VFB

20160613

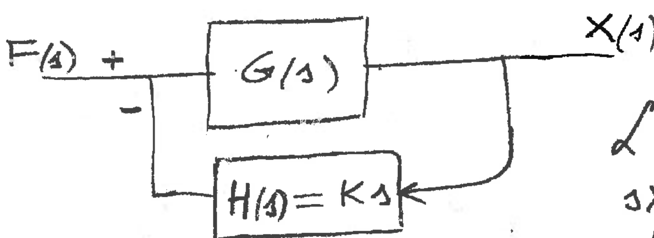
INSTABILITY SUPPRESSION WITH VELOCITY FEEBACK

Assume a 2nd order system with negative damping, $\zeta < 0$. Hence, the response will be unstable: $e^{-\zeta\omega_n t} = e^{|\zeta|\omega_n t}$.

$\zeta < 0$

$-\zeta\omega_n t = |\zeta|\omega_n$

We can use FB to suppress this instability.



$\mathcal{L}\,\dot{x} = sX$

sX is the LT of velocity!

$$G_{CL} = \frac{G}{1+GH} = \frac{\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}}{1+\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}Ks}$$

$$= \frac{\omega_n^2}{s^2+(2\zeta\omega_n+K\omega_n^2)s+\omega_n^2} \qquad (1)$$

$$G_{CL} = \frac{\omega_n^2}{s^2+2\zeta_{CL}\omega_n s+\omega_n^2} \qquad (2)$$

FB4
20181022 (1)&(2) :

$$2\zeta_{CL} = 2\zeta + K\omega_n$$

$$\zeta_{CL} = \zeta + \frac{K\omega_n}{2} \quad (3)$$

1) Velocity feedback increases damping.

2) If the initial system is unstable because it has -ve damping ($\zeta < 0$), then velocity feedback can give the CL system a positive damping, $\zeta_{CL} > 0$.

Solve Eq. (3) to get

$$K = \frac{2}{\omega_n}(\zeta_{CL} - \zeta) \quad (4)$$

Critical gain K_{cr} is the gain that makes $\zeta_{CL} = 0$.

Recall Eq (3) and set it to zero to get

$$\zeta + \frac{K\omega_n}{2} = 0 \longrightarrow K_{cr} = -\frac{2\zeta}{\omega_n} \quad (5)$$

<u>Example</u>:

$\zeta = -5\% = -0.05$ $\quad f_n = 4\ Hz$

$\omega_n = 2\pi \times 4 = 8\pi$ rad/s.

$$K_{cr} = -\frac{2\times(-0.05)}{8\pi} \cong 0.004$$

For $K > K_{cr}$ the system is stable

FB4a
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Calculation of FB gain K

Recall (4) $K = \frac{2}{\omega_n}(\zeta_{CL} - \zeta)$

Factor out $-\zeta$ to get $K = -\frac{2\zeta}{\omega_n}\left(1 - \frac{\zeta_{CL}}{\zeta}\right)$

Recall (5) and write

$$K = K_{cr}\left(1 - \frac{\zeta_{CL}}{\zeta}\right) \qquad (6)$$

Define ζ_{ratio} as

$$\zeta_{ratio} = \frac{\zeta_{CL}}{\zeta} \qquad (7)$$

(7) → (6):

