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MIL-DTL-9490E defines margin requirements for aircraft flight control systems (FCS) with feedback:

GM = gain margin
PM = phase margin

TABLE III. Gain and phase margin requirements (dB, degrees).

Air speed / Mode Frequency Hz	Below V_{oMIN}	V_{oMIN} to V_{oMAX}	At Limit Airspeed (V_L)	At 1.15 V_L
f_M <0.06	GM= 6 dB (No Phase Requirement Below V_{oMIN})	GM = ± 4.5 PM = ± 30	GM = ± 3.0 PM = ± 20	GM=0 PM=0 (Stable at Nominal Phase and Gain)
0.06≤ f_M <First Aero-elastic Mode		GM = ± 6.0 PM = ± 45	GM = ± 4.5 PM = ± 30	
f_M > First Aero-Elastic Mode		GM =± 8.0 PM = ± 60	GM =± 6.0 PM = ± 45	

Where:

V_L	=	Limit airspeed (MIL-A-8860).
V_{oMIN}	=	Minimum operational airspeed (MIL-F-8785)
V_{oMAX}	=	Maximum operational airspeed (MIL-F-8785)
Mode	=	A characteristics aeroelastic response of the aircraft as described by the an aeroelastic characteristic root of the coupled aircraft/FCS dynamic equation of motion.
GM (Gain Margin)	=	change The minimum chain in loop gain at normal phase, which results in instability beyond that allowed as a residual oscillation.
PM (Phase Margin)	=	The minimum change in phase at normal loop gain which results in instability.
f_M	=	Mode frequency in Hz
Nominal Phase and Gain	=	The contractor's best estimate or measurement of FCS and aircraft phase and gain characteristics available at the time of requirement verification.

3.1.3.6.2 Sensitivity analysis. Tolerances on feedback gain and phase shall be .established at the system level based on the anticipated range of gain and phase errors which will exist between nominal test values or predictions and in-service operation due to such factors as poorly defined nonlinear and higher order dynamics, anticipated manufacturing tolerances, aging, wear, maintenance and noncritical material failures. Gain and phase margins shall be defined, based on these tolerances, which will assure satisfactory operation in fleet usage. These gain and phase tolerances shall be established based on variations in system characteristics either anticipated or allowed by component or subsystem specification. The contractor shall establish, with the approval of the procuring agency, the range of variation to be considered based on a selected probability of exceedance for each type of variation. The contractor shall select the exceedance probability based on the criticality of the flight control function being provided. The stability requirements established through this sensitivity analysis shall not be less than 50 percent of the magnitude and phase requirements of 3.1.3.6.1.

3.1.3.7 Operation in turbulence. In Operational State I, while flying in the following applicable random and discrete turbulence environment, the FCS shall provide a safe level of operation and maintain mission accomplishment capabilities. For essential and flight phase commands by two aircrew members from causing any operation in opposing directions at the same time.

3.1.3.6 Stability. For FCS using feedback systems, the stability as specified in 3.1.3.6.1 shall be provided. Alternatively, when approved by the procuring activity, the stability defined by the contractor through the sensitivity analyses of 3.1.3.6.2 shall be provided. Where analysis is used to demonstrate compliance with these stability requirements, the effects of major system nonlinearities shall be included.

3.1.3.6.1 Stability margins. Required gain and phase margins about nominal are specified in table III for all aerodynamically closed loop FCS. With these gain or phase variations included, no oscillatory instabilities shall exist with amplitudes greater than those allowed for residual oscillations in 3.1.3.8, and any non oscillatory divergence of the aircraft shall remain within the applicable limits of MIL-F-8785 or MIL-F-83300. AFCS loops shall be stable with these gain or phase variations included for any amplitudes greater than those allowed for residual oscillations in 3.1.3.8. In multiple loop systems, variations shall be made with all gain and phase values in the feedback paths held at nominal values except for the path under investigation. A path is defined to include those elements connecting a sensor to a force or moment producer. For both aerodynamic and nonaerodynamic closed loops, at least 6 dB gain margin shall exist at zero airspeed. At the end of system wear tests, at least 4.5 dB gain margin shall exist for all loops at zero airspeed. The margins specified by table III shall be maintained under flight conditions of most adverse center-of-gravity, mass distribution, and external store configuration throughout the operational envelope and during ground operations.

