EE329 Project Report

Signals and Systems II

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1. Find the Fourier Series Coefficients a_k

Derivation of Fourier Series Coefficients a_0 and a_k

$$a_0 = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\,dt = \frac{1}{4}\int_{-2}^{2} 2t\,dt = \frac{1}{4}\left(t^2\right)_{-2}^{2} = \frac{1}{4}(4-4) = \boxed{0}$$

↳ this makes sense, as there's no DC offset for the signal.

$$a_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\,e^{-j2\pi f_0 k t}\,dt = \frac{1}{4}\int_{-2}^{2} 2t\,e^{-j2\pi\left(\frac{1}{4}\right)kt}\,dt$$

$$= \frac{1}{2}\int_{-2}^{2} t\,e^{-\frac{j\pi k}{2}t}\,dt \rightarrow \text{IBP} \qquad u = t \quad du = dt \qquad dv = e^{-\frac{j\pi k}{2}t} \quad v = -\frac{2}{j\pi k}e^{-\frac{j\pi k}{2}t}$$

$$= uv - \int v\,du = \frac{1}{2}\left[t\cdot\frac{-2}{j\pi k}e^{-\frac{j\pi k}{2}t} + \frac{+2}{(j\pi k)^2}e^{-\frac{j\pi k}{2}t}\right]\Bigg|_{-2}^{2}$$

$$= \frac{1}{2}\left[\frac{-4e^{-j\pi k} - 4e^{j\pi k}}{j\pi k} + \frac{2e^{-j\pi k} - 2e^{j\pi k}}{(j\pi k)^2}\right] \rightarrow \text{Eulers: } \frac{1}{2}\left(e^{j\pi k} - e^{-j\pi k}\right) = j\sin(\pi k) \qquad \frac{1}{2}\left(e^{j\pi k} + e^{-j\pi k}\right) = \cos(\pi k)$$

$$= \frac{1}{2}\left[\frac{-2\cos(\pi k)}{j\pi k} + \frac{-4\sin(\pi k)}{(\pi k)^2}\right] = \frac{-4\cos(\pi k)}{j\pi k} - \cancel{\frac{2j\sin(\pi k)}{(\pi k)^2}} \rightarrow \text{zero}$$

$$= -j\frac{4}{\pi k}\cos(\pi k) \Rightarrow \boxed{\frac{-j4(-1)^k}{\pi k}}$$

2. Magnitude and Phase Spectra of a_k

Calculations for Magnitude and Phase Spectra of a_k

$$|a_k|^2 = a_k \cdot a_k^* = -\frac{j4(-1)^k}{\pi k} \cdot \frac{j4(-1)^k}{\pi k} = \frac{4^2 \overbrace{\cancel{(-1)^{2k}}}^{\text{always } 1}}{(\pi k)^2} \rightarrow |a_k| = \boxed{\frac{4}{\pi|k|}}$$

$$\angle a_k = \tan^{-1}\left(\frac{4/\pi k}{\text{zero}}\right) = \boxed{\begin{cases} \pi/2, & k > 0 \\ -\pi/2, & k < 0 \end{cases}}$$

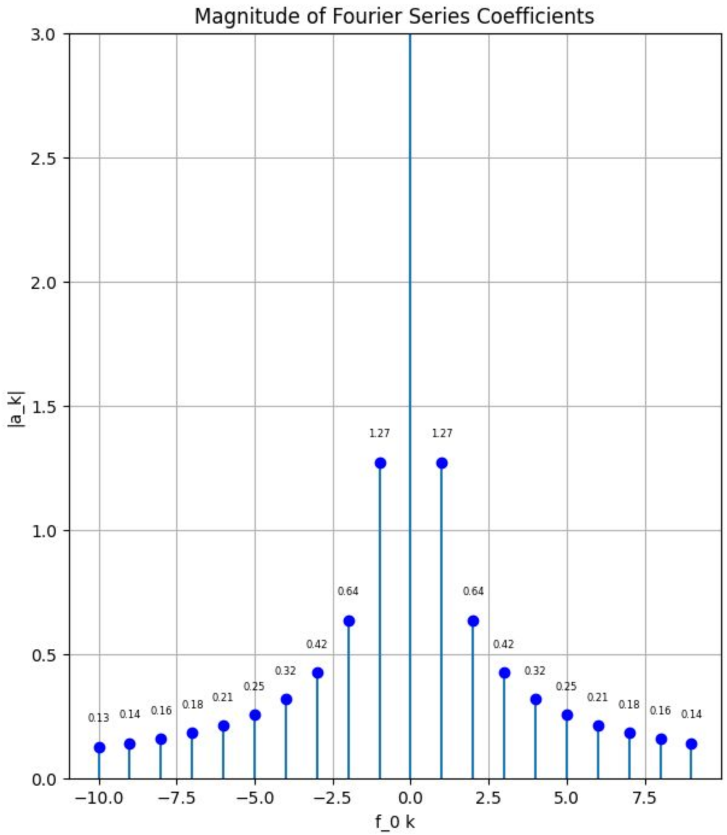
Calculations of a_k with k in range [-10,10]

