amerCall

April 1, 2018

In [1]:
```
# Austin Griffith
# Sf Values for American Call Option
# Python 3.6.5
# 4/1/2018

import pandas as pd
import numpy as np
from math import exp, log, sqrt, pi
import matplotlib.pyplot as plt
from scipy.stats import norm
import time
```

In [2]:
```
# motion of binomial tree
def direction(dt,mu,div=0.0):
    u = (1 + sigma*sqrt(dt))*(1 - div*dt)
    d = (1 - sigma*sqrt(dt))*(1 - div*dt)
    p = 0.5*(1 + mu*sqrt(dt)/sigma)
    return(u,d,p)

# create binomial tree of underlying asset value
# uses pandas to allow for easy intrinsic valuation for calls and puts
def tree(S0,m,up,down):
    bi = pd.DataFrame([S0])
    for step in range(1,m+1):
        temp = bi[step-1]
        bi0 = []
        size = len(temp)-1
        for i,t in enumerate(temp):
            if i == size:
                bi0.append(up*t)
                bi0.append(down*t)
            else:
                bi0.append(up*t)
        bi0 = pd.DataFrame(bi0)
        bi0.columns = [step]
        bi = pd.concat([bi,bi0],axis=1)
    return(bi)
```

```
# intrinsic for euro and american option valuation
class Intrinsic():
    def __init__(self,binomial):
        put = K - binomial
        put[put < 0] = 0
        self.intPut = put

        call = binomial - K
        call[call < 0] = 0
        self.intCall = call

# changes binomial dataframe tree to list tree
def to_list(data):
    treeList = []
    idx = data.columns
    for x in idx:
        temp = data[x]
        temp = temp.dropna(axis=0,how='all')
        temp = list(temp)
        treeList.append(temp)
    return(treeList)

# changes binomial list to dataframe data
def to_tree(data):
    treeFrame = pd.DataFrame([])
    for i in data:
        temp = pd.DataFrame(i)
        treeFrame = pd.concat([treeFrame,temp],axis=1)
    return(treeFrame)
```

