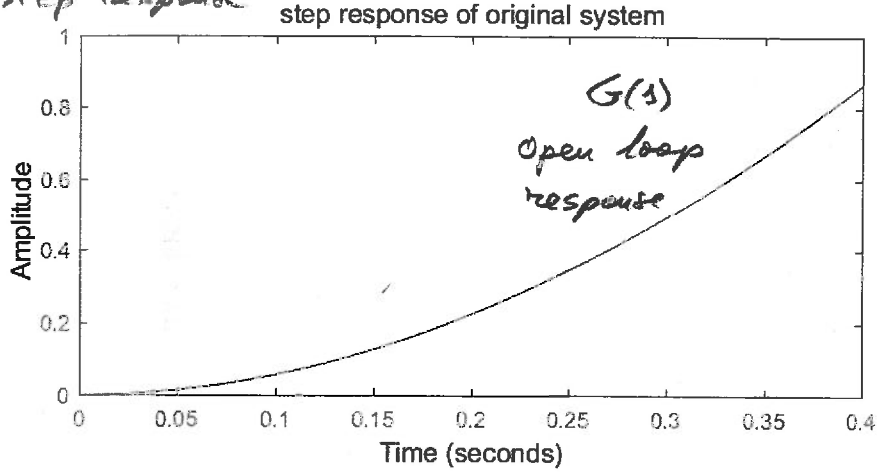
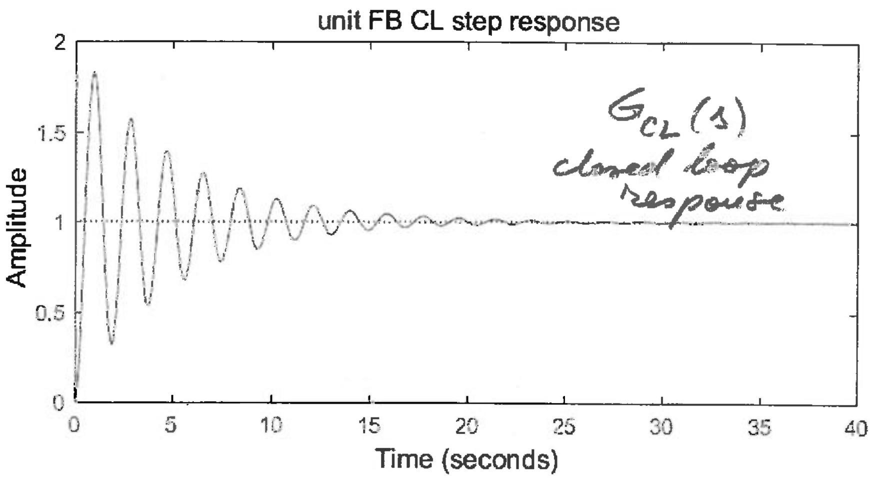
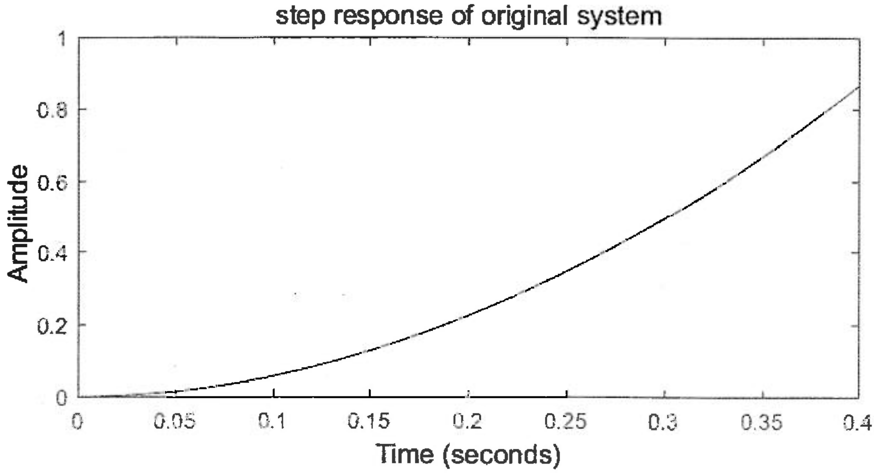
Aircraft roll step response

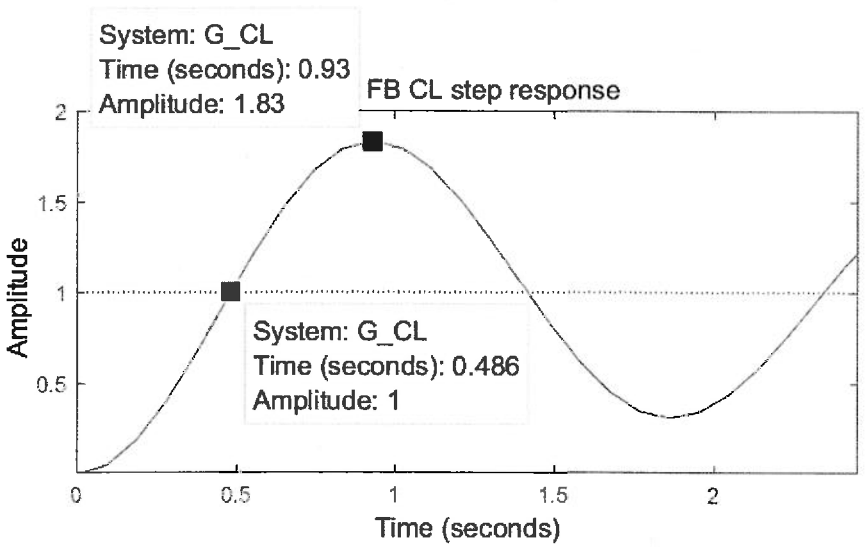
$$G(s) = \frac{114}{10s^2 + 4s}$$





D6c





Measured: $t_r = 0.486$ sec

$M_p = 83\%$

Design Specs

DS1: $t_r \leqslant 1.5$ sec

DS2: $M_p \leq 20\%$

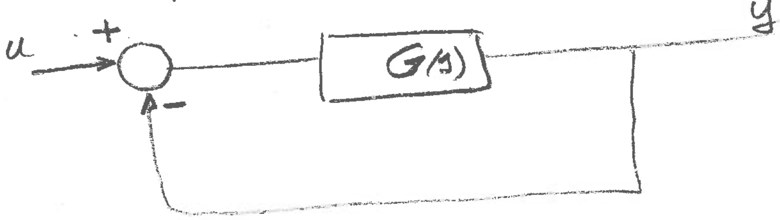
DS1: satisfied

DS2: NOT satisfied

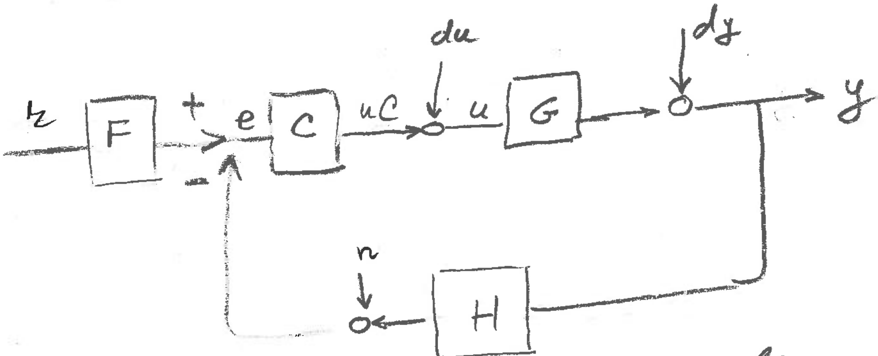
C
20140317

CONTROLLERS

Basic feedback system:



