

Machine Learning

Assignment 4 - Multi-Class Classification Spring 2023

1 Theory

1. Consider the following set of training examples for an unknown target function: $(x_1, x_2) \rightarrow y$:

Y	x_1	x_2	Count
+	T	T	3
+	T	F	4
+	F	T	4
+	F	F	1
-	T	T	0
-	T	F	1
-	F	T	3
-	F	F	5

- (a) positive:

$$-\frac{12}{21} * \log_2\left(\frac{12}{21}\right)$$

negative:

$$-\frac{9}{21} * \log_2\left(\frac{9}{21}\right)$$

$$\begin{aligned} H(Y) &= \text{positive} + \text{negative} \\ &= -\frac{12}{21} * \log_2\left(\frac{12}{21}\right) - \frac{9}{21} * \log_2\left(\frac{9}{21}\right) \\ &= 0.985 \end{aligned}$$

- (b) x_1 :

$$\begin{aligned} H(+|T) &= -\frac{7}{8} * \log_2\left(\frac{7}{8}\right) = 0.16, \\ H(-|T) &= -\frac{1}{8} * \log_2\left(\frac{1}{8}\right) = 0.37, \\ H(+|F) &= -\frac{5}{13} * \log_2\left(\frac{5}{13}\right) = 0.53, \\ H(-|F) &= -\frac{8}{13} * \log_2\left(\frac{8}{13}\right) = 0.43 \end{aligned}$$

$$\begin{aligned} x_1 &= \left(\frac{8}{21} * (H(+|T) + H(-|T))\right) + \left(\frac{13}{21} * (H(+|F) + H(-|F))\right) \\ &= \frac{8}{21} * (0.16 + 0.37) + \frac{13}{21} * (0.53 + 0.43) \\ &= 0.796 \end{aligned}$$

x2:

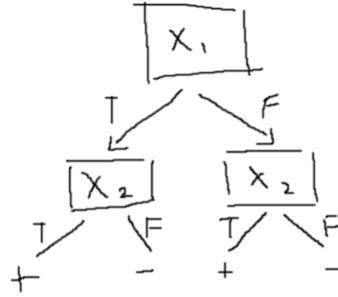
$$H(+|T) = -\frac{7}{10} * \log_2\left(\frac{7}{10}\right) = 0.36,$$

$$H(-|T) = -\frac{3}{10} * \log_2\left(\frac{3}{10}\right) = 0.52,$$

$$H(+|F) = -\frac{5}{11} * \log_2\left(\frac{5}{11}\right) = 0.51,$$

$$H(-|F) = -\frac{6}{11} * \log_2\left(\frac{6}{11}\right) = 0.47$$

$$\begin{aligned} x2 &= \left(\frac{10}{21} * (H(+|T) + H(-|T))\right) + \left(\frac{11}{21} * (H(+|F) + H(-|F))\right) \\ &= \frac{10}{21} * (0.36 + 0.52) + \frac{11}{21} * (0.51 + 0.47) \\ &= 0.932 \end{aligned}$$



(c)

(d) i. $p(-|x = [T, T]) = \frac{0}{3+0} = 0$
 $p(+|x = [T, T]) = \frac{3}{3+0} = 1$

ii. $p(-|x = [T, T]) = p(-) * p(x1 = T|-) * p(x2 = T|-) / p(-) + p(x1 = T|-) + p(x2 = T|-)$
 $= \frac{9}{21} * \frac{1}{8} * \frac{3}{8} / \frac{9}{21} + \frac{1}{8} + \frac{3}{8}$
 $= 0.021$

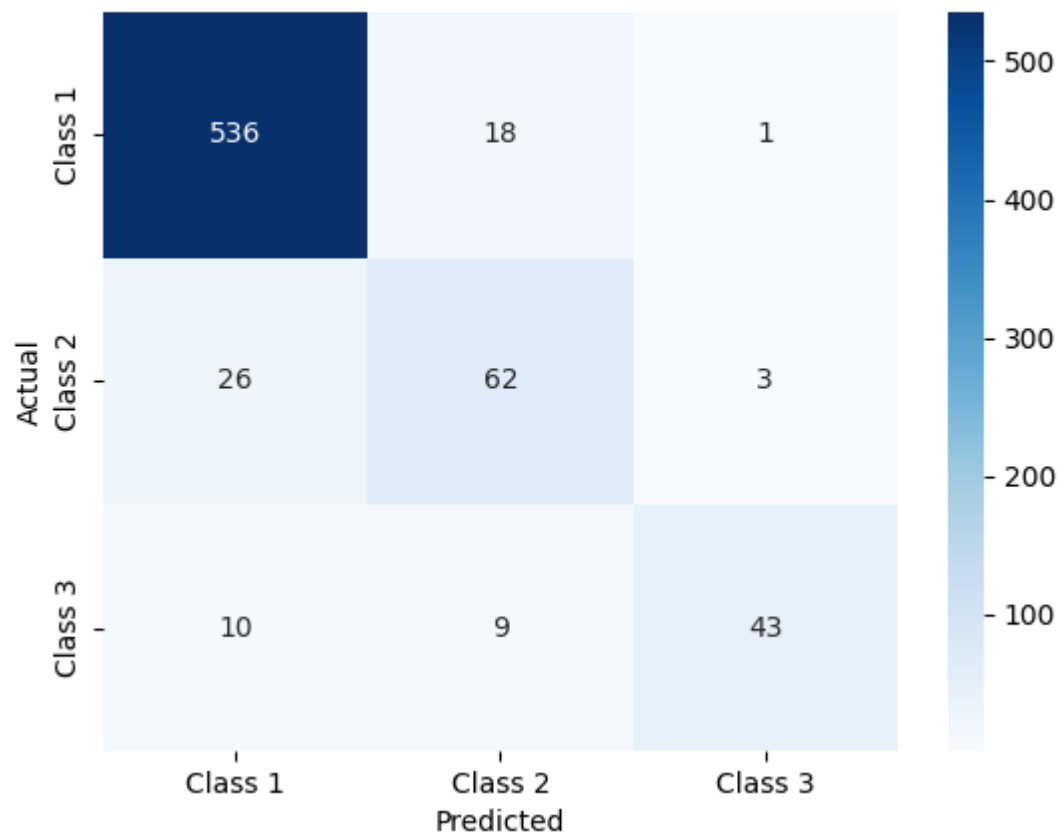
$p(+|x = [T, T]) = p(+) * p(x1 = T|+) * p(x2 = T|+) / p(+) + p(x1 = T|+) + p(x2 = T|+)$
 $= \frac{12}{21} * \frac{7}{12} * \frac{7}{12} / \frac{12}{21} + \frac{7}{12} + \frac{7}{12}$
 $= 0.111$

