Computation: Customer Arrivals

by : Austin Pesina

In[ ]:= A store opens at 8 am . From 8 am until 10 am, customers arrive at a Poisson rate of four per hour . Between 10 am and 12 pm, they arrive at a Poisson rate of eight per hour . From 12 pm to 2 pm, the arrival rate increases steadily from eight per hour at 12 pm to ten per hour at 2 pm; and from 2 pm to 5 pm, the arrival rate drops steadily from ten per hour at 2 pm to four per hour at 5 pm . Determine the probability distribution of the number of customers that enter the store on a given day .

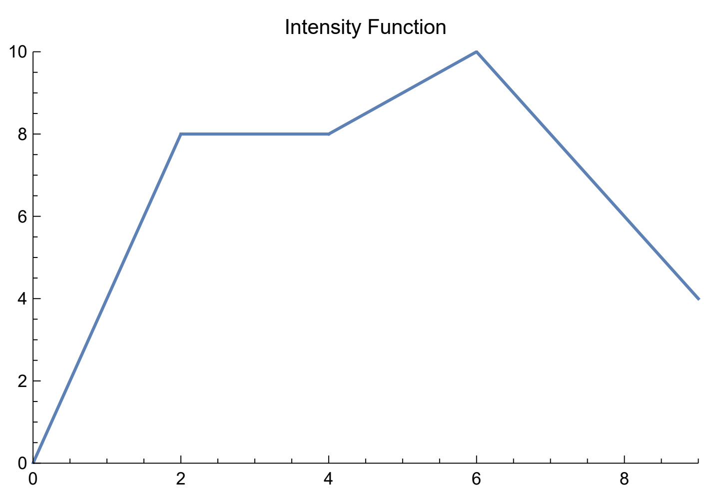
Abstract

In this analysis we are looking at an estimate of how many people enter a store on a given day using a Poisson distribution. By using a piecewise function using data from a given day, we can plug our function into an inhomogeneous Poisson process that gives us an estimate of 63 people entering the store on a daily basis. When we randomize data for 250 days, we still see that the mean of our distribution is around 63 people per day.

Intensity Function

In[12]:=
```
λ[t_] :=
 Piecewise[{{0 + 4 t, 0 ≤ t < 2}, {8, 2 ≤ t < 4}, {t + 4 , 4 ≤ t < 6}, {22 - 2 t, 6 ≤ t < 9}}]
Plot[λ[t], {t, 0, 9}, PlotLabel → "Intensity Function", PlotRange → {{0, 9}, {0, 10}}]
```

Out[13]=



Define the Process

```
In[30]:= 𝒯 = InhomogeneousPoissonProcess[λ[t], t]
```

Out[30]= InhomogeneousPoissonProcess$\left[\begin{cases} 4\,t & 0 \le t < 2 \\ 8 & 2 \le t < 4 \\ 4 + t & 4 \le t < 6 \\ 22 - 2\,t & 6 \le t < 9 \\ 0 & \text{True} \end{cases}, t\right]$

```
In[31]:= Mean[𝒯[t]]
```

Out[31]= $\begin{cases} 63 & t > 9 \\ 8 \times (-1 + t) & 2 < t \le 4 \\ 2\,t^2 & t \le 2 \\ -54 + 22\,t - t^2 & 6 < t \le 9 \\ \frac{1}{2} \times (8\,t + t^2) & \text{True} \end{cases}$

Answering the Questions

(a) What is the average number of arrivals by the end of the business day?

```
In[32]:= Mean[𝒯[9]]
```

Out[32]= 63

(b) What is the probability that no customers arrive before 10am?

```
In[42]:= PDF[PoissonDistribution[Mean[𝒯[2]] - Mean[𝒯[0]]], 0]
```

Out[42]= $e^{-8+\text{Mean}\left[\text{InhomogeneousPoissonProcess}\left[\begin{cases} 4\,t & 0\le t<2 \\ 8 & 2\le t<4 \\ 4+t & 4\le t<6 \\ 22-2\,t & 6\le t<9 \\ 0 & \text{True} \end{cases}, t\right][0]\right]}$

Because Mean[𝒯[0]] is not well defined, we will use a very small number that approaches 0.

```
In[43]:= PDF[PoissonDistribution[Mean[𝒯[2]] - Mean[𝒯[0.0001]]], 0]
```

Out[43]= 0.000335463

(c) What is the first quartile of the number of customers that arrive by the end of the business day?