TABLE III. Gain and phase margin requirements (dB, degrees).

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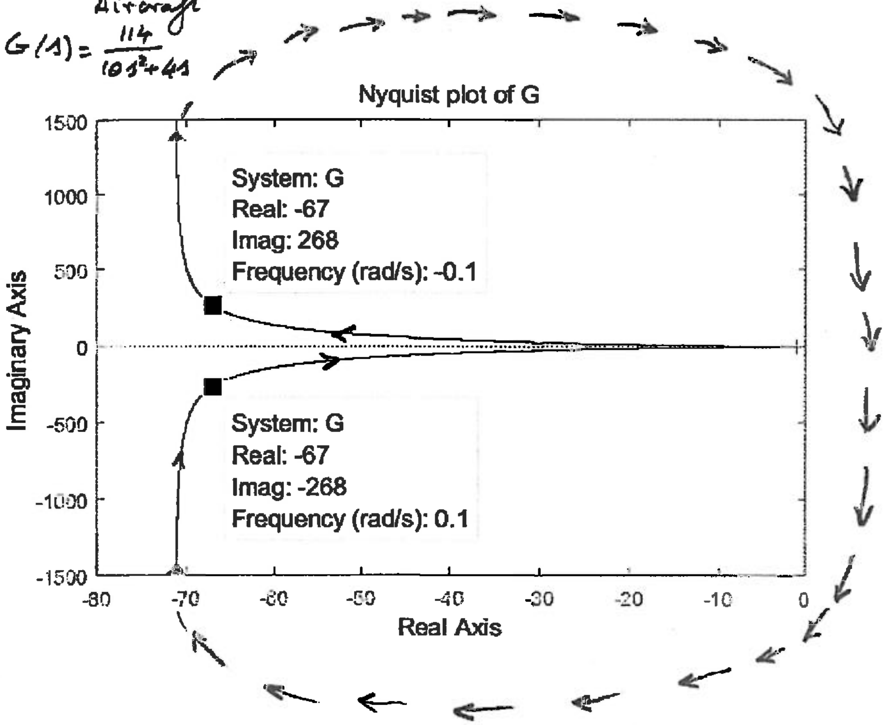
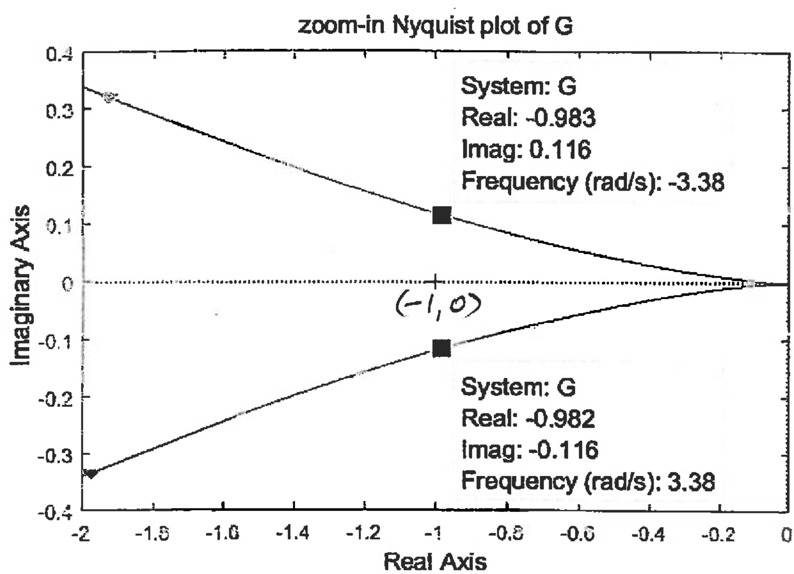
0.06 $\leq f_M$ <First Aero-elastic Mode		GM = $\pm$ 6.0 PM = $\pm$ 45	GM = $\pm$ 4.5 PM = $\pm$ 30	
f_M > First Aero-Elastic Mode		GM = $\pm$ 8.0 PM = $\pm$ 60	GM = $\pm$ 6.0 PM = $\pm$ 45	

