Jonathan Greco

Austin Glaser

ECEN 4636

Lab 8: Compensator Design

In order to design a compensator for our system, we began by choosing a compensator type. For our application, a lead-lag compensator appeared to be a good choice. In order to create a lead-lag compensator, we cascaded a lead compensator with a lag compensator.

$C(s) = \frac{1 + aTs}{1 + Ts} \qquad [a > 1]$	$C(s) = \frac{1}{a}\left(\frac{1 + aTs}{1 + Ts}\right) \qquad [a < 1]$
Figure X: Lead Compensator	Figure Y: Lag Compensator

We chose our poles and zeroes such that the frequency response of the compensator spanned our desired bandwidth, 100Hz to 10KHz. We also aimed for attenuation of the resonant peak present in the plant transfer function. In order to be able to physically realize said compensator transfer function using a passive network, we needed to ensure that the pole to zero ratio, α_{Lag}, for the lag compensator was inversely related to the pole to zero ratio, α_{Lead}, for the lead compensator. Using these constraints, the lead and lag compensator parameters were chosen, and the compensator transfer function was obtained. An iterative process was used to find the best α for the compensator. The open loop and closed loop transfer functions were then generated.

```
C_comp =

     2.534e-08 s^2 + 0.001215 s + 1
  -------------------------------------
  3.379e-10 s^2 + 2.144e-05 s + 0.01333
```

Figure X: Compensator Transfer Function

```
OL_sys =

              0.0005392 s^3 + 25.85 s^2 + 2.128e04 s
  --------------------------------------------------------------
  3.379e-10 s^4 + 2.173e-05 s^3 + 0.1846 s^2 + 9719 s + 6.036e06
```

Figure X: Open Loop System Transfer Function

```
sys =

             0.0005392 s^3 + 25.85 s^2 + 2.128e04 s
  -------------------------------------------------------------
  3.379e-10 s^4 + 0.000561 s^3 + 26.03 s^2 + 3.1e04 s + 6.036e06
```

Figure X: Closed Loop System Transfer Function

The open loop and closed loop response was examined using frequency response methods as well as root locus. The bode plots of the plant, compensator, and closed loop response were generated to ensure the compensator would have the desired effect on the system.

