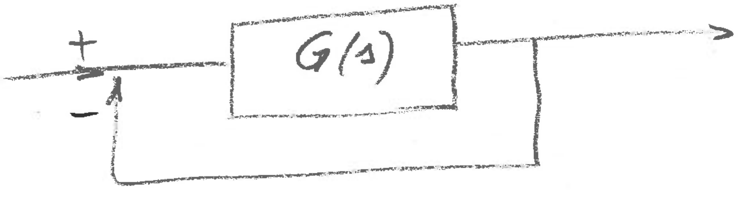
STABILITY ANALYSIS
IN FREQ. DOMAIN

Evaluate stability of CL system by analyzing OL system in freq. domain



Two methods:

- Nyquist criterion
- Gain & phase margins

NYQUIST ANALYSIS OF FEEDBACK STABILITY

Nyquist circuit

$$G(s) = \frac{1}{(s-p_1)(s-p_2)(s-p_3)}$$

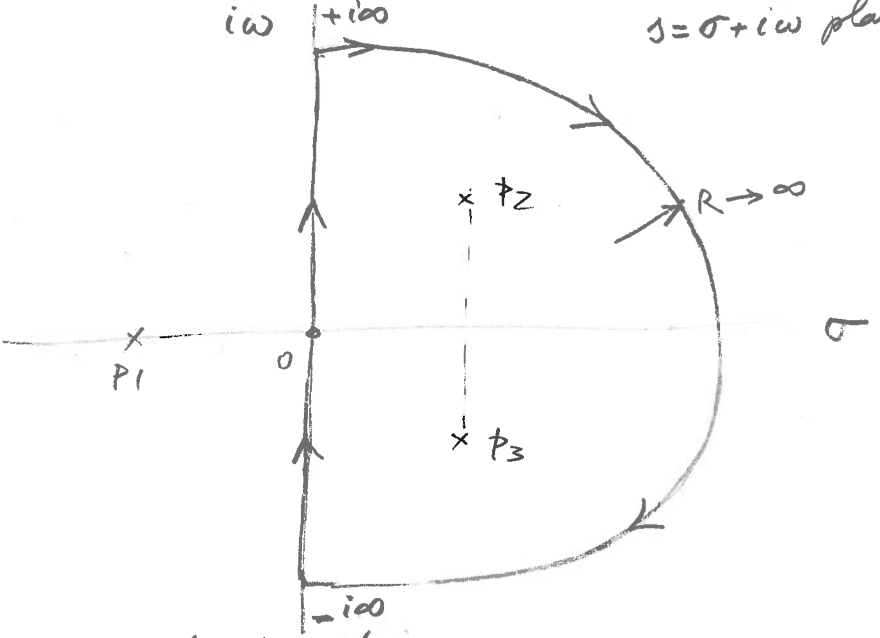
Assume

$p_1 \in \mathbb{R}$, $p_1 < 0$

$p_2, p_3 \in \mathbb{C}$ in RHS

$p_3 = \overline{p_2}$

$s = \sigma + i\omega$ plane

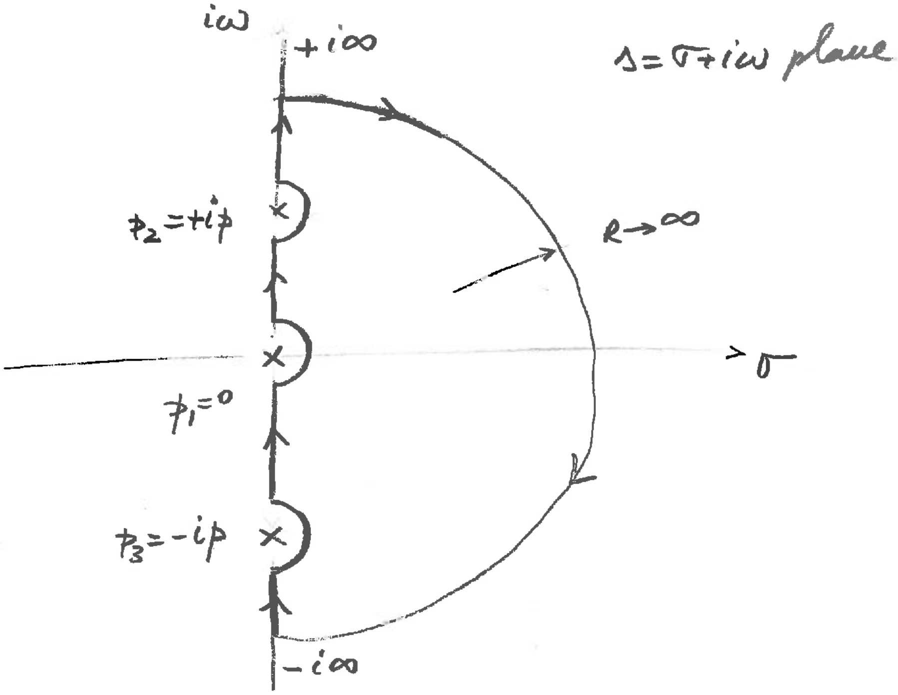


Nyquist circuit is:

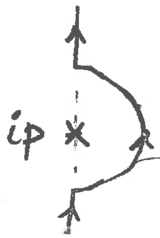
- semi-circle path with $R \to \infty$
- in the RHS of s-plane
- along vertical axis from $-i\infty$ to $+i\infty$
- traveled clockwise (CW)

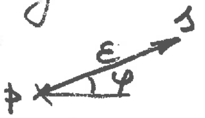
Nyquist circuit with poles on the vertical axis

$$G(s) = \frac{1}{s(s-ip)(s+ip)}$$



The poles on the vertical axis must be excluded. We travel around them in the RHS. To do so, we follow a small semi-circle with vanishing radius ε



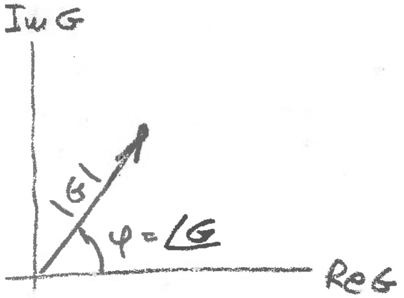


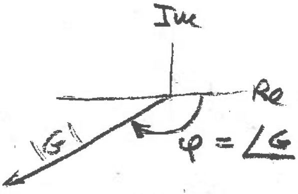
$$s = ip + \varepsilon e^{i\varphi}$$
$$-90° < \varphi < 90°$$

N1

POLAR PLOTS

$$G(i\omega) = \mathrm{Re}\,G(i\omega) + i\,\mathrm{Im}\,G(i\omega)$$



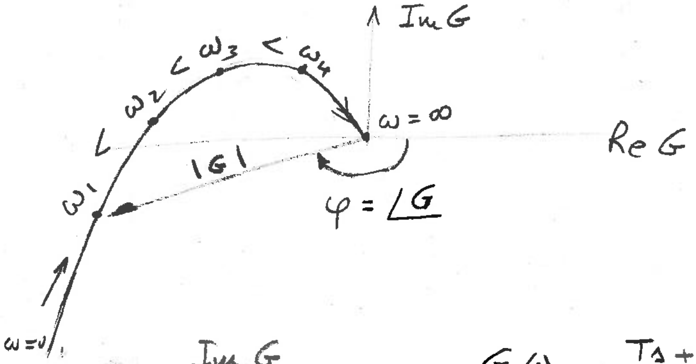


-ve value

$-\pi < \text{angle}(G) < \pi$

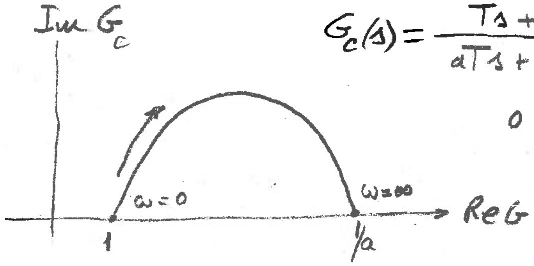
-180°   180°

(see MATLAB Help)



$$G_c(s) = \frac{Ts+1}{\alpha Ts+1}$$

$0 < \alpha < 1$



Run MATLAB examples.

1
NPP

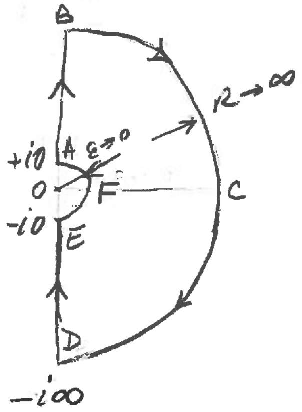
NYQUIST POLAR PLOTS

"s follows N-circuit ; G(s) follows N-polar plot"

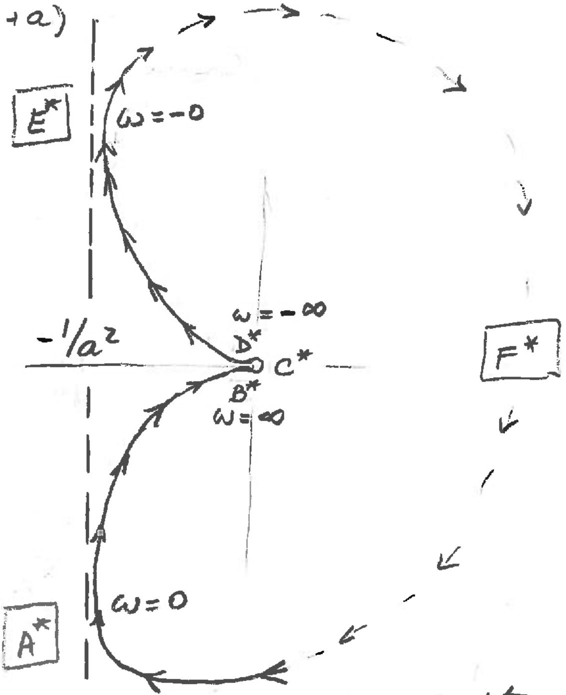
Example : $G(s) = \frac{1}{s(s+a)}$

2 poles : $p_1 = 0$, $p_2 = -a$

Nyquist circuit



Nyquist polar plot

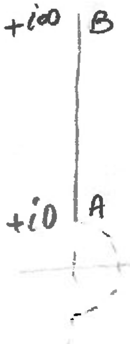


- The variable s follows the Nyquist circuit ABCDEFA clockwise in the s-plane
- The function G(s) follows the resulting circuit in the G-plane

In this example, we distinguish 4 segments to be analyzed individually and then assembled in one continuous circuit

2
NPP

Segment $(\overset{A}{\underset{\varepsilon\to 0}{+i\varepsilon}}, \overset{B}{\underset{R\to\infty}{+iR}})$



$$G(s) = \frac{1}{s(s+a)} = \frac{1}{as} - \frac{1}{a(s+a)}$$

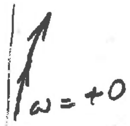
$$\boxed{A}\quad G(i\varepsilon) = \frac{1}{ai\varepsilon} - \frac{1}{a(\underset{\varepsilon \ll a}{\cancel{i\varepsilon}}+a)}$$

$$G(i\varepsilon) \simeq -\frac{1}{a^2} - i\frac{1}{a\varepsilon}$$

$$G(+i0) = \lim_{\varepsilon\to 0} G(i\varepsilon) = -\frac{1}{a^2} - i\infty$$

$$|G(+i0)| = \infty$$

$$\angle G(+i0) = \angle -i = -90^\circ$$

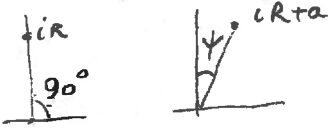


$$\boxed{B}\quad G(iR) = \frac{1}{iR(iR+a)}$$

$$|G(iR)| \underset{R \gg a}{\approx} \frac{1}{R^2} \xrightarrow[R\to\infty]{} 0$$

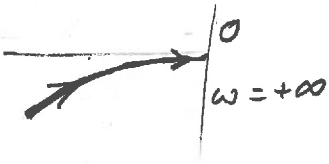
$$\angle G(iR) = -[\angle iR + \angle iR+a]$$

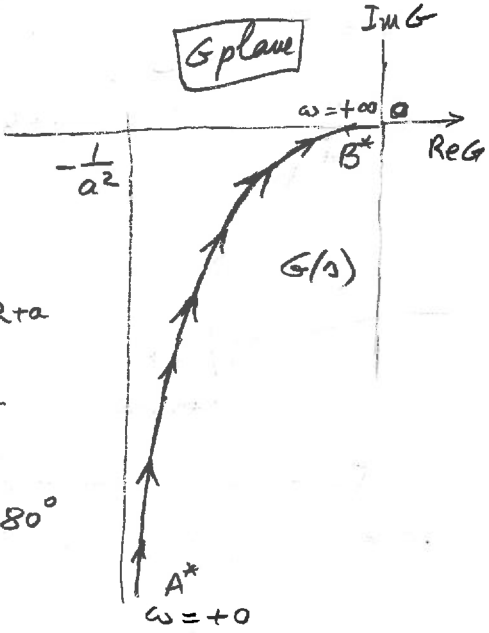
$$\varphi = \tan^{-1}\frac{a}{R}$$



$$\angle G(iR) = -(90^\circ + 90^\circ - \psi)$$

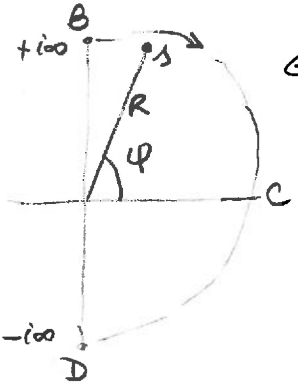
$$= -180^\circ + \psi \xrightarrow[\substack{R\to\infty \\ \psi\to 0}]{} -180^\circ$$





