Self-consistent, angular-momentum-dominated hadron beams for space charge mitigation

Austin Hoover

Space Charge Mini-Workshop

Knoxville, TN, USA

October 26, 2022

*hooveram@ornl.gov

Summary slide, 5th ICFA mini-workshop on Space Charge
Theme: Bridging the gap in space charge dynamics

In 1-2 sentences, summarize the content of this presentation
(If relevant, specify type of facility, species, tune shift):
I review several proposed techniques to control the density and angular momentum of hadron beams for the purpose of space charge mitigation.

From your perspective, where is the gap regarding space charge effects?
(understanding/control/mitigation/prediction/?)
How to increase the beam intensity in low-energy hadron rings

What is needed to bridge this gap?
New lattice design (nonlinear optics, strong coupling) and beam shaping techniques (painting)

Outline

- **Angular-momentum-dominated (AMD) beams**
- Self-consistent beams
- Beam shaping
- Questions/research directions

AMD beams have small 4D emittance — can exist in round or flat state

$$\boldsymbol{\Sigma} = \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle xx' \rangle & \langle x'x' \rangle & \langle yx' \rangle & \langle x'y' \rangle \\ \langle xy \rangle & \langle yx' \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'y' \rangle \end{bmatrix} = \boldsymbol{V} \begin{bmatrix} \varepsilon_1 & 0 & 0 & 0 \\ 0 & \varepsilon_1 & 0 & 0 \\ 0 & 0 & \varepsilon_2 & 0 \\ 0 & 0 & 0 & \varepsilon_2 \end{bmatrix} \boldsymbol{V^T}$$

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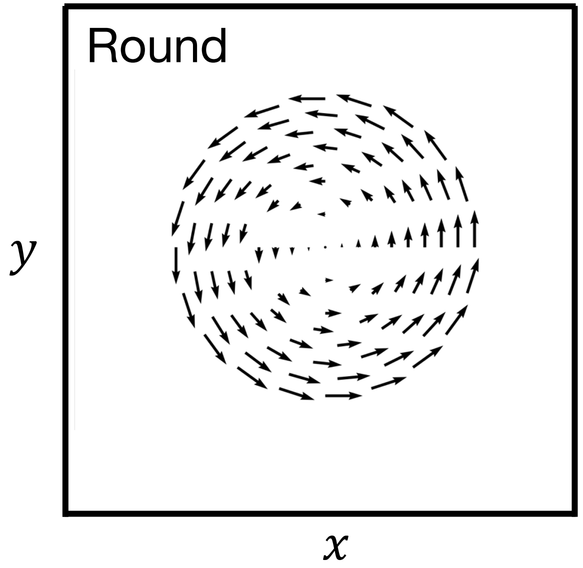

$$C = \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_x \varepsilon_y}} \to 0$$

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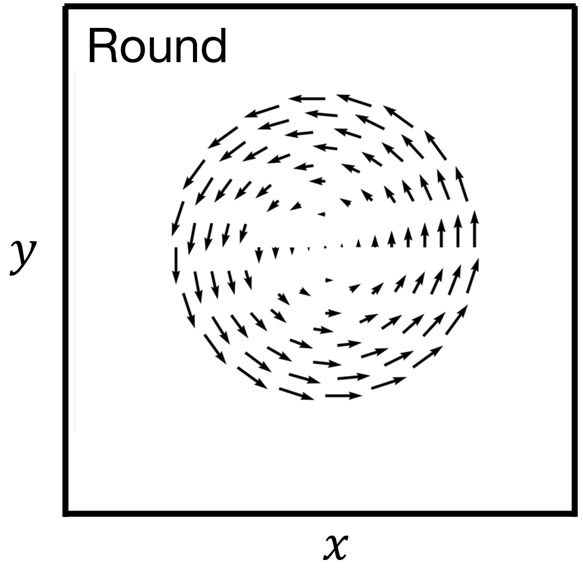
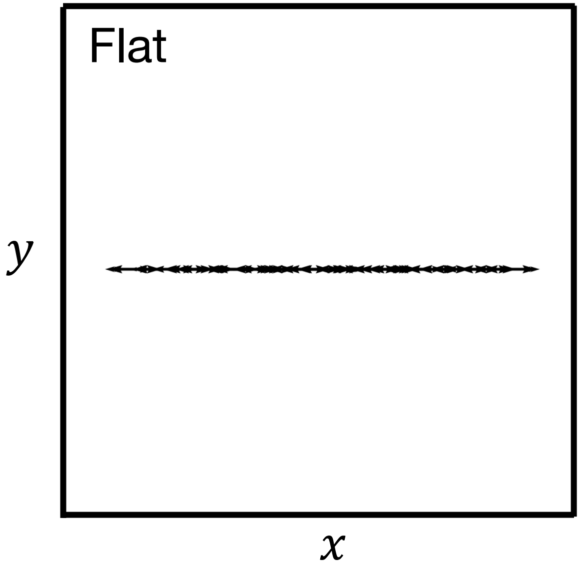
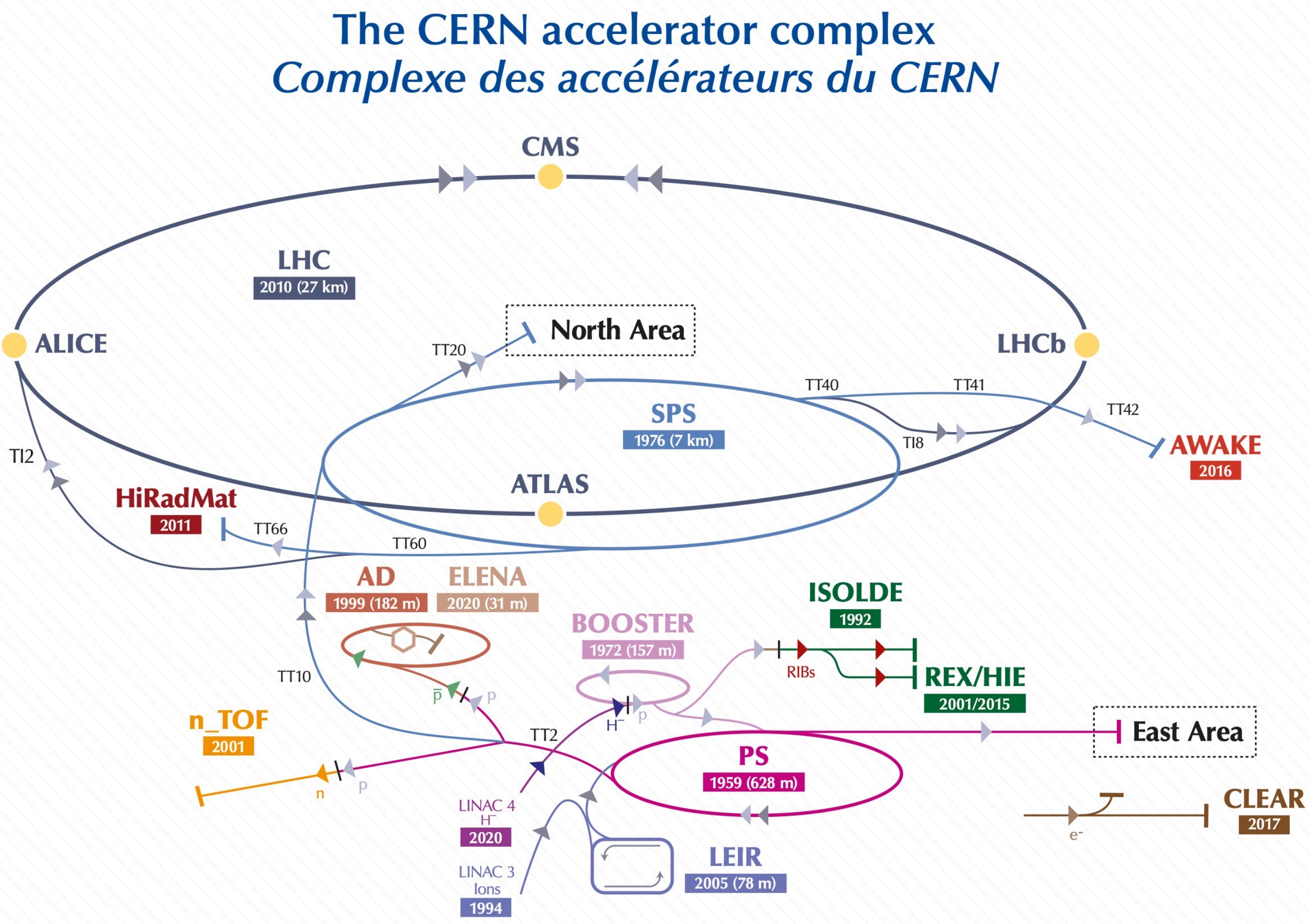


$$C = \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_x \varepsilon_y}} \to 0$$



$$C = \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_x \varepsilon_y}} \to 1$$

AMD beams could be useful in a hadron collider



Circular modes for flat beams in the LHC

A. Burov

Phys. Rev. ST Accel. Beams **16**, 061002 – Published 24 June 2013

https://cds.cern.ch/record/2693837/files/Poster-2019-858.pdf

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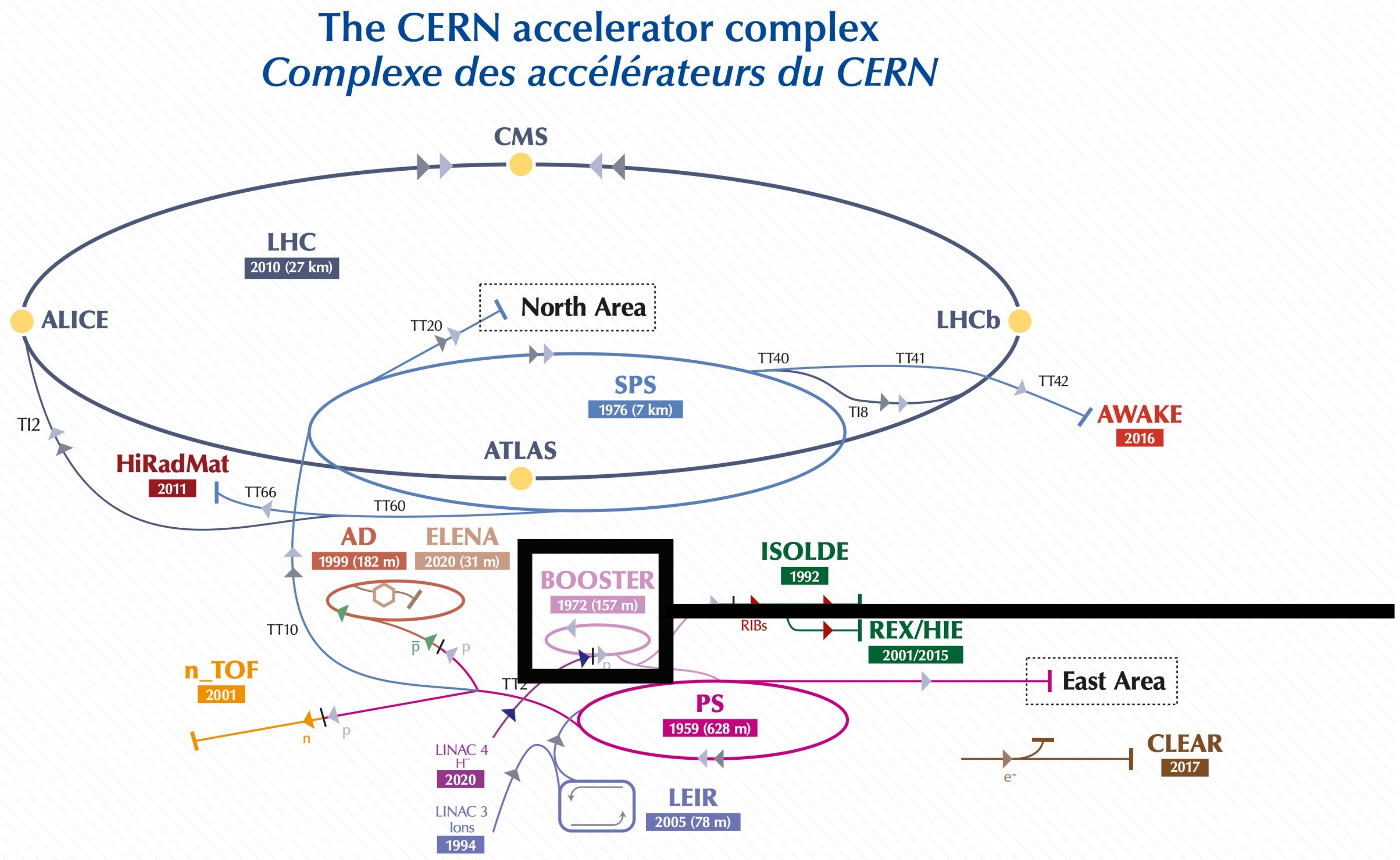

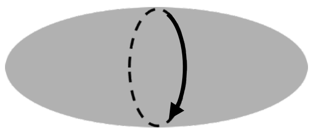

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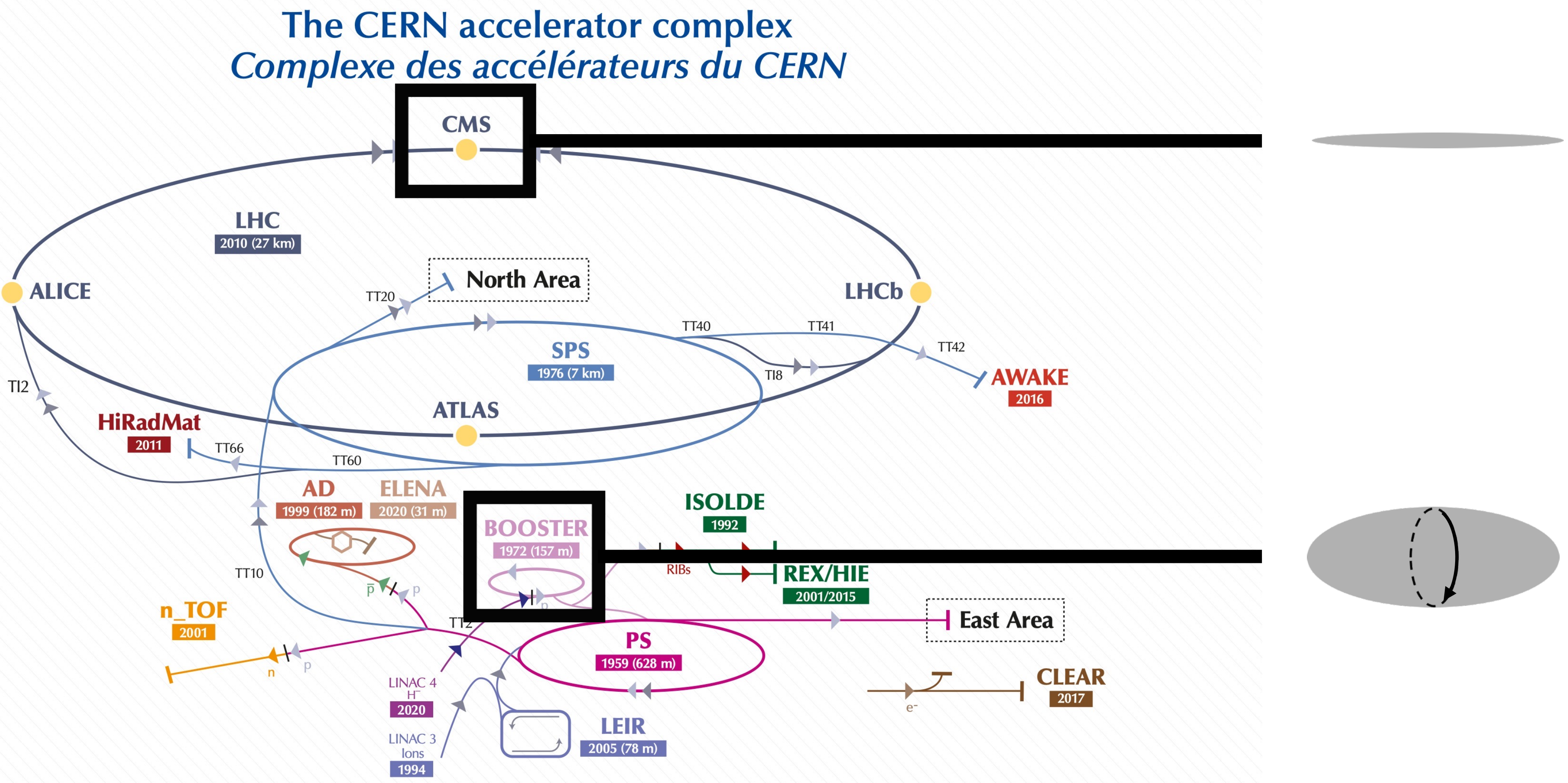

