2016 11 29

Time Response Calculation

$$X(s) = G(s)\,F(s)$$

Response LT — Transfer function — forcing function LT

$$x(t) = \mathcal{L}^{-1} X(s)$$

Step 

$$F(s) = \frac{1}{s}$$

$$x(t) = \mathcal{L}^{-1} G(s)\frac{1}{s}$$

MATLAB : $x = \mathrm{step}(G)$

---

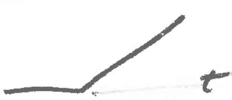
impulse 

$$F(s) = 1$$

$$x(t) = \mathcal{L}^{-1} G(s)$$

MATLAB : $x = \mathrm{impulse}(G)$

---

ramp 

$$F(s) = \frac{1}{s^2}$$

$$x(t) = \mathcal{L}^{-1} G(s)\frac{1}{s^2}$$

MATLAB: $x = \mathrm{impulse}(G/s^2)$

Note : we used the property

$$\underbrace{X(s) = G(s)\frac{1}{s^2}}_{\text{ramp LT}} = \left(G(s)/s^2\right)\cdot \underbrace{1}_{\text{impulse LT}}$$

1st Order System Time Response

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1^{st} order System time response

$$T\dot{x}(t) + x(t) = f(t), \quad x(0) = 0 \qquad (1)$$

time constant

Laplace transform

$$x \to X(s)$$
$$\dot{x} \to sX(s) \qquad (2)$$
$$f(t) \to F(s)$$

(2) → (1):

$$TsX(s) + X(s) = F(s) \qquad \text{s-domain equation} \qquad (3)$$

$$X(s) = \frac{F(s)}{Ts+1} = \frac{1}{Ts+1} F(s) \qquad (4)$$

$$G(s) = \frac{1}{Ts+1} \qquad \text{transfer function} \qquad (5)$$

$$X(s) = G(s)\, F(s) \qquad (6)$$

Transfer functⁿ

$$\underset{\text{input}}{F(s)} \longrightarrow \boxed{G = \frac{1}{Ts+1}} \longrightarrow \underset{\text{output}}{X(s)}$$

Typical inputs:

$F(s) = \frac{1}{s}$  step

$F(s) = 1$  impulse

$F(s) = \frac{1}{s^2}$  ramp.

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Step response of 1st order system

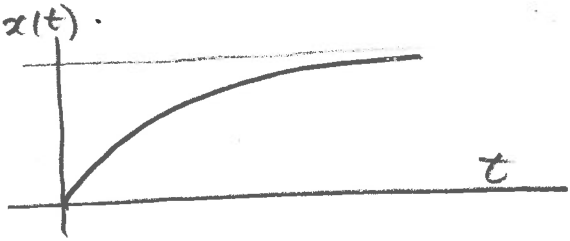
$$\begin{cases} f(t) = 1(t) \\ F(s) = \dfrac{1}{s} \end{cases} \qquad (1)$$

$$X(s) = G(s) \cdot F(s)$$

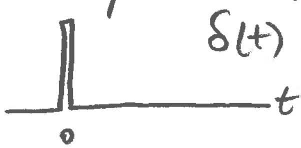
$$X(s) = \frac{1}{Ts+1} \cdot \frac{1}{s} = \frac{1}{s(Ts+1)}. \qquad (2)$$

ILT (Table 2.1, #14):

$$x(t) = 1 - e^{-t/T}$$



Impulse response of 1st order system

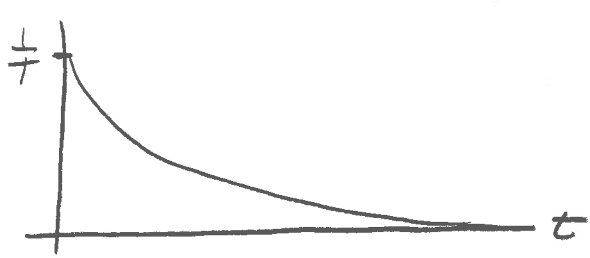


$$\begin{cases} f(t) = \delta(t) \\ F(s) = 1 \end{cases} \qquad (1)$$

$$X(s) = \frac{1}{Ts+1} \cdot 1 = \frac{1}{Ts+1}$$

ILT (Table 2.1, #6)

$$x(t) = \frac{1}{T} e^{-t/T}$$



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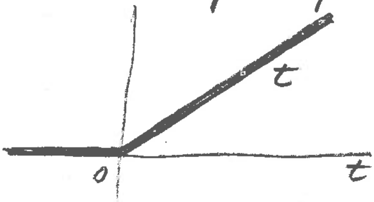
Ramp response of 1st order system



