Combined analysis

Purpose: use all the tools to evaluate the system stability

Time domain: Root locus of G
- Step response of G_{CL}

Frequency domain:
- Nyquist plot of G
- Bode plots of G: margins
  - gain K_g
  - phase γ

Given: $G(s)$

Find: combined analysis: Quad plot

Nyquist plot	Margins on Bode plot
Root locus for Gain=1	Step response

Examples:

open MATLAB codes

aircraft roll mode • Nyquist plot: STABLE • root locus —"— • step response —"— Bode plot: INSUFFICIENT phase margin	
B11.9 - STABLE: N-plot r locus step response - INSUFFICIENT phase margin	
B11.8 UNSTABLE	

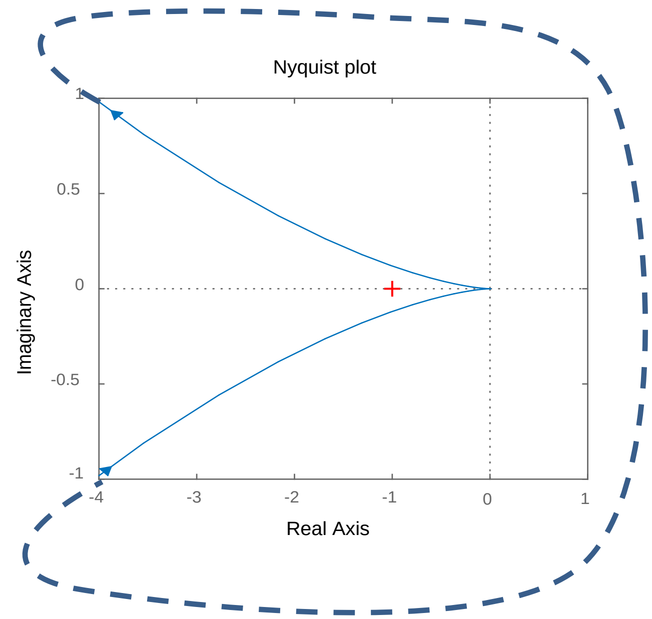
Combined analysis
of aircraft roll model.

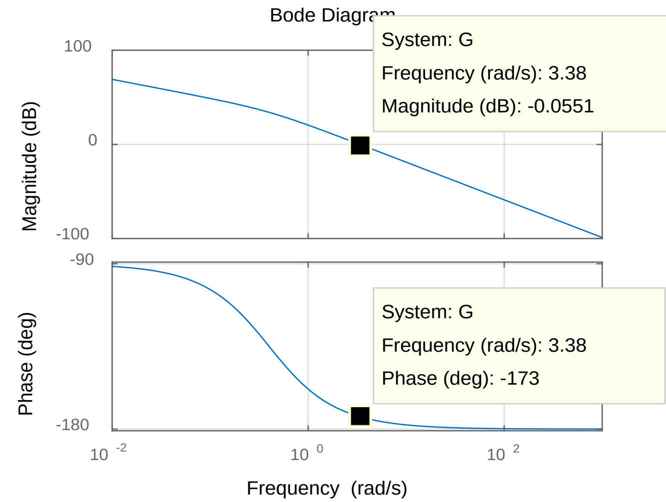
$$G(s) = \frac{K}{Js^2 + cs} = \frac{114}{10s^2 + 4s}$$