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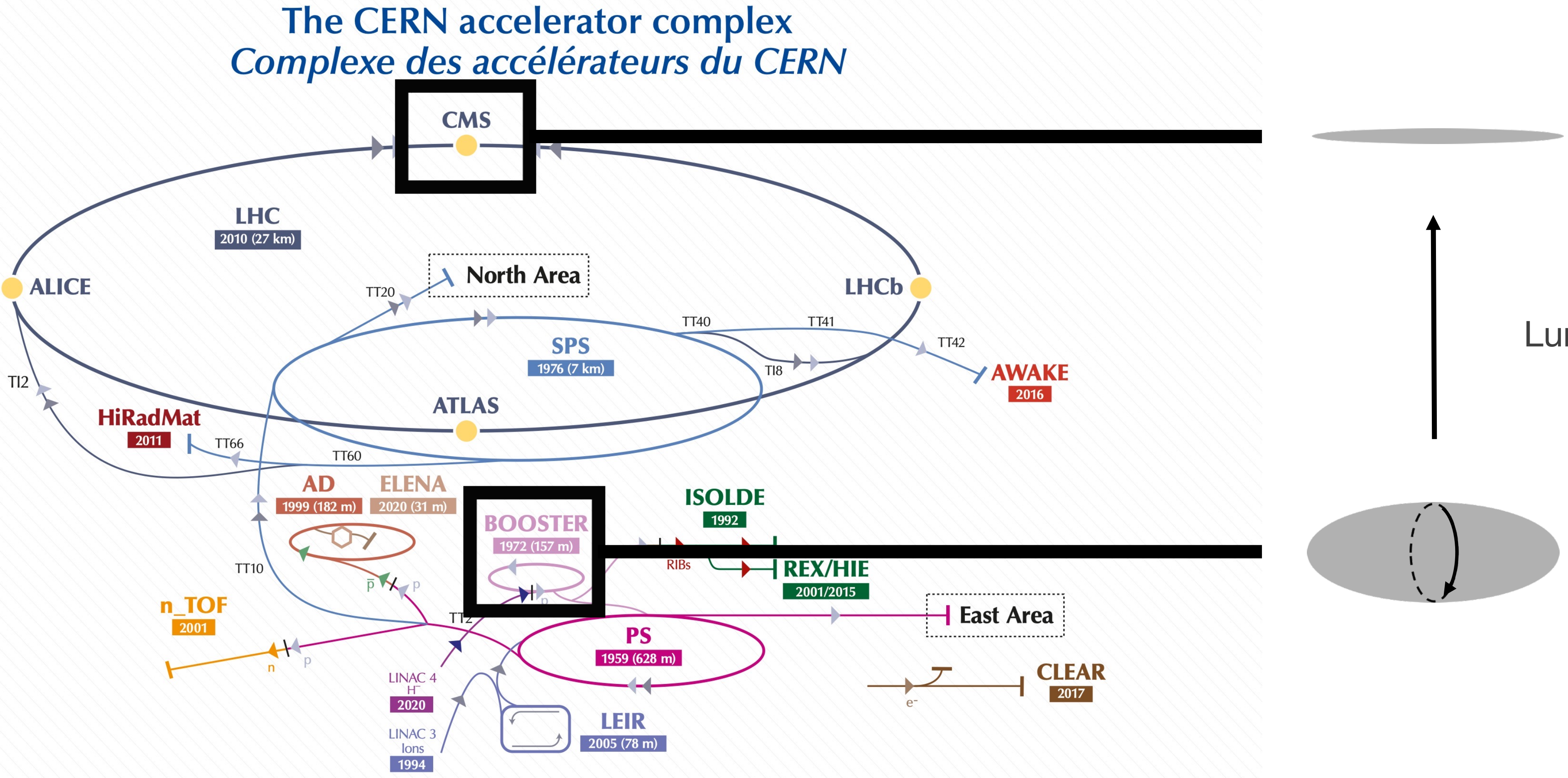

Luminosity $\sim \sqrt{\frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2}}$

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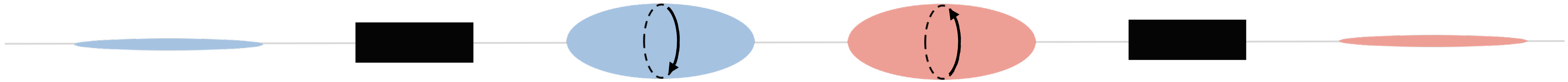
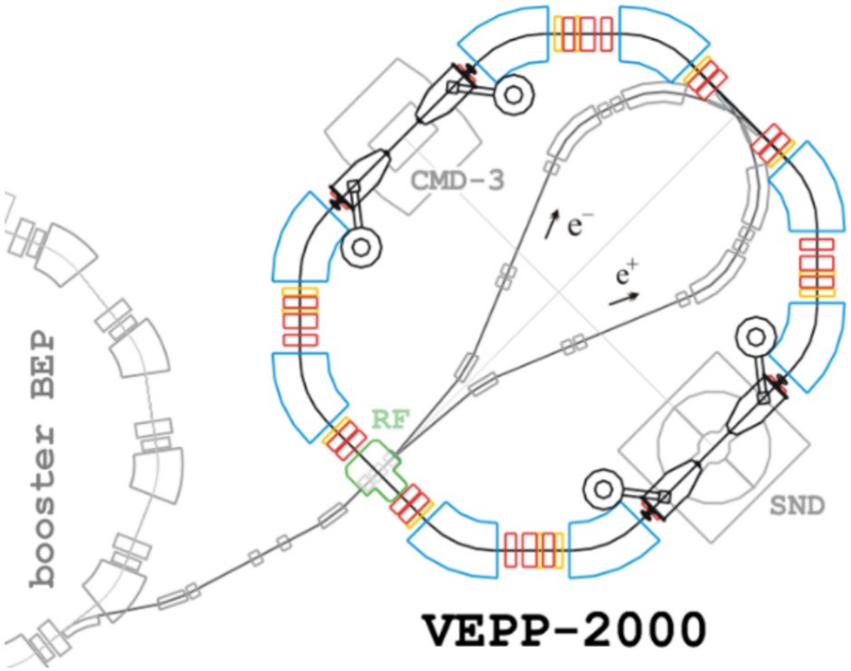




“Round Colliding Beams” As a Way to Integrability: Theory and Simulations for Tevatron

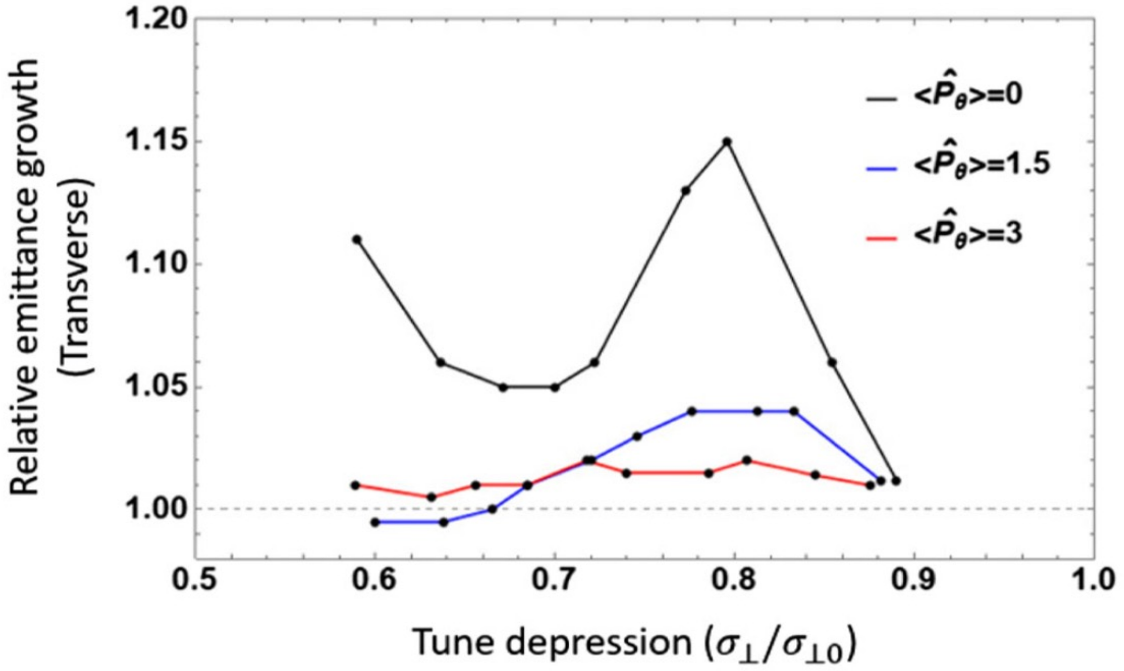
V.V. Danilov and V.D. Shiltsev

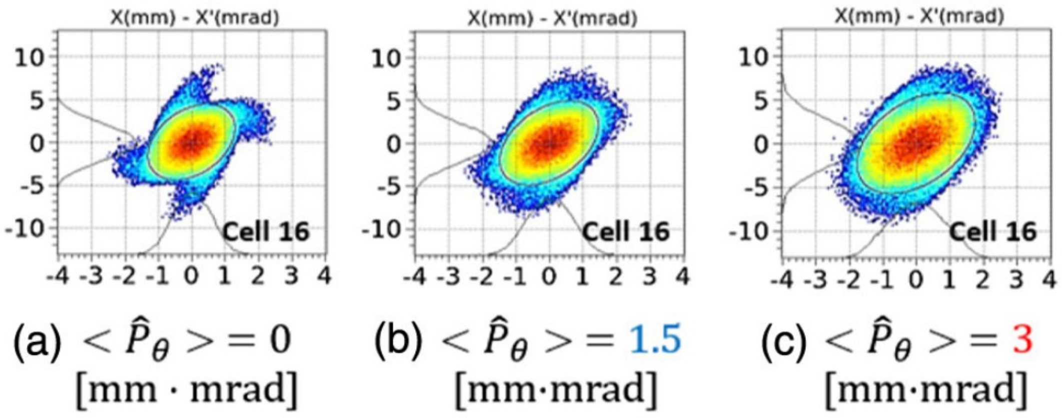
12th Int. Particle Acc. Conf. IPAC2021, Campinas, SP, Brazil JACoW Publishing
ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB002

ROUND COLLIDING BEAMS: SUCCESSFUL OPERATION EXPERIENCE

D. Shwartz†[1], O. Belikov, D. Berkaev, D. Burenkov, V. Denisov, A. Kasaev, A. Kirpotin, S. Kladov, I. Koop[1], A. Krasnov, A. Kupurzhanov, G. Kurkin, M. Lyalin, A. Lysenko, S. Motygin, E. Perevedentsev[1], V. Prosvetov, Yu. Rogovsky[1], A. Semenov, A. Senchenko, L. Serdakov, D. Shatilov, P. Shatunov, Yu. Shatunov[1], M. Timoshenko, I. Zemlyansky, Yu. Zharinov
Budker Institute of Nuclear Physics, Novosibirsk, 630090, Russia
[1]also at Novosibirsk State University, Novosibirsk, 630090, Russia

AMD beams could suppress particle-core resonances





$$r'' + k(s)r - P_\theta^2/r^3 = f(r)$$

Effects of beam spinning on the fourth-order particle resonance of 3D bunched beams in high-intensity linear accelerators

Yoo-Lim Cheon, Seok-Ho Moon, Moses Chung, and Dong-O Jeon
Phys. Rev. Accel. Beams **25**, 064002 – Published 10 June 2022

Outline

- Angular-momentum-dominated (AMD) beams
- **Self-consistent beams**
- Beam shaping
- Questions/research directions

Self-consistent beam = Vlasov equilibrium with linear internal space charge forces

Function of invariants: $$\frac{d}{ds}f(\{C_i\}) = \sum_i \frac{df}{dC_i}\frac{dC_i}{ds} = 0.$$

$\updownarrow$

Linear forces: $$\frac{\partial^2 \Phi}{\partial \mathbf{x}^2} = -\frac{q}{\epsilon_0}\int_{-\infty}^{\infty} f\, d\mathbf{x}'.$$

The KV distribution is not the only solution

{n, m} distributions: $$f(\boldsymbol{x}, \boldsymbol{x}') = g(H_b - H) \prod_{i=1}^{m} \delta(\boldsymbol{e_i} \cdot \boldsymbol{x} + \boldsymbol{e_i'} \cdot \boldsymbol{x}')$$

Self-consistent time dependent two dimensional and three dimensional space charge distributions with linear force

V. Danilov, S. Cousineau, S. Henderson, and J. Holmes

Phys. Rev. ST Accel. Beams **6**, 094202 – Published 29 September 2003; Erratum Phys. Rev. ST Accel. Beams **11**, 019901 (2008)

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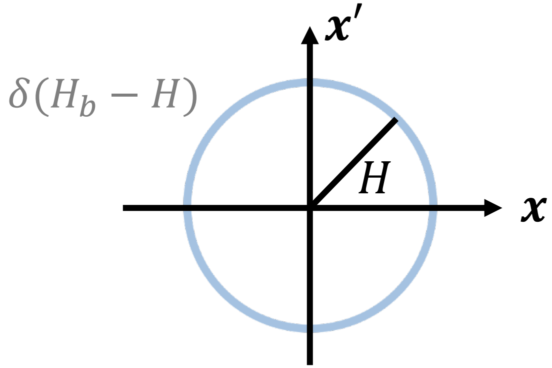
{n, 0} KV distributions



