

Homework 1

Due by 3:05 PM 9/5/2024

- (6 points) [Textbook Exercise 1.6] The data in Table 1.4 are the numbers of females and males in the progeny of 16 female light brown apple moths in Muswellbrook, New South Wales, Australia (from Lewis 1987).

Table 1.4 *Progeny of light brown apple moths.*

Progeny group	Females	Males
1	18	11
2	31	22
3	34	27
4	33	29
5	27	24
6	33	29
7	28	25
8	23	26
9	33	38
10	12	14
11	19	23
12	25	31
13	14	20
14	4	6
15	22	34
16	7	12

Let Y_i denote the number of females in each of the 16 groups of progeny in each group ($i = 1, \dots, 16$). Suppose the Y_i 's are independent random variables each with the Binomial distribution

$$f(y_i; \theta) = \binom{n_i}{y_i} \theta^{y_i} (1 - \theta)^{n_i - y_i}.$$

Find the maximum likelihood estimator (MLE) of θ using calculus and evaluate it for these data.

- (12 points) Let Y_1, \dots, Y_n be independent random variables from the exponential distribution

$$f(y_i; \lambda) = \lambda e^{-\lambda y_i}, \quad y_i > 0 \text{ and } \lambda > 0.$$

(a) What is the maximum likelihood estimator (MLE) of λ ? Show all the derivation details.

(b) Suppose $\lambda = e^\beta$. Find the MLE of β .

(c) Consider 150 observations y_i , ($i = 1, 2, \dots, 150$) from the exponential distribution with the sum of the 150 observations equal to 30. What is the numerical evaluation of the MLE of λ and β ?