Enhanced FB control systems



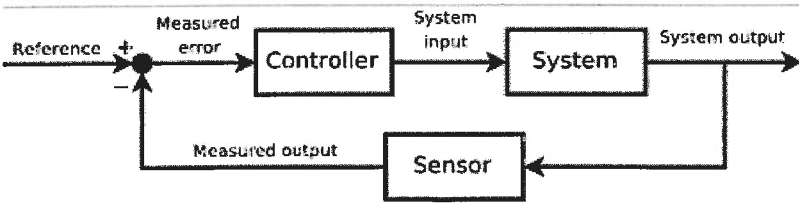
Addition "boxes" with specific transfer function may be added to improve performance

Examples:

- PID controllers
- Filters
- Compensators
- Pre-filters
- Post-filters

2
C

CONTROLLERS



Controllers modulate the feedback error to improve the performance of the feedback control system.

4c

Filters

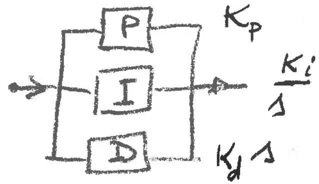
$$G_c(s) = \frac{1}{Ts+1} \quad \text{(low pass filter)}$$

$$G_c(s) = \frac{Ts}{Ts+1} \quad \text{(high pass filter)}$$

Compensators

$$G_c(s) = \frac{T_a s+1}{T_1 s+1}$$

PID controllers



$$\boxed{G_c = K_p + \frac{K_i}{s} + K_d s}$$

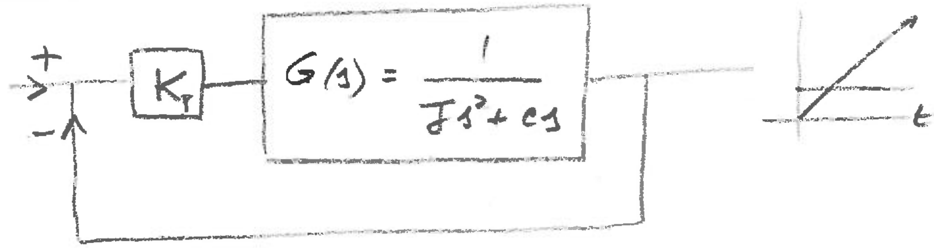
$$\boxed{G_c = K\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

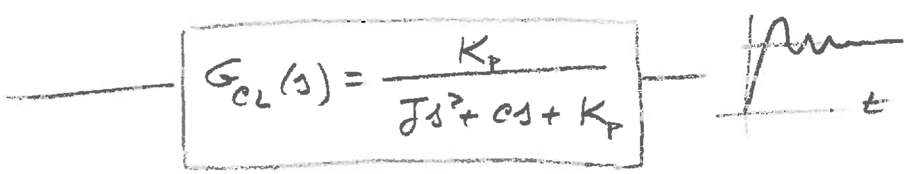
	t-domain	s-domain		comments	
P (proportional)	K	K		• simplest • adds stiffness, increases frequency	
I (integral)	$\int dt$	$\frac{K_i}{s}$	$\frac{1}{T_i s}$	• eliminates offsets • increases system order • may be unstable	"under compensator"
D (derivative)	d/dt	$K_d s$	$T_d s$	• adds damping • decreases system order • increases sensitivity • used as PD or PID	"anticipates & corrects the error"

Note: D control is not physically realizable. It is done as $\frac{Ns}{s+N} \xrightarrow[N \gg 1]{} s$

P control of Type 1 system

<u>Recall</u>:





$$G_{CL}(s) = \frac{K_p}{Js^2 + cs + K_p} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \frac{K_p}{J}, \quad \omega_n = \sqrt{\frac{K_p}{J}}$$

$$\zeta = \frac{c}{2\sqrt{JK_p}}$$

K = factor of proportionality ; "P-control"

Modification of K_p can modify frequency ω_n and damping ζ

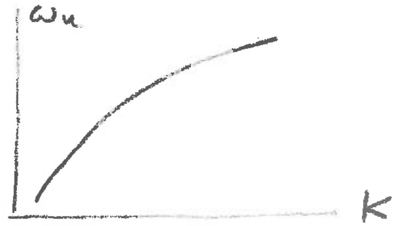
P

P-control of frequency and damping

K_p = factor of proportionality → P-control

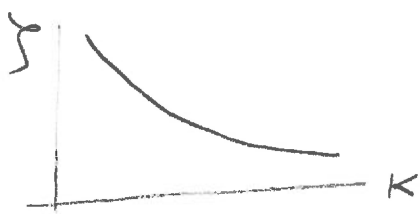
Modification of K_p can modify the frequency ω_n and the damping ζ

$$\omega_n^2 = \frac{K_p}{J}$$



Frequency increases with K

$$\zeta = \frac{c}{2\sqrt{J K_p}}$$

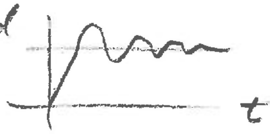


Damping decreases with K_p

Critical damping: $\zeta = 1 \rightarrow \frac{c}{2\sqrt{J K_{cr}}} = 1$: $K_{cr} = \frac{c^2}{4J}$

$0 < K_p \leq K_{cr}$. overdamped converging. exponential response.

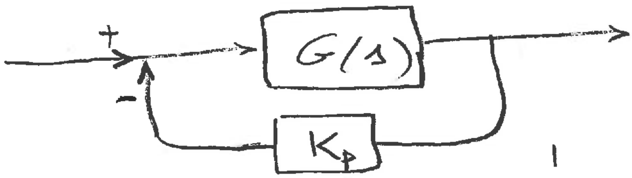
$K_{cr} < K_p$ underdamped oscillatory response