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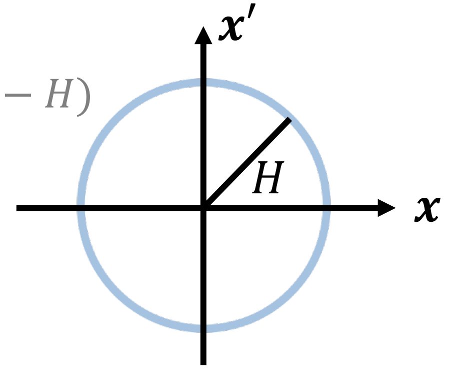
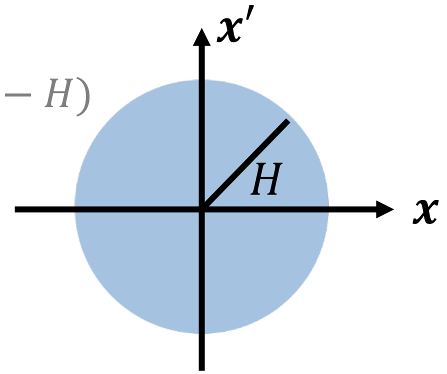
{n, 0} KV distributions

$\delta(H_b - H)$

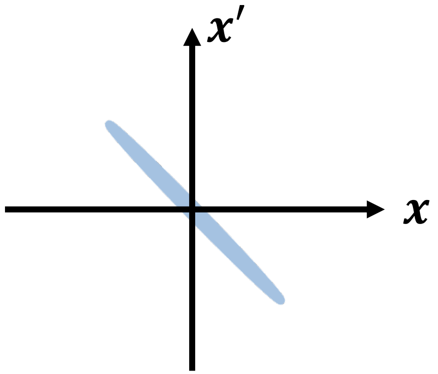


{n, n} vortex distributions

$\Theta(H_b - H)$



$$\prod_{i=1}^{m} \delta(\boldsymbol{e_i} \cdot \boldsymbol{x} + \boldsymbol{e}'_{\boldsymbol{i}} \cdot \boldsymbol{x}')$$



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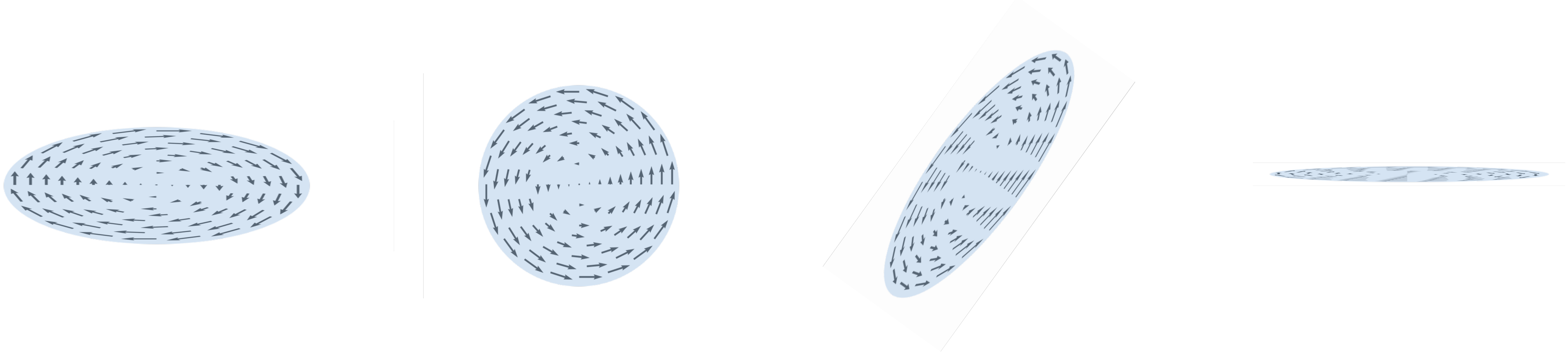
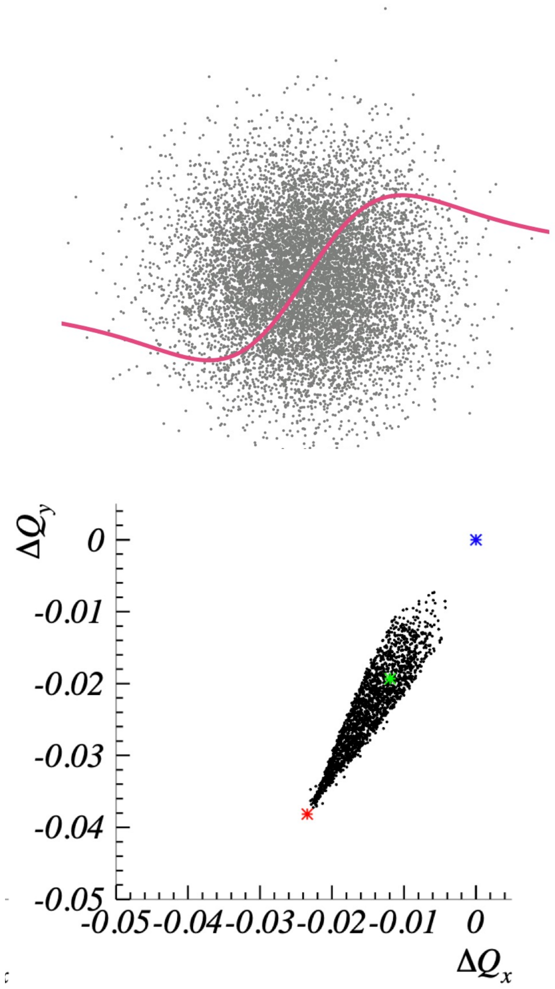
{2, 2} distribution — a uniform density, elliptical "vortex"

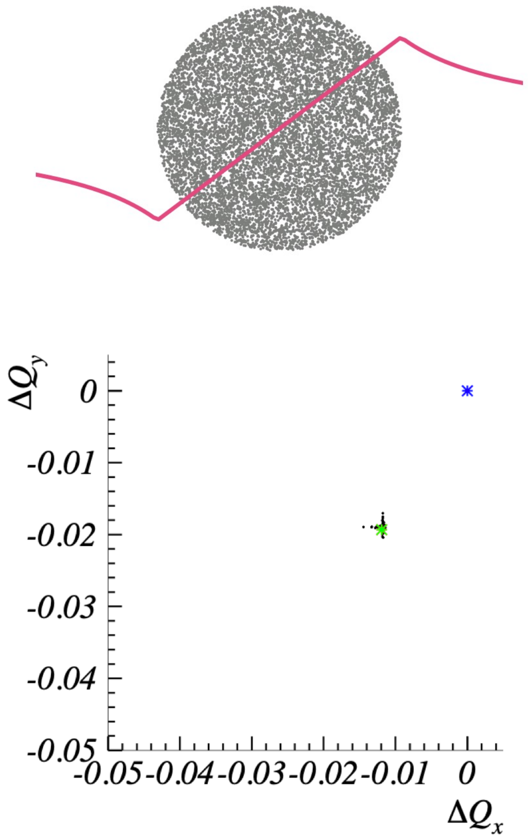
$$f = \rho\delta(X' - e_{xx}X - e_{xy}Y)\delta(Y' - e_{yx}X - e_{yy}Y),$$



- Inherits all properties of AMD beams
- Beam provides strong linear coupling in AG focusing

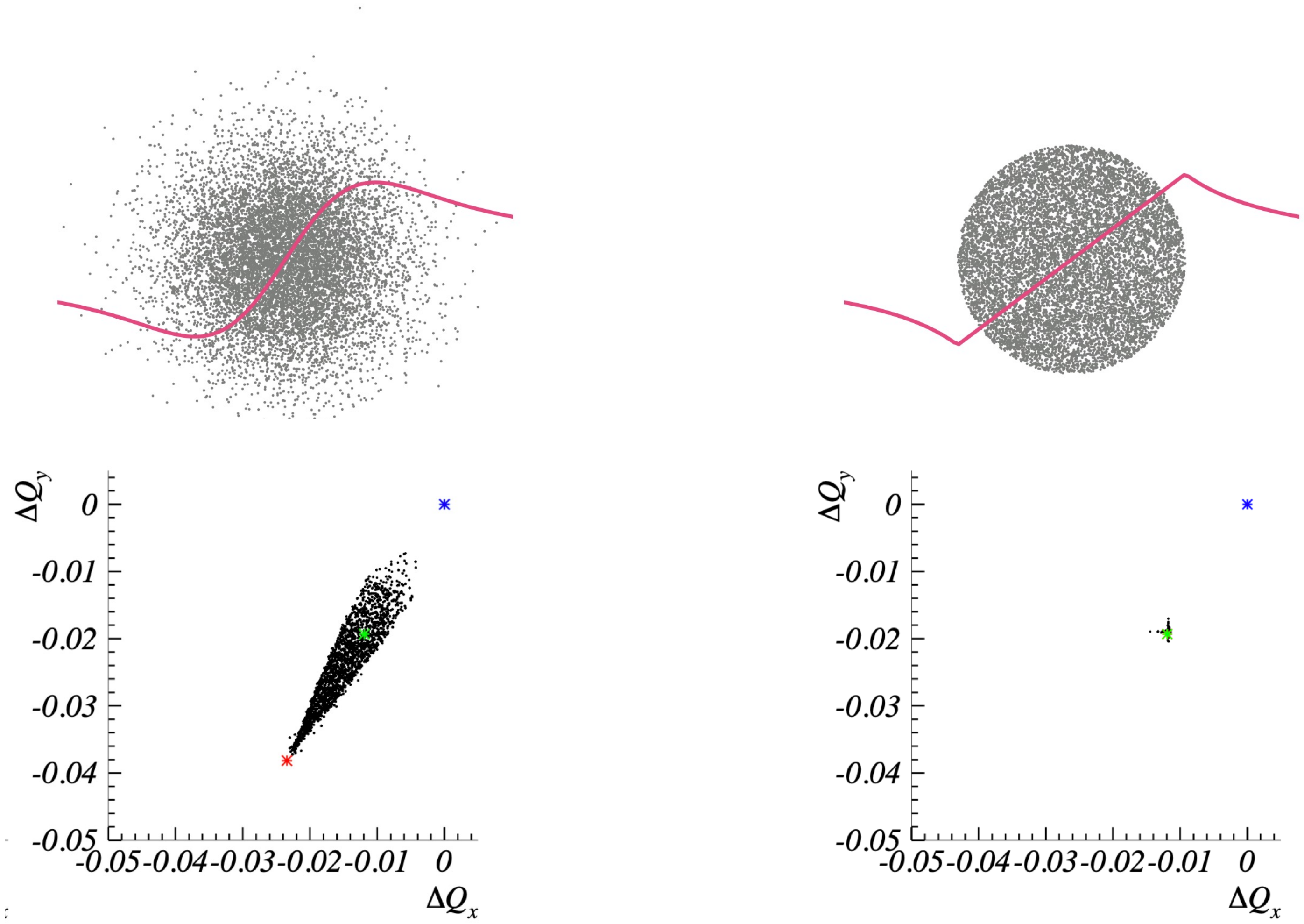
Physically attractive due to minimized space charge tune shift/spread





Source: G. Franchetti, *Space charge in circular machines,* CERN Accelerator School

Lack of higher-order multipoles may reduce nonlinear particle-core interactions



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Outline

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- **Beam shaping**
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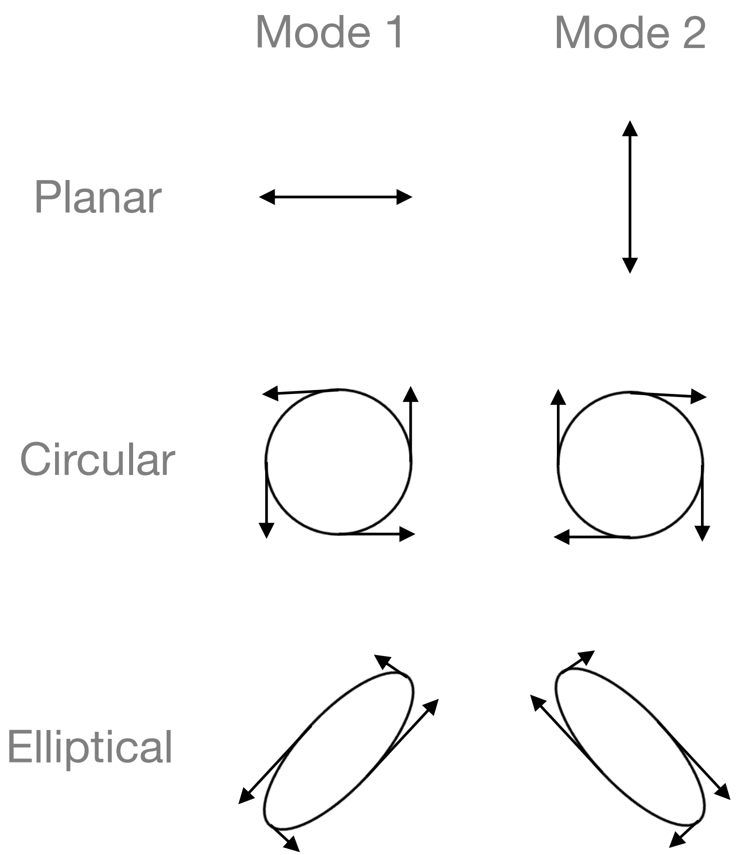
Elliptical painting: inject particles along eigenvector of ring transfer matrix



