

Summary of Main Equations in Mechanical Vibrations

The general response for the undamped case has the form:

$$x(t) = A \sin(\omega_n t + \phi) + X \cos(\omega t)$$

where:

free response:

$$A = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n}, \quad \phi = \tan^{-1} \left(\frac{x_0 \omega_n}{v_0} \right), \text{ and } X = 0$$

forced response:

$$A = \sqrt{\left(\frac{v_0}{\omega_n} \right)^2 + (x_0 - X)^2}, \quad \phi = \tan^{-1} \left(\frac{\omega_n(x_0 - X)}{v_0} \right), \text{ and } X = \frac{f_0}{\omega_n^2 - \omega^2}$$

The general response for the underdamped case has the form:

$$x(t) = A e^{-\zeta \omega_n t} \sin(\omega_d t + \phi) + X \cos(\omega t - \phi_p)$$

where:

free response:

$$A = \frac{\sqrt{(v_0 + \zeta \omega_n x_0)^2 + (x_0 \omega_d)^2}}{\omega_d}, \quad \phi = \tan^{-1} \left(\frac{x_0 \omega_d}{v_0 + \zeta \omega_n x_0} \right), \text{ and } X = 0$$

forced response:

$$A = \frac{x_0 - X \cos(\phi_p)}{\sin(\phi)}, \quad \phi = \tan^{-1} \left(\frac{\omega_d(x_0 - X \cos(\phi_p))}{v_0 + (x_0 - X \cos(\phi_p)) \zeta \omega_n - \omega X \sin(\phi_p)} \right),$$

$$X = \frac{f_0}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega_n \omega)^2}}, \text{ and } \phi_p = \tan^{-1} \left(\frac{2\zeta \omega_n \omega}{\omega_n^2 - \omega^2} \right)$$