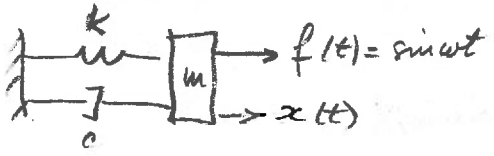
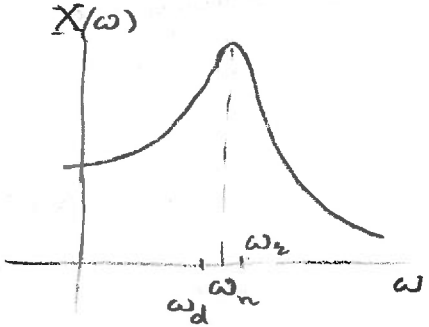
Time Response to harmonic excitation

One is interested in finding how the system responds to harmonic excitation of different frequency values, $\omega = 2\pi f$

Ex: For a 2nd order system, one is concerned with avoiding resonance where the respond may become very large





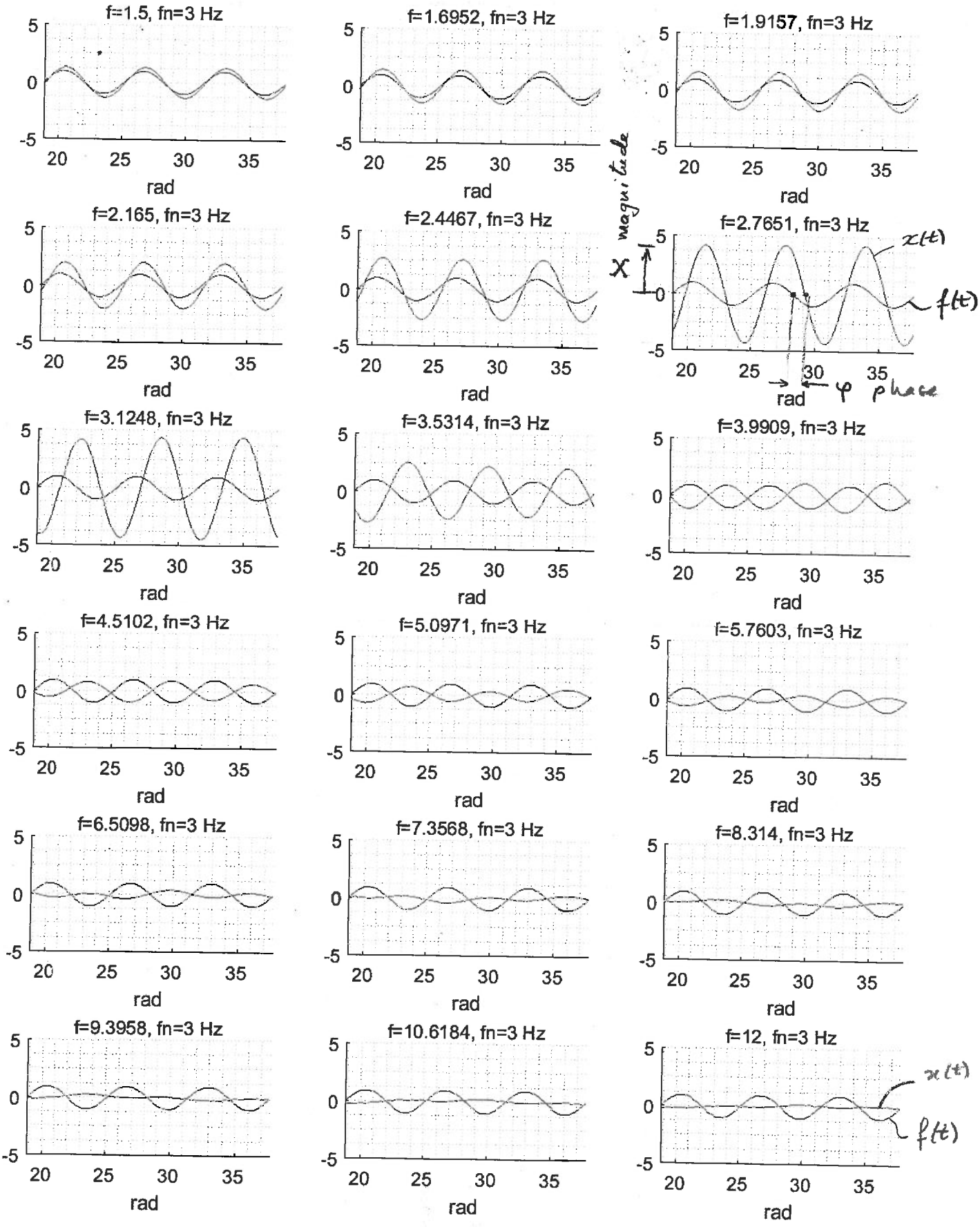
$$x(t) = X(\omega)\sin(\omega t + \varphi(\omega))$$

$$\zeta = \frac{c}{2\sqrt{km}}, \quad \omega_n = \sqrt{\frac{k}{m}}.$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