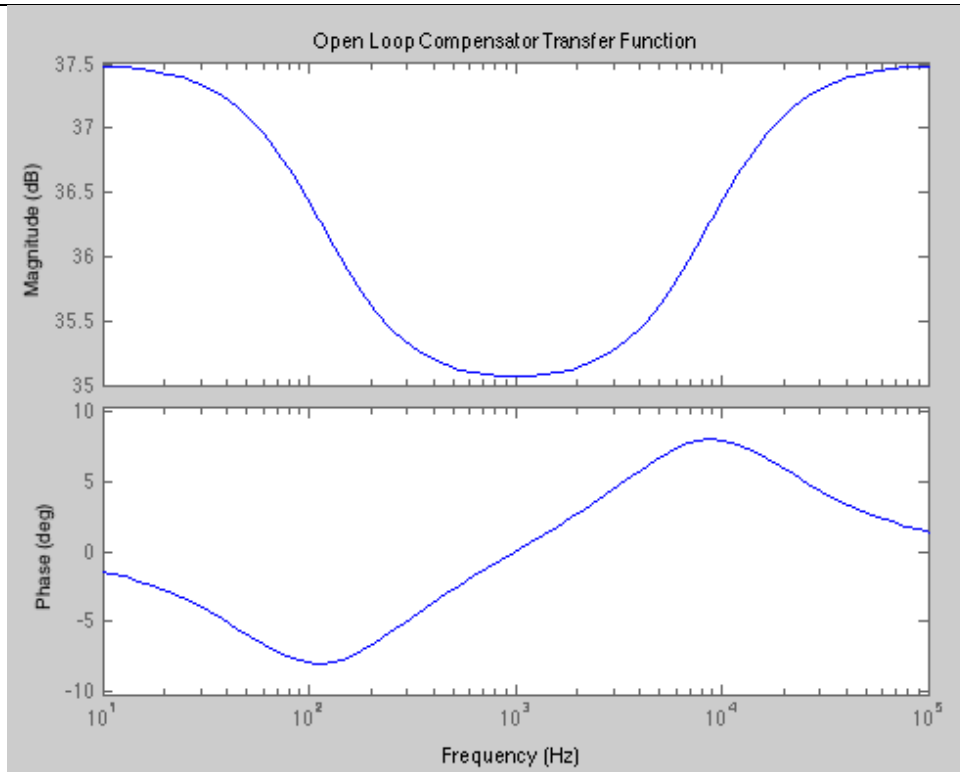


Figure X: Compensator Bode Plot

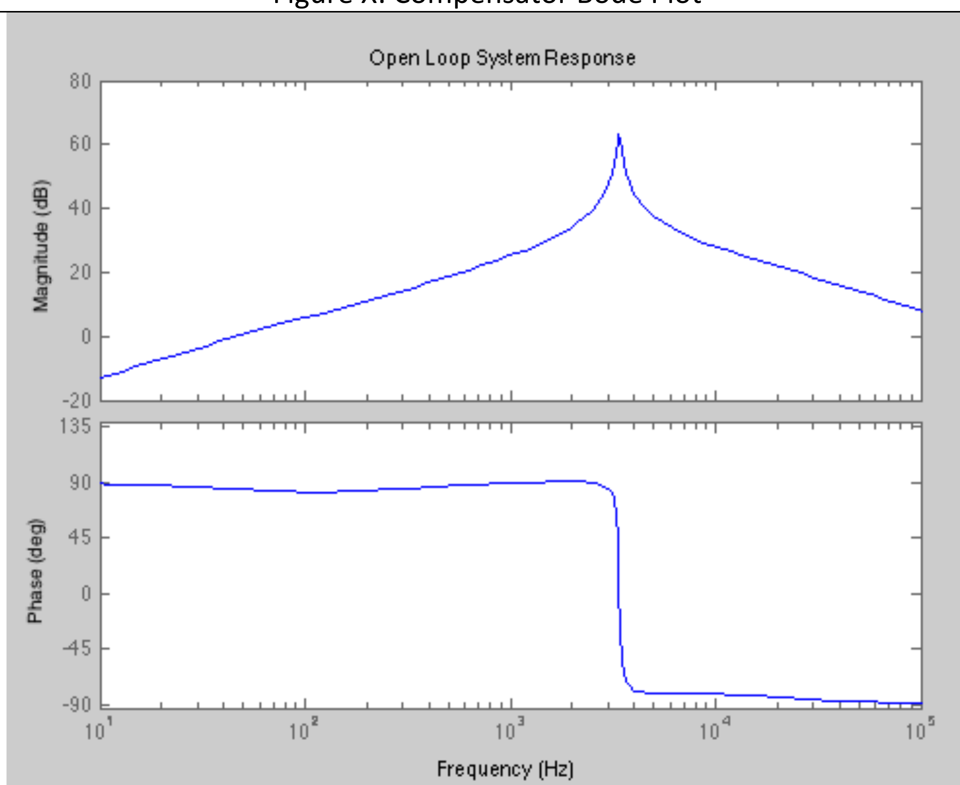


Figure X: Open Loop Bode Plot

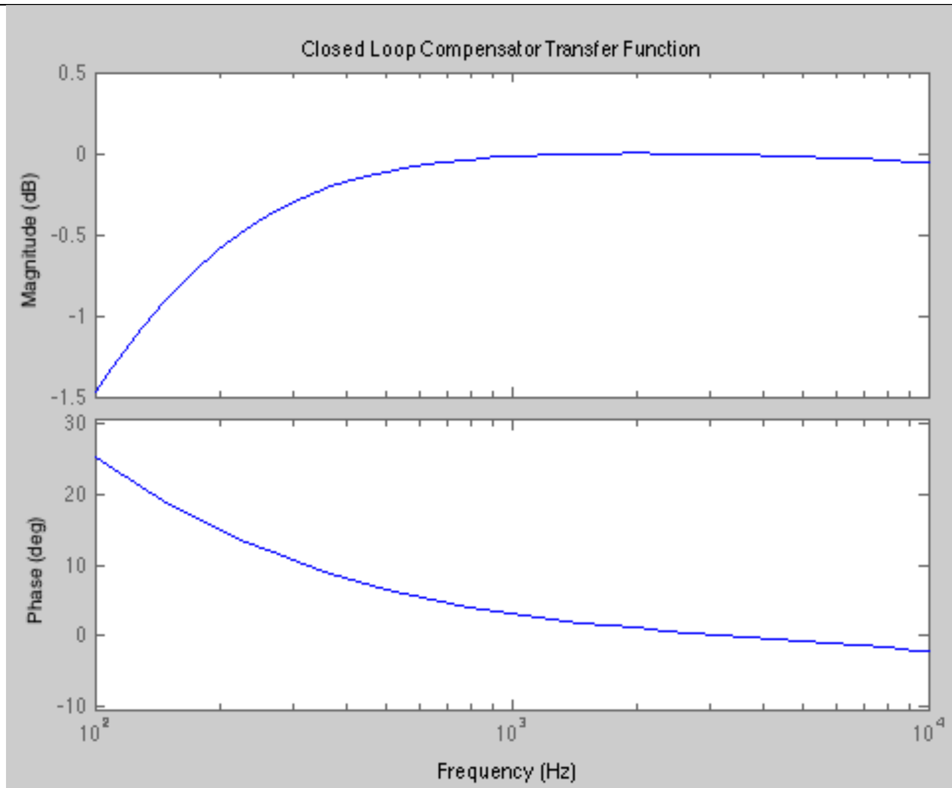


Figure Y: Closed Loop Bode Plot

Next, the compensator transfer function was exported from MATLAB in zero-pole-gain form using MATLAB's `zpk()` function.

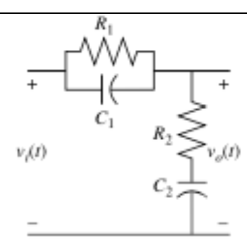
```
ans =

  75 (s+4.71e04) (s+837.7)
  ------------------------
    (s+6.283e04) (s+628)
```

Figure X: Compensator in Zero-Pole-Gain Form

The passive implementation of lag-lead compensators discussed on page 506 of *Control Systems Engineering, 6th Edition* by Nise was used to realize the compensator, specifically the last equation detailed in table 9.11.

Lag-lead compensation



$$\frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

Figure X: Physical Lag-Lead Compensator Transfer Function

The MATLAB generated transfer function was set equal to the given physical transfer function, and each component was solved for algebraically. C1 was first chosen to be 10nF. R1 was found to be 2123Ω, R2 was found to 6443Ω, and C2 was found to be 0.185μF, which can be implemented most realistically with a 0.2μF capacitor.