Recall Nyquist plot

$G(s) = \frac{114}{10s^2+4s}$ Aircraft





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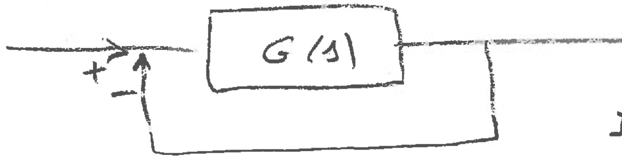
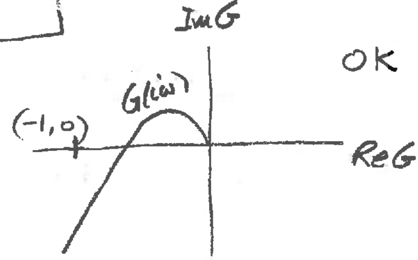
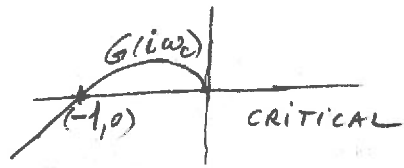
Margins analysis

- Gain margin
- Phase margin





$$G_{CL}(s) = \frac{G(s)}{1+G(s)}.$$



$G_{CL}(i\omega) \longrightarrow \infty$

if $1+G(i\omega) \to 0$

CRITICAL CONDITION ω_c

$\boxed{G(i\omega_c) = -1}$ $\Big<$ $|G(i\omega_c)| = 0 \text{ dB}$, $\angle G(i\omega_c) = -180°$

$-1 = e^{-i\pi}$

Objective : stay away from (-1,0) point !

Method

(1) Plot Bode diagram

(2) Determine margins :

- Gain margin : distance from 0dB line
- Phase margin : distance from -180° line

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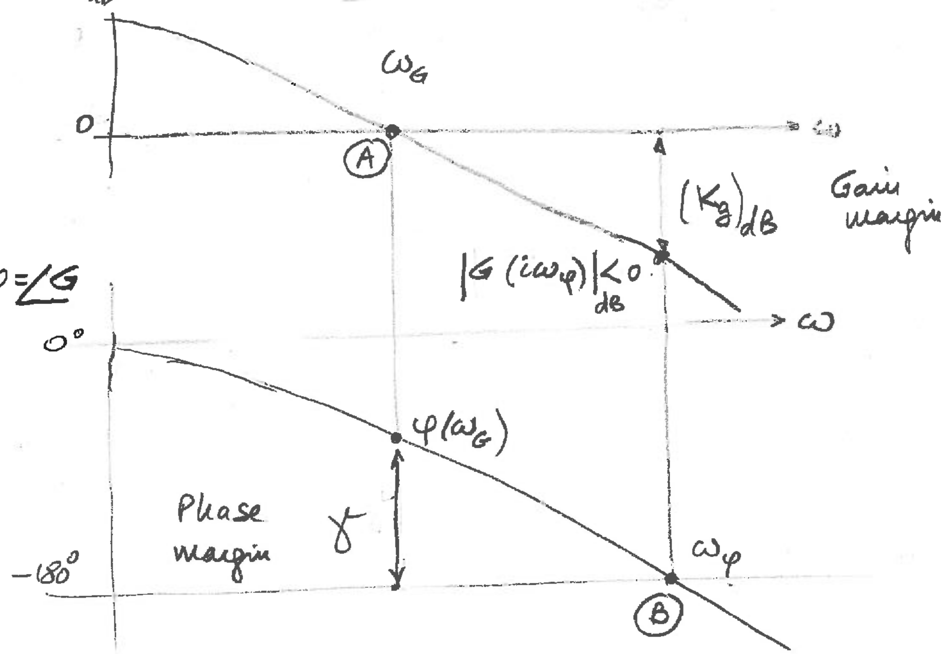
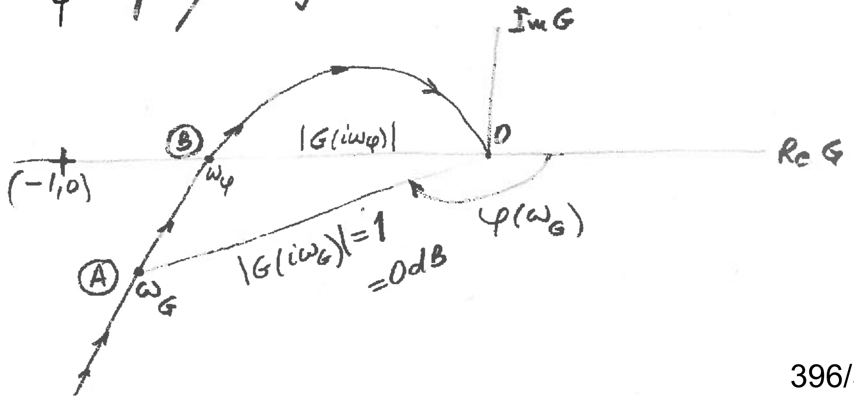