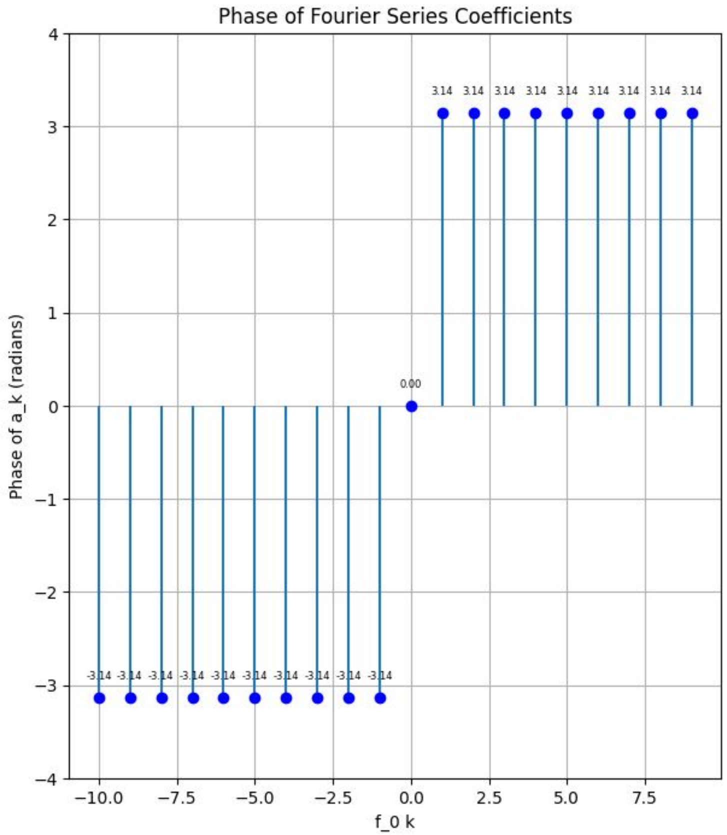
I calculated these results using the equations above in a for loop so that I didn't have to do it by hand:

```
# calculations
mag_of_ak = []
phase_of_ak = []
for i in range(-10, 10):
    # magnitude
    if i == 0:
        mag_of_ak.append(1000)  # its infinity but this is tall enough for the plot
    else:
        mag_of_ak.append(4 / (math.pi * abs(i)))

    # phase
    if i == 0:
        phase_of_ak.append(0.0)
    elif i > 0:
        phase_of_ak.append(math.pi)
    else:
        phase_of_ak.append(-math.pi)
```

Plots of a_k with k in range [-10,10]





3. Finding b_k and c_k

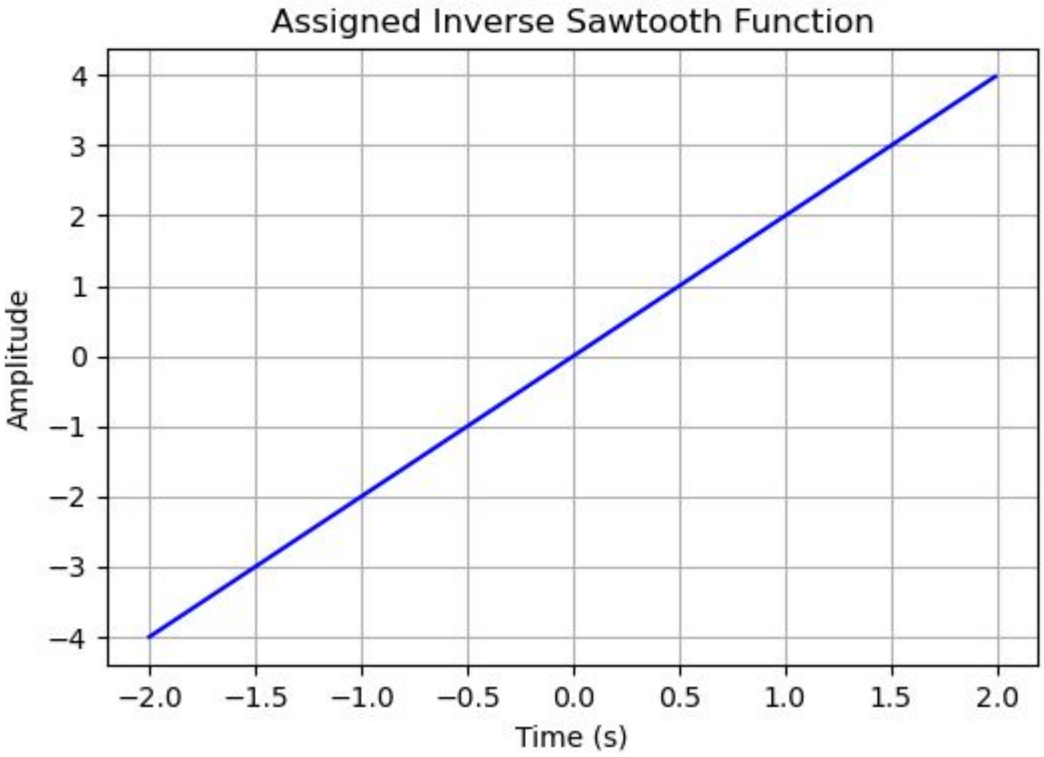
Finding b_k and c_k in terms of a_k

$b_0 = a_0 = \boxed{0} \rightarrow$ Since its an odd function,

$c_k = \boxed{0}$ and $\boxed{b_k = \frac{4(-1)^k}{\pi k}}$

4. Fourier Series Approximation

Original Assigned Inverse Sawtooth Function



I was assigned parameters:

- $a = 4$
- $b = 2$

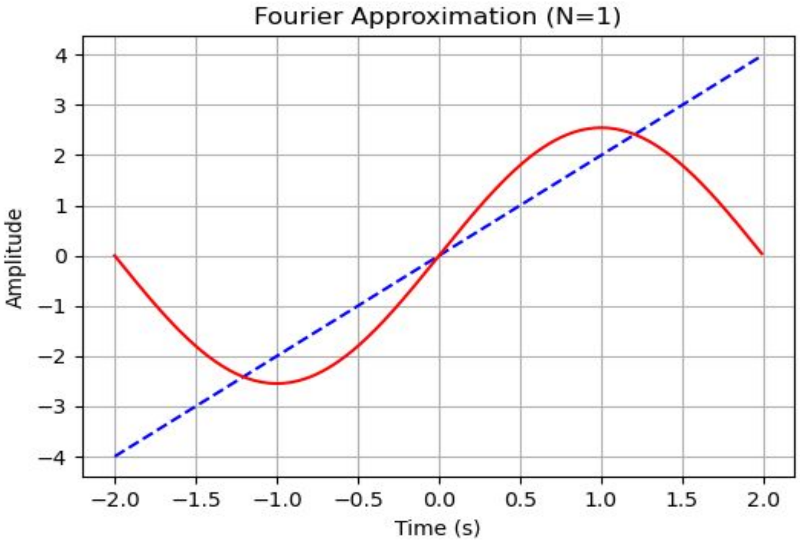
This results in:

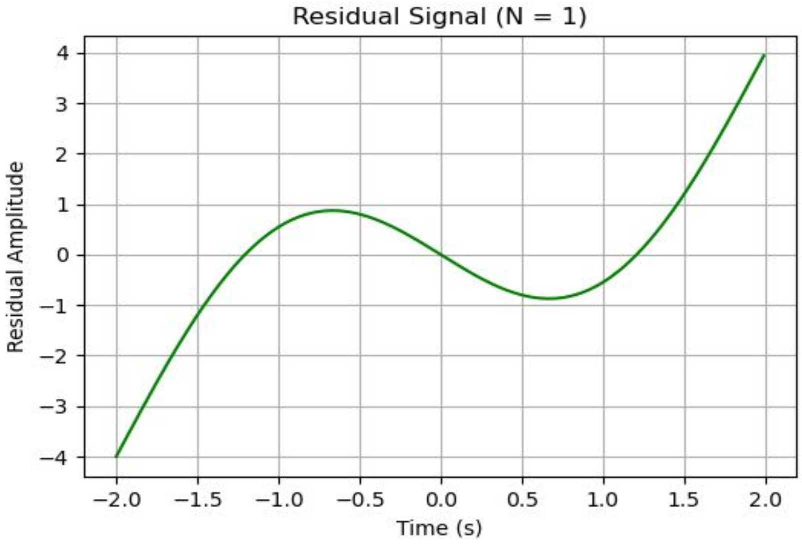
- amplitude = $a = 4$
- $T_0 = 2b = 4$

Additionally, I let:

- $t_0 = -2$
- $f_0 = 1/T_0 = 0.25$

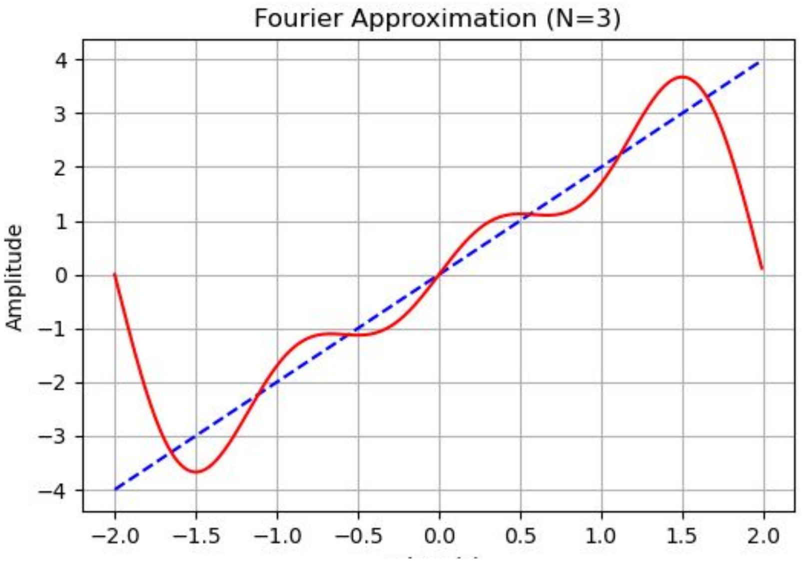
Order of Approximation N = 1

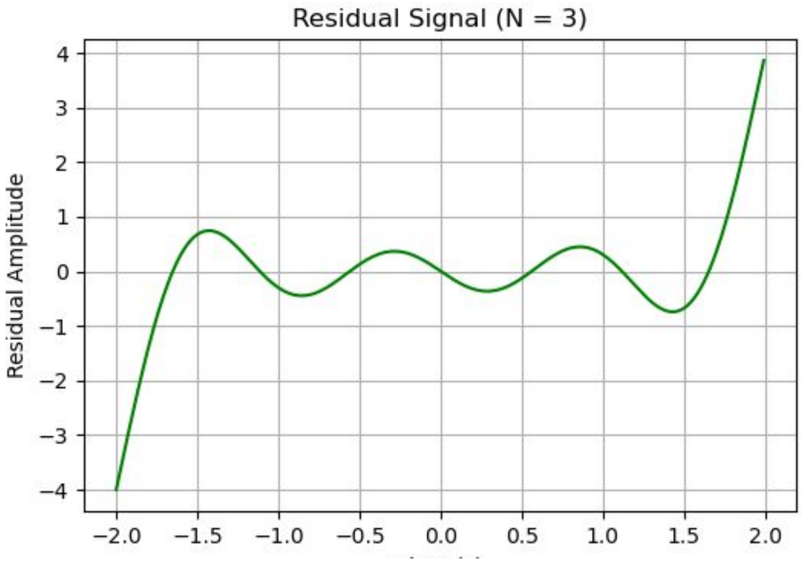




Power of residual signal = 2.0913 Watts

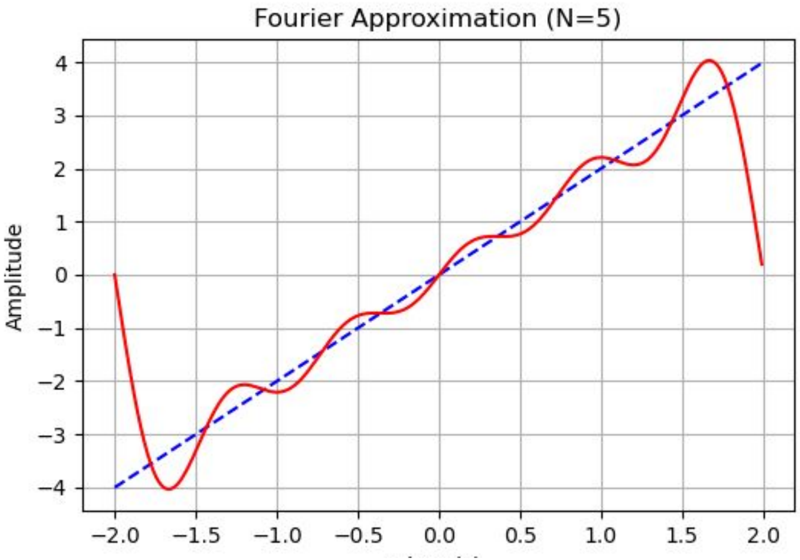
Order of Approximation N = 3

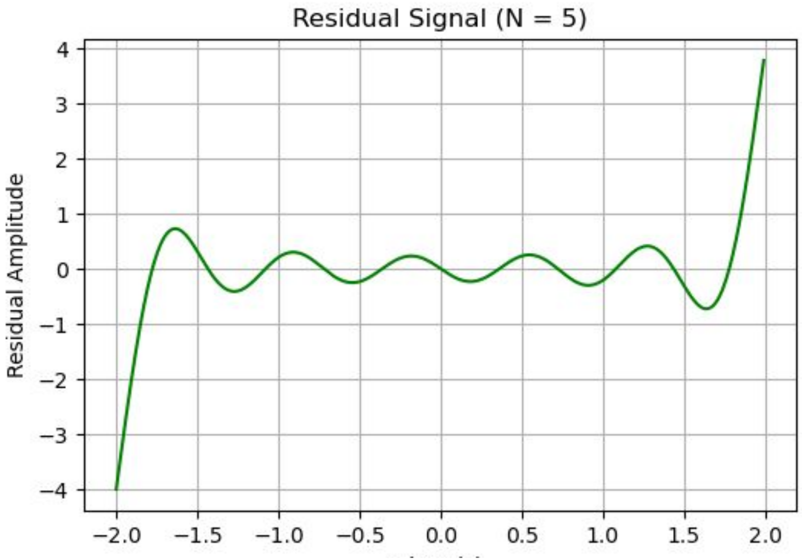




