

Table of Laplace Transforms for Vibrations

This is a partial lists of important Laplace transforms for vibrations that assumes zero initial conditions, $0 < t$, and $\zeta < 1$.

| $f(t)$ | $\mathcal{L}[f(t)] = F(s)$ | | $f(t)$ | $\mathcal{L}[f(t)] = F(s)$ | |
|---|--|------|---|---|------|
| $\delta(t)$ | 1 | (1) | $\frac{1}{\omega^3}(\omega t - \sin(\omega t))$ | $\frac{1}{s^2(s^2 + \omega^2)}$ | (17) |
| $\delta(t - t_0)$ | e^{-st_0} | (2) | $\frac{1}{2\omega^3}(\sin(\omega t) - \omega t \cos(\omega t))$ | $\frac{1}{(s^2 + \omega^2)^2}$ | (18) |
| 1 | $\frac{1}{s}$ | (3) | | | |
| e^{at} | $\frac{1}{s-a}$ | (4) | $\frac{t}{2\omega} \sin(\omega t)$ | $\frac{s}{(s^2 + \omega^2)^2}$ | (19) |
| $\sin(\omega t)$ | $\frac{\omega}{s^2 + \omega^2}$ | (5) | $t \sin(\omega t)$ | $\frac{2\omega s}{(s^2 + \omega^2)^2}$ | (20) |
| $\cos(\omega t)$ | $\frac{s}{s^2 + \omega^2}$ | (6) | $t \cos(\omega t)$ | $\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$ | (21) |
| $\sinh(\omega t)$ | $\frac{\omega}{s^2 - \omega^2}$ | (7) | $e^{at} \sin(\omega t)$ | $\frac{\omega}{(s-a)^2 + \omega^2}$ | (22) |
| $\cosh(\omega t)$ | $\frac{s}{s^2 - \omega^2}$ | (8) | $e^{at} \cos(\omega t)$ | $\frac{s-a}{(s-a)^2 + \omega^2}$ | (23) |
| $\frac{1}{\omega^2}(1 - \cos(\omega t))$ | $\frac{1}{s(s^2 + \omega^2)}$ | (9) | $e^{at} \sinh(\omega t)$ | $\frac{\omega}{(s-a)^2 - \omega^2}$ | (24) |
| $\frac{1}{\omega_d} e^{-\zeta \omega t} \sin(\omega_d t)$ | $\frac{1}{s^2 + 2\zeta \omega s + \omega^2}$ | (10) | $e^{at} \cosh(\omega t)$ | $\frac{s-a}{(s-a)^2 - \omega^2}$ | (25) |
| $1 - \frac{\omega}{\omega_d} e^{-\zeta \omega t} \sin(\omega_d t + \phi),$ $\phi = \cos^{-1}(\zeta)$ | $\frac{\omega^2}{s(s^2 + 2\zeta \omega s + \omega^2)}$ | (11) | $\frac{1}{\omega_2} \sin(\omega_2 t) - \frac{1}{\omega_1} \sin(\omega_1 t)$ $\frac{\omega_1^2 - \omega_2^2}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$ | | (26) |
| $\frac{t^{n-1}}{(n-1)!}, n = 1, 2, \dots$ | $\frac{1}{s^n}$ | (12) | $\cos(\omega_2 t) - \cos(\omega_1 t)$ | $\frac{s(\omega_1^2 - \omega_2^2)}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$ | (27) |
| $t^n, n = 1, 2, \dots$ | $\frac{n!}{s^{n+1}}$ | (13) | $e^{at} f(t)$ | $F(s-a)$ | (28) |
| $t^n e^{\omega t}, n = 1, 2, \dots$ | $\frac{n!}{(s-\omega)^{n+1}}$ | (14) | $f(t-a) \Phi(t-a)$ | $e^{-as} F(s)$ | (29) |
| $\frac{1}{\omega}(1 - e^{-\omega t})$ | $\frac{1}{s(s+\omega)}$ | (15) | $\Phi(t-a)$ | $\frac{e^{-as}}{s}$ | (30) |
| $\frac{1}{\omega^2}(e^{-\omega t} + \omega t - 1)$ | $\frac{1}{s^2(s+\omega)}$ | (16) | $f'(t)$ | $sF(s) - f(0)$ | (31) |