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ROUTH CRITERION

FOR HIGHER ORDER POLYNOMIALS

Given: $A(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0$

Find if any roots of $A(s)$ are in the RHS

Solution by Routh criterion

(1) If any of the coefficients $a_0, a_1, \ldots, a_n$ is negative, then at least one root is in RHS and the system is UNSTABLE. $\boxed{\text{STOP}}$

(2) If all coefficients are positive, do the Routh table:

R Table

$$A(s) = a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots \; a_1 s + a_0 = 0$$

$$\begin{array}{l|lll}
s^n & a_n & a_{n-2} & a_{n-4} \; \cdots \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} \; \cdots \\
s^{n-2} & b_1 & b_2 & \\
s^{n-3} & c_1 & c_2 & \\
\vdots & & &
\end{array}$$

$$b_1 = \frac{a_{n-1} a_{n-2} - a_n a_{n-3}}{a_{n-1}}$$

$$b_2 = \frac{a_{n-1} a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

$$c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1}$$

$$\vdots$$

Count the number of sign changes and zeros in the first column to get the number of roots in RHS

Use MATLAB file:
Routh_Hurwitz_Stability_criterion_modified_vg1.m

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R. criterion Example VG1

$$G(s) = \frac{4s+2}{s^3+3s^2+4s+2} = \frac{B(s)}{A(s)}$$

Examine $A(s) = s^3 + 3s^2 + 4s + 2$

$a_3 = 1 \qquad a_1 = 4$

$a_2 = 3 \qquad a_0 = 2$

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 3 & 2 \\ s^1 & 10/3 & \\ s^0 & 2 & \end{array}$$

$$b_1 = \frac{3 \times 4 - 1 \times 2}{3} = 10/3$$

$$b_2 = \frac{0-0}{3} = 0$$

$$c_1 = \frac{\frac{10}{3} \times 2 - 3 \times 0}{10/3} = 2$$

Routh criterion: number of sign changes = zero (0)

i.e. **NO** root in RHS $\longrightarrow$ STABLE!

Verification by MATLAB

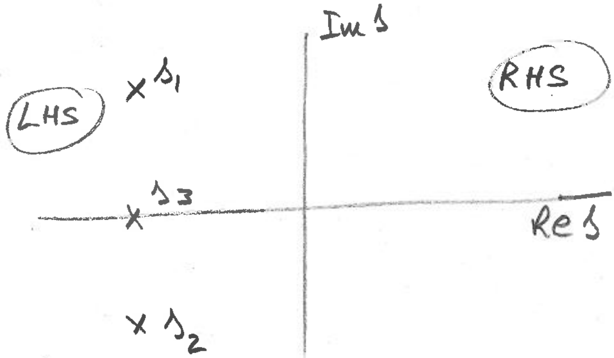
A = [1 3 4 2]

roots(A):

$s_1 = -1 + i$

$s_2 = -1 - i$

$s_3 = -1$



All roots are in LHS $\longrightarrow$ System is STABLE!

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R Criterion

Example VG2

$$G(s) = \frac{31}{s^3+5s^2+6s+31}$$

$a_3=1$ $a_2=5$ $a_1=6$ $a_0=31$.

s^3	1	6
s^2	5	31
s^1	-0.2	
s^0	31	

$$b_1 = \frac{5\times 6 - 1\times 31}{5} = -0.2$$

$$c_1 = \frac{-0.2\times 31}{-0.2} = 31$$

$\rightarrow$ 2 sign changes

2 roots in RHS $\rightarrow$ Unstable!

See Matlab print out on next page.

