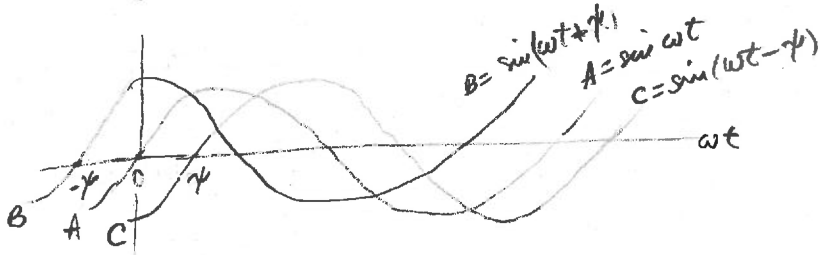
C1 PHASE COMPENSATORS

Recall signal phase definition:



A : $\sin(\omega t) = 0$ @ $t=0$

B: $\sin(\omega t+\psi)=0$ @ $\omega t = -\psi$ "ahead of time" it leads

C: $\sin(\omega t-\psi)=0$ @ $\omega t = \psi$ "delayed" it lags

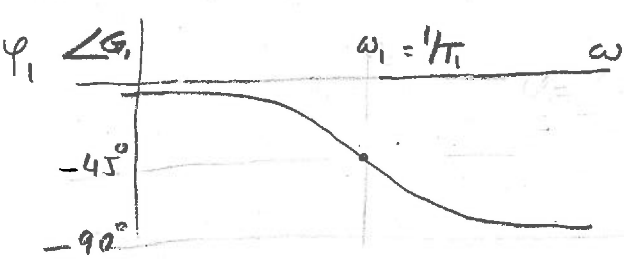
Phase compensator

$G_c(i\omega) = |G_c(i\omega)| e^{i\varphi_c}$, $\varphi_c = \angle G_c(i\omega)$

if $\varphi_c > 0$, then "lead compensator"

$\varphi_c < 0$ "lag compensator"

Example 1: 1^{st} order system $G_1(s) = \dfrac{1}{sT_1+1}$



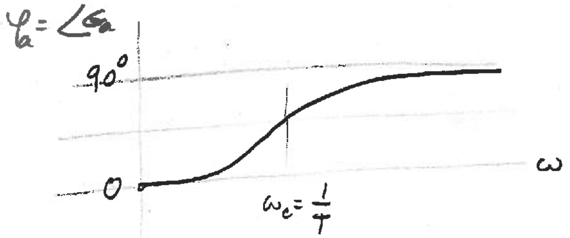
$\varphi_1 < 0$

"Lag"

C2

Example 2 : $G_a = sT_a + 1$

$$G_a(i\omega) = i\omega T_a + 1 \; ; \quad \varphi_a = \angle G_a = \tan^{-1} \omega T_a > 0$$



$\varphi_a > 0$

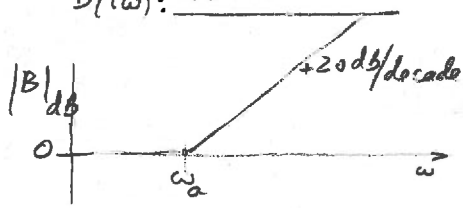
"Lead"

Example 3 $G_c = \dfrac{sT_a + 1}{sT_1 + 1} = \dfrac{B(s)}{A(s)}$

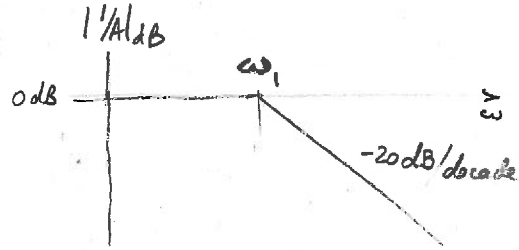
$$G_c(i\omega) = \frac{i\omega T_a + 1}{i\omega T_1 + 1} = \frac{B(i\omega)}{A(i\omega)}$$

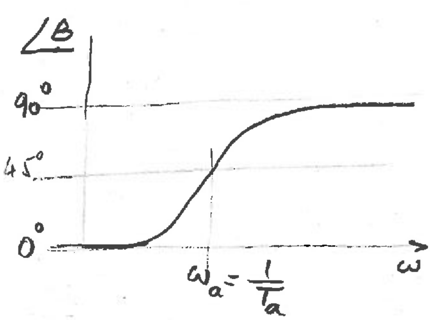
$$|G_c|_{dB} = |B|_{dB} - |A|_{dB} \quad ; \quad \angle G_c = \angle B - \angle A$$

$B(i\omega)$: NUMERATOR

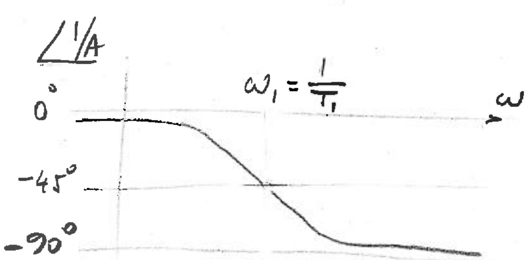


$A(i\omega)$: DENOMINATOR





"Lead"



"Lag"

C2a

LEAD COMPENSATOR

FOR PHASE MARGIN IMPROVEMENT

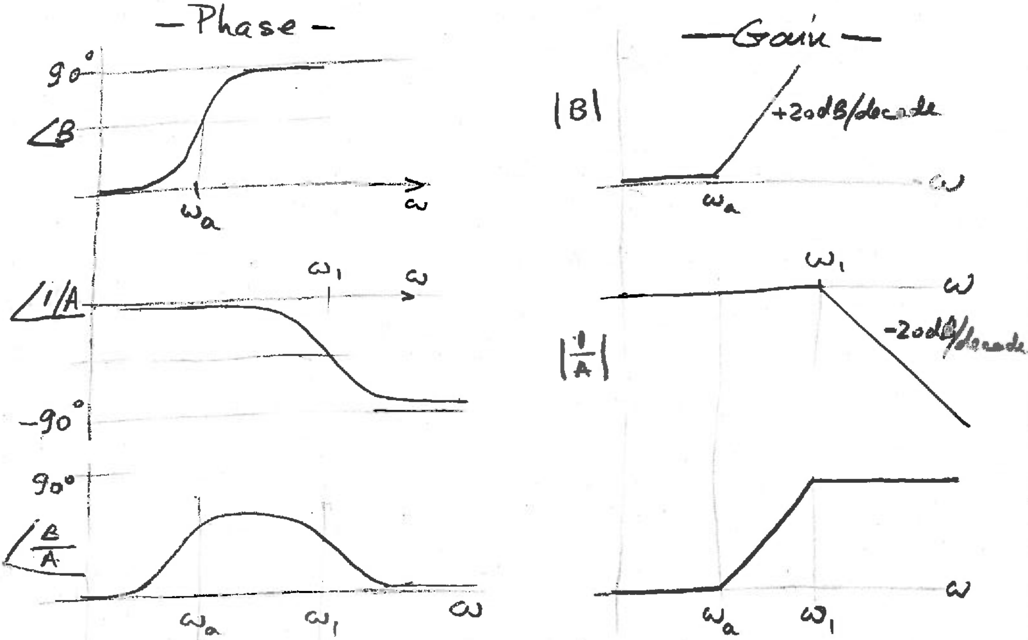
C3

LEAD COMPENSATOR

$$G_c = \frac{sT_a+1}{sT_1+1}$$

$\omega_a < \omega_1$, $T_1 < T_a$

$\omega_a = \frac{1}{T_a}$, $\omega_1 = \frac{1}{T_1}$

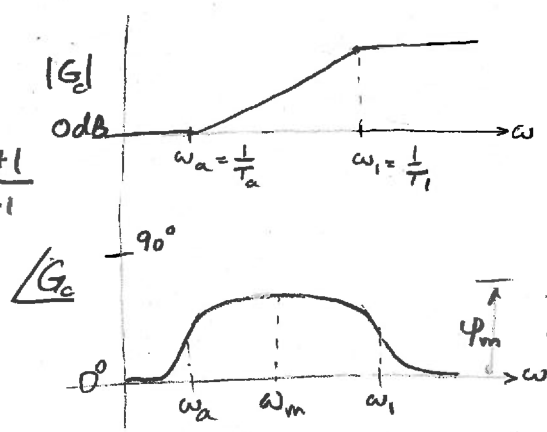


"phase lead" between ω_a & ω_1

High pass filter (higher frequencies are amplified)

$$G_c(s) = \frac{sT_a+1}{sT_1+1}$$

$T_1 < T_a$



HP filter

maximum phase lead

C3a

Lead Compensator Design

Use lead compensator to move phase plot upward and improve phase margin

$$G_c = \frac{sT_a+1}{sT_1+1} = \frac{sT+1}{s\alpha T+1} \quad \text{lead compensator} \qquad (1)$$

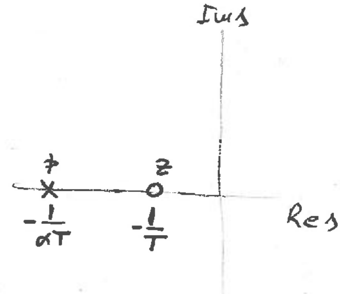
$\omega_a < \omega_1 \qquad 0 < \alpha < 1$

$T_a > T_1$

Poles & zeros :

$sT+1=0 \longrightarrow z = -\frac{1}{T}$

$s\alpha T+1=0 \longrightarrow p = -\frac{1}{\alpha T}$

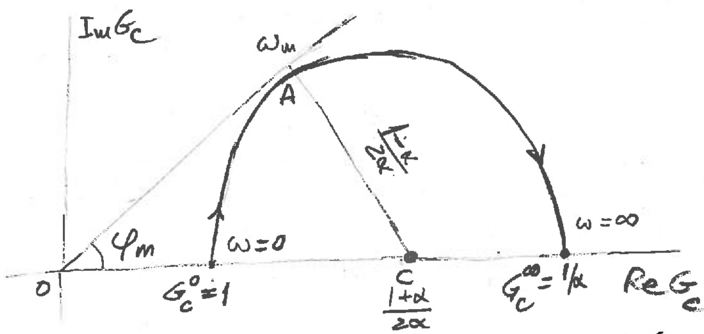


Nyquist plot

$G_c(i\omega) = \frac{i\omega T+1}{i\omega\alpha T+1}$

$G_c^0(i0) = 1$

$G_c^\infty(i\infty) = \frac{1}{\alpha}$



(2)

$$\sin\varphi_m = \frac{1-\alpha}{1+\alpha}$$

maximum phase lead $\boxed{\varphi_m = \sin^{-1}\frac{1-\alpha}{1+\alpha}}$

Proof : $OC = \frac{1}{2}\left(1+\frac{1}{\alpha}\right) = \frac{1+\alpha}{2\alpha}$

Radius $= \frac{1+\alpha}{2\alpha} - 1 = \frac{1-\alpha}{2\alpha} = AC$

$\sin\varphi_m = \frac{AC}{OC} = \frac{1-\alpha}{1+\alpha}$ QED.