In [3]:
```
# BS pricing of European Call and Put
class bs_price:
    def __init__(self,S,K,T,r,sigma,div=0.0):
        N = norm.cdf
        d1 = (log(S/K) + (r - div + 0.5*(sigma**2))*T)/(sigma*sqrt(T))
        d2 = d1 - sigma*sqrt(T)

        self.call = S*exp(-div*T)*N(d1) - K*exp(-r*T)*N(d2)
        self.put = K*exp(-r*T)*N(-d2) - S*exp(-div*T)*N(-d1)

# Calculating the values of options through binomial tree lattice
class Tree:
    def __init__(self,r,mu,m,dt,S0,div=0.0,option='put'):
        self.m = m
        self.r = r
        self.mu = mu
        self.dt = dt
        self.div = div
```

```
        self.S0 = S0
        self.option = option

        # set up directions for tree
        u,d,p = direction(dt,mu,div)
        self.prob = p
        self.up = u
        self.down = d

        # create binomial tree and intrinsic values
        self.binomial = tree(S0,m,u,d)
        intTree = Intrinsic(self.binomial)


        # set intrinsic to put or call
        # solve for black scholes of call or put
        if option == 'put':
            # put intrinsic
            self.intData = intTree.intPut
            self.intDataList = to_list(intTree.intPut)
        else:
            # call intrinsic
            self.intData = intTree.intCall
            self.intDataList = to_list(intTree.intCall)

        # intrinsic value at t = 0
        self.intValue = self.intDataList[0][0]

        # reversed trees
        self.euroRevTree = []
        self.amerRevTree = []

        # prices of each option from the binomial trees
        self.amerPrice = 0.0
        self.euroPrice = 0.0

        # Sf value list
        self.sfList = []

    # reverse valuation through European tree value
    def revEuro(self):
        M = list(range(0,self.m))
        endVals = self.intDataList[len(self.intDataList)-1]

        value = []
        value.append(endVals)

        for step in M:
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        v0 = value[step]
        vStep = []
        for i in range(0,len(v0)-1):
            temp = (v0[i]*self.prob + (1-self.prob)*v0[i+1])*(1 + self.r/self.m)**-s
            vStep.append(temp)
        value.append(vStep)

    # update reversed tree and price
    self.euroRevTree = value
    self.euroPrice = value[len(value)-1][0]

# reverse valuation through American tree value
def revAmer(self):
    M = list(range(0,self.m))
    endVals = self.intDataList[len(self.intDataList)-1]

    # reverse intrinsic data for time state comparison
    # required for American option
    intDataRev = self.intDataList*1
    intDataRev.reverse()

    value = []
    value.append(endVals)

    for step in M:
        v0 = value[step]
        v1 = intDataRev[step+1]
        vStep = []
        for i in range(0,len(v0)-1):
            temp0 = (v0[i]*self.prob + (1-self.prob)*v0[i+1])*(1 + self.r/self.m)**-
            temp1 = v1[i]
            vStep.append(max(temp0,temp1))
        value.append(vStep)

    # update reversed tree and price
    self.amerRevTree = value
    self.amerPrice = value[len(value)-1][0]

# calculate Sf values for American tree
def Sf(self,times):
    biList = to_list(self.binomial)
    amerList = self.amerRevTree*1
    amerList.reverse()

    # create a true/false check on whether intrinsic was used
    intUsed = []
    for t in range(0,len(biList)):
        check = np.equal(self.intDataList[t],[i+1 if i == 0 else i for i in amerList
```
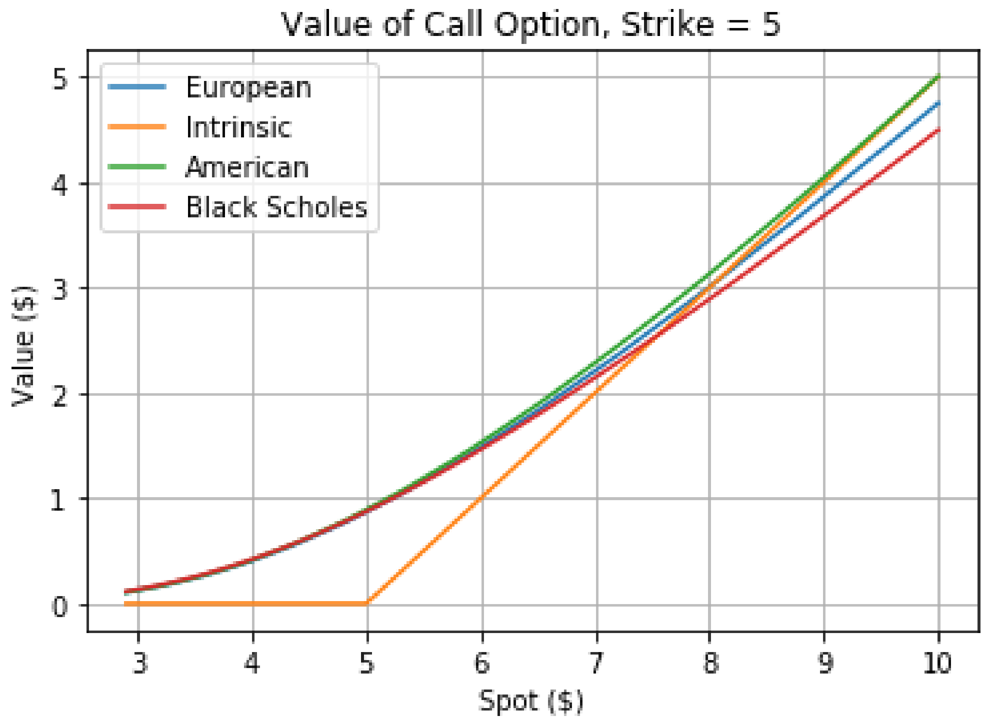

```
                checkValue = np.multiply(check,1)
                intUsed.append(checkValue)

            # solves for sf for put option
            if self.option == 'put':
                sf = []
                for t in times:
                    sft = max(np.multiply(biList[t],intUsed[t]))
                    sf.append(sft)
                self.sfList = sf

            # solves for sf for call option
            else:
                sf = []
                for t in times:
                    sf_check = np.multiply(biList[t],intUsed[t])
                    # change to array and check for nonzero elements
                    # if empty, then you know that the value is zero for that time
                    temp = np.array(sf_check)
                    sft = temp[temp.nonzero()]
                    if sft.size > 0:
                        sftList = list(sft)
                        sf.append(min(sftList))
                    else:
                        sf.append(0.0)
                self.sfList = sf
```

In [4]:
```
# initial values
sigma = 0.5
r = 0.15
mu = 0.1
T = 1.0

m = 100
dt = T/m
K = 5
q = 0.14

# set spot price range
steps = 0.1
S_start = 2.9
S_end = 10
S = np.arange(S_start,S_end+steps,steps)
S = np.around(S,1)
```

In [5]:
```
# time intervals to iterate with Sf
times = np.linspace(0,95,20)
times = times.astype(int)
```

