LaTeX Paper - MA200
Due Thursday, Dec. 12
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Theorem 4.8.1 (page 348)

The general solution of a given **linear first-order differential equation** $y' + F(x)y = G(x)$ can be found by:

$y = [e^{-\int F(x)dx}] * [\int G(x) * e^{\int F(x)dx}dx + C]$

(Assuming both F(x) and G(x) are continuous functions)

First we should understand the concepts and terms of what we are solving.

A **first order differential equation** is an equation where there exists a function F(x,y) consisting of two variables (x and y) defined on a region in the x-y plane.

A **linear differential equation** is a differential equation with a linear polynomial of the form:

$$a_0(x)y + a_1(x)y' + a_2(x)y'' + ... + a_n(x)y^n + b(x) = 0$$

$$a_0(x), ..., a_n(x), b(x) :$$

(Arbitrary differentiable functions, these do not have to be linear)

$$y', ..., y^n :$$

(Successive derivates of the function y with respect to the variable x)

This is an **ordinary differential equation (ODE)** meaning it contains one or more functions of an independent variable and its derivates. **Partial differential equations** also exist however these are with respect to more than one independent variable and will not be addressed here.

Now that we understand what we're working with, let's get into the numbers, or letters... I guess.

$y' + F(x)y = G(x)$ (Here's our linear first order ODE) **Equation 1**

We can use an **integrating factor** to solve for our first order ODE. An integrating factor is a function that can be multipled by an ODE for the purpose of making it integrable.
The integrating factor that will be used for linear first order ODE will be:

$e^{\int F(x)dx}$

Multiply this factor by all our terms to give:

$y'e^{\int F(x)dx} + F(x)ye^{\int F(x)dx} = G(x)e^{\int F(x)dx}$

That's a mess, so instead let's simplify our integrating factor:

I $= e^{\int F(x)dx}$

Now to clean everything up a little bit substitute I for our integrating factor from Equation 1. This is an **exact differential equation**.

$y'I + F(x)yI = G(x)I$

Both sides will be then integrated:
$\int[y'I + F(x)yI]dx = \int[G(x)I]dx$

From the product rule we get $(yI)' = y'I + F(x)yI$
This is why our integrating factor was chosen as $e^{\int F(x)dx}$! It can now be integrated easily and rewritten as:
$yI = \int[G(x)I]dx + C$ — **Equation 2**

Wait a second, remember how I $= e^{\int F(x)dx}$?

So let's expand Equation 2 with our integrating factor.

$ye^{\int F(x)dx} = \int[G(x)e^{\int F(x)dx}]dx + C$

Divide both sides by the integrating factor:
$y = [\int G(x) * e^{\int F(x)dx}dx + C]/[e^{\int F(x)dx}]$

$$y = [e^{-\int F(x)dx}] * [\int G(x) * e^{\int F(x)dx}dx + C]$$

$\blacksquare$