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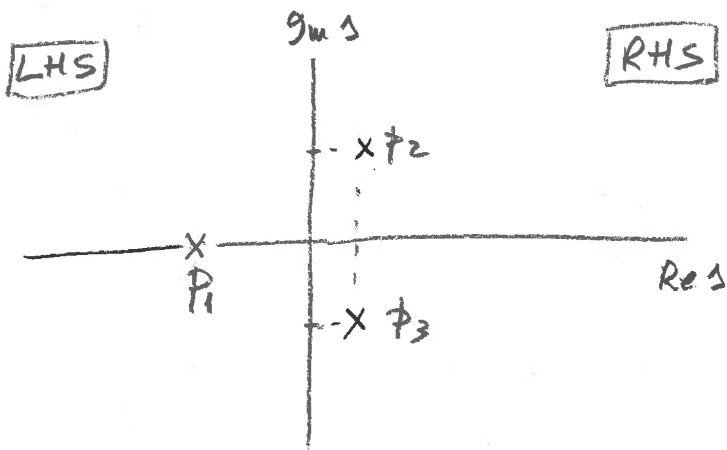
MATLAB Command Window

```
Input coefficients of characteristic equation
[an an-1 an-2 ... a0]= [1 5 6 31]

----------------------------------------
Roots of characteristic equation is:

ans =

p1   -5.0319
p2    0.0160 + 2.4820i
p3    0.0160 - 2.4820i

--------The R                    array is:--------

m =

    1.0000    6.0000
    5.0000   31.0000
   -0.2000         0
   31.0000         0

         ----> System is Unstable <----
>>
```



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R criterion Example VG3

$$G(s) = \frac{5s^2+8s+3}{s^6+3s^5-s^4-7s^3+10s^2+14s-20}$$

A(s) has some negative coefficients

At least one root of A(s) is in RHS

system is unstable

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R. criterion Example VG4

$$G(s) = \frac{5s^2+8s+3}{s^6+3s^5+s^4+7s^3+10s^2+14s+20}$$

$a_6=1$ $\quad a_5=3$ $\quad a_4=1$ $\quad a_3=7$ $\quad a_2=10$ $\quad a_1=14$ $\quad a_0=20$

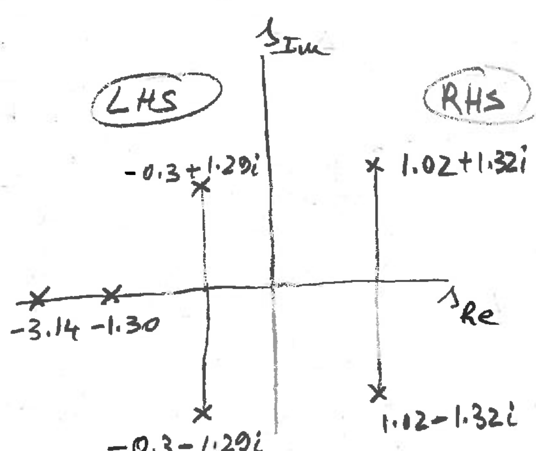
	s^6	1	1	10	20
	s^5	3	7	14	
1	s^4	-1.33	5.33	20	
2	s^3	19	59		
	s^2	9.4737	20		
	s^1	18.8889			
	s^0	20			

Two (2) sign changes
2 roots in RHS
UNSTABLE!

MATLAB
A=[1 3 1 7 10 14 20].
roots(A):

-3.14 + 0i
-1.30 + 0i
-0.30 + 1.29i
-0.30 - 1.29i
LHS

1.02 + 1.32i
1.02 - 1.32i
RHS



See book for more about Routh criterion

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Input coefficients of characteristic equation
[an an-1 an-2 ... a0]= [1 3 1 7 10 14 20]

Roots of characteristic equation are:

ans =

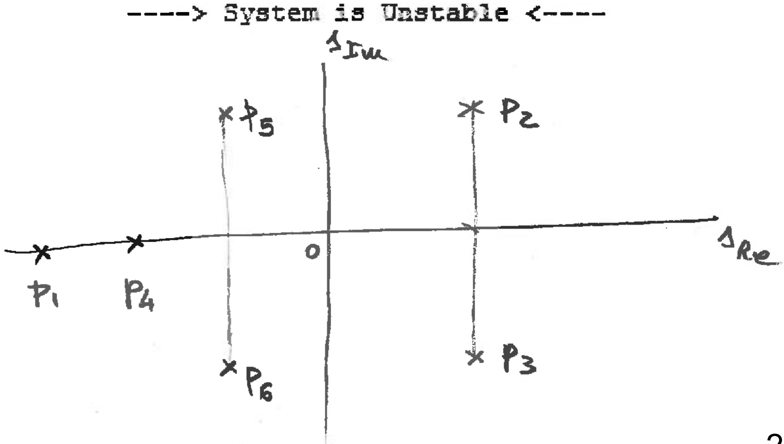
p1 -3.1462 + 0.0000i LHS
p2 1.0202 + 1.3244i RHS
p3 1.0202 - 1.3244i RHS
p4 -1.2962 + 0.0000i LHS
p5 -0.2990 + 1.2905i LHS
p6 -0.2990 - 1.2905i LHS