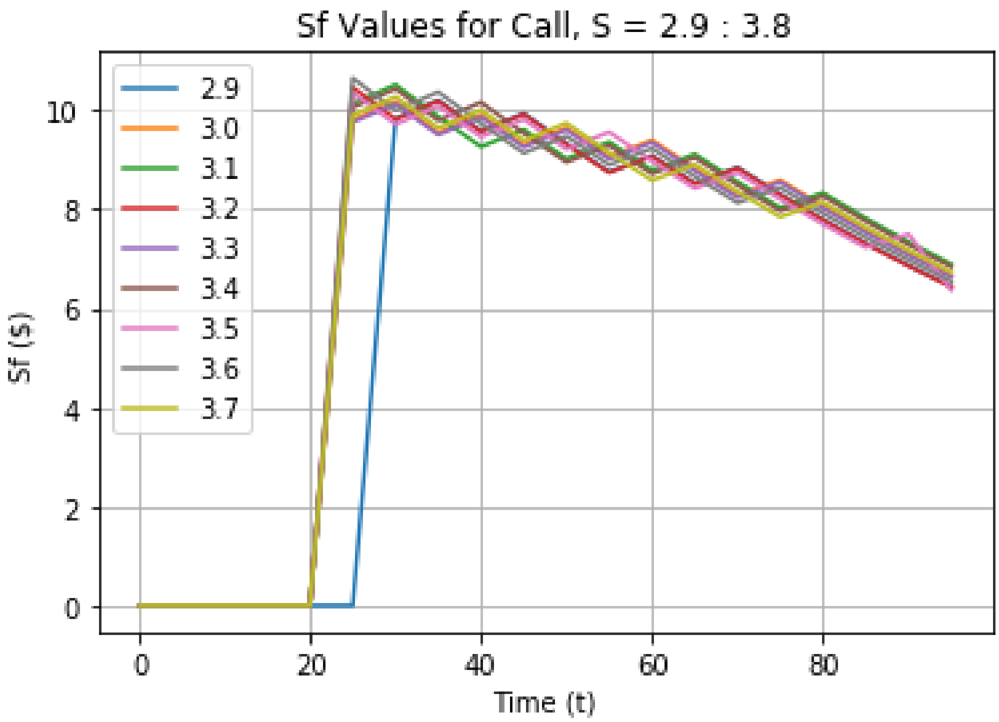
```
In [6]: # collection of call values across underlyings
        amerPutVals = []
        euroPutVals = []
        intVals = []
        sfVals = []
        bsVals = []
        optType = 'call'

        for stockVals in S:
            trees = Tree(r,mu,m,dt,stockVals,q,optType)
            trees.revEuro()
            trees.revAmer()
            euroPutVals.append(trees.euroPrice)
            amerPutVals.append(trees.amerPrice)
            intVals.append(trees.intValue)

            black = bs_price(stockVals,K,T,r,sigma,q)
            bsVals.append(black.call)

            trees.Sf(times)
            sfVals.append(trees.sfList)
```

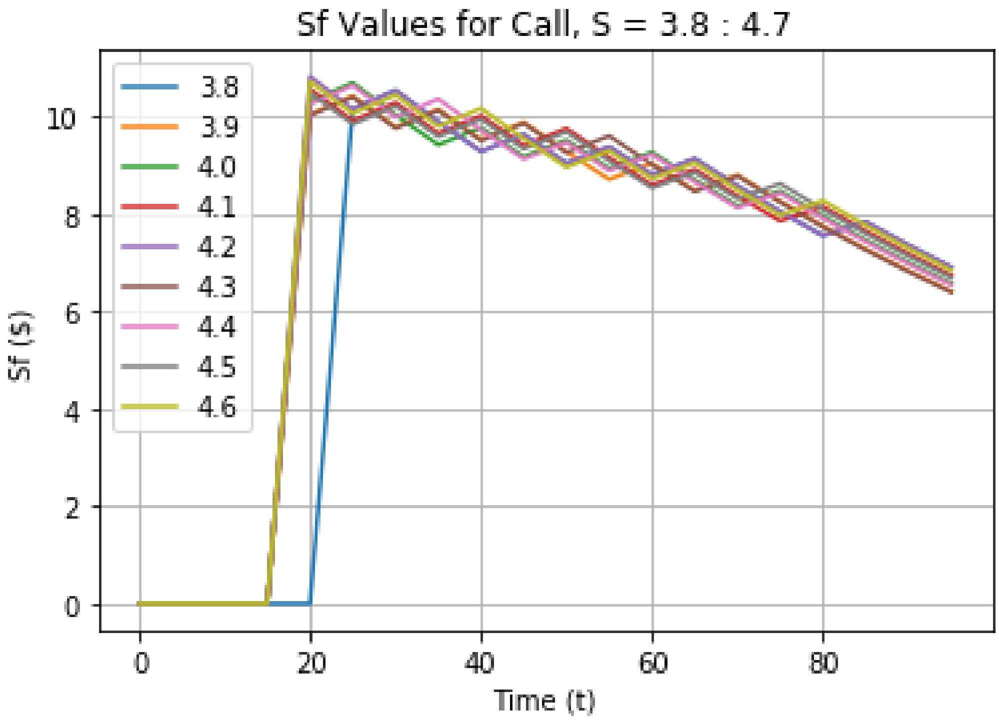
```
In [7]: # plot of American, European and Intrinsic put values
        plt.title('Value of Call Option, Strike = '+str(K))
        plt.xlabel('Spot ($)')
        plt.ylabel('Value ($)')
        plt.plot(S,euroPutVals,label='European')
        plt.plot(S,intVals,label='Intrinsic')
        plt.plot(S,amerPutVals,label='American')
        plt.plot(S,bsVals,label='Black Scholes')
        plt.legend()
        plt.grid()
        plt.show()
```