Power of residual signal = 0.9207 Watts

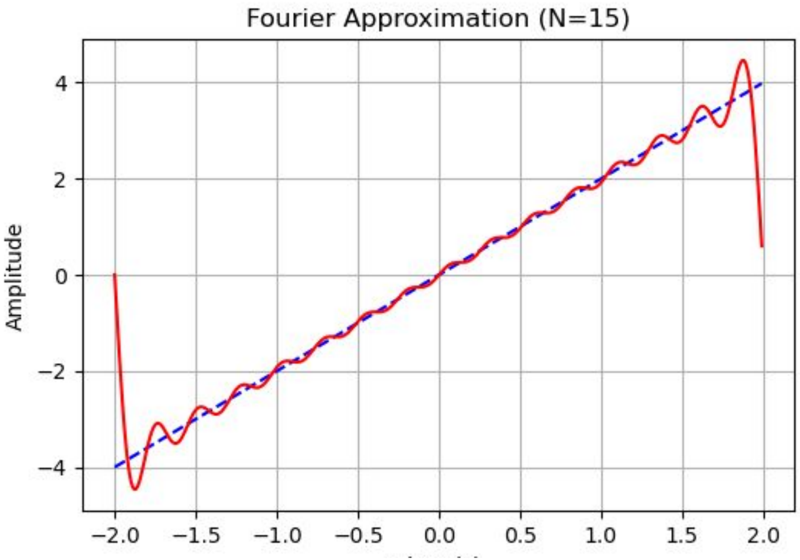
Order of Approximation N = 5

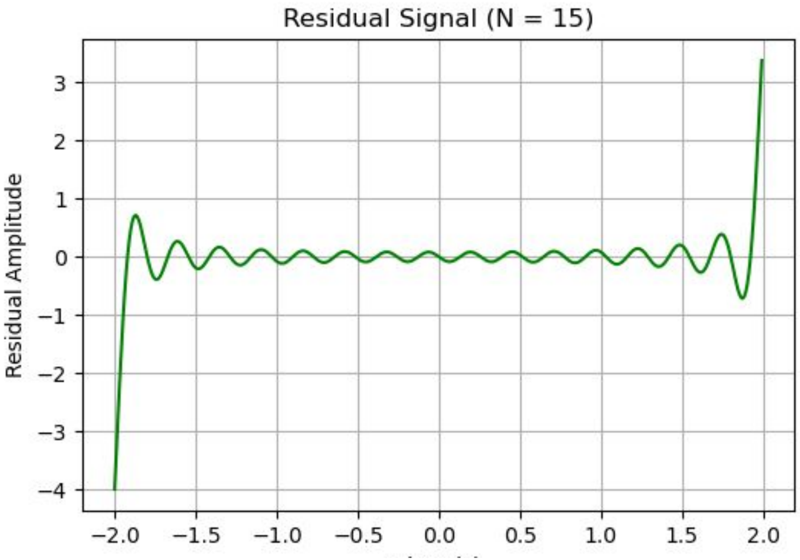




Power of residual signal = 0.5886 Watts

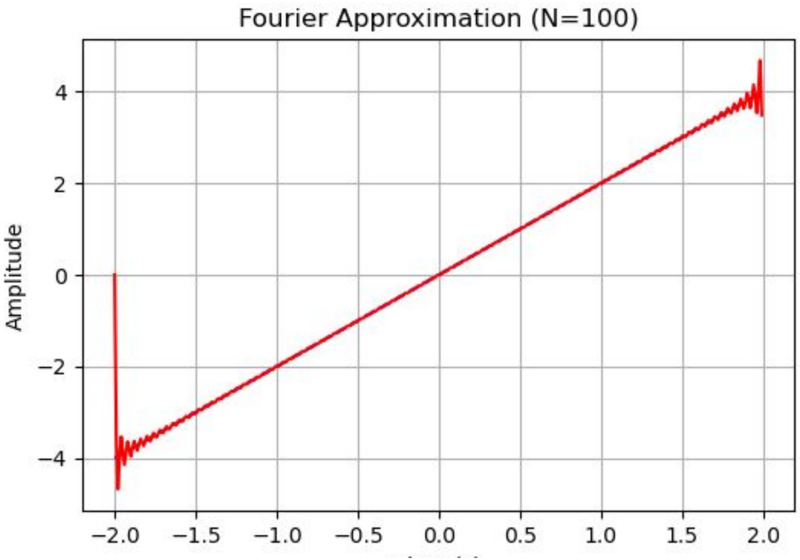
Order of Approximation N = 15

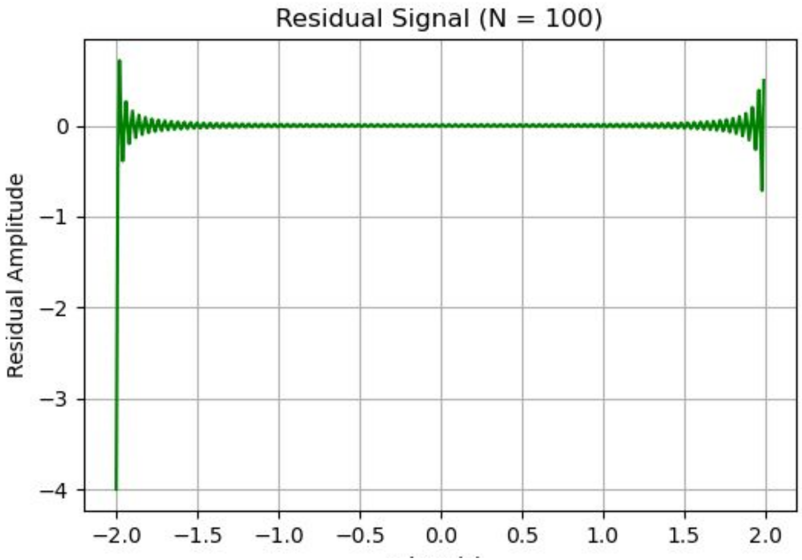