Gain margin $(K_g)_{dB} = 0 - |G(i\omega_\varphi)| = -|G(i\omega_\varphi)| > 0$

Phase margin $\gamma = \angle G(i\omega_G) - (-180^\circ) = 180^\circ + \angle G(i\omega_G) > 0$

$\omega_G =$ frequency when $|G(\omega_G)| = 0$ dB

$\omega_\varphi =$ frequency when $\varphi = -180^\circ$



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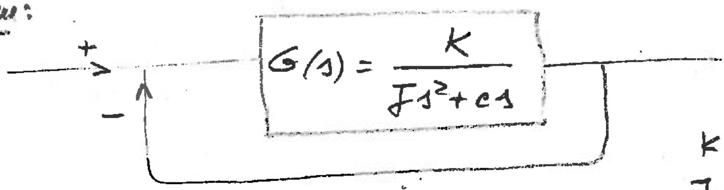
Example : aircraft roll model

Given:

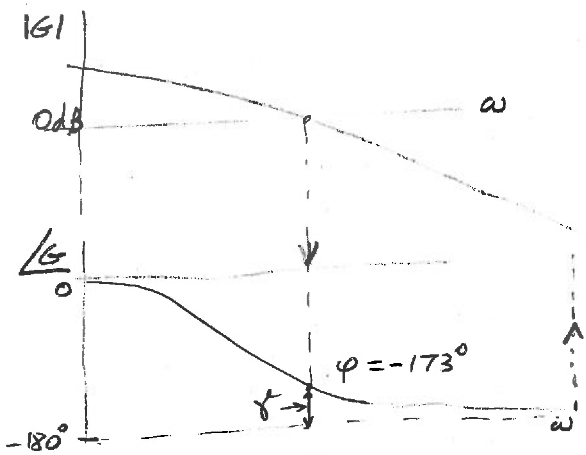


$K = 114$
$J = 10$
$c = 4$

Find

- gain margin, $(K_g)_{dB}$
- phase margin γ deg

Solution : Bode plot (see MATLAB plot) (Fig. 1)



$|G|_{\angle G = -180°} = -\infty$

∠G Never crosses −180° line

$\angle G \xrightarrow[\omega \to \infty]{} -180°$

$\gamma = \varphi + 180° = -173° + 180° = 7° \ll PM$

Very small phase margin!

NEED TO IMPROVE PHASE MARGIN!

