CS 435 - Computational Photography

Assignment 2 - Canny Edge Detection
Summer 2022

1 (26pts) Theory Questions

1. (5pts) Apply a 3×3 mean filter to the following 2D matrix. You may assume that the filter is only applied to areas of the data that have a full 9 samples to process. Feel free to use Matlab to help you compute this, however, realize that you may be asked to do this without a calculator on an exam.

$$I = \begin{bmatrix} 7 & 7 & 6 & 3 & 3 & 4 & 2 & 2 \\ 3 & 7 & 2 & 6 & 4 & 4 & 5 & 7 \\ 5 & 4 & 7 & 5 & 1 & 1 & 2 & 2 \\ 2 & 1 & 3 & 4 & 1 & 3 & 5 & 6 \\ 6 & 2 & 2 & 7 & 4 & 2 & 5 & 4 \\ 2 & 2 & 2 & 3 & 6 & 6 & 6 & 7 \\ 4 & 6 & 5 & 6 & 7 & 3 & 4 & 1 \\ 5 & 2 & 4 & 6 & 1 & 4 & 1 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & 7 & 6 & 3 & 3 & 4 & 2 & 2 \\ 3 & 2 & 5 & 4 & 3 & 3 & 3 & 7 \\ 5 & 4 & 4 & 4 & 3 & 3 & 3 & 2 \\ 2 & 4 & 4 & 4 & 3 & 3 & 3 & 6 \\ 6 & 3 & 3 & 4 & 4 & 4 & 5 & 4 \\ 2 & 3 & 4 & 5 & 5 & 5 & 4 & 7 \\ 4 & 4 & 4 & 5 & 5 & 4 & 4 & 1 \\ 5 & 2 & 4 & 6 & 1 & 4 & 1 & 4 \end{bmatrix}$$

2. (5pts) What is the kernel function for a 5×5 Gaussian function with $\sigma = 2$? Normalize the kernel so that its elements sum to one.

$$G2D(x, y, \sigma) = \frac{1}{2\pi\sigma^2} * e^{-(\frac{x^2+y^2}{2\sigma^2})}$$

$$G2D(5, 5, 2) = \frac{1}{2\pi(2)^2} * e^{-(\frac{5^2+5^2}{2(2)^2})}$$

$$G2D(5, 5, 2) = 0.0000768103$$

3. (5pts) Given the following 2D kernels, what is the magnitude and direction of the gradient at the center pixel in I? Feel free to use Matlab to help you compute this, however, realize that you may be asked to do this without a calculator on an exam.

$$\frac{\partial}{\partial x} = \begin{bmatrix} -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \end{bmatrix}, \frac{\partial}{\partial y} = \begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$I = \begin{bmatrix} 7 & 7 & 6 \\ 3 & 7 & 2 \\ 5 & 4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 7 & 7 & 6 \\ 3 & 7 & 2 \\ 5 & 4 & 7 \end{bmatrix} i + \begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 7 & 7 & 6 \\ 3 & 7 & 2 \\ 5 & 4 & 7 \end{bmatrix} j$$

$$\begin{bmatrix} -\frac{2}{3} & -1 & \frac{1}{3} \\ -\frac{2}{3} & -1 & \frac{1}{3} \\ -\frac{2}{3} & -1 & \frac{1}{3} \end{bmatrix} i + \begin{bmatrix} -5 & -6 & -5 \\ 0 & 0 & 0 \\ 5 & 6 & 5 \end{bmatrix} j$$

$$\begin{bmatrix} \frac{-2}{3} i \\ \frac{-2}{3} i \\ \frac{-2}{3} i \end{bmatrix} + \begin{bmatrix} -6j \\ 0 \\ -6j \end{bmatrix}$$

$$\begin{bmatrix} \frac{-2}{3} i - 6j \\ \frac{-2}{3} i \\ \frac{-2}{3} i - 6j \end{bmatrix}$$

Magnitude = 2/3
Direction = negative x direction

4. Imagine that the matrix below contains is for the magnitude of the *gradients* of an image. Which pixels would we consider edge pixels if we applied *hysteresis* with a low threshold of $T_L = 2$ and a high threshold of $T_H = 4$? Visualize this as a *binary* image/matrix such that a location has a value of one if it is an edge pixel. Do this using:

   (a) 4-way connectivity (2pts)

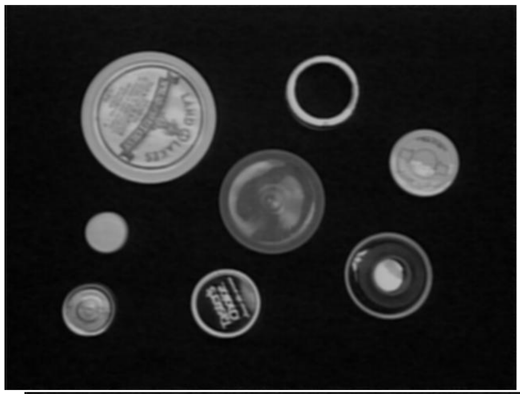
   (b) 8-way connectivity (3pts)

$$|G| = \begin{bmatrix} 2 & 3 & 4 & 5 & 1 \\ 1 & 0 & 2 & 2 & 1 \\ 4 & 3 & 5 & 1 & 2 \\ 4 & 4 & 4 & 4 & 6 \\ 4 & 5 & 2 & 0 & 2 \\ 2 & 3 & 3 & 0 & 3 \end{bmatrix}$$

