



? At Brookhaven National Laboratory in New York, atomic nuclei are accelerated to 99.995% of the ultimate speed limit of the universe—the speed of light,  $c$ . Compared to the kinetic energy of a nucleus moving at 99.000% of  $c$ , the kinetic energy of the same nucleus moving at 99.995% of  $c$  is about (i) 0.001% greater; (ii) 0.1% greater; (iii) 1% greater; (iv) 2% greater; (v) 16 times greater.

# 37 Relativity

In 1905, Albert Einstein—then an unknown 25-year-old assistant in the Swiss patent office—published four papers of extraordinary importance. One was an analysis of Brownian motion; a second (for which he was awarded the Nobel Prize) was on the photoelectric effect. In the last two, Einstein introduced his **special theory of relativity**, proposing drastic revisions in the Newtonian concepts of space and time.

Einstein based the special theory of relativity on two postulates. One states that the laws of physics are the same in all inertial frames of reference; the other states that the speed of light in vacuum is the same in all inertial frames. These innocent-sounding propositions have far-reaching implications. Here are three: (1) Events that are simultaneous for one observer may not be simultaneous for another. (2) When two observers moving relative to each other measure a time interval or a length, they may not get the same results. (3) For the conservation principles for momentum and energy to be valid in all inertial systems, Newton's second law and the equations for momentum and kinetic energy have to be revised.

Relativity has important consequences in *all* areas of physics, including electromagnetism, atomic and nuclear physics, and high-energy physics. Although many of the results derived in this chapter may run counter to your intuition, the theory is in solid agreement with experimental observations.

## 37.1 INVARIANCE OF PHYSICAL LAWS

Let's take a look at the two postulates that make up the special theory of relativity. Both postulates describe what is seen by an observer in an *inertial frame of reference*, which we introduced in Section 4.2. The theory is “special” in the sense that it applies to observers in such special reference frames.

### Einstein's First Postulate

Einstein's first postulate, called the **principle of relativity**, states: **The laws of physics are the same in every inertial frame of reference.** If the laws differed, that difference could distinguish one inertial frame from the others or make one frame more “correct” than another. As an example, suppose you watch two children playing catch with a ball while the three of you are aboard a train moving with constant velocity. Your observations

## LEARNING OUTCOMES

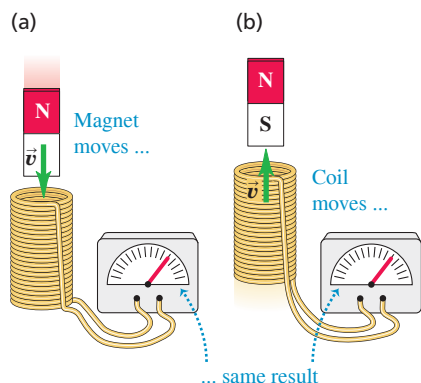
*In this chapter, you'll learn...*

- 37.1 The two postulates of Einstein's special theory of relativity, and what motivates these postulates.
- 37.2 Why different observers can disagree about whether two events are simultaneous.
- 37.3 How relativity predicts that moving clocks run slow, and what experimental evidence confirms this.
- 37.4 How the length of an object changes due to the object's motion.
- 37.5 How the velocity of an object depends on the frame of reference from which it is observed.
- 37.6 How the frequency of a light wave has different values for different observers.
- 37.7 How the theory of relativity modifies the relationship between velocity and momentum.
- 37.8 How to solve problems involving work and kinetic energy for particles moving at relativistic speeds.
- 37.9 Some of the key concepts of Einstein's general theory of relativity.

*You'll need to review...*

- 3.4, 3.5 Motion in a circle, relative velocity.
- 4.2 Inertial frames of reference.
- 16.8 Doppler effect for sound.
- 29.1 Electromagnetic induction.
- 32.2 Maxwell's equations and the speed of light.
- 35.5 Michelson-Morley experiment.

Figure 37.1 The same emf is induced in the coil whether (a) the magnet moves relative to the coil or (b) the coil moves relative to the magnet.



of the motion of the ball, no matter how carefully done, can't tell you how fast (or whether) the train is moving. This is because Newton's laws of motion are the same in every inertial frame.

Another example is the electromotive force (emf) induced in a coil of wire by a nearby moving permanent magnet. In the frame of reference in which the coil is stationary (Fig. 37.1a), the moving magnet causes a change of magnetic flux through the coil, and this induces an emf. In a different frame of reference in which the magnet is stationary (Fig. 37.1b), the motion of the coil through a magnetic field induces the emf. According to the principle of relativity, both of these frames of reference are equally valid. Hence the same emf must be induced in both situations shown in Fig. 37.1. As we saw in Section 29.1, this is indeed the case, so Faraday's law is consistent with the principle of relativity. Indeed, all of the laws of electromagnetism are the same in every inertial frame of reference.

Equally significant is the prediction of the speed of electromagnetic radiation, derived from Maxwell's equations (see Section 32.2). According to this analysis, light and all other electromagnetic waves travel in vacuum with a constant speed, now defined to equal exactly 299,792,458 m/s. (We often use the approximate value  $c = 3.00 \times 10^8$  m/s.) As we'll see, the speed of light in vacuum plays a central role in the theory of relativity.

## Einstein's Second Postulate

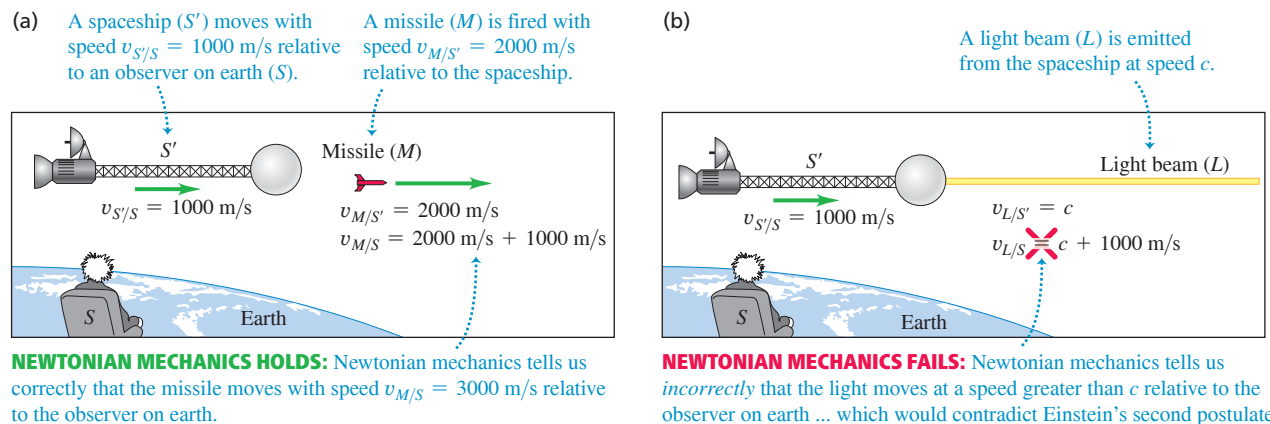
During the 19th century, most physicists believed that light traveled through a hypothetical medium called the *ether*, just as sound waves travel through air. If so, the speed of light measured by observers would depend on their motion relative to the ether and would therefore be different in different directions. The Michelson-Morley experiment, described in Section 35.5, was an effort to detect motion of the earth relative to the ether.

Einstein's conceptual leap was to recognize that if Maxwell's equations are valid in all inertial frames, then the speed of light in vacuum should also be the same in all frames and in all directions. In fact, Michelson and Morley detected *no* ether motion across the earth, and the ether concept has been discarded. Although Einstein may not have known about this negative result, it supported his bold hypothesis. We call this **Einstein's second postulate: The speed of light in vacuum is the same in all inertial frames of reference and is independent of the motion of the source.**

Let's think about what this means. Suppose two observers measure the speed of light in vacuum. One is at rest with respect to the light source, and the other is moving away from it. Both are in inertial frames of reference. According to the principle of relativity, the two observers must obtain the same result, despite the fact that one is moving with respect to the other.

If this seems too easy, consider the following situation. A spacecraft moving past the earth at 1000 m/s fires a missile straight ahead with a speed of 2000 m/s (relative to the spacecraft) (Fig. 37.2). What is the missile's speed relative to the earth? Simple, you say; this is an elementary problem in relative velocity (see Section 3.5). The correct answer,

Figure 37.2 (a) Newtonian mechanics makes correct predictions about relatively slow-moving objects; (b) it makes incorrect predictions about the behavior of light.



according to Newtonian mechanics, is 3000 m/s. But now suppose the spacecraft turns on a searchlight, pointing in the same direction in which the missile was fired. An observer on the spacecraft measures the speed of light emitted by the searchlight and obtains the value  $c$ . According to Einstein's second postulate, the motion of the light after it has left the source cannot depend on the motion of the source. So the observer on earth who measures the speed of this same light must also obtain the value  $c$ , *not*  $c + 1000$  m/s. This result contradicts our elementary notion of relative velocities, and it may not appear to agree with common sense. But "common sense" is intuition based on everyday experience, and this does not usually include measurements of the speed of light.

## The Ultimate Speed Limit

Einstein's second postulate immediately implies the following result: **It is impossible for an inertial observer to travel at  $c$ , the speed of light in vacuum.**

We can prove this by showing that travel at  $c$  implies a logical contradiction. Suppose that the spacecraft  $S'$  in Fig. 37.2b is moving at the speed of light relative to an observer on the earth, so that  $v_{S'/S} = c$ . If the spacecraft turns on a headlight, the second postulate now asserts that the earth observer  $S$  measures the headlight beam to be also moving at  $c$ . Thus this observer measures that the headlight beam and the spacecraft move together and are always at the same point in space. But Einstein's second postulate also asserts that the headlight beam moves at a speed  $c$  relative to the spacecraft, so they *cannot* be at the same point in space. This contradictory result can be avoided only if it is impossible for an inertial observer, such as a passenger on the spacecraft, to move at  $c$ . As we go through our discussion of relativity, you may find yourself asking the question Einstein asked himself as a 16-year-old student, "What would I see if I were traveling at the speed of light?" Einstein realized only years later that his question's basic flaw was that he could *not* travel at  $c$ .

## The Galilean Coordinate Transformation

Let's restate this argument symbolically, using two inertial frames of reference, labeled  $S$  for the observer on earth and  $S'$  for the moving spacecraft, as shown in **Fig. 37.3**. To keep things as simple as possible, we have omitted the  $z$ -axes. The  $x$ -axes of the two frames lie along the same line, but the origin  $O'$  of frame  $S'$  moves relative to the origin  $O$  of frame  $S$  with constant velocity  $u$  along the common  $x$ - $x'$ -axis. We on earth set our clocks so that the two origins coincide at time  $t = 0$ , so their separation at a later time  $t$  is  $ut$ .

**CAUTION Choose your inertial frame coordinates wisely** Many of the equations derived in this chapter are true *only* if you define your inertial reference frames as stated in the preceding paragraph. The positive  $x$ -direction must be the direction in which the origin  $O'$  moves relative to the origin  $O$ . In Fig. 37.3 this direction is to the right; if instead  $O'$  moves to the left relative to  $O$ , you must define the positive  $x$ -direction to be to the left. When solving problems with reference frames, you should *always* draw a diagram showing which reference frame goes with which observer. ■

Now think about how we describe the motion of a particle  $P$ . This might be an exploratory vehicle launched from the spacecraft or a pulse of light from a laser. We can describe the *position* of this particle by using the earth coordinates  $(x, y, z)$  in  $S$  or the spacecraft coordinates  $(x', y', z')$  in  $S'$ . Figure 37.3 shows that these are simply related by

$$x = x' + ut \quad y = y' \quad z = z' \quad (\text{Galilean coordinate transformation}) \quad (37.1)$$

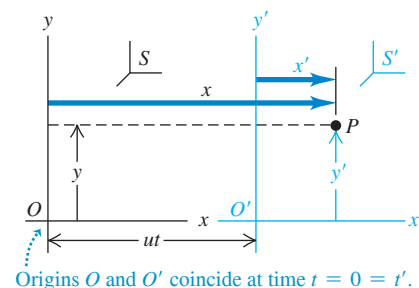
These equations, based on the familiar Newtonian notions of space and time, are called the **Galilean coordinate transformation**.

If particle  $P$  moves in the  $x$ -direction, its instantaneous velocity  $v_x$  as measured by an observer stationary in  $S$  is  $v_x = dx/dt$ . Its velocity  $v'_x$  as measured by an observer stationary in  $S'$  is  $v'_x = dx'/dt$ . We can derive a relationship between  $v_x$  and  $v'_x$  by taking the derivative with respect to  $t$  of the first of Eqs. (37.1):

$$\frac{dx}{dt} = \frac{dx'}{dt} + u$$

Figure 37.3 The position of particle  $P$  can be described by the coordinates  $x$  and  $y$  in frame of reference  $S$  or by  $x'$  and  $y'$  in frame  $S'$ .

Frame  $S'$  moves relative to frame  $S$  with constant velocity  $u$  along the common  $x$ - $x'$ -axis.





Now  $dx/dt$  is the velocity  $v_x$  measured in  $S$ , and  $dx'/dt'$  is the velocity  $v'_x$  measured in  $S'$ , so we get the *Galilean velocity transformation* for one-dimensional motion:

$$v_x = v'_x + u \quad (\text{Galilean velocity transformation}) \quad (37.2)$$

Although the notation differs, this result agrees with our discussion of relative velocities in Section 3.5.

Now here's the fundamental problem. Applied to the speed of light in vacuum, Eq. (37.2) says that  $c = c' + u$ . Einstein's second postulate, supported subsequently by a wealth of experimental evidence, says that  $c = c'$ . This is a genuine inconsistency, not an illusion, and it demands resolution. If we accept this postulate, we are forced to conclude that Eqs. (37.1) and (37.2) *cannot* be precisely correct, despite our convincing derivation. These equations have to be modified to bring them into harmony with this principle.

