# CASELLA AND BERGER NOTES AND EXERCISES - CHAPTER 1

AUSTIN DAVID BROWN

## Chapter 1 Exercises

Using this problem set: http://www.stat.ufl.edu/ jhobert/sta6326

Chapter 1: 1, 2, 4, 13, 14, 16, 18, 21, 22, 23, 24, 27, 33

Chapter 1: 35, 38, 39, 44, 46, 47, 49, 50, 51, 52, 54

**(1.1), (1.2), (1.4), (1.13), (1.14)**

These are straight forward.

**(1.16)**

These follow from multiplication rule.

**(1.18)**

I got

$$\frac{n\binom{n-1}{n-2}}{\binom{2n-1}{n}},$$

which I guess is incorrect.

**(1.21)**

TODO tough one.

**(1.22)**

(a)

Let $S \equiv 180$ days chosen from 366. Then $|S| = \binom{366}{180}$. Also, $180/12 = 15$. Let $E \equiv 180$ days distributed amongst the 12 months equally. We use Vandermonde's identity applied to each month as a group. By Vandermonde's identity 12 times with $366 = 31 + \cdots 31$ and $180 = k_1 + \cdots k_{12}$, we have

$$\binom{366}{180} = \sum_{k_1=0}^{31} \cdots \sum_{k_{12}=0}^{31} \binom{31}{k_1} \cdots \binom{31}{k_{12}}.$$

Hence,

$$P(E) = \frac{\binom{31}{15} \cdots \binom{31}{k_{15}}}{\binom{366}{180}}.$$

# CASELLA AND BERGER NOTES AND EXERCISES - CHAPTER 2

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## Chapter 2 Exercises

Using this problem set: http://www.stat.ufl.edu/ jhobert/sta6326

Chapter 2: 1, 2, 3, 4, 9, 11, 12, 13, 14, 16, 17

Chapter 2: 22, 23, 25, 26, 27, 30, 31, 33, 36, 38

**(2.1)**

Apply change of variables for each.

Check that the given PDF's are PDF's and then the result follows.

**(2.2)**

Apply change of variables again. Straight forward.

**(2.3)**

This one tripped me up for some reason. Let $B$ be a measurable set in the image of $Y$. Since $X$ is discrete $B = \{b\}$. Then $P(Y \in B) \iff P(Y = b) \iff P(X = b/(1-b)) = f_X(b/(1-b))$

It is much straight forward to just change the variables this way in my opinion instead of using distribution functions or something.

**(2.4)**

This is clear by the definition and additivity of the integral.

**(2.9)**

The distribution function.

**(2.11)**

TODO COME BACK TO THIS

**(2.12)**

**(2.14)**

Very nice application of Fubini's Theorem.

**(2.16)**

TODO

# CASELLA AND BERGER NOTES AND EXERCISES - CHAPTER 3

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## Chapter 3 Exercises

Using this problem set: http://www.stat.ufl.edu/ jhobert/sta6326

Chapter 3: 2, 4, 7, 10, 13, 14, 19, 20, 23, 24, 28, 37, 38, 42, 45, 46

**(3.2)**

REDO tricky.

**(3.4)**

(a) is geometric

(b) is hypergeometric?

**(3.7)**

REDO messed this one up bad

**(3.10)**

TODO

**(3.14)**

Skipping

**(3.19)**

**(3.38)**

TODO

**(3.42)**

(a) Pretty straight forward with set theory.

(b) I think this should say image space or just whatever space the scaling parameter is in.

**(3.45)**

Basically repeat the Chebyschev inequality proof.

**(3.46)**

Straight forward computation.

# CASELLA AND BERGER NOTES AND EXERCISES - CHAPTER 4

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## Exercises

Using this problem set: http://www.stat.ufl.edu/ jhobert/sta6326

Chapter 4: 1, 2, 4, 9, 10, 11, 15, 19(a), 22, 23, 24, 26, 27

Chapter 4: 30, 31, 33, 35, 39, 41, 45, 47, 51, 53, 55, 64

**(4.1)**

Straight forward.