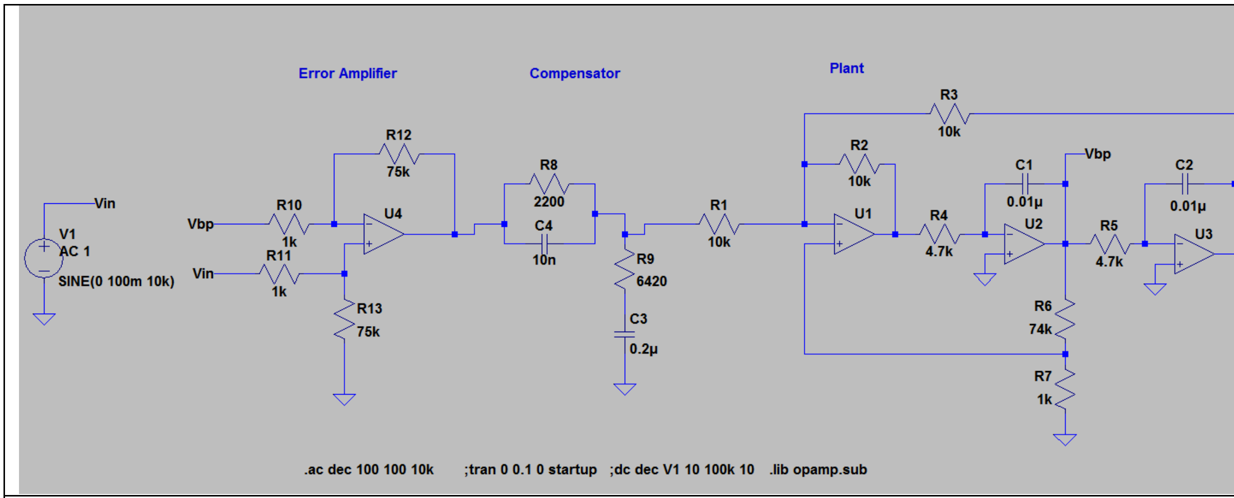


Figure X: LTSpice Circuit Simulation Setup

The circuit was then simulated in LTSpice alongside the plant. A single opamp subtractor circuit is needed in order to provide the negative feedback for the system. Using LTSpice's AC simulation, it can be seen that the compensator performs as expected and the calculated circuit elements closely enough implement the proper compensator design. Furthermore, the component values are both realistic and physically realizable. Finally, the closed loop response was generated and very well matches MATLAB's plotted response.

