

20/6/10/19

Stability of Oscillation

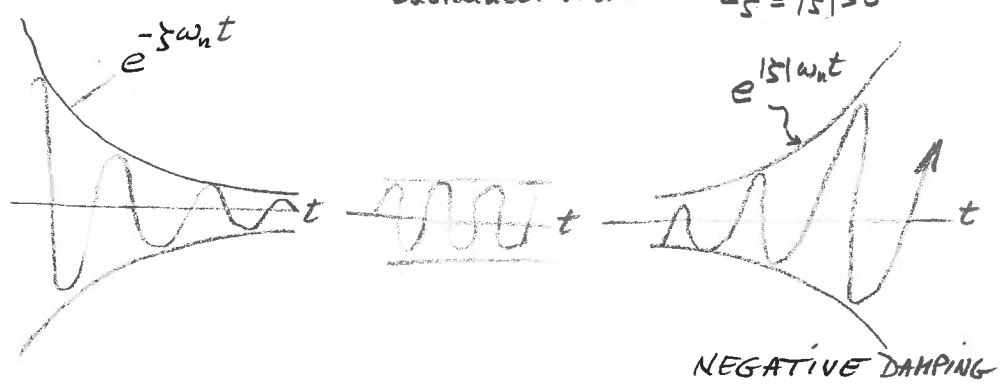
Recall damped free vibration response, i.e.,

$$x(t) = C e^{-\zeta \omega_n t} \sin(\omega_d t + \varphi) \quad (21)$$

$\zeta > 0$
stable
damped vibr.

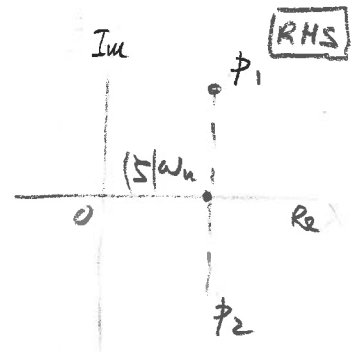
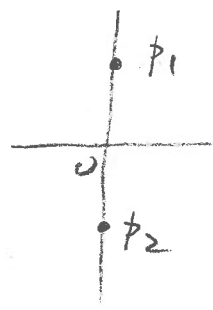
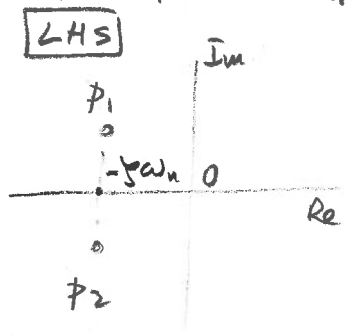
$\zeta = 0$
at the limit of
stability
sustained vibr.

$\zeta < 0$
unstable
"explosive" vibr.
 $-\zeta = |\zeta| > 0$



Complex poles: $p_{1,2} = -\zeta \omega_n \pm i \omega_d$

(poles)



$p_{1,2}$ in LHS
of complex
plane

$p_{1,2}$ in RHS
UNSTABLE!

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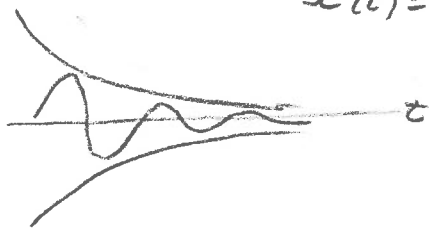
Underdamped, critically damped, overdamped 2nd order sys. and vibration response

(II) : $p_{1,2} = -\zeta \omega_n \pm i \omega_n \sqrt{1-\zeta^2}$

$0 < \zeta < 1$ underdamped vibration resp.

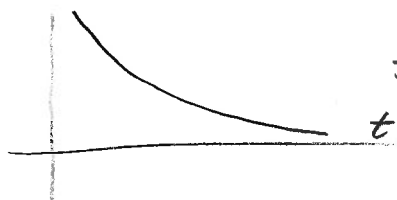
$$x(t) = C e^{-\zeta \omega_n t} \sin(\omega_d t + \varphi),$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$



$\zeta = 1$ critically damped response

$$p_1 = p_2 = -\omega_n$$



$$x(t) = (C_1 + C_2 t) e^{-\omega_n t}$$

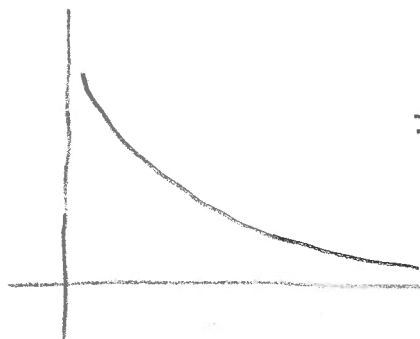
$1 < \zeta$

overdamped response

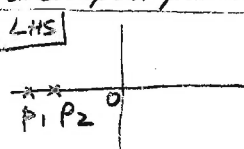
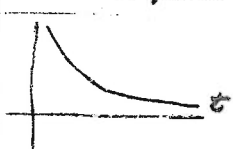

