Homework 3

Due by 3:05 PM 9/19/2024

1. (10 points) As in Homework 1, consider 150 observations y_i, $i = 1, \ldots, 150$, from the exponential distribution

$$f(y_i; \lambda) = \lambda e^{-\lambda y_i}, \quad y_i > 0, \lambda > 0$$

with the sum of these 150 observations equal to 30.

(a) Starting from $\lambda_{start} = 1$ obtain the next iteration of the Newton-Raphson algorithm. Show details of your work. Do NOT use computer.

(b) Starting from $\lambda_{start} = 1$ obtain the next iteration of the Method of Scoring algorithm. Show details of your work. Do NOT use computer.

(c) Are your answers in (1) and (2) different? Explain why.

2. (6 points) Suppose $Y_1, \ldots, Y_N$ are independent random variables following the normal distribution $Y_i \sim N(\log \beta, \sigma^2)$, where σ^2 is known. Derive matrix based formula for an iteratively re-weighted least squares (IRWLS) procedure for ML estimation of β.

[Hint: First, present matrices $\mathbf{X}$ and $\mathbf{W}$, and vector $\mathbf{z}$ for the formula $\mathbf{b}^{(m)} = \{\mathbf{X}^T \mathbf{W}^{(m-1)} \mathbf{X}\}^{-1} \mathbf{X}^T \mathbf{W}^{(m-1)} \mathbf{z}^{(m-1)}$, and then simplify this formula as feasible.]

3. (10 points) Consider Table 4.3 [Textbook page 67] discussed in class. We now use a **log link** function.

Table 4.3 Data for Poisson regression example

y_i	2	3	6	7	8	9	10	12	15
x_i	-1	-1	0	0	0	0	1	1	1

(a) Fit model $log(E(Y_i)) = \beta_1 + \beta_2 x_i$ with the glm function in R.

(b) We want to obtain

- Maximum likelihood estimates for β_1 and β_2
- Variance-covariance matrix of the MLE for β_1 and β_2

Write an iteratively reweighted least squares R function similar to the one covered in class but accounting for the **log link**. In addition, provide all the details of the algebraic derivations.

(c) Compare values obtained in (a) and (b).