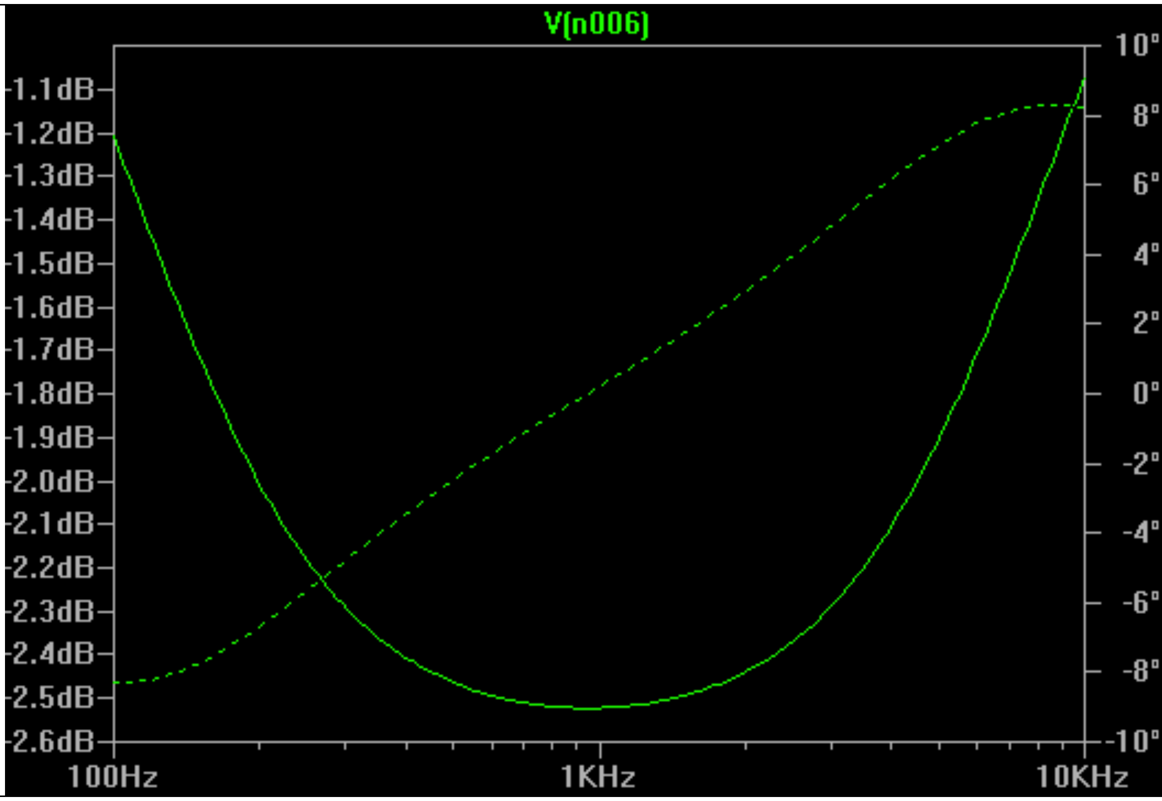


Figure X: Simulated Physical Compensator Response

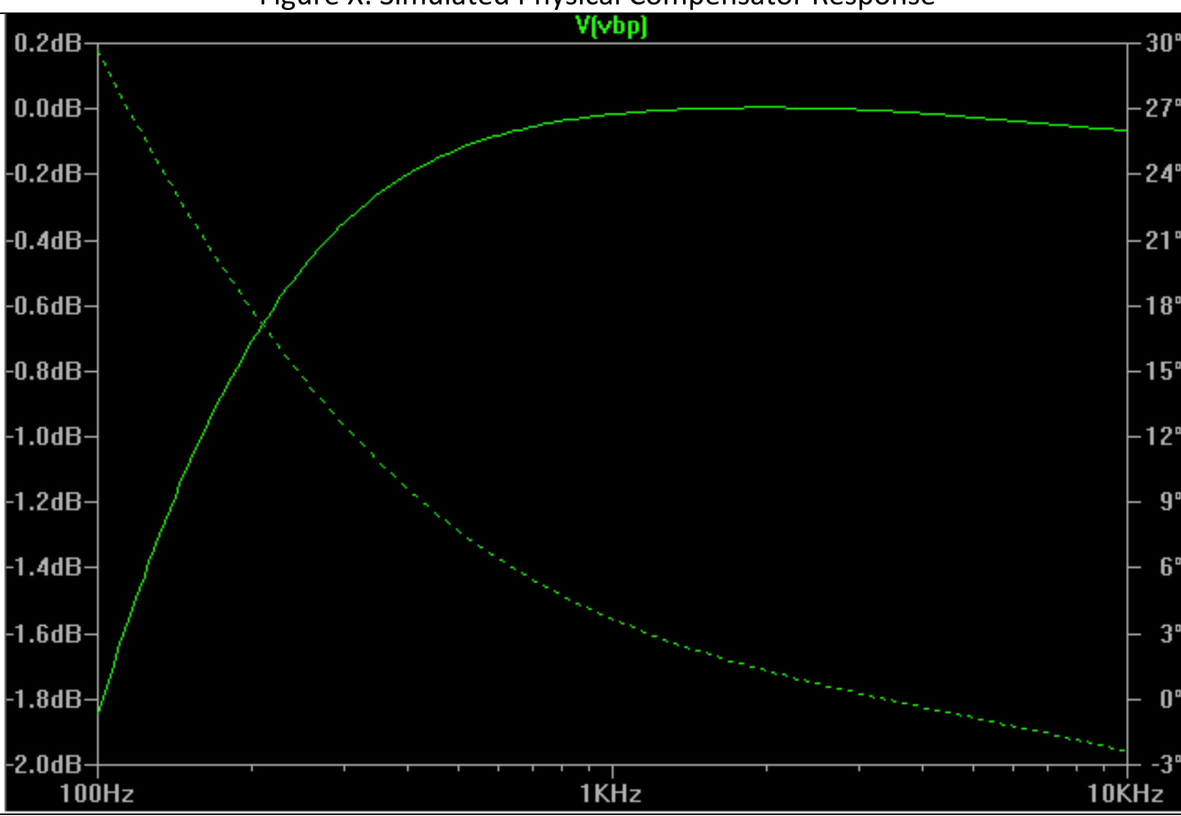


Figure X: Simulated Closed Loop System Response

Lastly, the list of physical circuit elements was created.

(1) LM741CN opamp
(1) 10nF ceramic capacitor
(2) 0.1uF ceramic capacitors, in parallel to create 0.2uF
(1) 2.2kΩ resistor
(2) 75kΩ resistors
(2) 1kΩ resistors
(1) 6.2kΩ resistor
(1) 220Ω resistor

Though the compensator in the LTSpice simulation does not exactly match the theoretical compensator designed in MATLAB, this is to be expected. Read circuit elements not only have tolerances but also are impossible to obtain in very specific values. Rather, they much be combined to come as close as reasonably possible to the desired value. With 10% tolerance components, even more error can be expected in the physical system, which is to be built as the next part of the lab.