3 NPP $\text{Segment } (\overset{B}{iR}, \overset{C}{R}, \overset{D}{-iR})_{R\to\infty}$ Big circle

$$s = Re^{i\varphi}, \quad \varphi \in (90^\circ \quad 0 \quad -90^\circ)$$

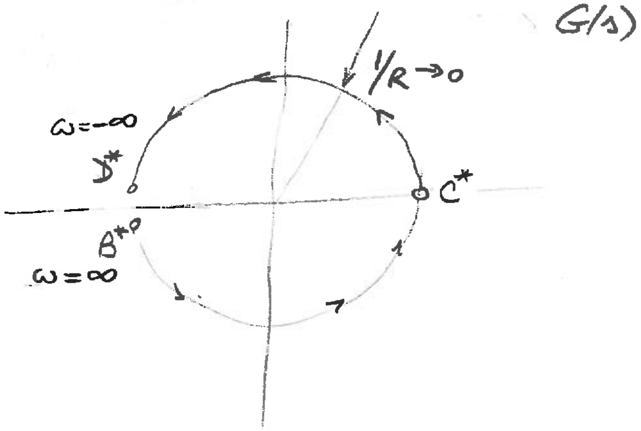


$$G(s) = \frac{1}{Re^{i\varphi}(Re^{i\varphi}+a)}$$

$$\underset{R\to\infty}{\approx} \frac{1}{R^2 e^{2i\varphi}} = \frac{1}{R^2} e^{-i2\varphi}$$

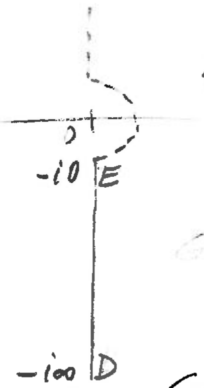
$$\angle G(s) = -2\varphi$$

$$\angle G(s) \in (\underset{B^*}{-180^\circ} \quad \underset{C^*}{0^\circ} \quad \underset{D^*}{180^\circ})$$



k
NPP

Segment $(\overset{D}{\underset{R\to\infty}{-iR}}, \overset{E}{\underset{\varepsilon\to 0}{-i\varepsilon}})$



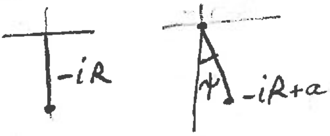
$$G(s) = \frac{1}{s(s+a)} = \frac{1}{as} - \frac{1}{a(s+a)}$$

$\boxed{D}$ $$G(s)\Big|_{s=-iR} = \frac{1}{-iR(-iR+a)}$$

$$|G(s)| = \frac{1}{R^2} \xrightarrow[R\to\infty]{} 0$$

$$\angle G(s) = -\left(\angle -iR + \angle -iR+a\right)$$

$$= -\left[-90^\circ + (-90^\circ+\psi)\right] = 180^\circ-\psi \xrightarrow[\substack{R\to\infty\\ \psi\to 0}]{} 180^\circ$$



$$\psi = \tan^{-1}\frac{a}{R}$$

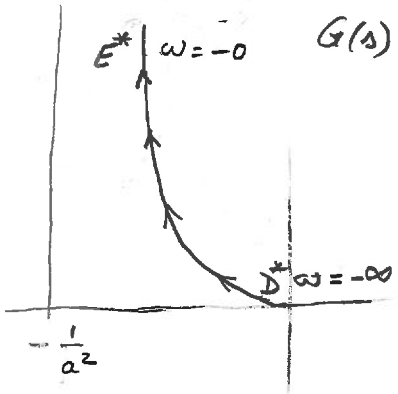
$\boxed{E}$ $$G(-i\varepsilon) = -\frac{1}{a(-i\varepsilon)} - \frac{1}{a(-i\varepsilon+a)} = \frac{i}{a\varepsilon} - \frac{1}{a^2}$$

$$G(-i0) = \lim_{\varepsilon\to 0} G(-i\varepsilon)$$

$$G(-i0) = -\frac{1}{a^2} + \frac{i}{a\varepsilon}\Big|_{\varepsilon\to 0} = -\frac{1}{a^2} + i\infty$$

$$|G(-i0)| = \infty$$

$$\angle G(-i0) = 90^\circ$$



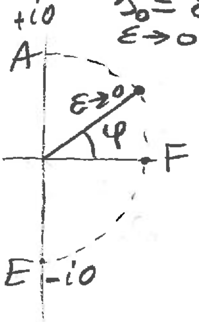
5
NPP

Segment $\left(\overset{E}{-i\varepsilon}, \overset{F}{\varepsilon}, \overset{A}{i\varepsilon}\right)$ small circle

$\varepsilon \to 0$

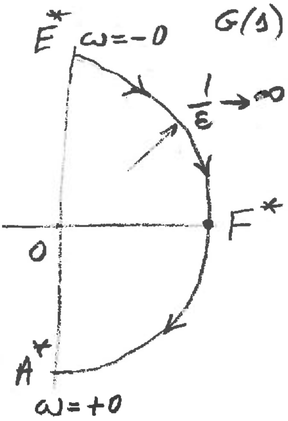
$s_0 = \varepsilon e^{i\varphi}$, $\varphi \in (\underset{D}{-90°}, \underset{E}{0}, \underset{F}{90°})$

$\varepsilon \to 0$



$$G(s) = \frac{1}{\varepsilon e^{i\varphi}(\varepsilon e^{i\varphi}+a)}$$

$$\underset{\varepsilon\to 0}{\approx} \frac{1}{a\varepsilon e^{i\varphi}} = \frac{1}{a\varepsilon} e^{-i\varphi}$$



$$|G(s_0)| = \frac{1}{a\varepsilon} \xrightarrow[\varepsilon\to 0]{} \infty$$

$$\angle G(s_0) = -\varphi \in (\underset{E^*}{90°} \quad \underset{F^*}{0°} \quad \underset{A^*}{-90°})$$

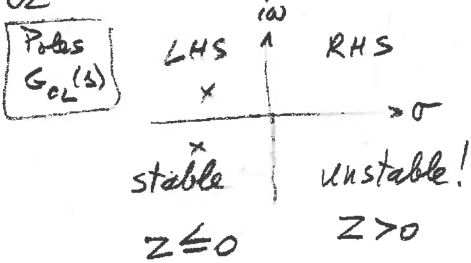
$\varphi = -90°$ $\varphi = 0°$ $\varphi = +90°$

NYQUIST STABILITY CRITERION

Find stability of $G_{CL} = \frac{G}{1+G}$ by analyzing the polar plot of $G(s)$ as s follow the Nyquist circuit.

$$G_{CL}(s) = \frac{G(s)}{1+G(s)}$$

Poles $G_{CL}(s) \Rightarrow$ zeros of $1+G(s)$



Nyquist criterion:

$$\boxed{Z = P + N \leqslant 0}$$

P = number of G(s) poles in RHS

$$G(s) = \frac{B(s)}{A(s)} \longrightarrow p_1, p_2, \dots$$

N = number of clockwise encirclements of the (−1, 0) point as s follows the Nyquist path (circuit)

Z = number of zeros of 1 + G(s) in RHS

STABLE if $Z \leq 0$.

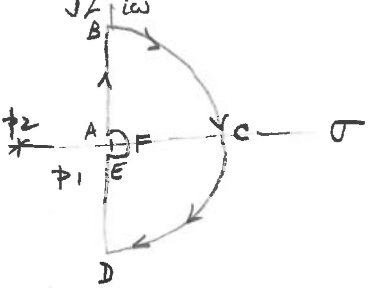
Example : aircraft roll model

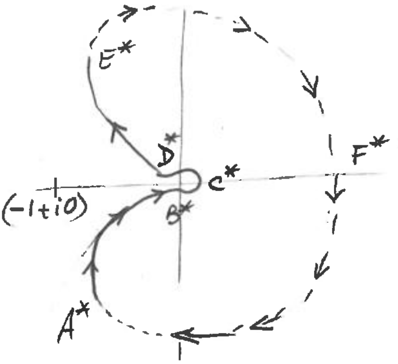
$$G(s) = \frac{114}{10s^2+4s} = \frac{114}{s(10s+4)}$$

$p_1 = 0$, $p_2 = -2/5$: $P = 0$

Recall $G(s) = \frac{1}{s(s+a)} = \frac{1}{s^2+sa}$ no poles in RHS

Nyquist circuit





$P = 0$ no poles in RHS

$N = 0$ no encirclements of $(-1+i0)$ point

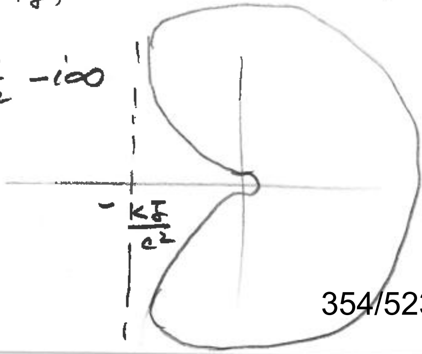
$Z = N + P = 0$ NO CL poles in RHS

Conclusion : system is unconditionally stable

Note : $G = \frac{K}{Js^2+cs} = \frac{K}{J}\frac{1}{s(s+c/J)} = B\frac{1}{s(s+a)}$ $B = K/J$ $a = c/J$

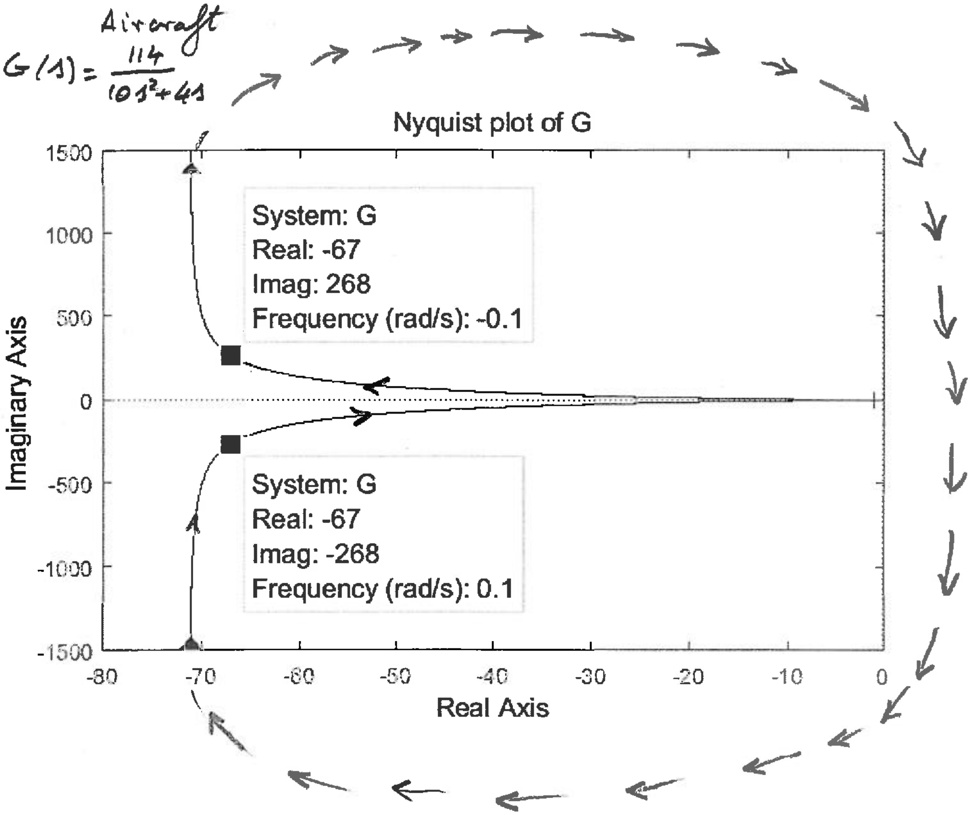
$$G(i\varepsilon) \underset{\varepsilon \to 0}{=} B\left(-\frac{1}{a^2} - i\infty\right) = -\frac{B}{a^2} - i\infty$$

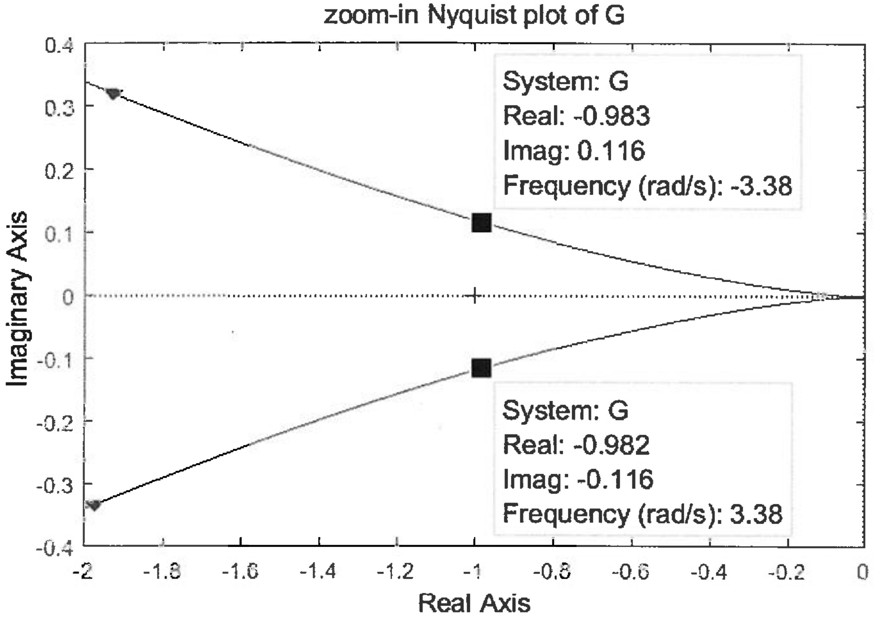
$$\frac{B}{a^2} = \frac{K}{J}\frac{J^2}{c^2} = \frac{KJ}{c^2}$$



Aircraft

$G(s) = \frac{114}{10s^2+4s}$





Example A11.8

Given: $G = K \frac{1}{s-1}$

Note: pole in RHS, $s=1$

Find: critical value of K for stability using Nyquist criterion

Solution: do Nyquist plot of $G(s)$

$G(0) = -\frac{K}{1} = -K$

$G(i\infty) = \lim_{\omega \to \infty} \frac{K}{i\omega - 1} \simeq \frac{K}{i\omega}$, $|G(i\omega)| = 0$