**(4.2)**

These all follow directly from properties of the integral.

**(4.4)**

Integrating over a rectangle but the density is a the perimeter. Kind of odd, but pretty straight forward.

**(4.9)**

Couple ways to do this, but pretty straight forward.

**(4.10)**

Straight forward.

**(4.11)**

Read the problem wrong. If you read it right, it is fairly obvious.

**(4.15)**

REDO I really made a mess of this one.

**(4.19)**

REDO I missed the easy way to do this.

**(4.22)**

A direct application of change of variable.s

**(4.23)**

Skipping because I am lazy.

**(4.24)**

# CASELLA AND BERGER NOTES AND PROBLEMS - CHAPTER 5

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Using this problem set: http://www.stat.ufl.edu/ jhobert/sta6327
Chapter 5: 3, 5, 6, 11, 12, 15, 16, 17, 19, 21, 22
Chapter 5: 23, 24, 25, 30, 32, 34, 42, 44, 47, 49, 50, 52, 53
Chapter 5: 58 (in part (a), assume $t \in (0, 1)$, 62, 63, 64, 66

## Chapter 5 Exercises

(5.3)
Since each $Y_i = 1_{X_i > \mu}$ is a characteristic function, and $(X_i)$ are iid, the sum is binomial.
(5.5)
I think by convolution $F_{\sum_{1 \leq k l \epsilon n} X_k}$ has a density, but suppose $F_{\sum_{1 \leq k l \epsilon n} X_k}$ exists with density $f_{\sum_{1 \leq k \leq n} X_k}$. Then

$$\begin{aligned} F_{\bar{X}}(x) &= P(y : \bar{X}(y) \leq x) \\ &= P(y : \sum_{1 \leq k \leq n} X_k(y) \leq nx) \\ &= F_{\sum_{1 \leq k \leq n} X_k}(nx) \\ &= \int_{\sum_{1 \leq k \leq n} X_k \leq nx} dF_{\sum_{1 \leq k \leq n} X_k}. \end{aligned}$$

By change of variables and since $F_{\sum_{1 \leq k \leq n} X_k}$ has a density,

$$\begin{aligned} &= \int_{\sum_{1 \leq k \leq n} X_k \leq x} n dF_{\sum_{1 \leq k \leq n} X_k} \\ &= \int_{\sum_{1 \leq k \leq n} X_k \leq x} n f_{\sum_{1 \leq k \leq n} X_k}(x) dx. \end{aligned}$$

By definition of the density being the integrand, we are done.
(5.6)

# CASELLA AND BERGER NOTES AND PROBLEMS - CHAPTER 6

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Chapter 6: 1, 2, 3, 5, 7, 10, 13, 14, 17, 19, 21, 22, 24, 30

## 1. Problems

(6.1)
Use Factorization Theorem.
(6.2) TODO
(6.3) TODO
(6.5) TODO
(6.7) TODO
(6.10) TODO
(6.13) TODO
(6.14) TODO
(6.17) TODO
(6.19)
Some algebra shows g is not constant in respect to $p$.
(6.21)
(a) Look at the map $F(v) = (p_{X(-1)}v_1, p_{X(0)}v_2, p_{X(1)}v_3)$ on the space $g(-1) \times g(0) \times g(1)$. The map is not injective, and hence $g$ is not 0.
(b) Look at the map $F(v) = (p_{X(0)}v_2, p_{X(1)}v_3)$ on the space $g(0) \times g(1)$. The map is injective, and hence $g$ is 0.
(c) Yes, by algebra. The exponential family in part (a) is not full rank in the sense of exponential families is where the problem lies.
(6.24) I believe this failed for $\lambda = 0$.
(6.30) TODO