$$4 - way = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

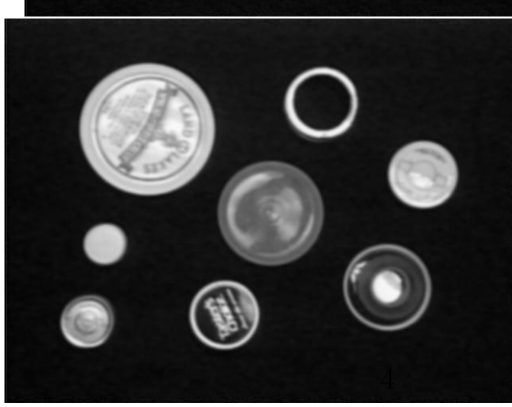
$$8 - way = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$


grey scale


5x5 Kernel 1.75 σ

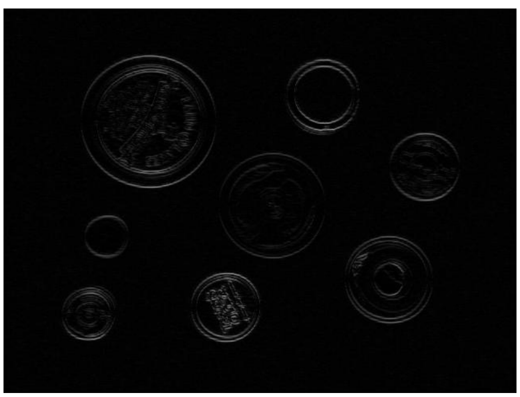

5x5 Kernel 1 σ


9x9 Kernel 1 σ

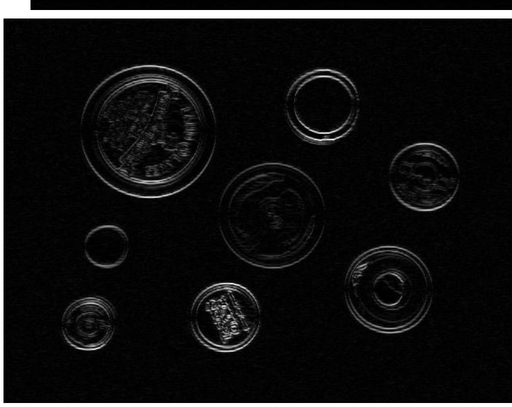

9x9 Kernel 1.75 σ

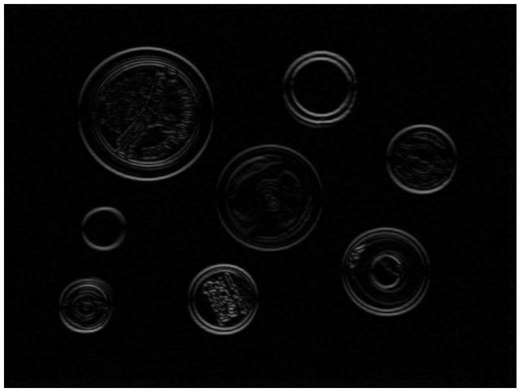

gradientX Without Smooth

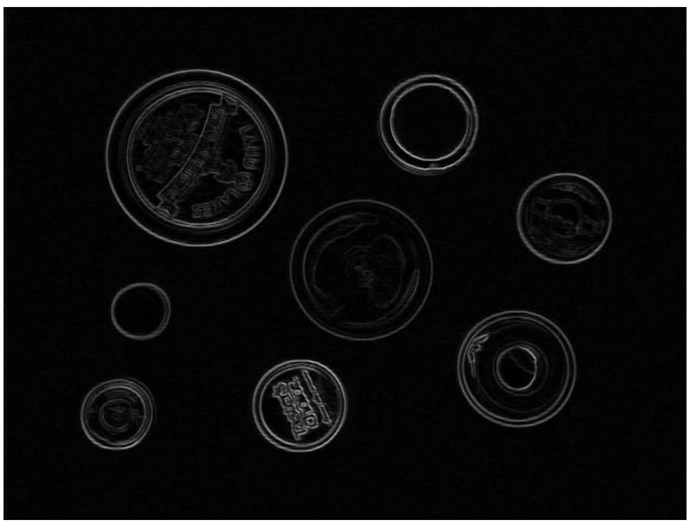

gradientX With Smooth

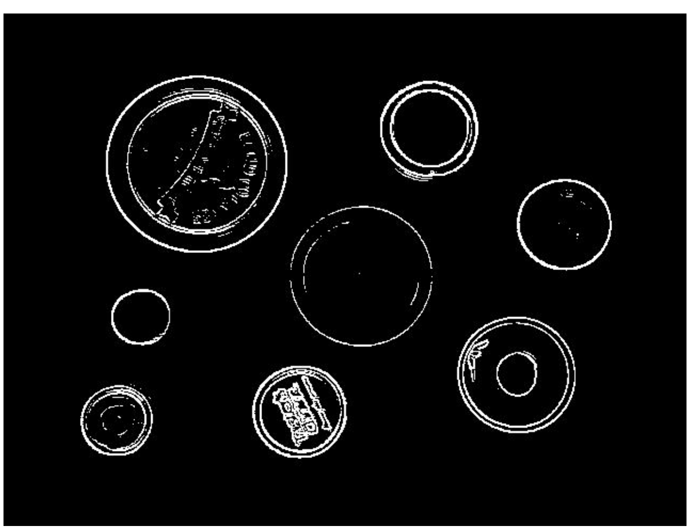

gradientY Without Smooth


gradientY With Smooth


magnitude Without Smooth


magnitude With Smooth


non-maximum suppression


hysteresis 9x9 Kernel 1.2 σ $T_h = 65$, $T_L = 45$

9x9 Kernel 1.2 σ

gradient magnitude

hysteresis 9x9 Kernel 1.2 σ $T_h = 65$, $T_L = 45$