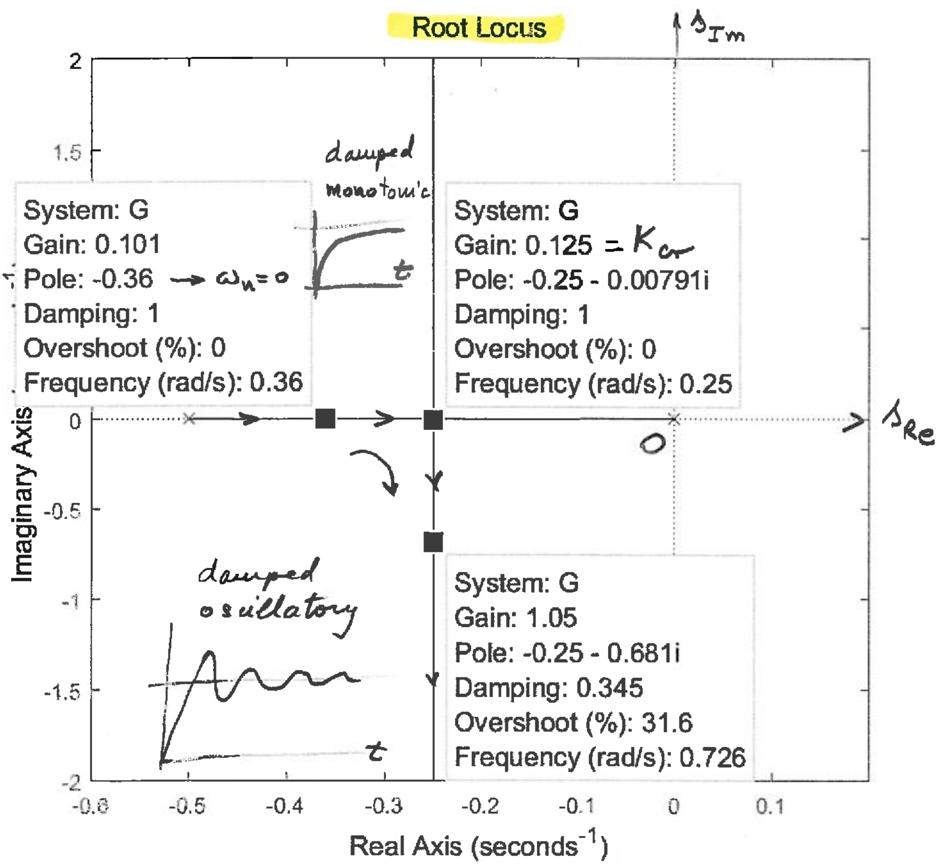


3 P

$$G(s) = \frac{1}{2s^2 + s}$$

$$K_{cr} = \frac{c^2}{4J} = \frac{1}{8} = 0.125$$

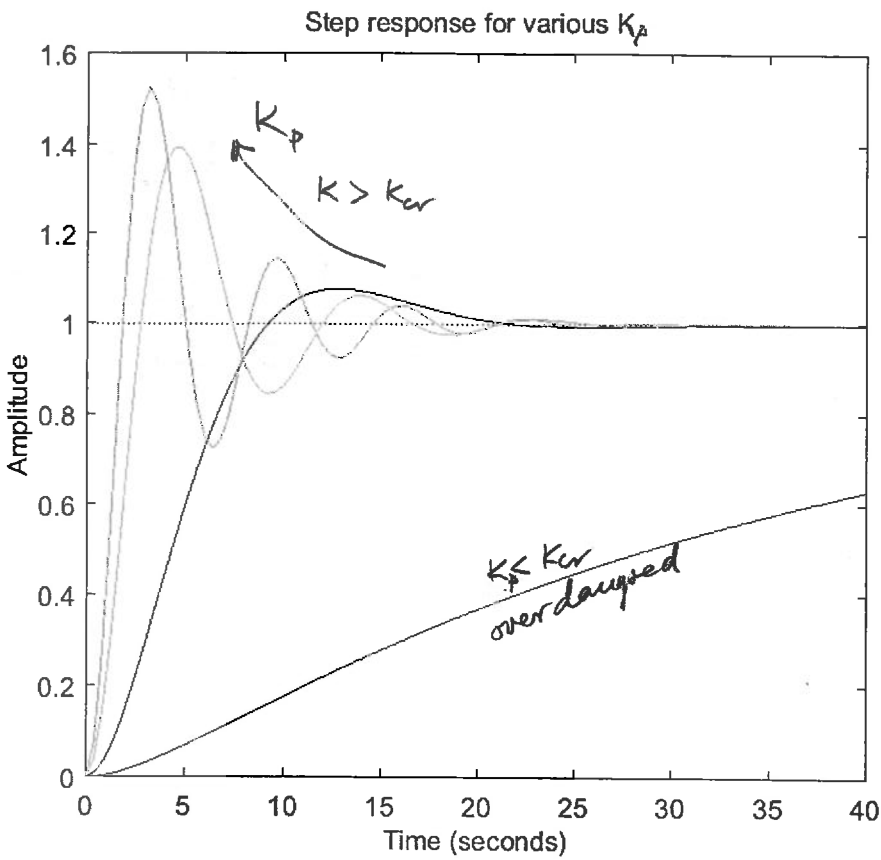




k_p

P-CONTROL

Type 1 sys



5p

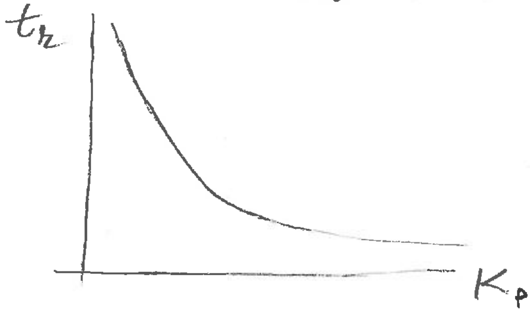
P-control of t_r and M_p

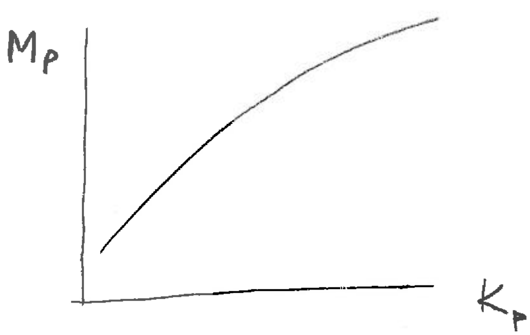
Recall:

rise time $$t_r = \frac{\pi - \varphi}{\omega_d} = \frac{\pi - \varphi}{\omega_n \sqrt{1-\zeta^2}} \quad , \quad \varphi = \sin^{-1}\sqrt{1-\zeta^2}$$

overshoot $$M_p = e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}}$$

Small overshoot M_p and small rise time t_r cannot be simultaneously met!