$$p_1 = 0$$
$$p_2 = -0.4.$$

See plot

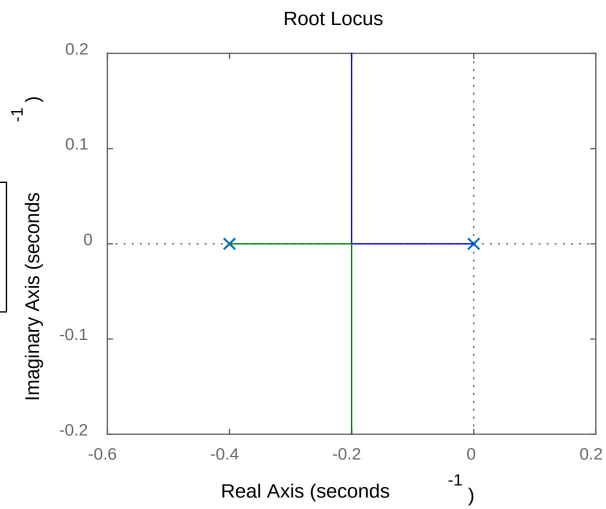
P=0
N=0
====
Z=0
STABLE

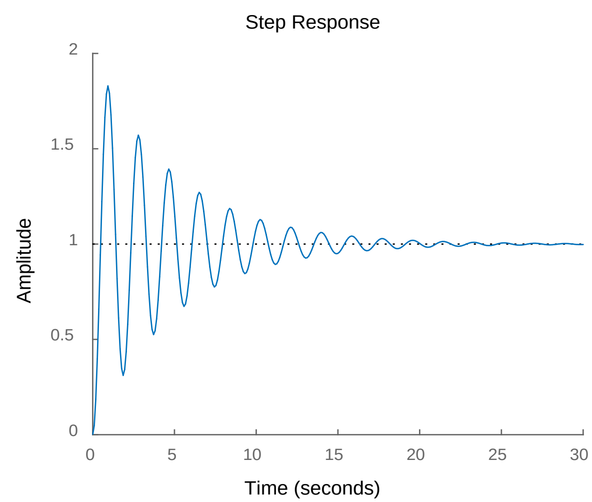




GM=inf
PM=7 deg
INSUFFICIENT PM

roots always
in LHS





Very lightly damped
step response
with many oscillations

1

20170417

Aircraft roll model combined analysis (cont.)

Nyquist.

✓ stable

$Z = 0$.

Bode

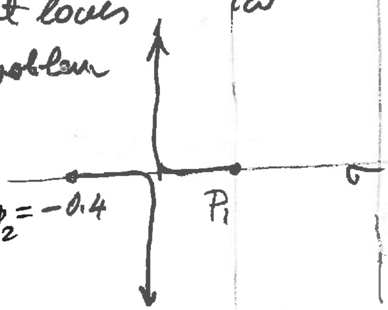
Margins

$(K_g)_{dB} = \infty$ ✓

$\gamma^\circ = 7^\circ$ Not sufficient

Root locus

No problem



step response

Conclusion

- Nyquist and root locus indicate stable system.
- Margin analysis warns about low phase margin; → need compensator
- step response — oscillatory

Problem B 11.9

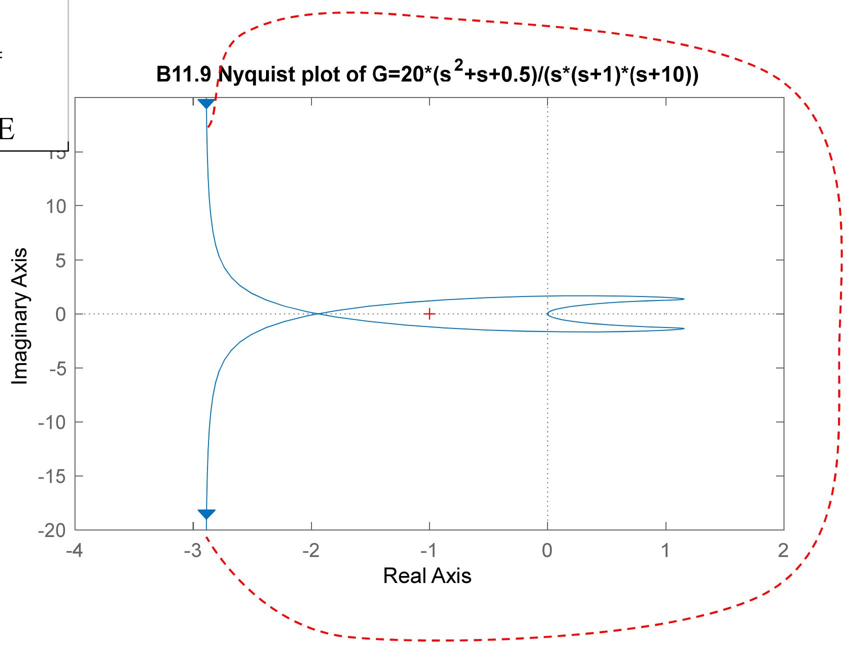
$$G(s) = \frac{20(s^2+s+0.5)}{s(s-1)(s+10)}$$

$$\left.\begin{array}{l} p_1 = 0 \\ p_2 = +1 \\ p_3 = -10 \end{array}\right\} \quad P = 1$$