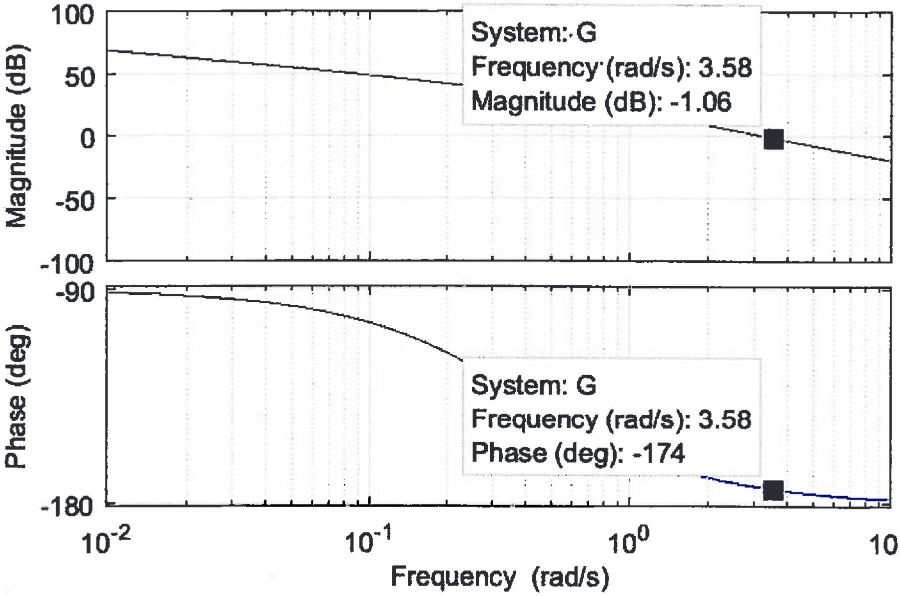
$K_g = \infty > GM$

OK

Plenty gain margin

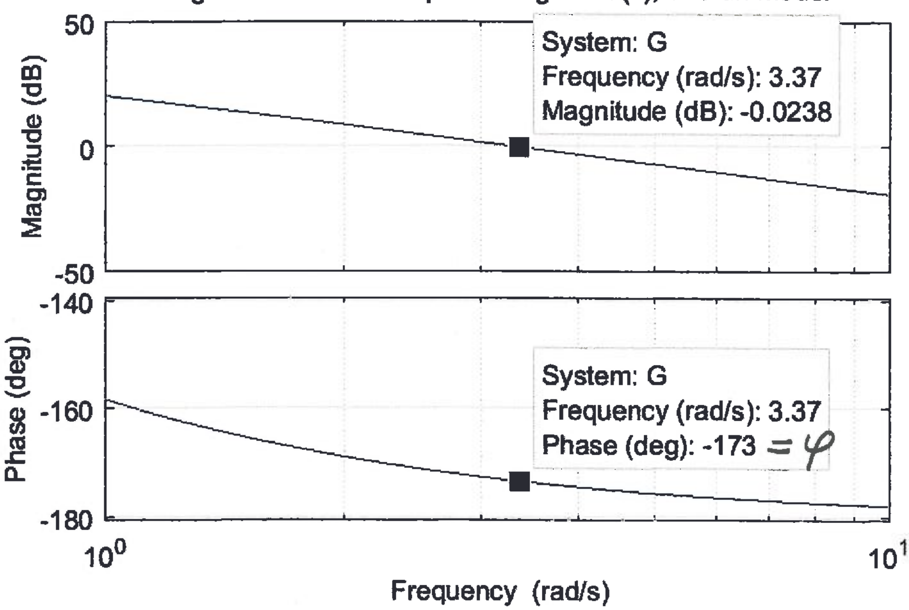
M Aircraft roll model $G(s) = \frac{114}{10s^2+4s}$

Fig.1a Bode plot of original G(s); aircraft model



Phase margin $\gamma^{o} = 180^{o} - 173^{o} = 7^{o}$ Gain margin $(K_g)_{dB} = \infty$
$= 180^{o} - \varphi$

Fig.1b zoom-in Bode plot of original G(s); aircraft model



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```
Phase margin: aircraft model
input data

    K   | J  |    c =

   114    10      4


G =

      114
  ------------
  10 s^2 + 4 s

Continuous-time transfer function.
```

```
GM, dB | PM, deg =

    10     60
```

```
phi = phase at |G|=0 dB point, deg =

  -173


gamma = phase margin, deg =

     7
```

---

margin

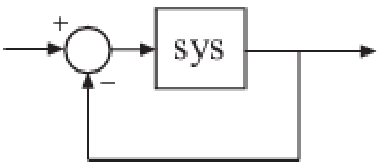
Gain margin, phase margin, and crossover frequencies

Syntax