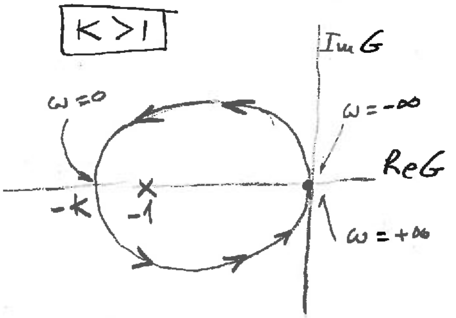
$\angle G(i\omega) = -90^\circ$

$G(-i\infty) \longrightarrow |G(-i\omega)| = 0$

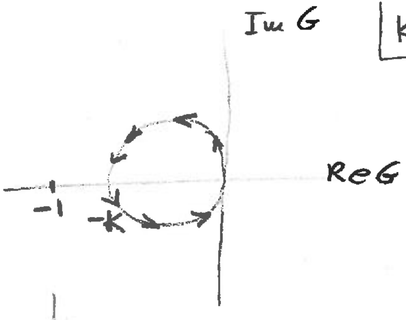
$\angle G(-i\omega) = 90^\circ$

Plot in MATLAB

$\boxed{K>1}$



$\boxed{K<1}$



N = −1 (counterclockwise)

P = 1 (s=1 pole of G in RHS)

Z = −1+1 = 0

STABLE for K>1

N = 0

P = 1

Z = 1

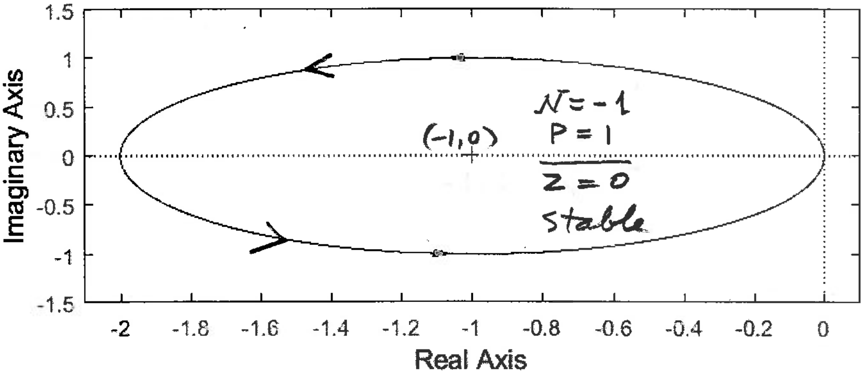
UNSTABLE for K<1

$\boxed{\text{CRITICAL VALUE: } K=1}$

A 11.8

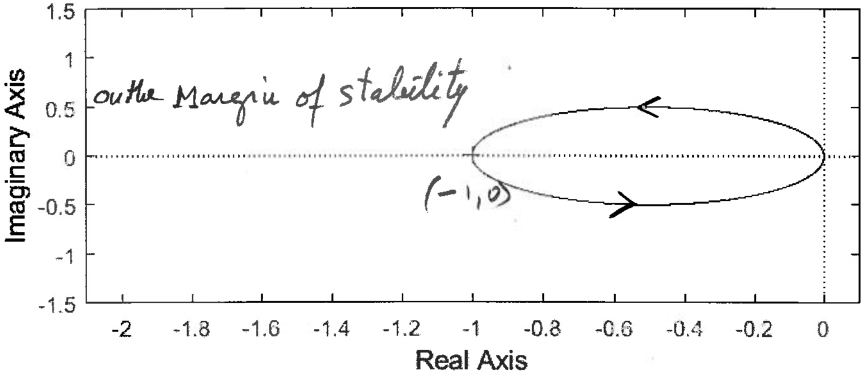
$G = \frac{2}{s-1}$

K=2 STABLE



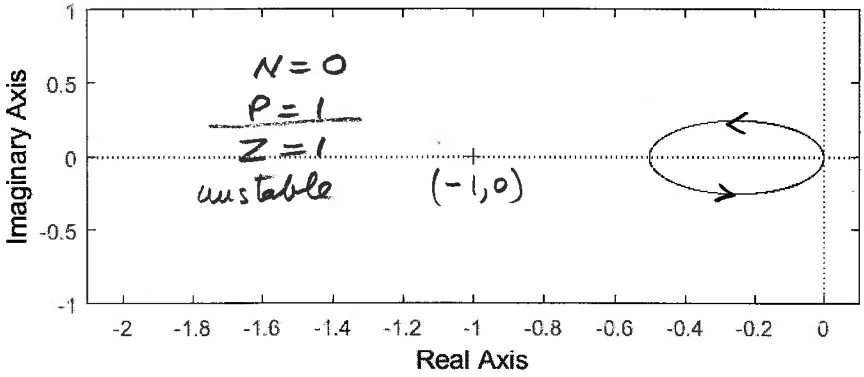
N = -1
P = 1
Z = 0
stable

K=1



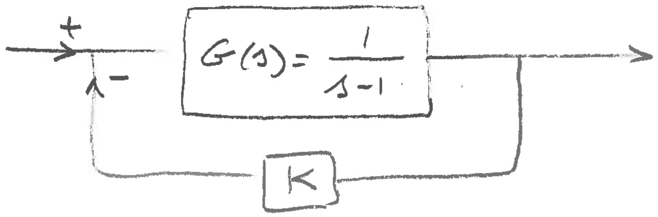
on the Margin of stability

K=0.5 UNSTABLE; Ex.A11-8



N = 0
P = 1
Z = 1
unstable

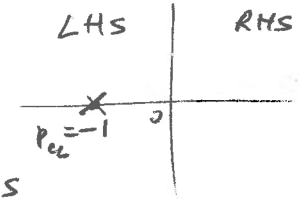
A11.8



$$G_{CL} = \frac{G}{1+GH} = \frac{1}{s-1+K} = \frac{1}{s-(1-K)}$$

$p_{CL} = 1-K$

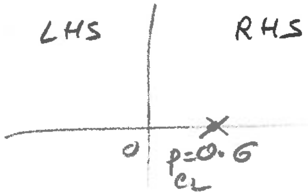
$K = 2$, $p_{CL} = -1$



STABLE

$p_{CL} \in LHS$

$K = 0.4$, $p_{CL} = 0.6$

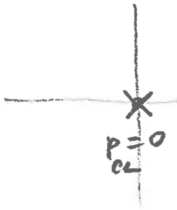


UNSTABLE

$p_{CL} \in RHS$

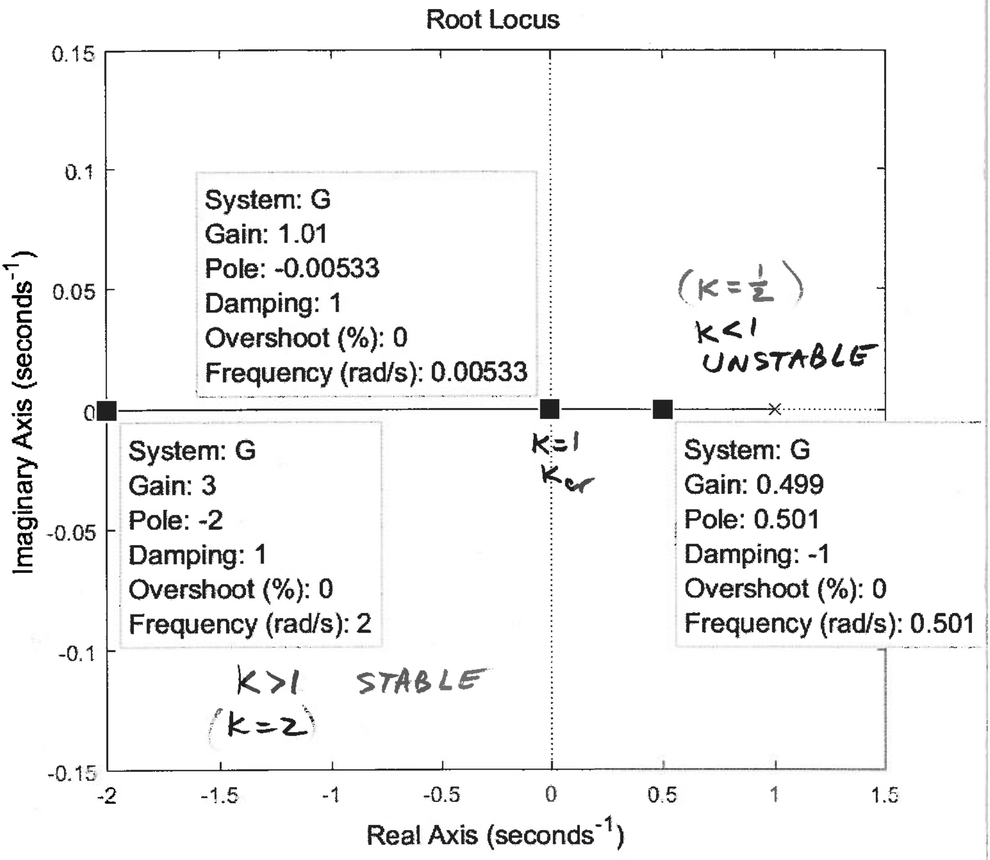
$p_{CL} = 1 - K_{cr} = 0$

$K_{cr} = 1$



critical K value

A11.8



$(K = \frac{1}{2})$

$K < 1$ UNSTABLE

$K = 1$

K_{cr}

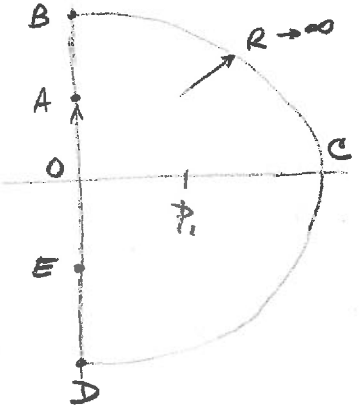
$K > 1$ STABLE

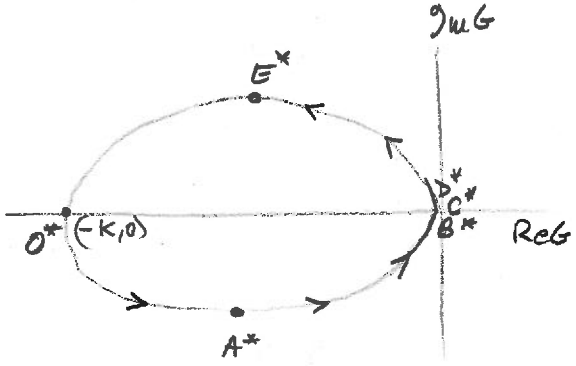
$(K = 2)$

All 8a

MANUAL PLOT

$G = k\frac{1}{s-1}$ $\quad p_1 = 1$





O: $\quad G(0) = K\frac{1}{0-1} = -K = Ke^{i\pi}$

$A = i \quad G(A) = K\frac{1}{i-1} = -K\frac{1+i}{2} = K\frac{\sqrt{2}}{2}e^{i(\pi+\frac{\pi}{4})}$

$\angle G(A) = \frac{5\pi}{4}$

$B = iR \quad G(B) = K\frac{1}{iR-1} = -K\frac{1+iR}{1+R^2} = \frac{K}{\sqrt{1+R^2}}e^{i(\pi+\varphi)}$

$\varphi = \tan^{-1}R \xrightarrow[R\to\infty]{} \frac{\pi}{2}$

$|G(B)| \xrightarrow[R\to\infty]{} 0$

$\angle G(B) \xrightarrow[R\to\infty]{} \frac{3\pi}{2}$

$C = R \quad G(C) = K\frac{1}{R-1} \xrightarrow[R\to\infty]{} 0$

$D = -iR \quad G(D) = K\frac{1}{-iR-1} = -K\frac{1-iR}{1+R^2} = \frac{K}{\sqrt{1+R^2}}e^{i(\pi-\varphi)}$

$\varphi = \tan^{-1}R \xrightarrow[R\to\infty]{} \frac{\pi}{2}$

$|G(D)| \xrightarrow[R\to\infty]{} 0$

$\angle G(D) \xrightarrow[R\to\infty]{} \pi - \frac{\pi}{2} = \frac{\pi}{2}$

A11.8b

$E = -i \qquad G(E) = K\frac{1}{-i-1} = -K\frac{1-i}{2} = K\frac{\sqrt{2}}{2}e^{i(\pi-\frac{\pi}{4})}$

$\angle G(E) = \frac{3\pi}{4}$

Nyquist criterion gives $K<1$ UNSTABLE

$K>1$ STABLE

To verify, calculate the poles of G_{CL}

$$G_{CL} = \frac{G}{1+G} = \frac{K}{s-1+K} = \frac{k}{s-(1-k)}$$

$p_{CL} = 1-K$

for $K<1$, $p_{CL}>0$, in RHS, UNSTABLE

$K>1$, $p_{CL}<0$, in LHS, STABLE

QED

1

p634

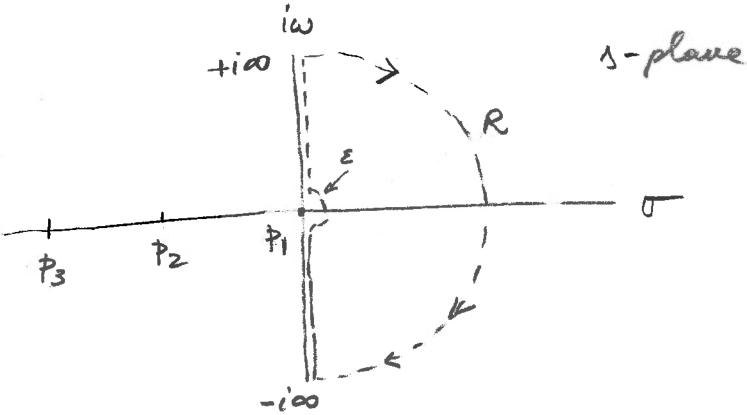
Example p634

$$G(s) = K \frac{1}{s(T_1 s+1)(T_2 s+1)} \qquad T_2 > T_1$$

Poles: $p_1 = 0$

$p_2 = -1/T_1$

$p_3 = -1/T_2$



Nyquist circuit will have to go around the origin on a small circle of radius $\varepsilon \to 0$. We study four segments:

- positive iω axis $s \in (+i0, +i\infty)$
- big circle $R \to \infty$
- negative iω axis $s \in (-i\infty, -i0)$
- small circle $\varepsilon \to 0$

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+iω axis.

