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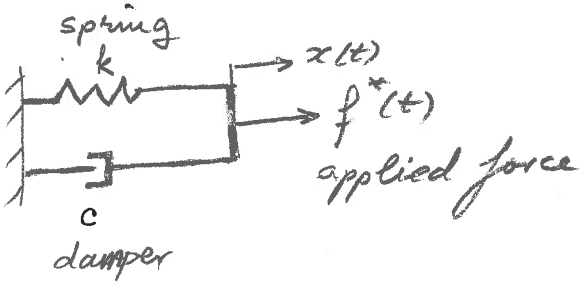
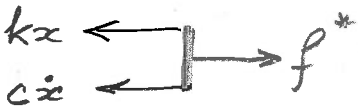
SPRING-DAMPER Mechanism



Free body diagram (FBD)



Force balance equation

$$-kx - c\dot{x} + f^* = 0$$

$$c\dot{x} + kx = f^* \quad \text{equation of motion} \qquad (1)$$

Divide Eq. (1) by k, i.e.,

$\frac{1}{k}(1)$: $$\frac{c}{k}\dot{x} + x = \frac{1}{k}f^* \qquad (2)$$

Define:

$$T = \frac{c}{k} \quad \text{Time constant} \qquad (3)$$

$$f(t) = \frac{1}{k}f^*(t) \quad \text{Normalized forcing function}$$

$(3) \rightarrow (2)$: $$T\dot{x}(t) + x(t) = f(t) \qquad (4)$$

1^{st} order ODE in standard form

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$$x(t) = x_c(t) + x_p(t) \qquad (5)$$

where

- $x_c(t)$ is called "complementary solution" and satisfies the homogeneous equation

$$T\dot{x}_c + x_c = 0 \quad \text{homogeneous eq}^{n} \qquad (6)$$

The homogeneous eq^{n} (5) is obtained from Eq (4) by making zero the right hand side (RHS).
The complementary sol^{n} $x_c(t)$ is the "free response."
- $x_p(t)$ is called "particular solution"; its role is the satisfy the RHS of Eq.(4).

Substitution of Eq.(5) into Eq.(4) yields

$$T(\dot{x}_c + \dot{x}_p) + x_c + x_p = f(t)$$

or

$$\underbrace{(T\dot{x}_c + x_c)}_{\substack{=0 \text{ because} \\ \text{of Eq.(6)}}} + T\dot{x}_p + x_p = f(t)$$

$$T\dot{x}_p + x_p = f(t) \qquad (7)$$

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Homogeneous Equation (Free Response)

The homogeneous eq^n is obtained from Eq. (4) be setting to zero the RHS, i.e.,

$$T\dot{x} + x = 0 \qquad (8)$$

To solve Eq. (8), assume

$$\left.\begin{aligned} x(t) &= C e^{pt} \\ \dot{x}(t) &= p C e^{pt} \end{aligned}\right\} \qquad (9)$$

(9) → (8) :

$$pTx + x = 0$$

or

$$(pT + 1)\, x = 0$$

or

$$pT + 1 = 0 \quad \text{characteristic eq}^n \qquad (10)$$

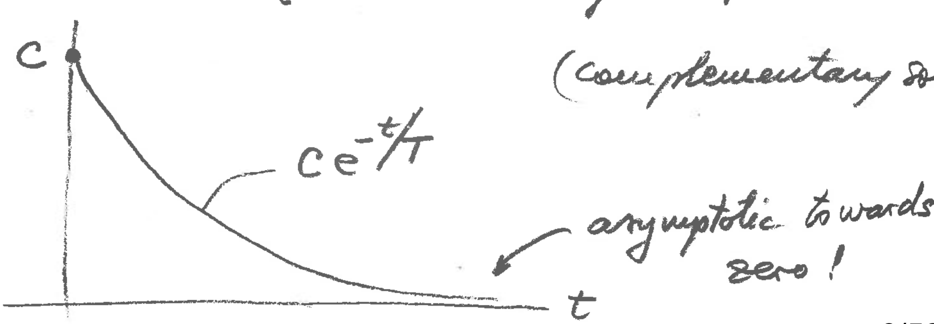
Hence,

$$p = -\frac{1}{T} \quad (\text{pole}) \qquad (11)$$

(11) → (9) :

$$x_c(t) = C e^{-t/T} \quad \text{free response} \qquad (12)$$

(complementary sol^n)



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Stability of 1st order Systems

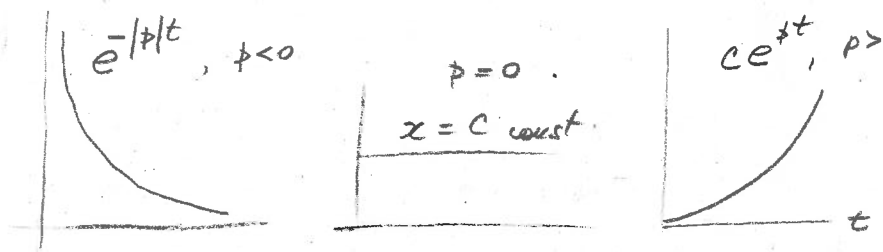
Recall general free response Eq.(9),

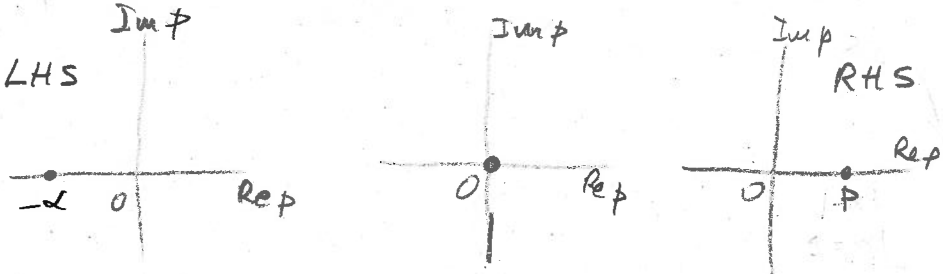
$$x(t) = C e^{pt} \qquad (9)$$

where p is a root of the characteristic equation (10) and is called "pole".

The stability is dictated by the sign of p, i.e. it's location in the complex p plane.

$p < 0$, STABLE system : if a disturbance is applied, then the system returns to initial state.
p in LHS





$p<0$	$p=0$	$P>0$
Negative pole		Positive pole
STABLE	Marginal	UNSTABLE

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Forced response

The total solⁿ of Eq.(4) is given by Eq.(5), i.e.,

$$x(t) = x_c(t) + x_p(t)$$

i.e.,

$$x(t) = Ce^{-t/T} + x_p(t)$$

The constant C and the function $x_p(t)$ have to be determined depending on initial conditions and form of $f(t)$.

In Control Theory, the initial condⁿˢ are usually assumed to be zero.