The resolution involves some very fundamental modifications in our kinematic concepts. The first idea to be changed is the seemingly obvious assumption that the observers in frames  $S$  and  $S'$  use the same *time scale*, formally stated as  $t = t'$ . Alas, we are about to show that this everyday assumption cannot be correct; the two observers *must* have different time scales. We must define the velocity  $v'$  in frame  $S'$  as  $v' = dx'/dt'$ , not as  $dx'/dt$ ; the two quantities are not the same. The difficulty lies in the concept of *simultaneity*, which is our next topic. A careful analysis of simultaneity will help us develop the appropriate modifications of our notions about space and time.

**TEST YOUR UNDERSTANDING OF SECTION 37.1** As a high-speed spaceship flies past you, it fires a strobe light that sends out a pulse of light in all directions. An observer aboard the spaceship measures a spherical wave front that spreads away from the spaceship with the same speed  $c$  in all directions. (a) What is the shape of the wave front that you measure? (i) Spherical; (ii) ellipsoidal, with the longest axis of the ellipsoid along the direction of the spaceship's motion; (iii) ellipsoidal, with the shortest axis of the ellipsoid along the direction of the spaceship's motion; (iv) not enough information is given to decide. (b) As measured by you, does the wave front remain centered on the spaceship?

**ANSWER** (a) (i), (b) no. You, too, will measure a spherical wave front that expands at the same speed  $c$  in all directions. This is a consequence of Einstein's second postulate. The wave front that you measure does *not* stay centered on the position of the moving spaceship; rather, it is centered on the point  $P$  where the spaceship was located at the instant that it emitted the light pulse. For example, suppose the spaceship is moving at speed  $c/2$ . When your watch shows that a time  $t$  has elapsed since the pulse of light was emitted, your measurements will show that the wave front is a sphere of radius  $ct$  centered on  $P$  and that the spaceship is a distance  $ct/2$  from  $P$ .

Figure 37.4 An event has a definite position and time—for instance, on the pavement directly below the center of the Eiffel Tower at midnight on New Year's Eve.



## 37.2 RELATIVITY OF SIMULTANEITY

Measuring times and time intervals involves the concept of **simultaneity**. In a given frame of reference, an **event** is an occurrence that has a definite position and time (**Fig. 37.4**). When you say that you awoke at seven o'clock, you mean that two events (your awakening and your clock showing 7:00) occurred *simultaneously*. The fundamental problem in measuring time intervals is this: In general, two events that are simultaneous in one frame of reference are *not* simultaneous in a second frame that is moving relative to the first, even if both are inertial frames.

### A Thought Experiment in Simultaneity

This may seem to be contrary to common sense. To illustrate the point, here is a version of one of Einstein's *thought experiments*—mental experiments that follow concepts to their logical conclusions. Imagine a train moving with a speed comparable to  $c$ , with uniform velocity (**Fig. 37.5**). Two lightning bolts strike a passenger car, one near each end. Each bolt leaves a mark on the car and one on the ground at the instant the bolt hits. The points on the ground are labeled  $A$  and  $B$  in the figure, and the corresponding points on the car are  $A'$  and  $B'$ . Stanley is stationary on the ground at  $O$ , midway between  $A$  and  $B$ .

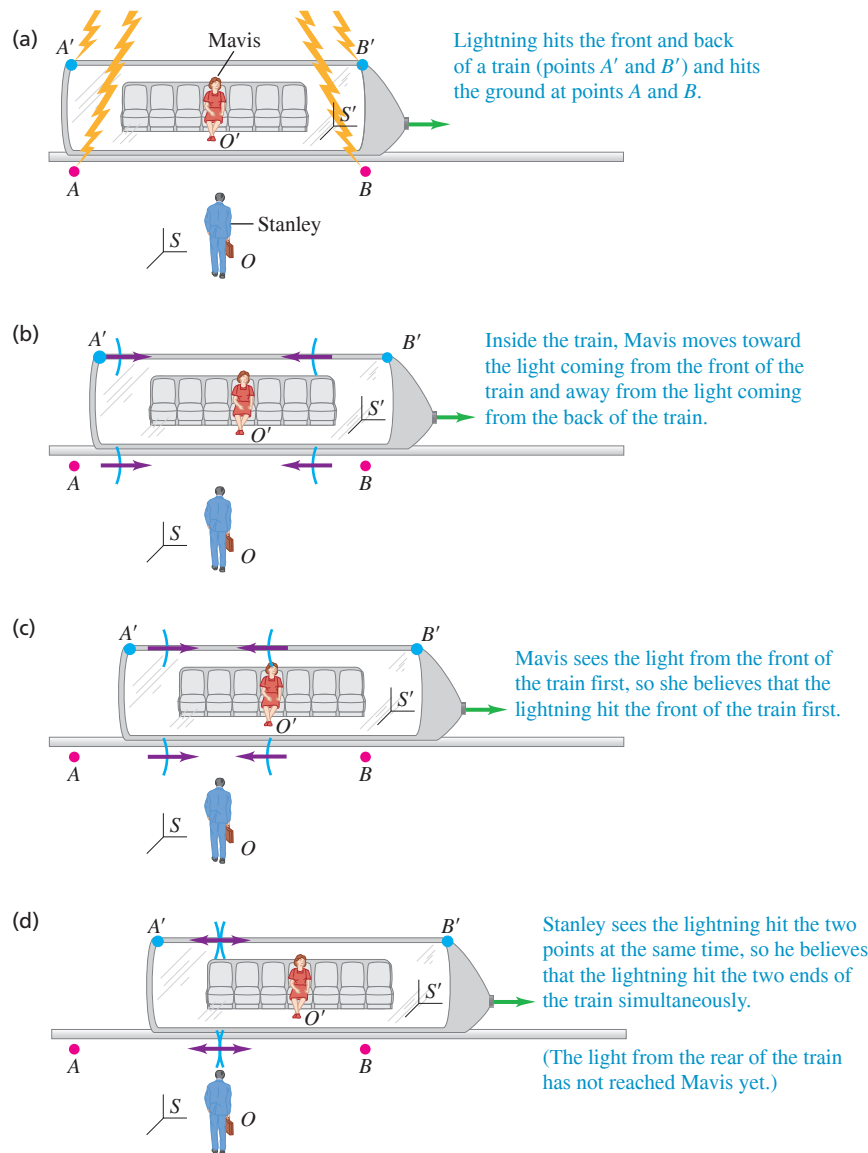


Figure 37.5 A thought experiment in simultaneity.

Mavis is moving with the train at  $O'$  in the middle of the passenger car, midway between  $A'$  and  $B'$ . Both Stanley and Mavis see both light flashes emitted from the points where the lightning strikes.

Suppose the two wave fronts from the lightning strikes reach Stanley at  $O$  simultaneously. He knows that he is the same distance from  $B$  and  $A$ , so Stanley concludes that the two bolts struck  $B$  and  $A$  simultaneously. Mavis agrees that the two wave fronts reached Stanley at the same time, but she disagrees that the flashes were emitted simultaneously.

Stanley and Mavis agree that the two wave fronts do not reach Mavis at the same time. Mavis at  $O'$  is moving to the right with the train, so she runs into the wave front from  $B'$  *before* the wave front from  $A'$  catches up to her. However, because she is in the middle of the passenger car equidistant from  $A'$  and  $B'$ , her observation is that both wave fronts took the same time to reach her because both moved the same distance at the same speed  $c$ . (Recall that the speed of each wave front with respect to *either* observer is  $c$ .) Thus she concludes that the lightning bolt at  $B'$  struck *before* the one at  $A'$ . Stanley at  $O$  measures the two events to be simultaneous, but Mavis at  $O'$  does not! *Whether or not two events at different  $x$ -axis locations are simultaneous depends on the state of motion of the observer.*

You may want to argue that in this example the lightning bolts really *are* simultaneous and that if Mavis at  $O'$  could communicate with the distant points without the time delay

caused by the finite speed of light, she would realize this. But that would be erroneous; the finite speed of information transmission is not the real issue. If  $O'$  is midway between  $A'$  and  $B'$ , then in her frame of reference the time for a signal to travel from  $A'$  to  $O'$  is the same as that from  $B'$  to  $O'$ . Two signals arrive simultaneously at  $O'$  only if they were emitted simultaneously at  $A'$  and  $B'$ . In this example they *do not* arrive simultaneously at  $O'$ , and so Mavis must conclude that the events at  $A'$  and  $B'$  were *not* simultaneous.

Furthermore, there is no basis for saying that Stanley is right and Mavis is wrong, or vice versa. According to the principle of relativity, no inertial frame of reference is more correct than any other in the formulation of physical laws. Each observer is correct *in his or her own frame of reference*. In other words, simultaneity is not an absolute concept. Whether two events are simultaneous depends on the frame of reference. As we mentioned at the beginning of this section, simultaneity plays an essential role in measuring time intervals. It follows that *the time interval between two events may be different in different frames of reference*. So our next task is to learn how to compare time intervals in different frames of reference.

**TEST YOUR UNDERSTANDING OF SECTION 37.2** Stanley, who works for the rail system shown in Fig. 37.5, has carefully synchronized the clocks at all of the rail stations. At the moment that Stanley measures all of the clocks striking noon, Mavis is on a high-speed passenger car traveling from Ogdenville toward North Haverbrook. According to Mavis, when the Ogdenville clock strikes noon, what time is it in North Haverbrook? (i) Noon; (ii) before noon; (iii) after noon.

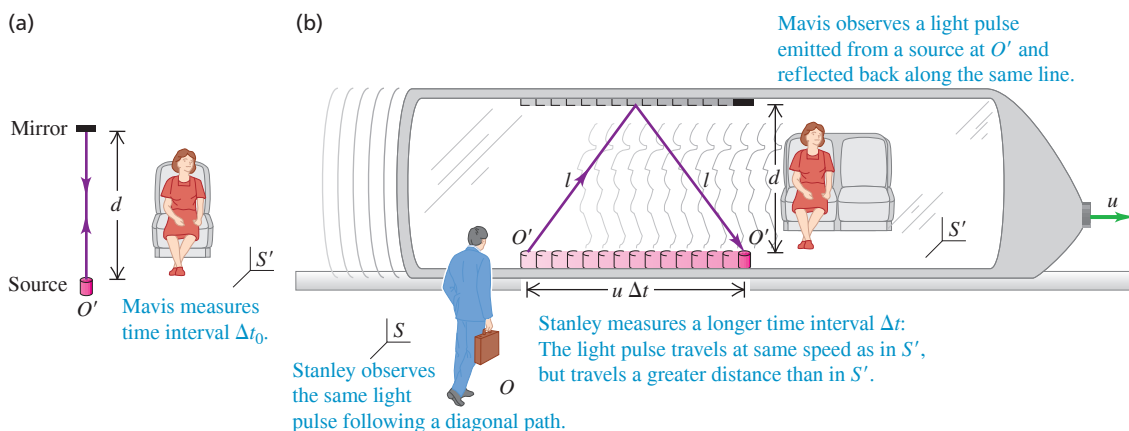
### ANSWER

(iii) In Mavis's frame of reference, the two events (the Ogdenville clock striking noon and the North Haverbrook clock striking noon) are not simultaneous. Figure 37.5 shows that the event toward the front of the rail car occurs first. Since the rail car is moving toward North Haverbrook, that clock struck noon before the one on Ogdenville. So, according to Mavis, it is after noon in North Haverbrook.

## 37.3 RELATIVITY OF TIME INTERVALS

We can derive a quantitative relationship between time intervals in different coordinate systems. To do this, let's consider another thought experiment. As before, a frame of reference  $S'$  moves along the common  $x$ - $x'$ -axis with constant speed  $u$  relative to a frame  $S$ . As discussed in Section 37.1,  $u$  must be less than the speed of light  $c$ . Mavis, who is riding along with frame  $S'$ , measures the time interval between two events that occur at the *same* point in space. Event 1 is when a flash of light from a light source leaves  $O'$ . Event 2 is when the flash returns to  $O'$ , having been reflected from a mirror a distance  $d$  away, as shown in **Fig. 37.6a**. We label the time interval  $\Delta t_0$ , using the subscript zero as a reminder

Figure 37.6 (a) Mavis, in frame of reference  $S'$ , observes a light pulse emitted from a source at  $O'$  and reflected back along the same line. (b) How Stanley (in frame of reference  $S$ ) and Mavis observe the same light pulse. The positions of  $O'$  at the times of departure and return of the pulse are shown.



that the apparatus is at rest, with zero velocity, in frame  $S'$ . The flash of light moves a total distance  $2d$ , so the time interval is

$$\Delta t_0 = \frac{2d}{c} \quad (37.3)$$

The round-trip time measured by Stanley in frame  $S$  is a different interval  $\Delta t$ ; in his frame of reference the two events occur at *different* points in space. During the time  $\Delta t$ , the source moves relative to  $S$  a distance  $u \Delta t$  (Fig. 37.6b). In  $S'$  the round-trip distance is  $2d$  perpendicular to the relative velocity, but the round-trip distance in  $S$  is the longer distance  $2l$ , where

$$l = \sqrt{d^2 + \left(\frac{u \Delta t}{2}\right)^2}$$

In writing this expression, we have assumed that both observers measure the same distance  $d$ . We'll justify this assumption in the next section. The speed of light is the same for both observers, so the round-trip time measured in  $S$  is

$$\Delta t = \frac{2l}{c} = \frac{2}{c} \sqrt{d^2 + \left(\frac{u \Delta t}{2}\right)^2} \quad (37.4)$$

We would like to have a relationship between  $\Delta t$  and  $\Delta t_0$  that is independent of  $d$ . To get this, we solve Eq. (37.3) for  $d$  and substitute the result into Eq. (37.4):

$$\Delta t = \frac{2}{c} \sqrt{\left(\frac{c \Delta t_0}{2}\right)^2 + \left(\frac{u \Delta t}{2}\right)^2} \quad (37.5)$$

Now we square this and solve for  $\Delta t$ ; the result is

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$$

Since the quantity  $\sqrt{1 - u^2/c^2}$  is less than 1,  $\Delta t$  is greater than  $\Delta t_0$ : Thus Stanley measures a *longer* round-trip time for the light pulse than does Mavis.