2 K-Nearest Neighbors Classifier

For pre-processing, I z-scored the observable data.

k	validation accuracy
3	90%
10	89%
30	88%

When $k = 3$:



3 Decision Trees

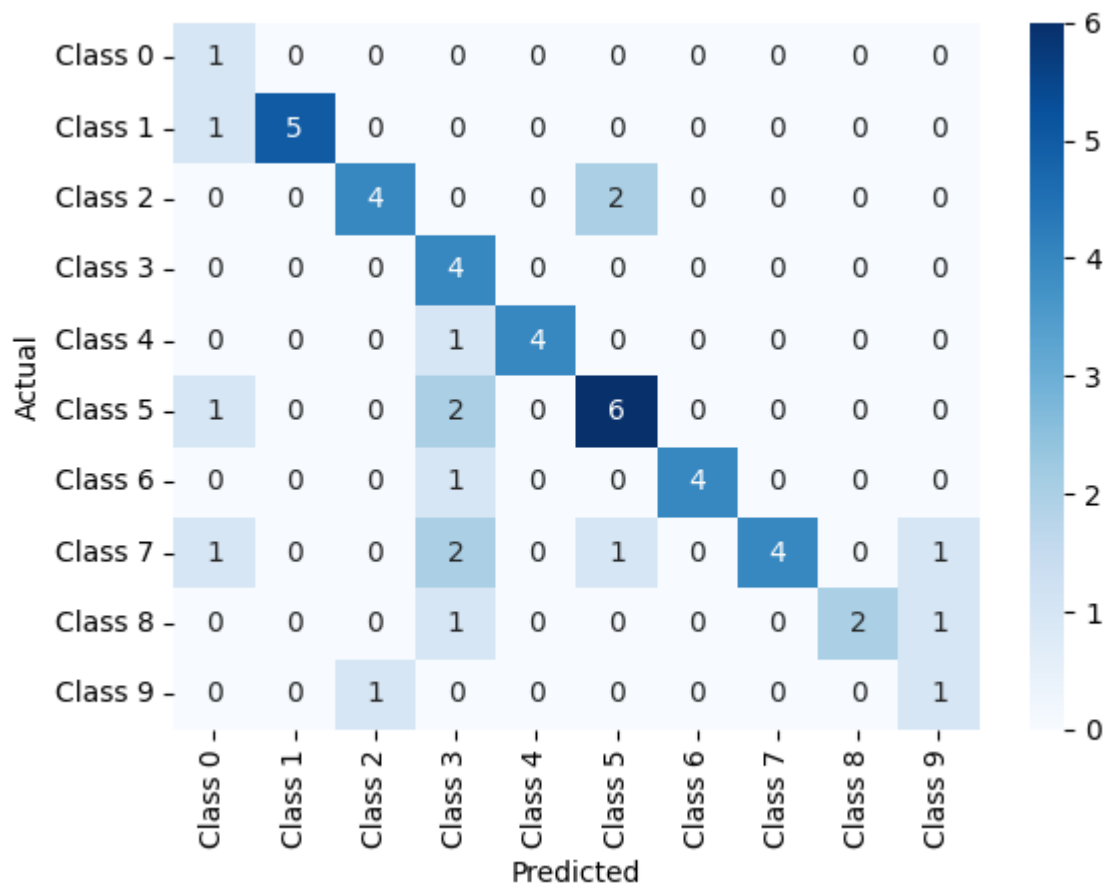
I worked on it, but was not able to get a working program.

4 Additional Dataset

KNN: For pre-processing, I z-scored the observable data.

k	validation accuracy
3	68%
10	45%
30	27%

When $k = 3$:



DT:

I worked on it, but was not able to get a working program.