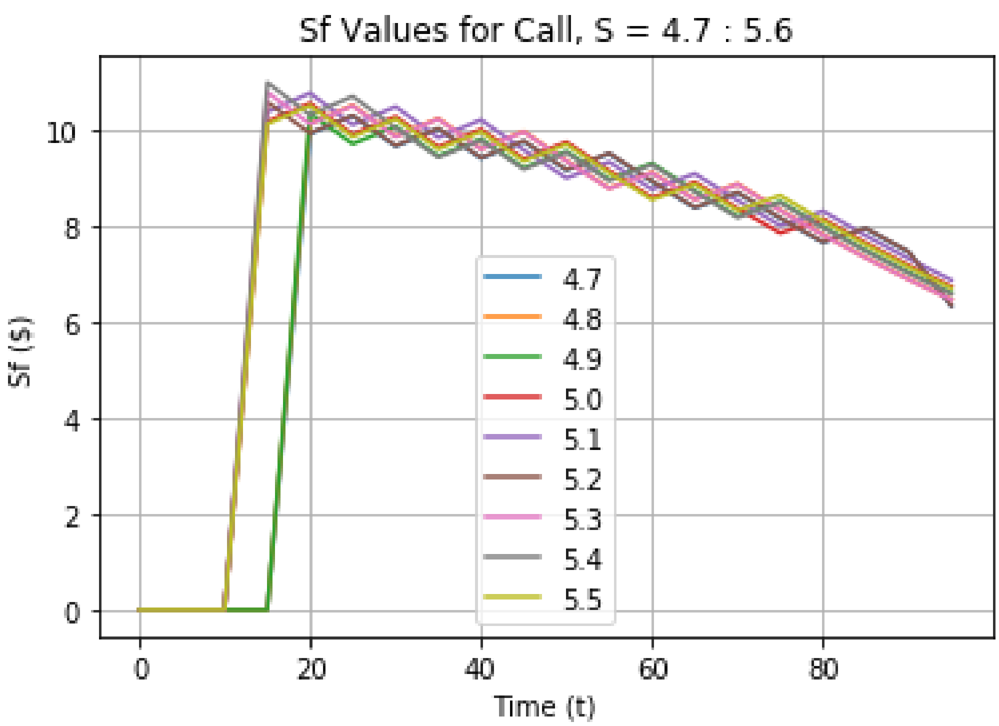


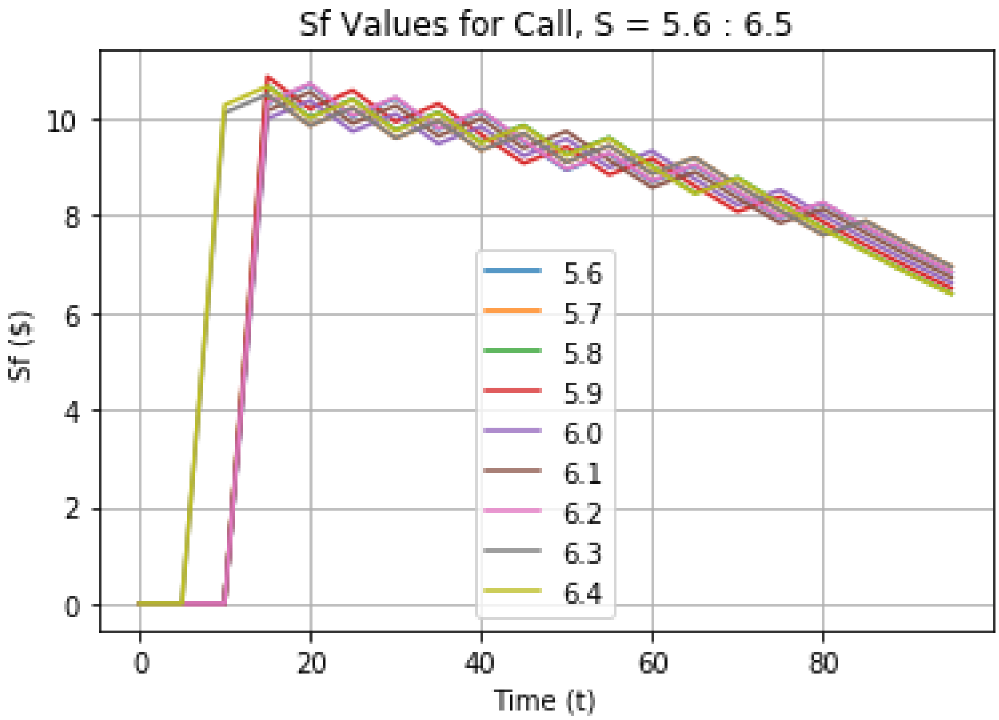
In [8]:
```
# plot of Sf over t, for varying underlying initial values
# for the sake of making the trend more readable
ind = 0
bunch = 9

sfPlot = plt.gca()
xlab = 'Time (t)'
ylab = 'Sf ($)'
for val in sfVals:
    sfPlot.plot(times, val, label=str(S[ind]))
    ind = ind + 1
    try:
        if ind % bunch == 0:
            sfPlot.set_xlabel(xlab)
            sfPlot.set_ylabel(ylab)
            plt.legend()