C36 <u>Calculation of α</u>

Eq (2) can be used to determine the value of α in Eq. (1), i.e.,

$$\sin\varphi_m = \frac{1-\alpha}{1+\alpha}$$

$$\frac{1+\sin\varphi_m}{1-\sin\varphi_m} = \frac{1-\alpha+1+\alpha}{1+\alpha-(1-\alpha)} = \frac{2}{2\alpha} = \frac{1}{\alpha}$$

$$\alpha = \frac{1-\sin\varphi_m}{1+\sin\varphi_m} \qquad \text{—————} \qquad (3)$$

<u>Calculation of T</u>

- ω_m is the geometric mean of the corner frequencies

$$\omega_m = \sqrt{\omega_a\omega_1} = \sqrt{\frac{1}{T_a}\cdot\frac{1}{T_1}}\Bigg|_{\substack{T_a=T\\ T_1=\alpha T}} = \sqrt{\frac{1}{T}\cdot\frac{1}{\alpha T}} = \frac{1}{T}\frac{1}{\sqrt{\alpha}}$$

$$T = \frac{1}{\omega_m\sqrt{\alpha}}, \quad \omega_m T = \frac{1}{\sqrt{\alpha}} \qquad \text{—————} \qquad (4)$$

- $$|G_c(\omega_m)| = \left|\frac{i\omega_m T+1}{i\omega_m\alpha T+1}\right| = \left|\frac{i\frac{1}{\sqrt{\alpha}}+1}{i\alpha\frac{1}{\sqrt{\alpha}}+1}\right| = \left|\frac{i+\sqrt{\alpha}}{i\sqrt{\alpha}+1}\right|\frac{1}{\sqrt{\alpha}}$$

$$= \frac{\sqrt{1+\alpha}}{\sqrt{1+\alpha}}\frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{\alpha}}$$

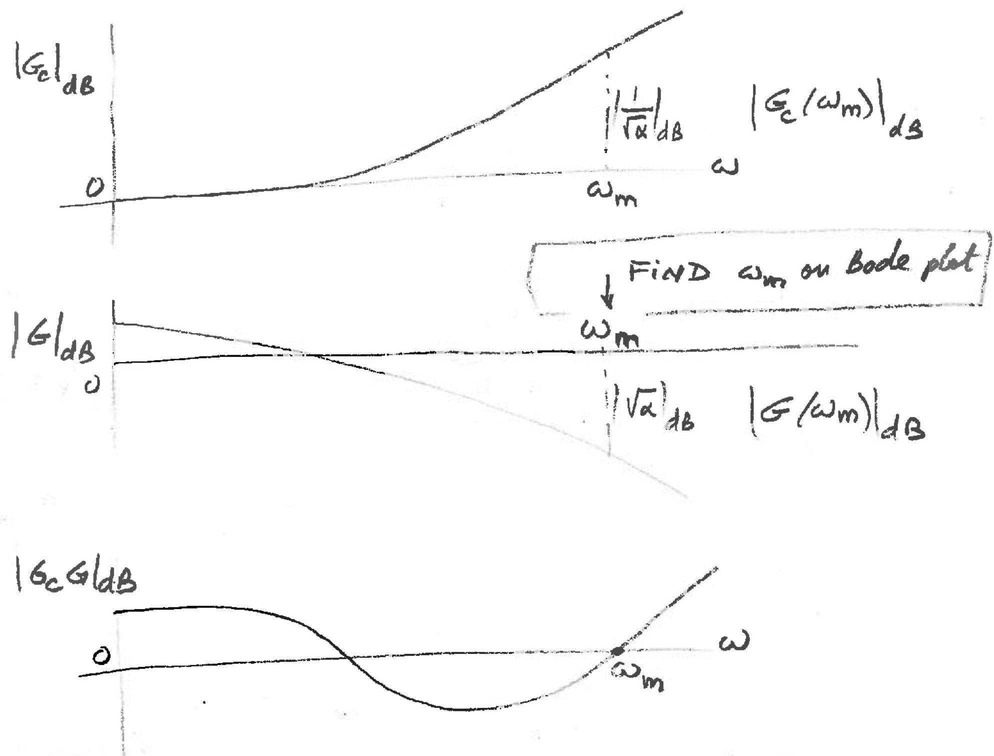
$$|G_c(\omega_m)| = \frac{1}{\sqrt{\alpha}}, \quad 0<\alpha<1 \qquad \text{—————} \qquad (5)$$

The phase compensator will produce a gain $\frac{1}{\sqrt{\alpha}}$.

To find ω_m, examine the Bode diagram of the original system G, and identify the freq. at which $|G(\omega_m)| = \sqrt{\alpha}$, such that

C3c

ω_m is selected at a value at which the gain drop $|G(\omega_m)|$ of the original system balances the gain addition $|G_c(\omega_m)|$ due to the compensator (balance condition)



- After determining ω_m graphically, calculate T with Eq.(4) i.e.,

$$T = \frac{1}{\omega_m} \cdot \frac{1}{\sqrt{\alpha}} \qquad (6)$$

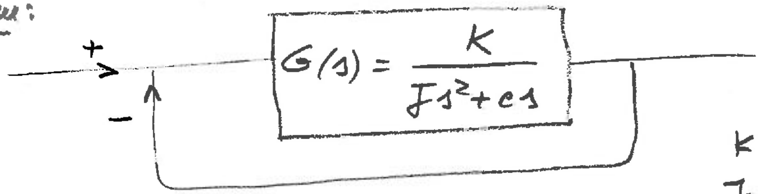
Thus:

$$G_c = \frac{sT+1}{s\alpha T+1} \qquad (7)$$

E

Example : aircraft roll model with lead comp.

Given:



$K = 114$
$J = 10$
$c = 4$

Find

(a) - gain margin, (K_g) dB

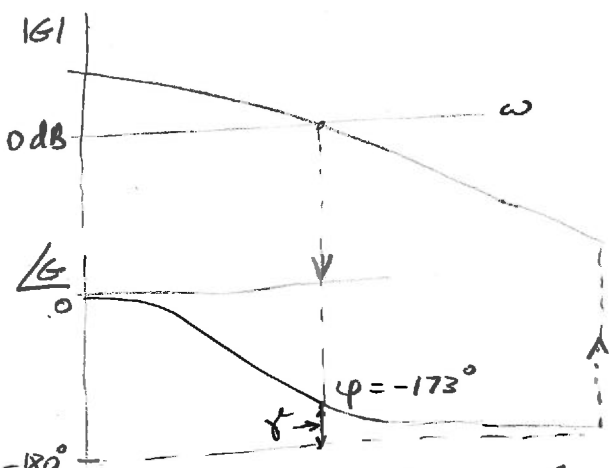
- phase margin γ deg

(b) design compensator to achieve

$$GM = 10 \text{ dB} \qquad (1)$$
$$PM = 60 \text{ deg}$$

(c) plot time response

Solution (a) Bode plot (see MATLAB plot, Fig. 1)



$$|G|_{\angle G=-180^\circ} = -\infty$$

$\angle G$ never crosses -180° line

$$\angle G \xrightarrow[\omega\to\infty]{} -180^\circ$$

$$\gamma = \varphi + 180^\circ = -173^\circ + 180^\circ = 7^\circ$$

Very small phase margin!

$$K_g = \infty > GM$$

OK

NEED TO IMPROVE PHASE MARGIN!

Plenty gain margin

2E

```
Phase margin: aircraft model
input data

    K    | J  |    c =

   114    10      4


G =

      114
  ------------
  10 s^2 + 4 s

Continuous-time transfer function.

=========================

GM, dB | PM, deg =

    10    60

=========================

phi = phase at |G|=0 dB point, deg =

  -173


gamma = phase margin, deg =

     7

=========================
```

Fig.1a Bode plot of original G(s); aircraft model

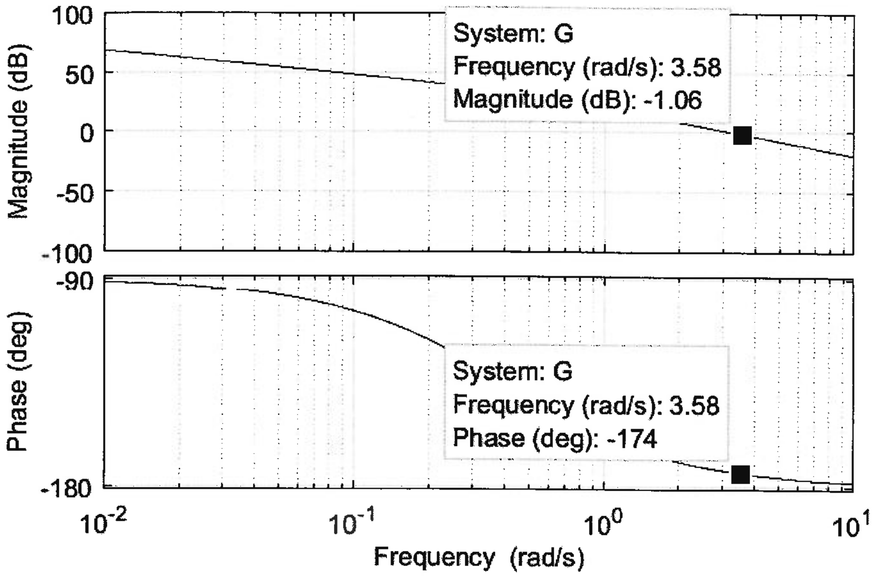
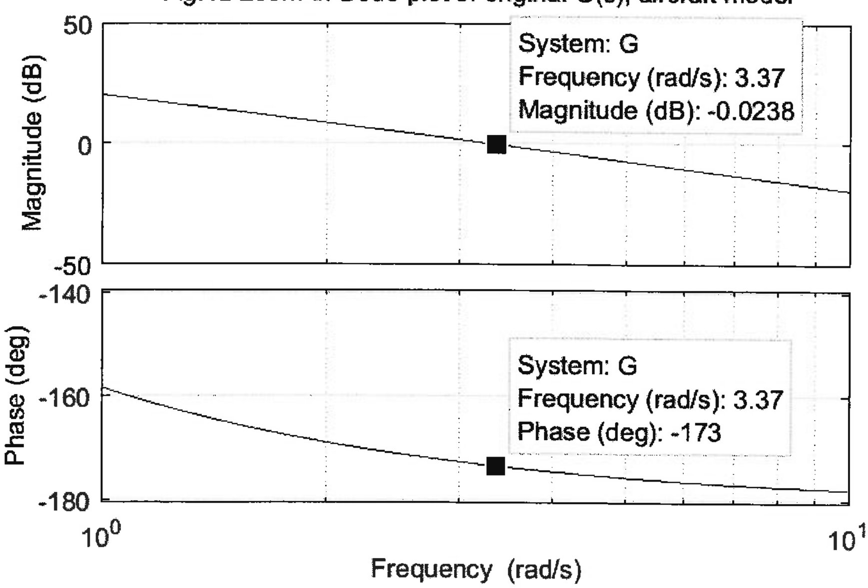
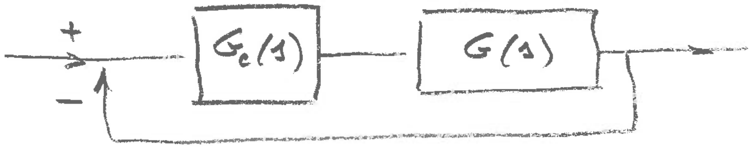


Fig.1b zoom-in Bode plot of original G(s); aircraft model



4E (b) Design phase compensator to improve phase margin.



$$G_c(s) = \frac{Ts+1}{\alpha Ts+1} \quad \text{lead compensator} \qquad (2)$$

(b1) We need to improve phase margin from $\gamma = 7^\circ$ to PM $= 60^\circ$, i.e., the phase compensator must add $\varphi_m = 53^\circ$.