```
[Gm,Pm,Wgm,Wpm] = margin(sys)
[Gm,Pm,Wgm,Wpm] = margin(mag,phase,w)
margin(sys)
```

Description

`margin` calculates the minimum gain margin, `Gm`, phase margin, `Pm`, and associated frequencies `Wgm` and `Wpm` of SISO open-loop models. The gain and phase margin of a system `sys` indicates the relative stability of the closed-loop system formed by applying unit negative feedback to `sys`, as in the following illustration.



The gain margin is the amount of gain increase or decrease required to make the loop gain unity at the frequency `Wgm` where the phase angle is –180° (modulo 360°). In other words, the gain margin is $1/g$ if g is the gain at the –180° phase frequency. Similarly, the phase margin is the difference between the phase of the response and –180° when the loop gain is 1.0. The frequency `Wpm` at which the magnitude is 1.0 is called the *unity-gain frequency* or *gain crossover frequency*. It is generally found that gain margins of three or more combined with phase margins between 30 and 60 degrees result in reasonable trade-offs between bandwidth and stability.

`[Gm,Pm,Wgm,Wpm] = margin(sys)` computes the gain margin `Gm`, the phase margin `Pm`, and the corresponding frequencies `Wgm` and `Wpm`, given the SISO open-loop dynamic system model `sys`. `Wgm` is the frequency where the gain margin is measured, which is a –180 degree phase crossing frequency. `Wpm` is the frequency where the phase margin is measured, which is a 0dB gain crossing frequency. These frequencies are expressed in radians/`TimeUnit`, where `TimeUnit` is the unit specified in the `TimeUnit` property of `sys`. When `sys` has several crossovers, `margin` returns the smallest gain and phase margins and corresponding frequencies.

The phase margin `Pm` is in degrees. The gain margin `Gm` is an absolute magnitude. You can compute the gain margin in dB by

```
Gm_dB = 20*log10(Gm)
```

`[Gm,Pm,Wgm,Wpm] = margin(mag,phase,w)` derives the gain and phase margins from Bode frequency response data (magnitude, phase, and frequency vector). `margin` interpolates between the frequency points to estimate the margin values. Provide the gain data `mag` in absolute units, and phase data `phase` in degrees. You can provide the frequency vector `w` in any units; `margin` returns `Wgm` and `Wpm` in the same units.

Note

When you use `margin(mag,phase,w)`, `margin` relies on interpolation to approximate the margins, which generally produces less accurate results. For example, if there is no 0 dB crossing within the `w` range, `margin` returns a phase margin of `Inf`. Therefore, if you have an analytical model `sys`, using `[Gm,Pm,Wgm,Wpm] = margin(sys)` is the most robust way to obtain the margins.

`margin(sys)`, without output arguments, plots the Bode response of `sys` on the screen and indicates the gain and phase margins on the plot. By default, gain margins are expressed in dB on the plot.

Examples

Gain and Phase Margins of Open-Loop Transfer Function

Create an open-loop discrete-time transfer function.