            title = 'Sf Values for Call, S = {:.1f} : {:.1f}'.format(S[ind-bunch],S[ind]
            plt.title(title)
            plt.grid()
            plt.show()
            sfPlot = plt.gca()
    except:
        sfPlot.set_xlabel(xlab)
        sfPlot.set_ylabel(ylab)
```

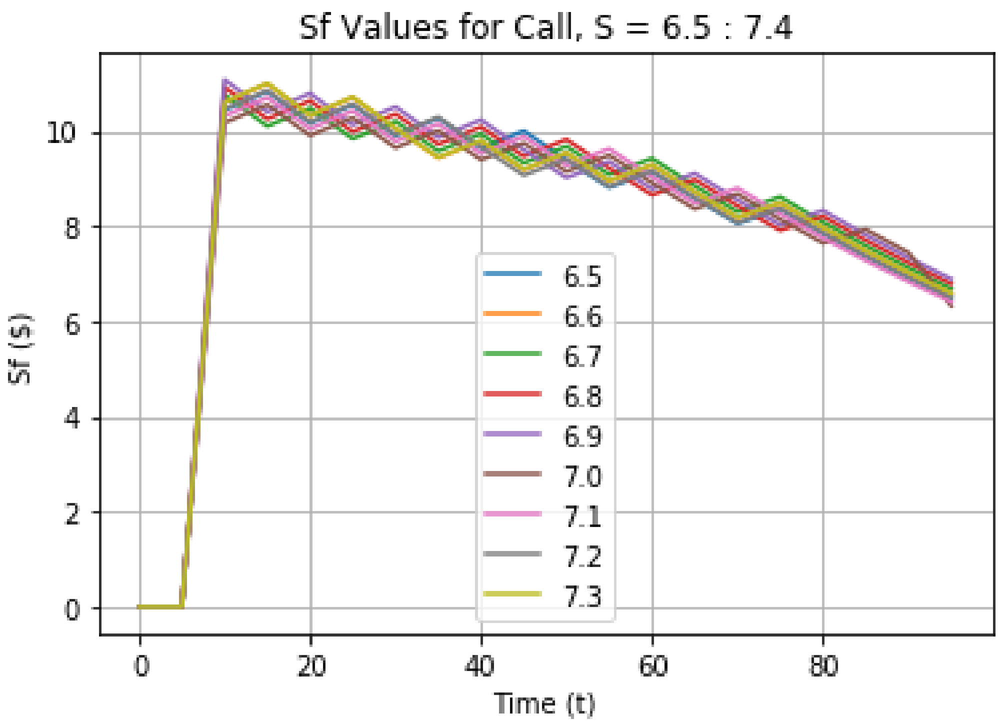
```
        plt.legend()
        title = 'Sf Values for Call, S = {:.1f} : {:.1f}'.format(S[ind-bunch],S[ind-1])
        plt.title(title)
        plt.grid()
        sfPlot = plt.gca()
plt.show()
```