## Time Dilation and Proper Time

We may generalize this important result. Suppose that in a particular frame of reference, two events occur at the same point in space. If these events are two ticks of a clock, then this is the frame of reference at which the clock is at rest. We call this the *rest frame* of the clock. There is only one frame of reference in which a clock is at rest, and there are infinitely many in which it is moving. Therefore the time interval measured between two events (such as two ticks of the clock) that occur at the same point in a particular frame is a more fundamental quantity than the interval between events at different points. We use the term **proper time** to describe the time interval between two events that occur *at the same point*.

Let  $\Delta t_0$  be the proper time between the two events—that is, the time as measured by an observer at rest in the frame in which the events occur at the same point. Then our above result says that an observer in a second frame moving with constant speed  $u$  relative to the rest frame will measure the time interval to be  $\Delta t$ , where

**Time dilation:**

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} \quad (37.6)$$

Proper time between two events (measured in rest frame)

Speed of light in vacuum

Speed of second frame relative to rest frame

Time interval between same events measured in second frame of reference

Figure 37.7 This image shows an exploding star, called a *supernova*, within a distant galaxy. The brightness of a typical supernova decays at a certain rate. But supernovae that are moving away from us at a substantial fraction of the speed of light decay more slowly, in accordance with Eq. (37.6). The decaying supernova is a moving “clock” that runs slow.

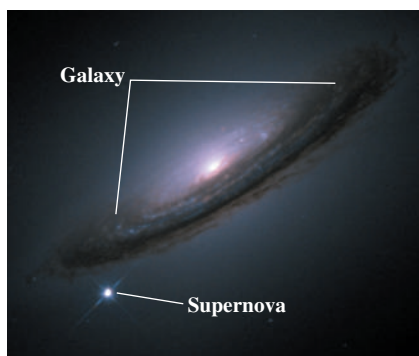


Figure 37.8 The Lorentz factor  $\gamma = 1/\sqrt{1 - u^2/c^2}$  as a function of the relative speed  $u$  of two frames of reference.

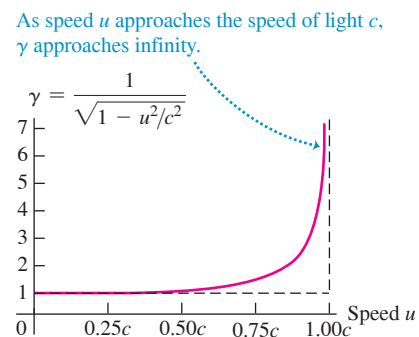
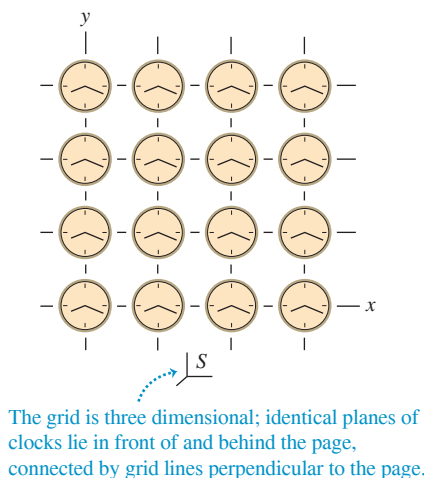


Figure 37.9 A frame of reference pictured as a coordinate system with a grid of synchronized clocks.



We recall that no inertial observer can travel at  $u = c$  and we note that  $\sqrt{1 - u^2/c^2}$  is imaginary for  $u > c$ . Thus Eq. (37.6) gives sensible results only when  $u < c$ . The denominator of Eq. (37.6) is always smaller than 1, so  $\Delta t$  is always *larger* than  $\Delta t_0$ . Thus we call this effect **time dilation**.

Think of an old-fashioned pendulum clock that has one second between ticks, as measured by Mavis in the clock’s rest frame; this is  $\Delta t_0$ . If the clock’s rest frame is moving relative to Stanley, he measures a time between ticks  $\Delta t$  that is longer than one second. In brief, *observers measure any clock to run slow if it moves relative to them (Fig. 37.7)*. Note that this conclusion is a direct result of the fact that the speed of light in vacuum is the same in both frames of reference.

The quantity  $1/\sqrt{1 - u^2/c^2}$  in Eq. (37.6) is called the **Lorentz factor**. It appears often in relativity and is denoted by the symbol  $\gamma$  (the Greek letter gamma):

$$\text{Lorentz factor } \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \quad (37.7)$$

Speed of light in vacuum  
Speed of one frame of reference relative to another

In terms of this symbol, we can express the time dilation formula, Eq. (37.6), as

$$\text{Time dilation: } \Delta t = \gamma \Delta t_0 \quad (37.8)$$

Proper time between two events (measured in rest frame)  
Lorentz factor relating the two frames  
Time interval between same events measured in second frame of reference

As a further simplification,  $u/c$  is sometimes given the symbol  $\beta$  (the Greek letter beta); then  $\gamma = 1/\sqrt{1 - \beta^2}$ .

Figure 37.8 shows a graph of  $\gamma$  as a function of the relative speed  $u$  of two frames of reference. When  $u$  is very small compared to  $c$ ,  $u^2/c^2$  is much smaller than 1 and  $\gamma$  is very nearly *equal* to 1. In that limit, Eqs. (37.6) and (37.8) approach the Newtonian relationship  $\Delta t = \Delta t_0$ , corresponding to the same time interval in all frames of reference.

If the relative speed  $u$  is great enough that  $\gamma$  is appreciably greater than 1, the speed is said to be *relativistic*; if the difference between  $\gamma$  and 1 is negligibly small, the speed  $u$  is called *nonrelativistic*. Thus  $u = 6.00 \times 10^7 \text{ m/s} = 0.200c$  (for which  $\gamma = 1.02$ ) is a relativistic speed, but  $u = 6.00 \times 10^4 \text{ m/s} = 0.000200c$  (for which  $\gamma = 1.00000002$ ) is a nonrelativistic speed.

**CAUTION Measuring time intervals** It is important to note that the time interval  $\Delta t$  in Eq. (37.6) involves events that occur *at different space points* in the frame of reference  $S$ . Note also that any differences between  $\Delta t$  and the proper time  $\Delta t_0$  are *not* caused by differences in the times required for light to travel from those space points to an observer at rest in  $S$ . We assume that our observer can correct for differences in light transit times, just as an astronomer who’s observing the sun understands that an event seen now on earth actually occurred 500 s ago on the sun’s surface. Alternatively, we can use *two* observers, one stationary at the location of the first event and the other at the second, each with his or her own clock. We can synchronize these two clocks without difficulty, as long as they are at rest in the same frame of reference. For example, we could send a light pulse simultaneously to the two clocks from a point midway between them. When the pulses arrive, the observers set their clocks to a prearranged time. (But clocks that are synchronized in one frame of reference *are not* in general synchronized in any other frame.)

In thought experiments, it’s often helpful to imagine many observers with synchronized clocks at rest at various points in a particular frame of reference. We can picture a frame of reference as a coordinate grid with lots of synchronized clocks distributed around it, as suggested by Fig. 37.9. Only when a clock is moving relative to a given frame of reference do we have to watch for ambiguities of synchronization or simultaneity.

Throughout this chapter we’ll frequently use phrases like “Stanley *observes* that Mavis passes the point  $x = 5.00 \text{ m}$ ,  $y = 0$ ,  $z = 0$  at time  $2.00 \text{ s}$ .” This means that Stanley is using



a grid of clocks in his frame of reference, like the grid shown in Fig. 37.9, to record the time of an event. We could restate the phrase as “When Mavis passes the point at  $x = 5.00$  m,  $y = 0$ ,  $z = 0$ , the clock at that location in Stanley’s frame of reference reads 2.00 s.” We’ll avoid using phrases like “Stanley *sees* that Mavis is at a certain point at a certain time,” because there is a time delay for light to travel to Stanley’s eye from the position of an event.

### PROBLEM-SOLVING STRATEGY 37.1 Time Dilation

**IDENTIFY** *the relevant concepts:* The concept of time dilation is used whenever we compare the time intervals between events as measured by observers in different inertial frames of reference.

**SET UP** *the problem* using the following steps:

1. First decide what two events define the beginning and the end of the time interval. Then identify the two frames of reference in which the time interval is measured.
2. Identify the target variable.

**EXECUTE** *the solution* as follows:

1. In many problems, the time interval as measured in one frame of reference is the *proper* time  $\Delta t_0$ . This is the time interval

between two events in a frame of reference in which the two events occur at the same point in space. In a second frame of reference that has a speed  $u$  relative to that first frame, there is a longer time interval  $\Delta t$  between the same two events. In this second frame the two events occur at different points. You’ll need to decide in which frame the time interval is  $\Delta t_0$  and in which frame it is  $\Delta t$ .

2. Use Eq. (37.6) or (37.8) to relate  $\Delta t_0$  and  $\Delta t$ , and then solve for the target variable.

**EVALUATE** *your answer:* Note that  $\Delta t$  is never smaller than  $\Delta t_0$ , and  $u$  is never greater than  $c$ . If your results suggest otherwise, you need to rethink your calculation.

### EXAMPLE 37.1 Time dilation at 0.990c

#### WITH VARIATION PROBLEMS

High-energy subatomic particles coming from space interact with atoms in the earth’s upper atmosphere, in some cases producing unstable particles called *muons*. A muon decays into other particles with a mean lifetime of  $2.20 \mu\text{s} = 2.20 \times 10^{-6}$  s as measured in a reference frame in which it is at rest. If a muon is moving at  $0.990c$  relative to the earth, what will an observer on earth measure its mean lifetime to be?

**IDENTIFY and SET UP** The muon’s lifetime is the time interval between two events: the production of the muon and its subsequent decay. Our target variable is the lifetime in your frame of reference on earth, which we call frame  $S$ . We are given the lifetime in a frame  $S'$  in which the muon is at rest; this is its *proper* lifetime,  $\Delta t_0 = 2.20 \mu\text{s}$ . The relative speed of these two frames is  $u = 0.990c$ . We use Eq. (37.6) to relate the lifetimes in the two frames.

**EXECUTE** The muon moves relative to the earth between the two events, so the two events occur at different positions as measured in

$S$  and the time interval in that frame is  $\Delta t$  (the target variable). From Eq. (37.6),

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \frac{2.20 \mu\text{s}}{\sqrt{1 - (0.990)^2}} = 15.6 \mu\text{s}$$

**EVALUATE** Our result predicts that the mean lifetime of the muon in the earth frame ( $\Delta t$ ) is about seven times longer than in the muon’s frame ( $\Delta t_0$ ). This prediction has been verified experimentally; indeed, this was the first experimental confirmation of the time dilation formula, Eq. (37.6).

**KEYCONCEPT** The time interval between two events depends on the frame of reference of the observer. The time interval is shortest in an inertial frame  $S$  where the two events occur at the same point; this is the proper time  $\Delta t_0$ . In any other inertial frame that is moving with respect to  $S$ , the time interval is longer (it is *dilated*) [Eq. (37.6)].

### EXAMPLE 37.2 Time dilation at airliner speeds

#### WITH VARIATION PROBLEMS

An airplane flies from San Francisco to New York (about 4800 km, or  $4.80 \times 10^6$  m) at a steady speed of 300 m/s (about 670 mi/h). How much time does the trip take, as measured by an observer on the ground? By an observer in the plane?

**IDENTIFY and SET UP** Here we’re interested in the time interval between the airplane departing from San Francisco and landing in New York. The target variables are the time intervals as measured in the frame of reference of the ground  $S$  and in the frame of reference of the airplane  $S'$ .

**EXECUTE** As measured in  $S$  the two events occur at different positions (San Francisco and New York), so the time interval measured by ground observers corresponds to  $\Delta t$  in Eq. (37.6). To find it, we simply divide the distance by the speed  $u = 300$  m/s:

$$\Delta t = \frac{4.80 \times 10^6 \text{ m}}{300 \text{ m/s}} = 1.60 \times 10^4 \text{ s} \quad (\text{about } 4\frac{1}{2} \text{ hours})$$

In the airplane’s frame  $S'$ , San Francisco and New York passing under the plane occur at the same point (the position of the plane). Hence the time interval in the airplane is a proper time, corresponding to  $\Delta t_0$  in Eq. (37.6). We have

$$\frac{u^2}{c^2} = \frac{(300 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2} = 1.00 \times 10^{-12}$$

From Eq. (37.6),

$$\Delta t_0 = (1.60 \times 10^4 \text{ s})\sqrt{1 - 1.00 \times 10^{-12}}$$

The square root can’t be evaluated with adequate precision with an ordinary calculator. But we can approximate it using the binomial theorem (see Appendix B):

$$(1 - 1.00 \times 10^{-12})^{1/2} = 1 - \left(\frac{1}{2}\right)(1.00 \times 10^{-12}) + \dots$$

*Continued*

The remaining terms are of the order of  $10^{-24}$  or smaller and can be discarded. The approximate result for  $\Delta t_0$  is

$$\Delta t_0 = (1.60 \times 10^4 \text{ s})(1 - 0.50 \times 10^{-12})$$

The proper time  $\Delta t_0$ , measured in the airplane, is very slightly less (by less than one part in  $10^{12}$ ) than the time measured on the ground.