- To calculate α, recall

$$\alpha = \left.\frac{1-\sin\varphi_m}{1+\sin\varphi_m}\right|_{\varphi_m = 53^\circ} = 0.1120 \qquad (3)$$

- To calculate T, recall $T = \dfrac{1}{\omega_m\sqrt{\alpha}}$

We need ω_n. We find ω_m from the balance condition, i.e.,

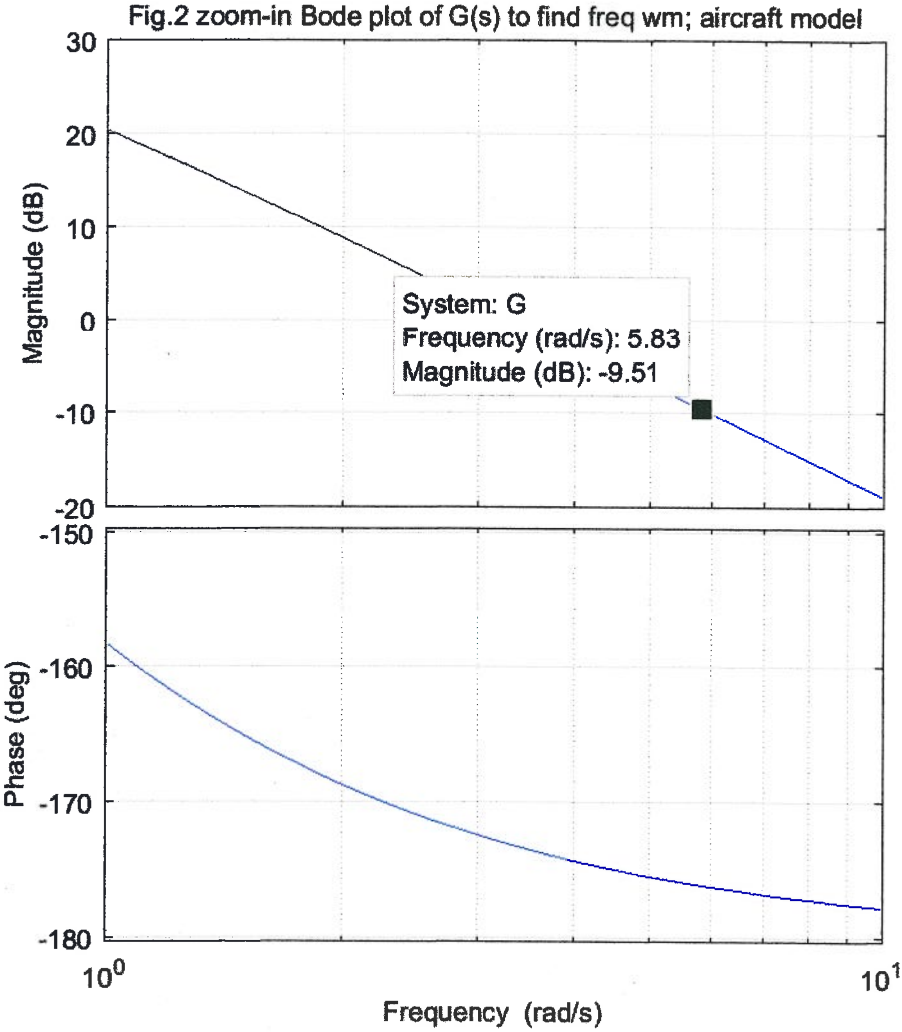
$$|G_c(\omega_m)| = \frac{1}{\sqrt{\alpha}} = 2.9887 = 9.5096\,dB \qquad (4)$$

$$|G(\omega_m)| = -|G_c(\omega_m)| = -9.5096\ dB \qquad (5)$$

From Bode plot Fig. 2, we find $\omega_n = 5.83$ rad/sec.

Hence $T = 0.5126$ sec. (6)

5 E



$\omega_m = 5.83$ rad/s

$|G(\omega_m)|_{dB} = -9.51$ dB.

6 (b2): Test compensated system G_cG

Fig. 3 shows the performance of the compensated system G_cG in comparison to the original system G. The compensated system G_cG has

$\varphi_c = -123^\circ$ @ $\omega_c = 5.83$ rad/s where $|G_cG| = 0\,dB$

The phase margin of the compensated system is

$$\gamma_c = \varphi_c + 180^\circ = -123^\circ + 180^\circ = 57^\circ < \underset{PM}{60^\circ}$$

The system has improved, but the phase margin is still less than PM.

(b3). Need to adjust the compensator to add a little more phase shift,

$$\Delta\varphi = PM - \gamma_c = 60^\circ - 57^\circ = 3^\circ \quad (7)$$

The new φ_m is

$$\varphi_{m_1} = \varphi_m + \Delta\varphi = 53^\circ + 3^\circ = 56^\circ \quad (8)$$

The new α is

$$\alpha_1 = \frac{1 - \sin\varphi_{m_1}}{1 + \sin\varphi_{m_1}} = 0.0935 \quad (9)$$

We keep same time constant, $T_1 = T$ (10)

7
E

(b)

```
===========================
Phase compensator design: aircraft model
phi_m =
    53
alpha =
    0.1120
1/sqrt(alpha) =
    2.9887
Gc_wm, dB =
    9.5096
wm =
    5.8300
T =
    0.5126

Gc =

  0.5126 s + 1
  -------------
  0.05739 s + 1

Continuous-time transfer function.

phi_c = phase at |GcG|=0 dB point, deg =
  -123
gamma_c = phase margin of GcG, deg =
    57
===========================
```

8E

(62)

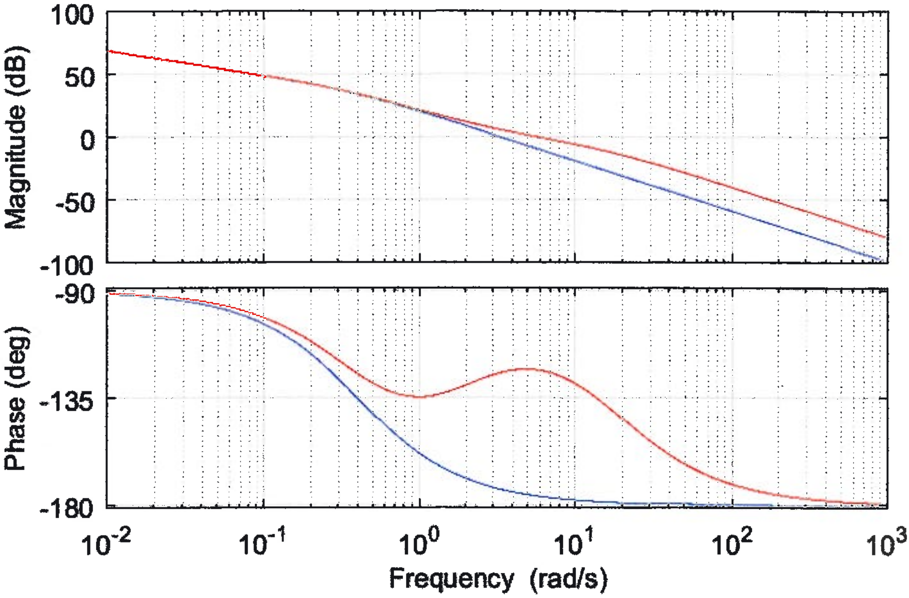


Fig.3a Bode plot of G & GcG(s); aircraft model

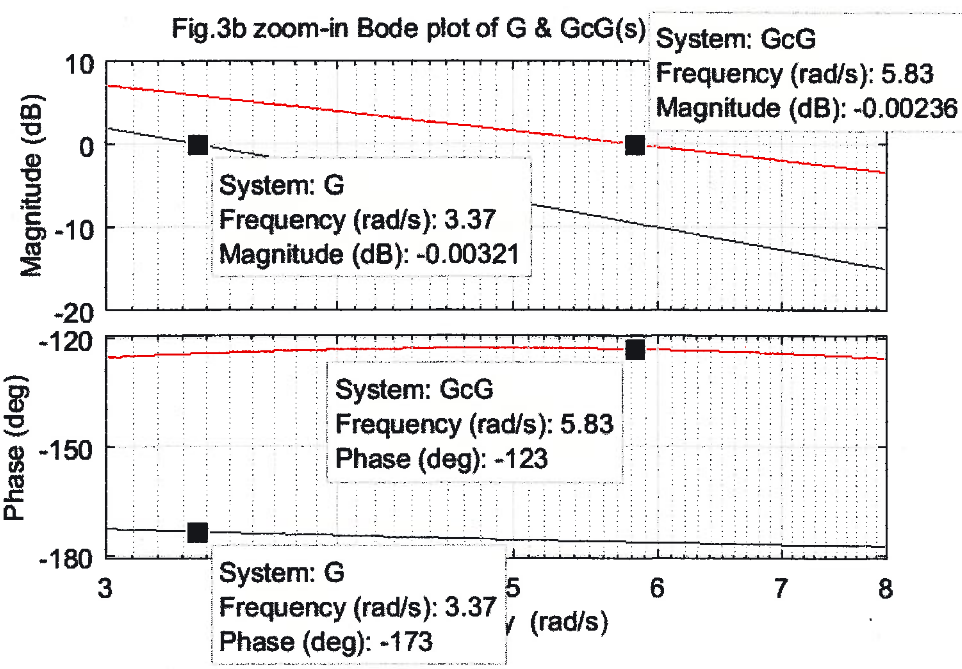


Fig.3b zoom-in Bode plot of G & GcG(s)

9
E

(63)

```
==========================================
Adjustment of phase compensator: aircraft model
dphi =
     3
phi_m1 =
    56
alpha1 =
    0.0935
T1 =
    0.5126

Gc1 =

  0.5126 s + 1
  -------------
  0.04792 s + 1

Continuous-time transfer function.
```

10
E (b4) Test adjusted compensator

Fig. 4 shows the performance of the system with adjusted compensator $G_{c_1}G$ compared to the original system G.

The new phase value at $|G_{c_1}G| = 0\,dB$ is:

$$\varphi_{c_1} = -120° \text{ @ } 5.91\,rad/s \text{ where } |G_{c_1}G| = 0\,dB.$$

The new phase margin is

$$\gamma_{c_1} = \varphi_{c_1} + 180° = -120° + 180° = 60° = PM \quad (11).$$

The adjusted system meets the phase margin specification $PM = 60°$.