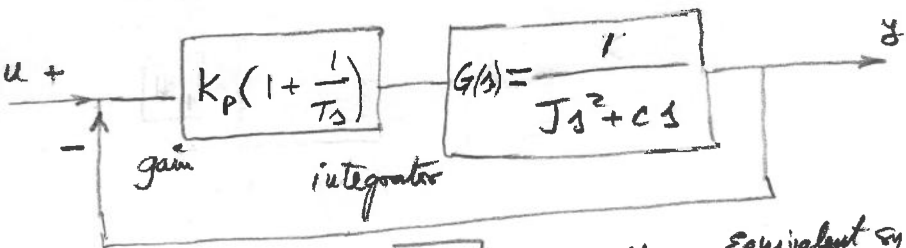


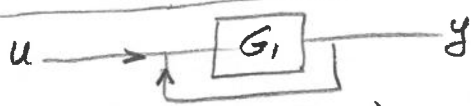


Run MATLAB program

PI

PI Control Principle





Equivalent sys.

$$G_1(s)=K_p\left(1+\frac{1}{Ts}\right)\left(\frac{1}{Js^2+cs}\right)=\frac{K_p(Ts+1)}{Ts(Js^2+cs)}$$

$$G_{CL}=\frac{G_1}{1+G_1}=\frac{K_p(Ts+1)}{(TJ)s^3+(Tc)s^2+(K_pT)s+K_p}$$

- 3rd order system
- May become unstable
- One could use R–H criterion to predict critical T for instability, i.e.,

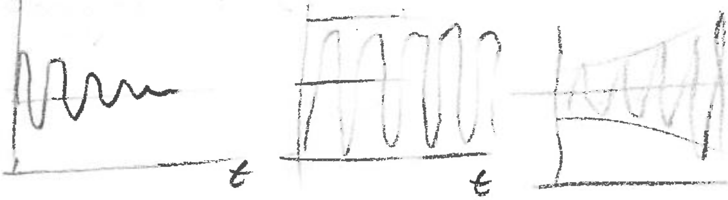
$$T > T_{cr} = J/c$$

(see next page.

<u>Example</u> : $J=2$, $c=1$, $T_{cr}=\frac{2}{1}=2$

(See results next page)

$T=4$ stable　　　$T=T_{cr}=2$　　　$T=1$ unstable



10
P5

RH (Routh-Hurwitz) Criterion Table.

$$\overset{a_0}{(TJ)}s^3+\overset{a_1}{(Tc)}s^2+\overset{a_2}{(K_pT)}s+\overset{a_3}{K}=0$$

s^3	TJ	K_pT
s^2	Tc	K_p
s^1	b_1	
s^0	K_p	

$$b_1=\frac{(\cancel{T}c)(K_pT)-(\cancel{T}J)K_p}{\cancel{T}c}=\frac{(cT-J)K_p}{c}$$

$$c_1=\frac{\cancel{b_1}K_p}{\cancel{b_1}}=K_p$$

Discussion

s^3	TJ	+ve
s^2	Tc	+ve
s^1	b_1	may be +ve or -ve.
s^0	K_p	+ve

If b_1 is -ve, then sign change, i.e., INSTABILITY

For stability, $b_1>0$, i.e. $cT-J>0$.

Need $cT>J$

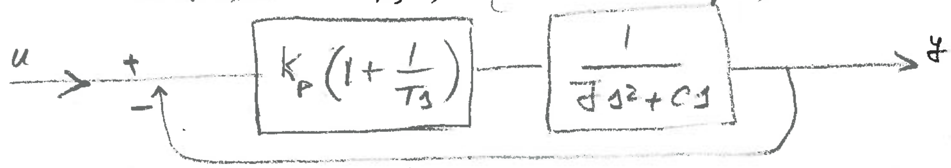
$$T>\frac{J}{c}$$

Denote $T_{cr}=\frac{J}{c}$

Need $T>T_{cr}$ for stability.

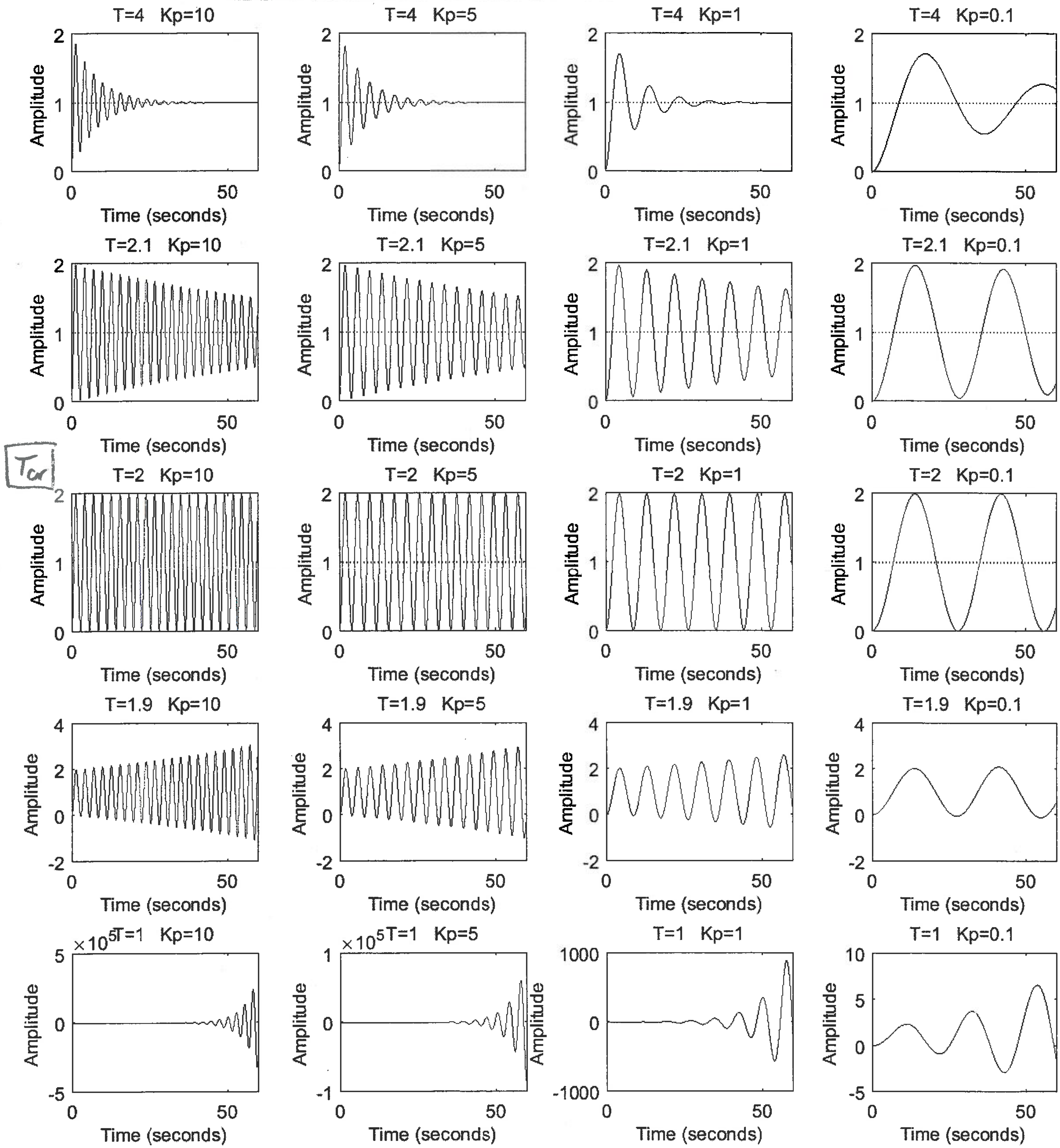
PI 1b

PI controller (K_p, T) (class example).