## 2. Notes

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# CASELLA AND BERGER NOTES AND PROBLEMS - CHAPTER 7

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## 1. Problems

Chapter 7: 1, 2, 6, 7, 8, 11, 12, 13, 14, 19, 20, 23, 24, 26, 33, 37, 38, 39, 41, 42, 44, 46
Chapter 7: 49, 50, 52(a,b), 57, 58, 59, 60

## 2. Notes

8 / 11 ______________________________________________
8 / 12 ______________________________________________

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I think this is a good problem to work out.
Find the MLE for $\mu, \sigma^2$ for a normal distribution.
Let $X = (X_k)_{k=1}^n$ be iid random vector with $X_k \sim N(\mu, \sigma^2)$.
Since $Int\mathbb{R} = \mathbb{R}$, the max, min must won't be on the boundary and the derivative will be 0.
Let $v = (\mu, \sigma^2)$.
The likelihood map $L_X : \mathbb{R}^2 \to \mathbb{R}$ is $C^\infty$ and

$$L_X(v) = \frac{1}{(\pi 2\sigma^2)^{n/2}} \exp(\frac{-1}{2\sigma^2}\sum_{k=1}^n (x_k - \mu)^2).$$

Then

$$\log L_X(v) = \frac{-n}{2}\log(\pi 2\sigma^2) - \frac{1}{2\sigma^2}\sum_{k=1}^n (x_k - \mu)^2,$$

and

$$\begin{aligned} D(\log L_X(v))(v) &= \begin{pmatrix} \partial_\mu \log L_X(v) & \partial_{\sigma^2} \log L_X(v) \end{pmatrix} \\ &= \begin{pmatrix} \frac{n}{\sigma^2}(\overline{X_n} - \mu) & \frac{n}{2(\sigma^2)^2}(-\sigma^2 + \frac{1}{n}\sum_{k=1}^n (x_k - \mu)^2). \end{pmatrix} \end{aligned}$$

# CASELLA AND BERGER NOTES AND PROBLEMS - CHAPTER 8

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## 1. Problems

Chapter 8: 1, 2, 5, 6, 9, 13, 16, 18, 19, 20, 22, 25, 26, 28, 31, 34, 35(a,b), 39, 41

(1) Use CLT

(2) Calculate the density or the distribution function at 15.

(5) Ya got stuck

(6)

(9)

(13)

(16) This follows from the definitions. $\alpha = 1$ for (a), $\alpha = 0$ for (b).

(18) algebra problem

(19) Use NP Lemma

(20) Not sure about this one Cannot figure out how to precisely pick the correct values.

(22)

## 2. Notes

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THIS IS THE EXAMPLE I SHOULD USE: (See Wasserman)

Comparing algorithms is a great example for statistics. Coin flip

Hypothesis Testing in Stats is a proof by contradiction.

This entire theory is a way around proof by contradiction for some reason.

statistical power is alpha.

Mathematical way to think about this:

Statistical power is just $P_\theta(X \in R)$ Suppose $\theta \in \Theta_0$.

Then the "power" is $\alpha = P_\theta(X \in R)$

Suppose $\theta \in \Theta_A$.

# CASELLA AND BERGER NOTES AND PROBLEMS - CHAPTER 9

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## 1. Problems

Problem set: http://sites.stat.psu.edu/ drh20/514/practiceProblems.html Chapter 9 Exercises (pp. 451 to 463): 9.2, 9.9, 9.12,
(9.2)

## 2. Notes

8 / 14 ____________________________________________

These are really the cont distributions worth knowing: Uniform Gamma Normal
Know these functions: Gamma Beta

## References

[1] Casella and Berger. *Statistical Inference*.

# CASELLA AND BERGER NOTES AND EXERCISES - CHAPTER 10

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## Exercises

Using this problem set: http://www.stat.ufl.edu/ jhobert/sta6326

## Notes

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Consistency

Consistent estimators are really the only ones worth considering.

Under some conditions, consistent estimators have nice limiting distributions.

Efficiency

Review Order Statistics

Median is an important estimator actually

prove median minimizes that formula