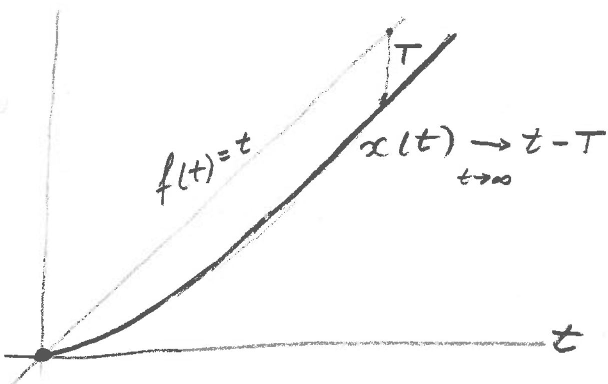
$$\begin{cases} f(t) = t \\ F(s) = \frac{1}{s^2} \end{cases} \quad (1)$$

$$X(s) = \frac{1}{Ts+1} \cdot \frac{1}{s^2} = \frac{1}{s^2(Ts+1)} \quad (2)$$

ILT (Table 2.1, #19)

$$x(t) = t - T + Te^{-t/T}$$

$$x(t) = t - T(1 - e^{-t/T}) \quad (3)$$



$$x(t) = t - T + Te^{-t/T} \xrightarrow[t\to\infty]{} t - T \quad (4)$$

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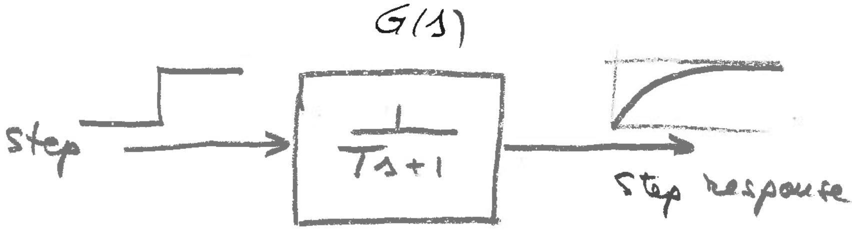
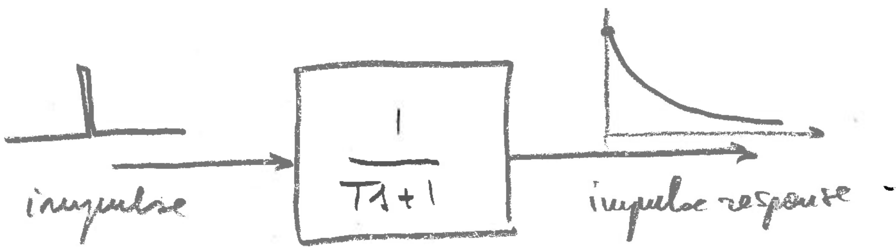
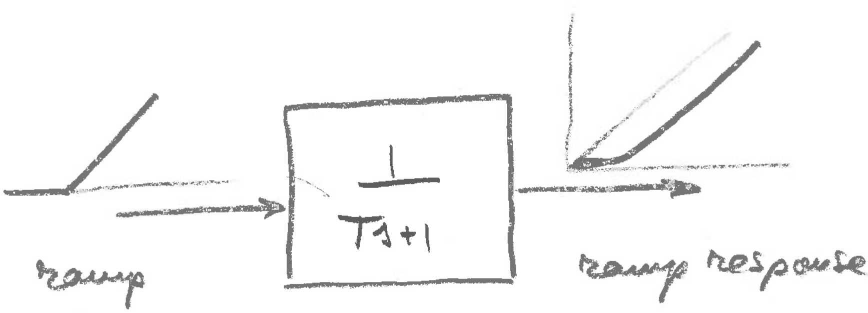
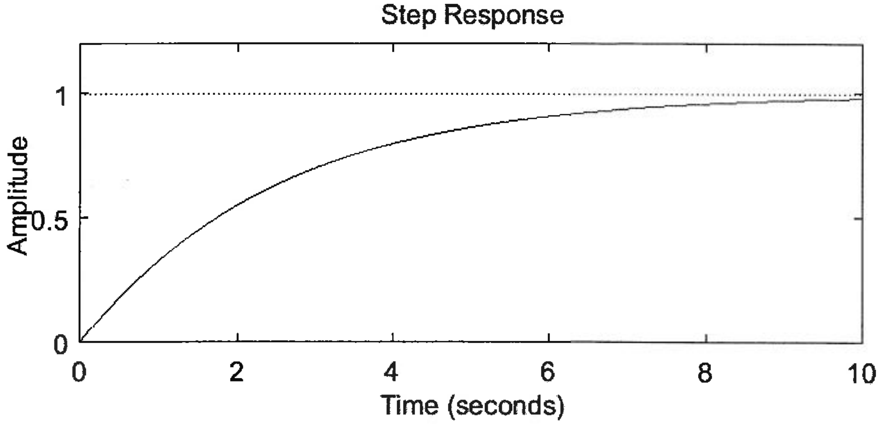
SUMMARY