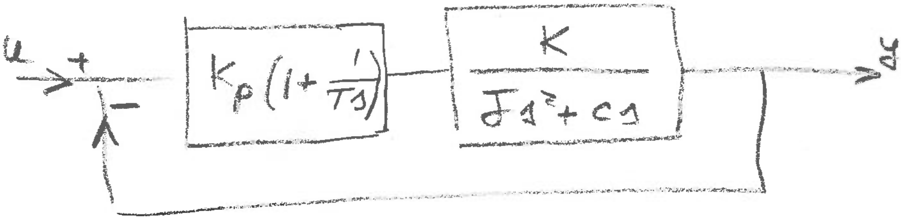
$J = 2$

$c = 1$



2 PI

PI Controller (K_p, T)



$$G(s) = K_p\left(1+\frac{1}{Ts}\right)\left(\frac{K}{Js^2+cs}\right) = \frac{K_pK(Ts+1)}{Ts(Js^2+cs)}$$

$$G_{CL} = \frac{G}{1+G} = \frac{K_pK(Ts+1)}{TJs^3+Tcs^2+K_pKTs+K_pK}$$

- 3^{rd} order system
- May become unstable
- R-H stability criterion requires $T > T_{cr}$,

  $T_{cr} = \frac{J}{c}$, for stability (next page)

Example : $K=114$, $J=10$, $c=4$, $T_{cr}=\frac{10}{4}=2.5$
(aircraft)

20
P1

R-H Table

$$(TJ)s^3 + (Tc)s^2 + (K_pKT)s + (K_pK) = 0$$

$$\begin{array}{c|cc}
s^3 & TJ & K_pKT \\
s^2 & Tc & K_pK \\
s^1 & b_1 & \\
s^0 & K_pK &
\end{array}$$

$$b_1 = \frac{\cancel{T}cK_pKT - \cancel{T}JK_pK}{\cancel{T}c} = \frac{Tc - J}{c} K_pK$$

Discussion

$$\begin{array}{c|ll}
s^3 & TJ & +ve \\
s^2 & Tc & +ve \\
s^1 & b_1 & \text{may be } +ve \text{ or } -ve \\
s^0 & K_pK & +ve
\end{array}$$

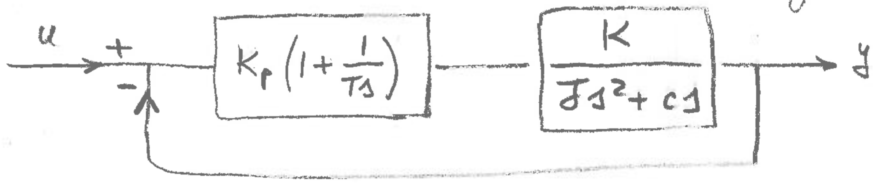
INSTABILITY happens if $b_1 < 0$, i.e., $Tc < J$.

For stability, need $Tc > J$ or

$T > T_{cr}$, $T_{cr} = \frac{J}{c}$

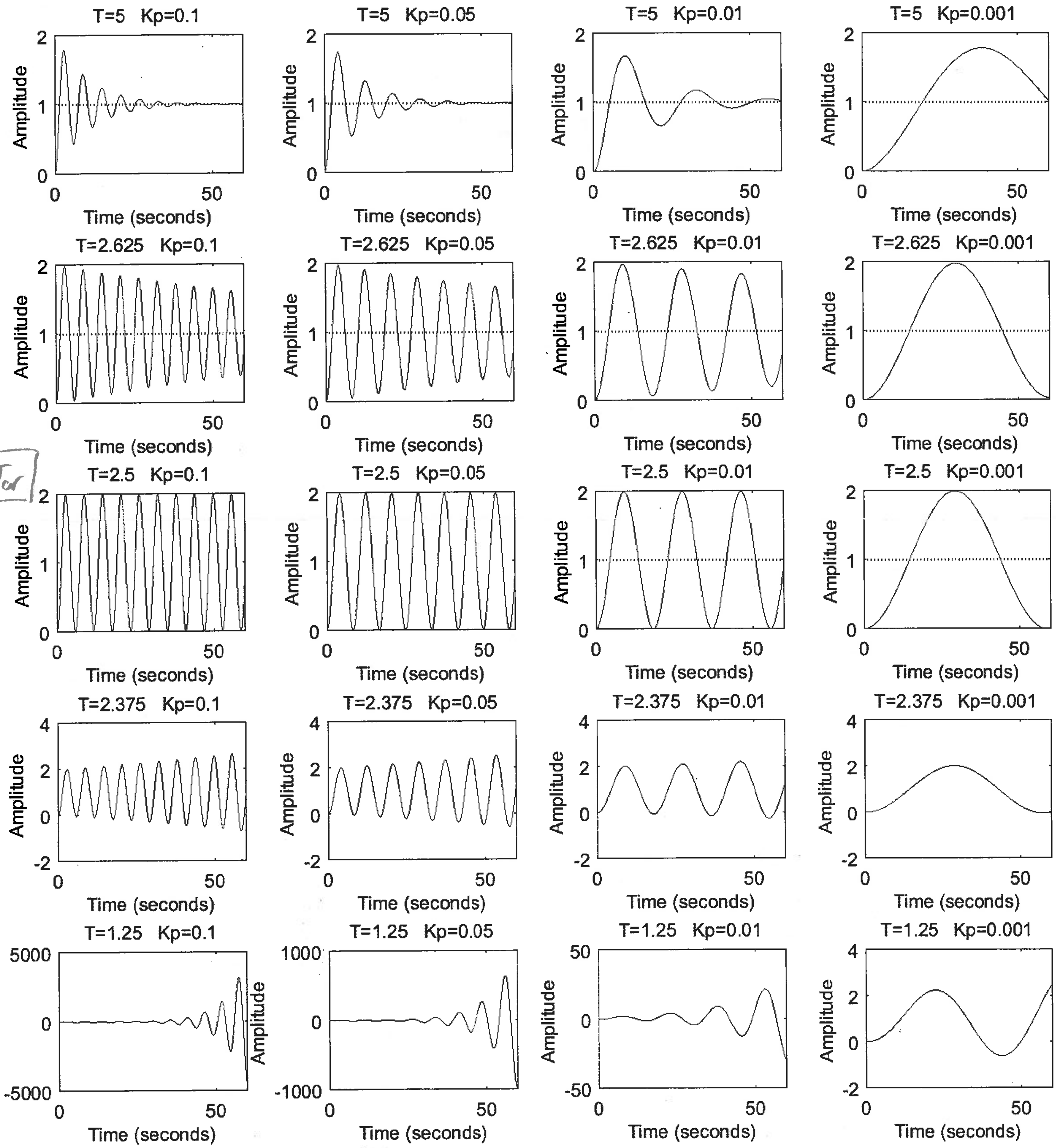
PI
9b

PI controller (Kp, T) aircraft model



$K = 114$
$J = 10$
$c = 4$

T_{cr}



2C
PI

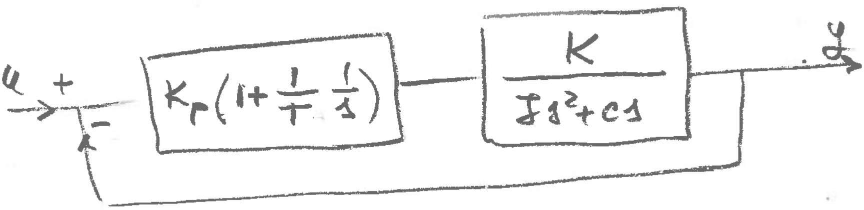
Aircraft roll response
with (K_p, T) PI controller