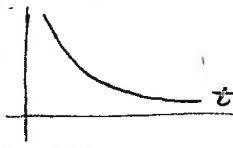
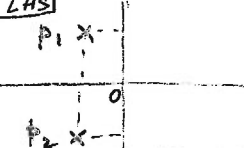
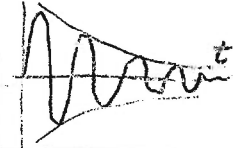

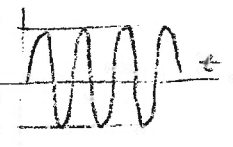

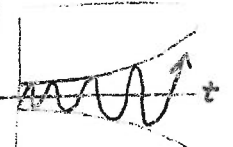


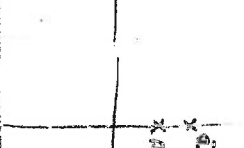

$$p_1 = -\zeta \omega_n + \omega_n \sqrt{\zeta^2 - 1}$$

$$= (-\zeta + \sqrt{\zeta^2 - 1}) \omega_n < 0$$

$$p_2 = (-\zeta - \sqrt{\zeta^2 - 1}) \omega_n < 0$$



$$x(t) = C_1 e^{-|\lambda_1|t} + C_2 e^{-|\lambda_2|t}$$

	pole location in complex plane	time response	
1 $p_1, p_2 < 0$ negative real poles in LHS			↑ monotonic stable
2 $p_1 = p_2 < 0$ negative real double pole in LHS			
3 $p_{1,2} = \sigma \pm i\omega$ $\sigma < 0$ complex poles in LHS			↑ oscillatory at the limit of stability
4 $p_{1,2} = \pm i\omega$ imaginary poles ($\sigma = 0$)			
5 $p_{1,2} = \sigma \pm i\omega$ $\sigma > 0$ complex poles in RHS			↑ unstable
6 $p_1 = p_2 > 0$ positive real double pole in RHS			
7 $p_1, p_2 > 0$ positive real poles in RHS			↑ monotonic

Overdamped 2nd order system

z =
1.5000
p =
-10.4721
-1.5279

=====

Critically damped 2nd order system

z =
1
p =
-4
-4

=====

Underdamped 2nd order system

z =
0.0250
p =
-0.1000 + 3.9987i
-0.1000 - 3.9987i

=====

Undamped 2nd order system

z =
0
p =
0.0000 + 4.0000i
0.0000 - 4.0000i

=====

Negatively underdamped 2nd order system

z =
-0.0150
p =
0.0600 + 3.9995i
0.0600 - 3.9995i

=====

Negatively critically damped 2nd order system

z =
-1
p =
4
4

=====

Negatively overdamped 2nd order system

z =
-1.1500
p =
6.8716
2.3284

Stability - 2nd-order - sys. m

C:\Mydata\1_USC\courses\EMCH516\EMCH516 Core\EMC.

```
1 %{
2 examples of 2nd order system stability
3 %}
4 %% initialization
5 clc
6 clear
7 format compact
8 %% initial data
9 wn=4; % natural frequency wn, rad/sec
10 %% Overdamped 2nd order system
11 display('Overdamped 2nd order system')
12 z=150e-2 % damping z%
13 A=[1 2*z*wn wn^2];
14 p=roots(A)
15 display('=====')
16 %% Critically damped 2nd order system
17 display('Critically damped 2nd order system')
18 z=100e-2 % damping z%
19 A=[1 2*z*wn wn^2];
20 p=roots(A)
21 display('=====')
22 %% Underdamped 2nd order system
23 display('Underdamped 2nd order system')
24 z=2.5e-2 % damping z%
25 A=[1 2*z*wn wn^2];
26 p=roots(A)
27 display('=====')
28 %% Undamped 2nd order system
29 display('Undamped 2nd order system')
30 z=0 % damping z%
31 A=[1 2*z*wn wn^2];
32 p=roots(A)
33 display('=====')
34 %% Negatively underdamped 2nd order system
35 display('Negatively underdamped 2nd order system')
36 z=-1.5e-2 % damping z%
37 A=[1 2*z*wn wn^2];
```

C:\Mydata\1_USC\courses\EMCH516\EMCH516 Core\EMC.

```
38 p=roots(A)
39 display('=====)
40 %% Negatively critically damped 2nd order system
41 display('Negatively critically damped 2nd order system)
42 z=-100e-2 % damping z%
43 A=[1 2*z*wn wn^2];
44 p=roots(A)
45 display('=====)
46 %% Negatively overdamped 2nd order system
47 display('Negatively overdamped 2nd order system)
48 z=-115e-2 % damping z%
49 A=[1 2*z*wn wn^2];
50 p=roots(A)
51 display('=====)
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