---------The Routh array is:---------

m =

1.0000	1.0000	10.0000	20.0000
3.0000	7.0000	14.0000	0
-1.3333	5.3333	20.0000	0
19.0000	59.0000	0	0
9.4737	20.0000	0	0
18.8889	0	0	0
20.0000	0	0	0

----> System is Unstable <----



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5.4 HURWITZ STABILITY CRITERION

The Hurwitz criterion is another method for determining whether all the roots of the characteristic equation of a continuous system have negative real parts. This criterion is applied using determinants formed from the coefficients of the characteristic equation. It is assumed that the first coefficient, a_n, is positive. The determinants Δ_i, $i=1,2,\ldots,n-1$, are formed as the principal minor determinants of the determinant

$$\Delta_n = \begin{vmatrix} a_{n-1} & a_{n-3} & \cdots & \begin{bmatrix} a_0 & \text{if } n \text{ odd} \\ a_1 & \text{if } n \text{ even} \end{bmatrix} & 0 & \cdots & 0 \\ a_n & a_{n-2} & \cdots & \begin{bmatrix} a_1 & \text{if } n \text{ odd} \\ a_0 & \text{if } n \text{ even} \end{bmatrix} & 0 & \cdots & 0 \\ 0 & a_{n-1} & a_{n-3} & \cdots & \cdots & \cdots & 0 \\ 0 & a_n & a_{n-2} & \cdots & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & a_0 \end{vmatrix}$$

The determinants are thus formed as follows:

$$\Delta_1 = a_{n-1}$$

$$\Delta_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = a_{n-1}a_{n-2} - a_n a_{n-3}$$

$$\Delta_3 = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix} = a_{n-1}a_{n-2}a_{n-3} + a_n a_{n-1} a_{n-5} - a_n a_{n-3}^2 - a_{n-4} a_{n-1}^2$$

and so on up to Δ_{n-1}.

Hurwitz Criterion: *All the roots of the characteristic equation have negative real parts if and only if* $\Delta_i > 0$, $i = 1,2,\ldots,n$.

EXAMPLE 5.5. For $n=3$,

$$\Delta_3 = \begin{vmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{vmatrix} = a_2 a_1 a_0 - a_0^2 a_3, \qquad \Delta_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = a_2 a_1 - a_0 a_3, \qquad \Delta_1 = a_2$$

Thus all the roots of the characteristic equation have negative real parts if

$$a_2 > 0 \qquad a_2 a_1 - a_0 a_3 > 0 \qquad a_2 a_1 a_0 - a_0^2 a_3 > 0$$

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Q : What is R criterion ?

A : R criterion is a tabular method to determine if a polynomial

$A(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0$

has roots in the RHS

Q : What is R criterion good for ?

A : R criterion is used to evaluate the stability of a system $G(s) = B(s)/A(s)$

If $A(s)$ has roots in RHS, then the system is UNSTABLE

Q : How do I use R criterion ?

A : - If $A(s)$ has at least one -ve coefficient, then the system is UNSTABLE. $\boxed{\text{STOP}}$

- If all coefficients of $A(s)$ are +ve, then do R Table to find if any roots are in RHS

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Q: What is H criterion?

A: H criterion is another method of using the coefficients of a polynomial equation to determine whether all the roots have negative real parts

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Q: How is H criterion different from R criterion?

A: H criterion uses determinants, whereas R criterion uses a table