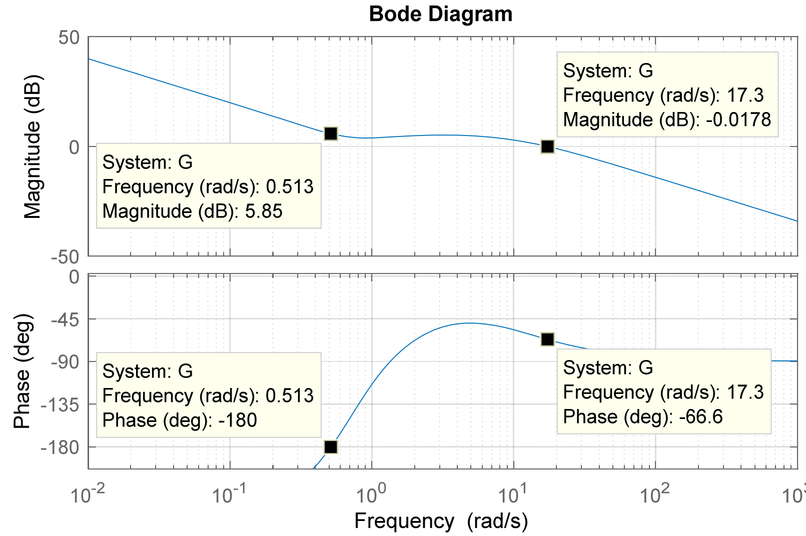
Nyquist	Bode
root locus	step response

see plot

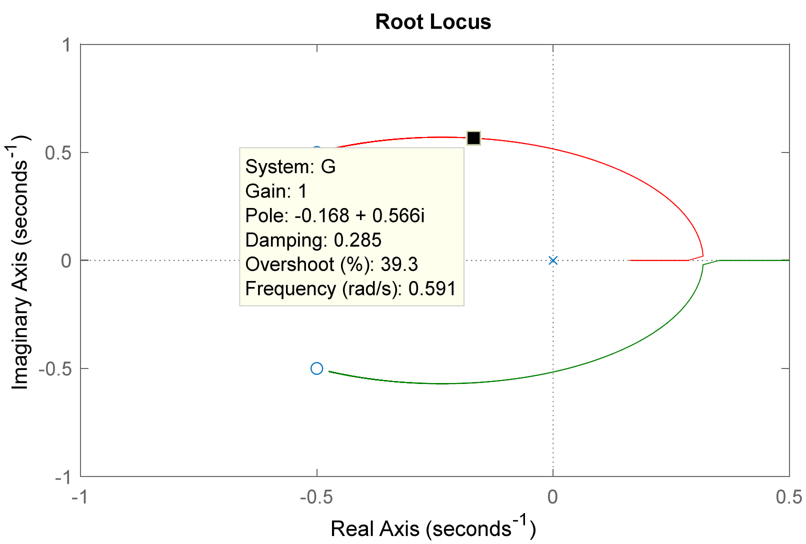
N= –2
P = +1
======
Z = –1
STABLE

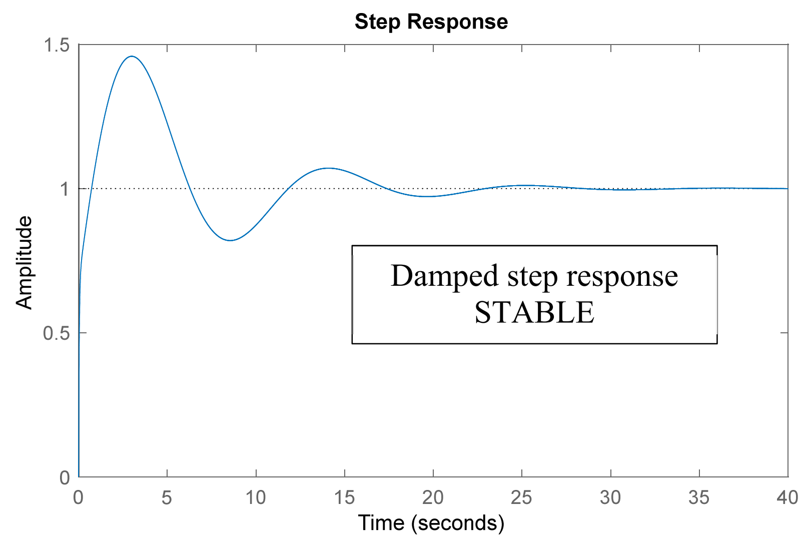


Good phase margin PM = ~114 deg
Bad gain margin GM = ~ 5.85 dB



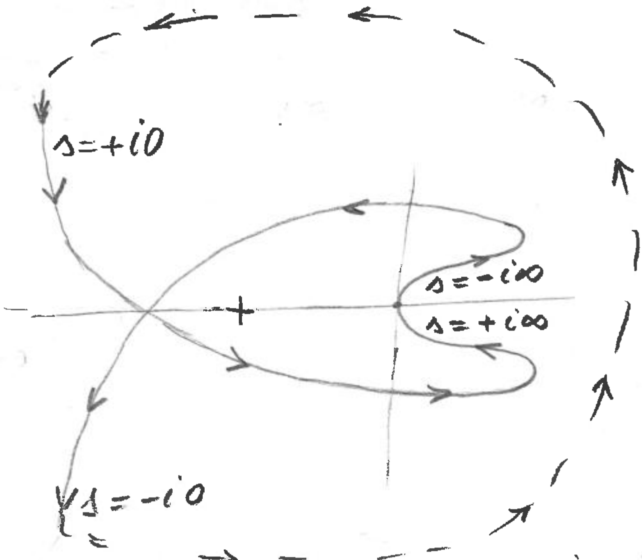
K = 1 point
is in LHS
STABLE

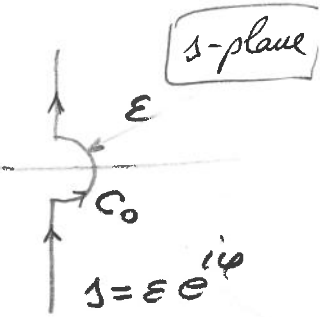




Problem B11.9 (cont.)

Nyquist plot





$-90° < \varphi < 90°$

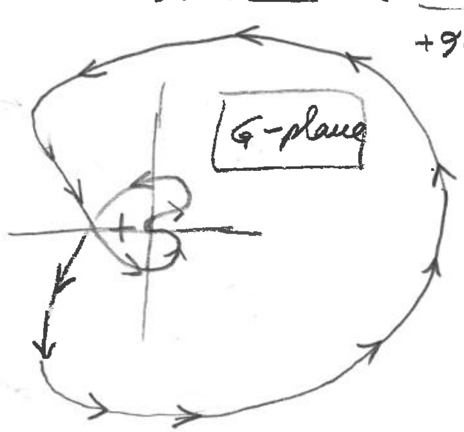
$$G^0 = \frac{20}{-s} = -2\frac{1}{\varepsilon}e^{-i\varphi}$$

$$G^0 = \frac{2}{\varepsilon}e^{i(-180°-\varphi)}$$

$-90° < \varphi < 90°$

$-90° < \angle G^0 < -270°$

$+90°$



$N = -2$

$P = 1$

$Z = -1$

$\boxed{\text{Stable}}$

(B11.9 cont.)

Bode plot

Good phase margin: $\gamma \approx 114°$ @ $16.6 \frac{rad}{sec}$

$\boxed{\text{Bad}}$ gain margin: $K_g = -5.3\,dB$ @ $0.55 \frac{rad}{sec}$
at low freq.

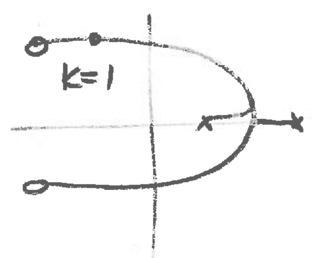
Good gain margin at higher $\omega > 16 \frac{rad}{sec}$, $\angle$

(φ does not cross $-180°$)

There could be problems at low ω
($\omega \sim 0.5$ rad/sec) due to imperfections