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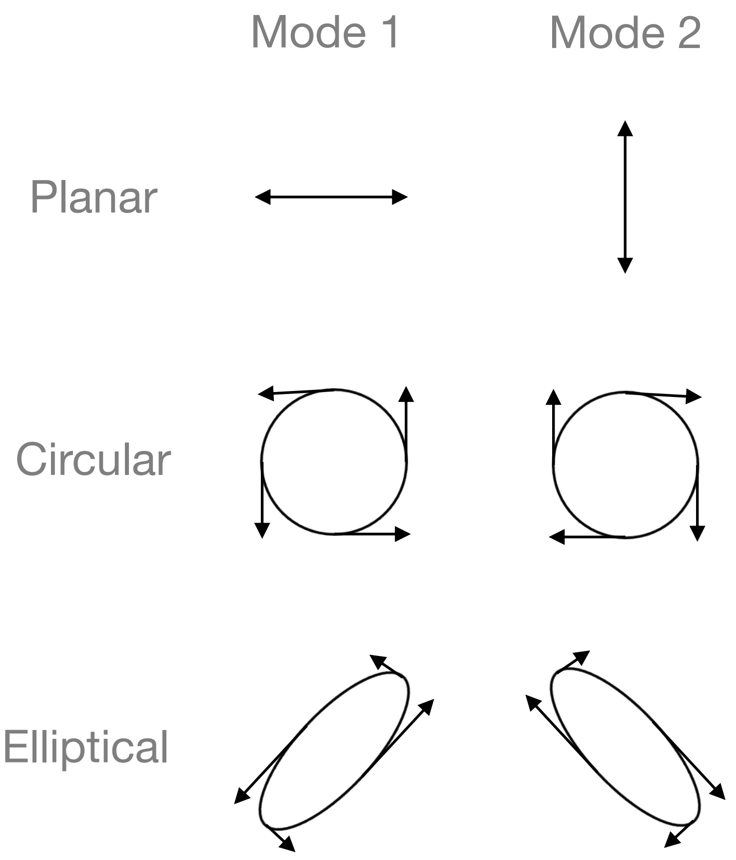
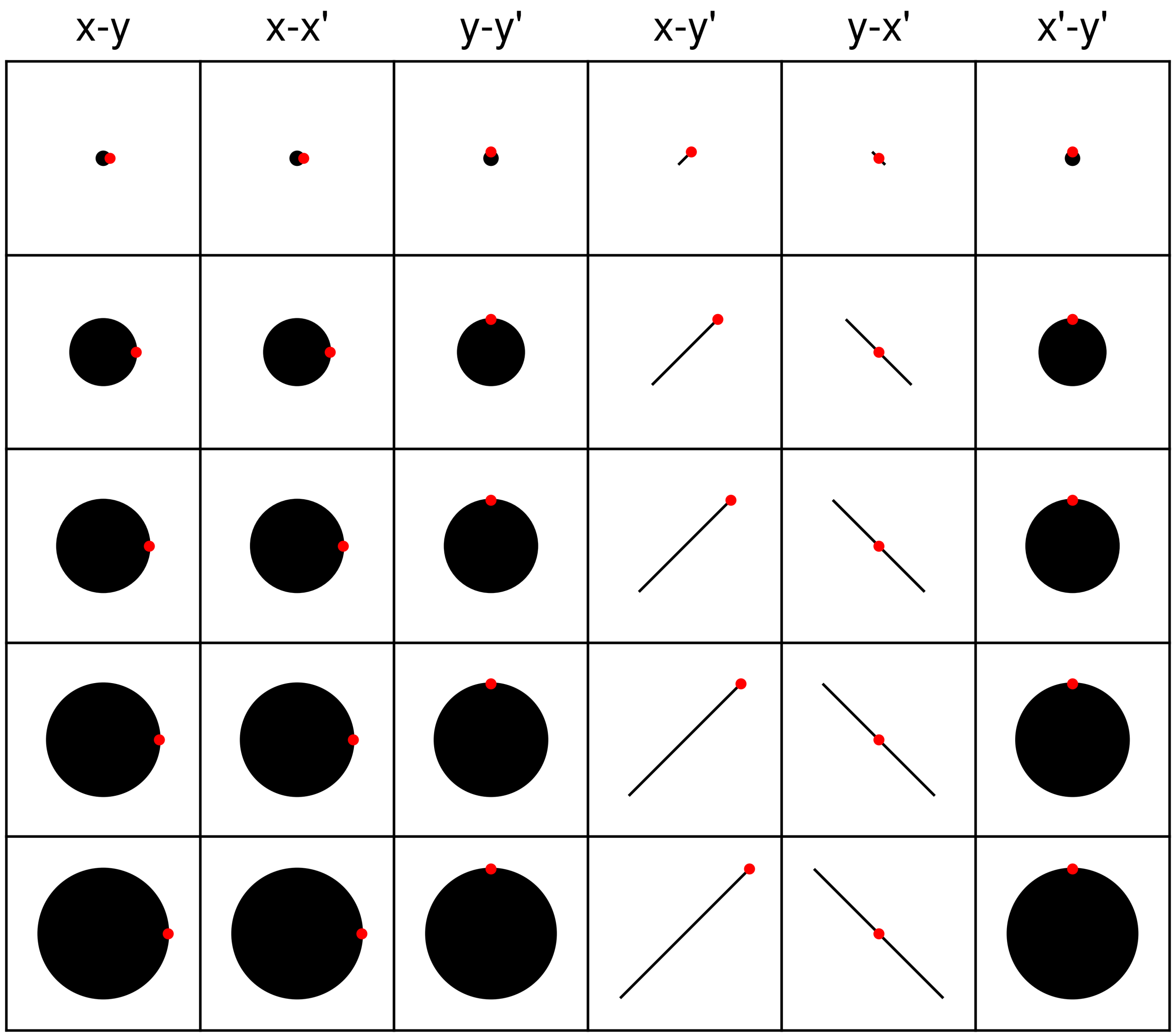
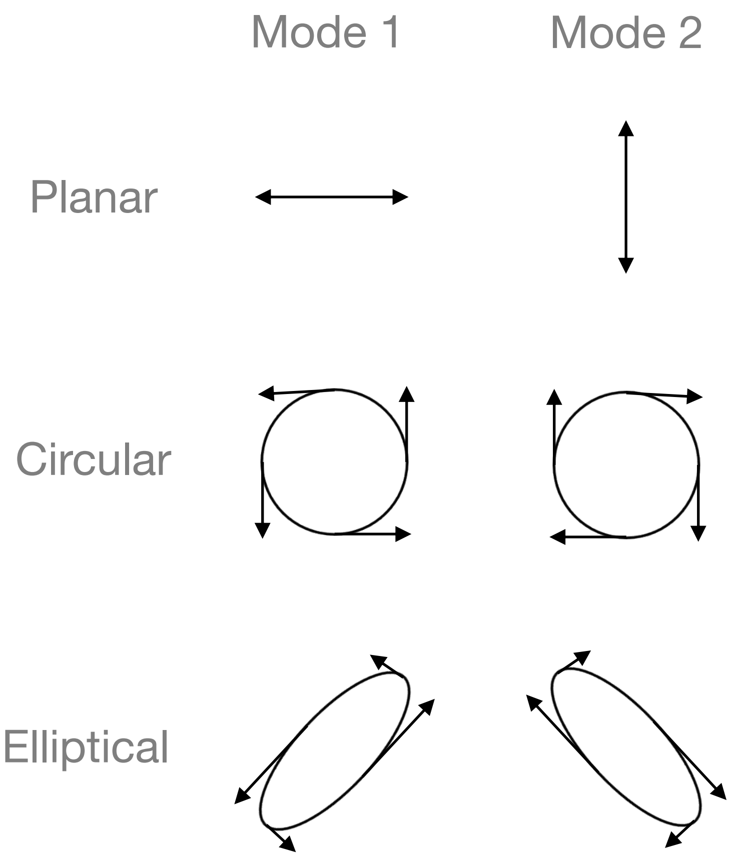


$$\boldsymbol{x} = Re\{\sqrt{t}e^{-i\mu_l}\boldsymbol{v_l}\}$$



{2, 2} Danilov distribution maintained throughout injection **(with space charge)**



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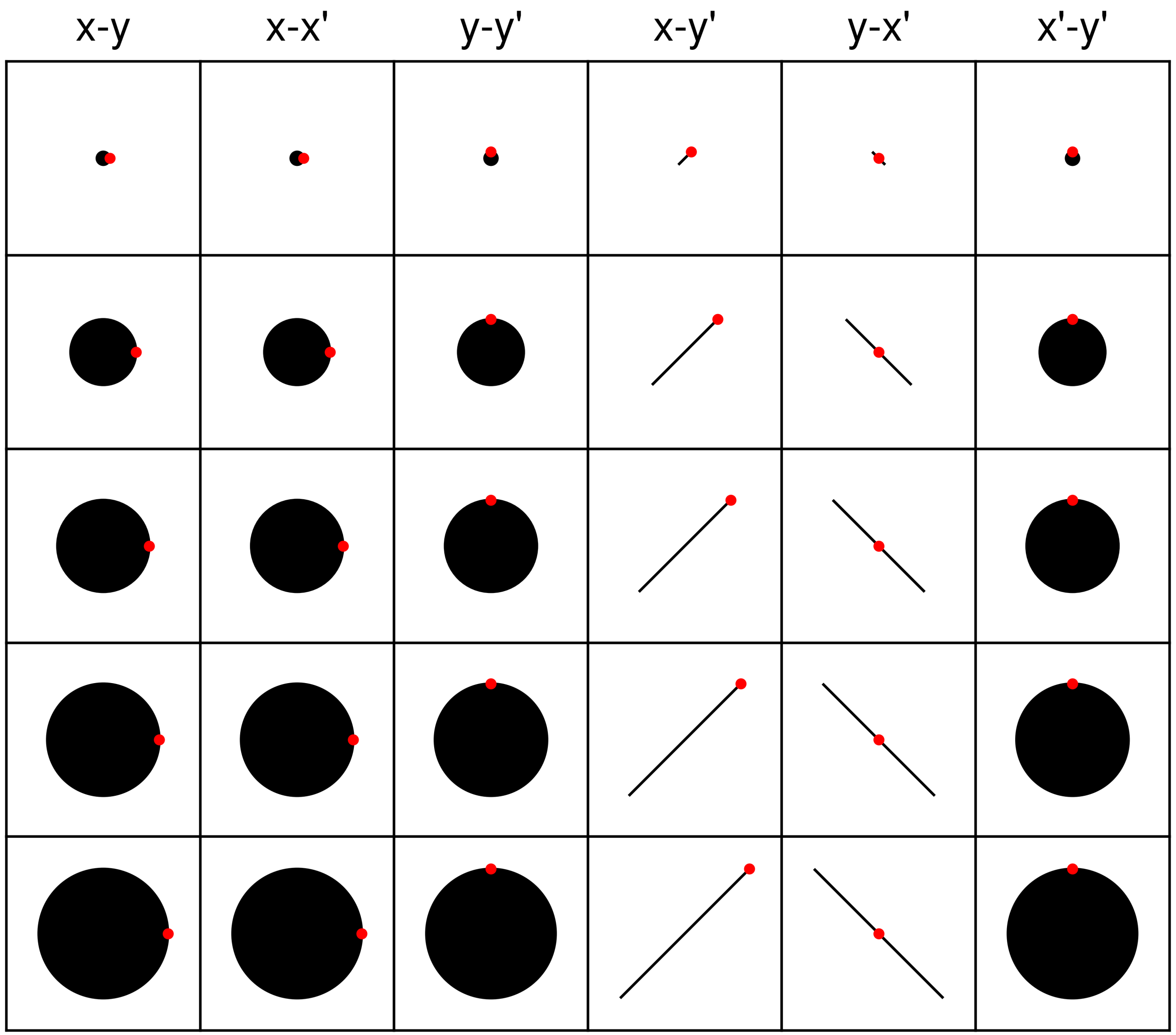
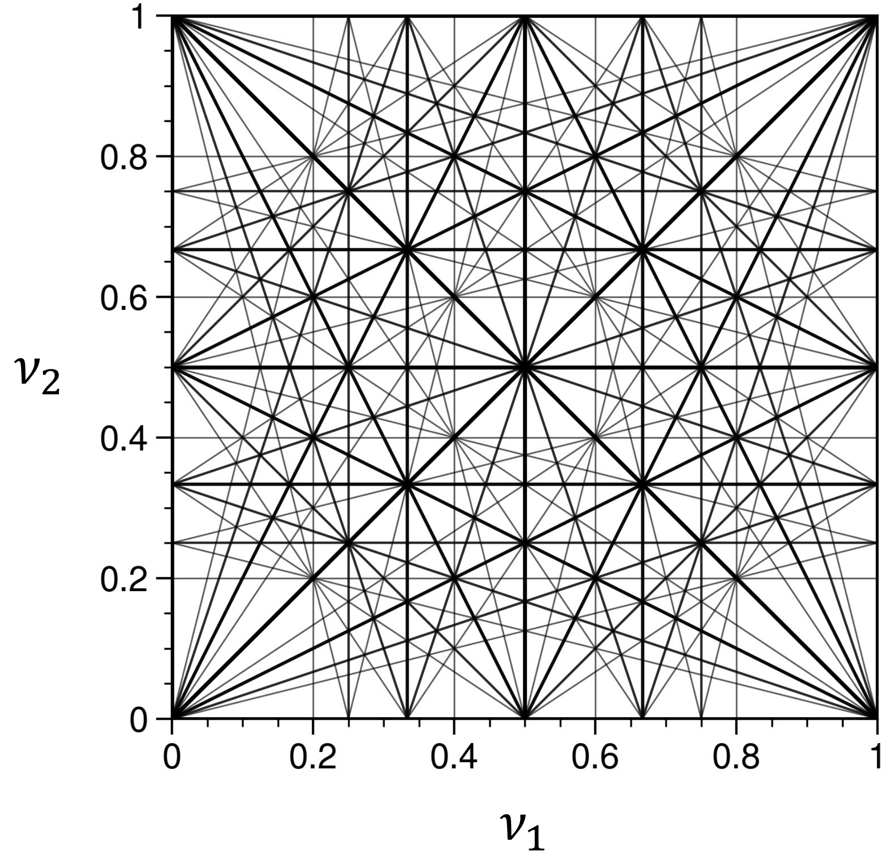


