2016 10 19

Stability of Oscillation

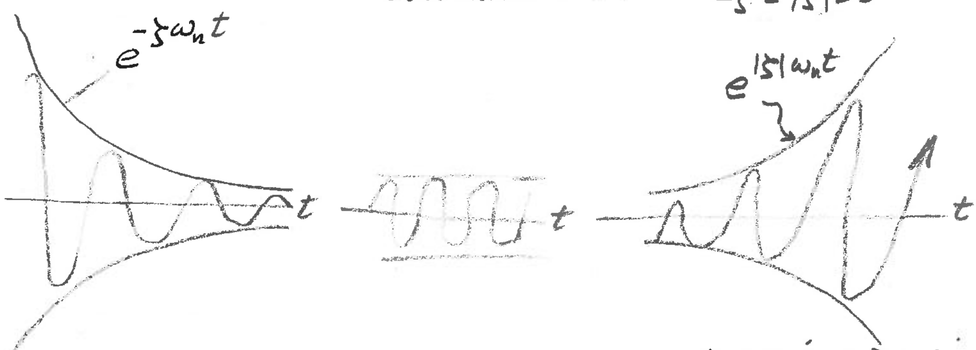
Recall damped free vibration response, i.e.,

$$x(t) = C e^{-\zeta \omega_n t} \sin(\omega_d t + \varphi) \qquad (21)$$

$\zeta > 0$
stable
damped vibr.

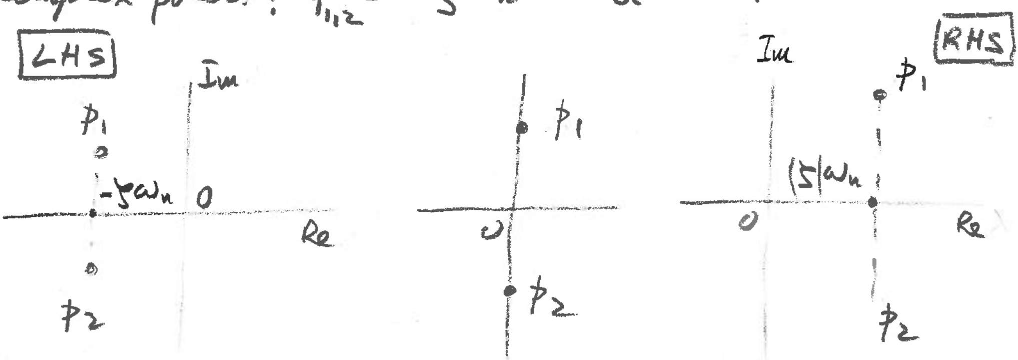
$\zeta = 0$
at the limit of stability
sustained vibr.

$\zeta < 0$
unstable
"explosive" vibr.
$-\zeta = |\zeta| > 0$



NEGATIVE DAMPING

Complex poles : $p_{1,2} = -\zeta\omega_n \pm i\omega_d$ (poles)



$p_{1,2}$ in LHS of complex λ plane

$p_{1,2}$ in RHS UNSTABLE!

2016/10/19

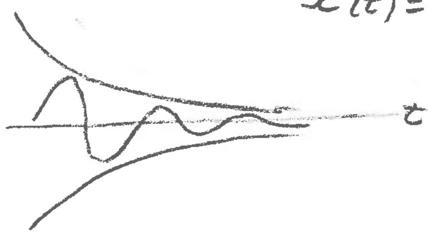
Underdamped, critical damped, overdamped 2nd order sys. and vibration response

(II) : $p_{1,2} = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$

$0<\zeta<1$ underdamped vibration resp.