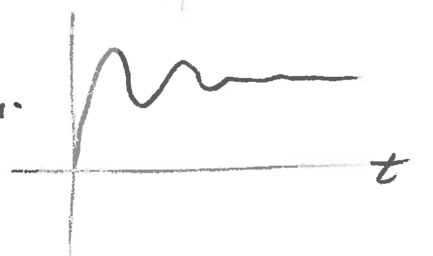
Root locus

K=1 in LHS
Stable



Step response

damped oscillation.
STABLE



Problem B11.8

$$G(s) = \frac{320\,(s+2)}{s(s+1)(s^2+8s+44)}$$

4 Poles: 0

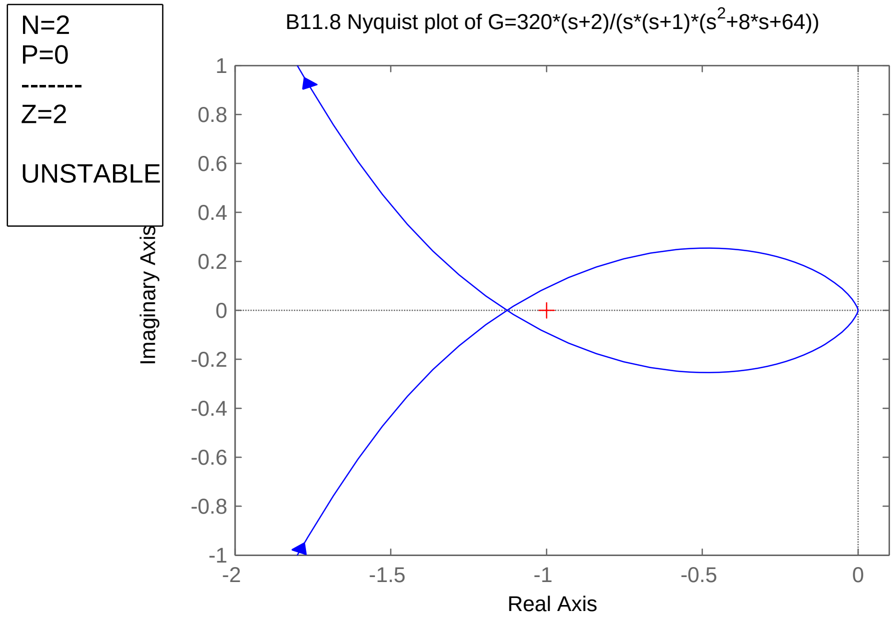
-1

$-4 \pm i5.3$

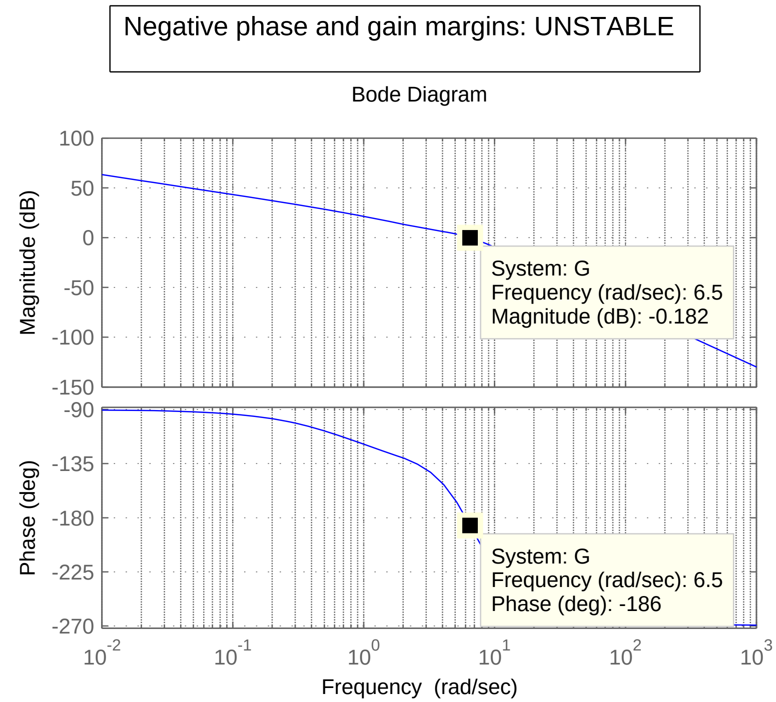
all in LHS

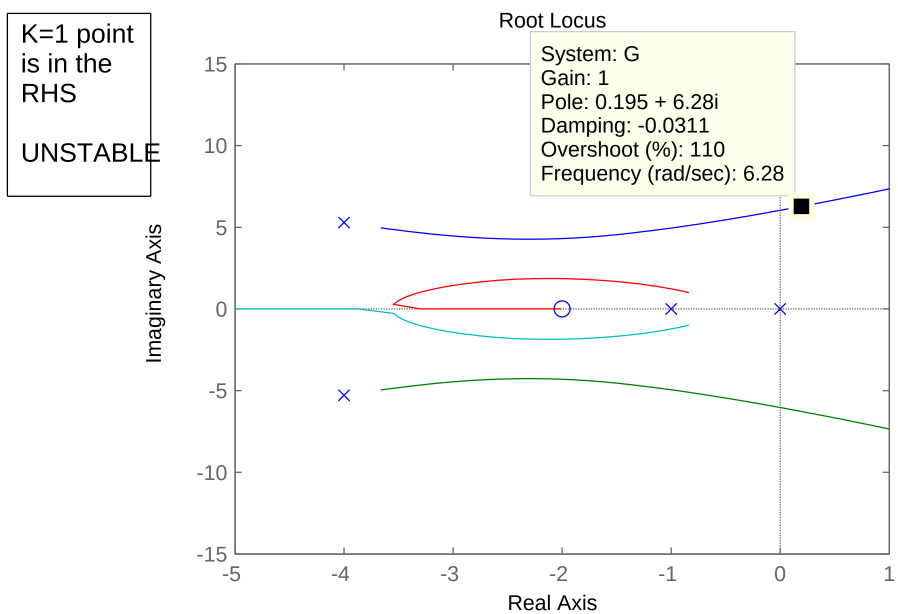
N	B
R	S

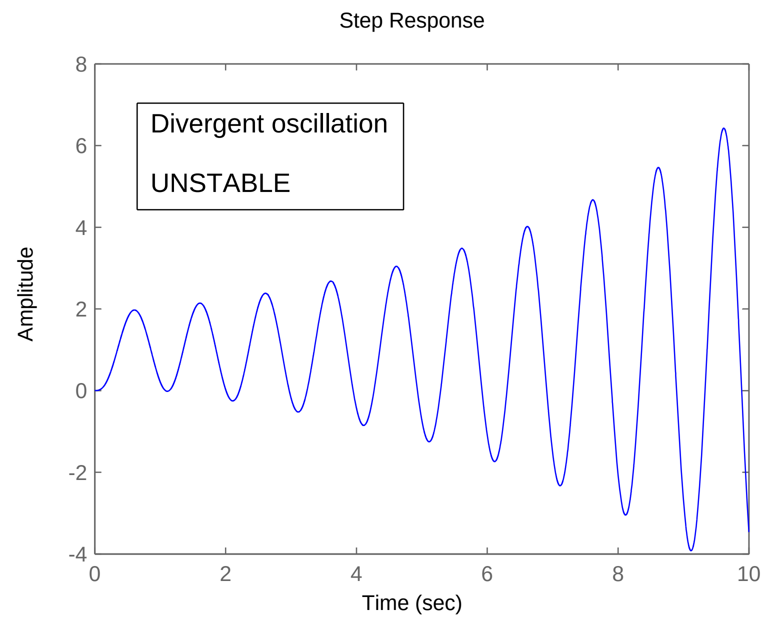
see plot


N=2
P=0

Z=2
UNSTABLE
B11.8 Nyquist plot of G=320*(s+2)/(s*(s+1)*(s^2+8*s+64))
Imaginary Axis
Real Axis

