# 1 Theory Questions

1. Given the grayscale image, $I$, below, if the top left location in our image is $(x = 1, y = 1)$, what would be value of a pixel at location $x = 2.7, y = 3.1$, if we were to:

   (a) Use the nearest neighbor in $I$. (3pts)
   I(3,3) = 5

   (b) Perform interpolation (using the Manhattan distance as our distance function) using $I$. Leave your answer in terms of fractions (7pts)

   I will be using the coordinate format (y,x)

   | | |
   |---|---|
   | floor = (3,2) | ceil = (4,3) |
   | d((3,2), (3.1,2.7)) | d((4,3), (3.1,2.7)) |
   | $\|x_1 - x_2\| + \|y_1 - y_2\|$ | $\|x_1 - x_2\| + \|y_1 - y_2\|$ |
   | $\|3 - 3.1\| + \|2 - 2.7\|$ | $\|4 - 3.1\| + \|3 - 2.7\|$ |
   | 0.1 + 0.7 | 0.9 + 0.3 |
   | $\frac{4}{5}$ | $\frac{6}{5}$ |
   | $\frac{5}{4}$ | $\frac{5}{6}$ |

   $\frac{5}{4}$ is more dominant. So,

   $$\frac{\frac{5}{4}}{\frac{5}{4}+\frac{5}{6}}$$

   $$\frac{3}{5}$$

2. If the matrix below is the gradient magnitude image

$$G = \begin{bmatrix} 2 & 3 & 4 & 5 & 1 \\ 1 & 0 & 2 & 2 & 1 \\ 4 & 3 & 5 & 1 & 2 \\ 4 & 4 & 4 & 4 & 6 \\ 4 & 5 & 2 & 0 & 2 \\ 2 & 3 & 3 & 0 & 3 \end{bmatrix}$$

   (a) Construct the cost matrix if we assume vertical seams (10pts).

$$CostMatrix = \begin{bmatrix} 2 & 3 & 4 & 5 & 1 \\ 3 & 2 & 5 & 3 & 2 \\ 6 & 5 & 7 & 3 & 4 \\ 9 & 9 & 7 & 7 & 9 \\ 13 & 12 & 9 & 7 & 9 \\ 14 & 12 & 10 & 7 & 10 \end{bmatrix}$$

(b) What is the optimal seam (2pts)?
In bold from bottom to top

$$CostMatrix = \begin{bmatrix} 2 & 3 & 4 & 5 & \mathbf{1} \\ 3 & 2 & 5 & 3 & \mathbf{2} \\ 6 & 5 & 7 & \mathbf{3} & 4 \\ 9 & 9 & 7 & \mathbf{7} & 9 \\ 13 & 12 & 9 & \mathbf{7} & 9 \\ 14 & 12 & 10 & \mathbf{7} & 10 \end{bmatrix}$$

$7 \to 7 \to 7 \to 3 \to 2 \to 1$

3. In lecture we discussed applying Poisson Blending to a simple 1D case. Now let's extend this for the 2D case (after all, we're dealing with images!)!.

For simplicity, let's assume a $1 \times 1$ square mask. Figure **??** shows two images, the one we're copying *to* and the one that we're copying *from*. The masked area is indicated in green.

(a) (5pts) First let's establish the cost function $J$ for this 2D case. We'll once again use the squared error. To avoid confusion between the unknowns $x$ and the direction $x$, we'll use $t$ for the values in our *target* image (as opposed to $x$ in the 1D example).

The cost with regards to pixel $(i, j)$ and the *x-gradient* is as follows. It's basically the same as the 1D case in the lecture slides:

$$J_{i,j}^x = ((t_{i,j} - t_{i-1,j}) - (s_{i,j} - s_{i-1,j}))^2$$

The cost for this same pixel, with regards to the *y-gradient* is:

$$J_{i,j}^y = ((t_{i,j} - t_{i,j-1}) - (s_{i,j} - s_{i,j-1}))^2$$

Write out the terms of $J_{i,j}^x$ and $J_{i,j}^y$ that include at least one unknown in the target image.

$J_{i,j}^x = ((t_{i,j} - 1) - (3 - 4))^2 + ((5 - t_{i,j}) - (3 - 5))^2$
$J_{i,j}^x = ((t_{i,j} - 1) - (-1))^2 + ((5 - t_{i,j}) - (-2))^2$

$J_{i,j}^y = ((t_{i,j} - 5) - (3 - 0))^2 + ((3 - t_{i,j}) - (3 - 4))^2$
$J_{i,j}^y = ((t_{i,j} - 5) - (3))^2 + ((3 - t_{i,j}) - (-1))^2$

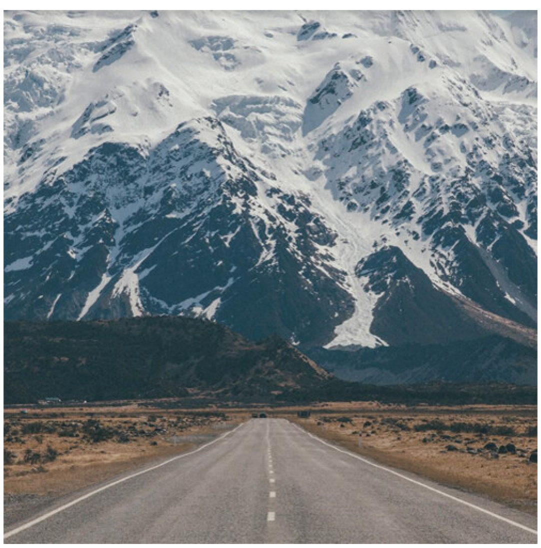
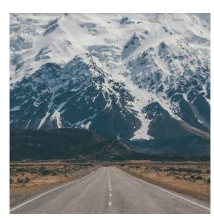
Full cost function
$J = ((t_{i,j} - 1) - (-1))^2 + ((5 - t_{i,j}) - (-2))^2 + ((t_{i,j} - 5) - (3))^2 + ((3 - t_{i,j}) - (-1))^2$

(b) (5pts) Using your answer from the previous part, what is the coeeficient matrix $A$, the unknown vector $t$ and the known vector $b$ so that we can write $J$ as $J = (At - b)^T(At - b)$?

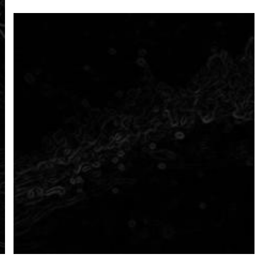
A $= \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ t $= \begin{bmatrix} t_{i,j} \end{bmatrix}$ b $= \begin{bmatrix} 0 \\ -7 \\ 8 \\ -4 \end{bmatrix}$

(c) (3pts) Finally, what are the values for the new pixel within the target mask? For partial credit, make sure it's clear where you got your values from.

Original Images  500x500  225x225

Nearest Neighbor  1000x1000  250x250

Nearest Neighbor  450x450  113x113

Energy Image: $\sigma = 1$, central difference gradient  

Optimal Seam