Fig.4a Bode plot of G & adjusted Gc1G(s); aircraft model

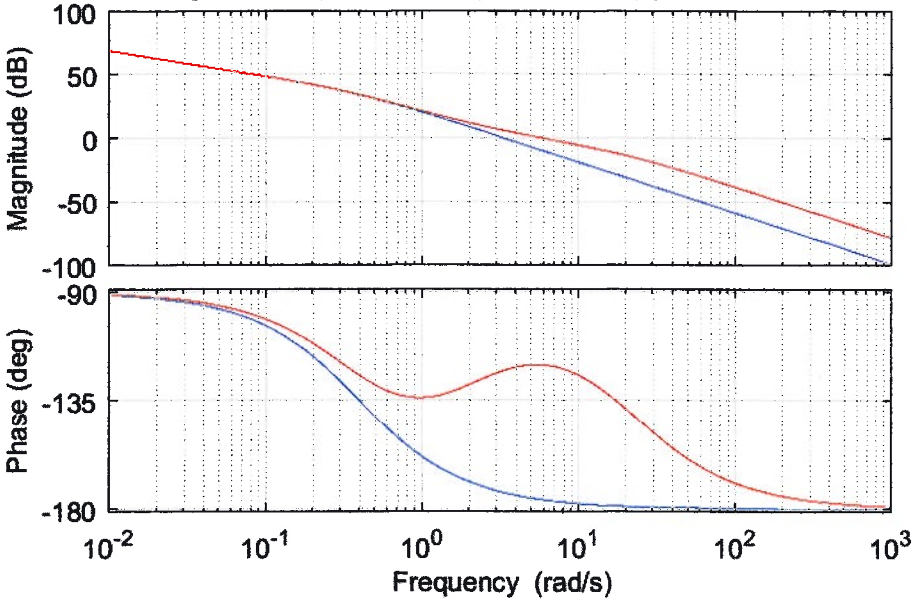
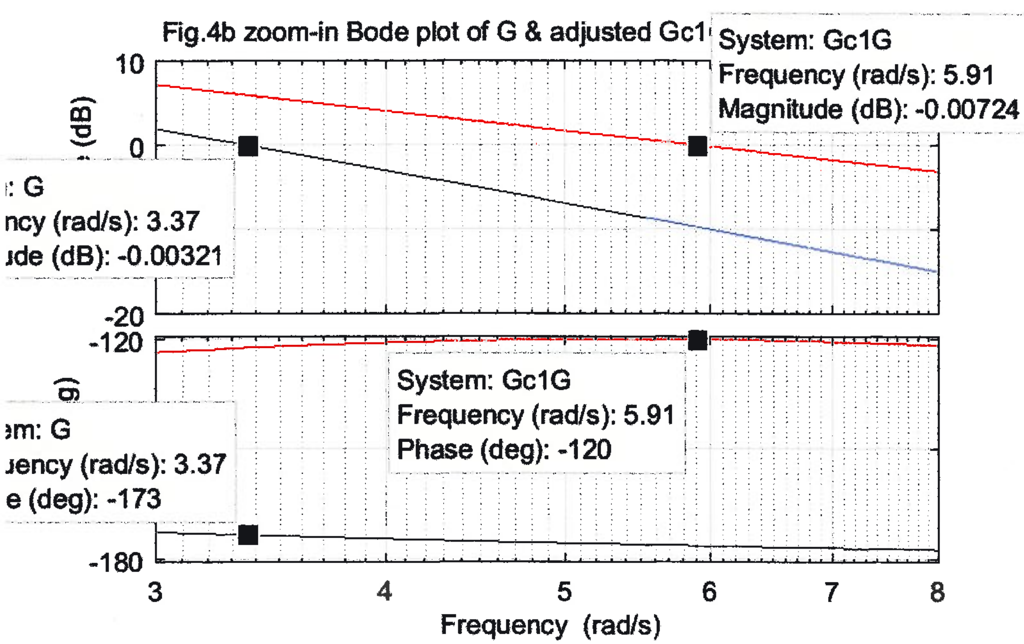


Fig.4b zoom-in Bode plot of G & adjusted Gc1



12
E

(c) Time response behavior of the compensated system.

Fig 5 displays the time response of the compensated system $(G_c G)_{CL}$ compared with the response of the original system G_{CL}. It can be observed that the compensated system has a fast rise time and a small overshoot

$$t_r = 0.296 \text{ sec} \qquad (12)$$

$$M_p = 17\%$$

The original system had a longer rise time, a much higher overshoot, and took a long time to settle.

13 E

Fig.5a Step response aircraft model G_{CL}, $Gc1G_{CL}$

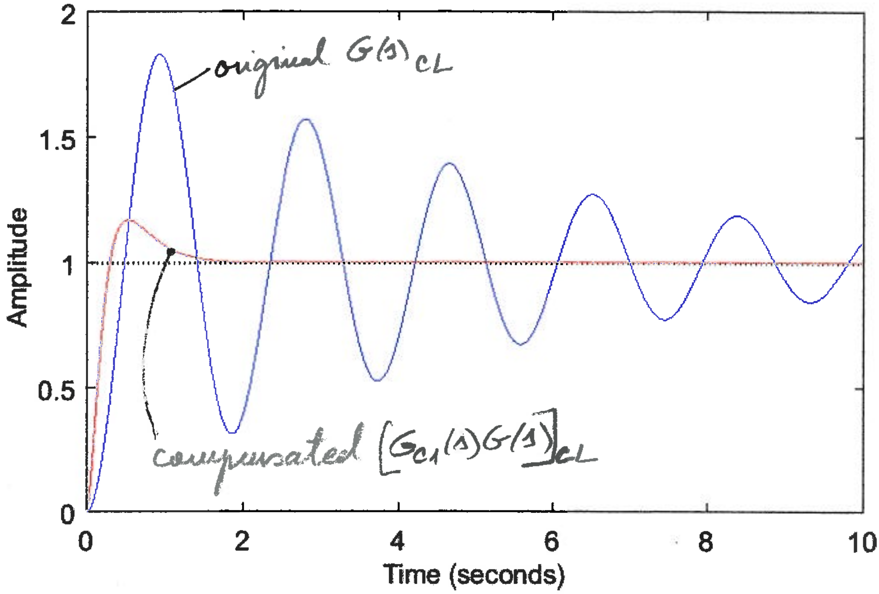
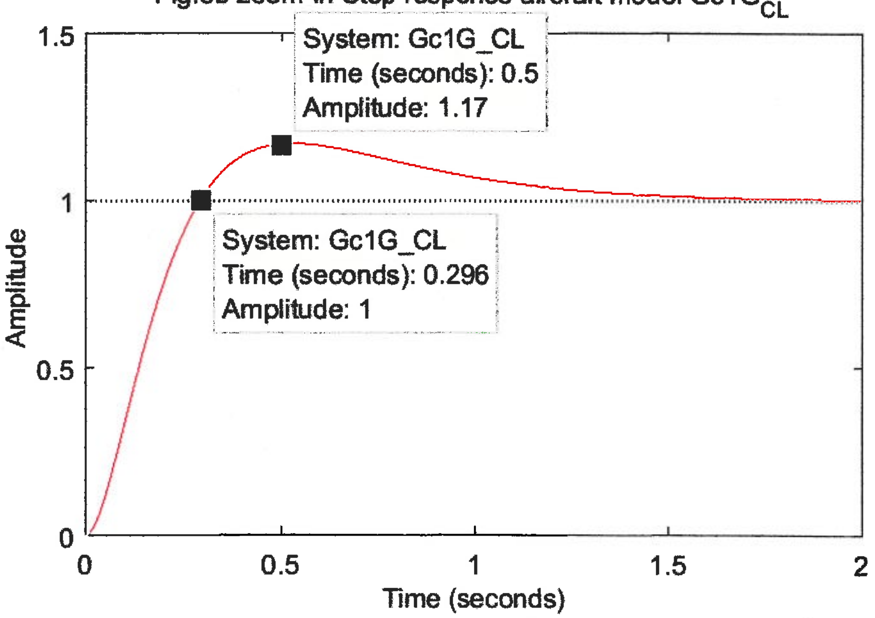


