CS 477/677 Analysis of Algorithms

Homework 5

Due November 3, 2016

For the programming problem below use the new submission instructions at:
https://source2.cse.unr.edu/w/cse/submit/
You will need to download a new submission script.

1. (U & G-required) [100 points]

Suppose you are consulting for a company that manufactures PC equipment and ships it to distributors all over the country. For each of the n next weeks, they have a projected supply s_i of equipment (measured in pounds), which has to be shipped by an air freight carrier. Each week's supply can be carried by one of two air freight companies, A or B.

- Company A charges a fixed rate r per pound, so it costs $r * s_i$ to ship a week's supply (s_i)
- Company B makes contracts for a fixed amount c per week, independent of the weight. However, contracts with company B must be made in blocks of four consecutive weeks at a time.

A *schedule*, for the PC company, is a choice of air freight company (A or B) for each of the n weeks with the restriction that company B, whenever it is chosen, must be chosen for blocks of four contiguous weeks at a time. The cost of the schedule is the total amount paid to companies A and B, according to the description above.

You are asked to give a polynomial time algorithm that takes a sequence of supply values $s_1, s_2, \ldots, s_n$ and returns a schedule of minimum cost. In order to achieve this, you need to answer the following questions:

(a) [20 points] Determine and **prove** the optimal substructure of the problem and write a recursive formula of an optimal solution (i.e., define the variable that you wish to optimize and explain how a solution to computing it can be obtained from solutions to

subproblems). **Submit**: the recursive formula, along with definitions and explanations on what is computed.

(b) [30 points] Write an algorithm that computes an optimal solution to this problem, based on the recurrence above. Implement your algorithm in C/C++ and run it on the following values:

- `r` = 1, `c` = 10, the sequence of s_i values: 11, 9, 9, 12, 12, 12, 12, 9, 9, 11.

Submit:

- A printed version of the algorithm (name your algorithm schedule.c or schedule.cpp).
- A printout of the table that contains the solutions to the subproblems, run on the values given above (print the entire table!)

(c) [20 points] Update the algorithm you developed at point (b) to enable the reconstruction of the optimal solution, i.e., which company was used in an optimal solution for shipping. (Hint: use an auxiliary table like we did in the examples in class.) Include these updates in your algorithm implementation from point (b).

Submit:

- A printed version of the algorithm (name your algorithm schedule_1.c or schedule_1.cpp).
- A printout of the values that you obtain in the table containing the additional information needed to reconstruct the optimal solution, run on the values given above (print the entire table!)

(d) [30 points] Using the additional information computed at point (c), write an algorithm that outputs which company was used for shipping in the optimal schedule. Implement this algorithm in C/C+.

Submit:

- A printed version of the algorithm (name your algorithm schedule_2.c or schedule_2.cpp).

- A printout of the **solution** to the problem, i.e., the optimal *schedule*. (e.g., A, A, B, A, B)

2. **(G-required)** [20 points] Show how the algorithm MATRIX-CHAIN-ORDER discussed in class computes the number of scalar multiplications for the product of the following three matrices (i.e., give the values in table "**m**" as computed by the algorithm):

A: size 4x3

B: size 3x5

C: size 5x2

Extra Credit

3. [20 points] Indicate whether the following statements are true or false and justify your answers.

(a) If X and Y are sequences that both begin with the character A, every longest common subsequence of X and Y begins with A.

(b) If X and Y are sequences that both end with the character A, some longest common subsequence of X and Y ends with A.