$$s = (+i\varepsilon\ ,\quad iR)$$

$\varepsilon \to 0$
$R \to \infty$

$$G(+i\varepsilon) = -k(T_1+T_2) - i\frac{k}{\varepsilon}$$

$$G(+i\varepsilon) \xrightarrow[\varepsilon\to 0]{} -K(T_1+T_2) - i\infty$$

$$|G(i\varepsilon)| \xrightarrow[\varepsilon\to 0]{} \infty$$

$$\angle G(i\varepsilon) \xrightarrow[\varepsilon\to 0]{} -90^\circ$$

Proof

$$\frac{K}{i\varepsilon(i\epsilon T_1+1)(i\varepsilon T_2+1)} = \frac{K}{i\varepsilon[\cancel{i^2\varepsilon^2 T_1 T_2} + i\varepsilon(T_1+T_2) + 1]} =$$

$$= \frac{K}{i\varepsilon}\ \frac{1}{i\varepsilon(T_1+T_2)+1}$$

$$= \frac{K}{i\varepsilon}\ \frac{-i\varepsilon(T_1+T_2)+1}{\cancel{\varepsilon^2(T_1+T_2)}+1}$$

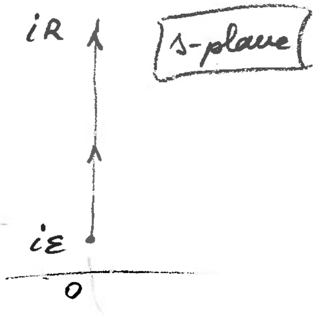
$$= -K(T_1+T_2) - i\frac{K}{\varepsilon}$$

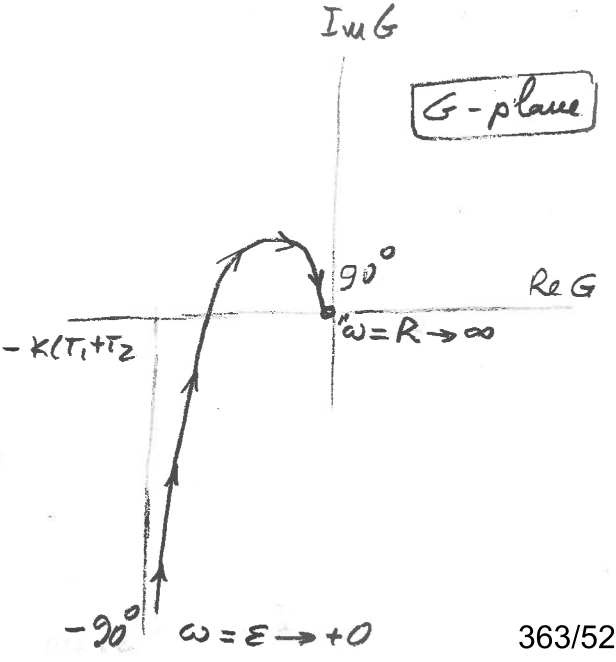
$$G(iR) = \frac{K}{iR(T_1 iR+1)(T_2 iR+1)} \approx \frac{K}{T_1T_2R^3 i^3} = \frac{K}{T_1T_2R^3} e^{i\frac{\pi}{2}}$$

$$\frac{1}{i^3} = \frac{i^4}{i^3} = i = e^{i\frac{\pi}{2}}$$

$$|G(iR)| = \frac{K}{T_1T_2R^3} \xrightarrow[R\to\infty]{} 0$$

$$\angle G(iR) = \frac{\pi}{2} = 90^\circ$$





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p634

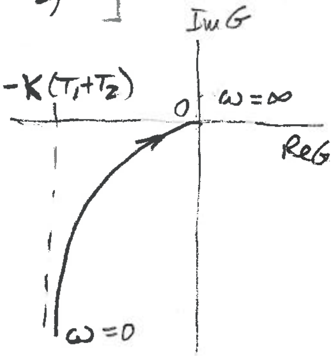
$$G(i\varepsilon) = \frac{K}{i\varepsilon(i\varepsilon T_1+1)(i\omega T_2+1)}$$

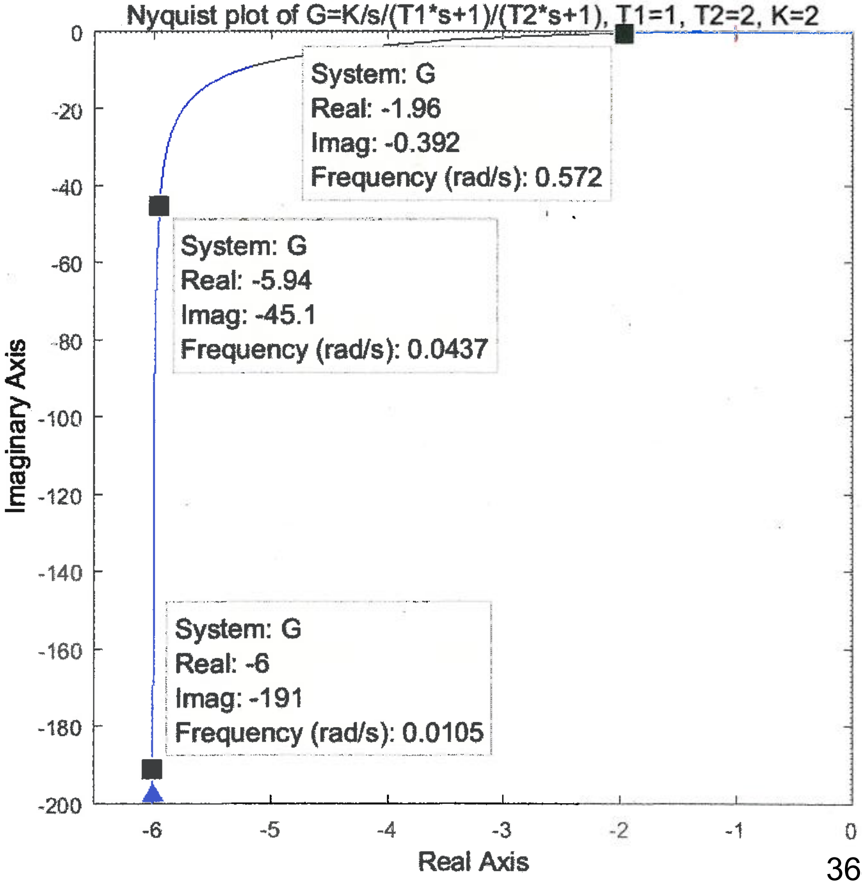
$$= \frac{K}{i\varepsilon\left(\cancel{i^2\varepsilon^2 T_1T_2} + i\varepsilon(T_1+T_2)+1\right)} = \frac{K}{\varepsilon}\,\frac{1}{-\varepsilon(T_1+T_2)+i}$$

$$= \frac{K}{\varepsilon}\,\frac{-\varepsilon(T_1+T_2)-i}{\cancel{\varepsilon^2(T_1+T_2)^2}+1} = -\frac{K}{\varepsilon}\left[\varepsilon(T_1+T_2)+i\right]$$

$$G(i\varepsilon) = -K(T_1+T_2) - i\frac{K}{\varepsilon}$$

$$\xrightarrow[\varepsilon\to 0]{} \underbrace{-K(T_1+T_2)}_{ReG = const} - i\infty$$





4
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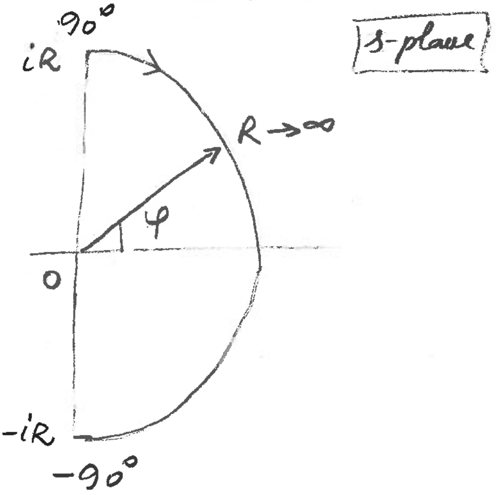
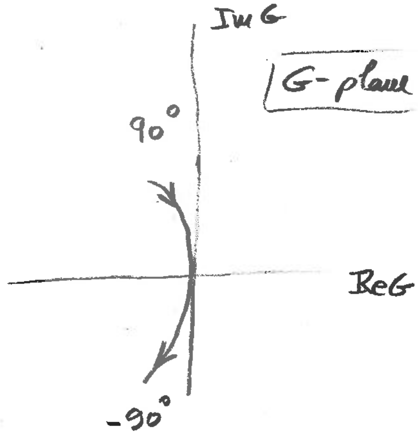
Big circle





$$G(s) = \frac{K}{Re^{i\varphi}(T_1Re^{i\varphi}+1)(T_2Re^{i\varphi}+1)}$$

$$G(s) \underset{R>>1}{\approx} \frac{K}{T_1T_2R^3e^{3i\varphi}} = \frac{K}{T_1T_2R^3}e^{-3i\varphi}$$

$$|G(s)| = \frac{K}{T_1T_2R^3} \xrightarrow[R\to\infty]{} 0$$

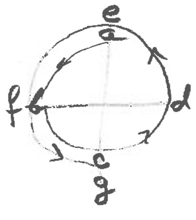
$$\angle G(s) = -3\varphi = (-270^\circ \quad +270^\circ)$$

$\varphi = 90 \quad -90$

$(-270+360=90^\circ;\ 270-360=-90)$

$$\angle G(s) = (90^\circ, -90^\circ)$$

$\varphi =$	90°	60°	30°	0°	-30°	-60°	-90°
$\angle G$	-270	-180	-90	0	90	180	270
	90	180	-90	0	90	180	-90
	a	b	c	d	e	f	g



G(s) goes round origin 1.5 times

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$-i\omega$ axis

$$s = (-iR \quad -i\varepsilon)$$

$\varepsilon \to 0$

$R \to \infty$

$$G(-i\varepsilon) \cong -K(T_1+T_2) + i\frac{K}{\varepsilon}$$

$$G(-i\epsilon) \xrightarrow[\varepsilon\to 0]{} -K(T_1+T_2) + i\infty$$

$$|G(-i\varepsilon)| \xrightarrow[\varepsilon\to 0]{} \infty$$

$$\angle G(-i\varepsilon) \xrightarrow[\varepsilon\to 0]{} 90^\circ$$

Proof

$$\frac{K}{-i\varepsilon(-i\varepsilon T_1+1)(-i\varepsilon T_2+1)} =$$

$$\frac{K}{-i\varepsilon[\cancel{i^2\varepsilon^2T_1T_2} - i\varepsilon(T_1+T_2)+1]}$$

$$= i\frac{K}{\varepsilon}\,\frac{i\varepsilon(T_1+T_2)+1}{\cancel{\varepsilon^2(T_1+T_2)^2}+1}$$

$$= i\frac{K}{\varepsilon}[i\varepsilon(T_1+T_2)+1]$$

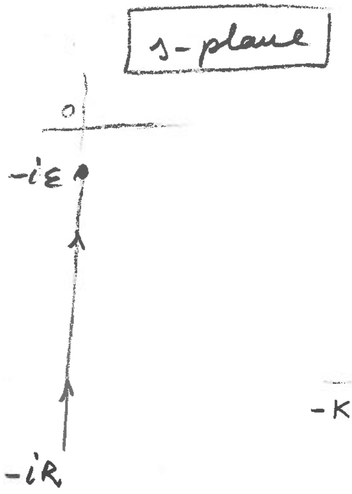
$$= -K(T_1+T_2) + iK/\varepsilon$$

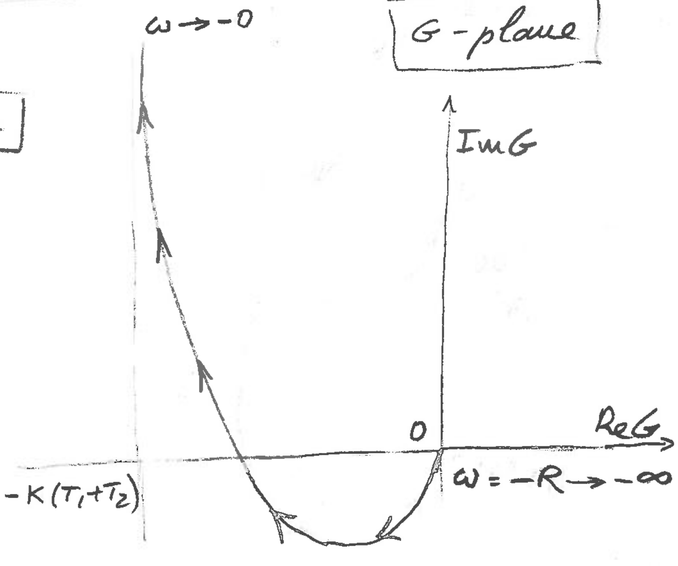
$$G(-iR) = \frac{K}{-iR(-T_1iR+1)(-T_2iR+1)} \approx \frac{K}{-T_1T_2R^3i^3} = \frac{K}{T_1T_2R^3}e^{-i\frac{\pi}{2}}$$

$$\frac{-1}{i^3} = \frac{i^2}{i^3} = \frac{1}{i} = -i$$

$$|G(-iR)| = \frac{K}{T_1T_2R^3} \xrightarrow[R\to\infty]{} 0$$

$$\angle G(iR) = -\frac{\pi}{2} = -90^\circ$$





exists 0 at -90°

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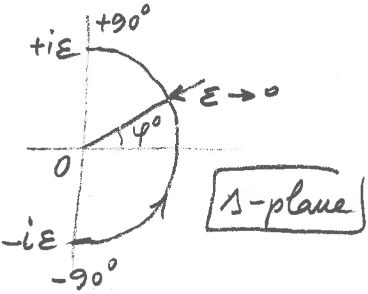
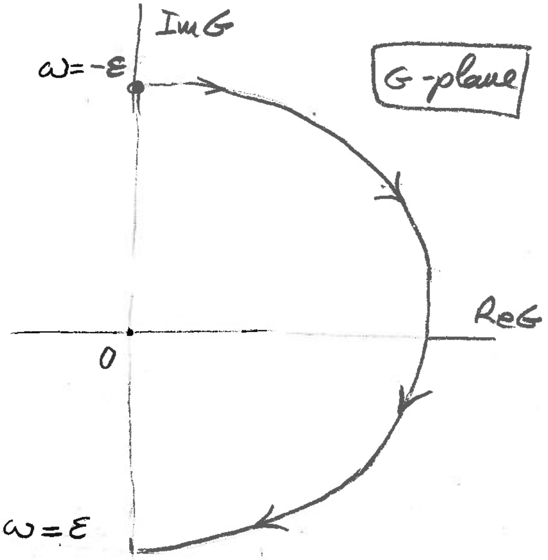
Small circle

$s_0 = \varepsilon e^{i\varphi_0}$

$\varphi_0 = (-90^\circ, 90^\circ)$





$$\varphi_0 = (-90^\circ, 90^\circ)$$

$$G(s_0) = \frac{K}{\varepsilon e^{i\varphi_0}(T_1 \varepsilon e^{i\varphi_0}+1)(T_2 \varepsilon e^{i\varphi_0}+1)}$$

$$G(s_0) \approx \frac{K}{\varepsilon e^{i\varphi_0}} = \frac{K}{\varepsilon} e^{-i\varphi_0}$$

$$|G(s_0)| = \frac{K}{\varepsilon} \xrightarrow[\varepsilon \to 0]{} \infty$$

$$\angle G(s_0) = -\varphi_0 \quad = (90^\circ, -90^\circ)$$

$$\varphi_0 = \underset{-i\varepsilon}{-90^\circ}, \underset{i\varepsilon}{90^\circ})$$

$G(s_0)$ travels a big semicircle from 90° to -90°.