Fig.5b zoom-in Step response aircraft model $Gc1G_{CL}$

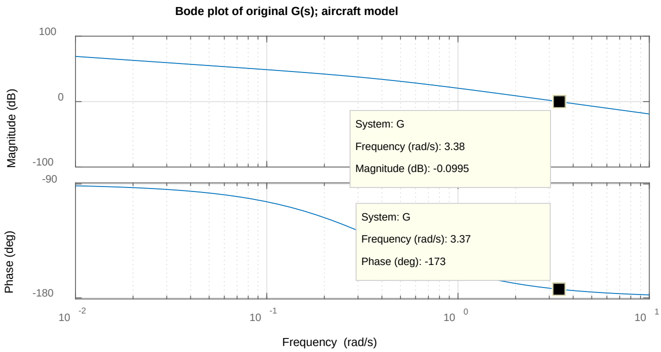


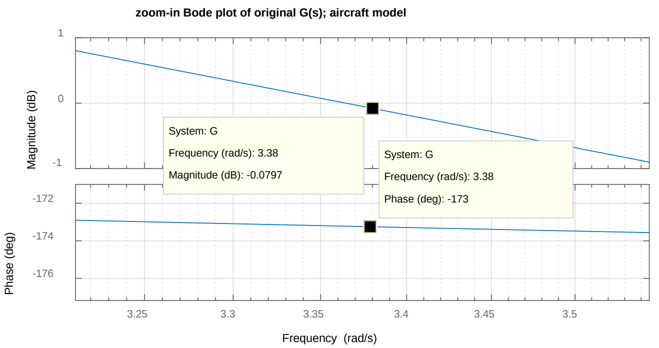
Lead_compensator_aircraft_20180510.m

```matlab
% Phase compensator design: aircraft model
%% initialization
clc %clear command window
clear %removes all variables from workspace; release memory
format compact
close all %closes all figures
s=tf('s');
%% original aircraft model
display('Phase margin: aircraft model')
K=114;              % gain
J=10;               % inertia
c=4;                % damping
display('input data')
display([K J c],'    K    | J  |    c')
G=K/(J*s^2+c*s)     % G(s)
display('=========================')
%% specs: MIL-DTL-9490E margin requirements
GM=10;    % gain margin spec, dB
PM=60;    % phase margin spec, deg
display([GM PM],'GM, dB | PM, deg')
figure(1)
subplot(2,1,1)
bode(G)
grid
title('Bode plot of original G(s); aircraft model')
subplot(2,1,2)
d1=0;d2=1;N=1e3; w=logspace(d1,d2,N);
bode(G,w)
grid
title('zoom-in Bode plot of original G(s); aircraft model')
display('=========================')
% READ ON PLOT: phase in deg for |G|=0 dB
% phi=-173;
phi=input('Input phase in deg when |G|=0 dB, phi=');
gamma=phi-(-180);    % gamma = phase margin, deg
% display([phi],'phi = phase at |G|=0 dB point, deg')
display([gamma],'gamma = phase margin, deg')
display('=========================')
%% add compensator Gc(s)
display('Phase compensator design: aircraft model')
phi_m=PM-gamma % maximum compensator phase that need to be obtained
alpha=(1-sind(phi_m))/(1+sind(phi_m)) % compensator attenuation factor alpha
display(1/sqrt(alpha),'1/sqrt(alpha)') % expected gain rise from compensator
Gc_wm=mag2db(1/sqrt(alpha)); % expected gain rise from compensator, dB
G_wm=-Gc_wm;
display(G_wm,'G_wm, dB')
figure(2) % Bode plot to find the freq wm
d1=log10(3);d2=log10(12);N=1e3; w=logspace(d1,d2,N);
bode(G,w)
grid
title('zoom-in Bode plot of |G| to find freq wm; aircraft model')
% READ ON PLOT: frequecy for which |G|=G_wm
% wm=5.83
wm=input('Input frequency in rad/s when |G|=G_wm, wm=');
T=1/wm/sqrt(alpha) % time constant of the compensator
```

```
%% Test compensated system
figure(3)
subplot(2,1,1)
Gc=(s*T+1)/(s*alpha*T+1)
GcG=Gc*G;
bode(G,GcG)
grid
title('Bode plot of G & GcG(s); aircraft model')
subplot(2,1,2)
d1=log10(3);d2=log10(8);N=1e3; w=logspace(d1,d2,N);
bode(G,GcG,w)
grid
title('zoom-in Bode plot of G & GcG(s); aircraft model')
% READ ON PLOT: phase in deg for |GcG|=0 dB
% phi_c=-123;
phi_c=input('Input phase in deg when |GcG|=0 dB, phi_c=');
gamma_c=phi_c-(-180);   % gamma_c = phase margin of GcG, deg
display([phi_c],'phi_c = phase at |GcG|=0 dB point, deg')
display([gamma_c],'gamma_c = phase margin of GcG, deg')
display('========================')
%% Adjust compensator to reach PM requirements
display('Adjustment of phase compensator: aircraft model')
dphi=PM-gamma_c  % additional phase shift needed
phi_m1=phi_m+dphi   % adjusted phase shift
alpha1=(1-sind(phi_m1))/(1+sind(phi_m1)) % adjusted alpha
Gc1_wm=mag2db(1/sqrt(alpha1)); % expected gain rise from compensator, dB
G_wm1=-Gc1_wm;
display(G_wm1,'G_wm1, dB')
figure(4) % Bode plot to find the freq wm for |G_wm|dB=-|Gc_wm|dB
d1=0;d2=1;N=1e3; w=logspace(d1,d2,N);
bode(G,w)
grid
title('zoom-in Bode plot of G(s) to find new freq wm1; aircraft model')
% READ ON PLOT: frequecy for which |G|=-Gc_wm1, dB
% wm1=6.1;
wm1=input('Input frequency rad/s when |G|=G_wm1, wm1=');
T1=1/wm1/sqrt(alpha1) % updated time constant of the compensator
%% Final compensator design
Gc1=(s*T1+1)/(s*alpha1*T1+1)
Gc1G=Gc1*G;
figure(5)
subplot(2,1,1)
bode(G,Gc1G)
grid
title('Bode plot of G & adjusted Gc1G(s); aircraft model')
subplot(2,1,2)
d1=log10(3);d2=log10(8);N=1e3; w=logspace(d1,d2,N);
bode(G,Gc1G,w)
grid
title('zoom-in Bode plot of G & adjusted Gc1G(s); aircraft model')
phi_c1=input('Input phase in deg for |Gc1G|=0 dB, phi_c1=');
gamma_c1=phi_c1-(-180);   % gamma_c = phase margin of GcG, deg
display([phi_c1],'phi_c1 = phase at |Gc1G|=0 dB point, deg')
display([gamma_c1],'gamma_c = phase margin of GcG, deg')
%% Plot step response response of the original and compensated systems
figure(6)
Tfinal=10; dt=0.01; t=0:dt:Tfinal;
G_CL=feedback(G,1);
Gc1G_CL=feedback(Gc1G,1);
subplot(2,1,1)
step(G_CL,Gc1G_CL,t)
title('step response aircraft model G_C_L, Gc1G_C_L')
subplot(2,1,2)
Tzoom=2; Nt=1e3; dt=Tzoom/Nt; tz=0:dt:Tzoom;
step(Gc1G_CL,tz,'r')
title('zoom-in step response aircraft model Gc1G_C_L')
ylim([0 1.5])
```

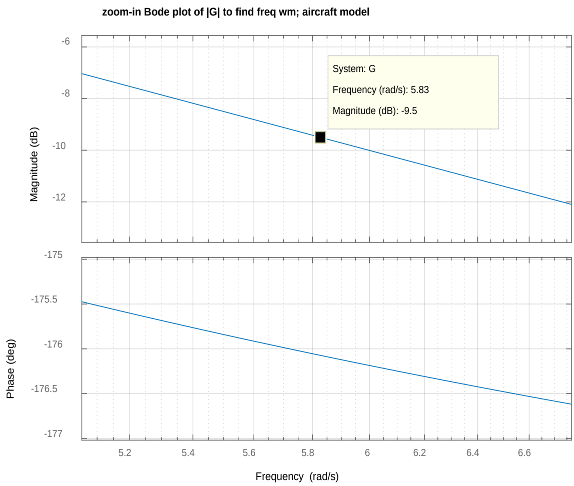
```
Phase margin: aircraft model
input data
    K    | J  |    c =
   114     10      4

G =

      114
  ------------
  10 s^2 + 4 s

Continuous-time transfer function.

=========================
GM, dB | PM, deg =
    10     60
=========================
```





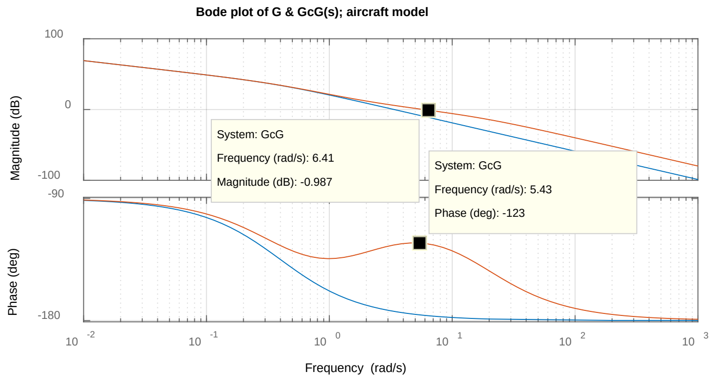

```
Input phase in deg when |G|=0 dB, phi=-173
gamma = phase margin, deg =
     7
```

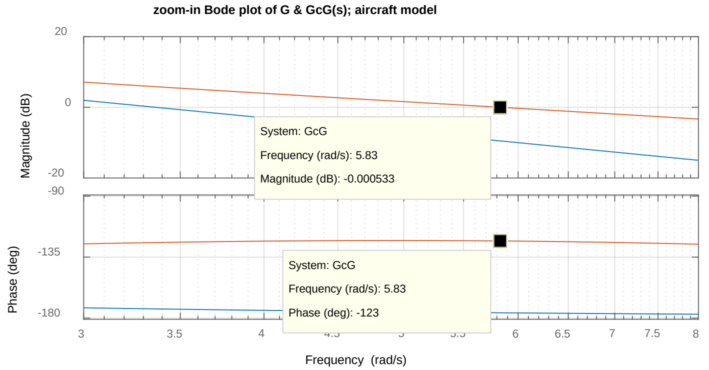
```
Phase compensator design: aircraft model
phi_m =
    53
alpha =
    0.1120
1/sqrt(alpha) =
    2.9887
G_wm, dB =
   -9.5096
```




```
Input frequency in rad/s when |G|=G_wm, wm=5.83
T =
    0.5126
```

```
Gc =

  0.5126 s + 1
  -------------
  0.05739 s + 1
```

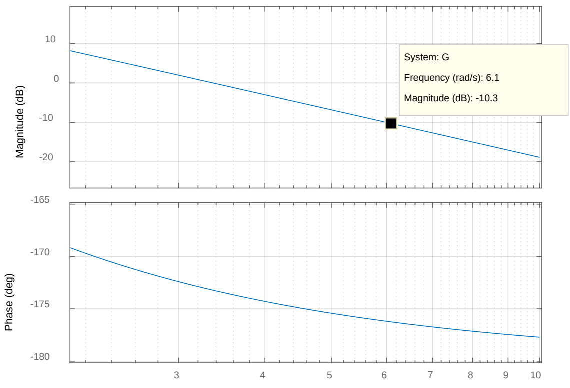





```
Input phase in deg when |GcG|=0 dB, phi_c=-123
phi_c = phase at |GcG|=0 dB point, deg =
  -123
gamma_c = phase margin of GcG, deg =
    57
-------------------------
```

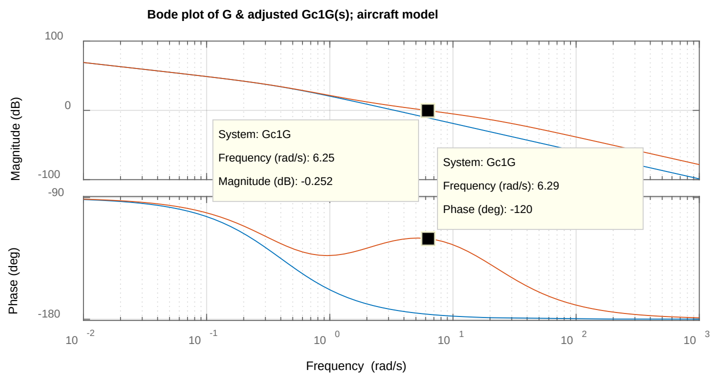
```
Adjustment of phase compensator: aircraft model
dphi =
     3
phi_m1 =
    56
alpha1 =
    0.0935
G_wm1, dB =
  -10.2932
```

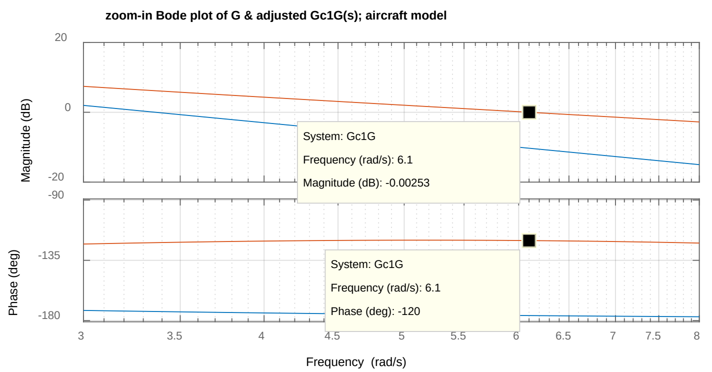
zoom-in Bode plot of G(s) to find new freq wm1; aircraft model