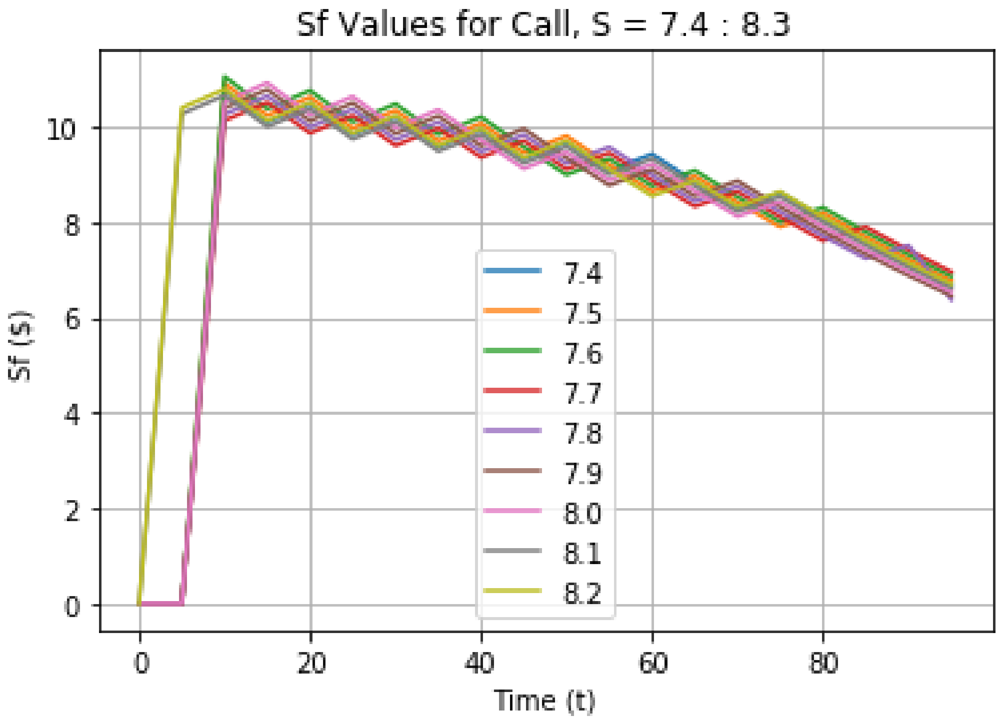


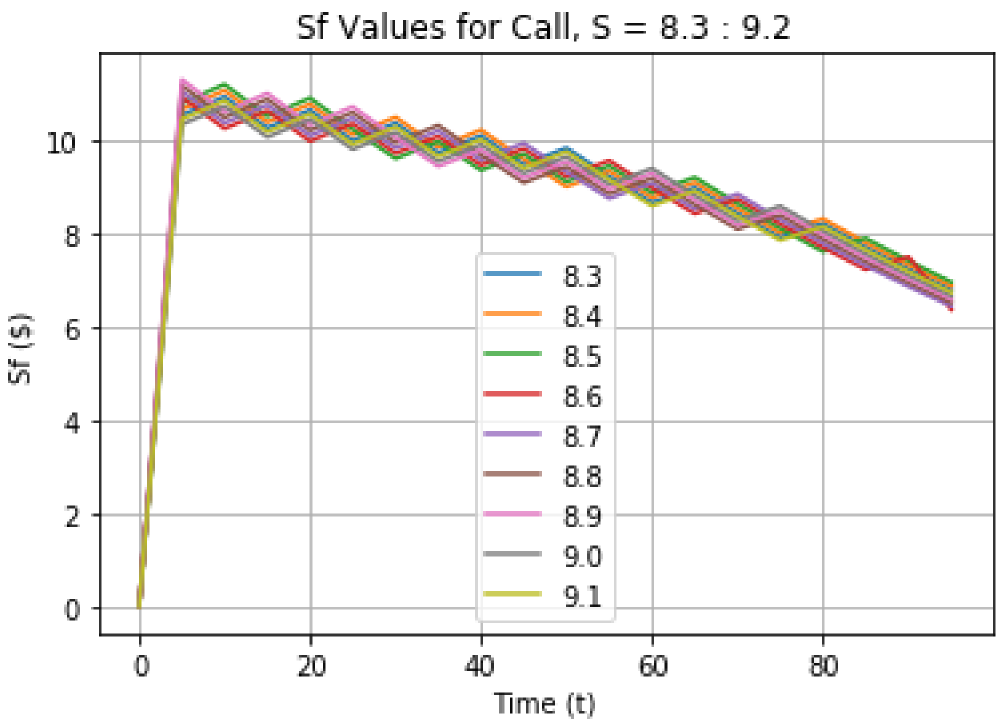

Sf Values for Call, S = 3.8 : 4.7
Sf ($)
Time (t)
0
2
4
6
8
10
0
20
40
60
80
3.8
3.9
4.0
4.1
4.2
4.3
4.4
4.5
4.6