$$K = K_{cr}(1 - \zeta_{ratio}) \qquad (8)$$

Eq(8) gives the FB gain K as a function of ζ_{ratio}

Gain ratio K_{ratio}

Divide Eq(8) by K_{cr} to get

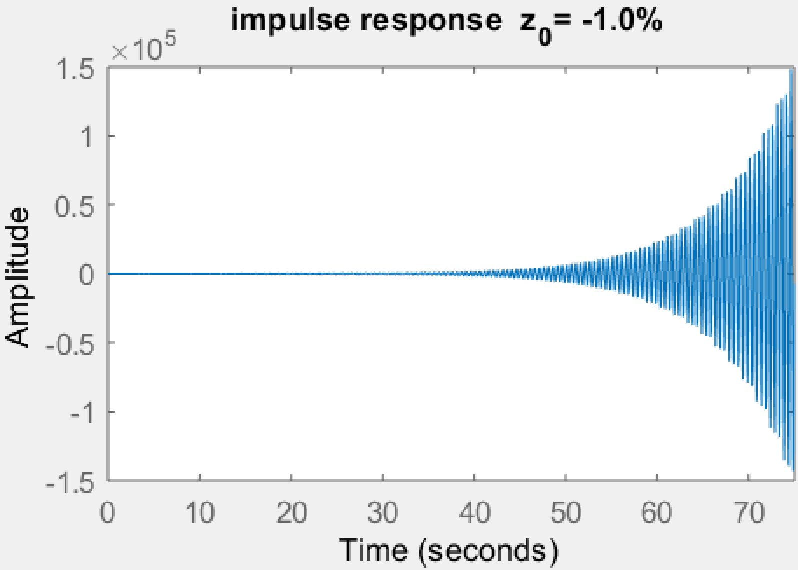
$$K_{ratio} = 1 - \zeta_{ratio} \qquad (9)$$

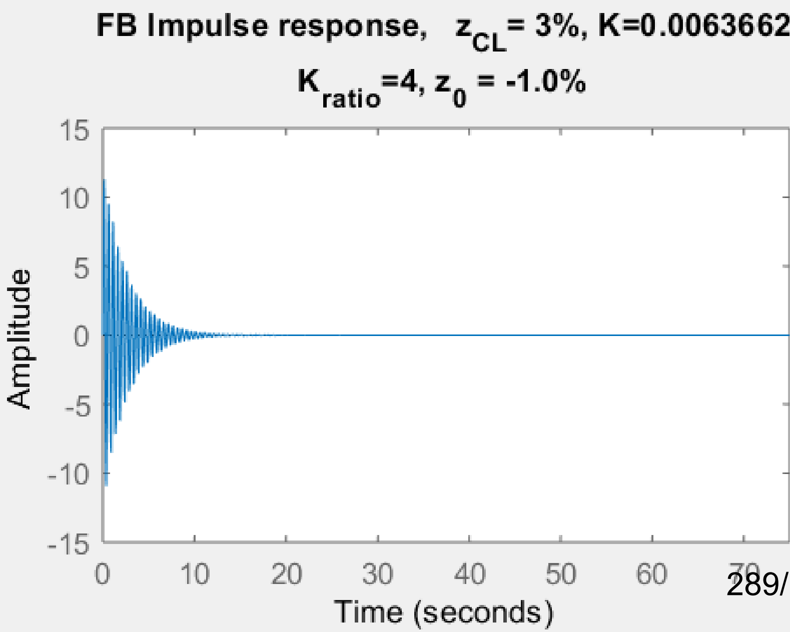
Discussion of Eq.(8)

(a) if $\zeta < 0$, then $\zeta_{ratio} < 0$, $K_{cr} > 0$ → $\boxed{K > 0}$

(b) if $0 < \zeta < \zeta_{CL}$, then $K_{cr} < 0$, $\zeta_{ratio} > 1$, $(1 - \zeta_{ratio}) < 0$, $\boxed{K > 0}$

(c) $\zeta_{CL} < \zeta$, then $K_{cr} < 0$, $\zeta_{ratio} < 1$, $1 - \zeta_{ratio} < 0$, $\boxed{K < 0}$
reduce ζ_{CL}


impulse response z_0= -1.0%
×10^5
1.5
1
0.5
0
-0.5
-1
-1.5
0 10 20 30 40 50 60 70
Amplitude
Time (seconds)


FB Impulse response, z_{CL}= 3%, K=0.0063662
K_{ratio}=4, z_0 = -1.0%
15
10
5
0
-5
-10
-15
0 10 20 30 40 50 60 70
Amplitude
Time (seconds)