Eigenvectors must self-consistently include linear coupling from space charge

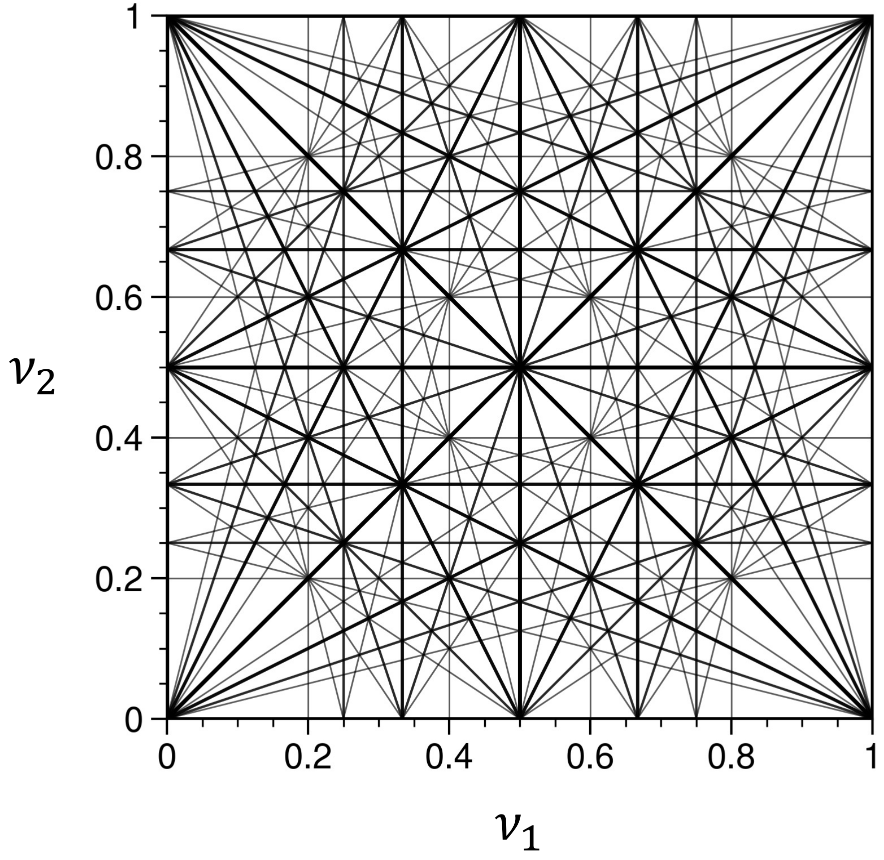


Linear lattice

Nonlinear perturbation from lattice/beam

$$x'' + k(s)x = \sum_{n,m} C_{nm}(s)x^n y^m$$

Eigenvectors must self-consistently include linear coupling from space charge



Linear lattice

Nonlinear perturbation from lattice/beam

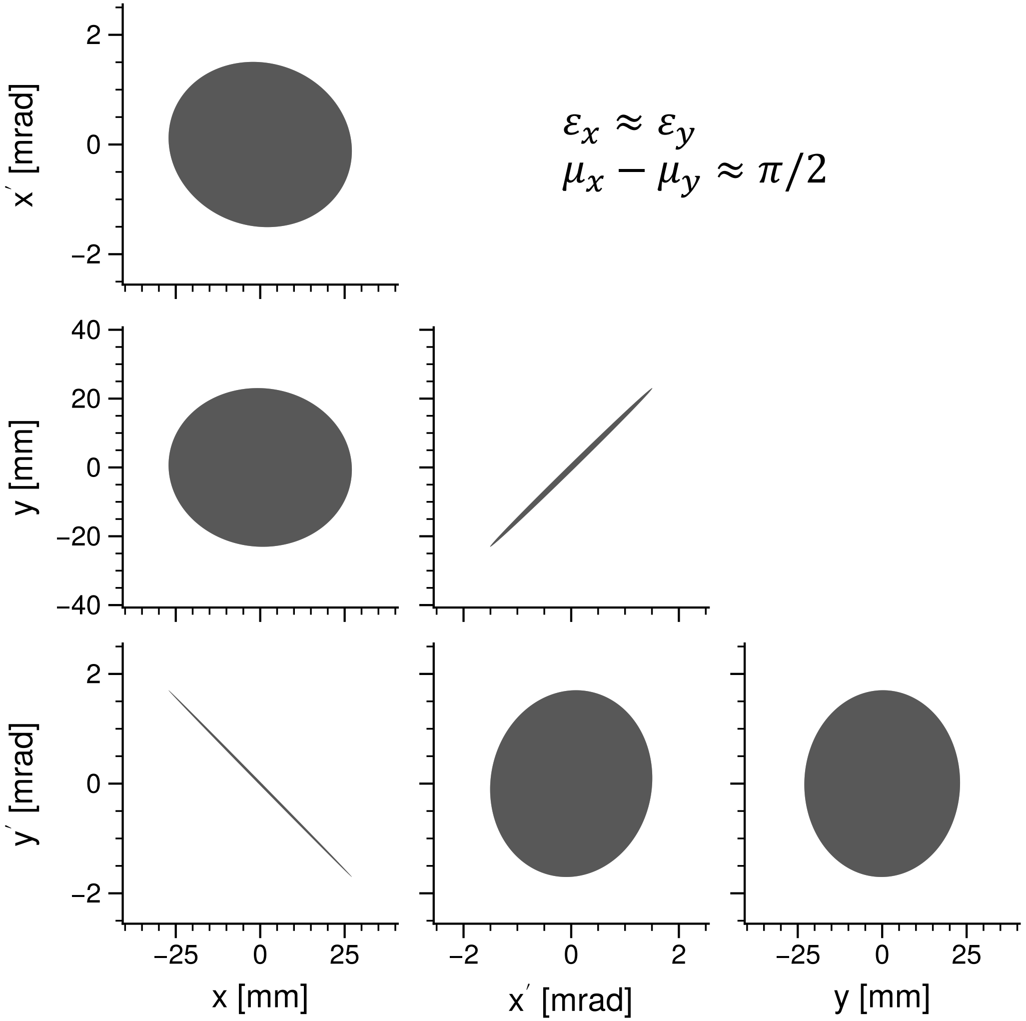
$$x'' + k(s)x = \sum_{n,m} C_{nm}(s)x^n y^m$$

Beam changes eigenvectors

$$x'' + k_{11}(s)x + k_{13}(s)y = \sum_{n,m} C_{nm}(s)x^n y^m$$

Eigenvectors must self-consistently include linear coupling from space charge

Matched envelope at injection



$\varepsilon_x \approx \varepsilon_y$
$\mu_x - \mu_y \approx \pi/2$

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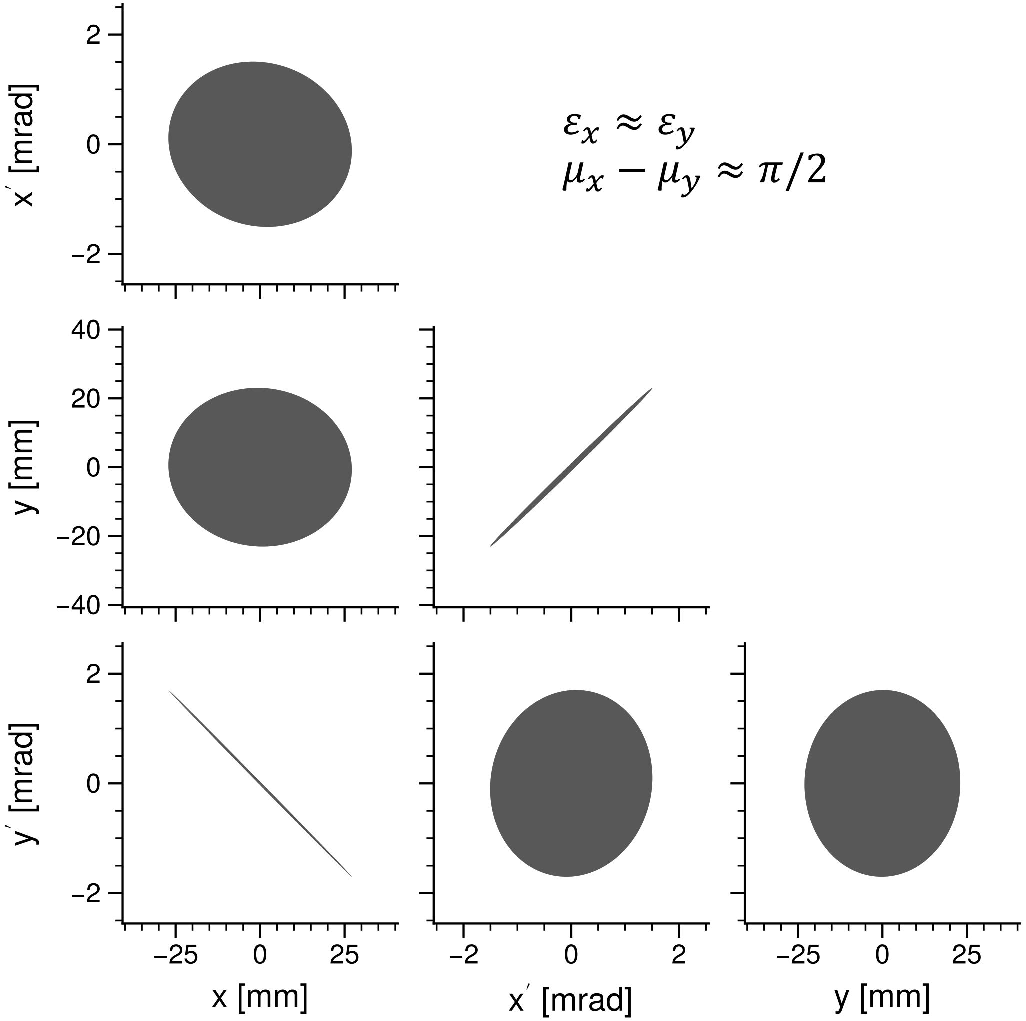

$\varepsilon_x \approx \varepsilon_y$
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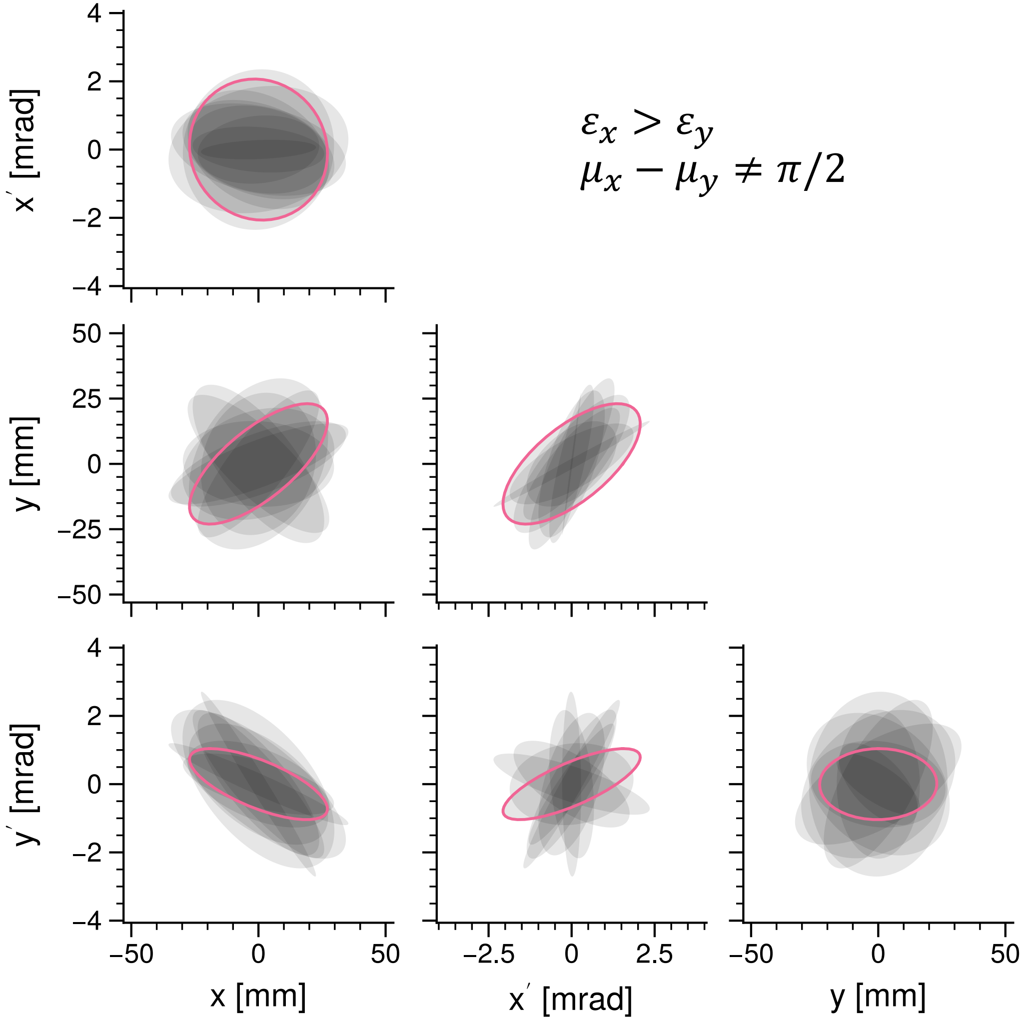
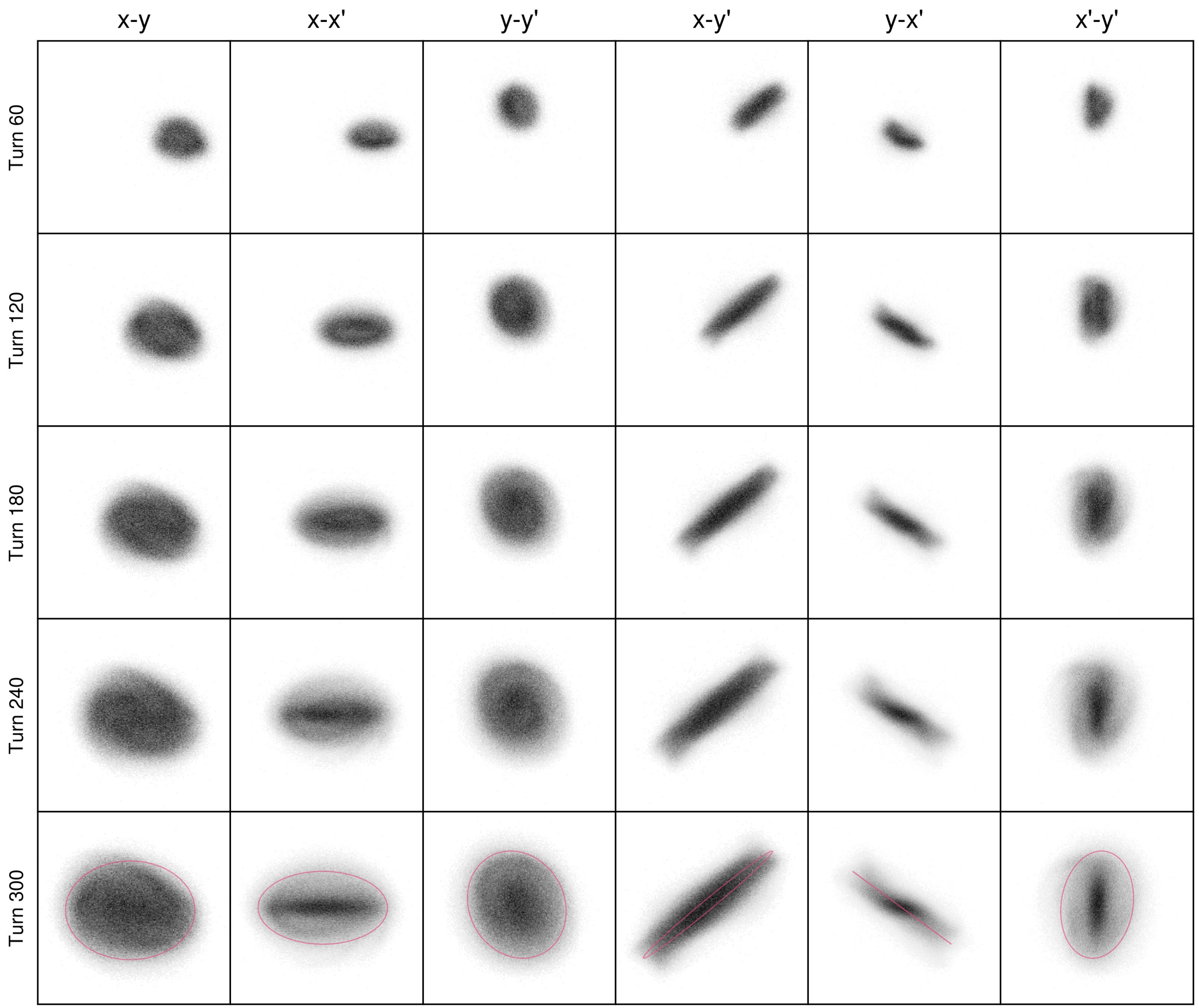
Mismatched envelope at injection



$\varepsilon_x > \varepsilon_y$
$\mu_x - \mu_y \neq \pi/2$

Surprisingly, realistic SNS injection simulations preserve leading-order features



Injection of a self-consistent beam with linear space charge force into a ring

J. A. Holmes, T. Gorlov, N. J. Evans, M. Plum, and S. Cousineau
Phys. Rev. Accel. Beams **21**, 124403 – Published 17 December 2018