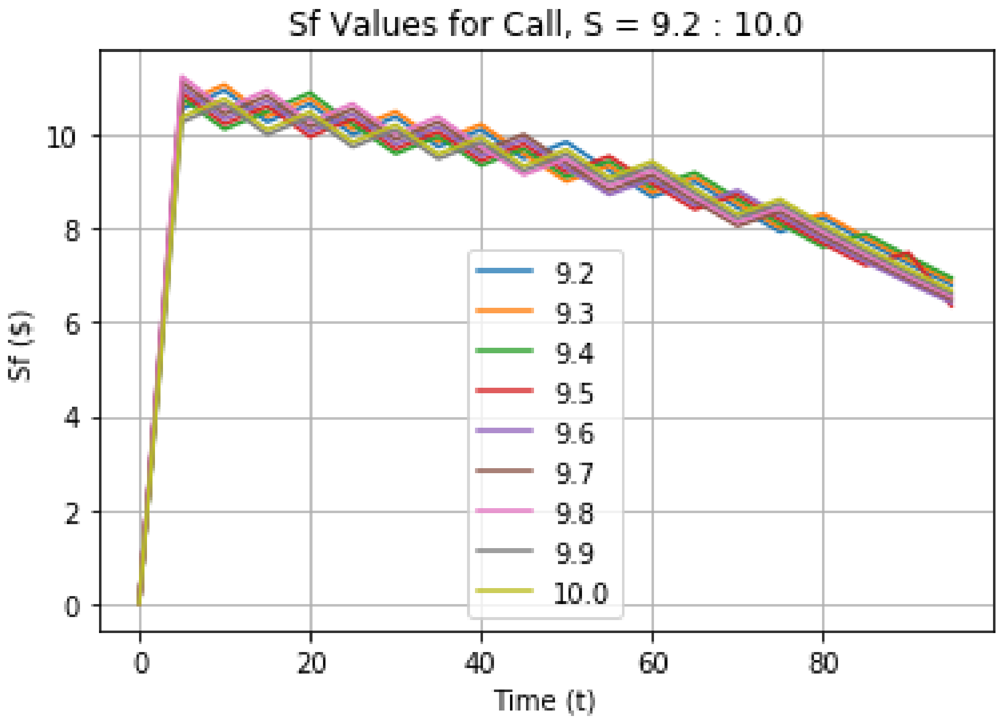

Sf Values for Call, S = 4.7 : 5.6
Sf ($)
Time (t)
0
2
4
6
8
10
0
20
40
60
80
4.7
4.8
4.9
5.0
5.1
5.2
5.3
5.4
5.5


Sf Values for Call, S = 5.6 : 6.5
Sf ($)
Time (t)
5.6
5.7
5.8
5.9
6.0
6.1
6.2
6.3
6.4

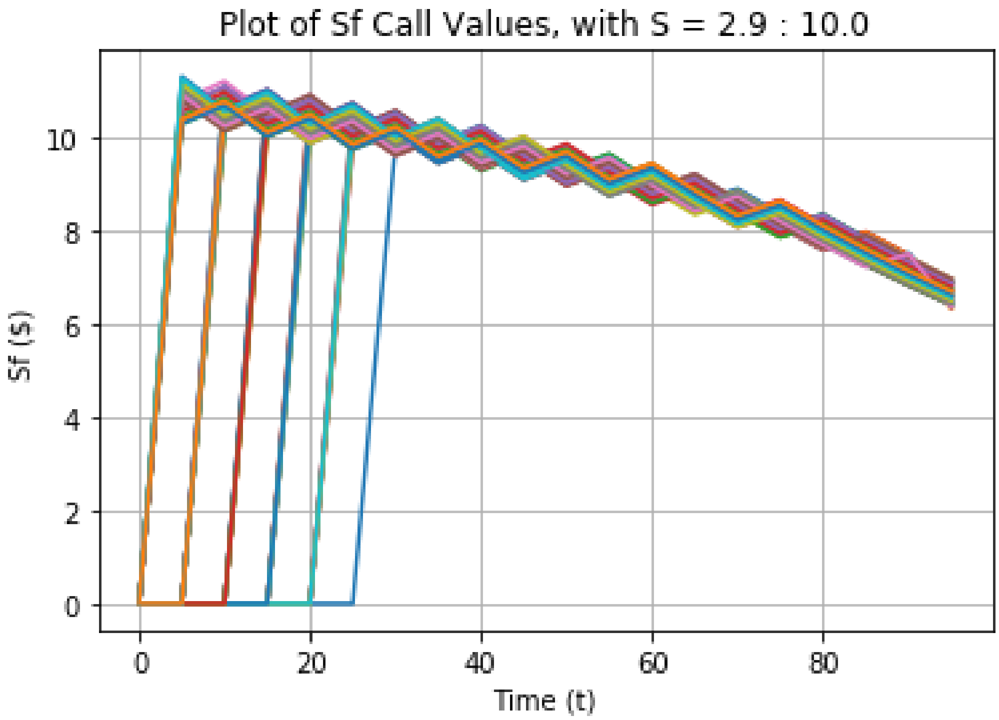

Sf Values for Call, S = 6.5 : 7.4
Sf ($)
Time (t)
6.5
6.6
6.7
6.8
6.9
7.0
7.1
7.2
7.3


