ID 1
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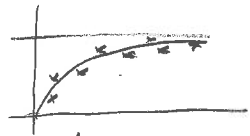
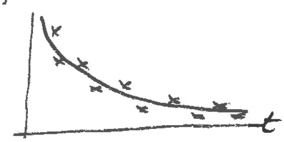
System ID for 1st order systems

Given: experimental signal



step response



Impulse response

Find: System parameters

TF: $G(s) = \dfrac{1}{Ts+1}$; T = time const. 1 parameter to find

Solution

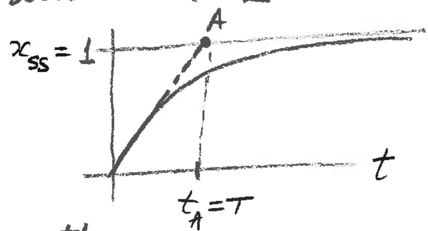
Option 1 : Curve fitting (optimization)

$$x(t,T) = 1 - e^{-t/T}$$

$$\left\{ \begin{array}{l} x_{exp} = x_1, x_2, x_3, \ldots \\ t_{exp} = t_1, t_2, t_3, \ldots \end{array} \right.$$

use curve fitting software to find T

Option 2 Graphical methods (quick estimates)



• tangent in origin

$$x(t) = 1 - e^{-t/T}$$

$$\dot{x} = \frac{dx}{dt} = \left(-\frac{1}{T}\right)\left(-e^{-t/T}\right) = \frac{1}{T}e^{-t/T}$$

$$\dot{x}_0 = \left.\frac{dx}{dt}\right|_{t=0} = \frac{1}{T}$$; tangent in origin $y(t) = \dot{x}_0 t = \frac{1}{T} t$

$y(t)$ intersects $x_{ss} = 1$ at $\frac{1}{T} t_A = 1$

$$\boxed{T = t_A}$$

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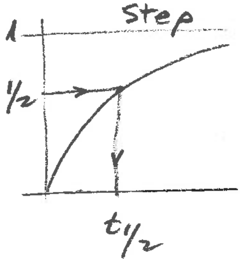
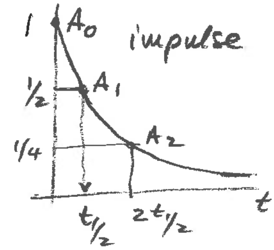
• Half time





Step

$$x = 1 - e^{-t/T}$$

$$x(t_{1/2}) = 1 - e^{-t_{1/2}/T} = \frac{1}{2}$$

$$e^{-t_{1/2}/T} = \frac{1}{2} \longrightarrow \frac{t_{1/2}}{T} = \ln 2$$

$$T = \frac{t_{1/2}}{\ln 2} = \frac{t_{1/2}}{0.693} \approx 1.4\, t_{1/2}$$

$$\boxed{T \approx 1.4\, t_{1/2}}$$

Impulse

$$x = \frac{1}{T} e^{-t/T}$$

$$x(t_{1/2}) = \frac{1}{T} e^{-t_{1/2}/T} = \frac{1}{2}$$

$$T = \frac{t_{1/2}}{\ln 2} \approx 1.4\, t_{1/2}$$

May use consecutive points A_1, A_2 if A_0 not easy to determine. Important is that signal halves between A_1 and A_2

4.4 ESTIMATE SYSTEM PARAMETERS FROM MEASURED PERFORMANCE INDICATORS

4.4.1 Estimation of time period T.

Two methods can be used to estimate the time period:

Method 1: estimate time period using t_d, i.e.,

$t_d = -T \ln 0.5$ and hence $T = -\frac{t_d}{\ln 0.5}$

This method gives $T_1 = 2.513 \text{ sec}$ with an error $\Delta T_1^{\exp} = -0.53\%$

Method 2: estimate time period using t_s

$t_s^{2\%} = -T \ln 0.02$ and hence $T = -\frac{t_s^{2\%}}{\ln 0.02}$

This method gives $T_2 = 2.4985 \text{ sec}$ with an error $\Delta T_2^{\exp} = -0.06\%$

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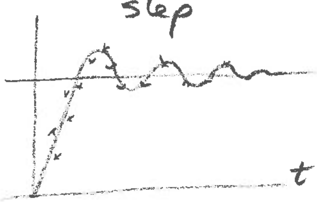
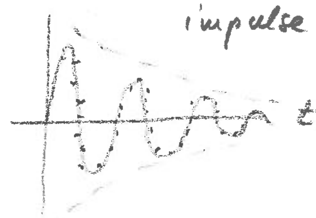
System ID for 2^{nd} order systems

Given: experimental signal





Find: System parameters

$$TF: \quad G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2 parameters to be found: ω_n – natural frequency; ζ – damping ratio