**EVALUATE** We don't notice such effects in everyday life. But present-day atomic clocks (see Section 1.3) can attain a precision of about one

part in  $10^{13}$ . A cesium clock traveling a long distance in an airliner has been used to measure this effect and thereby verify Eq. (37.6) even at speeds much less than  $c$ .

**KEYCONCEPT** If two events take place at the same point in an inertial frame  $S$ , an observer in a frame  $S'$  moving relative to  $S$  observes a time interval between these events that is greater than in  $S$  by the Lorentz factor  $\gamma$  [Eqs. (37.7) and (37.8)]. This factor is substantially different from 1 only if the relative speed of  $S'$  and  $S$  is a substantial fraction of the speed of light.

### EXAMPLE 37.3 Just when is it proper?

### WITH VARIATION PROBLEMS

Mavis boards a spaceship and then zips past Stanley on earth at a relative speed of  $0.600c$ . At the instant she passes him, they both start timers. (a) A short time later Stanley measures that Mavis has traveled  $9.00 \times 10^7 \text{ m}$  beyond him and is passing a space station. What does Stanley's timer read as she passes the space station? What does Mavis's timer read? (b) Stanley starts to blink just as Mavis flies past him, and Mavis measures that the blink takes  $0.400 \text{ s}$  from beginning to end. According to Stanley, what is the duration of his blink?

**IDENTIFY and SET UP** This problem involves time dilation for two *different* sets of events measured in Stanley's frame of reference (which we call  $S$ ) and in Mavis's frame of reference (which we call  $S'$ ). The two events of interest in part (a) are when Mavis passes Stanley and when Mavis passes the space station; the target variables are the time intervals between these two events as measured in  $S$  and in  $S'$ . The two events in part (b) are the start and finish of Stanley's blink; the target variable is the time interval between these two events as measured in  $S$ .

**EXECUTE** (a) The two events, Mavis passing the earth and Mavis passing the space station, occur at different positions in Stanley's frame but at the same position in Mavis's frame. Hence Stanley measures time interval  $\Delta t$ , while Mavis measures the *proper* time  $\Delta t_0$ . As measured by Stanley, Mavis moves at  $0.600c = 0.600(3.00 \times 10^8 \text{ m/s}) = 1.80 \times 10^8 \text{ m/s}$  and travels  $9.00 \times 10^7 \text{ m}$  in time  $\Delta t = (9.00 \times 10^7 \text{ m})/(1.80 \times 10^8 \text{ m/s}) = 0.500 \text{ s}$ . From Eq. (37.6), Mavis's timer reads an elapsed time of

$$\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = 0.500 \text{ s} \sqrt{1 - (0.600)^2} = 0.400 \text{ s}$$

Since Stanley measures  $0.500 \text{ s}$  elapsed on his timer but only  $0.400 \text{ s}$  elapsed on Mavis's timer, Stanley concludes that Mavis's timer runs slow.

(b) It is tempting to answer that Stanley's blink lasts  $0.500 \text{ s}$  in his frame. But this is wrong, because we are now considering a *different* pair of events than in part (a). The start and finish of Stanley's blink occur at the same point in his frame  $S$  but at different positions in Mavis's frame  $S'$ , so the time interval of  $0.400 \text{ s}$  that she measures between these events is equal to  $\Delta t$ . The duration of the blink measured on Stanley's timer is the proper time  $\Delta t_0$ :

$$\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = 0.400 \text{ s} \sqrt{1 - (0.600)^2} = 0.320 \text{ s}$$

Since Mavis measures  $0.400 \text{ s}$  elapsed on her timer for the duration of Stanley's blink but only  $0.320 \text{ s}$  elapsed on Stanley's timer, Mavis concludes that Stanley's timer runs slow. *Both* observers measure that a moving clock runs slow!

**EVALUATE** This example illustrates both time dilation and the relativity of simultaneity. In Mavis's frame she passes the space station at the same instant that Stanley finishes his blink,  $0.400 \text{ s}$  after she passed Stanley. Hence these two events are simultaneous to Mavis in frame  $S'$ . But these two events are *not* simultaneous to Stanley in his frame  $S$ : According to his timer, he finishes his blink after  $0.320 \text{ s}$  and Mavis passes the space station after  $0.500 \text{ s}$ .

**KEYCONCEPT** No matter which inertial frame of reference you are in, you observe dilated time intervals for timekeepers (clocks) of any kind that move relative to you. If  $S$  and  $S'$  are two inertial frames in relative motion, an observer in  $S$  measures clocks in  $S'$  to run slow, and an observer in  $S'$  measures clocks in  $S$  to run slow.

## The Twin Paradox

Equations (37.6) and (37.8) for time dilation suggest an apparent paradox called the **twin paradox**. Consider identical twin astronauts named Eartha and Astrid. Eartha remains on earth while her twin Astrid takes off on a high-speed trip through the galaxy. Because of time dilation, Eartha observes Astrid's heartbeat and all other life processes proceeding more slowly than her own. Thus to Eartha, Astrid ages more slowly; when Astrid returns to earth she is younger (has aged less) than Eartha.

Here is the paradox: All inertial frames are equivalent. Can't Astrid make exactly the same arguments to conclude that Eartha is in fact the younger? Then each twin measures the other to be younger when they're back together, and that's a paradox.

To resolve the paradox, note that the twins are *not* identical in all respects. While Eartha remains in an approximately inertial frame at all times, Astrid must *accelerate* with respect to that frame during parts of her trip in order to leave, turn around, and return to earth. Eartha's reference frame is always approximately inertial; Astrid's is often far from inertial. Thus there is a real physical difference between the circumstances of the two twins. Careful analysis shows that Eartha is correct; when Astrid returns, she *is* younger than Eartha.

**TEST YOUR UNDERSTANDING OF SECTION 37.3** Samir (who is standing on the ground) starts his stopwatch at the instant that Maria flies past him in her spaceship at a speed of  $0.600c$ . At the same instant, Maria starts her stopwatch. (a) As measured in Samir's frame of reference, what is the reading on Maria's stopwatch at the instant that Samir's stopwatch reads  $10.0$  s? (i)  $10.0$  s; (ii) less than  $10.0$  s; (iii) more than  $10.0$  s. (b) As measured in Maria's frame of reference, what is the reading on Samir's stopwatch at the instant that Maria's stopwatch reads  $10.0$  s? (i)  $10.0$  s; (ii) less than  $10.0$  s; (iii) more than  $10.0$  s.

**ANSWER** (a) (ii), (b) (iii) The statement that moving clocks run slow refers to any clock that is moving relative to an observer. Maria and her stopwatch are moving relative to Samir, so Samir measures Maria's stopwatch to be running slow and to have ticked off fewer seconds than his own stopwatch. Samir and his stopwatch are moving relative to Maria, so she likewise measures Samir's stopwatch to be running slow. Each observer's measurement is correct for his or her own frame of reference. Both observers conclude that a moving stopwatch runs slow. This is consistent with the principle of relativity (see Section 37.1), which states that the laws of physics are the same in all inertial frames of reference.

## 37.4 RELATIVITY OF LENGTH

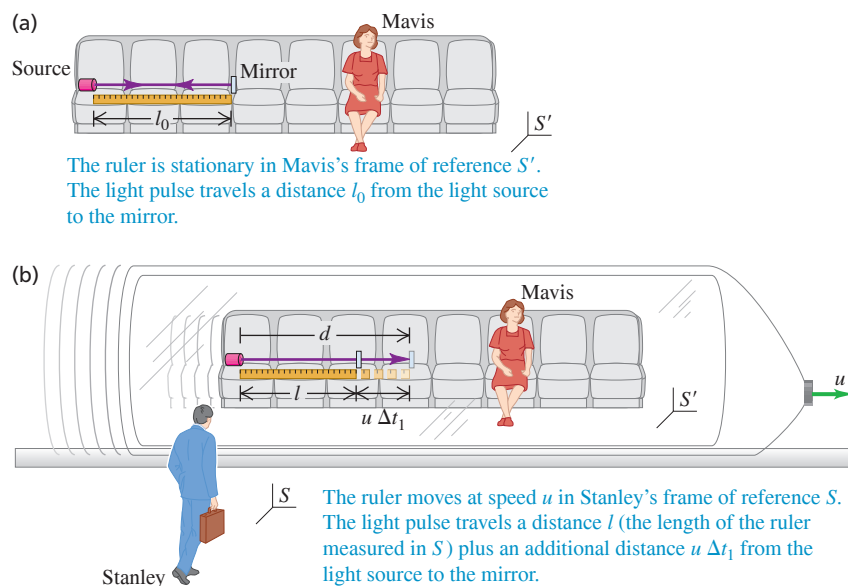
Not only does the time interval between two events depend on the observer's frame of reference, but the *distance* between two points may also depend on the observer's frame of reference. The concept of simultaneity is involved. Suppose you want to measure the length of a moving car. One way is to have two assistants make marks on the pavement at the positions of the front and rear bumpers. Then you measure the distance between the marks. But your assistants have to make their marks *at the same time*. If one marks the position of the front bumper at one time and the other marks the position of the rear bumper half a second later, you won't get the car's true length. Since we've learned that simultaneity isn't an absolute concept, we have to proceed with caution.

### Lengths Parallel to the Relative Motion

To develop a relationship between lengths that are measured parallel to the direction of motion in various coordinate systems, we consider another thought experiment. We attach a light source to one end of a ruler and a mirror to the other end. The ruler is at rest in reference frame  $S'$ , and its length in this frame is  $l_0$  (**Fig. 37.10a**). Then the time  $\Delta t_0$  required for a light pulse to make the round trip from source to mirror and back is

$$\Delta t_0 = \frac{2l_0}{c} \quad (37.9)$$

This is a *proper* time interval because departure and return occur at the same point in  $S'$ .



**APPLICATION Which One's the Grandmother?** The answer to this question may seem obvious, but it could depend on which person had traveled to a distant destination at relativistic speeds. Imagine that a 20-year-old woman had given birth to a child and then immediately left on a 100 light-year trip (50 light-years out and 50 light-years back) at 99.5% the speed of light. Because of time dilation for the traveler, only 10 years would pass for her, and she would be 30 years old when she returned, even though 100 years had passed by for people on earth. Meanwhile, the child she left behind at home could have had a baby 20 years after her departure, and this grandchild would now be 80 years old!



Figure 37.10 (a) A ruler is at rest in Mavis's frame  $S'$ . A light pulse is emitted from a source at one end of the ruler, reflected by a mirror at the other end, and returned to the source position. (b) Motion of the light pulse as measured in Stanley's frame  $S$ .

In reference frame  $S$  the ruler is moving to the right with speed  $u$  during this travel of the light pulse (Fig. 37.10b). The length of the ruler in  $S$  is  $l$ , and the time of travel from source to mirror, as measured in  $S$ , is  $\Delta t_1$ . During this interval the ruler, with source and mirror attached, moves a distance  $u \Delta t_1$ . The total length of path  $d$  from source to mirror is not  $l$ , but rather

$$d = l + u \Delta t_1 \quad (37.10)$$

The light pulse travels with speed  $c$ , so it is also true that

$$d = c \Delta t_1 \quad (37.11)$$

Combining Eqs. (37.10) and (37.11) to eliminate  $d$ , we find

$$\begin{aligned} c \Delta t_1 &= l + u \Delta t_1 \quad \text{or} \\ \Delta t_1 &= \frac{l}{c - u} \end{aligned} \quad (37.12)$$

(Dividing the distance  $l$  by  $c - u$  does *not* mean that light travels with speed  $c - u$ , but rather that the distance the pulse travels in  $S$  is greater than  $l$ .)

In the same way we can show that the time  $\Delta t_2$  for the return trip from mirror to source is

$$\Delta t_2 = \frac{l}{c + u} \quad (37.13)$$

The *total* time  $\Delta t = \Delta t_1 + \Delta t_2$  for the round trip, as measured in  $S$ , is

$$\Delta t = \frac{l}{c - u} + \frac{l}{c + u} = \frac{2l}{c(1 - u^2/c^2)} \quad (37.14)$$

We also know that  $\Delta t$  and  $\Delta t_0$  are related by Eq. (37.6) because  $\Delta t_0$  is a proper time in  $S'$ . Thus Eq. (37.9) for the round-trip time in the rest frame  $S'$  of the ruler becomes

$$\Delta t \sqrt{1 - \frac{u^2}{c^2}} = \frac{2l_0}{c} \quad (37.15)$$

Finally, we combine Eqs. (37.14) and (37.15) to eliminate  $\Delta t$  and simplify:

$$\begin{aligned} \text{Length contraction:} \quad l &= l_0 \sqrt{1 - \frac{u^2}{c^2}} = \frac{l_0}{\gamma} \end{aligned} \quad (37.16)$$

Proper length of object (measured in rest frame)   
 Speed of second frame relative to rest frame   
 Lorentz factor relating the two frames   
 Speed of light in vacuum

Figure 37.11 The speed at which electrons traverse the 3 km beam line of the SLAC National Accelerator Laboratory is slower than  $c$  by only 10 cm/s. As measured in the reference frame of such an electron, the beam line (which extends from the top to the bottom of this photograph) is only about 8 cm long!



**CAUTION** Length contraction is real. This is *not* an optical illusion! The ruler really is shorter in reference frame  $S$  than it is in  $S'$ . ■

[We have used the Lorentz factor  $\gamma$  defined in Eq. (37.7).] Thus the length  $l$  measured in  $S$ , in which the ruler is moving, is *shorter* than the length  $l_0$  measured in its rest frame  $S'$ .