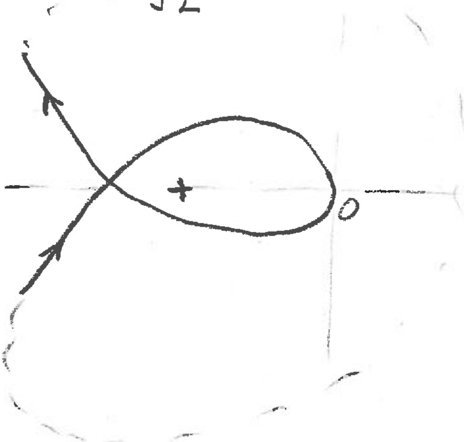

Negative phase and gain margins: UNSTABLE
Bode Diagram
Magnitude (dB)
System: G
Frequency (rad/sec): 6.5
Magnitude (dB): -0.182
Phase (deg)
System: G
Frequency (rad/sec): 6.5
Phase (deg): -186
Frequency (rad/sec)


K=1 point
is in the
RHS
UNSTABLE
Root Locus
System: G
Gain: 1
Pole: 0.195 + 6.28i
Damping: -0.0311
Overshoot (%): 110
Frequency (rad/sec): 6.28
Imaginary Axis
Real Axis


Step Response
Divergent oscillation
UNSTABLE
Amplitude
Time (sec)

Problem B11.8 (cont.)