Sf Values for Call, S = 7.4 : 8.3
Sf ($)
Time (t)
0
20
40
60
80
0
2
4
6
8
10
7.4
7.5
7.6
7.7
7.8
7.9
8.0
8.1
8.2


Sf Values for Call, S = 8.3 : 9.2
Sf ($)
Time (t)
0
20
40
60
80
0
2
4
6
8
10
8.3
8.4
8.5
8.6
8.7
8.8
8.9
9.0
9.1

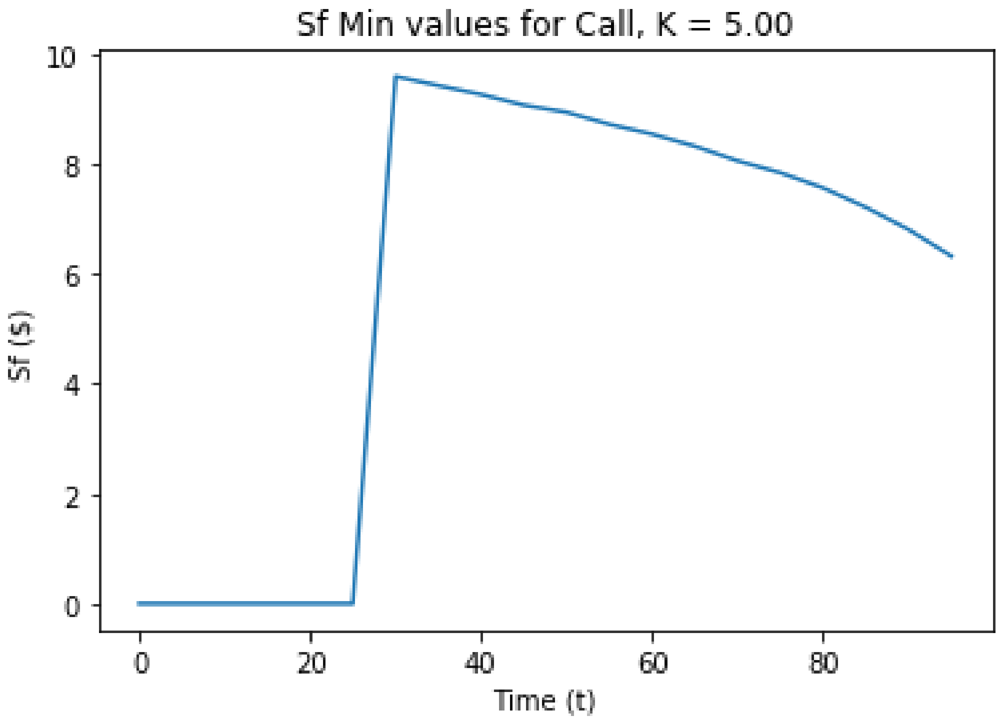



```
In [9]: # show entire Sf values together
        sfPlot = pd.DataFrame(sfVals).transpose()
        sfPlot.columns = S
        sfPlot['Time'] = times
        sfPlot = sfPlot.set_index('Time')
        title = 'Plot of Sf Call Values, with S = {:.1f} : {:.1f}'.format(S[0],S[-1])

        plt.figure()
        for i in sfPlot.columns:
            plt.plot(sfPlot[i],label=i)
        plt.title(title)
        plt.xlabel(xlab)
        plt.ylabel(ylab)
        plt.grid()
        plt.show()
```



In [10]:
```
# find minimum Sft for each time t (for call)
sfMatrix = pd.DataFrame([])
for i in sfVals:
    sfMatrix = pd.concat([sfMatrix,pd.DataFrame(i)],axis=1)
sfFinal = sfMatrix.min(axis=1)

# plot Sft max values
title = 'Sf Min values for Call, K = {:.2f}'.format(K)
plt.title(title)
plt.xlabel(xlab)
plt.ylabel(ylab)
plt.plot(times,sfFinal)
```

Out[10]:
```
[<matplotlib.lines.Line2D at 0x2846970edd8>]
```


Sf Min values for Call, K = 5.00
Sf ($)
Time (t)
0
2
4
6
8
10
0
20
40
60
80