A length measured in the frame in which the object is at rest (the rest frame of the object) is called a **proper length**; thus  $l_0$  is a proper length in  $S'$ , and the length measured in any other frame moving relative to  $S'$  is *less than*  $l_0$ . This effect is called **length contraction**.

When  $u$  is very small in comparison to  $c$ ,  $\gamma$  approaches 1. Thus in the limit of small speeds we approach the Newtonian relationship  $l = l_0$ . This and the corresponding result for time dilation show that Eqs. (37.1), the Galilean coordinate transformation, are usually sufficiently accurate for relative speeds much smaller than  $c$ . If  $u$  is a reasonable fraction of  $c$ , however, the quantity  $\sqrt{1 - u^2/c^2}$  can be appreciably less than 1. Then  $l$  can be substantially smaller than  $l_0$ , and the effects of length contraction can be substantial (Fig. 37.11).



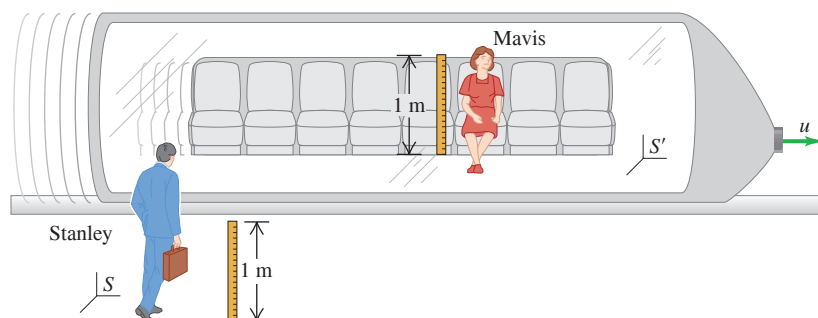


Figure 37.12 The meter sticks are perpendicular to the relative velocity. For any value of  $u$ , both Stanley and Mavis measure either meter stick to have a length of 1 meter.

## Lengths Perpendicular to the Relative Motion

We have derived Eq. (37.16) for lengths measured in the direction *parallel* to the relative motion of the two frames of reference. Lengths that are measured *perpendicular* to the direction of motion are *not* contracted. To prove this, consider two identical meter sticks. One stick is at rest in frame  $S$  and lies along the positive  $y$ -axis with one end at  $O$ , the origin of  $S$ . The other is at rest in frame  $S'$  and lies along the positive  $y'$ -axis with one end at  $O'$ , the origin of  $S'$ . Frame  $S'$  moves in the positive  $x$ -direction relative to frame  $S$ . Observers Stanley and Mavis, at rest in  $S$  and  $S'$  respectively, station themselves at the 50 cm mark of their sticks. At the instant the two origins coincide, the two sticks lie along the same line. At this instant, Mavis makes a mark on Stanley's stick at the point that coincides with her own 50 cm mark, and Stanley does the same to Mavis's stick.

Suppose for the sake of argument that Stanley observes Mavis's stick as longer than his own. Then the mark Stanley makes on her stick is *below* its center. In that case, Mavis will think Stanley's stick has become shorter, since half of its length coincides with *less* than half her stick's length. So Mavis observes moving sticks getting shorter and Stanley observes them getting longer. But this implies an asymmetry between the two frames that contradicts the basic postulate of relativity that tells us all inertial frames are equivalent. We conclude that consistency with the postulates of relativity requires that both observers measure the rulers as having the *same* length, even though to each observer one of them is stationary and the other is moving (**Fig. 37.12**). So *there is no length contraction perpendicular to the direction of relative motion of the coordinate systems*. We used this result in our derivation of Eq. (37.6) in assuming that the distance  $d$  is the same in both frames of reference.

For example, suppose a moving rod of length  $l_0$  makes an angle  $\theta_0$  with the direction of relative motion (the  $x$ -axis) as measured in its rest frame. Its length component in that frame parallel to the motion,  $l_0 \cos \theta_0$ , is contracted to  $(l_0 \cos \theta_0)/\gamma$ . However, its length component perpendicular to the motion,  $l_0 \sin \theta_0$ , remains the same.

## PROBLEM-SOLVING STRATEGY 37.2 Length Contraction

**IDENTIFY** the relevant concepts: The concept of length contraction is used whenever we compare the length of an object as measured by observers in different inertial frames of reference.

**SET UP** the problem using the following steps:

1. Decide what defines the length in question. If the problem describes an object such as a ruler, it is just the distance between the ends of the object. If the problem is about a distance between two points in space, it helps to envision an object like a ruler that extends from one point to the other.
2. Identify the target variable.

**EXECUTE** the solution as follows:

1. Determine the reference frame in which the object in question is at rest. In this frame, the length of the object is its proper length  $l_0$ .

In a second reference frame moving at speed  $u$  relative to the first frame, the object has contracted length  $l$ .

2. Keep in mind that length contraction occurs only for lengths parallel to the direction of relative motion of the two frames. Any length that is perpendicular to the relative motion is the same in both frames.
3. Use Eq. (37.16) to relate  $l$  and  $l_0$ , and then solve for the target variable.

**EVALUATE** your answer: Check that your answers make sense:  $l$  is never larger than  $l_0$ , and  $u$  is never greater than  $c$ .

**EXAMPLE 37.4** How long is the spaceship?**WITH VARIATION PROBLEMS**

A spaceship flies past earth at a speed of  $0.990c$ . A crew member on board the spaceship measures its length, obtaining the value 400 m. What length do observers measure on earth?

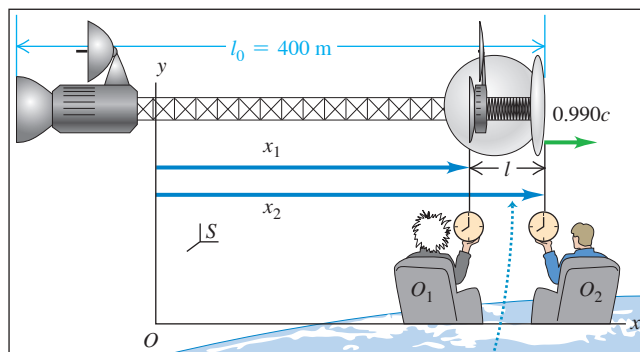
**IDENTIFY and SET UP** This problem is about the nose-to-tail length of the spaceship as measured on the spaceship and on earth. This length is along the direction of relative motion (**Fig. 37.13**), so there will be length contraction. The spaceship's 400 m length is the *proper* length  $l_0$  because it is measured in the frame in which the spaceship is at rest. Our target variable is the length  $l$  measured in the earth frame, relative to which the spaceship is moving at  $u = 0.990c$ .

**EXECUTE** From Eq. (37.16), the length in the earth frame is

$$l = l_0 \sqrt{1 - \frac{u^2}{c^2}} = (400 \text{ m}) \sqrt{1 - (0.990)^2} = 56.4 \text{ m}$$

**EVALUATE** The spaceship is shorter in a frame in which it is in motion than in a frame in which it is at rest. To measure the length  $l$ , two earth observers with synchronized clocks could measure the positions of the two ends of the spaceship simultaneously in the earth's reference frame, as shown in **Fig. 37.13**. (These two measurements will *not* appear simultaneous to an observer in the spaceship.)

**Figure 37.13** Measuring the length of a moving spaceship.



The two observers on earth ( $S$ ) must measure  $x_2$  and  $x_1$  simultaneously to obtain the correct length  $l = x_2 - x_1$  in their frame of reference.

**KEYCONCEPT** The dimensions of an object depend on the frame of reference of the observer. If the object is moving at constant velocity relative to your inertial frame, its length is contracted *along* the direction of relative motion [Eq. (37.16)]. There is no change in length *perpendicular* to the direction of relative motion.

**EXAMPLE 37.5** How far apart are the observers?**WITH VARIATION PROBLEMS**

Observers  $O_1$  and  $O_2$  in **Fig. 37.13** are 56.4 m apart on the earth. How far apart does the spaceship crew measure them to be?

**IDENTIFY and SET UP** In this example the 56.4 m distance is the *proper* length  $l_0$ . It represents the length of a ruler that extends from  $O_1$  to  $O_2$  and is at rest in the earth frame in which the observers are at rest. Our target variable is the length  $l$  of this ruler measured in the spaceship frame, in which the earth and ruler are moving at  $u = 0.990c$ .

**EXECUTE** As in Example 37.4, but with  $l_0 = 56.4 \text{ m}$ ,

$$l = l_0 \sqrt{1 - \frac{u^2}{c^2}} = (56.4 \text{ m}) \sqrt{1 - (0.990)^2} = 7.96 \text{ m}$$

**EVALUATE** This answer does *not* say that the crew measures their spaceship to be both 400 m long and 7.96 m long. As measured on earth,

the tail of the spacecraft is at the position of  $O_1$  at the same instant that the nose of the spacecraft is at the position of  $O_2$ . Hence the length of the spaceship measured on earth equals the 56.4 m distance between  $O_1$  and  $O_2$ . But in the spaceship frame  $O_1$  and  $O_2$  are only 7.96 m apart, and the nose (which is 400 m in front of the tail) passes  $O_2$  before the tail passes  $O_1$ .

**KEYCONCEPT** No matter which inertial frame of reference you are in, you observe objects that move relative to you to be contracted along the direction of relative motion. If  $S$  and  $S'$  are two inertial frames in relative motion, an observer in  $S$  measures objects in  $S'$  to have contracted lengths, and an observer in  $S'$  measures objects in  $S$  to have contracted lengths.

**How an Object Moving Near  $c$  Would Appear**

Let's think a little about the visual appearance of a moving three-dimensional object. If we could see the positions of all points of the object simultaneously, it would appear to shrink only in the direction of motion. But we *don't* see all the points simultaneously; light from points farther from us takes longer to reach us than does light from points near to us, so we see the farther points at the positions they had at earlier times.

Suppose we have a rectangular rod with its faces parallel to the coordinate planes. When we look end-on at the center of the closest face of such a rod at rest, we see only that face. (See the center rod in computer-generated **Fig. 37.14a**.) But when that rod is moving past us toward the right at an appreciable fraction of the speed of light, we may also see its left side because of the earlier-time effect just described. That is, we can see some points that we couldn't see when the rod was at rest because the rod moves out of the way of the light rays from those points to us. Conversely, some light that can get to us when the rod is at rest is blocked by the moving rod. Because of all this, the rods in **Figs. 37.14b** and **37.14c** appear rotated and distorted.

**TEST YOUR UNDERSTANDING OF SECTION 37.4** A miniature spaceship is flying past you, moving horizontally at a substantial fraction of the speed of light. At a certain instant, you observe that the nose and tail of the spaceship align exactly with the two ends of a meter stick that you hold in your hands. Rank the following distances in order from longest to shortest: (i) The proper length of the meter stick; (ii) the proper length of the spaceship; (iii) the length of the spaceship measured in your frame of reference; (iv) the length of the meter stick measured in the spaceship's frame of reference.

**ANSWER**

**ANSWER**

(i), (ii), (iii) and (iv) You measure both the rest length of the stationary meter stick and the contracted length of the moving spaceship to be 1 meter. The rest length of the spaceship is greater than the contracted length that you measure, and so must be greater than 1 meter. A miniature observer on board the spaceship would measure a contracted length for the meter stick of less than 1 meter. Note that in your frame of reference the nose and tail of the spaceship can simultaneously align with the two ends of the meter stick, since in your frame of reference they have the same length of 1 meter. In the spaceship's frame these two alignments cannot happen simultaneously because the meter stick is shorter than the spaceship. Section 37.2 tells us that this shouldn't be a surprise; two events that are simultaneous to one observer may not be simultaneous to a second observer moving relative to the first one.

## 37.5 THE LORENTZ TRANSFORMATIONS

In Section 37.1 we discussed the Galilean coordinate-transformation equations, Eqs. (37.1). They relate the coordinates  $(x, y, z)$  of a point in frame of reference  $S$  to the coordinates  $(x', y', z')$  of the point in a second frame  $S'$ . The second frame moves with constant speed  $u$  relative to  $S$  in the positive direction along the common  $x$ - $x'$ -axis. This transformation also assumes that the time scale is the same in the two frames of reference, so  $t = t'$ . This Galilean transformation, as we have seen, is valid only in the limit when  $u$  approaches zero. We are now ready to derive more general transformations that are consistent with the principle of relativity. The more general relationships are called the **Lorentz transformations**.

## The Lorentz Coordinate Transformation

Our first question is this: When an event occurs at point  $(x, y, z)$  at time  $t$ , as observed in a frame of reference  $S$ , what are the coordinates  $(x', y', z')$  and time  $t'$  of the event as observed in a second frame  $S'$  moving relative to  $S$  with constant speed  $u$  in the  $+x$ -direction?

To derive the coordinate transformation, we refer to **Fig. 37.15**, (next page) which is the same as Fig. 37.3. As before, we assume that the origins coincide at the initial time  $t = 0 = t'$ . Then in  $S$  the distance from  $O$  to  $O'$  at time  $t$  is still  $ut$ . The coordinate  $x'$  is a *proper length* in  $S'$ , so in  $S$  it is contracted by the factor  $1/\gamma = \sqrt{1 - u^2/c^2}$ , as in Eq. (37.16). Thus the distance  $x$  from  $O$  to  $P$ , as measured in  $S$ , is not simply  $x = ut + x'$ , as in the Galilean coordinate transformation, but

$$x = ut + x' \sqrt{1 - \frac{u^2}{c^2}} \quad (37.17)$$

Solving this equation for  $x'$ , we obtain

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} \quad (37.18)$$

Equation (37.18) is part of the Lorentz coordinate transformation; another part is the equation giving  $t'$  in terms of  $x$  and  $t$ . To obtain this, we note that the principle of relativity requires that the *form* of the transformation from  $S$  to  $S'$  be identical to that from  $S'$  to  $S$ . The only difference is a change in the sign of the relative velocity component  $u$ . Thus from Eq. (37.17) it must be true that

$$x' = -ut' + x\sqrt{1 - \frac{u^2}{c^2}} \quad (37.19)$$

Figure 37.14 Computer simulation of the appearance of an array of 25 rods with square cross section. The center rod is viewed end-on. The simulation ignores color changes in the array caused by the Doppler effect (see Section 37.6).