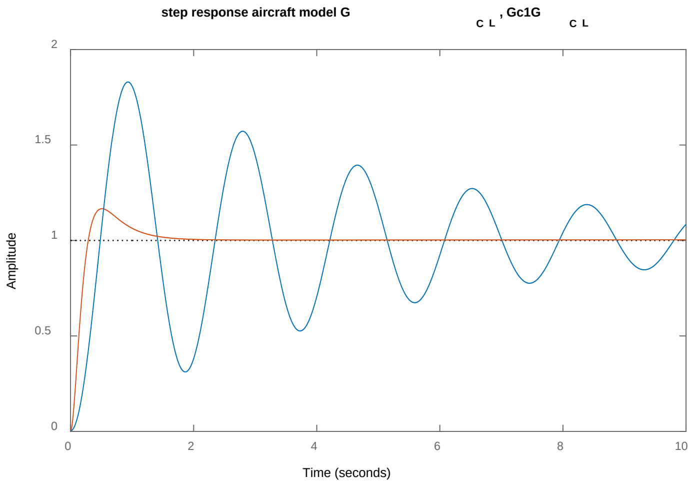

```
Input frequency rad/s when |G|=G_wm1, wm1=6.1
T1 =
    0.5362
```

```
Gc1 =

  0.5362 s + 1
  -------------
  0.05012 s + 1
```

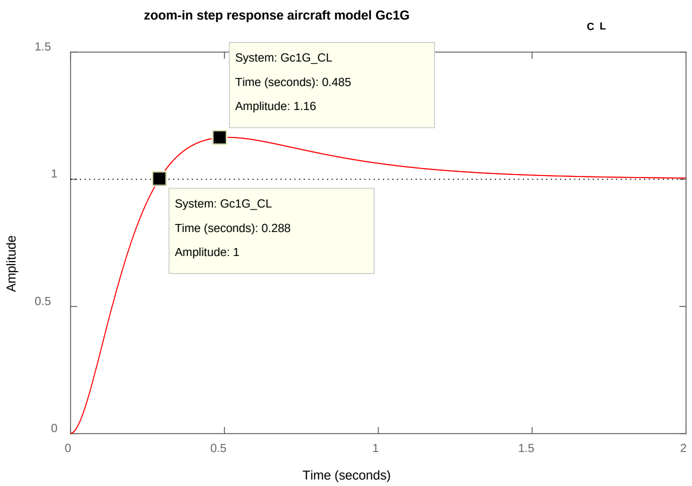





```
Input phase in deg for |Gc1G|=0 dB, phi_c1=-120
phi_c1 = phase at |Gc1G|=0 dB point, deg =
  -120
gamma_c = phase margin of GcG, deg =
    60
```



step response aircraft model G_CL, Gc1G_CL
Amplitude
Time (seconds)
0
0.5
1
1.5
2
0
2
4
6
8
10

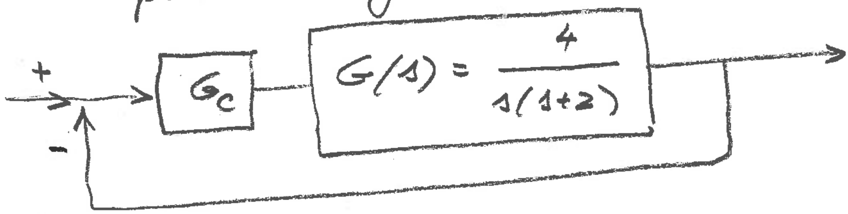


zoom-in step response aircraft model Gc1G_CL
System: Gc1G_CL
Time (seconds): 0.485
Amplitude: 1.16
System: Gc1G_CL
Time (seconds): 0.288
Amplitude: 1
Amplitude
Time (seconds)
0
0.5
1
1.5
0
0.5
1
1.5
2

C7

Ex. 11.6

Design a lead compensator to reduce servomotor ramp error and meet phase and gain margin requirements.



<u>Given</u> : $G(s) = \dfrac{4}{s(s+2)}$

<u>Find</u> :

(a) static velocity error const. K_V

phase margin, γ

gain margine, $(K_g)_{dB}$

(b) design compensator to achieve:

$K_V \geq 20 /\text{sec}$

$\gamma \geq 50°$ (PM = 50°)

$(K_g)_{dB} \geq 10 dB$ (GM = 10 dB)

<u>Solution</u>

(a) characterize current system

- $K_V = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} s \dfrac{4}{s(s+2)} = \dfrac{4}{2} = 2/\text{sec}.$
- $\varphi(G = 0dB) = -128°$; $\gamma = -128° + 180° = 52°$
- $(K_g)_{dB} = \infty$ because $\angle G$ never crosses $-180°$ line

C7a

Fig.1a Bode plot of original G(s); Example 11.6

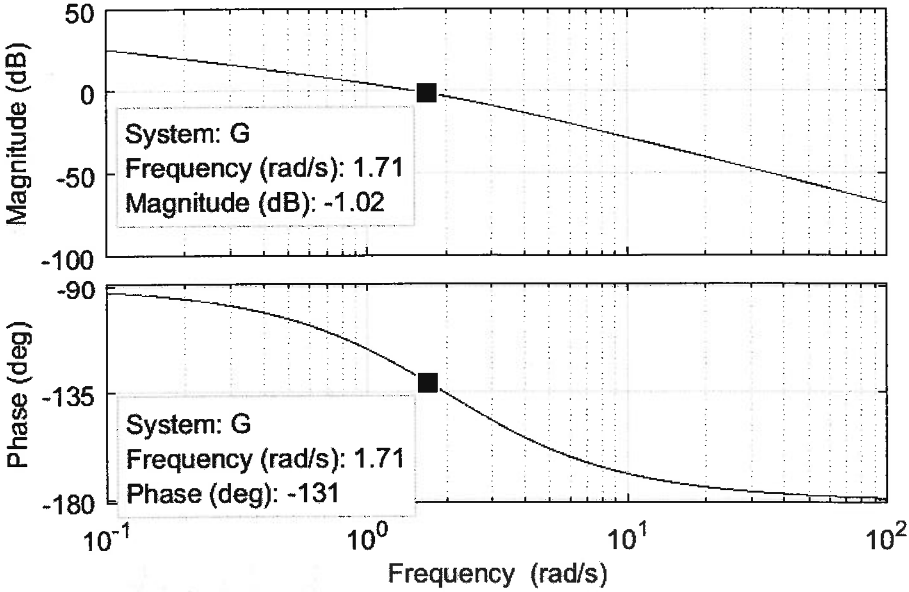
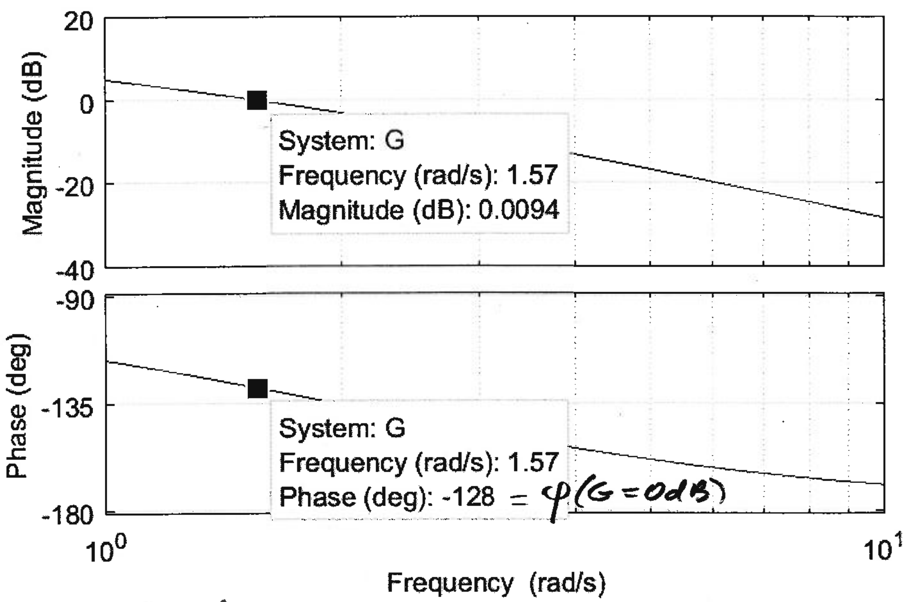
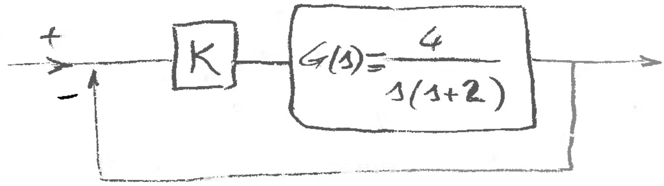


Fig.1b zoom-in Bode plot of original G(s); Example 11.6



$\gamma = -128 + 180 = 52°$

C8/ (b) Design P-controller to improve K_v



<u>Roadmap</u>

(1) add K to improve K_V to $K_V^* = 20/\text{sec}$

(2) Notice that adding K shifts magnitude plot upward and spoils the phase margin.

(3) add compensator G_c to improve phase and bring phase back to $\gamma^* \cong 50°$

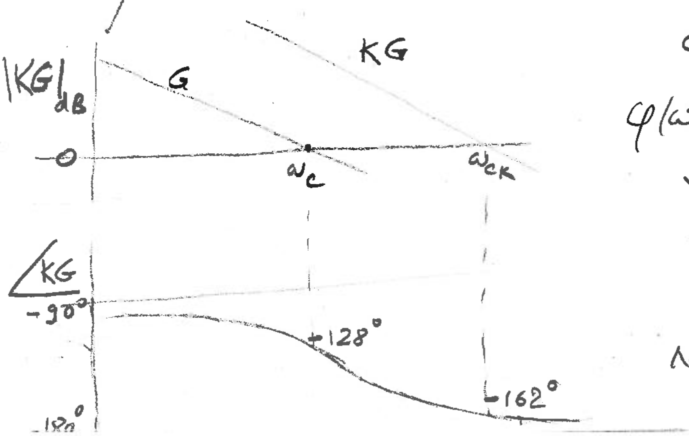
<u>Design</u>

D1. Calculate K needed to bring K_v to value $K_v^* = 20/s$

$$K_v^* = \lim_{s\to 0} s\, G^*(s) = \lim_{s\to 0} K \cdot \frac{4}{s(s+2)} = 2K = 20$$

$\longrightarrow$ $\boxed{K = 10}$ $\qquad K_{dB} = 20\,dB$

D2. Draw Bode diagram for new system. We find that gain crossover freq ω_c has moved to the right and the phase margin has decreased



$\omega_{cK} = 6.16$ rad/sec.

