Arwen 0.1.0
500 N methanol / gox Liquid Rocket Engine

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Contents

0.1 Overview

A brief overview of the target-metrics for the engine.

- **Thrust:** 500 N
- **Isp:** 248 s
- **Chamber Pressure:** 2.07 MPa
- **Propellants:**
  - **Mixture Ratio:** 1.2
  - **Total Mass Flow Rate:** 0.205518 kg s-1
  - **Fuel:**
    - **Type:** methanol
    - **Mass Flow Rate:** 0.0934172 kg s-1
  - **Oxidizer:**
    - **Type:** gox
    - **Mass Flow Rate:** 0.112101 kg s-1

0.2 Throat

Defining characteristics and calculations for the throat joining the nozzle and combustion chamber.

0.2.1 Critical Constants

Underlying constants that form the throat profile:

- **Gamma (prop heat capacity ratio):** 1.2
- **Propellant Combustion Temp (Kelvin):** 3322.04 K
- **Chamber Pressure:** 2.07 MPa

0.2.2 Temperature Profile

The following equation gives the temperature to be expected at the throat plane of the nozzle.

$$\begin{aligned} \boldsymbol{T_{throat}} &= \boldsymbol{T_{coefficient}} * \boldsymbol{T_{chamber}} \\ &= 3322.04K * \frac{1}{1 + \frac{\gamma-1}{2}} \\ &= 3020.04K \end{aligned} \tag{1}$$

0.2.3 Pressure Profile

The following equation gives the pressure to be expected at the throat plane of the nozzle.

$$\begin{aligned} \boldsymbol{P_{throat}} &= \boldsymbol{P_{coefficient}} * \boldsymbol{P_{chamber}} \\ &= \left(1 + \frac{\gamma - 1}{2}\right)^{-1*\frac{\gamma}{\gamma-1}} * 2.068e+06Pa \\ &= 1.16733e+06Pa \end{aligned} \tag{2}$$

0.2.4 Throat Geometry

The following equations define the geometrical/physical dimensions of the throat.

First, we find $\boldsymbol{R'}$ using the following equation:

$$\begin{aligned}\boldsymbol{R'} &= \frac{\boldsymbol{R}}{0.02668kgmol-1} \\ &= 311.636m+2s-2K-1\end{aligned} \tag{3}$$

where:

$$\boldsymbol{R} = 8.31446m+2kgs-2K-1mol-1 \tag{4}$$

This allows us to find the area of the throat cross-section via:

$$\begin{aligned}\boldsymbol{A}_{\boldsymbol{throat}} &= \frac{0.205518kgs-1}{1.16733e+06Pa} * \sqrt{\frac{\boldsymbol{R'} * 3020.04K}{\boldsymbol{\gamma}}} \\ &= (1.76058e-07ms) * (885.604m/s) \\ &= 155.917mm+2\end{aligned} \tag{5}$$

From here, basic geometry gets us the diameter/radius:

$$
\begin{aligned}
\boldsymbol{r_{throat}} &= \sqrt{\frac{0.000155917m+2}{\pi}} \\
&= 7.04486mm
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
\boldsymbol{d_{throat}} &= \boldsymbol{r_{throat}} * 2 \\
&= 14.0897mm
\end{aligned}
\tag{7}
$$

0.2.5 Summary

- **Throat Pressure:** 1.17 MPa
- **Throat Temperature:** 3020.04 K
- **Throat Area:** 155.917 mm+2
- **Throat Radius:** 7.04486 mm

0.3 Combustion Chamber

Defining characteristics and calculations for the combustion chamber.

0.3.1 Critical Constants

Underlying constants that form the combustion chamber profile:

- **Throat Area:** 155.917 mm+2
- **Throat Dimeter:** 14.0897 mm
- **Converging Angle (θ):** 30 °
- **L*:** 600 mm

0.3.2 Combustion Chamber Geometry

The following equations define the geometrical/physical dimensions of the chamber and converging portion of the nozzle.

$$\begin{aligned} \boldsymbol{V_{chamber}} &= \boldsymbol{A_{throat}} * \boldsymbol{L*} \\ &= (0.000155917m + 2) * (0.6m) \\ &= 93550.4mm + 3 \end{aligned} \tag{8}$$

We achieve an initial approximation of the chamber length from the following formula:

$$\begin{aligned} \boldsymbol{L_{estimate}} &= e^{\left(0.029*(\ln(\boldsymbol{D_{throat}}))^2 + 0.047*\ln(\boldsymbol{D_{throat}}) + 1.94\right)} \\ &= 96.458mm \end{aligned} \tag{9}$$

$$\boldsymbol{D_{estimate}} = 35.1406mm \tag{10}$$

Which we can further refine by solving the following via iteration:

$$\boldsymbol{D_{estimate}} = \sqrt{\frac{(\boldsymbol{D_{throat}})^3 + \frac{24}{\pi} * \tan(\theta) * \boldsymbol{V_{chamber}}}{0.0351406 + 6 * \tan(\theta) * \boldsymbol{L_{estimate}}}} \tag{11}$$