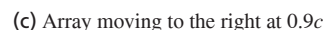
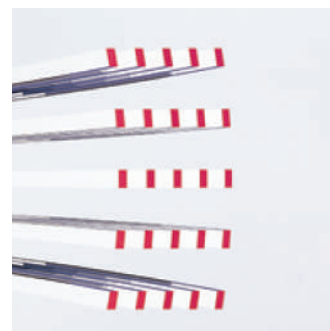
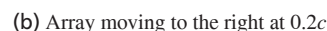
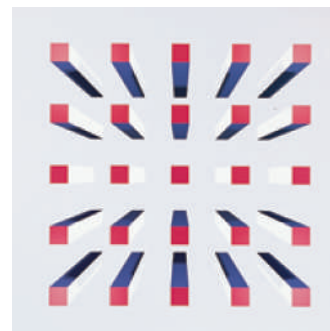
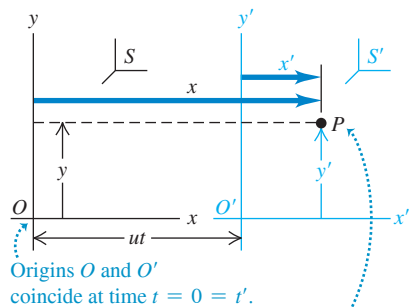


Figure 37.15 As measured in frame of reference  $S$ ,  $x'$  is contracted to  $x'/\gamma$ , so  $x = ut + (x'/\gamma)$  and  $x' = \gamma(x - ut)$ .

Frame  $S'$  moves relative to frame  $S$  with constant velocity  $u$  along the common  $x$ - $x'$ -axis.



The Lorentz coordinate transformation relates the spacetime coordinates of an event as measured in the two frames:  $(x, y, z, t)$  in frame  $S$  and  $(x', y', z', t')$  in frame  $S'$ .

We now equate Eqs. (37.18) and (37.19) to eliminate  $x'$ . This gives us an equation for  $t'$  in terms of  $x$  and  $t$ . You can do the algebra to show that

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} \quad (37.20)$$

As we discussed previously, lengths perpendicular to the direction of relative motion are not affected by the motion, so  $y' = y$  and  $z' = z$ .

Collecting our results, we have the *Lorentz coordinate transformation*:

**Lorentz coordinate transformation:**  
Spacetime coordinates of an event are  $x, y, z, t$  in frame  $S$  and  $x', y', z', t'$  in frame  $S'$ .

Velocity of  $S'$  relative to  $S$  in positive direction along  $x$ - $x'$ -axis

$$\begin{aligned} x' &= \frac{x - ut}{\sqrt{1 - u^2/c^2}} = \gamma(x - ut) \\ y' &= y \\ z' &= z \\ t' &= \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} = \gamma(t - ux/c^2) \end{aligned} \quad (37.21)$$

Lorentz factor relating the two frames  
Speed of light in vacuum

These equations are the relativistic generalization of the Galilean coordinate transformation, Eqs. (37.1) and  $t = t'$ . For values of  $u$  that approach zero,  $\sqrt{1 - u^2/c^2}$  and  $\gamma$  approach 1, and the  $ux/c^2$  term approaches zero. In this limit, Eqs. (37.21) become identical to Eqs. (37.1) along with  $t = t'$ . In general, though, both the coordinates and time of an event in one frame depend on its coordinates and time in another frame. *Space and time have become intertwined; we can no longer say that length and time have absolute meanings independent of the frame of reference.* For this reason, we refer to time and the three dimensions of space collectively as a four-dimensional entity called **spacetime**, and we call  $(x, y, z, t)$  together the **spacetime coordinates** of an event.

## The Lorentz Velocity Transformation

We can use Eqs. (37.21) to derive the relativistic generalization of the Galilean velocity transformation, Eq. (37.2). We consider only one-dimensional motion along the  $x$ -axis and use the term “velocity” as being short for the “ $x$ -component of the velocity.” Suppose that in a time  $dt$  a particle moves a distance  $dx$ , as measured in frame  $S$ . We obtain the corresponding distance  $dx'$  and time  $dt'$  in  $S'$  by taking differentials of Eqs. (37.21):

$$\begin{aligned} dx' &= \gamma(dx - u dt) \\ dt' &= \gamma(dt - u dx/c^2) \end{aligned}$$

We divide the first equation by the second and then divide the numerator and denominator of the result by  $dt$  to obtain

$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - u}{1 - \frac{u}{c^2} \frac{dx}{dt}}$$

Now  $dx/dt$  is the velocity  $v_x$  in  $S$ , and  $dx'/dt'$  is the velocity  $v'_x$  in  $S'$ , so

**Lorentz velocity transformation**  
(velocity in  $S'$  in terms of velocity in  $S$ ):

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} \quad (37.22)$$

Velocity of  $S'$  relative to  $S$  in positive direction along  $x$ - $x'$ -axis  
Speed of light in vacuum



When  $u$  and  $v_x$  are much smaller than  $c$ , the denominator in Eq. (37.22) approaches 1, and we approach the nonrelativistic result  $v'_x = v_x - u$ . The opposite extreme is the case  $v_x = c$ ; then we find

$$v'_x = \frac{c - u}{1 - uc/c^2} = \frac{c(1 - u/c)}{1 - u/c} = c$$

This says that anything moving with velocity  $v_x = c$  measured in  $S$  also has velocity  $v'_x = c$  measured in  $S'$ , despite the relative motion of the two frames. So Eq. (37.22) is consistent with Einstein's postulate that the speed of light in vacuum is the same in all inertial frames of reference.

The principle of relativity tells us there is no fundamental distinction between the two frames  $S$  and  $S'$ . Thus the expression for  $v_x$  in terms of  $v'_x$  must have the same form as Eq. (37.22), with  $v_x$  changed to  $v'_x$ , and vice versa, and the sign of  $u$  reversed. Carrying out these operations with Eq. (37.22), we find

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} \quad (37.23)$$

Lorentz velocity transformation (velocity in  $S$  in terms of velocity in  $S'$ ):
x-velocity of object in frame  $S'$ 
x-velocity of object in frame  $S$ 
Velocity of  $S'$  relative to  $S$  in positive direction along  $x$ - $x'$ -axis
Speed of light in vacuum

You can also obtain this equation by solving Eq. (37.22) for  $v_x$ . Both Eqs. (37.22) and (37.23) are *Lorentz velocity transformations* for one-dimensional motion.

When  $u$  is less than  $c$ , the Lorentz velocity transformations show us that an object moving with a speed less than  $c$  in one frame of reference always has a speed less than  $c$  in *every other* frame of reference. This is one reason for concluding that no material object may travel with a speed equal to or greater than  $c$  relative to *any* inertial frame of reference. Later we'll see that the relativistic generalizations of energy and momentum give further support to this hypothesis.

**CAUTION Use the correct reference frame coordinates** The Lorentz transformation equations given by Eqs. (37.21), (37.22), and (37.23) assume that frame  $S'$  is moving in the positive  $x$ -direction with velocity  $u$  relative to frame  $S$ . Always set up your coordinate system to follow this convention. ■

**APPLICATION Relative Velocity and Reference Frames** A relay race illustrates the frames of reference used in Eqs. (37.22) and (37.23). The runner in purple is the particle, and the two frames of reference in which the particle's motion is observed are our rest frame  $S$  (we are spectators standing next to the track) and the rest frame  $S'$  of the runner in red, who has speed  $u$  relative to us. Since the runner in red is moving to the left relative to us, we must take the positive  $x$ -direction to the left. The runner in purple has positive velocity  $v_x$  relative to us (she is moving to the left); if the runner in red is moving faster ( $u > v_x$ ), then from Eq. (37.22) the runner in purple has a *negative* velocity  $v'_x$  relative to the runner in red.

Reference frame  $S$ : Our rest frame as we stand next to the track

Reference frame  $S'$ : Rest frame of runner in red. She moves with speed  $u$  relative to us.



Runner in purple has velocity  $v_x$  in  $S$  and velocity  $v'_x$  in  $S'$ .

### PROBLEM-SOLVING STRATEGY 37.3 Lorentz Transformations

**IDENTIFY** *the relevant concepts:* The Lorentz coordinate-transformation equations relate the spacetime coordinates of an event in one inertial reference frame to the coordinates of the same event in a second inertial frame. The Lorentz velocity-transformation equations relate the velocity of an object in one inertial reference frame to its velocity in a second inertial frame.

**SET UP** *the problem* using the following steps:

1. Identify the target variable.
2. Define the two inertial frames  $S$  and  $S'$ . Remember that  $S'$  moves relative to  $S$  at a constant velocity  $u$  in the  $+x$ -direction.
3. If the coordinate-transformation equations are needed, make a list of spacetime coordinates in the two frames, such as  $x_1, x'_1, t_1, t'_1$ , and so on. Label carefully which of these you know and which you don't.
4. In velocity-transformation problems, clearly identify  $u$  (the relative velocity of the two frames of reference),  $v_x$  (the velocity of the object relative to  $S$ ), and  $v'_x$  (the velocity of the object relative to  $S'$ ).

**EXECUTE** *the solution* as follows:

1. In a coordinate-transformation problem, use Eqs. (37.21) to solve for the spacetime coordinates of the event as measured in  $S'$  in terms of the corresponding values in  $S$ . (If you need to solve for the spacetime coordinates in  $S$  in terms of the corresponding values in  $S'$ , you can easily convert the expressions in Eqs. (37.21): Replace all of the primed quantities with unprimed ones, and vice versa, and replace  $u$  with  $-u$ .)
2. In a velocity-transformation problem, use either Eq. (37.22) or Eq. (37.23), as appropriate, to solve for the target variable.

**EVALUATE** *your answer:* Don't be discouraged if some of your results don't seem to make sense or if they disagree with "common sense." It takes time to develop intuition about relativity; you'll gain it with experience.

**EXAMPLE 37.6** Was it received before it was sent?**WITH VARIATION PROBLEMS**

Winning an interstellar race, Mavis pilots her spaceship across a finish line in space at a speed of  $0.600c$  relative to that line. A “hooray” message is sent from the back of her ship (event 2) at the instant (in her frame of reference) that the front of her ship crosses the line (event 1). She measures the length of her ship to be 300 m. Stanley is at the finish line and is at rest relative to it. When and where does he measure events 1 and 2 to occur?

**IDENTIFY and SET UP** This example involves the Lorentz coordinate transformation. Our derivation of this transformation assumes that the origins of frames  $S$  and  $S'$  coincide at  $t = 0 = t'$ . Thus for simplicity we fix the origin of  $S$  at the finish line and the origin of  $S'$  at the front of the spaceship so that Stanley and Mavis measure event 1 to be at  $x = 0 = x'$  and  $t = 0 = t'$ .

Mavis in  $S'$  measures her spaceship to be 300 m long, so she has the “hooray” sent from 300 m behind her spaceship’s front at the instant she measures the front to cross the finish line. That is, she measures event 2 at  $x' = -300$  m and  $t' = 0$ .

Our target variables are the coordinate  $x$  and time  $t$  of event 2 that Stanley measures in  $S$ .

**EXECUTE** To solve for the target variables, we modify the first and last of Eqs. (37.21) to give  $x$  and  $t$  as functions of  $x'$  and  $t'$ . We do so in the same way that we obtained Eq. (37.23) from Eq. (37.22). We remove

the primes from  $x'$  and  $t'$ , add primes to  $x$  and  $t$ , and replace each  $u$  with  $-u$ . The results are

$$x = \gamma(x' + ut') \quad \text{and} \quad t = \gamma(t' + ux'/c^2)$$

From Eq. (37.7),  $\gamma = 1.25$  for  $u = 0.600c = 1.80 \times 10^8$  m/s. We also substitute  $x' = -300$  m,  $t' = 0$ ,  $c = 3.00 \times 10^8$  m/s, and  $u = 1.80 \times 10^8$  m/s in the equations for  $x$  and  $t$  to find  $x = -375$  m at  $t = -7.50 \times 10^{-7}$  s =  $-0.750 \mu\text{s}$  for event 2.

**EVALUATE** Mavis says that the events are simultaneous, but Stanley says that the “hooray” was sent *before* Mavis crossed the finish line. This does not mean that the effect preceded the cause. The fastest that Mavis can send a signal the length of her ship is  $300 \text{ m} / (3.00 \times 10^8 \text{ m/s}) = 1.00 \mu\text{s}$ . She cannot send a signal from the front at the instant it crosses the finish line that would cause a “hooray” to be broadcast from the back at the same instant. She would have to send that signal from the front at least  $1.00 \mu\text{s}$  before then, so she had to slightly anticipate her success.

**KEYCONCEPT** Given the position and time of an event as measured in one inertial frame of reference, you can use the Lorentz coordinate transformation to find the position and time of that same event as measured in a second inertial frame moving relative to the first one.

**EXAMPLE 37.7** Relative velocities**WITH VARIATION PROBLEMS**

(a) A spaceship moving away from the earth at  $0.900c$  fires a robot space probe in the same direction as its motion at  $0.700c$  relative to the spaceship. What is the probe’s velocity relative to the earth? (b) A scoutship is sent to catch up with the spaceship by traveling at  $0.950c$  relative to the earth. What is the velocity of the scoutship relative to the spaceship?

