

Consider the following historical (and classic) data set concerning Canadian lynx and snowshoe hare populations from 1845 to 1903.

Year	Snowshoe Hare Pelts (thousands)	Canada Lynx Pelts (thousands)
1845	20	32
1847	20	50
1849	52	12
1851	83	10
1853	64	13
1855	68	36
1857	83	15
1859	12	12
1861	36	6
1863	150	6
1865	110	65
1867	60	70
1869	7	40
1871	10	9
1873	70	20

Year	Snowshoe Hare Pelts (thousands)	Canada Lynx Pelts (thousands)
1875	100	34
1877	92	45
1879	70	40
1881	10	15
1883	11	15
1885	137	60
1887	137	80
1889	18	26
1891	22	18
1893	52	37
1895	83	50
1897	18	35
1899	10	12
1901	9	12
1903	65	25

Figure 1: Population data.

1. Develop a DMD model to forecast the future population states
2. Do a time-delay DMD model to produce a forecast and compare with regular DMD. Determine if it is likely that there are latent variables.
3. Empirical Predator-Prey models such as Lotka-Volterra are commonly used to models such phenomenon. Consider the model $\dot{x} = (b - py)x$ and $\dot{y} = (rx - d)y$. Use the data to fit values of b, p, r and d .
4. Find the best fit nonlinear, dynamical systems model to the data using sparse regression.
5. Compute the KL divergence of the best model fit to the data between all the above models.
6. Retain three of your best fit models and compare their AIC and BIC scores.

Download the data set BZ.mat (which is a snippet from a Belousov-Zhabotinsky chemical oscillator movie – check them out on youtube). Repeat the first two steps above to see how DMD and delay embedded DMD work in producing forecasts of the chemical reaction.

The following code may be helpful to view the data.

```
[m,n,k]=size(BZ_tensor); % x vs y vs time data
for j=1:k
    A=BZ_tensor(:,:,j);
    pcolor(A), shading interp, pause(0.2)
end
```