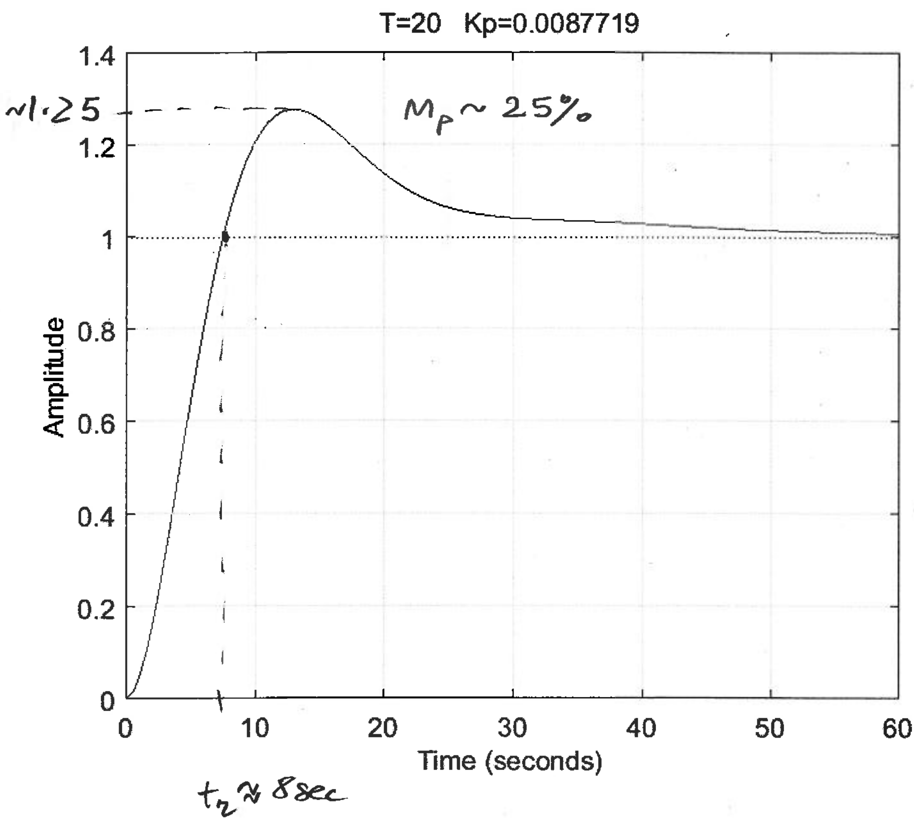
$$\boxed{\begin{aligned} K &= 114 \\ J &= 10 \\ c &= 4 \end{aligned}}$$