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Finally, assemble the four segments:

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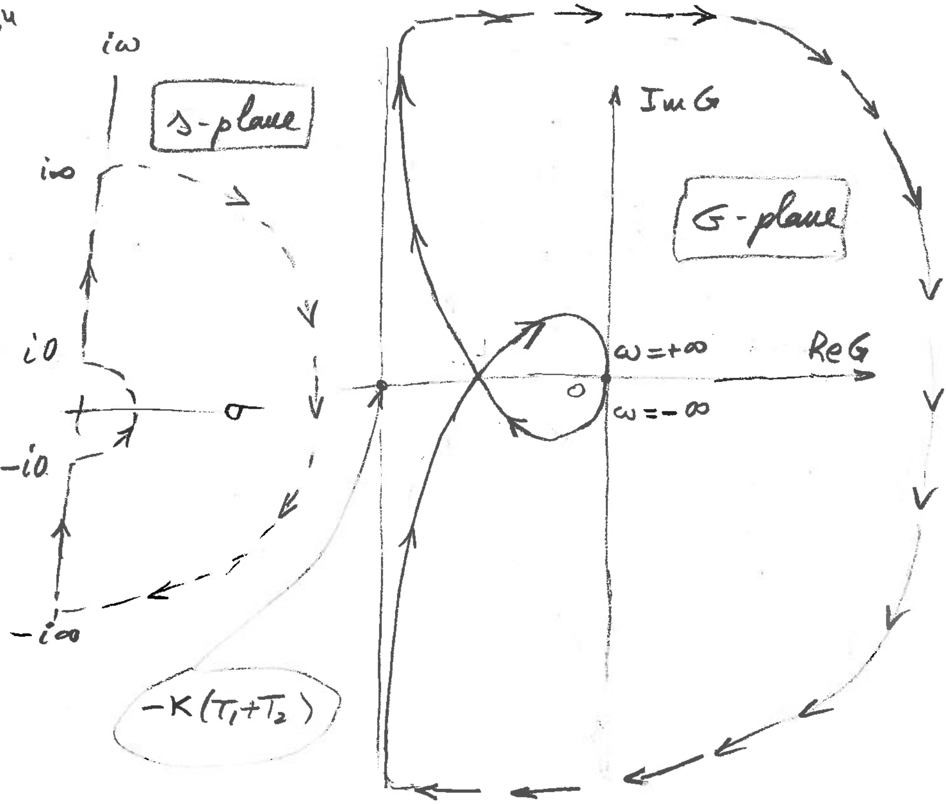
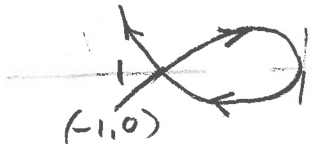
p634



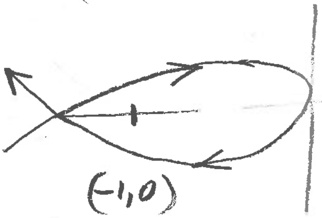
Note that G(s) goes around two times!

Depending on K, it may or may not enclose the point (-1,0).



STABLE

$$\begin{array}{c} N=0 \\ P=0 \\ \hline Z=0 \end{array}$$



UNSTABLE

$$\begin{array}{c} N=2 \\ P=0 \\ \hline Z=2 \end{array}$$

(see MATLAB plot next page)

Ex. p634

STABLE
N = 0
P = 0
Z = N + P = 0

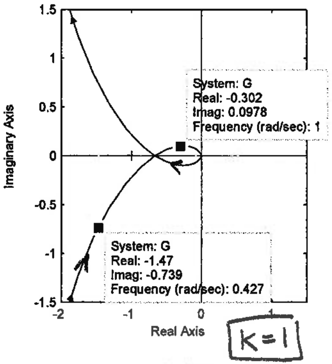
STABLE
N=0
P=0
Z=N+P=0

UNSTABLE
N = 2
P = 0
Z = N+P = 2 UNSTABLE

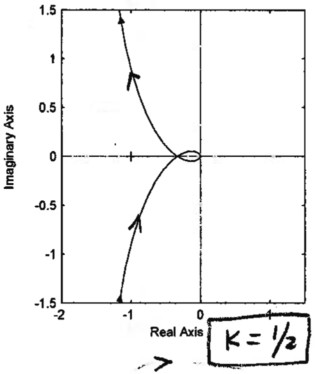
p364

Zoom view

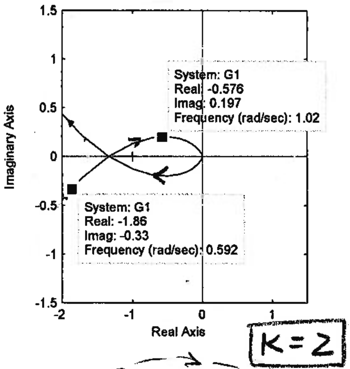
K=1 Nyquist plot; Example pp 634



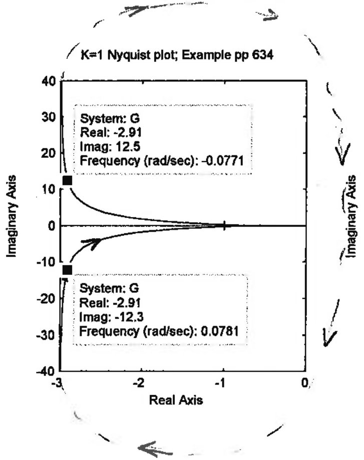
K=0.5 Nyquist plot; Example pp 634

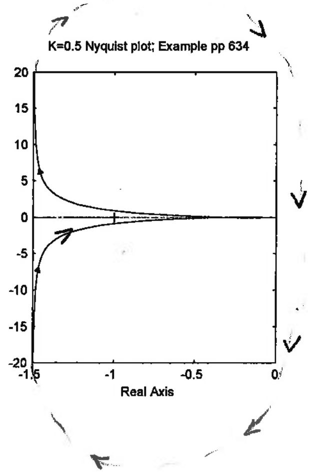


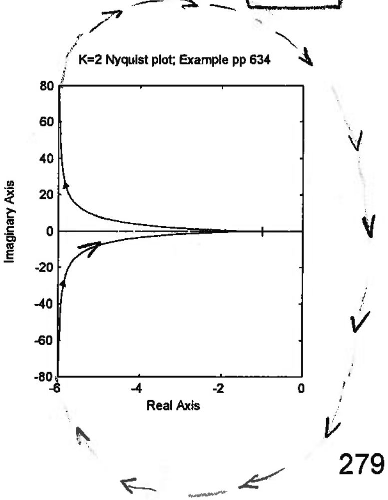
K=2 Nyquist plot; Example pp 634



far view


K=1 Nyquist plot; Example pp 634


K=0.5 Nyquist plot; Example pp 634


K=2 Nyquist plot; Example pp 634

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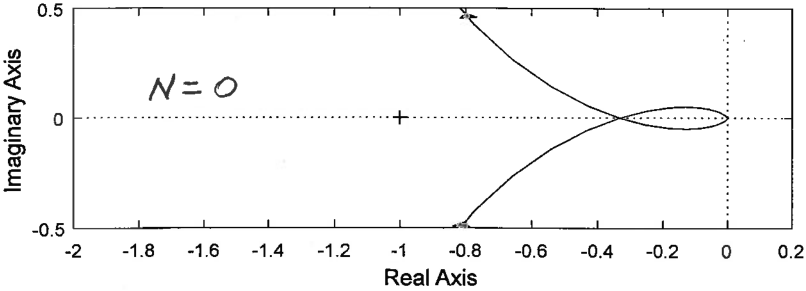
p634

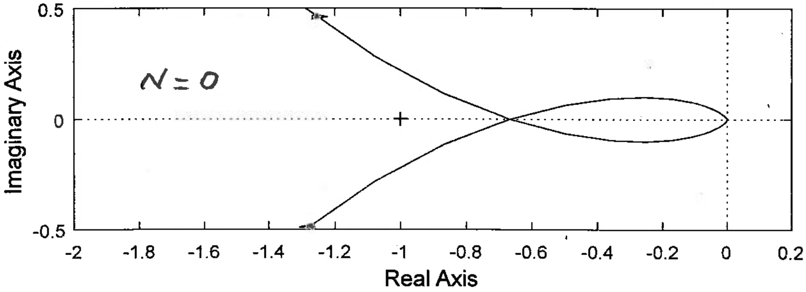
Nyquist zoom-in K=0.5 STABLE



N=0
P=0

Z=0
stable

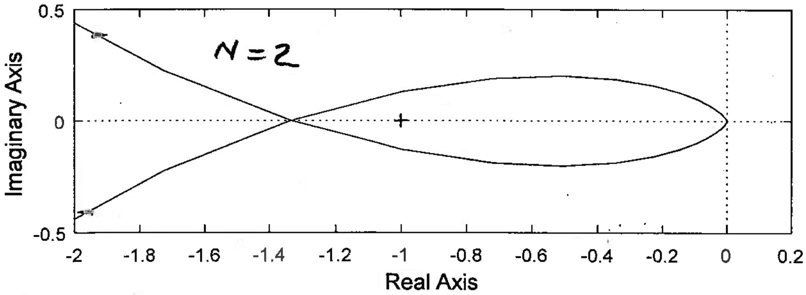
Nyquist zoom-in K=1 STABLE

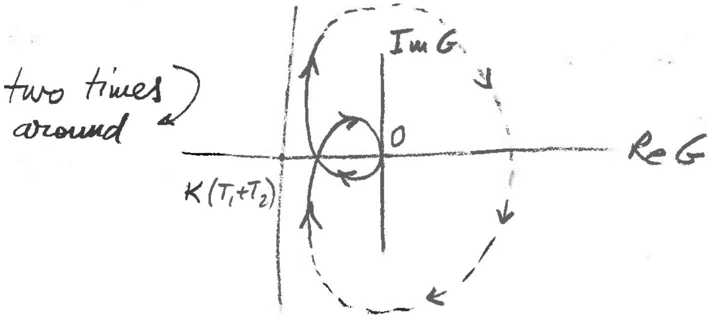


Nyquist zoom-in K=2 UNSTABLE

N=2
P=0

Z=2
unstable



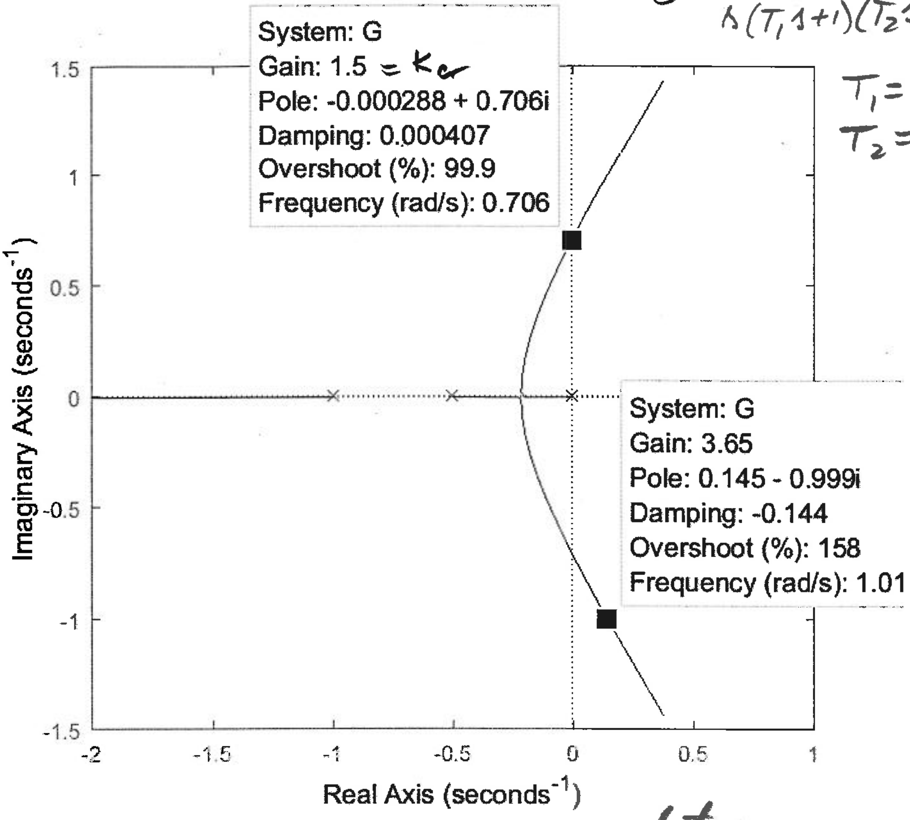


ROOT LOCUS

$$G = \frac{K}{s(T_1 s+1)(T_2 s+1)}$$

$T_1 = 1$

$T_2 = 2$



$K_{cr} = 1.5$ according to r locus plot

verify using $G_{CL} = \frac{G}{1+G} = \frac{K}{s(T_1 s+1)(T_2 s+1)+K}$

$$s(T_1 s+1)(T_2 s+1)+K\Big|_{\substack{T_1=1\\T_2=2}} = s(s+1)(2s+1)+K$$

$$s(2s^2+3s+1)+K = 2s^3+3s^2+s+K$$

Routh criterion

s^3	2	1
s^2	3	K
s^1	$\frac{3-2K}{3}$	
s^0	K	

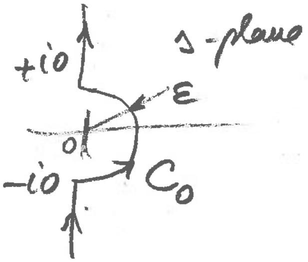
For sign change:
$3-2K=0$

$K_{cr} = \frac{3}{2} = 1.5$

QED

N3d

Tips on Nyquist Plot



$$s_0 = \varepsilon e^{i\varphi}$$

$$-90^\circ < \varphi < 90^\circ$$

$G = \frac{1}{s}$, $G(s_0) = \frac{1}{\varepsilon e^{i\varphi}} = \frac{1}{\varepsilon} e^{-i\varphi}$

(Type 1 system)

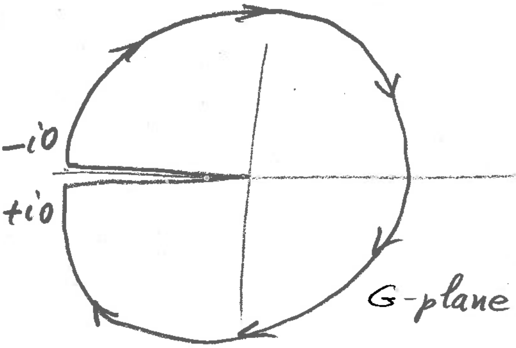
$$+90^\circ > \angle G(s_0) > -90^\circ$$

Half-circle, clockwise.