**IDENTIFY and SET UP** This example uses the Lorentz velocity transformation. Let the earth and spaceship reference frames be  $S$  and  $S'$ , respectively (**Fig. 37.16**); their relative velocity is  $u = 0.900c$ . In part (a) we are given the probe velocity  $v'_x = 0.700c$  with respect to  $S'$ , and the target variable is the velocity  $v_x$  of the probe relative to  $S$ . In part (b) we are given the velocity  $v_x = 0.950c$  of the scoutship relative to  $S$ , and the target variable is its velocity  $v'_x$  relative to  $S'$ .

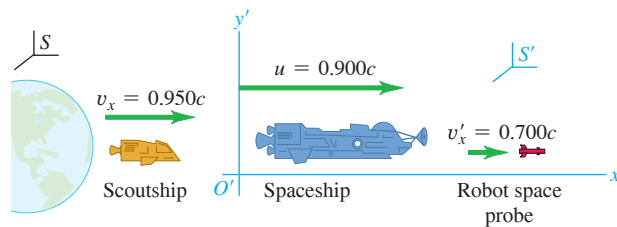
**EXECUTE** (a) We use Eq. (37.23) to find the probe velocity relative to the earth:

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = \frac{0.700c + 0.900c}{1 + (0.900c)(0.700c)/c^2} = 0.982c$$

(b) We use Eq. (37.22) to find the scoutship velocity relative to the spaceship:

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} = \frac{0.950c - 0.900c}{1 - (0.900c)(0.950c)/c^2} = 0.345c$$

Figure 37.16 The spaceship, robot space probe, and scoutship.



**EVALUATE** What would the Galilean velocity-transformation formula, Eq. (37.2), say? In part (a) we would have found the probe’s velocity relative to the earth to be  $v_x = v'_x + u = 0.700c + 0.900c = 1.600c$ , which is greater than  $c$  and hence impossible. In part (b), we would have found the scoutship’s velocity relative to the spaceship to be  $v'_x = v_x - u = 0.950c - 0.900c = 0.050c$ ; the relativistically correct value,  $v'_x = 0.345c$ , is almost seven times greater than the incorrect Galilean value.

**KEYCONCEPT** Given the velocity of an object as measured in one inertial frame of reference, you can use the Lorentz velocity transformation to find the velocity of that same object as measured in a second inertial frame moving relative to the first one. If an object is moving slower than  $c$  as measured in one inertial frame, its speed is less than  $c$  as measured in any other inertial frame.

**TEST YOUR UNDERSTANDING OF SECTION 37.5** (a) In frame  $S$  events  $P_1$  and  $P_2$  occur at the same  $x$ -,  $y$ -, and  $z$ -coordinates, but event  $P_1$  occurs before event  $P_2$ . In frame  $S'$ , which event occurs first? (b) In frame  $S$  events  $P_3$  and  $P_4$  occur at the same time  $t$  and the same  $y$ - and  $z$ -coordinates, but event  $P_3$  occurs at a less positive  $x$ -coordinate than event  $P_4$ . In frame  $S'$ , which event occurs first?



Using  $c^2 - u^2 = (c - u)(c + u)$  gives

$$f = \sqrt{\frac{c + u}{c - u}} f_0 \quad (37.25)$$

Doppler effect, electromagnetic waves, source approaching observer:  
 Frequency measured by observer  
 Frequency measured in rest frame of source  
 Speed of light in vacuum  
 Speed of source relative to observer

**Figure 37.18** This handheld radar gun emits a radio beam of frequency  $f_0$ , which in the frame of reference of an approaching car has a higher frequency  $f$  given by Eq. (37.25). The reflected beam also has frequency  $f$  in the car's frame, but has an even higher frequency  $f'$  in the police officer's frame. The radar gun calculates the car's speed by comparing the frequencies of the emitted beam and the doubly Doppler-shifted reflected beam. (Compare Example 16.18 in Section 16.8.)



This shows that when the source moves *toward* the observer, the observed frequency  $f$  is *greater* than the emitted frequency  $f_0$ . The difference  $f - f_0 = \Delta f$  is called the Doppler frequency shift. When  $u/c$  is much smaller than 1, the fractional shift  $\Delta f/f$  is also small and is approximately equal to  $u/c$ :

$$\frac{\Delta f}{f} = \frac{u}{c}$$

When the source moves *away from* the observer, we change the sign of  $u$  in Eq. (37.25) to get

$$f = \sqrt{\frac{c - u}{c + u}} f_0 \quad (\text{Doppler effect, electromagnetic waves, source moving away from observer}) \quad (37.26)$$

This agrees with Eq. (16.30) with minor notation changes.

With light, unlike sound, there is no distinction between motion of source and motion of observer; only the *relative* velocity of the two is significant. The last four paragraphs of Section 16.8 discuss several practical applications of the Doppler effect with light and other electromagnetic radiation; we suggest you review those paragraphs now. **Figure 37.18** shows one common application.

### EXAMPLE 37.8 A jet from a black hole

Many galaxies have supermassive black holes at their centers (see Section 13.8). As material swirls around such a black hole, it is heated, becomes ionized, and generates strong magnetic fields. The resulting magnetic forces steer some of the material into high-speed jets that blast out of the galaxy and into intergalactic space (**Fig. 37.19**). The light we observe from the jet in Fig. 37.19 has a frequency of  $6.66 \times 10^{14}$  Hz (in the far ultraviolet; see Fig. 32.4), but in the reference frame of the jet material the light has a frequency of  $5.55 \times 10^{13}$  Hz (in the infrared). What is the speed of the jet material with respect to us?

**IDENTIFY and SET UP** This problem involves the Doppler effect for electromagnetic waves. The frequency we observe is  $f = 6.66 \times 10^{14}$  Hz, and the frequency in the frame of the source is  $f_0 = 5.55 \times 10^{13}$  Hz. Since  $f > f_0$ , the jet is approaching us and we use Eq. (37.25) to find the target variable  $u$ .

**EXECUTE** We need to solve Eq. (37.25) for  $u$ . We'll leave it as an exercise for you to show that the result is

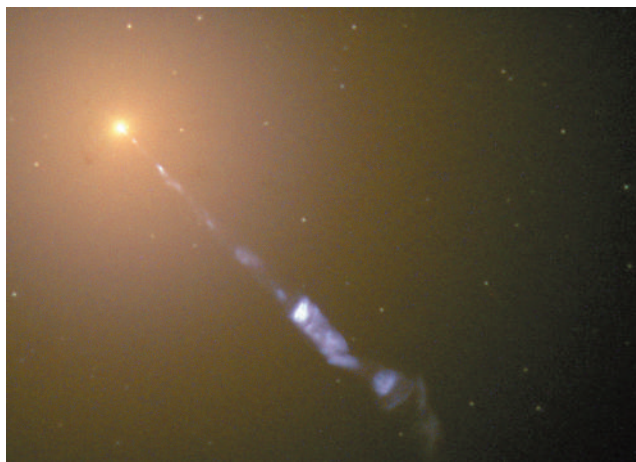
$$u = \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1} c$$

We have  $f/f_0 = (6.66 \times 10^{14} \text{ Hz}) / (5.55 \times 10^{13} \text{ Hz}) = 12.0$ , so

$$u = \frac{(12.0)^2 - 1}{(12.0)^2 + 1} c = 0.986c$$

**EVALUATE** Because the frequency shift is quite substantial, it would have been erroneous to use the approximate expression  $\Delta f/f = u/c$ . Had you done so, you would have found that  $u = c(\Delta f/f_0) = c(6.66 \times 10^{14} \text{ Hz} - 5.55 \times 10^{13} \text{ Hz}) / (5.55 \times 10^{13} \text{ Hz}) = 11.0c$ .

**Figure 37.19** This image shows a fast-moving jet 5000 light-years in length emanating from the center of the galaxy M87. The light from the jet is emitted by fast-moving electrons spiraling around magnetic field lines (see Fig. 27.18).



This result cannot be correct because the jet material cannot travel faster than light.

**KEYCONCEPT** The frequency of electromagnetic waves as measured by an observer moving relative to their source is different from the frequency measured in the rest frame of the source. The measured frequency is greater than the source frequency if the observer and source are approaching, and less than the source frequency if they are receding [Eq. (37.25)].



### 37.7 RELATIVISTIC MOMENTUM

Newton's laws of motion have the same form in all inertial frames of reference. When we use transformations to change from one inertial frame to another, the laws should be *invariant* (unchanging). But we have just learned that the principle of relativity forces us to replace the Galilean transformations with the more general Lorentz transformations. As we'll see, this requires corresponding generalizations in the laws of motion and the definitions of momentum and energy.

The principle of conservation of momentum states that *when two objects interact, the total momentum is constant*, provided that the net external force acting on the objects in an inertial reference frame is zero (for example, if they form an isolated system, interacting only with each other). If conservation of momentum is a valid physical law, it must be valid in *all* inertial frames of reference. Now, here's the problem: Suppose we look at a collision in one inertial coordinate system  $S$  and find that momentum is conserved. Then we use the Lorentz transformation to obtain the velocities in a second inertial system  $S'$ . We find that if we use the Newtonian definition of momentum ( $\vec{p} = m\vec{v}$ ), momentum is *not* conserved in the second system! The only way to make momentum conservation consistent with relativity is to generalize the *definition* of momentum.

We won't derive the correct relativistic generalization of momentum, but here is the result. Suppose we measure the mass of a particle to be  $m$  when it is at rest relative to us: We often call  $m$  the **rest mass**. We'll use the term "material particle" for a particle that has a nonzero rest mass. When such a particle has a velocity  $\vec{v}$ , its **relativistic momentum**  $\vec{p}$  is

$$\text{Relativistic momentum } \vec{p} = \frac{\text{Rest mass of particle } m \times \text{Velocity of particle } \vec{v}}{\sqrt{1 - \frac{\text{Speed of particle } v^2}{\text{Speed of light in vacuum } c^2}}} \quad (37.27)$$

When the particle's speed  $v$  is much less than  $c$ , this is approximately equal to the Newtonian expression  $\vec{p} = m\vec{v}$ , but in general the momentum is greater in magnitude than  $mv$  (Fig. 37.20). In fact, as  $v$  approaches  $c$ , the momentum approaches infinity.

#### Relativity, Newton's Second Law, and Relativistic Mass

What about the relativistic generalization of Newton's second law? In Newtonian mechanics the most general form of the second law is

$$\vec{F} = \frac{d\vec{p}}{dt} \quad (37.28)$$

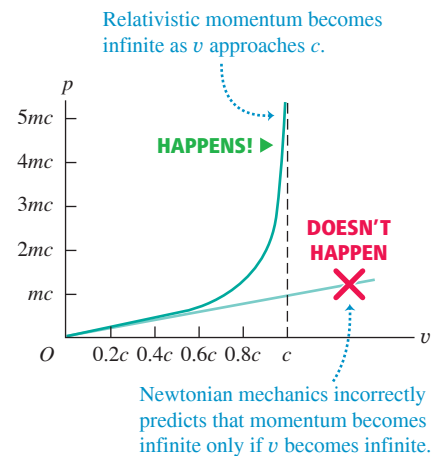
That is, the net force  $\vec{F}$  on a material particle equals the time rate of change of its momentum. Experiments show that this result is still valid in relativistic mechanics, provided that we use the relativistic momentum given by Eq. (37.27). That is, the relativistically correct generalization of Newton's second law is

$$\vec{F} = \frac{d}{dt} \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} \quad (37.29)$$

Because momentum is no longer directly proportional to velocity, the rate of change of momentum is no longer directly proportional to the acceleration. As a result, *constant force does not cause constant acceleration*. For example, when the net force and the velocity are both along the  $x$ -axis, Eq. (37.29) gives

$$F = \frac{m}{(1 - v^2/c^2)^{3/2}} a \quad (\vec{F} \text{ and } \vec{v} \text{ along the same line}) \quad (37.30)$$

Figure 37.20 Graph of the magnitude of the momentum of a particle of rest mass  $m$  as a function of speed  $v$ . Also shown is the Newtonian prediction, which gives correct results only at speeds much less than  $c$ .



where  $a$  is the acceleration, also along the  $x$ -axis. Solving Eq. (37.30) for the acceleration  $a$  gives

$$a = \frac{F}{m} \left( 1 - \frac{v^2}{c^2} \right)^{3/2}$$

We see that as a particle's speed increases, the acceleration caused by a given force continuously *decreases*. As the speed approaches  $c$ , the acceleration approaches zero, no matter how great a force is applied. Thus it is impossible to accelerate a particle with nonzero rest mass to a speed equal to or greater than  $c$ . We again see that the speed of light in vacuum represents an ultimate speed limit.

Equation (37.27) for relativistic momentum is sometimes interpreted to mean that a rapidly moving material particle undergoes an increase in mass. If the mass at zero velocity (the rest mass) is denoted by  $m$ , then the “relativistic mass”  $m_{\text{rel}}$  is

$$m_{\text{rel}} = \frac{m}{\sqrt{1 - v^2/c^2}}$$

Indeed, when we consider the motion of a system of particles (such as rapidly moving ideal-gas molecules in a stationary container), the total rest mass of the system is the sum of the relativistic masses of the particles, not the sum of their rest masses.

However, if blindly applied, the concept of relativistic mass has its pitfalls. As Eq. (37.29) shows, the relativistic generalization of Newton's second law is *not*  $\vec{F} = m_{\text{rel}}\vec{a}$ , and we'll show in Section 37.8 that the relativistic kinetic energy of a particle is *not*  $K = \frac{1}{2}m_{\text{rel}}v^2$ . The use of relativistic mass has its supporters and detractors, some quite strong in their opinions. We'll mostly deal with individual particles, so we'll sidestep the controversy and use Eq. (37.27) as the generalized definition of momentum with  $m$  as a constant for each particle, independent of its state of motion.