Power of residual signal = 0.2112 Watts

Order of Approximation N = 100





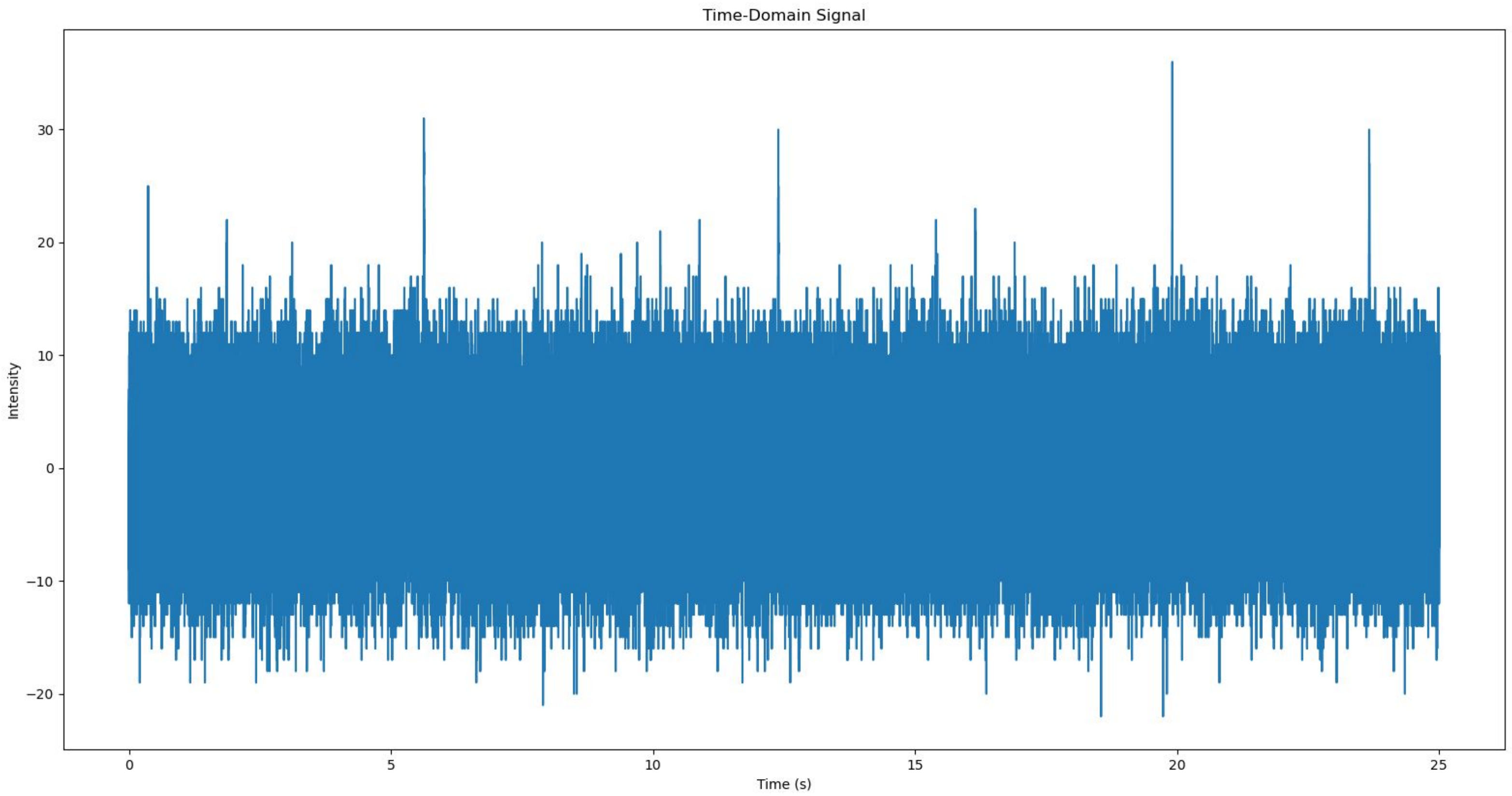
Power of residual signal = 0.0459 Watts

Fourier Series Approximation Summary

Approximation (N)	Power of residual signal (Watts)
1	2.0913
3	0.9207
5	0.5886
15	0.2112
100	0.0459