$$K_p = 1/K = 1/114$$

$$T_{cr} = \frac{J}{c} = 2.5$$

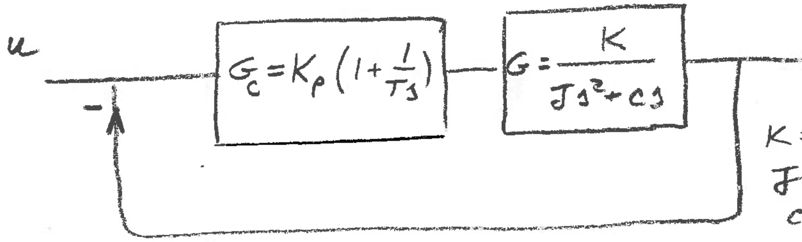
$$T = 20 > T_{cr}$$





3

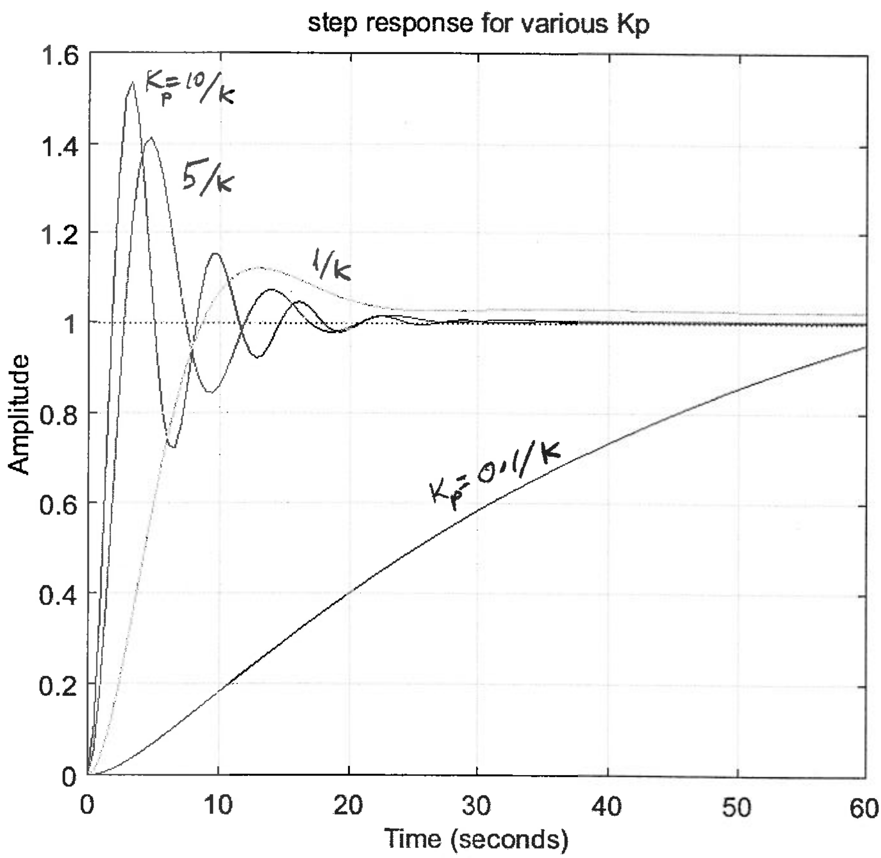
PI - control



$K = 114$
$J = 10$
$c = 4$

$T_{cr} = \frac{10}{4} = 2.5$

$T = 100 > T_{cr}$



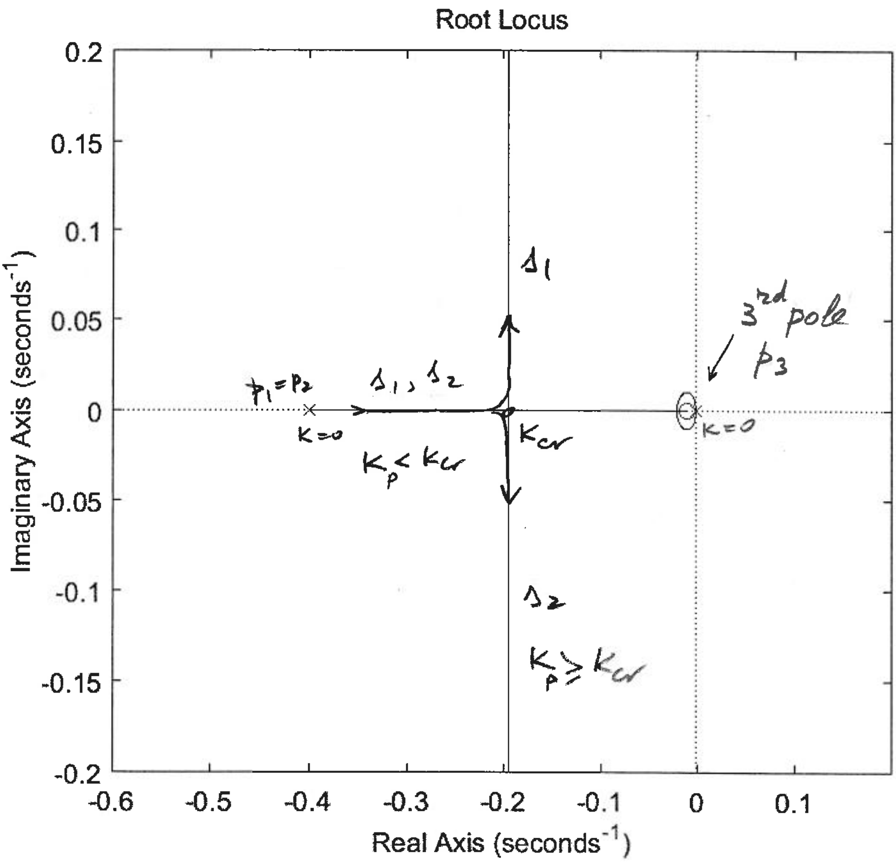
4

PI control

Root locus

aircraft $\begin{cases} K=114 \\ J=10 \\ c=4 \\ T_{cr}=2.5 \end{cases}$

$T=100 > T_{cr}$



5

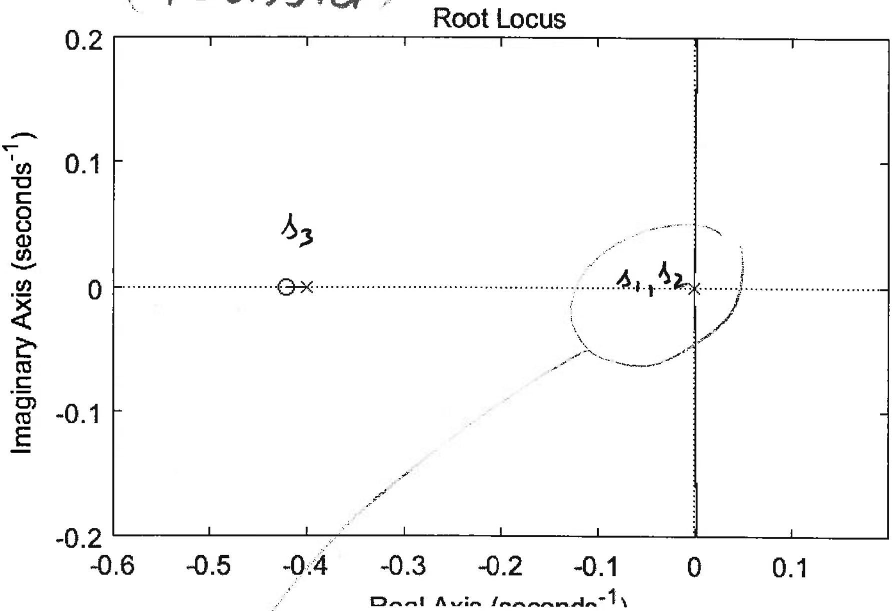
UNSTABLE PI Control
Root locus

aircraft $\begin{cases} K = 114 \\ J = 10 \\ c = 4 \\ T_{cr} = 2.5 \end{cases}$

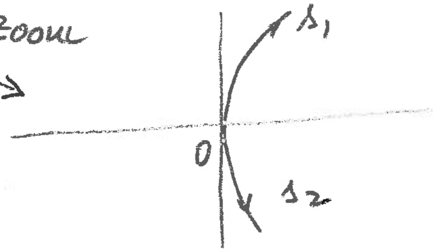
UNSTABLE

$T < T_{cr}$

$(T = 0.95\, T_{cr})$



Zoom



unstable

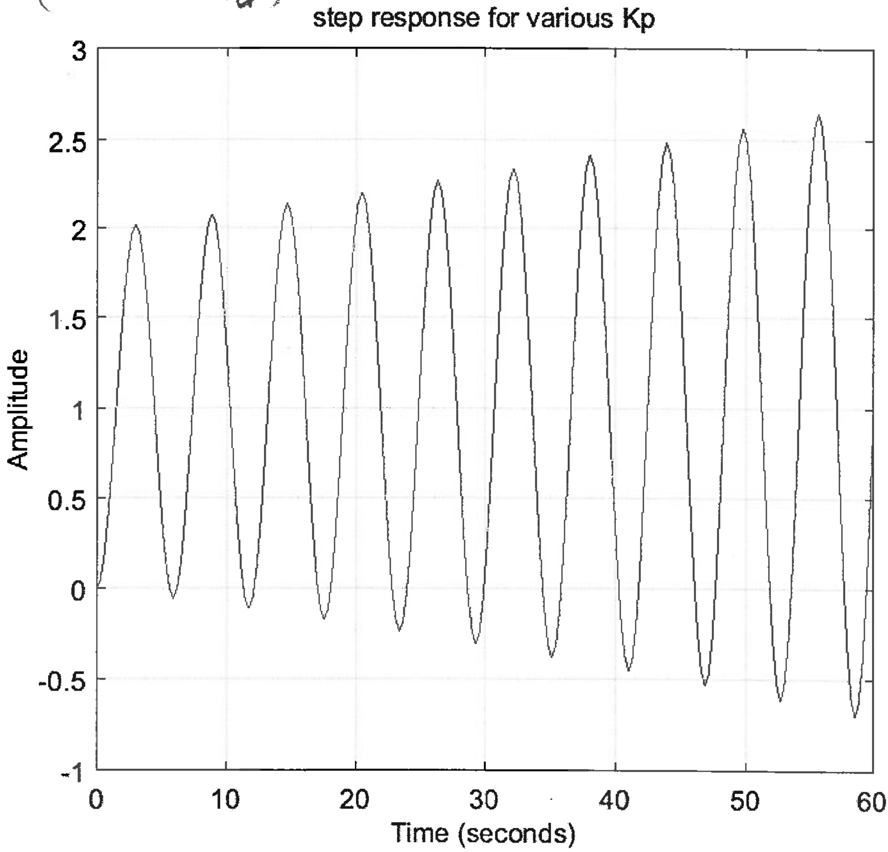
s_1, s_2 in RHS

6

UNSTABLE PI Control
Step response