Solution

Option 1: Curve fitting (optimization)

$$x(t;\omega_n,\zeta) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \varphi)$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}, \quad \varphi = \sin^{-1}\sqrt{1-\zeta^2}$$

$$\begin{cases} x_{exp} = x_1, x_2, x_3, \dots \\ t_{exp} = t_1, t_2, t_3, \dots \end{cases}$$

- Use curve fitting software
- 2 unknowns to be determined: ω_n & ζ

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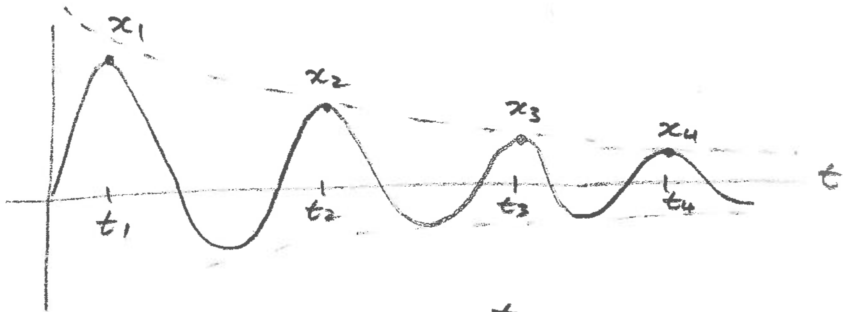
Option 2 Graphical methods (quick estimates)

Impulse response analysis for $\zeta << 1$



Logarithmic decrement

$$x(t) = x_0 e^{-\zeta\omega_n t} \sin \omega_d t \, , \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \approx \omega_n$$
$\zeta << 1$

$\omega_d t_1 = \frac{\pi}{2}$ $\qquad x_1 = x(t_1) = x_0 e^{-\zeta\frac{\pi}{2}}$

$\omega_d t_2 = 2\pi + \frac{\pi}{2}$ $\qquad x_2 = x(t_2) = x_0 e^{-\zeta(2\pi+\frac{\pi}{2})}$

⋮

$\omega_d t_n = (n-1)2\pi + \frac{\pi}{2}$ $\qquad x_n = x(t_n) = x_0 e^{-\zeta[(n-1)2\pi+\frac{\pi}{2}]}$

$$\frac{x_1}{x_n} = \frac{x_0 e^{-\zeta\frac{\pi}{2}}}{x_0 e^{-\zeta[(n-1)2\pi+\frac{\pi}{2}]}} = e^{(n-1)2\pi\zeta}$$

Logarithmic decrement $\ln \frac{x_1}{x_n} = (n-1)2\pi\zeta$

$$\zeta = \frac{1}{2\pi(n-1)} \ln\frac{x_1}{x_n}$$ damping ratio

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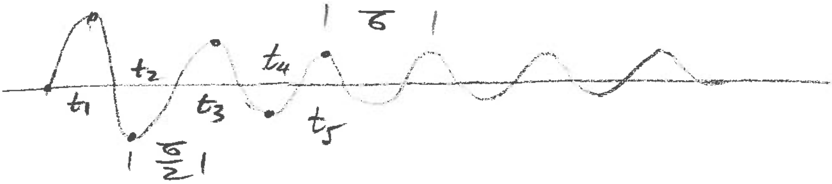
Frequency estimation

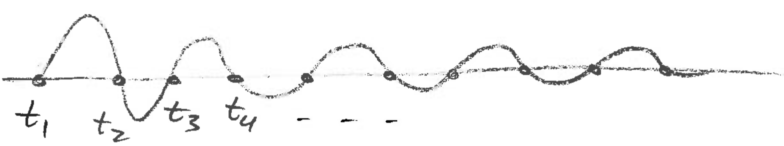
- Peak detection



Average half period:

$$\frac{\tau}{2} = \text{avg}\left[(t_2 - t_1), (t_3 - t_2), \cdots\right]$$

- zero crossing detection



$$\frac{\tau}{2} = \text{avg}\left[(t_2 - t_1), (t_3 - t_2), \cdots\right]$$

$$f_d = \frac{2}{\tau} \qquad \omega_d = 2\pi f_d.$$

$$f_n = \frac{f_d}{\sqrt{1-\zeta^2}}$$

$$\omega_n = 2\pi f_n$$

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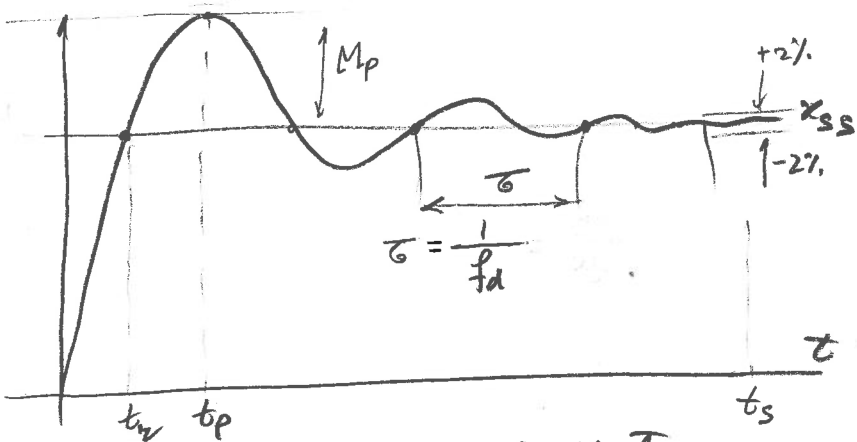
Step response analysis ($\zeta \ll 1$)



Recall performance indicators:

$$t_r = \frac{\pi - \varphi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi - \varphi}{\omega_d}$$

$\omega_d = \omega_n \sqrt{1-\zeta^2}$

$\varphi = \sin^{-1}\sqrt{1-\zeta^2}$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$

$$M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} = \frac{x_p - x_{ss}}{x_{ss}} 100\%$$

$$t_s = \frac{4}{\zeta \omega_n}$$

There are only 2 unknowns: ω_n, ζ

There is more information than minimally required: $t_r, t_p, M_p, t_s, \ldots$

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ζ, ω_n from t_r, t_p

$$t_p = \frac{\pi}{\omega_d} \longrightarrow \omega_d = \frac{\pi}{t_p} \qquad (1)$$

$$t_r = \frac{\pi - \varphi}{\omega_d} \longrightarrow \varphi = \pi - t_r \omega_d$$

$$= \pi - \frac{t_r}{t_p}\pi$$

$$\varphi = \pi\left(1 - \frac{t_r}{t_p}\right) \qquad (2)$$

Recall $\varphi = \sin^{-1}\sqrt{1-\zeta^2}$

$$1 - \zeta^2 = \sin^2\varphi$$

$$\zeta = \sqrt{1 - \sin^2\varphi} \qquad (3)$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} \qquad (4)$$

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ζ from M_p

$$M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}$$

$$-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi = \ln M_p$$

$$\zeta^2\pi^2 = (1-\zeta^2)(\ln M_p)^2$$

$$\zeta^2\pi^2 = (\ln M_p)^2 - \zeta^2(\ln M_p)^2$$

$$\zeta^2\left[\pi^2 + (\ln M_p)^2\right] = (\ln M_p)^2$$

$$\zeta^2 = \frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}$$

$$\zeta = \frac{|\ln M_p|}{\sqrt{\pi^2 + (\ln M_p)^2}}$$

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ζ, ω_n from t_p, t_s

$$t_p = \frac{\pi}{\omega_d} \longrightarrow \omega_d = \frac{\pi}{t_p} \qquad (1)$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{\pi}{t_p\sqrt{1-\zeta^2}}$$

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4\, t_p\sqrt{1-\zeta^2}}{\zeta\ \pi}$$

$$\zeta\pi t_s = 4\, t_p\sqrt{1-\zeta^2}$$

$$\zeta^2\pi^2 t_s^2 = 16 t_p^2(1-\zeta^2).$$

$$(16\, t_p^2 - \pi^2 t_s^2)\,\zeta^2 = 16\, t_p^2$$

$$\zeta^2 = \frac{16 t_p^2}{16 t_p^2 - \pi^2 t_s^2} = \frac{1}{1-\left(\frac{\pi}{4}\frac{t_s}{t_p}\right)^2}$$

$$\boxed{\zeta = \frac{1}{\sqrt{1-\left(\frac{\pi}{4}\frac{t_s}{t_p}\right)^2}}} \qquad (2)$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \boxed{\frac{\pi/t_p}{\sqrt{1-\zeta^2}}} \qquad (3)$$

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ζ, ω_n from M_p, t_r

Use M_p to get $\zeta = \dfrac{|\ln M_p|}{\sqrt{\pi^2 + (\ln M_p)^2}}$ (1)

Calculate $\varphi = \sin^{-1}\sqrt{1-\zeta^2}$ (2)

Recall $t_r = \dfrac{\pi - \varphi}{\omega_d} = \dfrac{\pi - \varphi}{\omega_n\sqrt{1-\zeta^2}}$ (3)

Solve (3): $\omega_n = \dfrac{\pi - \varphi}{t_r\sqrt{1-\zeta^2}}$ (4)

