CASELLA AND BERGER NOTES AND EXERCISES - CHAPTER 1

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Chapter 1 Exercises

Using this problem set: http://www.stat.ufl.edu/ jhobert/sta6326

Chapter 1: 1, 2, 4, 13, 14, 16, 18, 21, 22, 23, 24, 27, 33

Chapter 1: 35, 38, 39, 44, 46, 47, 49, 50, 51, 52, 54

(1.1), (1.2), (1.4), (1.13), (1.14)

These are straight forward.

(1.16)

These follow from multiplication rule.

(1.18)

I got

$$\frac{n\binom{n-1}{n-2}}{\binom{2n-1}{n}},$$

which I guess is incorrect.

(1.21)

TODO tough one.

(1.22)

(a)

Let $S \equiv 180$ days chosen from 366. Then $|S| = \binom{366}{180}$. Also, $180/12 = 15$. Let $E \equiv 180$ days distributed amongst the 12 months equally. We use Vandermonde's identity applied to each month as a group. By Vandermonde's identity 12 times with $366 = 31 + \cdots 31$ and $180 = k_1 + \cdots k_{12}$, we have

$$\binom{366}{180} = \sum_{k_1=0}^{31} \cdots \sum_{k_{12}=0}^{31} \binom{31}{k_1} \cdots \binom{31}{k_{12}}.$$

Hence,

$$P(E) = \frac{\binom{31}{15} \cdots \binom{31}{k_{15}}}{\binom{366}{180}}.$$

(b) TODO not sure how to count this one.

(1.23)

I think this works.

Let $S \equiv 2n$ flips from the 2 people. Then $|S| = 2^n 2^n$. Let $E \equiv$ both people flip the same number of heads. There are 2n total heads and we want to choose all n-subsets of them. So, $\binom{n+n}{n}$ is the count of n-subsets of heads from a total of 2n heads. Hence,

$$P(E) = \frac{\binom{n+n}{n}}{2^n 2^n}.$$

(1.24)

This is a geometric distribution problem.

Need some more practice with this distribution.

(1.27)

TODO Skipping these until I study more combinatorics.

(1.33)

Partition the space by Males and Females and then use the definition of conditional probability.

$$\begin{aligned}
P(\text{Male} \mid \text{Colorblind}) &= \frac{P(\{\text{Male}\} \bigcap \{\text{Colorblind}\})}{P(\{\text{Colorblind}\})} \\
&= \frac{P(\{\text{Male}\} \bigcap \{\text{Colorblind}\})}{P(\{\text{Male}\} \bigcap \{\text{Colorblind}\}) + P(\{\text{Female}\} \bigcap \{\text{Colorblind}\})}.
\end{aligned}$$

(1.35)

This follows straight from the definition of a measure. Let $P : \Omega \to [0, 1]$ be a probability measure. Let $B \in \Omega$ such that $B > 0$. Let $(E_k)_{k\in\mathbb{N}}$ be disjoint sets in Ω. Then $P(\varnothing \mid B) = \varnothing$. Since P is a measure, we have

$$P(\bigcup_{k\in\mathbb{N}} E_k \mid B) = \frac{P(\bigcup_{k\in\mathbb{N}} E_k \bigcap B)}{P(B)} = \sum_{k\in\mathbb{N}} \frac{P(E_k \bigcap B)}{P(B)} = \sum_{k\in\mathbb{N}} P(E_k \mid B).$$

Therefore, it is a measure.

(1.38)

All of these follow straight from the definitions.

(1.39)

Follows straight from the definitions

(1.44) ---

Let X be an RV that measures # of correct answers. Then X is bin, and

$$P(10 \le X) = P(10 \le X \le 20) = \sum_{k=10}^{20} \binom{20}{k} (1/4)^k (3/4)^{20-k}$$

(1.46) ---

TODO tough one.

(1.47) ---

These are all straight forward unless you think it said pdf like I did.

(1.49) ---

Follows directly from definitions using compliments.
Let $t \in \mathbb{R}$. If $F_X(t) \le F_Y(t)$,

$$P(Y \in \mathbb{R}) - F_Y(t) \le P(X \in \mathbb{R}) - F_Y(t) \iff P(Y > t) \le P(X > t).$$

The second part follows from the second part of the definition.

(1.50) ---

Using discrete calculus,

$$\sum_{k=1}^{n} t^{k-1} = \sum_{1 \le k \le n+1} \Delta_k \frac{1}{t-1} t^{k-1} = \frac{t^n - 1}{t-1}.$$

Discrete solutions are so neat.

(1.51) ---

Since we are choosing without replacement from 2 groups, this is a hypergeometric problem.
By Vandermode's identity,

$$\binom{30}{5} = \sum_{k=1}^{5} \binom{5}{k} \binom{25}{5-k}$$

iff

$$1 = \binom{30}{5}^{-1} \sum_{k=1}^{5} \binom{5}{k} \binom{25}{5-k}$$

yields the pmf and cdf.

(1.52) ---

Follows directly from definitions.

(1.54) ---

Both of these are easy.

(a) Use Riemann-Lebesgue for the endpoints, and FTC.

(b) Use additivity of the integral.

Notes

7 / 30 ____________________

8 / 1 ____________________

On section 1.4 currently

discrete distributions: characteristic functions geometric binomial

8 / 2 ____________________

8 / 3 ____________________

8 / 4 ____________________

Difference operators properties are awesome!

`en.wikipedia.org/wiki/Finite_difference#Rules_for_calculus_of_finite_difference_operators`

In fact, this is much more enlightening than just using the geometric series.

Finite difference calculus

Hypergeometric and Binomial distributions are closely related.

8 / 19 ____________________

Ok I read chapter 1 completely now.

Need to do the problem set and move on to chapter 2

8 / 20 ____________________

Problem set

Memorize and understand the main Discrete Distributions: ——— Discrete Uniform Bournoulli, Binomial, Poisson, Multinomial Hypergeometric, Multivariate Hypergeometric distribution Geometric, Negative Binomial

These are really fundamental:: ——— Multinomial and Multivariate Hypergeometric. Multinomial - replacement Multivariate Hypergeometric - no replacement See Wikipedia for Hypergeometric distribution