```
hd = tf([0.04798 0.0464],[1 -1.81 0.9048],0.1)
```

```
hd =

   0.04798 z + 0.0464
  ---------------------
  z^2 - 1.81 z + 0.9048

Sample time: 0.1 seconds
Discrete-time transfer function.
```

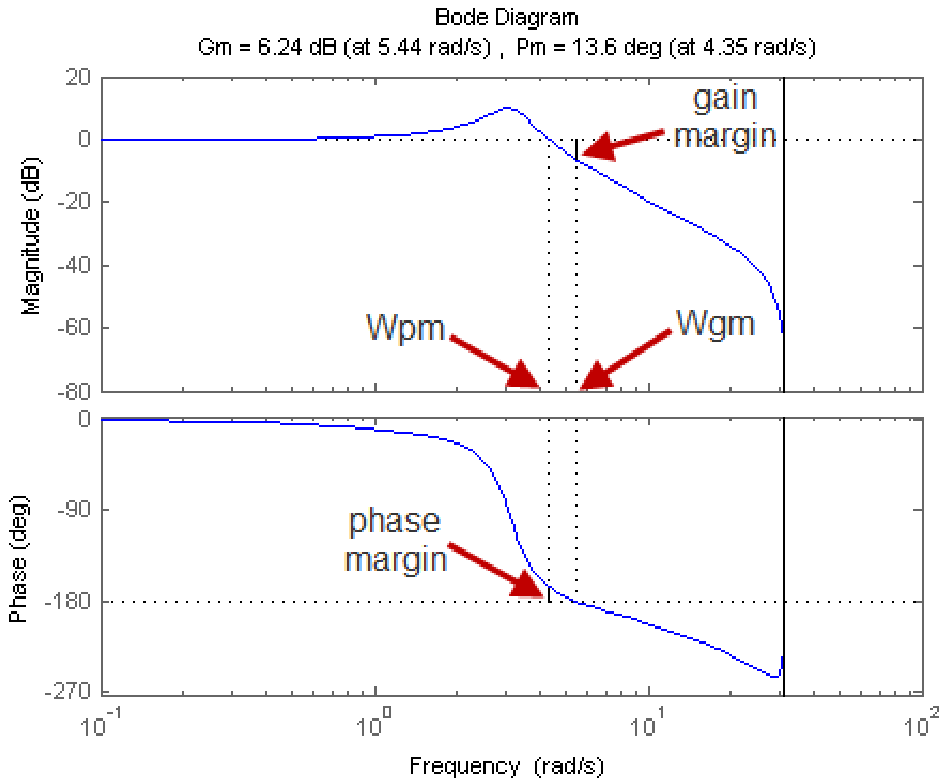
Compute the gain and phase margins.

```
[Gm,Pm,Wgm,Wpm] = margin(hd)
```

```
Gm =

    2.0517


Pm =

   13.5711


Wgm =

    5.4374


Wpm =

    4.3544
```

Display the gain and phase margins graphically.

```
margin(hd)
```



Solid vertical lines mark the gain margin and phase margin. The dashed vertical lines indicate the locations of `Wpm`, the frequency where the phase margin is measured, and `Wgm`, the frequency where the gain margin is measured.

Algorithms

The phase margin is computed using H_∞ theory, and the gain margin by solving $H(j\omega) = \overline{H(j\omega)}$ for the frequency ω.

See Also

Linear System Analyzer | `bode`

Introduced before R2006a

margins_aircraft.m

```
%% initialization
clc %clear command window
clear %removes all variables from workspace; release memory
format compact
close all %closes all figures
s=tf('s');
%% original aircraft model
display('Phase margin: aircraft model')
K=114;                % gain
J=10;                 % inertia
c=4;                  % damping
display('input data')
display([K J c],'    K    | J   |    c')
G=K/(J*s^2+c*s)       % G(s)
figure(1)
subplot(2,1,1)
bode(G)
grid
title('Fig.1a Bode plot of aircraft model')
subplot(2,1,2)
d1=0;d2=1;N=1e3; w=logspace(d1,d2,N);
bode(G,w)
grid
title('Fig.1b zoom-in Bode plot of aircraft model')
% READ ON PLOT: phase at |G|=0 dB point, deg
% phi=-173;
phi=input('Input phase read on Bode plot in deg, phi=');
gamma=phi-(-180);   % gamma = phase margin, deg
display([gamma],'gamma = phase margin, deg')
figure(2)
margin(G)
```

```
Phase margin: aircraft model
input data
    K    | J   |    c =
   114     10      4

G =

      114
  ------------
  10 s^2 + 4 s

Continuous-time transfer function.