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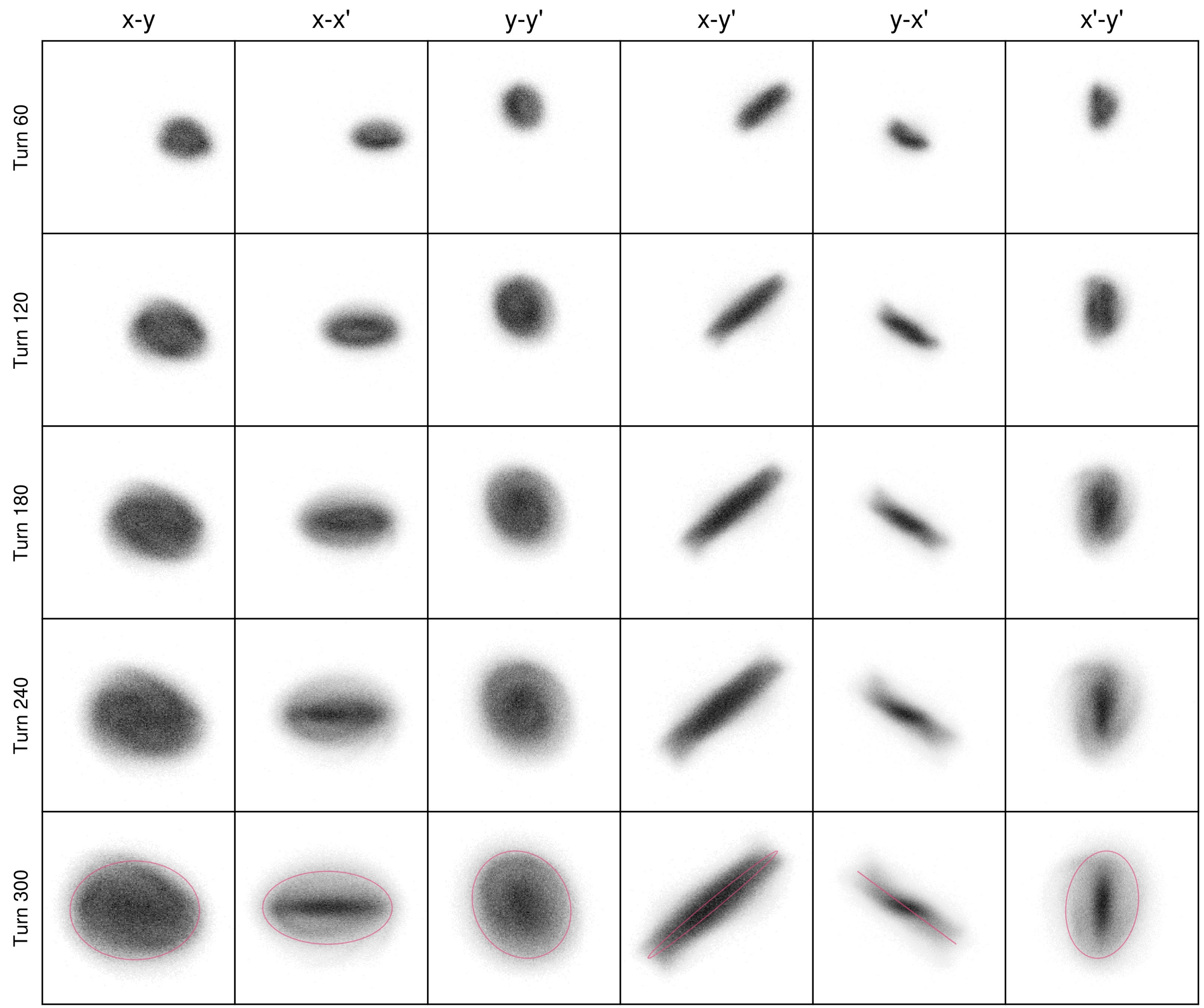

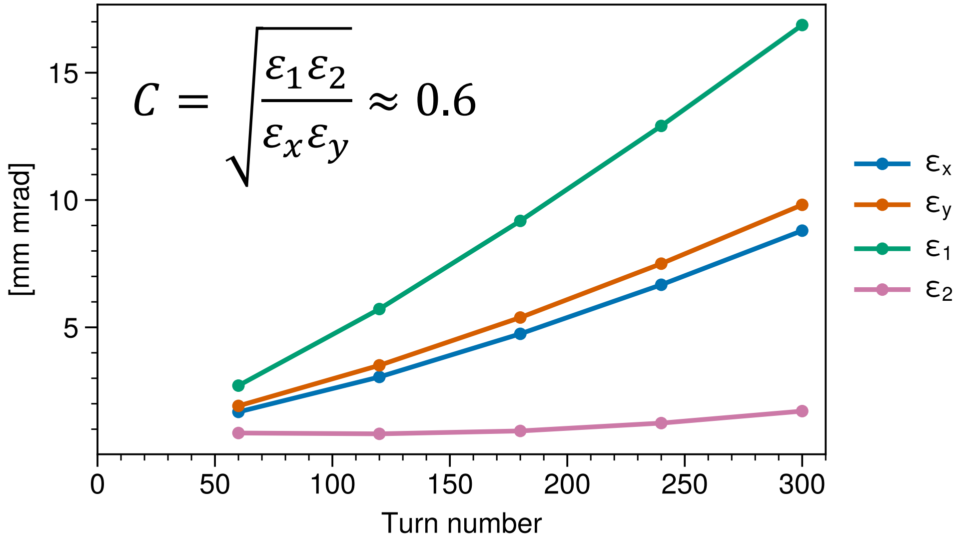

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Questions/research directions

- Best-case performance of elliptical painting
  - Reduction in tune shift/spread
  - Minimum $C = \sqrt{\varepsilon_1 \varepsilon_2 / \varepsilon_x \varepsilon_y}$
- Practicality
  - Proof-of-principle experiment at the SNS
  - Feasibility in collider complex
- Long-term beam stability
  - Halo formation
  - Acceleration, IBS
  - Necessity of circular mode optics
- {3, 3} distribution?
- Theoretical studies?

Extra slides

AMD beams can be produced within a solenoid *at the source*

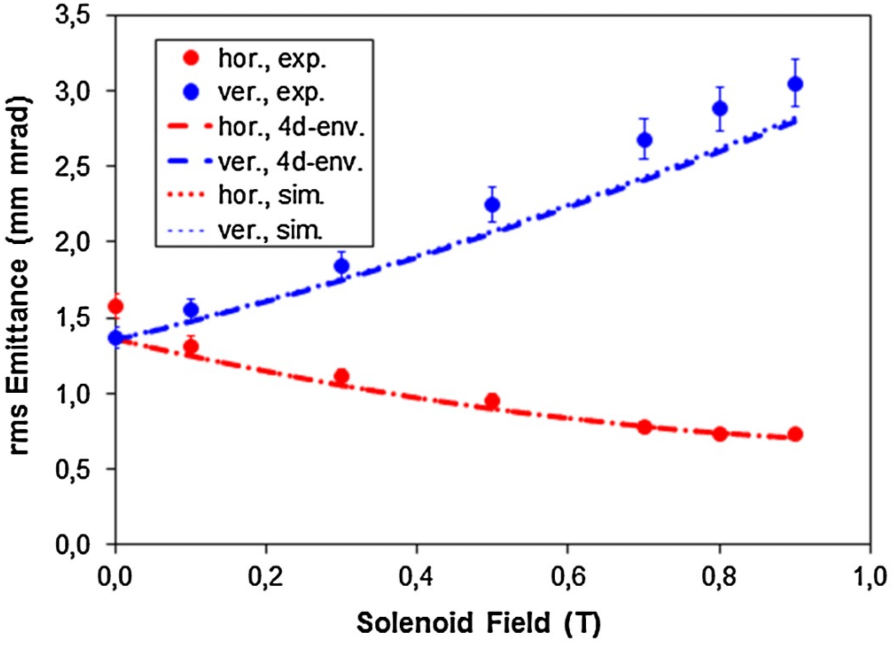
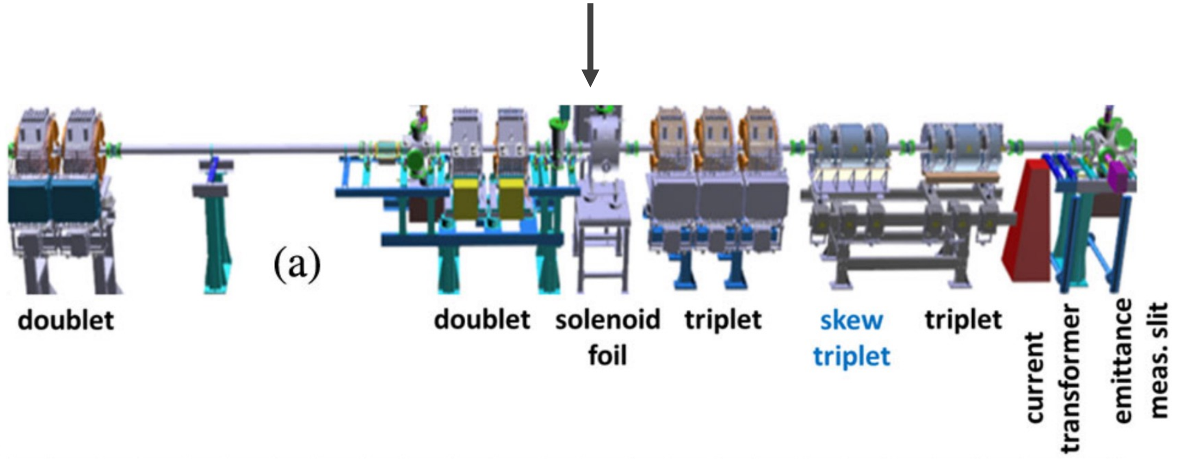


FIG. 4 (color online). Vertical (blue) and horizontal (red) rms emittances at the exit of the EMTEX beam line as functions of the solenoid field strength. All other settings were kept constant. Shown are results from measurements (dots), from application of the 4d-envelope model for coupled lattices (dashed line), and from tracking simulations (dotted line). With respect to Fig. 2, the gradients of the skew quadrupole triplet are inverted.

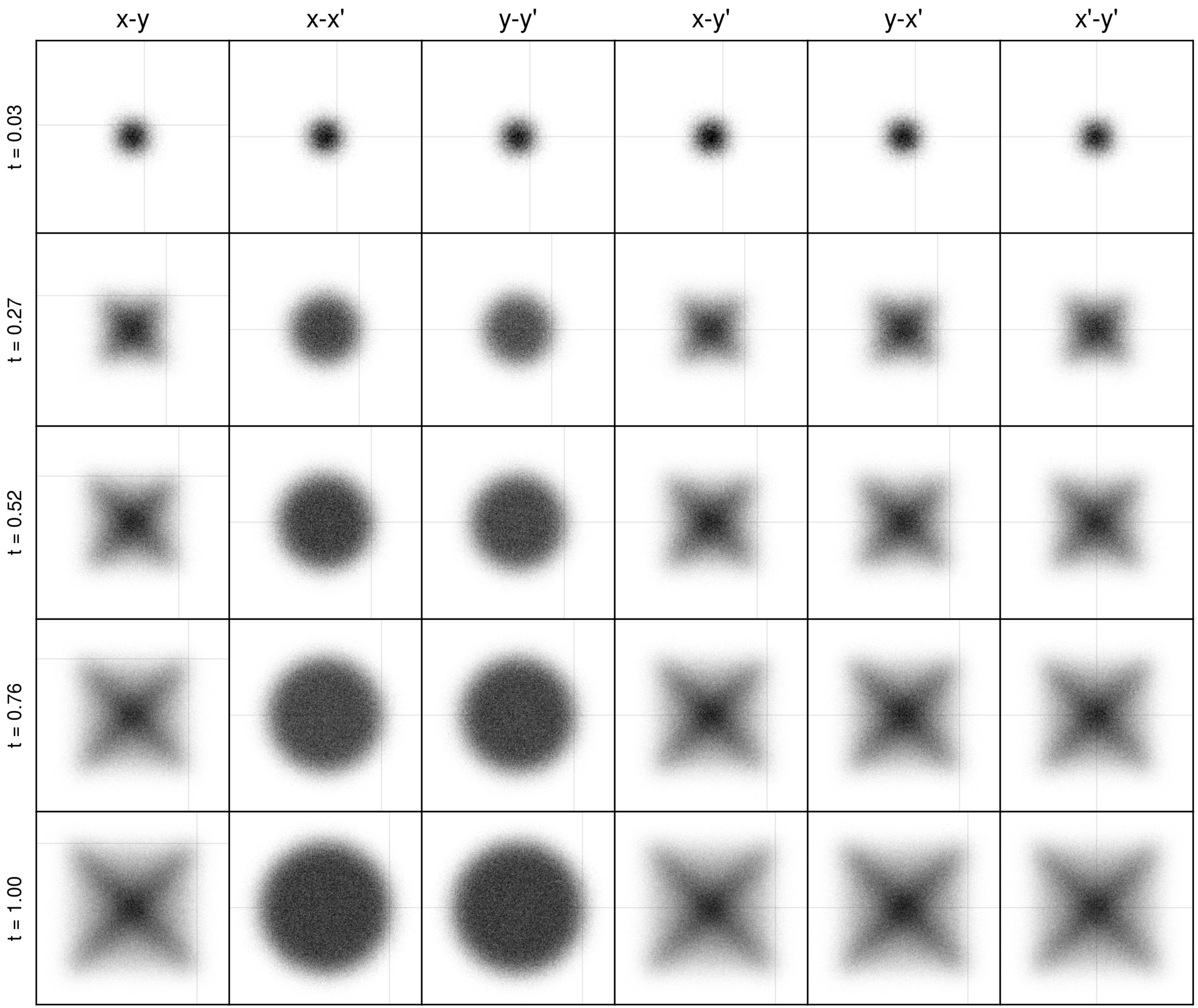
Strip in solenoid



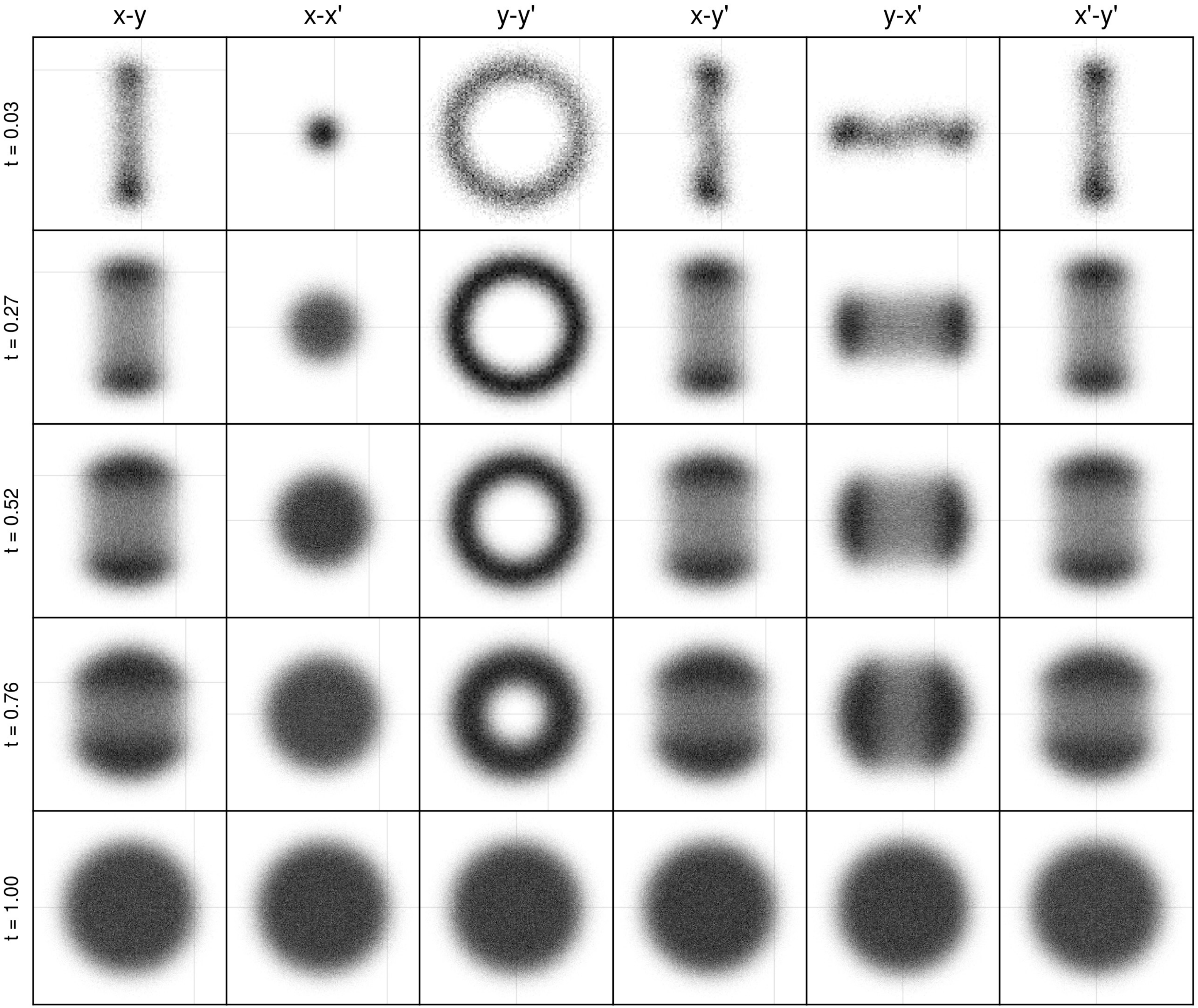
Experimental Proof of Adjustable Single-Knob Ion Beam Emittance Partitioning