$G = \frac{1}{s^2}$, $G(s_0) = \frac{1}{(\varepsilon e^{i\varphi})^2} = \frac{1}{\varepsilon^2} e^{-i2\varphi}$

(Type 2 system)

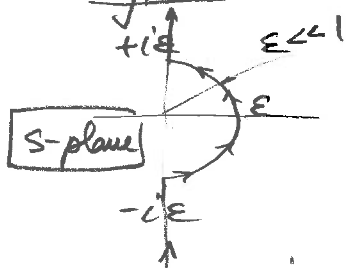
$$+180^\circ > \angle G > -180^\circ .$$



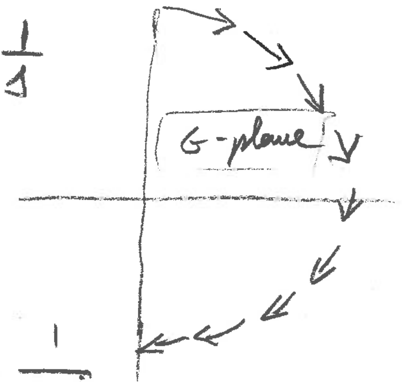
Full circle, clockwise

Details on behavior of common functions on the Nyquist circuit

Type 1



$G(s) = \frac{1}{s}$



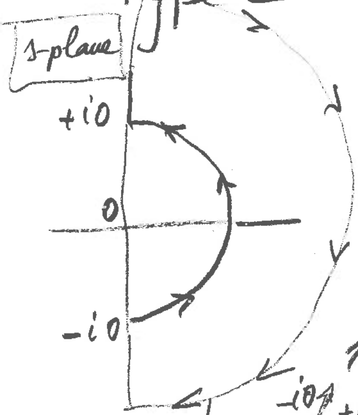
$s_0 = \varepsilon e^{i\varphi}$

$-90° < \varphi < 90°$

$$G(s_0) = \frac{1}{\varepsilon e^{i\varphi}} = \frac{1}{\varepsilon} e^{-i\varphi}$$

$$90° > \angle G(s_0) > -90°$$

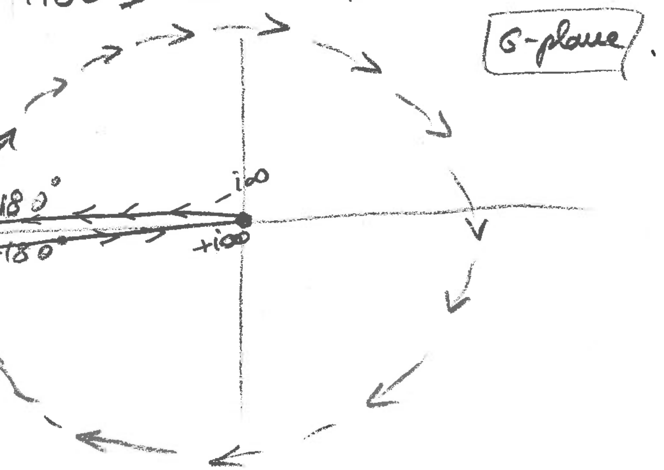
Type 2



$G(s) = \frac{1}{s^2}$

$G(s_0) = \frac{1}{\varepsilon^2} e^{-i2\varphi}$, $-90° < \varphi < +90°$

$$+180° > \angle G(s_0) > -180°$$



$G(-i0) = \frac{1}{(-i0)^2}$

$G(+i0) = \frac{1}{(i0)^2}$

№1a

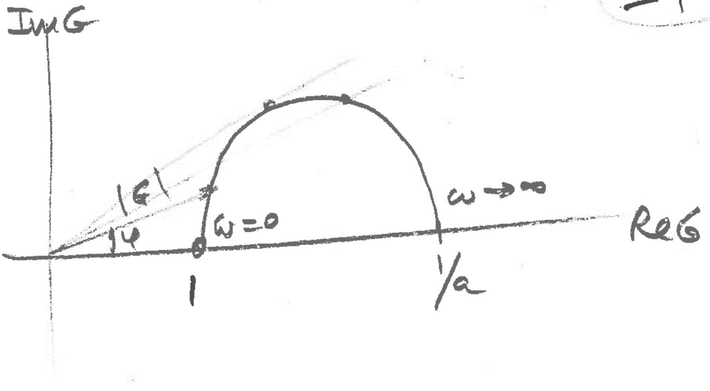
$$G(s) = \frac{Ts+1}{aTs+1} \qquad 0<a<1$$

$$G(i\omega) = \frac{T\cdot(i\omega)+1}{a\cdot T\cdot(i\omega)+1}$$

$\omega \to 0$, $G(i0)\boxed{=1}$

$\omega \to \infty$, $G(i\infty) = \lim\limits_{\omega\to\infty} \frac{T(i\omega)+1}{a\cdot T\cdot(i\omega)+1} = \frac{T}{aT} = \frac{1}{a}$

$\boxed{>1}$



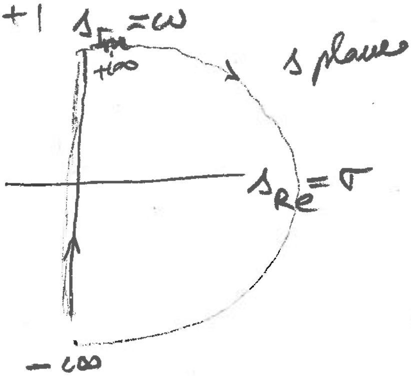
N16. Ex 11.4 , p 641

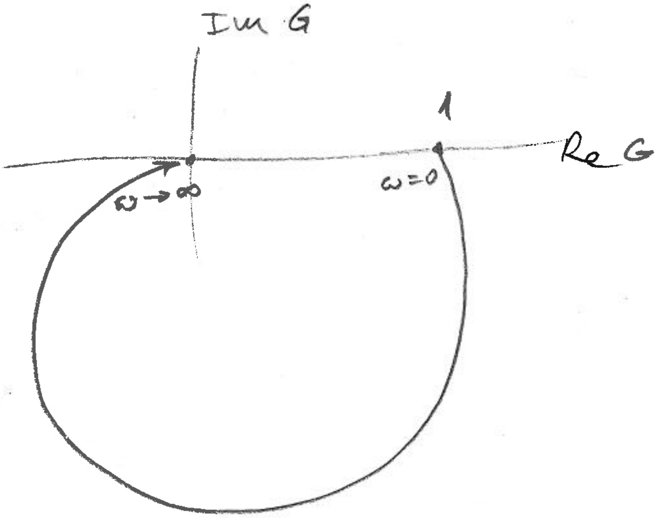
$$G(s) = \frac{1}{s^2+0.8s+1}$$

$G = 1/$— — —

nyquist(G)

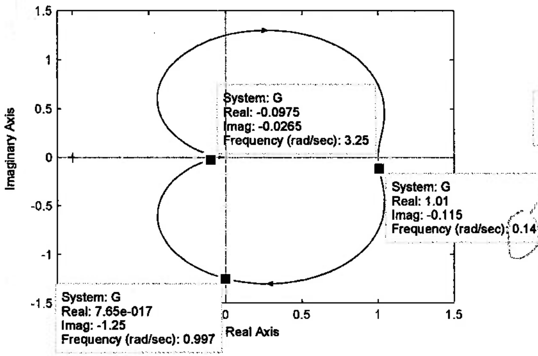
Nyquist circuit for ω



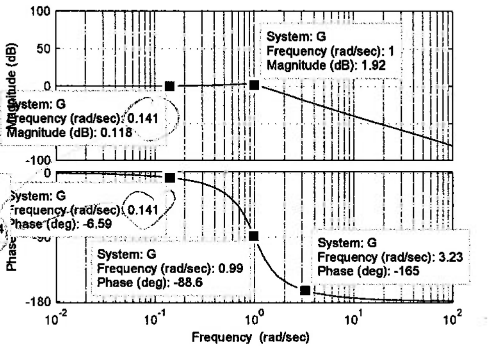


Ex. 11.5 p 642

$$G(s) = \frac{1}{s(s+1)}.$$

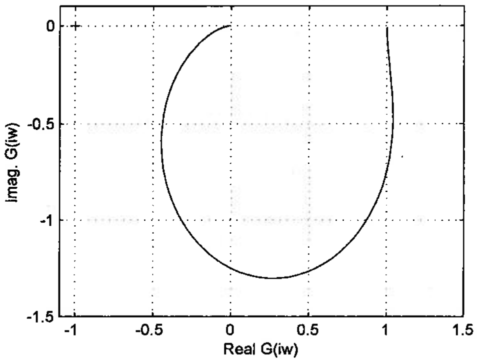


Nyquist plot; Example 11.4 pp 641

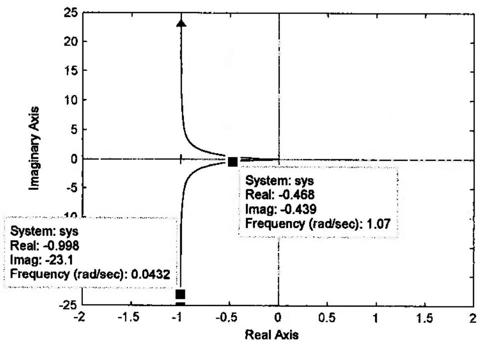


Bode plot; Example 11.4 pp 641

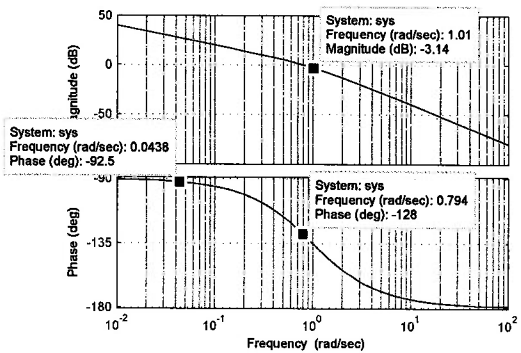
Re-Im plot; Example 11.4 pp 641



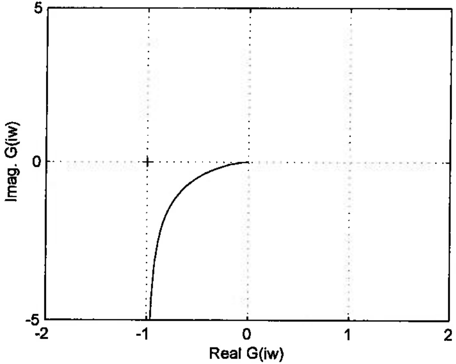
$$G(s) = \frac{1}{s^2+0{,}8s+1}$$



Nyquist plot; Example 11.5 pp 642



Bode plot; Example 11.5 pp 642



Re-Im plot; Example 11.5 pp 642

$$G(s) = \frac{1}{s(s+1)}.$$

N2

Nyquist plot trends

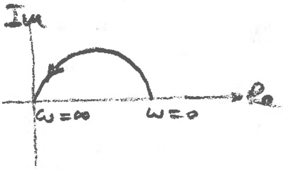
$$G(s) = \frac{K}{s^N} \frac{(T_a s+1)(T_b s+1)\cdots}{(T_1 s+1)(T_2 s+1)\cdots}$$

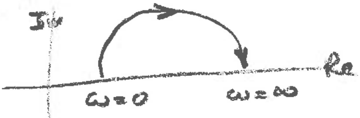
Type 0 systems $(N = 0)$, $G(i\omega) = K \dfrac{\overbrace{(T_a i\omega+1)(T_b i\omega+1)\cdots}^{\text{order } m}}{\underbrace{(T_1 i\omega+1)(T_2 i\omega+1)\cdots}_{\text{order } n}}$

$\omega = 0$ $\quad G(i0) = K$

$\omega \to \infty$

$$G(i\infty) = \begin{cases} 0 & \text{for } n > m \\ \text{const}, & n = m \\ \infty, & n < m \end{cases}$$

