1
PID

(See SIMULINK first)

PID Control

Aircraft model

$K = 114$

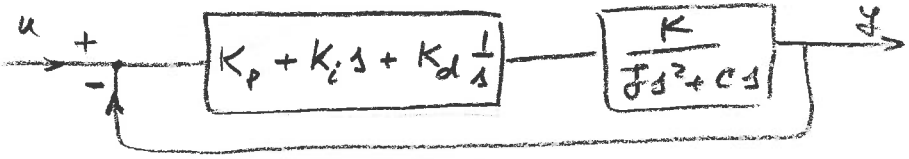
$J = 10$

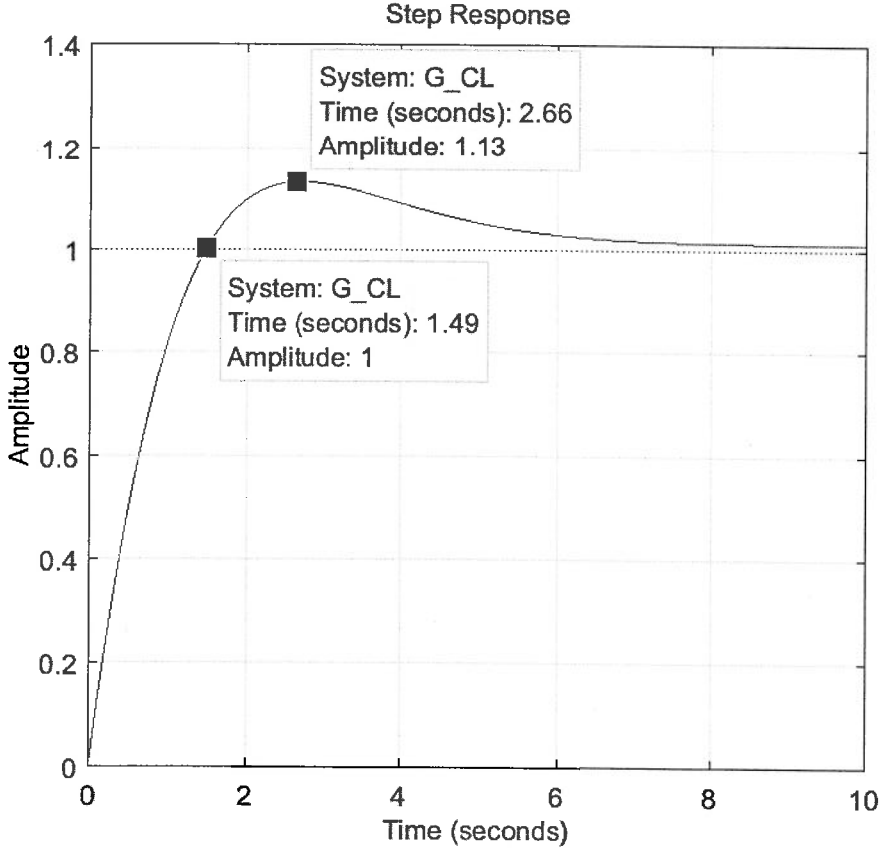
$c = 4$

$K_p = 0.094$

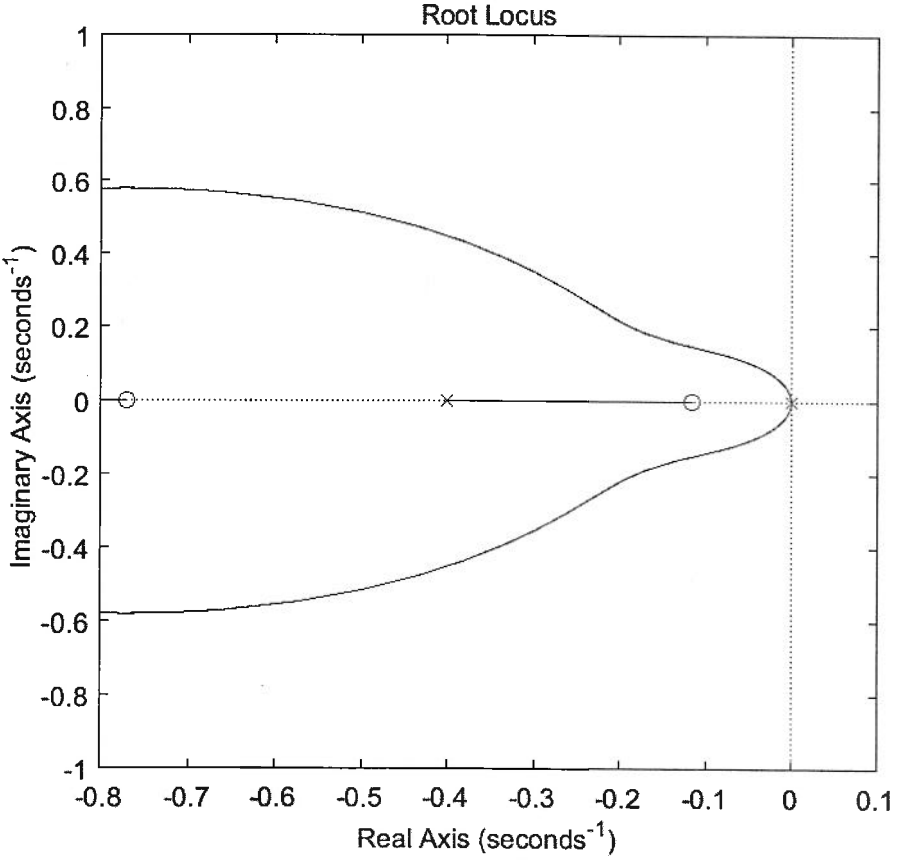
$K_i = 0.0094241$

$K_d = 0.10623$



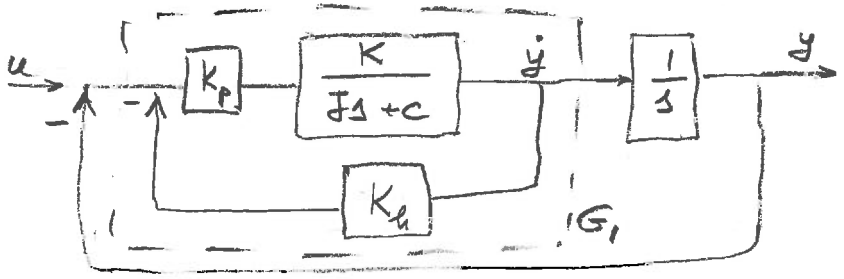


PID control
Root locus

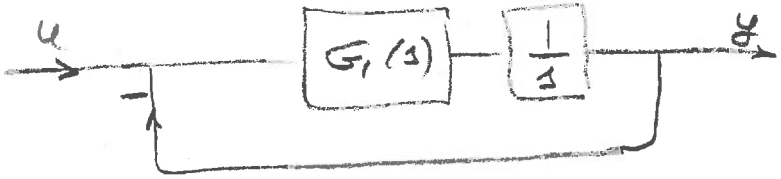


Velocity Feedback P-Controller

$$G(s) = \frac{K}{Js^2 + cs} = \frac{K}{Js + c} \cdot \frac{1}{s}$$



$$G_1(s) = \frac{\frac{K_p K}{Js + c}}{1 + \frac{K_p K}{Js + c} K_h} = \frac{K_p K}{Js + c + K_p K K_h} \qquad (1)$$



$$G_2 = G_1(s)\frac{1}{s} = \frac{K_p K}{Js^2 + (c + K_p K K_h)s} \qquad (2)$$

$$G_{CL} = \frac{G_2}{1 + G_2} = \frac{K_p K}{Js^2 + (c + K_p K K_h)s + K_p K}$$

c^*

$$G_{CL} = \frac{K_p K}{Js^2 + c^* s + K_p K} \qquad (3)$$

2

$$c^* = c + K_p K K_h \qquad (4)$$

$$G_{CL} = \frac{\frac{K_p K}{J}}{s^2 + \frac{c^*}{J} s + \frac{K_p K}{J}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \frac{K_p K}{J} \rightarrow \omega_n = \sqrt{\frac{K_p K}{J}} \qquad (5)$$

$$2\zeta\omega_n = \frac{c^*}{J} \rightarrow \zeta = \frac{c^*}{2\omega_n J} = \frac{c^*}{2\sqrt{K_p K J}}$$

$$\zeta = \frac{c + K_p K K_h}{2\sqrt{K_p K J}}$$

- Velocity feedback gain K_h increases ζ (damping)
- P-control gain K_p modifies both freq. ω_n and damping ζ