L. Groening, M. Maier, C. Xiao, L. Dahl, P. Gerhard, O. K. Kester, S. Mickat, H. Vormann, M. Vossberg, and M. Chung

Phys. Rev. Lett. **113**, 264802 – Published 30 December 2014



x-y
x-x'
y-y'
x-y'
y-x'
x'-y'
t = 0.03
t = 0.27
t = 0.52
t = 0.76
t = 1.00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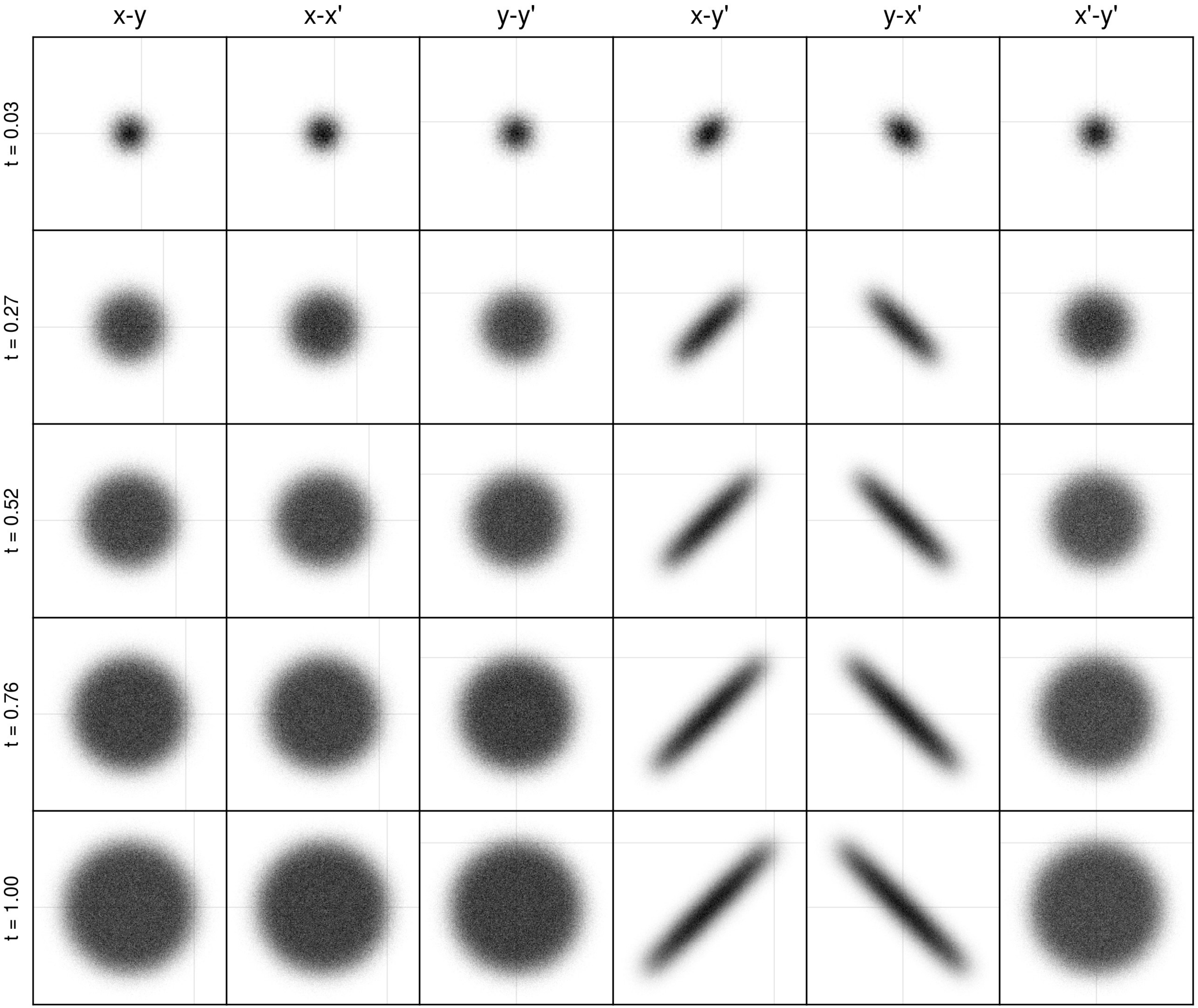
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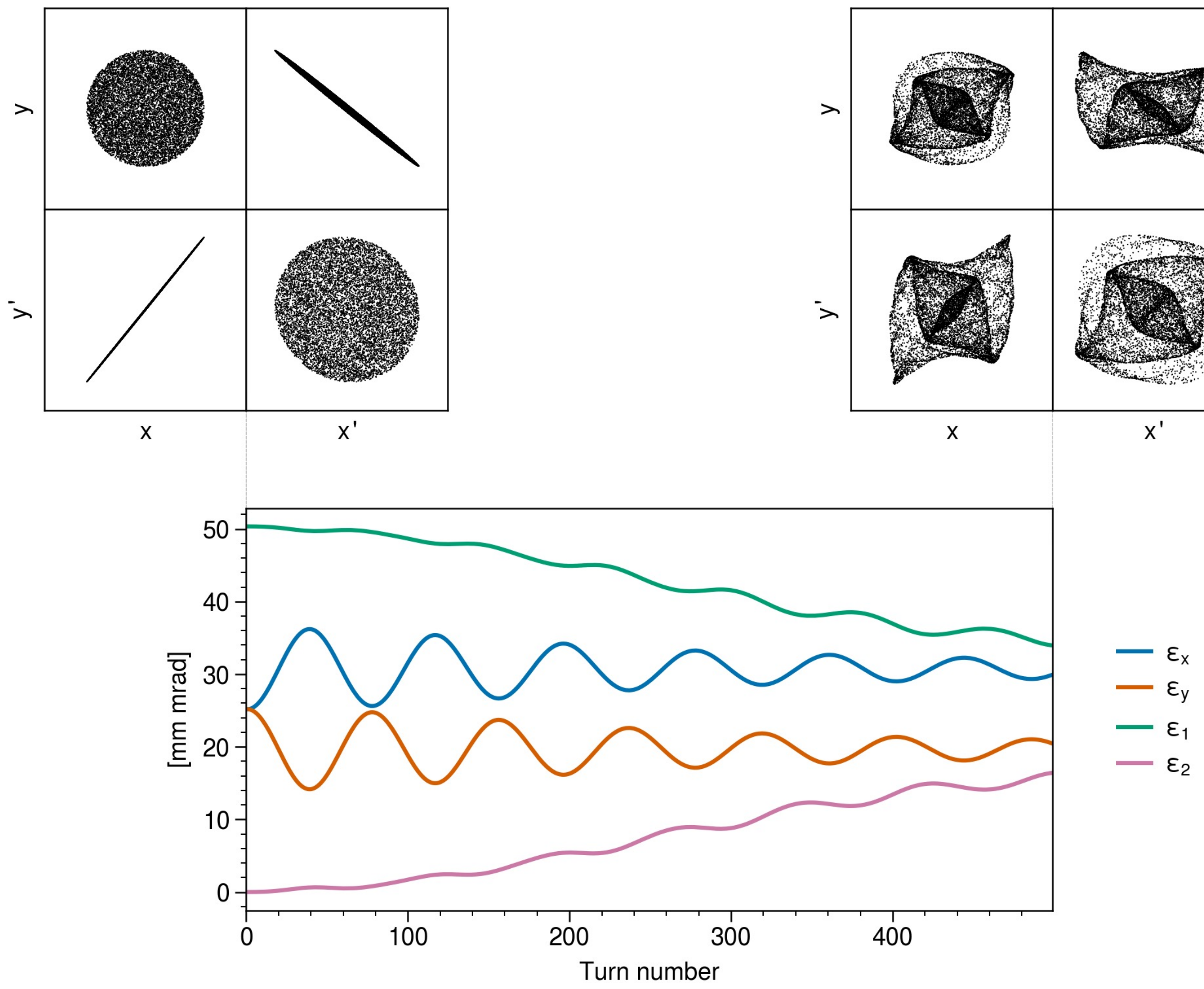
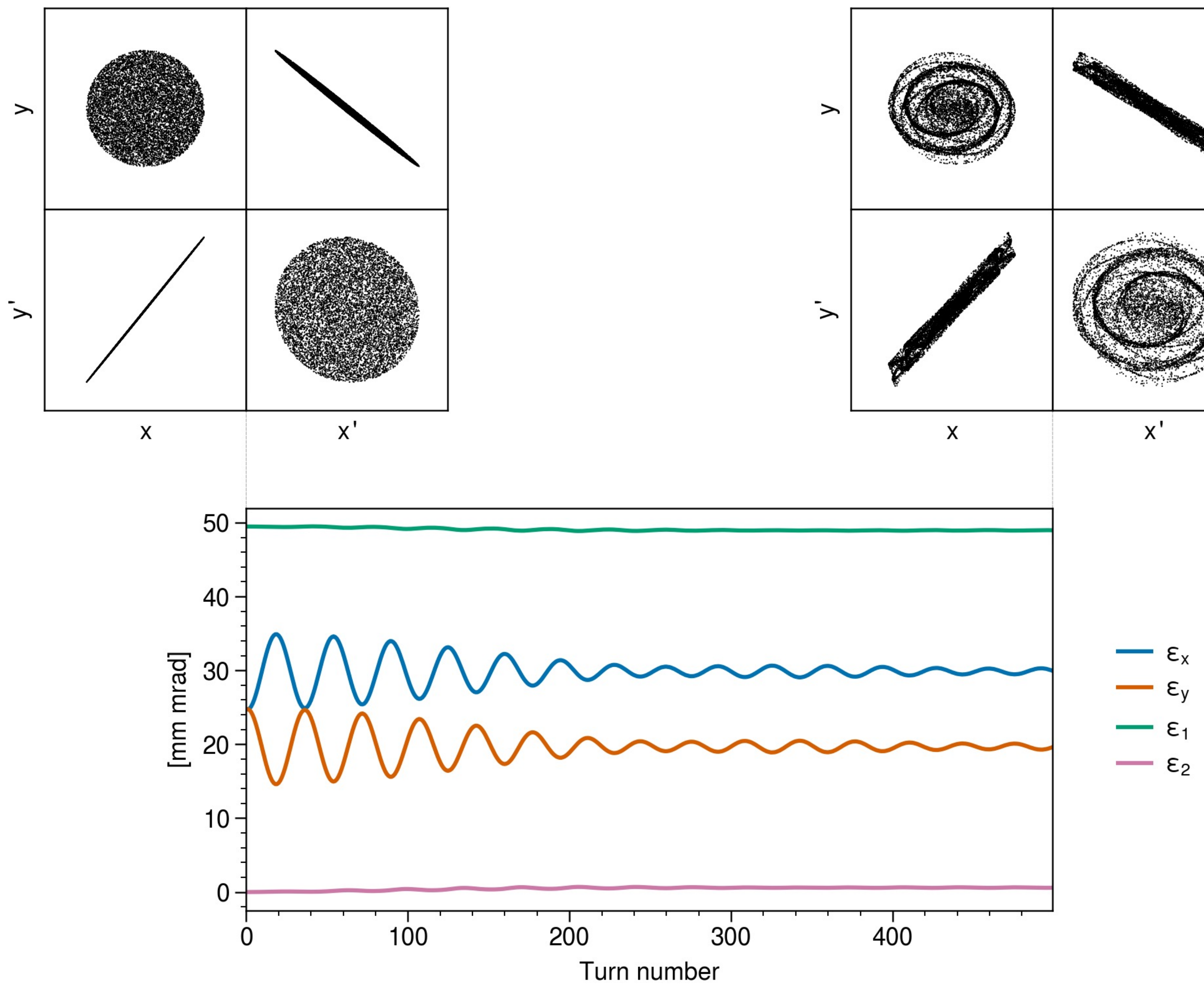
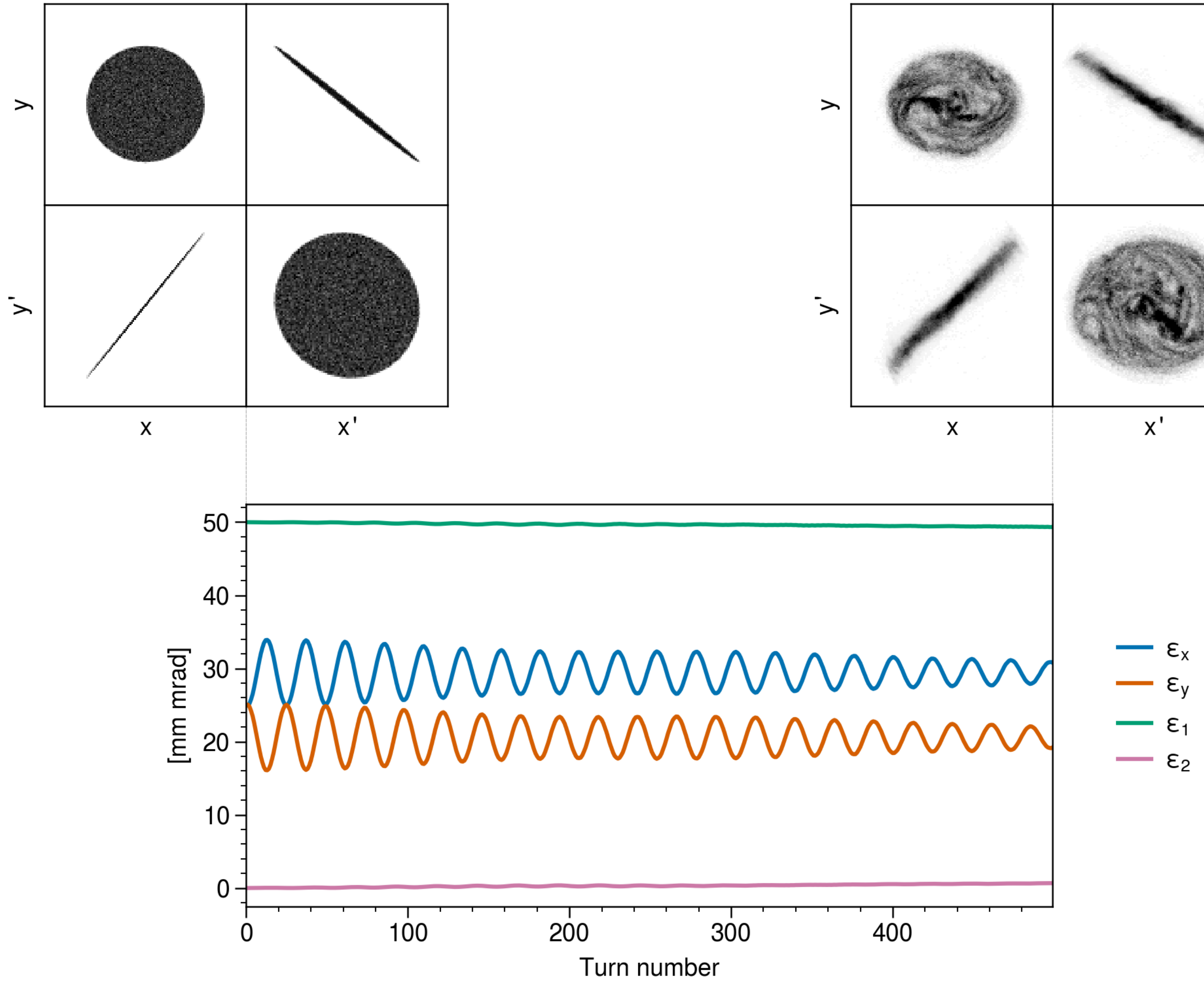
Nonlinear fringe fields can degrade beam quality near difference resonance