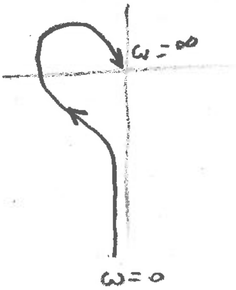


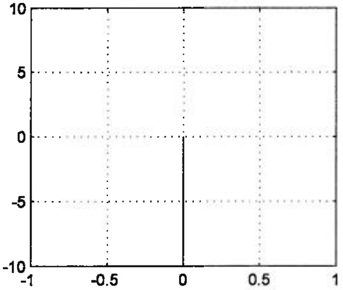


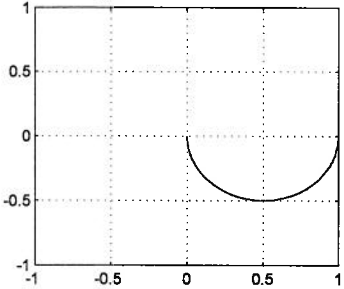
Type 1 systems $(N=1)$, $G(i\omega) = \dfrac{K}{i\omega} \dfrac{\overbrace{(\quad)(\quad)}^{m}}{\underbrace{(\quad)(\quad)}_{n}}$

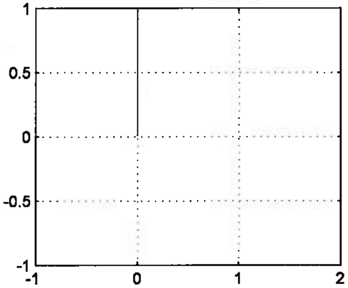
for $n = m$, $G(i\infty) = 0$

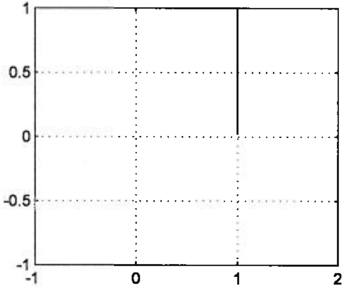
$G(i0) = \dfrac{1}{i0}$, $\angle G(i0) = -90^\circ$

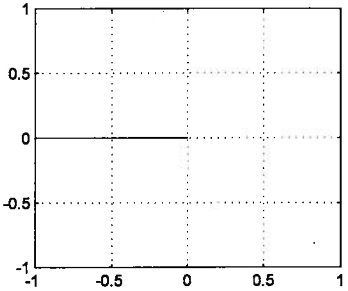


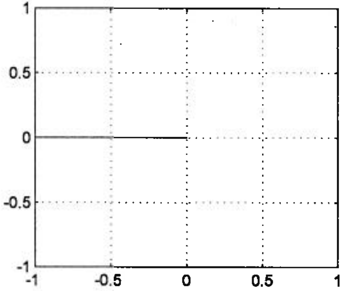

Nyquist plot of G=1/s

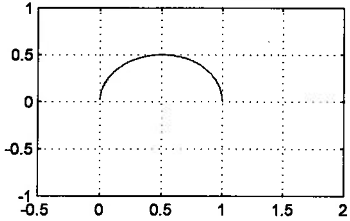

Nyquist plot of G=1/(Ts+1)

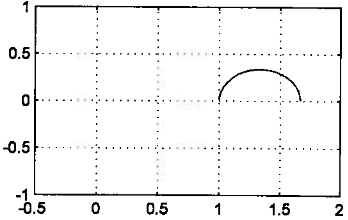

Nyquist plot of G=s

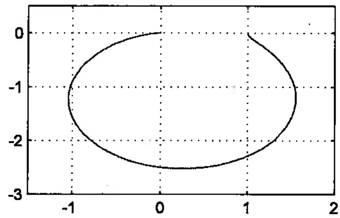

Nyquist plot of G=Ts+1

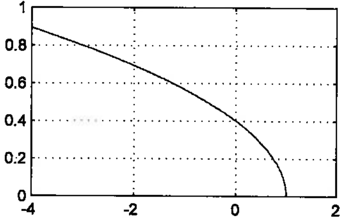

Nyquist plot of $G=1/s^2$

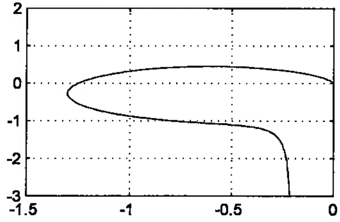

Nyquist plot of $G=1/s^2$

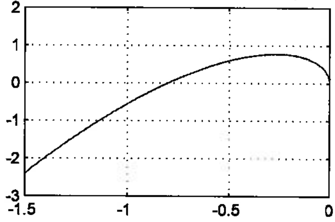

Nyquist plot of $G=Ts/(Ts+1)$

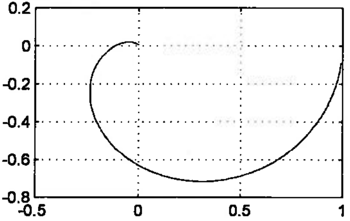

Nyquist plot of $G=(Ts+1)/(aTs+1)$

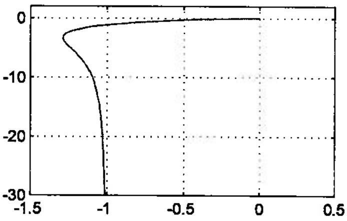

Nyquist plot of $G=wn^2/(s^2+2*z*wn*s+wn^2)$

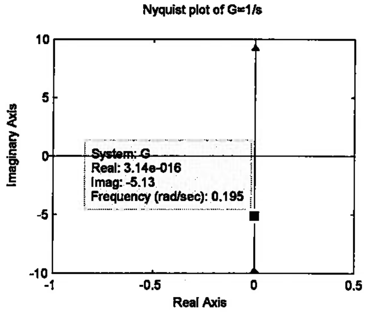

Nyquist plot of $G=(s^2+2*z*wn*s+wn^2)/wn^2$

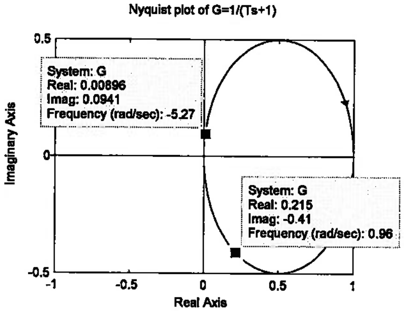

Nyquist plot of $G=wn^2/s/(s^2+2*z*wn*s+wn^2)$

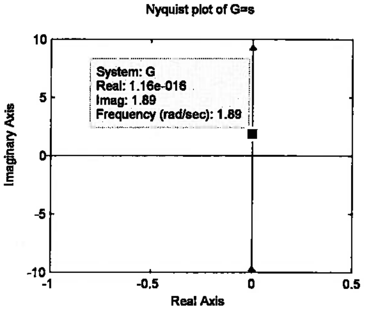

Nyquist plot of $G=s*(s^2+2*z*wn*s+wn^2)/wn^2$

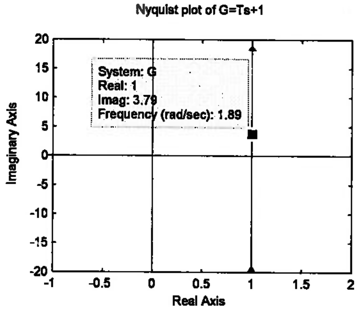

Nyquist plot of $G=1/(T1*s+1)/(T2*s+1)/(T3*s+1)$

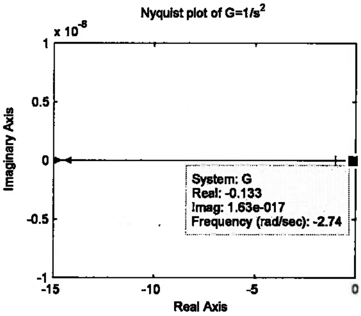

Nyquist plot of $G=(T1*s+1)/s/(T2*s+1)/(T3*s+1)$

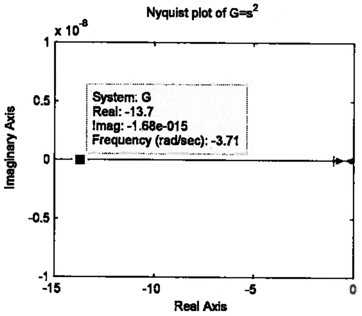

Nyquist plot of G=1/s

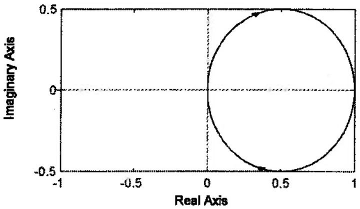

Nyquist plot of G=1/(Ts+1)


Nyquist plot of G=s

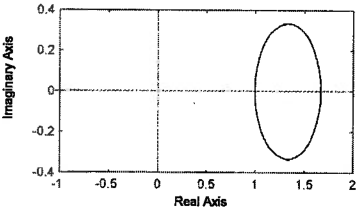

Nyquist plot of G=Ts+1


Nyquist plot of G=1/s^2

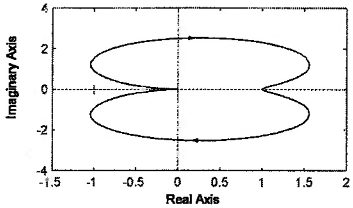

Nyquist plot of G=s^2



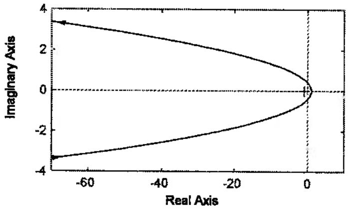
Nyquist plot of $G=Ts+1$



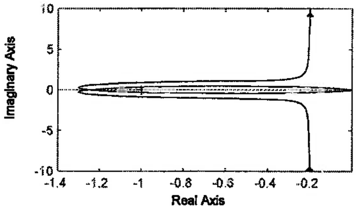
Nyquist plot of $G=(Ts+1)/(aTs+1)$



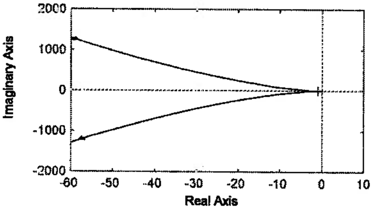
Nyquist plot of $G=wn^2/(s^2+2*z*wn*s+wn^2)$



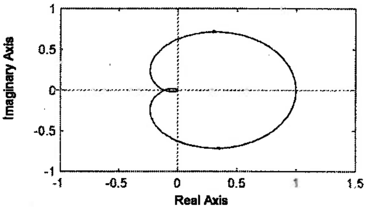
Nyquist plot of $G=(s^2+2*z*wn*s+wn^2)/wn^2$



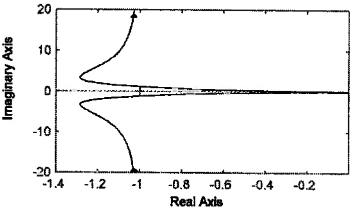
Nyquist plot of $G=wn^2/s/(s^2+2*z*wn*s+wn^2)$



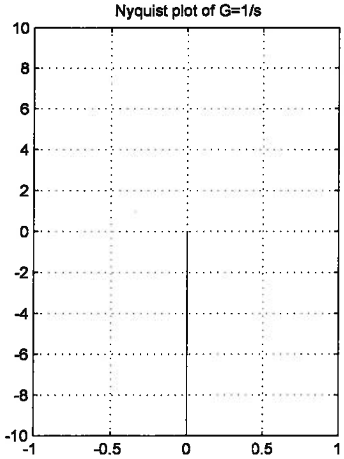
Nyquist plot of $G=s*(s^2+2*z*wn*s+wn^2)/wn^2$

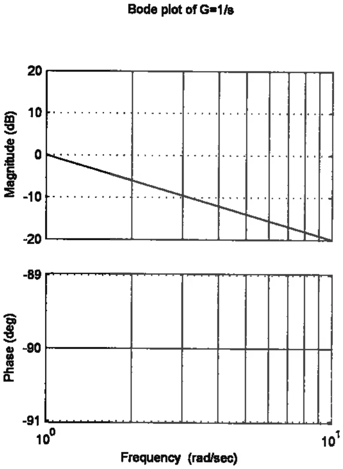


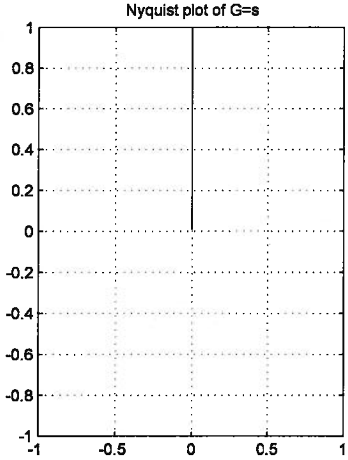
Nyquist plot of $G=1/(T1*s+1)/(T2*s+1)/(T3*s+1)$

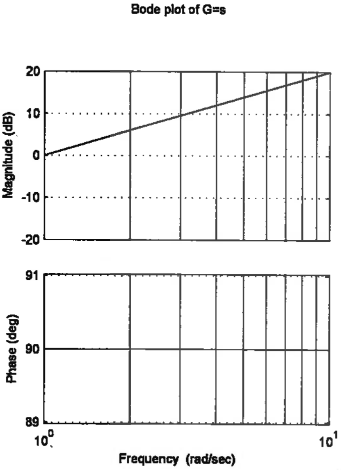


Nyquist plot of $G=(T1*s+1)/s/(T2*s+1)/(T3*s+1)$

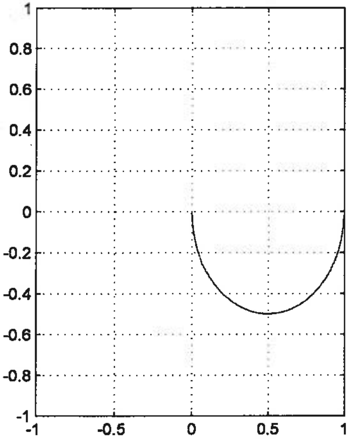

Nyquist plot of G=1/s


Bode plot of G=1/s

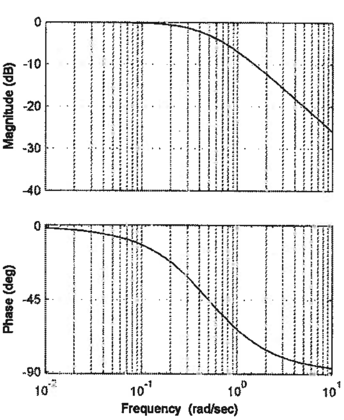

Nyquist plot of G=s


Bode plot of G=s

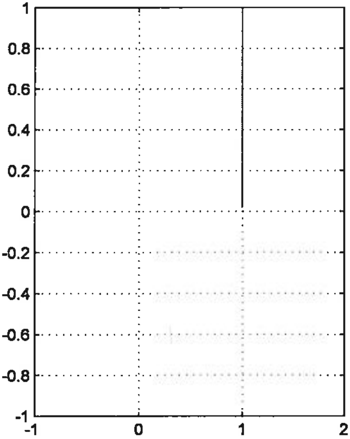
Nyquist plot of G=1/(Ts+1)