$\varphi(\omega_{cK}) = -162°$

$\gamma_K = 18° < 50°$

$(-162° + 180° = 18°)$

Need to add 32°!

c8a

Fig.2a Bode plot of G & KG(s);

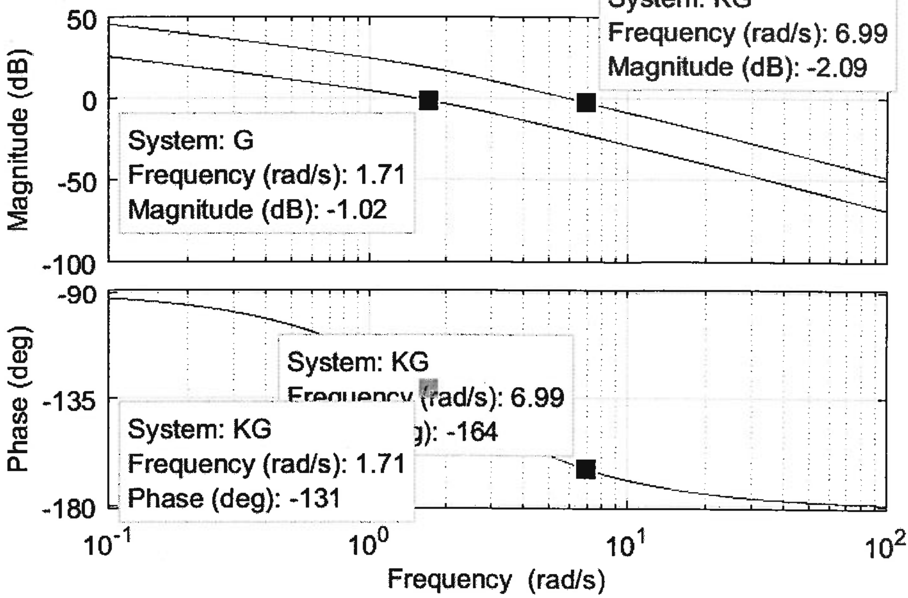
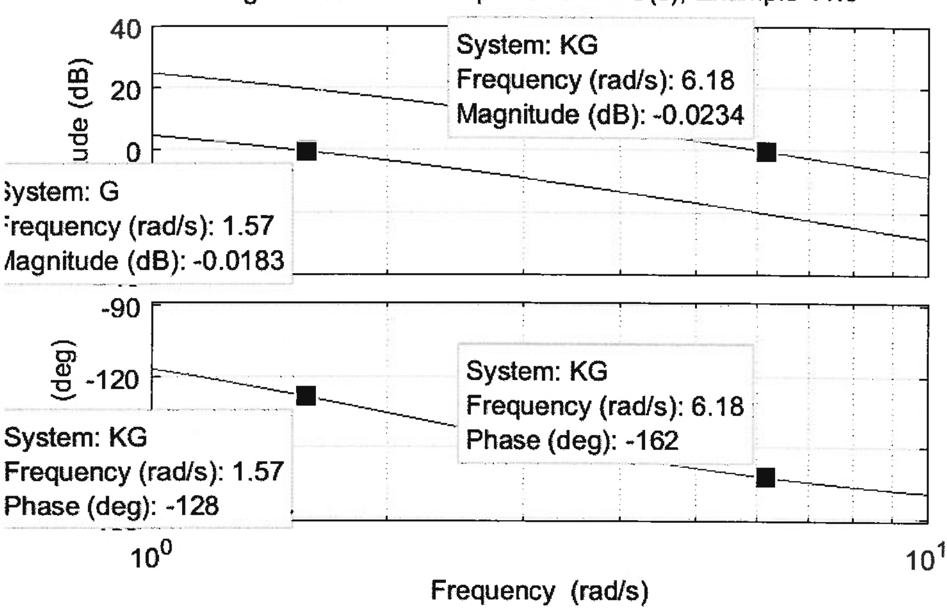
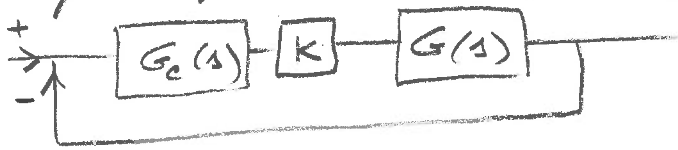


Fig.2b zoom-in Bode plot of G & KG(s); Example 11.6



C9 Design compensator to improve γ.



We need lead compensator

$$G_c(s) = \frac{Ts+1}{\alpha Ts+1}$$

We need to improve the phase margin from $\gamma = 18^\circ$ to $\gamma_M = 50^\circ$, i.e., we need the phase compensator to add $\varphi_m = 32^\circ$

Recall $\alpha = \left.\dfrac{1-\sin\varphi_m}{1+\sin\varphi_m}\right|_{\varphi_m = 32^\circ} = 0.3073$

To calculate ω_m, recall

$$|G_c(\omega_m)| = \frac{1}{\sqrt{\alpha}} = 1.8040 = 5.125\,dB$$

We identify ω_u as the point on the Bode plot of $KG(\omega)$ where

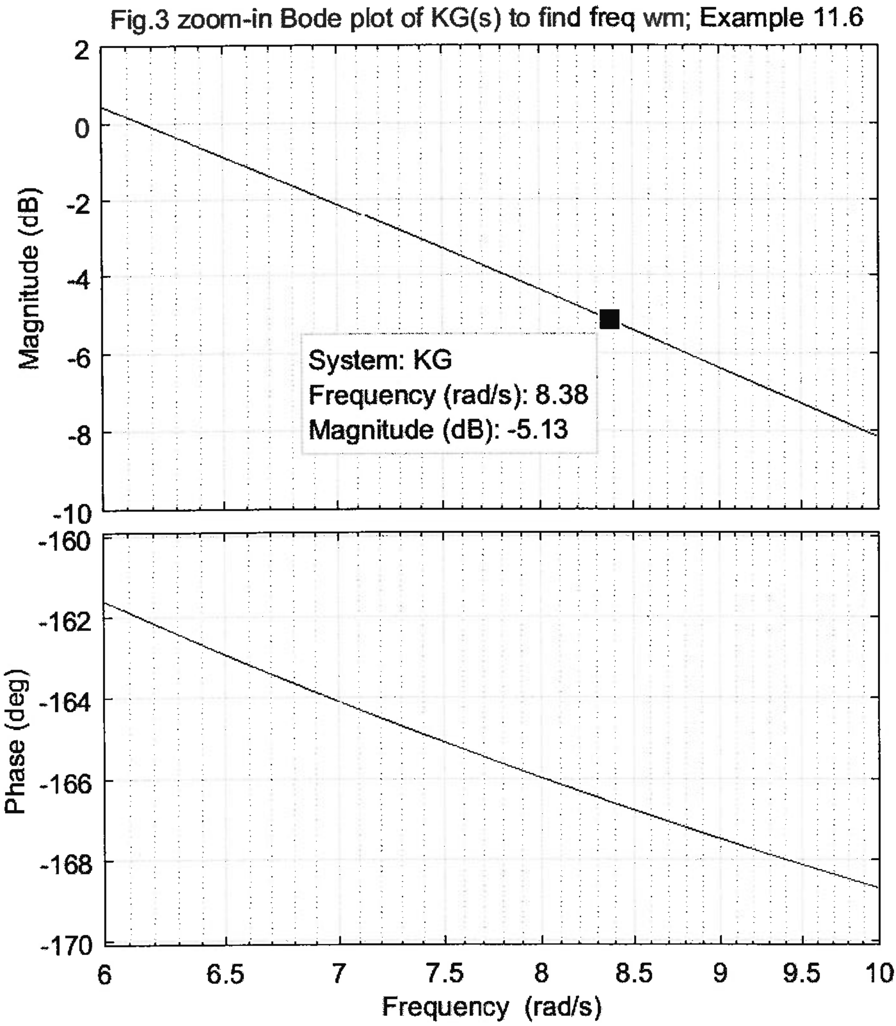
$$|KG(\omega_m)|_{dB} = -|G_c(\omega_n)|_{dB} = -5.125\,dB.$$

From Bode plot, we read

$$\omega_m = 8.38 \text{ rad/sec for } |KG|_{dB} = -5.13dB$$

Hence $T = \dfrac{1}{\omega_m\sqrt{\alpha}} = 0.2153$ sec.

C9a



$\omega_m = 8.38$ rad/sec for $|KG|_{dB} = -5.13$dB

C96

```
Phase compensator design: Exampe 11.6

phi =

  -128


gamma =

    52


alpha =

    0.3073


1/sqrt(alpha =

    1.8040


Gc_wm, dB =

    5.1250


T =

    0.2153


Gc =

  0.2153 s + 1
  ------------
  0.06615 s + 1
```

cgc

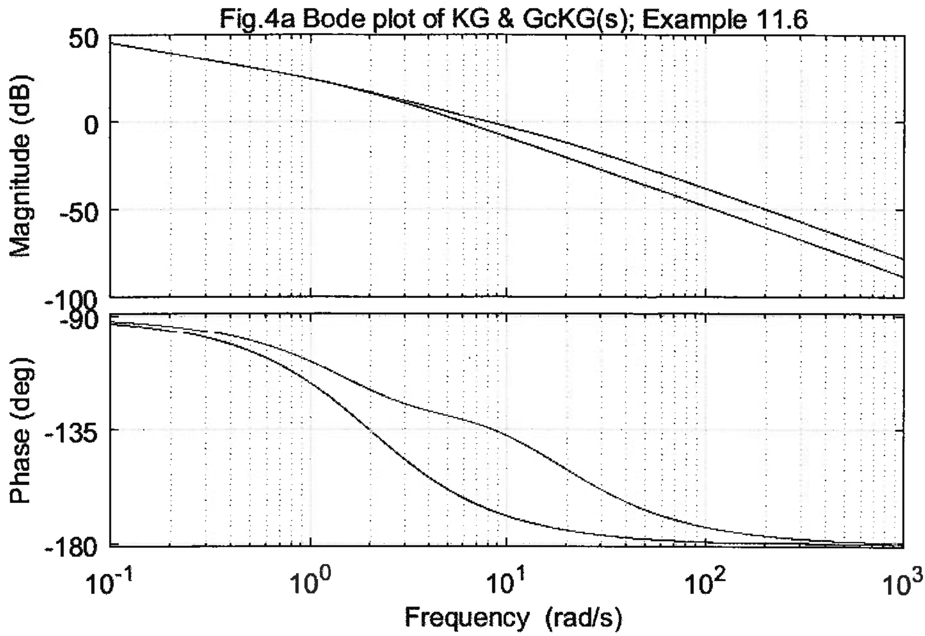


Fig.4a Bode plot of KG & GcKG(s); Example 11.6
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