We notice that the amplitude and phase of the time response varies with excitation freq. ω

$f(x) = \sin(\omega t)$, $\omega = 2\pi f$ $x(t)$ = 2nd order system response to $f(x)$



$f(x) = \sin \omega t$

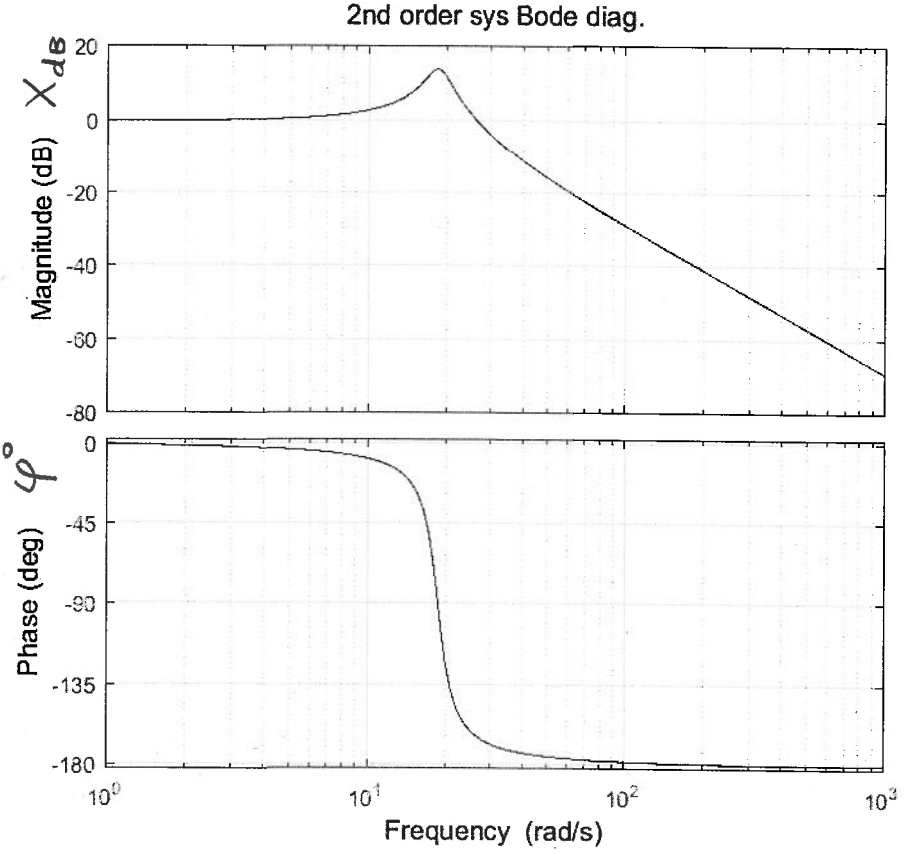
$x(t) = X(\omega) \sin[\omega t + \varphi(\omega)]$

$X(\omega)$ = magnitude of response

$\varphi(\omega)$ = phase of response

Magnitude X and phase φ vary with excitation freq. ω.

Bode diagram plots: variation of $X(\omega)$ & $\varphi(\omega)$



Note that φ is -ve, i.e. the response lags behind the excitation.

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% Amplitude_and_phase_2ndOrderSys.m
% AMPLITUDE AND PHASE IN FREQUENCY RESPONSE
clc
clear
close all
s=tf('s');
% f=input('f=');
figure (1);
%% 2nd order system
fn=3; z=10e-2; wn=2*pi*fn; G2=wn^2/(s^2+2*z*wn*s+wn^2);
M=6; N=3; Nplots=M*N;
fmin=fn/2; fmax=fn*4;
a=log10(fmin); b=log10(fmax); f=logspace(a,b,Nplots);
%% plotting setup
Na=1e3; amax=10*2*pi; da=amax/Na; angle=0:da:amax;
xmin=0.3*Na*da; xmax=0.6*Na*da;
%% plot respose at various frequencies
for i=1:Nplots
w=2*pi*f(i);    % excitation frequency
t=angle/w;      % time steps at this excitation freq.
A=1;  % forcing function amplitude
F=A*w/(s^2+w^2);    % Laplace transform of sine forcing function
fe=impulse(F,t);    % time response of forcing function
X2=G2*F;            % Laplace transform of 2nd order system response
subplot(M,N,i);
x2=impulse(X2,t);   % time response of 2nd order system
plot(angle,fe,angle,x2); hold  on
title(['f=' num2str(f(i)) ', fn=' num2str(fn) ' Hz' ],...
    'FontSize', 10,'FontWeight','normal')
xlabel('rad'); xlim([xmin xmax]); ylim([-5*A 5*A]);
% grid on
grid minor; box off
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```
33 end
34 %% FRF Bode plots
35 figure(2)
36 bode(G2); grid on; title('2nd order sys Bode diag.' );
37 aw=log10(2*pi*fn/2); bw=log10(2*pi*fn*2); N=1e4; wBode=logspace(aw,bw,N);
38 figure(3)
39 bode(G2,wBode); grid on; title('zoom 2nd order sys Bode diag.' );
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