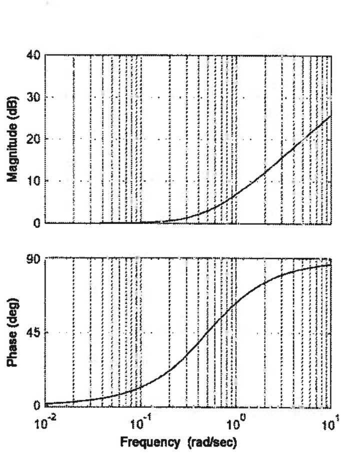
Bode plot of G=1/(T*s+1)

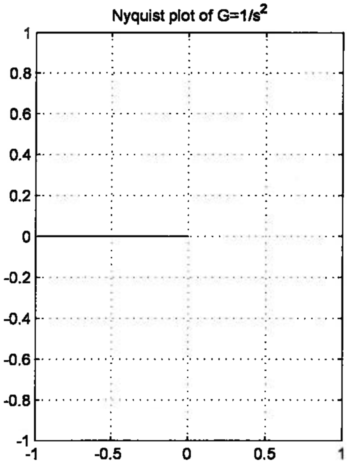


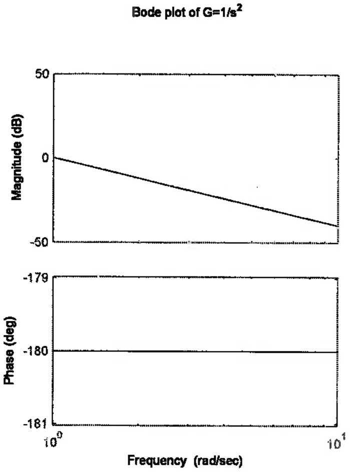
Nyquist plot of G=Ts+1

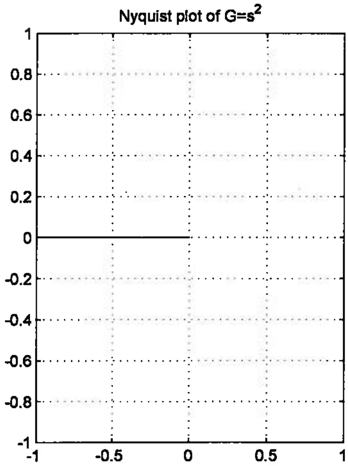


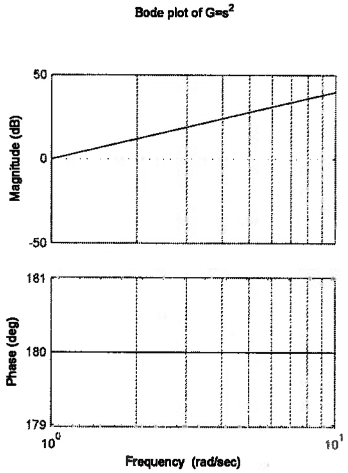
Bode plot of G=T*s+1

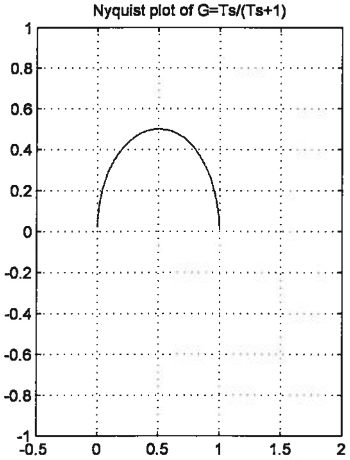


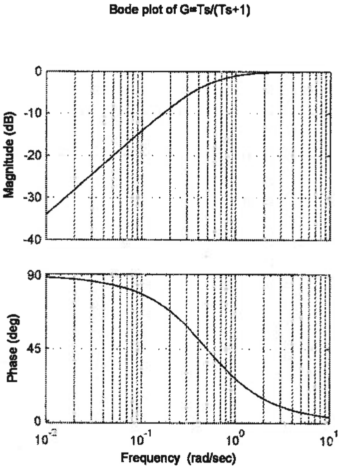

Nyquist plot of $G=1/s^2$

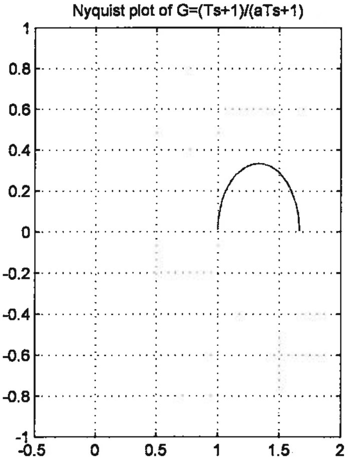

Bode plot of $G=1/s^2$

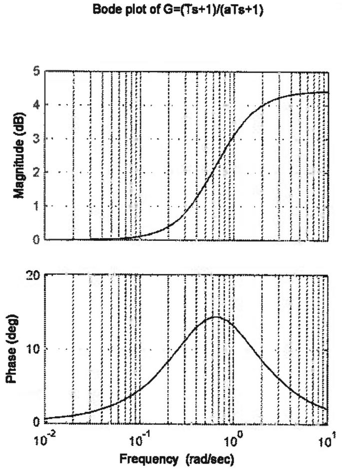

Nyquist plot of $G=s^2$

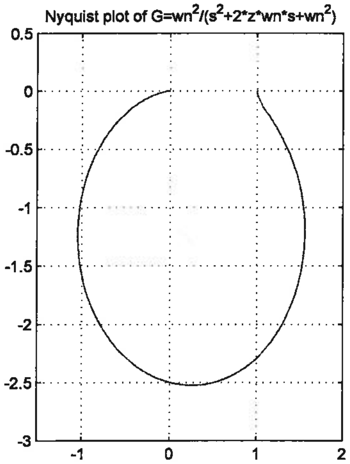

Bode plot of $G=s^2$


Nyquist plot of G=Ts/(Ts+1)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0
0.5
1
1.5
2

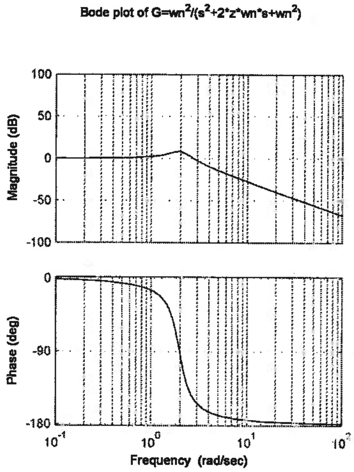

Bode plot of G=Ts/(Ts+1)
Magnitude (dB)
0
-10
-20
-30
-40
Phase (deg)
90
45
0
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)


Nyquist plot of G=(Ts+1)/(aTs+1)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0
0.5
1
1.5
2

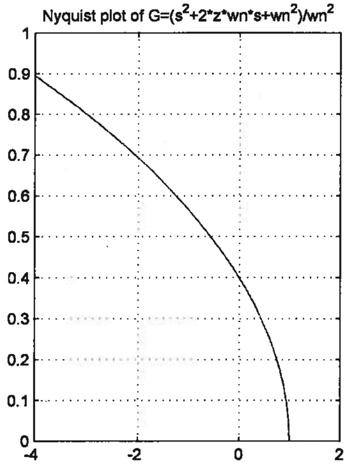

Bode plot of G=(Ts+1)/(aTs+1)
Magnitude (dB)
5
4
3
2
1
0
Phase (deg)
20
10
0
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)



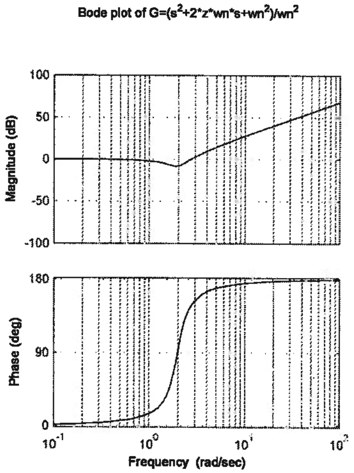
Nyquist plot of $G=wn^2/(s^2+2*z*wn*s+wn^2)$



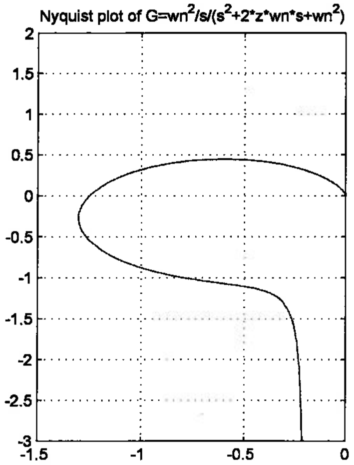
Bode plot of $G=wn^2/(s^2+2*z*wn*s+wn^2)$

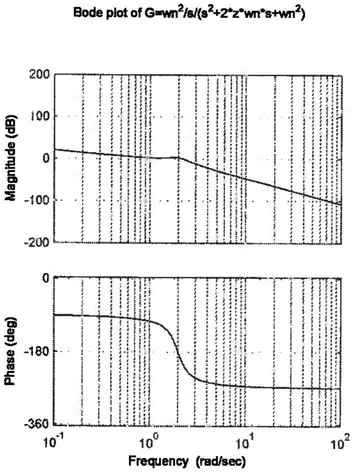


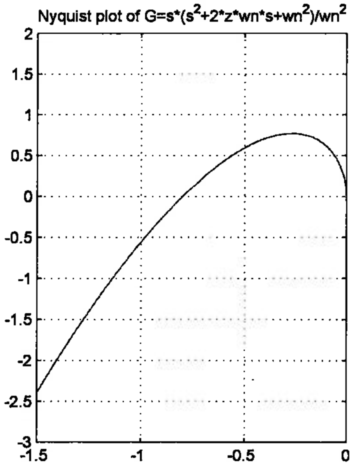
Nyquist plot of $G=(s^2+2*z*wn*s+wn^2)/wn^2$

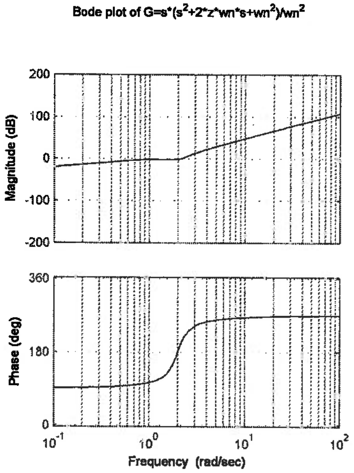


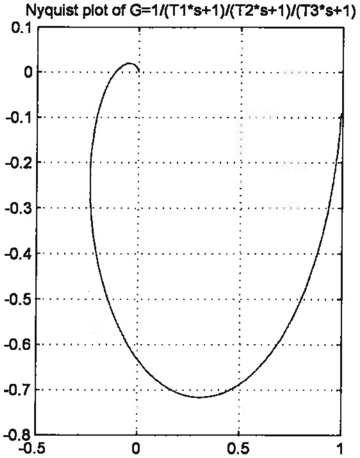
Bode plot of $G=(s^2+2*z*wn*s+wn^2)/wn^2$

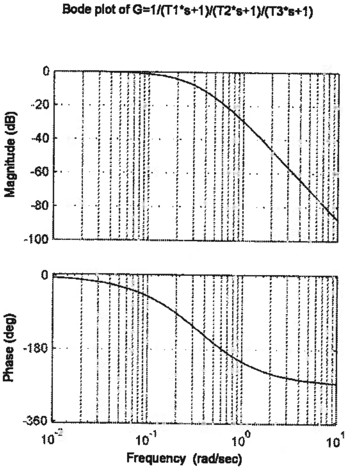

Nyquist plot of G=wn^2/s/(s^2+2*z*wn*s+wn^2)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-1.5
-1
-0.5
0

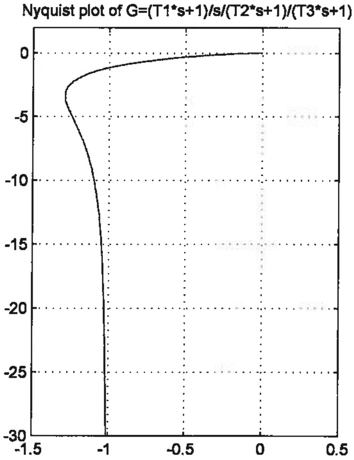

Bode plot of G=wn^2/s/(s^2+2*z*wn*s+wn^2)
Magnitude (dB)
200
100
0
-100
-200
Phase (deg)
0
-180
-360
10^-1
10^0
10^1
10^2
Frequency (rad/sec)

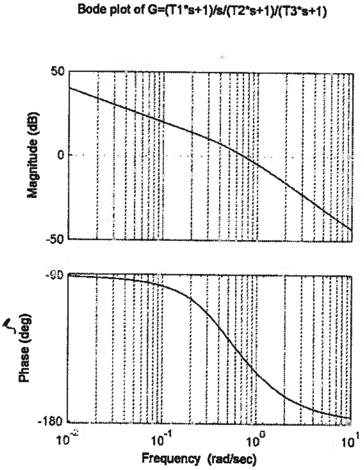

Nyquist plot of G=s*(s^2+2*z*wn*s+wn^2)/wn^2
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-1.5
-1
-0.5
0


Bode plot of G=s*(s^2+2*z*wn*s+wn^2)/wn^2
Magnitude (dB)
200
100
0
-100
-200
Phase (deg)
360
180
0
10^-1
10^0
10^1
10^2
Frequency (rad/sec)


Nyquist plot of G=1/(T1*s+1)/(T2*s+1)/(T3*s+1)


Bode plot of G=1/(T1*s+1)/(T2*s+1)/(T3*s+1)


Nyquist plot of G=(T1*s+1)/s/(T2*s+1)/(T3*s+1)


Bode plot of G=(T1*s+1)/s/(T2*s+1)/(T3*s+1)