Strategy

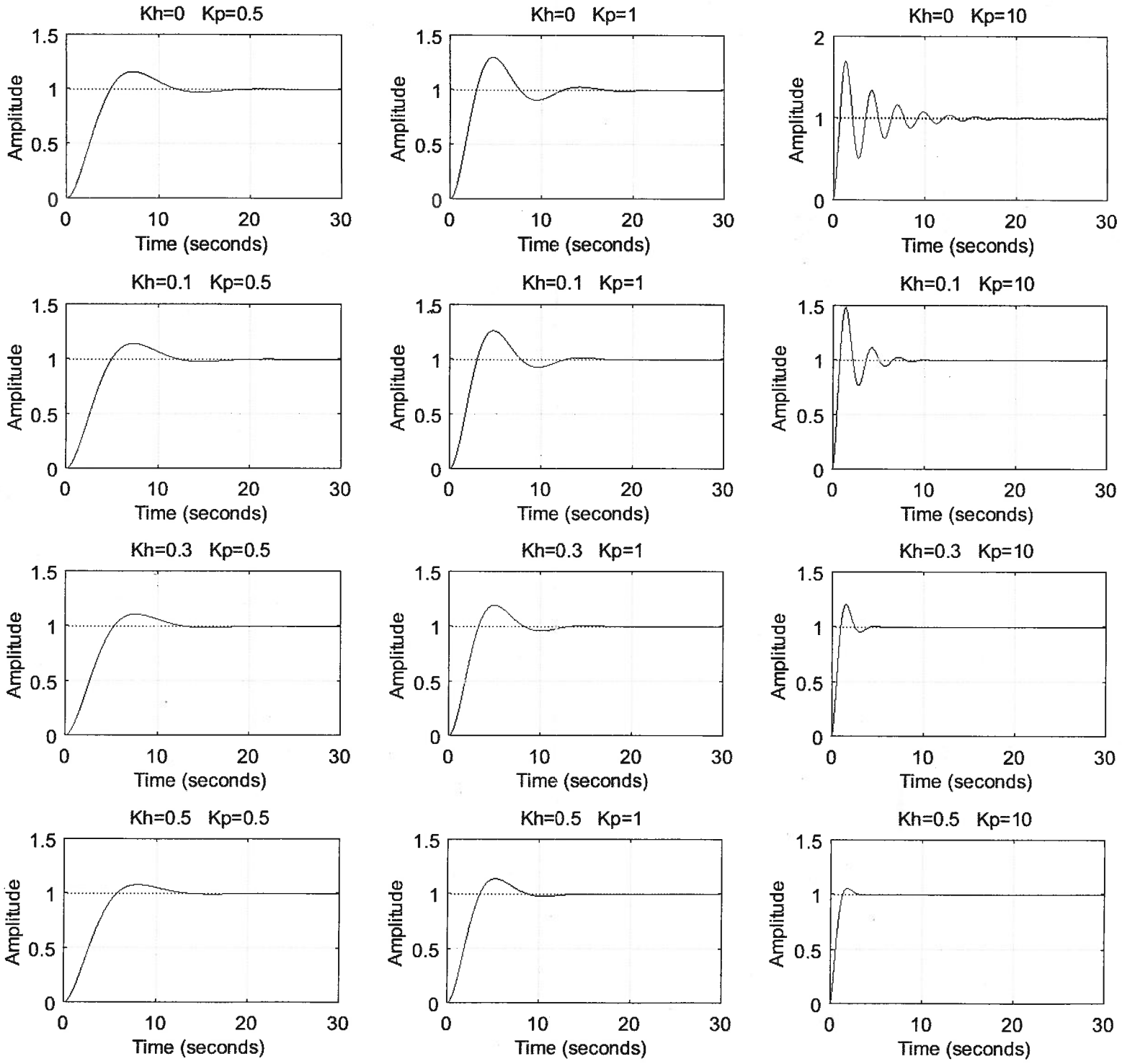
- Modify frequency with K_p
- add more damping with K_h

