The Three - Stop Taxi

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1. The Process

A taxi alternates among three different locations, cleverly labeled 1, 2, and 3. Whenever it reaches location *i*, it stops and spends a random amount of time having mean t_i before obtaining another passenger. A passenger entering the cab at location *i* will want to go to location *j* with probability P_{ij}. The time to travel from *i* to *j* is a random variable with mean m_{ij}.

2. Questions

Suppose that:

$t_1 = 1$ $t_2 = 2$ $t_3 = 4$

$P_{12} = 1$ $P_{23} = 1$ $P_{31} = 2/3 = 1 - P_{32}$

$m_{12} = 10$ $m_{23} = 20$ $m_{31} = 15$ $m_{32} = 25$

Define an appropriate semi-Markov model and determine:

In[1]:=
```
𝒯 = {{0, 1/2, 1/2}, {1/3, 0, 2/3}, {1, 0, 0}};
```

In[2]:=
```
𝒮 = DiscreteMarkovProcess[{1/3, 1/3, 1/3}, 𝒯];
```

1) the proportion of time the tax is waiting at location i

In[6]:=
```
StationaryDistribution[𝒮]
```

Out[6]=
$$\text{ProbabilityDistribution}\left[\frac{3}{7}\,\text{Boole}[x == 1] + \frac{3}{14}\,\text{Boole}[x == 2] + \frac{5}{14}\,\text{Boole}[x == 3], \{x, 1, 3, 1\}\right]$$

In[7]:=
```
πs = Table[PDF[StationaryDistribution[𝒮], i], {i, 1, 3}]
```

Out[7]=
$$\left\{\frac{3}{7}, \frac{3}{14}, \frac{5}{14}\right\}$$

2) the proportion of time the taxi is on the road for *i* to *j*, *i,j=1,2,3*.

In[8]:=
```
μ = {μ1, μ2, μ3}
```

Out[8]=
```
{μ1, μ2, μ3}
```

In[9]:=
```
P = Simplify[Table[πs[[i]] × μ[[i]] / πs.μ, {i, 1, 3}]]
```

Out[9]=
$$\left\{\frac{6\,\mu 1}{6\,\mu 1+3\,\mu 2+5\,\mu 3},\ \frac{3\,\mu 2}{6\,\mu 1+3\,\mu 2+5\,\mu 3},\ \frac{5\,\mu 3}{6\,\mu 1+3\,\mu 2+5\,\mu 3}\right\}$$

If μ_1=6, μ_2=3, and μ_3=5, then the proportion of times in each state would be:

In[10]:=
```
P /. {μ1 → 6, μ2 → 3, μ3 → 5}
```

Out[10]=
$$\left\{\frac{18}{35},\ \frac{9}{70},\ \frac{5}{14}\right\}$$