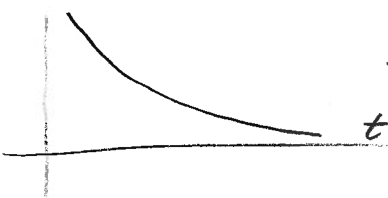
$x(t) = Ce^{-\zeta\omega_n t}\sin(\omega_d t+\varphi)$,

$\omega_d = \omega_n\sqrt{1-\zeta^2}$



$\zeta=1$ critically damped response

$p_1 = p_2 = -\omega_n$



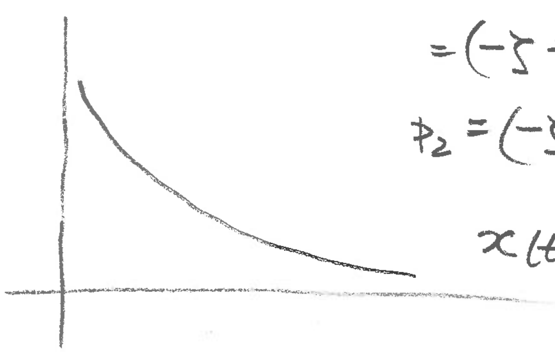
$x(t) = (C_1 + C_2 t)e^{-\omega_n t}$

$1<\zeta$ overdamped response

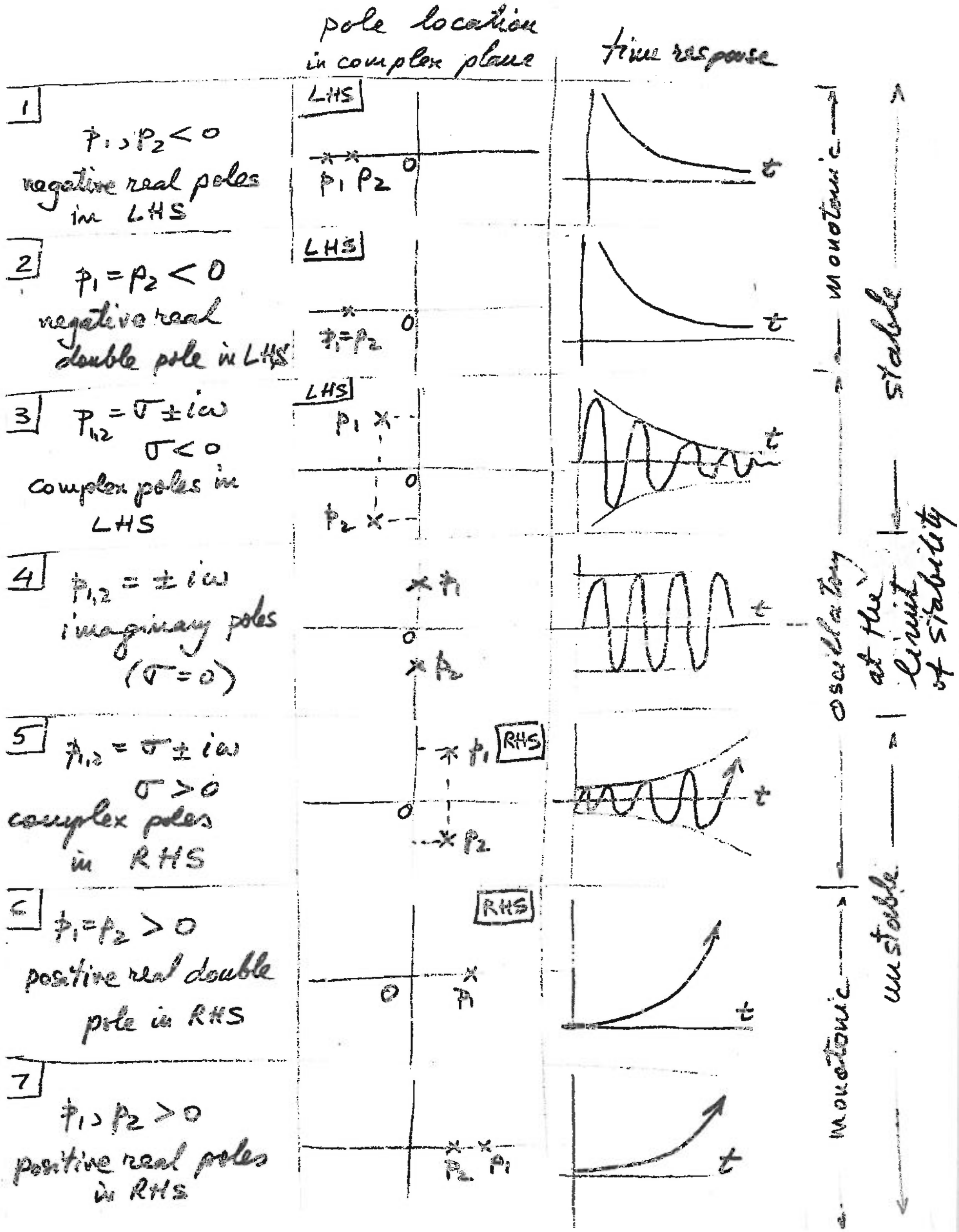
$p_1 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$

