ISyE 6673 HW5 (Due: 8:00 am on 04/24/2018)

Please turn in a hard copy of your solutions to all problems. **Your hardcopy should be properly stapled/clipped** - loose/folded sheets will not be accepted - and should have your name and GA Tech id clearly indicated. For each of the modeling problems, clearly indicate your decision variables, constraints, and objectives. All notation should be clearly explained. For the computational problems, submit a printout of your code and results. Note that only a subset of the problems may be graded. Please abide by the Georgia Tech Honor Code.

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1. Mr. Riches owns a portfolio of N assets containing B_i units of asset i ($i = 1, \ldots, N$). He wishes to liquidate (convert to cash) his portfolio within the next T years. It is known with certainty that the value of asset i in year t is V_{it} \$ per unit. There is a fixed transaction cost of C_t \$ associated with any sale of assets in period t. You should assume that there is at most one transaction in a year, but this might involve selling several different assets at the same time. Any cash earned from liquidating assets (minus the transaction costs) is taxed annually according to the following schedule:

Earning $\times$1000(\$)	Tax rate (%)
0-25	0
25-50	15
50-75	28
75-∞	31

   Assume that the cash earned (minus taxes and costs) in a year is consumed in the same year, so earnings from previous year cannot be used to pay taxes or costs in a later year. You have to guarantee that the cash earned (minus taxes and costs) in each year is non-negative. Ignore interests, loans and any other factor not mentioned explicitly.

   Mr.Riches wishes to maximize the total cash earned minus taxes and transaction costs over all T years.

   - Formulate an optimization model to determine the optimum liquidation schedule (how much of asset i should be liquidated in year t?). Clearly state the decision variables, objective and constraints of your model.
   - State the type of the optimization model formulated above, and state which optimization algorithm you would apply.

2. Dunwoody baseball little league has to schedule 6 games over a 3 day period. The following table indicates the games and the teams that play in each game.

Game	Team A	Team B	Team C	Team D	Team E
1	x	x			
2	x		x		
3	x				x
4		x		x	
5			x	x	
6			x		x

   For example, game 4 is between teams B and D.

   The concession revenues for having game i on day t are given as r_{it} for $i = 1, \ldots, 6$ and $t = 1, \ldots, 3$. Formulate an integer program to help Dunwoody little league decide which game should be played on which day so as to maximize concession revenues subject to the following restrictions:

   - Each game has to be scheduled.
   - No team can play more than 2 games in a day.

- Team D cannot play on day 1.
- There has to be at least 2 games assigned on day 3.
- Game 1 can only be assigned on day 1 if game 5 is assigned on day 2 or if game 6 is not assigned on day 3.

You have to indicate in words, the constraint(s) of your model corresponding to each of the above restrictions.
(Hint: Use binary variables x_{it} which take value 1 if game i is scheduled on day t.)

3. The data file `monthly_returns.csv` contains last 24 month returns for 97 stocks. Use python and Gurobi to implement the following.

   (a) Solve the following portfolio optimization problem for the provided data:

$$\begin{aligned} \min \quad & x^\top Q x \\ \text{s.t.} \quad & r^\top x = 0.01 \\ & e^\top x = 1.0 \\ & x \geq 0 \end{aligned}$$

   where r is the vector of expected returns and Q is the variance covariance of returns. State the optimal portfolio, the associated risk (standard deviation), and the expected return.

   (b) Add a constraint that you can invest in at most 10 stocks, and resolve the problem. State the optimal portfolio, the associated risk (standard deviation), and the expected return.

   (c) Add the following additional constraints:
   - If you invest in PYPL then you must invest at least 0.5 and at most 0.7 units.
   - If you invest in both AMZN and T, the total amount invested in DHR, FB, MON have to be at least 2 times the amount invested in RTN.

   Resolve the problem. State the optimal portfolio, the associated risk (standard deviation), and the expected return.

4. We are given a directed graph $G = (V, E)$. Let $s \in V$ and $t \in V$ be two special vertices. The nominal travel time on edge (ij) is c_{ij}. If the edge fails (gets congested) then the travel time becomes $c_{ij} + d_{ij}$. We know that at most Γ of the edges can fail simultaneously. The parameters c_{ij}, d_{ij} for each edge (i, j), and Γ are given.

   (a) Formulate a mixed integer linear optimization problem to find a path from s to t to minimize the worst-case travel time, i.e. a robust optimization model. Show full work in deriving your model.

   (b) The data file `edge_data.csv` lists c_{ij} and d_{ij} for a graph with 50 nodes and 2209 edges. The nodes are labelled as $\{0, 1, 2, \ldots, 49\}$ and the edges are listed in the data file (an edge goes from node i to node j). Suppose that $s = 0$ and $t = 49$. Solve your robust shortest path model for $\Gamma = 0, 1, 2, 3$ and list the corresponding paths (an ordered sequence of nodes) and lengths.