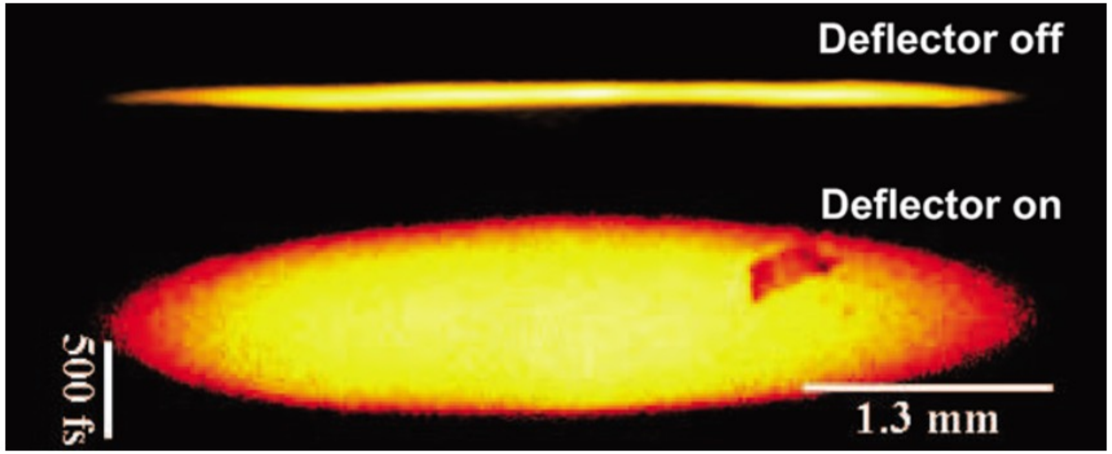
Linear coupling (split tunes) alleviates this problem



Linear coupling can also be provided by the beam

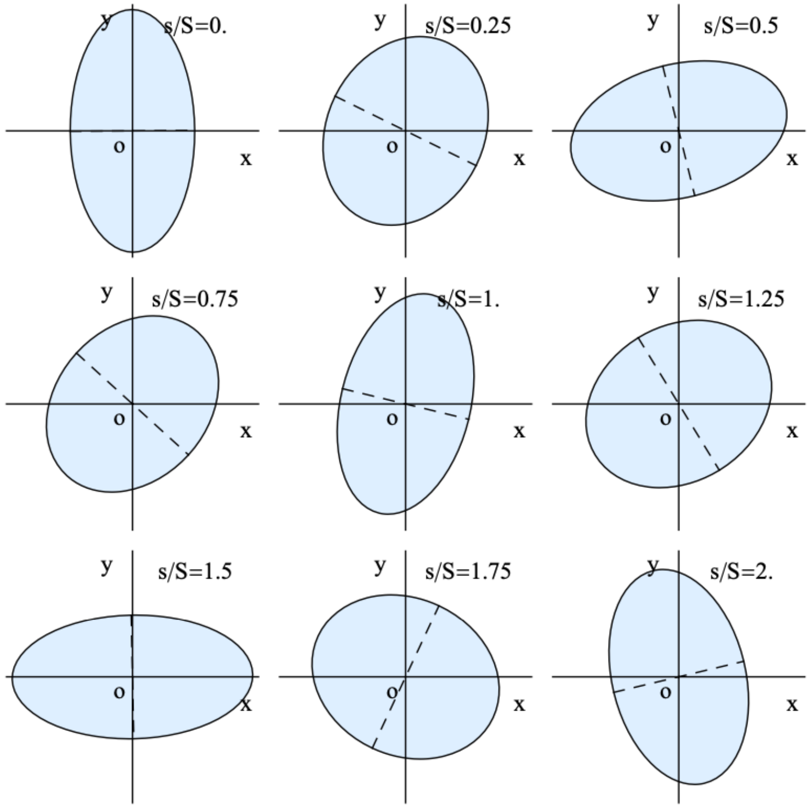


Uniform density ellipsoidal *electron* distributions can be generated with properly shaped laser pulses (the pancake regime)



[Citation]

A more general KV distribution

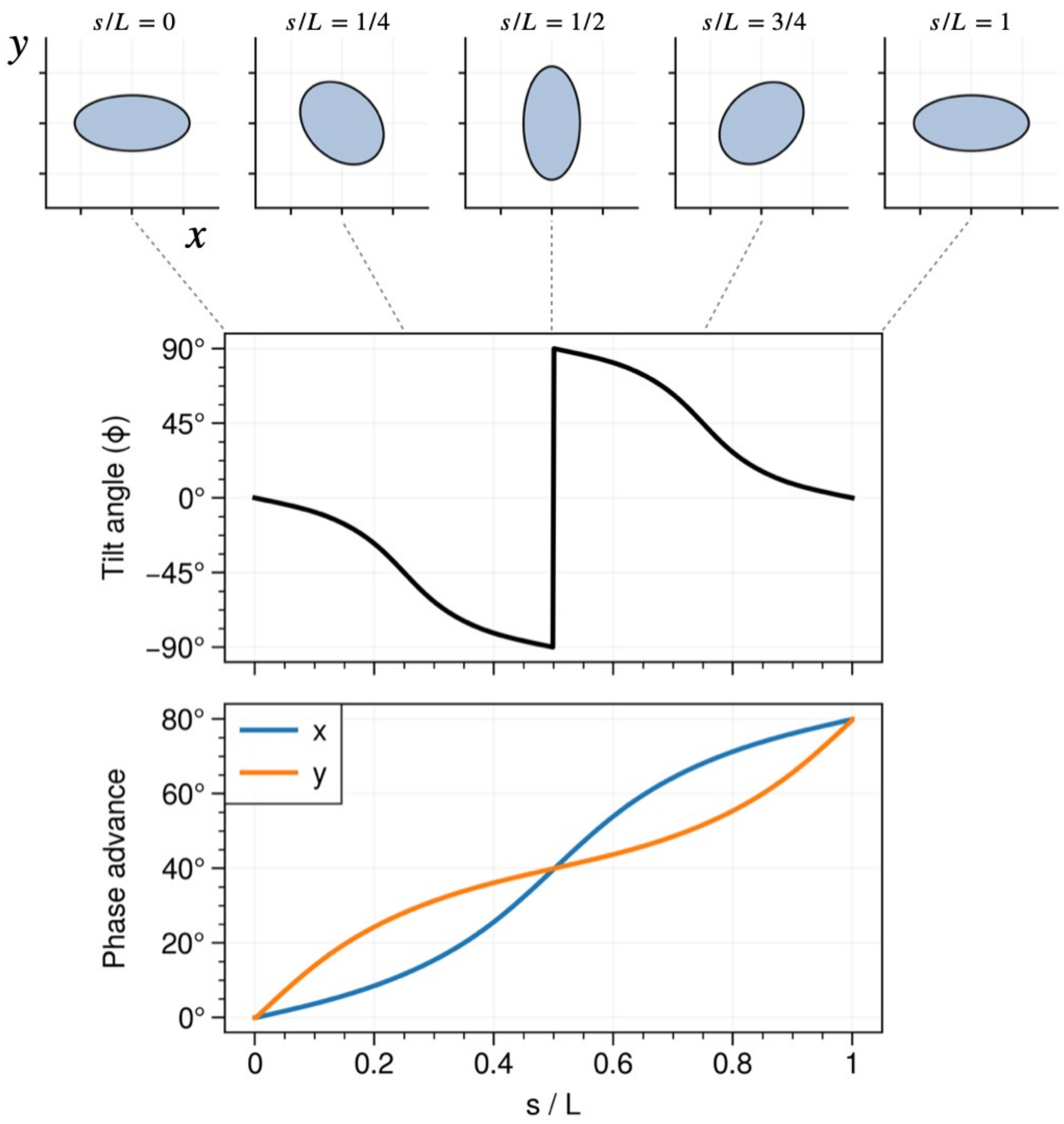


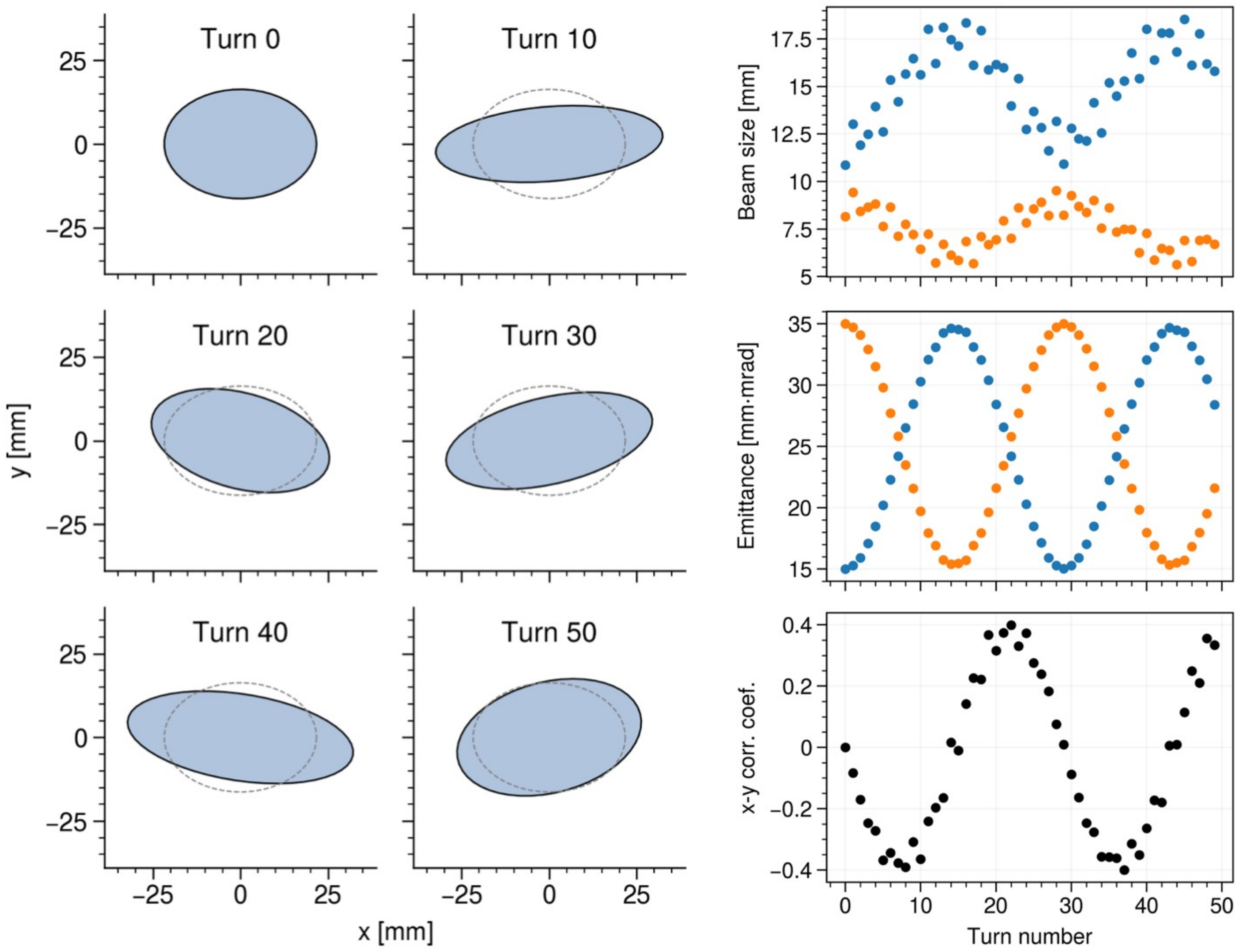
$$f = \frac{N_b \sqrt{|\xi|}}{\pi^2} \delta(I_\xi - 1).$$

$$I_\xi = \mathbf{z}^T Q^T P^T \xi P Q \mathbf{z},$$

Class of Generalized Kapchinskij-Vladimirskij Solutions and Associated Envelope Equations for High-Intensity Charged-Particle Beams

Hong Qin and Ronald C. Davidson
Phys. Rev. Lett. **110**, 064803 – Published 5 February 2013


$s/L = 0$
$s/L = 1/4$
$s/L = 1/2$
$s/L = 3/4$
$s/L = 1$
y
x
90°
45°
0°
−45°
−90°
Tilt angle (ϕ)
80°
60°
40°
20°
0°
Phase advance
x
y
0
0.2
0.4
0.6
0.8
1
s / L



Turn 0
Turn 10
Turn 20
Turn 30
Turn 40
Turn 50
y [mm]
x [mm]
25
0
−25
Beam size [mm]
Emittance [mm·mrad]
x-y corr. coef.
Turn number
0
10
20
30
40
50
17.5
15
12.5
10
7.5
5
35
30
25
20
15
0.4
0.2
0
−0.2
−0.4