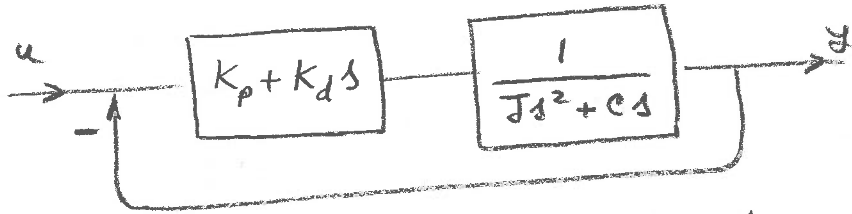
aircraft
$K = 114$
$J = 10$
$c = 4$
$T_{cr} = 2.5$

UNSTABLE
$T < T_{cr}$
$(T = 0.95 T_{cr})$



PD

PD - Control



$$G(s) = \frac{1}{Js^2 + cs}$$ (assume $K=1$ for ease)

$$G_c(s) = K_p + K_d s$$

$$G_1(s) = \frac{K_p + K_d s}{Js^2 + cs}$$

$$G_{CL}(s) = \frac{K_p + K_d s}{Js^2 + cs + K_p + K_d s}$$

$$= \frac{K_p + K_d s}{Js^2 + \underline{(c + K_d)} s + K_p}$$

damping is augmented by K_d

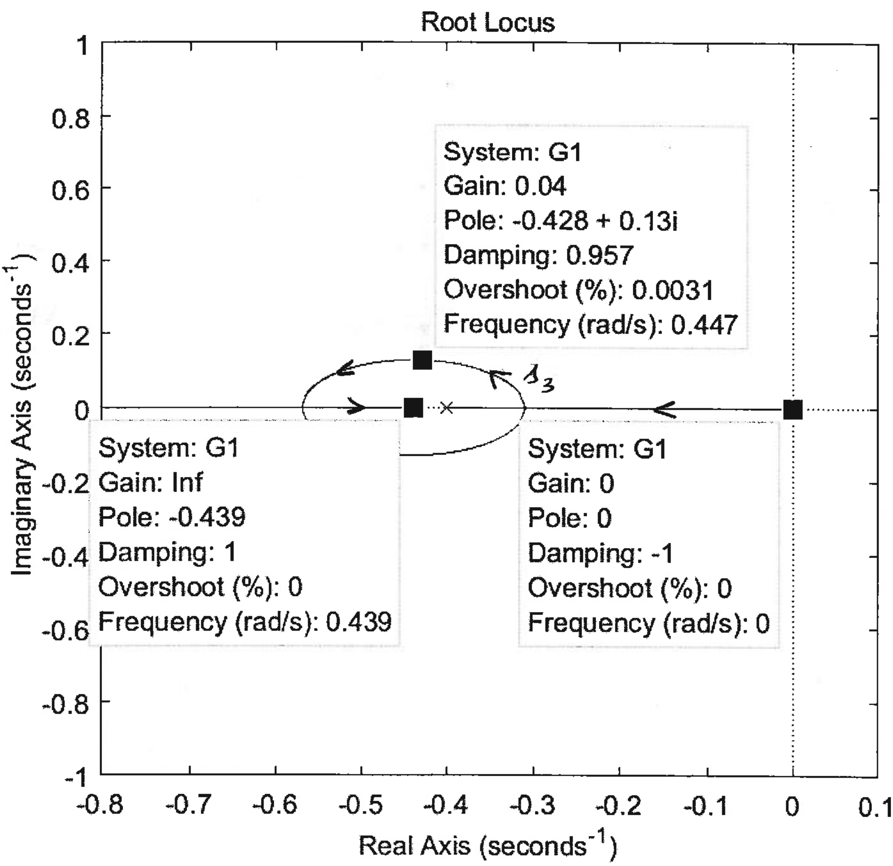
Strategy

- Adjust frequency with K_p to get small t_r
- Reduce overshoot by increasing damping with K_d

2 PD

PD controller

Interesting behavior of the poles s_1, s_2 in the root locus

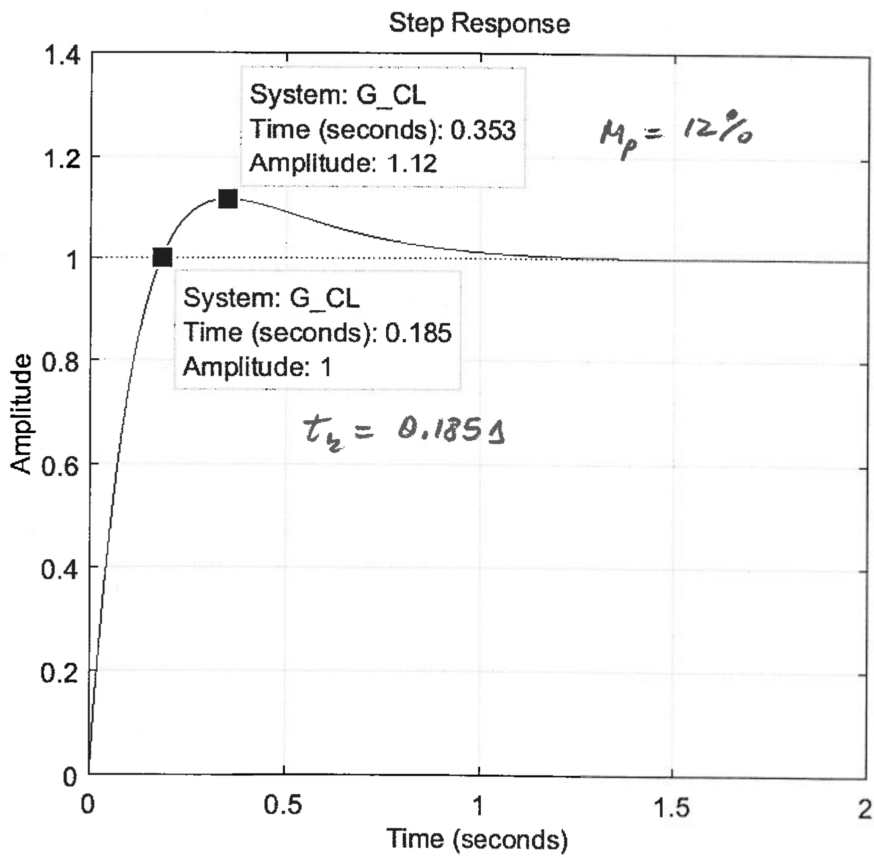


3
PD

PD control

$K = 114$
$J = 10$
$c = 4$

$K_p = 3$
$K_d = 1$



u
PD

PD control