$= (-\zeta + \sqrt{\zeta^2-1})\omega_n < 0$

$p_2 = (-\zeta - \sqrt{\zeta^2-1})\omega_n < 0$



$x(t) = C_1 e^{-|\lambda_1|t} + C_2 e^{-|\lambda_2|t}$


pole location in complex plane
time response
1
$p_1, p_2 < 0$
negative real poles in LHS
2
$p_1 = p_2 < 0$
negative real double pole in LHS
3
$p_{1,2} = \sigma \pm i\omega$
$\sigma < 0$
complex poles in LHS
4
$p_{1,2} = \pm i\omega$
imaginary poles
$(\sigma = 0)$
5
$p_{1,2} = \sigma \pm i\omega$
$\sigma > 0$
complex poles in RHS
6
$p_1 = p_2 > 0$
positive real double pole in RHS
7
$p_1, p_2 > 0$
positive real poles in RHS
LHS
RHS
p_1
p_2
$p_1 = p_2$
0
t
monotonic
oscillatory
stable
at the limit of stability
unstable

```
Overdamped 2nd order system
z =
    1.5000
p =
  -10.4721
   -1.5279
=============================================
Critically damped 2nd order system
z =
     1
p =
    -4
    -4
=============================================
Underdamped 2nd order system
z =
    0.0250
p =
  -0.1000 + 3.9987i
  -0.1000 - 3.9987i
=============================================
Undamped 2nd order system
z =
     0
p =
   0.0000 + 4.0000i
   0.0000 - 4.0000i
=============================================
Negatively underdamped 2nd order system
z =
   -0.0150
p =
   0.0600 + 3.9995i
   0.0600 - 3.9995i
=============================================
Negatively critically damped 2nd order system
z =
    -1
p =
     4
     4
=============================================
Negatively overdamped 2nd order system
z =
   -1.1500
p =
    6.8716
    2.3284
```

stability_2nd_order_sys.m

C:\Mydata\1_USC\courses\EMCH516\EMCH516 Core\EMC.

```
%{
examples of 2nd order system stability
%}
%% initialization
clc
clear
format compact
%% initial data
wn=4;   % natural frequency wm, rad/sec
%% Overdamped 2nd order system
display('Overdamped 2nd order system')
z=150e-2   % damping z%
A=[1 2*z*wn wn^2];
p=roots(A)
display('=============================================')
%% Critically damped 2nd order system
display('Critically damped 2nd order system')
z=100e-2   % damping z%
A=[1 2*z*wn wn^2];
p=roots(A)
display('=============================================')
%% Underdamped 2nd order system
display('Underdamped 2nd order system')
z=2.5e-2   % damping z%
A=[1 2*z*wn wn^2];
p=roots(A)
display('=============================================')
%% Undamped 2nd order system
display('Undamped 2nd order system')
z=0        % damping z%
A=[1 2*z*wn wn^2];
p=roots(A)
display('=============================================')
%% Negatively underdamped 2nd order system
display('Negatively underdamped 2nd order system)
z=-1.5e-2   % damping z%
A=[1 2*z*wn wn^2];
```

```
p=roots(A)
display('=============================================')
%% Negatively critically damped 2nd order system
display('Negatively critically damped 2nd order system')
z=-100e-2   % damping z%
A=[1 2*z*wn wn^2];
p=roots(A)
display('=============================================')
%% Negatively overdamped 2nd order system
display('Negatively overdamped 2nd order system')
z=-115e-2   % damping z%
A=[1 2*z*wn wn^2];
p=roots(A)
display('=============================================')
```