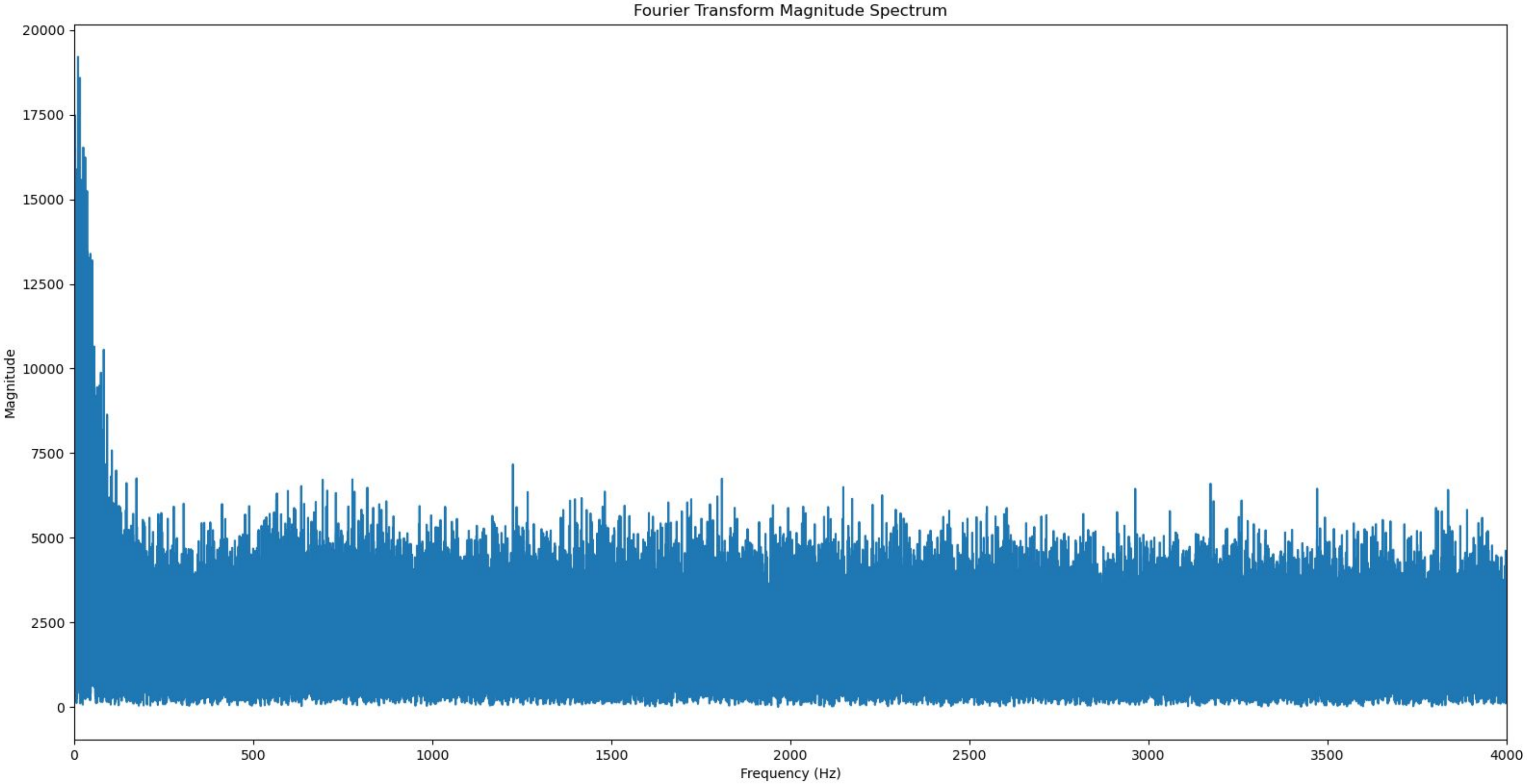
5. Fourier Transform Analysis

Time Series

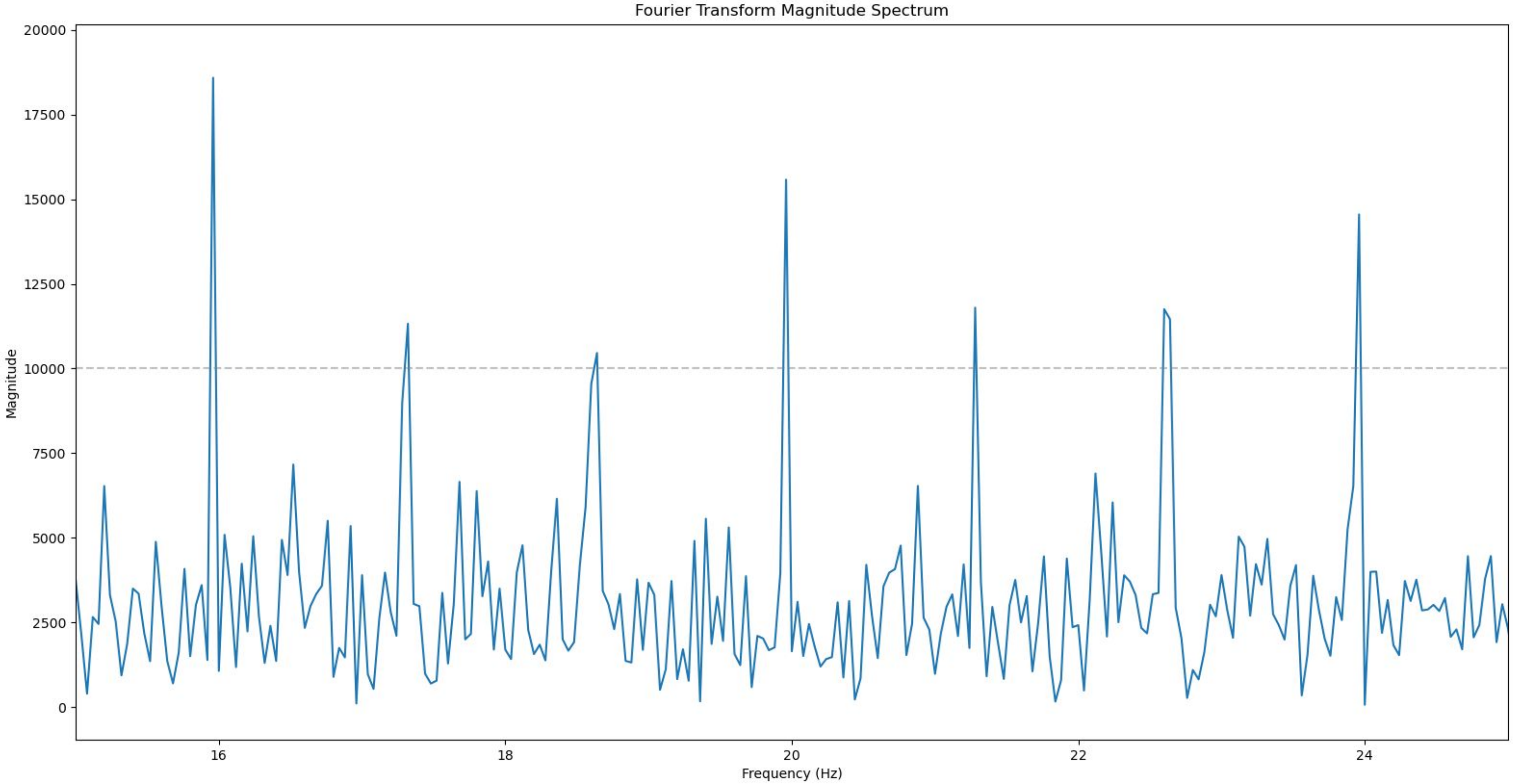


Time-Domain Signal
Intensity
Time (s)
30
20
10
0
−10
−20
0
5
10
15
20
25

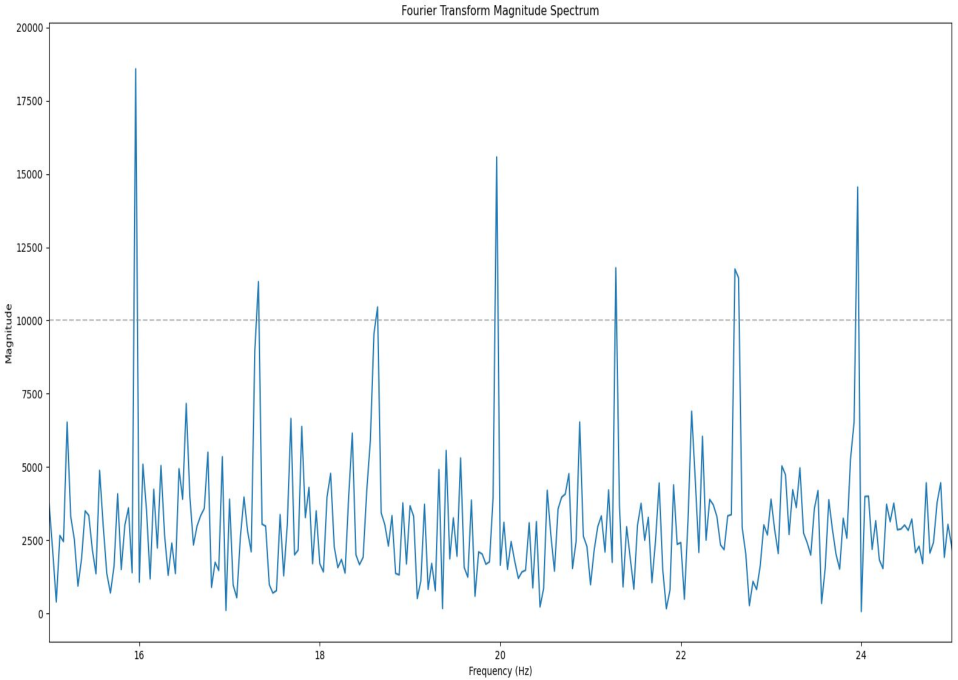
Fourier Transform



Further Illustration between 15 to 25 Hz



FT Analysis - Fundamental Frequency and Period



Zooming in on the data allows us to see the distance between two adjacent peaks, which is the fundamental frequency. I defined the peaks as values above the horizontal dashed line at y = 10,000.

The fundamental frequency is:

- $f_0 \cong 1.32$ Hz

The fundamental period is:

- $T_0 = 1/f_0 = 0.758$ seconds