Which yields (after 100 iterations):

$$\boldsymbol{D_{chamber}} = 33.4026mm \tag{12}$$

$$\boldsymbol{R_{chamber}} = 16.7013mm \tag{13}$$

Giving:

$$\begin{aligned} \boldsymbol{A_{chamber}} &= 0.000278934m + 2 * \pi \\ &= 876.296mm + 2 \end{aligned} \tag{14}$$

Giving a contraction ratio of:

$$\frac{\boldsymbol{A_{chamber}}}{\boldsymbol{A_{throat}}} = 5.62026 \tag{15}$$

We can now find a length for the chamber:

$$\begin{aligned} \boldsymbol{L_{chamber}} &= \frac{\boldsymbol{V_{chamber}}}{\boldsymbol{A_{chamber}}} \\ &= \frac{93550.4mm + 3}{876.296mm + 2} \\ &= 106.757mm \end{aligned} \tag{16}$$

Via trigonometry we find the converging section length:

$$\begin{aligned} \boldsymbol{R_{diff}} &= \boldsymbol{R_{conv}} - \boldsymbol{R_{throat}} \\ &= 9.65645mm \end{aligned} \tag{17}$$

$$\begin{aligned} \boldsymbol{L_{conv}} &= \frac{\boldsymbol{R_{diff}}}{\sin(\theta)} * \sin(\mathit{90}^\circ - \theta) \\ &= 16.7255mm \end{aligned} \tag{18}$$

Which then yields:

$$\begin{aligned} \boldsymbol{L_{flatwall}} &= \boldsymbol{L_{chamber}} - \boldsymbol{L_{conv}} \\ &= 90.0311mm \end{aligned} \tag{19}$$

0.3.3 Summary

- **Flatwall Section Diameter:** 33.4026 mm
- **Flatwall Section Area:** 876.296 mm+2

- **Flatwall Section Length:** 90.0311 mm
- **Flatwall Lateral Area:** 112.028 cm+2
- **Flatwall Volume:** 93.5504 cm+3

- **Converging Section Height:** 9.65645 mm
- **Converging Section Length:** 16.7255 mm
- **Converging Lateral Area:** 14.4076 cm+2
- **Converging Section Volume:** 7.81552 cm+3

- **Length/Width Ratio:** 3.19605
- **Contraction Ratio:** 5.62026

- **Total Chamber Length:** 106.757 mm
- **Total Surface Area:** 126.435 cm+2
- **Combustible Volume:** 101.366 cm+3

0.4 Nozzle

Defining characteristics and calculations for the nozzle.

0.4.1 Critical Constants

Underlying constants that form the nozzle profile:

- $P_{ambient}$**:** 101 kPa
- $P_{chamber}$**:** 2.07 MPa
- A_{throat}**:** 155.917 mm
- **Gamma:** 1.2

0.4.2 Nozzle Geometry

The following equations define the geometrical/physical dimensions of the nozzle and exit area.

First we solve for the mach number:

$$\begin{aligned} \boldsymbol{N_{mach}} &= \sqrt{\frac{2}{\boldsymbol{\gamma} - 1} * \left(\frac{\boldsymbol{P_{chamber}}}{\boldsymbol{P_{ambient}}}\right)^{\frac{\gamma-1}{\gamma}} - 1} \\ &= 2.55548 \end{aligned} \tag{20}$$

We can now find the appropriate exit area:

$$\begin{aligned} \boldsymbol{A_{exit}} &= \frac{\boldsymbol{A_{throat}}}{2.55548} * \left(\frac{1 + \frac{\gamma-1}{2} * 6.53049}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2*\gamma-1}} \\ &= 573.236mm + 2 \end{aligned} \tag{21}$$

Just as in the throat calculaions, basic geometry gives us radius and diameter.

$$\begin{aligned} \boldsymbol{r_{exit}} &= \sqrt{\frac{0.000573236m+2}{\pi}} \\ &= 13.508mm \end{aligned} \tag{22}$$

$$\begin{aligned} \boldsymbol{d_{exit}} &= \boldsymbol{r} * 2 \\ &= 27.016mm \end{aligned} \tag{23}$$

Via trigonometry we find the nozzle length:

$$\begin{aligned} \boldsymbol{R_{diff}} &= \boldsymbol{R_{nozzle}} - \boldsymbol{R_{throat}} \\ &= 6.46316mm \end{aligned} \tag{24}$$

$$\begin{aligned} \boldsymbol{L_{nozzle}} &= \frac{\boldsymbol{R_{diff}}}{\sin(\theta)} * \sin(90^\circ - \theta) \\ &= 24.1208mm \end{aligned} \tag{25}$$

For the given diverging angle of: 15 °

0.4.3 Summary

- **Gas Mach Number:** 2.55548
- **Nozzle Length:** 24.1208 mm

- **Nozzle Radius:** 13.508 mm
- **Nozzle Diameter:** 27.016 mm
- **Diverging Angle:** 15 °