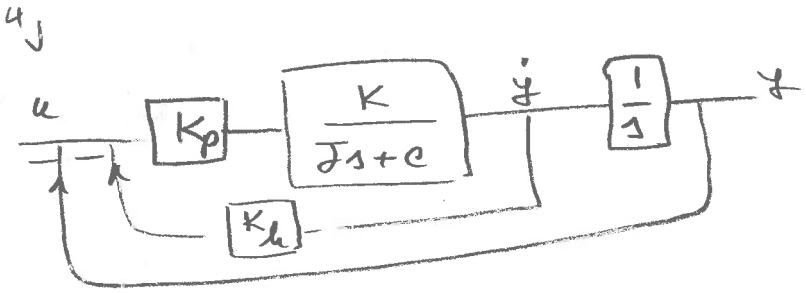
3J

class example.

K=1
J=2
c=1

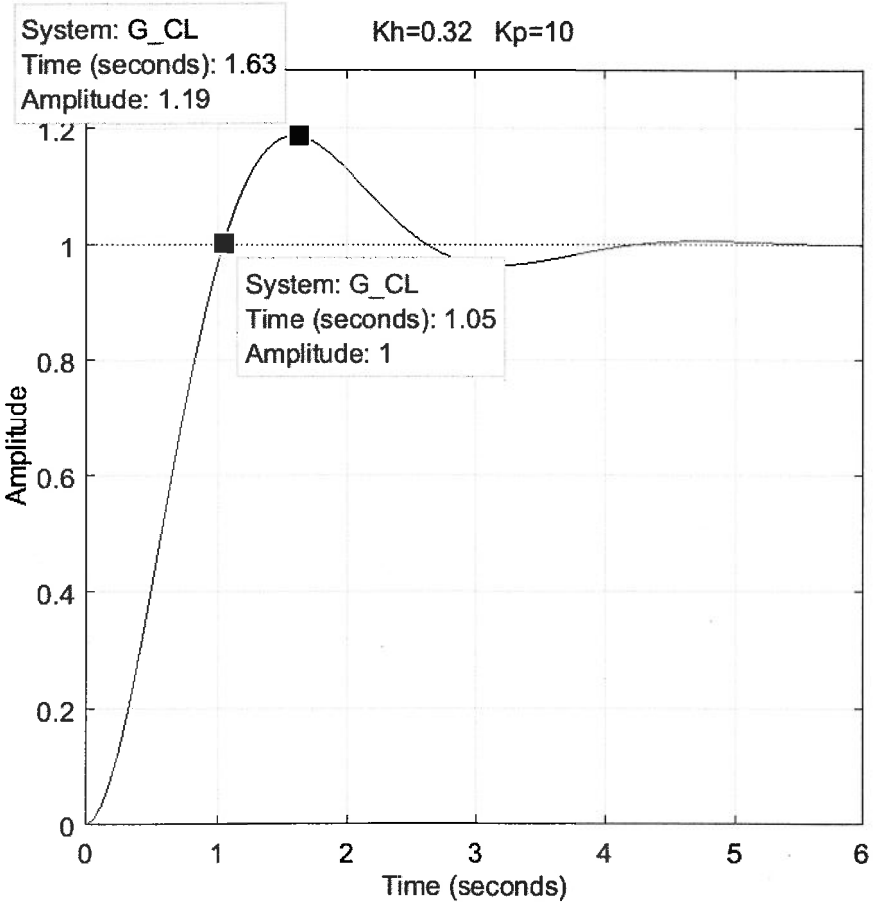
$\frac{K}{Js+c} \cdot \frac{1}{s}$

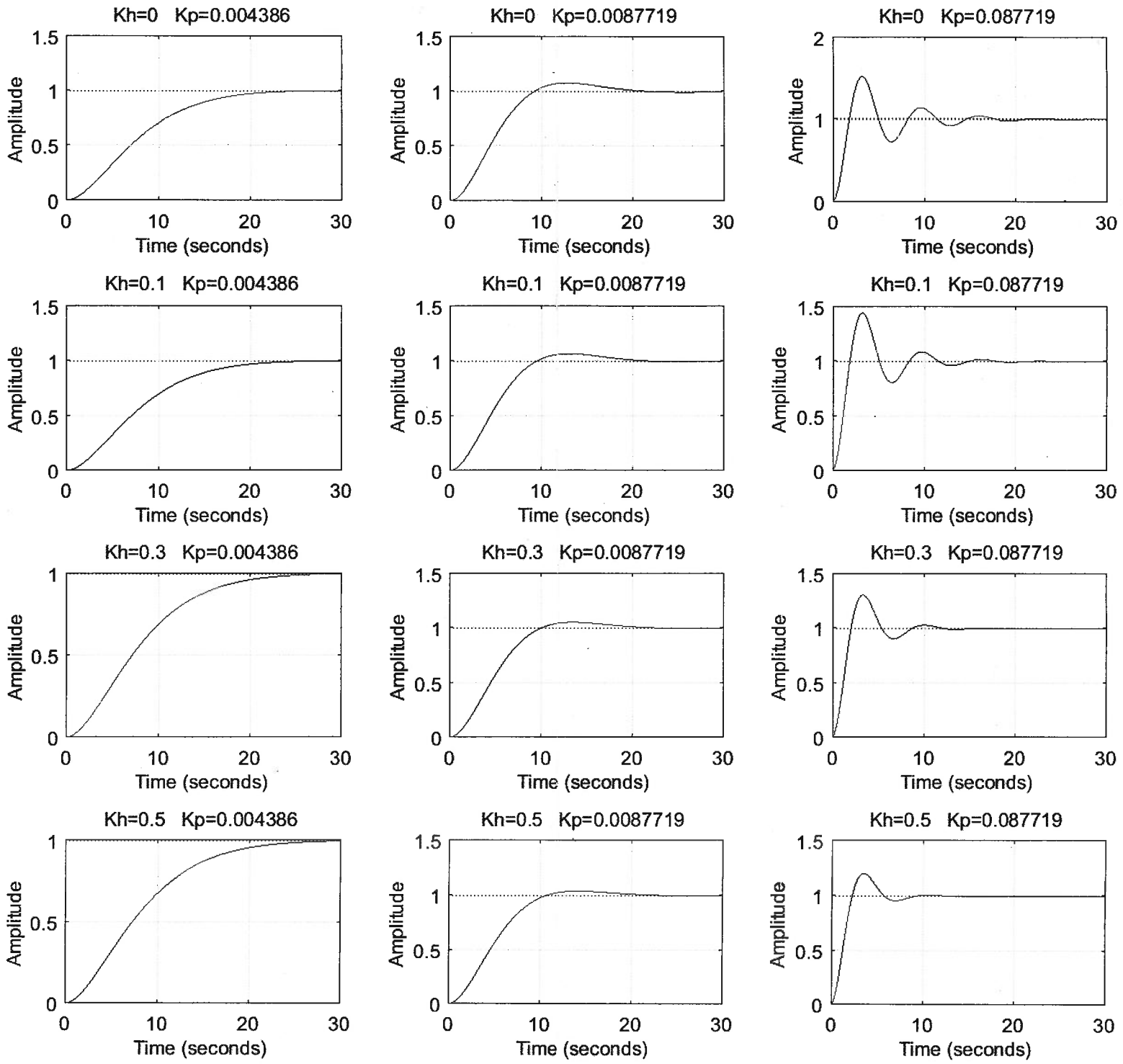




K = 1
J = 2
c = 1

class example.





aircraft

$\frac{K}{Js+c} \cdot \frac{1}{s}$

$K = 114$

$J = 10$

$c = 4$

6)

Velocity feedback $\frac{K}{Js+C} \cdot \frac{1}{s}$

$K = 114$ aircraft
$J = 10$ roll
$C = 4$ model