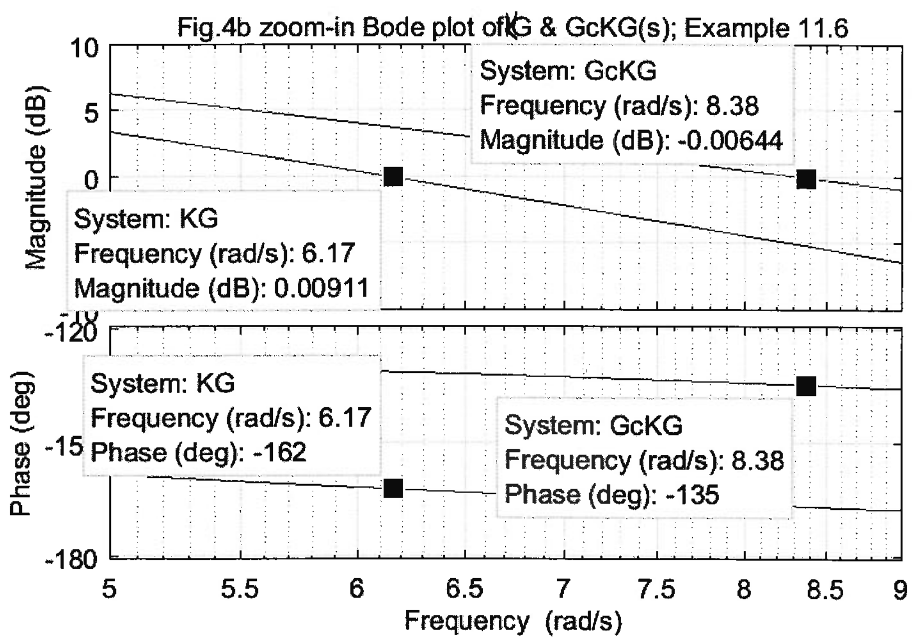


Fig.4b zoom-in Bode plot of KG & GcKG(s); Example 11.6
System: GcKG
Frequency (rad/s): 8.38
Magnitude (dB): -0.00644
System: KG
Frequency (rad/s): 6.17
Magnitude (dB): 0.00911
System: KG
Frequency (rad/s): 6.17
Phase (deg): -162
System: GcKG
Frequency (rad/s): 8.38
Phase (deg): -135
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

C10 Test the compensator

Fig 4 shows that the compensator is not sufficient because the compensated phase margin is

$$\gamma_c = -135 + 180 = 45^\circ$$

We need 50°; thus, we need to adjust the compensator to get to 50°. We need an additional 5° of compensation, i.e., $\Delta\varphi = 5^\circ$. The new φ_m is

$$\varphi_{m_1} = \varphi_m + \Delta\varphi = 32 + 5 = 37^\circ$$

The new α is $\alpha_1 = \dfrac{1 - \sin\varphi_{m_1}}{1 + \sin\varphi_{m_1}} = 0.2486$

We keep $T_1 = T$ and calculate new compensator

$$G_{c_1} = \frac{T_1 s + 1}{\alpha_1 T_1 s + 1}$$

With this new compensator, the Bode plot is as shown in Fig. 5. The phase at $G_{c_1}KG = 0\,dB$ is -130°

The margin is $\gamma_{c_1} = -130^\circ + 180^\circ = 50^\circ = \gamma_M$

We have met the specification!

C10a

```
dphi =

     5


phi_m1 =

    37


alpha1 =

    0.2486


T1 =

    0.2153


Gc1 =

  0.2153 s + 1
  -------------
  0.05352 s + 1
```

c10b

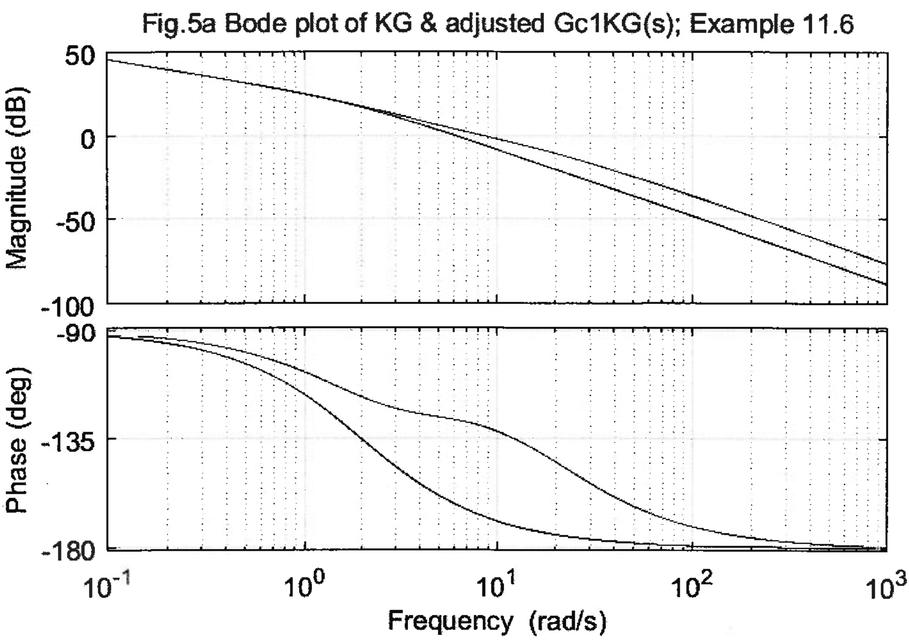


Fig.5a Bode plot of KG & adjusted Gc1KG(s); Example 11.6
Magnitude (dB)
50
0
-50
-100
Phase (deg)
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)

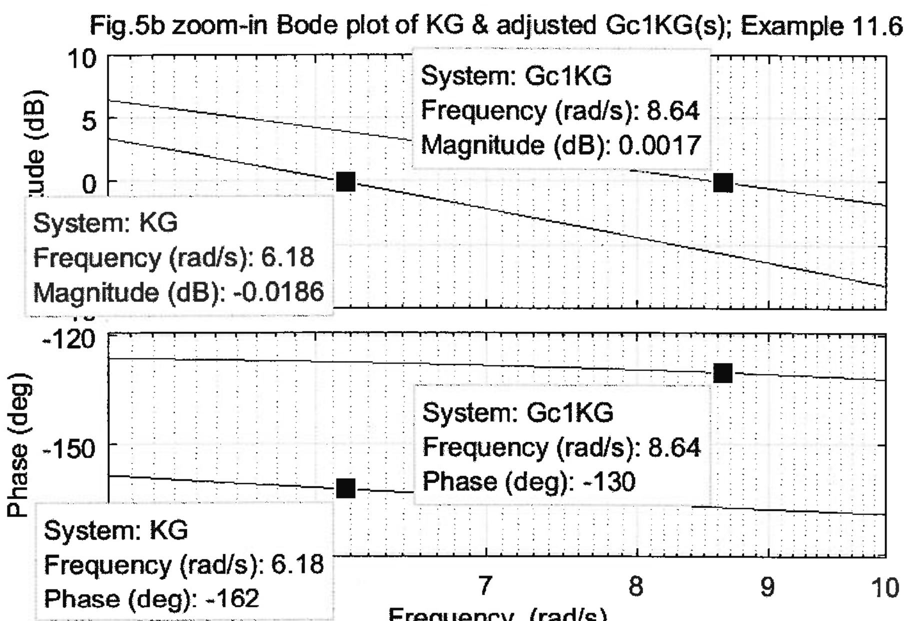
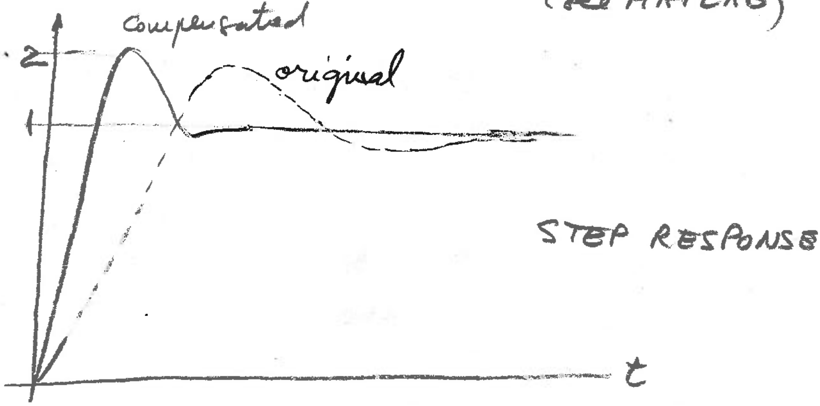
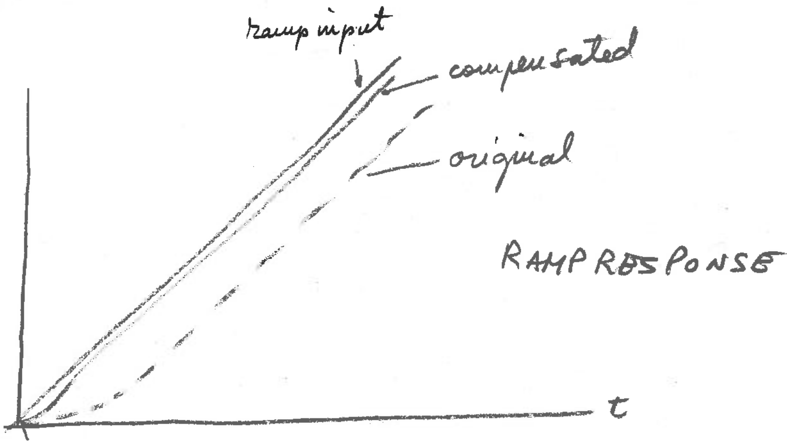


Fig.5b zoom-in Bode plot of KG & adjusted Gc1KG(s); Example 11.6
10
5
0
System: Gc1KG
Frequency (rad/s): 8.64
Magnitude (dB): 0.0017
System: KG
Frequency (rad/s): 6.18
Magnitude (dB): -0.0186
Phase (deg)
-120
-150
System: Gc1KG
Frequency (rad/s): 8.64
Phase (deg): -130
System: KG
Frequency (rad/s): 6.18
Phase (deg): -162
7
8
9
10
Frequency (rad/s)

C11) Plot step response and ramp response of the original and compensated systems

(see MATLAB)





Discussion

Ramp response has improved dramatically

Step response has a faster rise time and a faster settling time, but it has a slightly higher overshoot

C11a

Fig.6a Step response Example 11.6

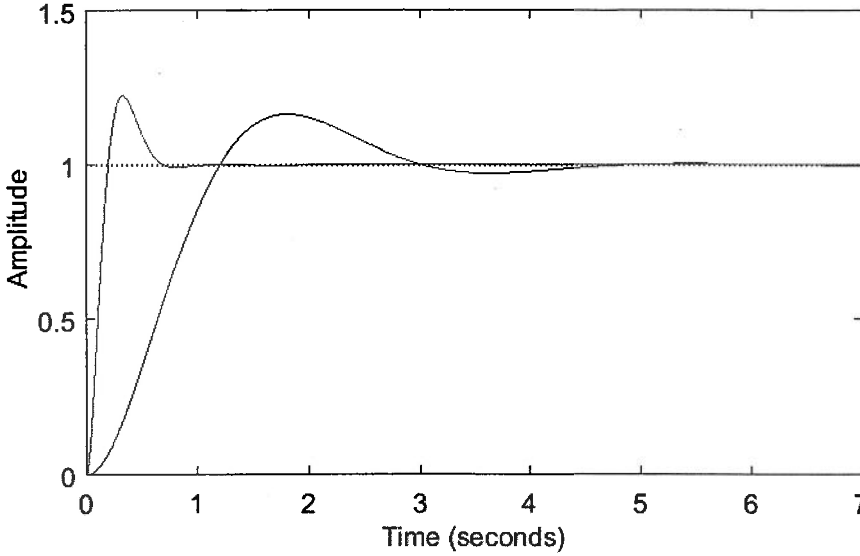


Fig.6b Ramp response Example 11.6