Nyquist plot



$P = 0$ no poles in RHS

$N = 2$

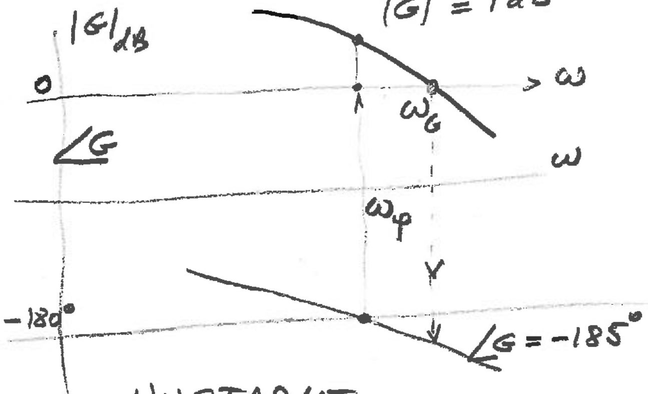
$Z = P + N = 2$ zeros in RHS of $1 + G(s)$

UNSTABLE

because

$$G_{CL} = \frac{G(s)}{1+G(s)}$$

Bode plot : margins



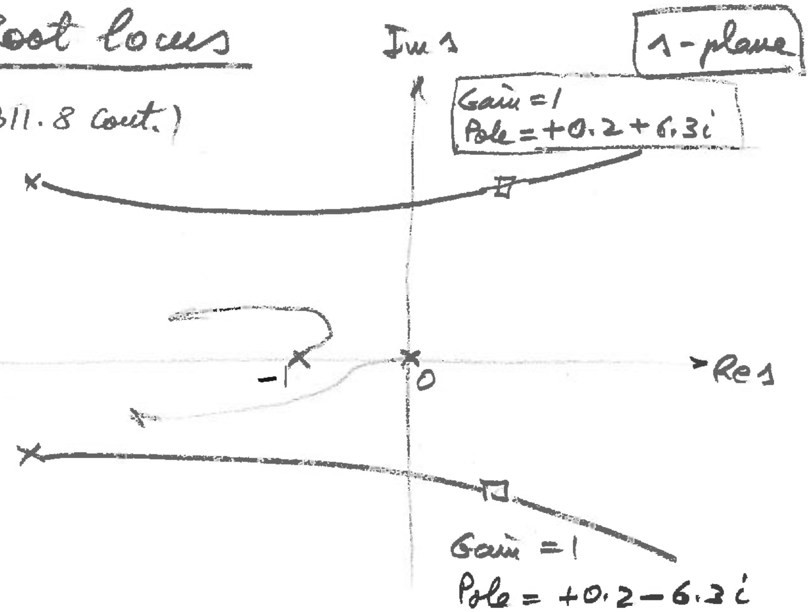
$K_g = -1dB < 0\,dB$

$\gamma = -5° < 0$

UNSTABLE

Root locus

Problem B11.8 cont.)

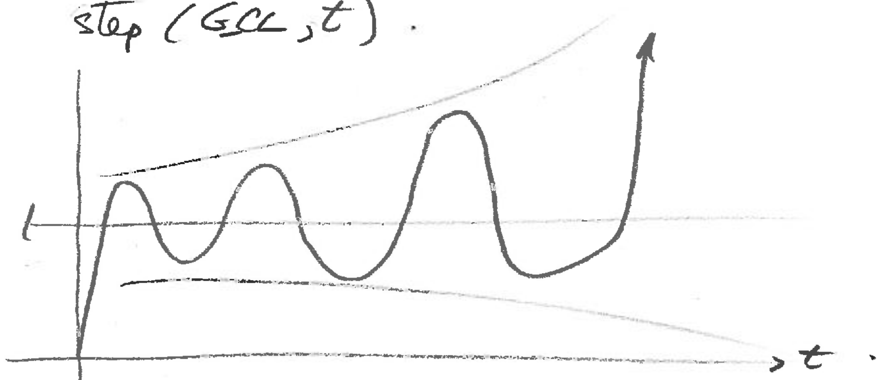


Roots in RHS → UNSTABLE.

Step response

G_{CL} = feedback(G,1).

step (GCL, t).



Divergent response → UNSTABLE

All four criteria predicted an UNSTABLE closed loop response.

ACTION: DO NOT close the loop!