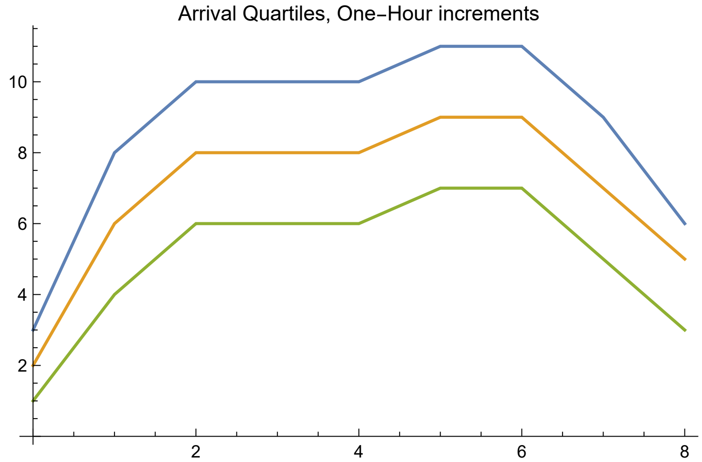
```
In[39]:= qTable[𝒫_, t0_, tmax_, dt_, q_] :=
          Table[{x, InverseCDF[PoissonDistribution[(Mean[𝒫[t + 1]] - Mean[𝒫[t]])], q] /. {t → x}},
           {x, t0, tmax, dt}];

In[44]:= qTable[𝒯, 0.0001, 9, 1, 0.75]
```

```
Out[44]= {{0.0001, 3}, {1.0001, 8}, {2.0001, 10}, {3.0001, 10},
          {4.0001, 10}, {5.0001, 11}, {6.0001, 11}, {7.0001, 9}, {8.0001, 6}}
```

In[46]:=
```
ListLinePlot[{qTable[𝒯, 0.0001, 9, 1, 0.75], qTable[𝒯, 0.0001, 9, 1, 0.5],
  qTable[𝒯, 0.0001, 9, 1, 0.25]}, PlotLabel → "Arrival Quartiles, One-Hour increments"]
```

Out[46]=

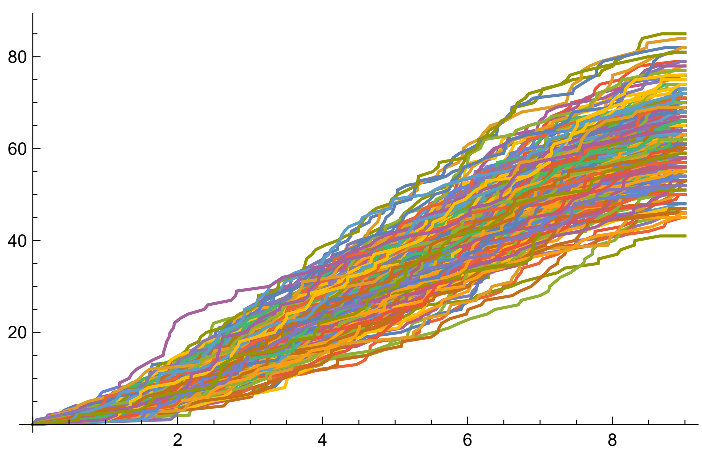


Simulate 250 days of arrivals :

In[47]:=
```
data = RandomFunction[𝒯, {0, 9}, 250];
```

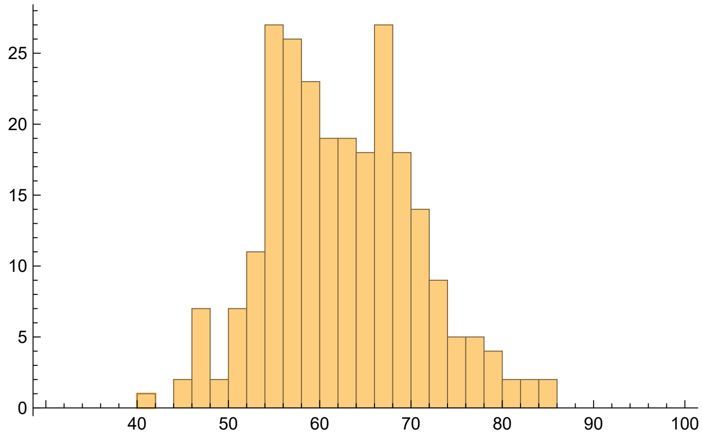
In[48]:=
```
ListLinePlot[data, PlotRange → All]
```

Out[48]=



In[54]:=
```
Histogram[data["SliceData", 9], 20, PlotRange → {{30, 100}, All}]
```

Out[54]=



In[55]:=
```
DiscretePlot[PDF[𝒯[9], x], {x, 30, 100}]
```

Out[55]=