Input phase read on Bode plot in deg, phi=-173
gamma = phase margin, deg =
     7
```

Fig.1a Bode plot of aircraft model

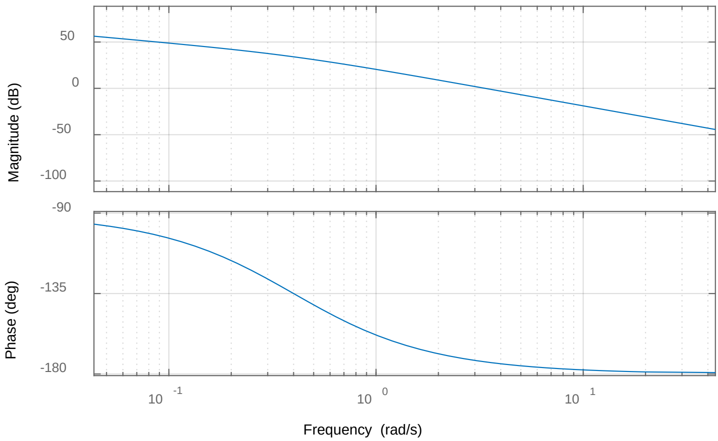
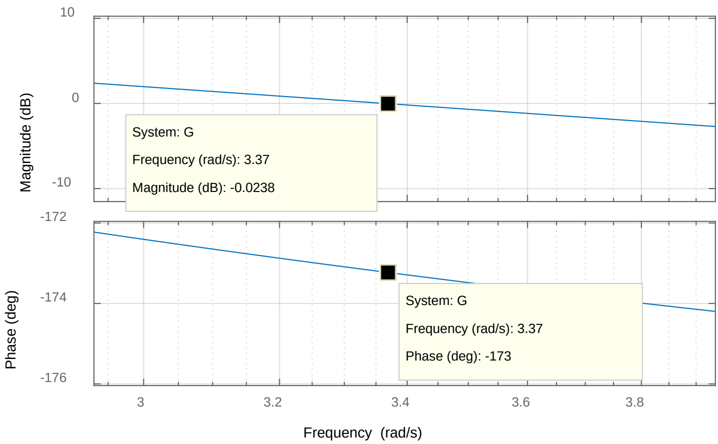
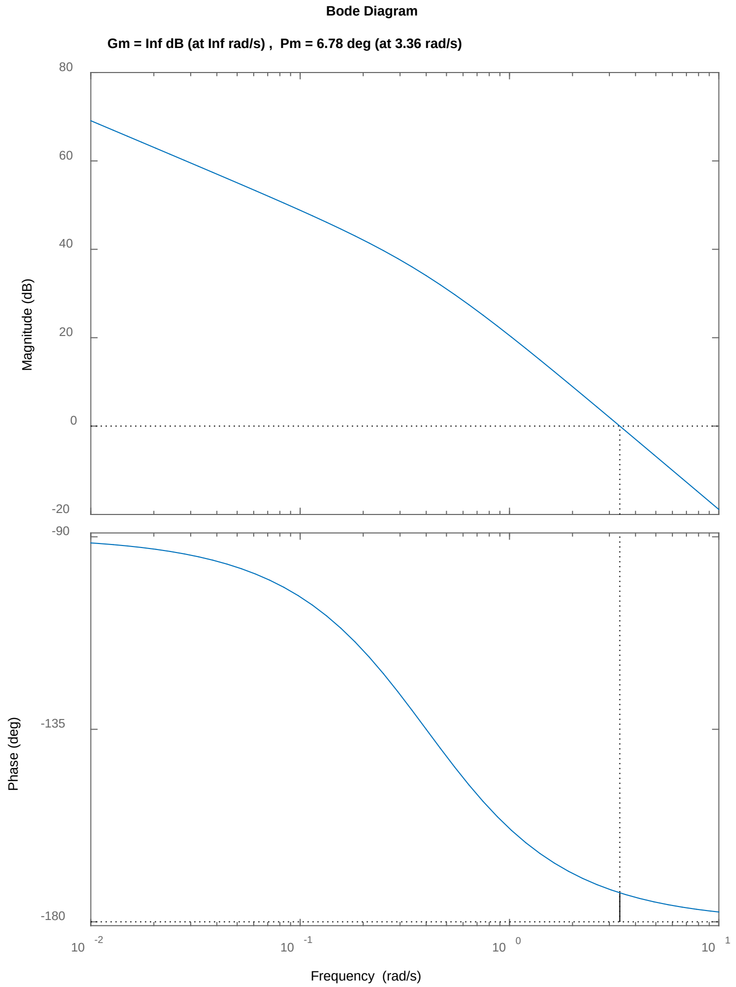


Fig.1b zoom-in Bode plot of aircraft model




Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 6.78 deg (at 3.36 rad/s)
Magnitude (dB)
80
60
40
20
0
-20
Phase (deg)
-90
-135
-180
10 -2
10 -1
10 0
10 1
Frequency (rad/s)