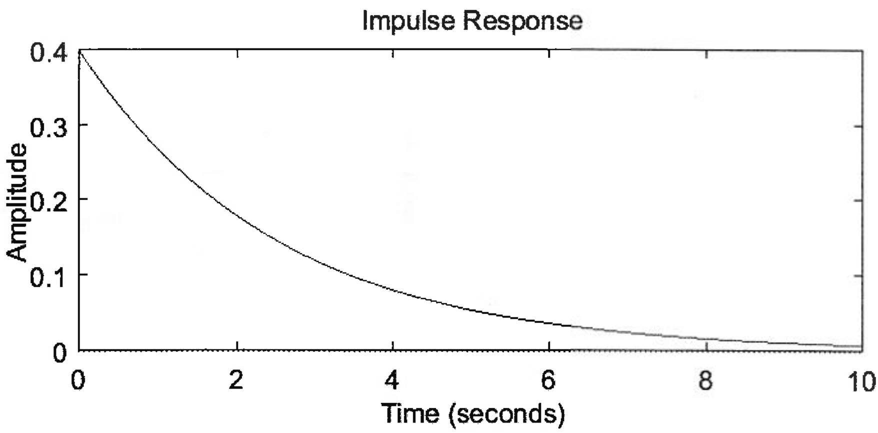
1st order system response

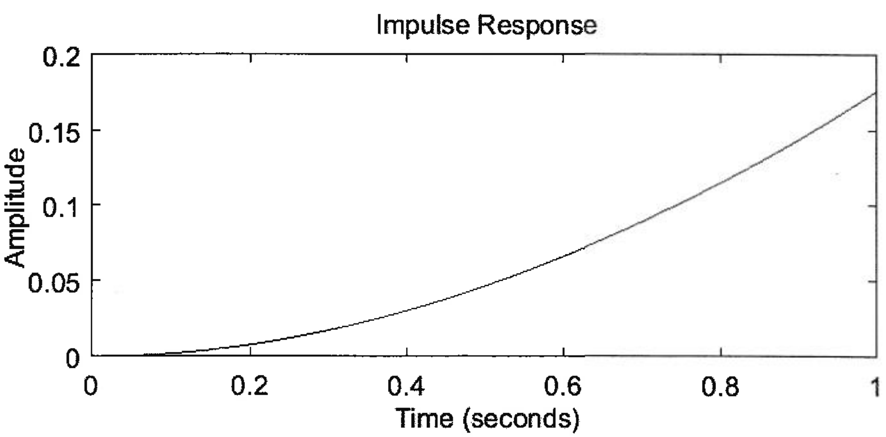







Step Response
Amplitude
1
0.5
0
0
2
4
6
8
10
Time (seconds)


Impulse Response
Amplitude
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
Time (seconds)


Impulse Response
Amplitude
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
Time (seconds)

```
%{
% This program studies time response of 1st order systems
%}
%% Initialization
clc
clear
% close all
format compact
%% Given data
T=2.5;    % time response
%% time range setup
Tmax=10;
dt=Tmax*1e-4; t=0:dt:Tmax;  % time range
%% Define system
B=[1]; A=[T 1];  G=tf(B,A)
%% Step response
figure(1)
subplot(3,1,1)
step(G,t)
ylim([0 1.2])
%% Impulse response
subplot(3,1,2)
impulse(G,t)
%% Ramp response
F_ramp=tf([1],[1 0 0])
subplot(3,1,3)
impulse(G*F_ramp,t)
xlim([0 1])
```

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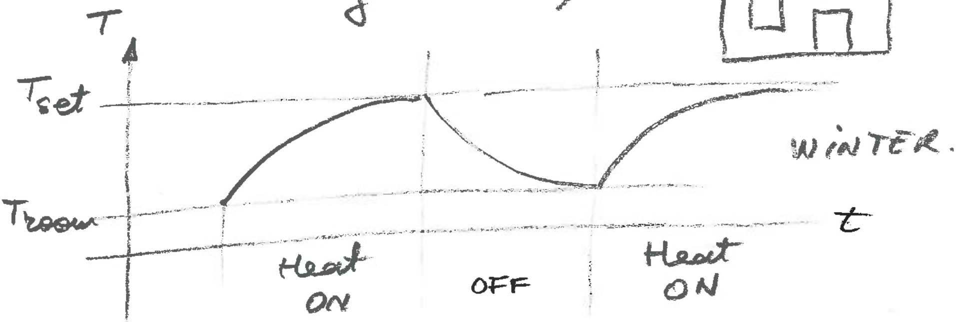
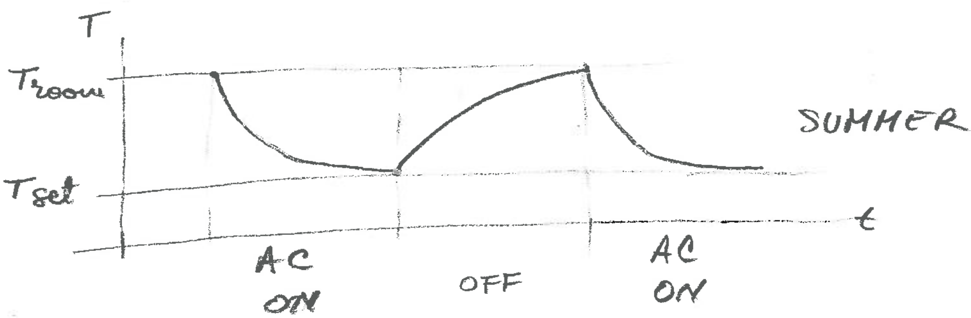
Other 1^{st} Order Systems in the Physical World

• Heating – Cooling





• Radioactive decay : half life time