq f_i + \beta_i^T(x_{i'} - x_i) \text{ for all } i, i' \tag{1.2}$$

The optimization for a convex additive model is

$$\min_{f,\beta} \quad \sum_{i=1}^{n}\Big(Y_i - \sum_{j=1}^{p} f_{ij}\Big)^2 \tag{1.3}$$

$$\text{such that} \quad \text{for all } j = 1, \dots, p: \tag{1.4}$$

$$f_{i'j} \geq f_{ij} + \beta_{ij}(x_{i'j} - x_{ij}) \text{ for all } i, i' \tag{1.5}$$

$$\sum_{i=1}^{n} f_{ij} = 0. \tag{1.6}$$

In the convexity-pattern estimation problem, we want to estimate the regression function as $m(x) = \sum_{j=1}^{p}\big\{f_j(x_j) + g_j(x_j)\big\}$ where f_j is convex, g_j is concave, and at most one of f_j and g_j is nonzero.

Formulate this as

$$\min_{f,g,\beta,\gamma,z,w} \quad \sum_{i=1}^{n}\Big(Y_i - \sum_{j=1}^{p}(f_{ij} + g_{ij})\Big)^2 \tag{1.7}$$

$$\text{such that} \quad \text{for all } j = 1, \dots, p: \tag{1.8}$$

$$f_{i'j} \geq f_{ij} + \beta_{ij}(x_{i'j} - x_{ij}) \text{ for all } i, i' \tag{1.9}$$

$$g_{i'j} \leq g_{ij} + \gamma_{ij}(x_{i'j} - x_{ij}) \text{ for all } i, i' \tag{1.10}$$

$$\sum_{i=1}^{n} f_{ij} = 0 \tag{1.11}$$

$$\sum_{i=1}^{n} g_{ij} = 0 \tag{1.12}$$

$$\sqrt{\sum_{i=1}^{n} f_{ij}^2} \leq z_j B \tag{1.13}$$

$$\sqrt{\sum_{i=1}^{n} g_{ij}^2} \leq w_j B \tag{1.14}$$

$$z_j + w_j \leq 1 \tag{1.15}$$

$$z_j, w_j \in \{0, 1\}. \tag{1.16}$$

Without the last nonconvex zero/one constraint, this is a second-order cone program (SOCP). A natural relaxation of this is to $z_j, w_j \in [0, 1]$. With the constraint it's a mixed integer second-order cone program (MISOCP). While still a relatively new and specialized topic, there is work studying relaxations of MISOCPs that parallels the semi-definite hierarchies studied for integer quadratic programming. An overview article is www.pages.drexel.edu/~us26/Tutorial.pdf