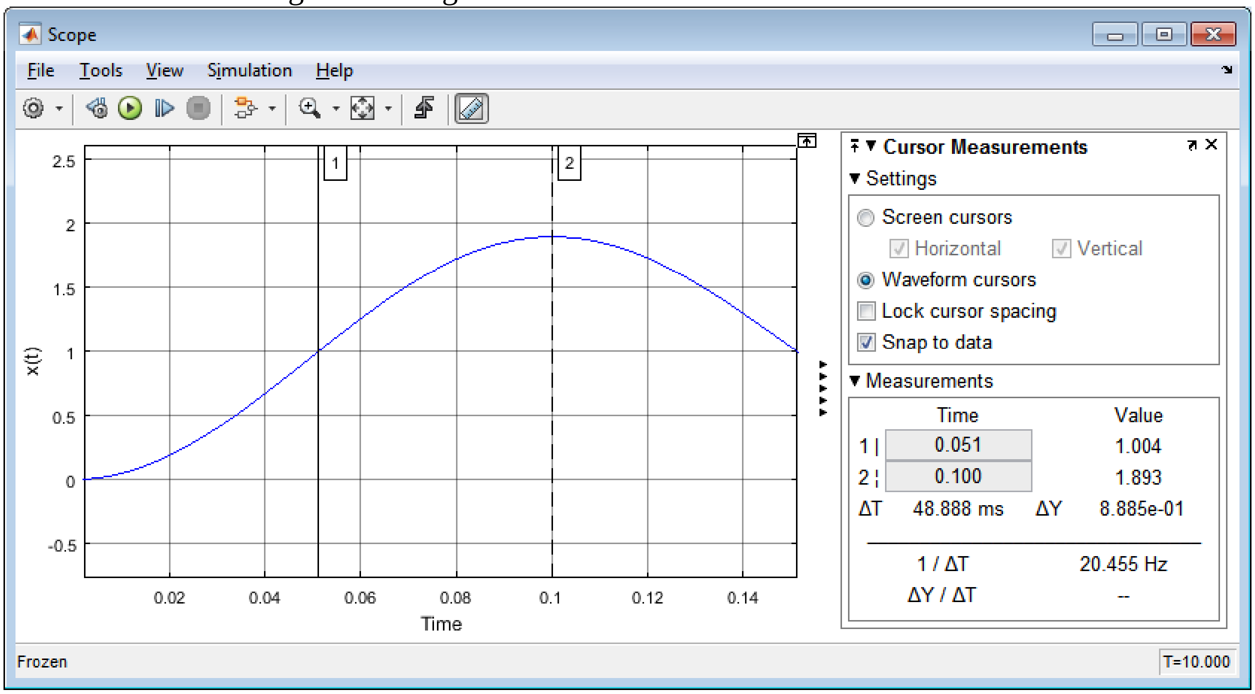
where φ is given by Eq. (2).

4.4 Use Trace to Measure Performance Indicators

Release the 'Zoom' button and press the 'Cursor Measurements' button to activate the cursors. In the 'Cursor Measurements → Settings' check 'Snap to data' box.

4.4.1 Measurement of t_r, t_p, x_p, M_p

Zoom closer to the origin in the region $t < 0.15$.



Use the first cursor to find the first crossing of $x_{ss} = 1$. Read the time as $t_r = 0.051$ sec

Place the second cursor at peak value. Read the peak time $t_p = 0.100$ sec and peak amplitude $x_p = 1.893$. Calculate $M_p = 89.3\%$.

4.5 Estimate System Parameters from Measured Performance Indicators and Other Measurements

4.5.1 Estimation of Damping Ratio ζ

Two methods can be used to estimate the damping ratio:

<u>Method 1</u>: estimate damping ratio using t_r, t_p, i.e.,

$$\varphi = \pi\left(1 - t_r / t_p\right) \text{ and } \zeta = \sqrt{1 - \sin\varphi^2}$$

This method gives $\zeta_1^{\exp} = 3.1\%$ with an error $\Delta\zeta_1^{\exp} = 10.3\%$

<u>Method 2</u>: estimate damping ratio using M_p

$$\zeta = \frac{\left|\ln M_p\right|}{\sqrt{\pi^2 + (\ln M_p)^2}}$$

This method gives $\zeta_2^{\exp} = 3.6\%$ with an error $\Delta\zeta_2^{\exp} = -2.9\%$

4.5.2 Natural frequency estimation

Place the first cursor at the second rising crossing of $x_{ss} = 1$; place the second cursor at next rising crossing of x_{ss}. Try to get as close as possible to the value $x_{ss} = 1$. The 'Cursor Measurements' shows $\Delta T = 200.100\,\text{ms}$ with a corresponding frequency $f_d^{\exp} = 4.998\ \text{Hz}$, which is close to the theoretical damped frequency $f_d = 4.9969\ \text{Hz}$.

The frequency estimation error is $\Delta f = -0.02\%$