**TABLE 2–1** Laplace Transform Pairs

| | $f(t)$ | $F(s)$ |
|---|---|---|
| 1 | Unit impulse $\delta(t)$ | $1$ |
| 2 | Unit step $1(t)$ | $\frac{1}{s}$ |
| 3 | $t$ | $\frac{1}{s^2}$ |
| 4 | $\frac{t^{n-1}}{(n-1)!}$ $(n = 1, 2, 3, \ldots)$ | $\frac{1}{s^n}$ |
| 5 | $t^n$ $(n = 1, 2, 3, \ldots)$ | $\frac{n!}{s^{n+1}}$ |
| 6 | $e^{-at}$ \| $\frac{1}{T}e^{-t/T}$ \| $e^{pt}$ | $\frac{1}{s+a}$ \| $\frac{1}{Ts+1}$ \| $\frac{1}{s-p}$ |
| 7 | $te^{-at}$ | $\frac{1}{(s+a)^2}$ |
| 8 | $\frac{1}{(n-1)!}t^{n-1}e^{-at}$ $(n = 1, 2, 3, \ldots)$ | $\frac{1}{(s+a)^n}$ |
| 9 | $t^n e^{-at}$ $(n = 1, 2, 3, \ldots)$ | $\frac{n!}{(s+a)^{n+1}}$ |
| 10 | $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ |
| 11 | $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ |
| 12 | $\sinh \omega t$ | $\frac{\omega}{s^2-\omega^2}$ |
| 13 | $\cosh \omega t$ | $\frac{s}{s^2-\omega^2}$ |
| 14 | $\frac{1}{a}(1-e^{-at})$ \| $(1-e^{-t/T})$ | $\frac{1}{s(s+a)}$ \| $\frac{1}{s(Ts+1)}$ |
| 15 | $\frac{1}{b-a}(e^{-at}-e^{-bt})$ | $\frac{1}{(s+a)(s+b)}$ |
| 16 | $\frac{1}{b-a}(be^{-bt}-ae^{-at})$ | $\frac{s}{(s+a)(s+b)}$ |
| 17 | $\frac{1}{ab}\left[1+\frac{1}{a-b}(be^{-at}-ae^{-bt})\right]$ | $\frac{1}{s(s+a)(s+b)}$ |

$t - T(1 - e^{-t/T})$

**TABLE 2–1** *(continued)*

| | $f(t)$ | $F(s)$ |
|---|---|---|
| 18 | $\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$ | $\frac{1}{s(s + a)^2}$ |
| 19 | $\frac{1}{a^2}(at - 1 + e^{-at})$ | $\frac{1}{s^2(s + a)}$ |
| 20 | $e^{-at} \sin \omega t$ | $\frac{\omega}{(s + a)^2 + \omega^2}$ |
| 21 | $e^{-at} \cos \omega t$ | $\frac{s + a}{(s + a)^2 + \omega^2}$ |
| 22 | $\frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2}\, t$ | $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ |
| 23 | $-\frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t - \phi)$ <br> $\phi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta}$ | $\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ |
| 24 | $1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t + \phi)$ <br> $\phi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta} = \sin^{-1} \sqrt{1 - \zeta^2}$ | $\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$ |
| 25 | $1 - \cos \omega t$ | $\frac{\omega^2}{s(s^2 + \omega^2)}$ |
| 26 | $\omega t - \sin \omega t$ | $\frac{\omega^3}{s^2(s^2 + \omega^2)}$ |
| 27 | $\sin \omega t - \omega t \cos \omega t$ | $\frac{2\omega^3}{(s^2 + \omega^2)^2}$ |
| 28 | $\frac{1}{2\omega} t \sin \omega t$ | $\frac{s}{(s^2 + \omega^2)^2}$ |
| 29 | $t \cos \omega t$ | $\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$ |
| 30 | $\frac{1}{\omega_2^2 - \omega_1^2}(\cos \omega_1 t - \cos \omega_2 t) \qquad (\omega_1^2 \neq \omega_2^2)$ | $\frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$ |
| 31 | $\frac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$ | $\frac{s^2}{(s^2 + \omega^2)^2}$ |

$\frac{1}{s^2(Ts+1)}$

Impulse

$\frac{s^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$

step

**TABLE 2–2** Properties of Laplace Transforms

| | |
|---|---|
| 1 | $\mathcal{L}[Af(t)] = AF(s)$ |
| 2 | $\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$ |
| 3 | $\mathcal{L}_{\pm}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$ |
| 4 | $\mathcal{L}_{\pm}\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$ |
| 5 | $\mathcal{L}_{\pm}\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k} f^{(k-1)}(0\pm)$ where $f^{(k-1)}(t) = \frac{d^{k-1}}{dt^{k-1}}f(t)$ |
| 6 | $\mathcal{L}_{\pm}\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \frac{[\int f(t)\,dt]_{t=0\pm}}{s}$ |
| 7 | $\mathcal{L}_{\pm}\left[\iint f(t)\,dt\,dt\right] = \frac{F(s)}{s^2} + \frac{[\int f(t)\,dt]_{t=0\pm}}{s^2} + \frac{[\iint f(t)\,dt\,dt]_{t=0\pm}}{s}$ |
| 8 | $\mathcal{L}_{\pm}\left[\int \cdots \int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^{n} \frac{1}{s^{n-k+1}}\left[\int \cdots \int f(t)(dt)^k\right]_{t=0\pm}$ |
| 9 | $\mathcal{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$ |
| 10 | $\int_0^\infty f(t)\,dt = \lim_{s\to 0} F(s)$ if $\int_0^\infty f(t)\,dt$ exists |
| 11 | $\mathcal{L}[e^{-at}f(t)] = F(s + a)$ |
| 12 | $\mathcal{L}[f(t - \alpha)1(t - \alpha)] = e^{-\alpha s}F(s) \quad \alpha \geq 0$ |
| 13 | $\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$ |
| 14 | $\mathcal{L}[t^2f(t)] = \frac{d^2}{ds^2}F(s)$ |
| 15 | $\mathcal{L}[t^nf(t)] = (-1)^n\frac{d^n}{ds^n}F(s) \quad n = 1, 2, 3, \ldots$ |
| 16 | $\mathcal{L}\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s)\,ds$ if $\lim_{t\to 0}\frac{1}{t}f(t)$ exists |
| 17 | $\mathcal{L}\left[f\left(\frac{t}{a}\right)\right] = aF(as)$ |