The quantity  $1/\sqrt{1 - v^2/c^2}$  in Eqs. (37.27) and (37.29) is the Lorentz factor  $\gamma$  from Eq. (37.7) (Section 37.3), but with a difference: We've replaced  $u$ , the relative speed of two coordinate systems, by  $v$ , the speed of a particle in a particular coordinate system—that is, the speed of the particle's *rest frame* with respect to that system. In terms of  $\gamma$ , Eqs. (37.27) and (37.30) become

$$\text{Relativistic momentum } \vec{p} = \gamma m \vec{v} \quad \begin{array}{l} \text{Rest mass of particle} \\ \text{Velocity of particle} \\ \text{Lorentz factor relating} \\ \text{rest frame of particle} \\ \text{and frame of observer} \end{array} \quad (37.31)$$

$$F = \gamma^3 m a \quad (\vec{F} \text{ and } \vec{v} \text{ along the same line}) \quad (37.32)$$

In linear accelerators (used in medicine as well as nuclear and elementary-particle physics; see Fig. 37.11) the net force  $\vec{F}$  and the velocity  $\vec{v}$  of the accelerated particle are along the same straight line. But for much of the path in most *circular* accelerators the particle moves in uniform circular motion at constant speed  $v$ . Then the net force and velocity are perpendicular, so the force can do no work on the particle and the kinetic energy and speed remain constant. Thus the denominator in Eq. (37.29) is constant, and we obtain

$$F = \frac{m}{(1 - v^2/c^2)^{1/2}} a = \gamma m a \quad (\vec{F} \text{ and } \vec{v} \text{ perpendicular}) \quad (37.33)$$

Recall from Section 3.4 that if the particle moves in a circle, the net force and acceleration are directed inward along the radius  $r$ , and  $a = v^2/r$ .

What about the general case in which  $\vec{F}$  and  $\vec{v}$  are neither along the same line nor perpendicular? Then we can resolve the net force  $\vec{F}$  at any instant into components parallel to and perpendicular to  $\vec{v}$ . The resulting acceleration will have corresponding components obtained from Eqs. (37.32) and (37.33). Because of the different  $\gamma^3$  and  $\gamma$  factors, the acceleration components will not be proportional to the net force components. That is, *unless the net force on a relativistic particle is either along the same line as the particle's velocity or perpendicular to it, the net force and acceleration vectors are not parallel.*

**EXAMPLE 37.9 Relativistic dynamics of an electron****WITH VARIATION PROBLEMS**

An electron (rest mass  $9.11 \times 10^{-31}$  kg, charge  $-1.60 \times 10^{-19}$  C) is moving opposite to an electric field of magnitude  $E = 5.00 \times 10^5$  N/C. All other forces are negligible in comparison to the electric-field force. (a) Find the magnitudes of momentum and of acceleration at the instants when  $v = 0.010c$ ,  $0.90c$ , and  $0.99c$ . (b) Find the corresponding accelerations if a net force of the same magnitude is perpendicular to the velocity.

**IDENTIFY and SET UP** In addition to the expressions from this section for relativistic momentum and acceleration, we need the relationship between electric force and electric field from Chapter 21. In part (a) we use Eq. (37.31) to determine the magnitude of momentum; the force acts along the same line as the velocity, so we use Eq. (37.32) to determine the magnitude of acceleration. In part (b) the force is perpendicular to the velocity, so we use Eq. (37.33) rather than Eq. (37.32).

**EXECUTE** (a) For  $v = 0.010c$ ,  $0.90c$ , and  $0.99c$  we have  $\gamma = \sqrt{1 - v^2/c^2} = 1.00$ ,  $2.29$ , and  $7.09$ , respectively. The values of the momentum magnitude  $p = \gamma mv$  are

$$\begin{aligned} p_1 &= (1.00)(9.11 \times 10^{-31} \text{ kg})(0.010)(3.00 \times 10^8 \text{ m/s}) \\ &= 2.7 \times 10^{-24} \text{ kg} \cdot \text{m/s at } v_1 = 0.010c \\ p_2 &= (2.29)(9.11 \times 10^{-31} \text{ kg})(0.90)(3.00 \times 10^8 \text{ m/s}) \\ &= 5.6 \times 10^{-22} \text{ kg} \cdot \text{m/s at } v_2 = 0.90c \\ p_3 &= (7.09)(9.11 \times 10^{-31} \text{ kg})(0.99)(3.00 \times 10^8 \text{ m/s}) \\ &= 1.9 \times 10^{-21} \text{ kg} \cdot \text{m/s at } v_3 = 0.99c \end{aligned}$$

From Eq. (21.4), the magnitude of the force on the electron is

$$F = |q|E = (1.60 \times 10^{-19} \text{ C})(5.00 \times 10^5 \text{ N/C}) = 8.00 \times 10^{-14} \text{ N}$$

From Eq. (37.32),  $a = F/\gamma^3 m$ . For  $v = 0.010c$  and  $\gamma = 1.00$ ,

$$a_1 = \frac{8.00 \times 10^{-14} \text{ N}}{(1.00)^3(9.11 \times 10^{-31} \text{ kg})} = 8.8 \times 10^{16} \text{ m/s}^2$$

The accelerations at the two higher speeds are smaller than the nonrelativistic value by factors of  $\gamma^3 = 12.0$  and  $356$ , respectively:

$$a_2 = 7.3 \times 10^{15} \text{ m/s}^2 \quad a_3 = 2.5 \times 10^{14} \text{ m/s}^2$$

(b) From Eq. (37.33),  $a = F/\gamma m$  if  $\vec{F}$  and  $\vec{v}$  are perpendicular. When  $v = 0.010c$  and  $\gamma = 1.00$ ,

$$a_1 = \frac{8.00 \times 10^{-14} \text{ N}}{(1.00)(9.11 \times 10^{-31} \text{ kg})} = 8.8 \times 10^{16} \text{ m/s}^2$$

Now the accelerations at the two higher speeds are smaller by factors of  $\gamma = 2.29$  and  $7.09$ , respectively:

$$a_2 = 3.8 \times 10^{16} \text{ m/s}^2 \quad a_3 = 1.2 \times 10^{16} \text{ m/s}^2$$

These accelerations are larger than the corresponding ones in part (a) by factors of  $\gamma^2$ .

**EVALUATE** Our results in part (a) show that at higher speeds, the relativistic values of momentum differ more and more from the nonrelativistic values calculated from  $p = mv$ . The momentum at  $0.99c$  is more than three times as great as at  $0.90c$  because of the increase in the factor  $\gamma$ . Our results also show that the acceleration drops off very quickly as  $v$  approaches  $c$ .

**KEYCONCEPT** The expression for momentum [Eq. (37.31)] and the relationship between net force and acceleration [Newton's second law, Eqs. (37.32) and (37.33)] are both modified in the special theory of relativity. As the speed of a material particle approaches  $c$ , its momentum approaches infinity and the acceleration caused by a net force approaches zero.

**TEST YOUR UNDERSTANDING OF SECTION 37.7** According to relativistic mechanics, when you double the speed of a particle, the magnitude of its momentum increases by (i) a factor of 2; (ii) a factor greater than 2; (iii) a factor between 1 and 2 that depends on the mass of the particle.

**ANSWER**

(ii) Equation (37.27) tells us that the magnitude of momentum of a particle with mass  $m$  and speed  $v$  is  $p = mv/\sqrt{1 - v^2/c^2}$ . If  $v$  increases by a factor of 2, the numerator  $mv$  increases by a factor of 2 and the denominator  $\sqrt{1 - v^2/c^2}$  decreases. Hence  $p$  increases by a factor greater than 2. (Note that in order to double the speed, the initial speed must be less than  $c/2$ . That's because the speed of light is the ultimate speed limit.)

**37.8 RELATIVISTIC WORK AND ENERGY**

When we developed the relationship between work and kinetic energy in Chapter 6, we used Newton's laws of motion. Since we have generalized these laws according to the principle of relativity, we need a corresponding generalization of the equation for kinetic energy.

**Relativistic Kinetic Energy**

We use the work–energy theorem, beginning with the definition of work. When the net force and displacement are in the same direction, the work done by that force is  $W = \int F dx$ . We substitute the expression for  $F$  from Eq. (37.30), the relativistic version of Newton's second law for straight-line motion. In moving a particle of rest mass  $m$  from point  $x_1$  to point  $x_2$ ,

$$W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} \frac{ma dx}{(1 - v_x^2/c^2)^{3/2}} \quad (37.34)$$

We've replaced  $v$  in Eq. (37.34) with  $v_x$  because the motion is along the  $x$ -axis only. So  $v_x$  is the varying  $x$ -component of the particle's velocity as the net force accelerates it. To derive the generalized expression for kinetic energy  $K$ , first remember that the kinetic energy of a particle equals the net work done on it in moving it from rest to speed  $v$ :  $K = W$ . Thus we let the speeds be zero at point  $x_1$  and  $v$  at point  $x_2$ . It's useful to convert Eq. (37.34) to an integral on  $v_x$ . To do this, note that  $dx$  and  $dv_x$  are the infinitesimal changes in  $x$  and  $v_x$ , respectively, in the time interval  $dt$ . Because  $v_x = dx/dt$  and  $a = dv_x/dt$ , we can rewrite  $a dx$  in Eq. (37.34) as

$$a dx = \frac{dv_x}{dt} dx = dx \frac{dv_x}{dt} = \frac{dx}{dt} dv_x = v_x dv_x$$

Making these substitutions gives us

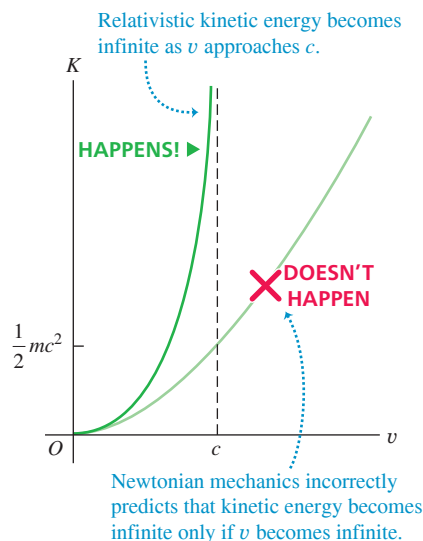
$$K = W = \int_0^v \frac{mv_x dv_x}{(1 - v_x^2/c^2)^{3/2}} \quad (37.35)$$

We can evaluate this integral by a simple change of variable; the final result is

$$\text{Relativistic kinetic energy } K = \frac{\text{Rest mass of particle } mc^2}{\sqrt{1 - \frac{\text{Speed of particle } v^2}{\text{Speed of light in vacuum } c^2}}} - mc^2 = (\gamma - 1)mc^2 \quad (37.36)$$

Lorentz factor relating rest frame of particle and frame of observer

Figure 37.21 Graph of the kinetic energy of a particle of rest mass  $m$  as a function of speed  $v$ . Also shown is the Newtonian prediction, which gives correct results only at speeds much less than  $c$ .



As  $v$  approaches  $c$ , the kinetic energy approaches infinity. If Eq. (37.36) is correct, it must also approach the Newtonian expression  $K = \frac{1}{2}mv^2$  when  $v$  is much smaller than  $c$  (Fig. 37.21). To verify this, we expand the radical, using the binomial theorem in the form

$$(1 + x)^n = 1 + nx + n(n-1)x^2/2 + \dots$$

In our case,  $n = -\frac{1}{2}$  and  $x = -v^2/c^2$ , and we get

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \dots$$

Combining this with  $K = (\gamma - 1)mc^2$ , we find

$$K = \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \dots - 1\right)mc^2 = \frac{1}{2}mv^2 + \frac{3}{8}\frac{mv^4}{c^2} + \dots \quad (37.37)$$

When  $v$  is much smaller than  $c$ , all the terms in the series in Eq. (37.37) except the first are negligibly small, and we obtain the Newtonian expression  $\frac{1}{2}mv^2$ .

## Rest Energy and $E = mc^2$

Equation (37.36) for the kinetic energy of a moving particle includes a term  $mc^2/\sqrt{1 - v^2/c^2}$  that depends on the motion and a second energy term  $mc^2$  that is independent of the motion. It seems that the kinetic energy of a particle is the difference between some **total energy**  $E$  and an energy  $mc^2$  that it has even when it is at rest. Thus we can rewrite Eq. (37.36) as

$$\text{Total energy of a particle } E = K + mc^2 = \frac{\text{Kinetic energy } K + \text{Rest energy } mc^2}{\sqrt{1 - \frac{\text{Speed of particle } v^2}{\text{Speed of light in vacuum } c^2}}} = \gamma mc^2 \quad (37.38)$$

Lorentz factor relating rest frame of particle and frame of observer

For a particle at rest ( $K = 0$ ), we see that  $E = mc^2$ . The energy  $mc^2$  associated with rest mass  $m$  rather than motion is called the **rest energy** of the particle.



There is abundant experimental evidence that rest energy really does exist. The simplest example is the decay of a neutral *pion*. This is an unstable subatomic particle of rest mass  $m_\pi$ ; when it decays, it disappears and electromagnetic radiation appears. If a neutral pion has no kinetic energy before its decay, the total energy of the radiation after its decay is found to equal exactly  $m_\pi c^2$ . In many other fundamental particle transformations the sum of the rest masses of the particles changes. In every case there is a corresponding energy change, consistent with the assumption of a rest energy  $mc^2$  associated with a rest mass  $m$ .

Historically, the principles of conservation of mass and of energy developed quite independently. The theory of relativity shows that they are actually two special cases of a single broader conservation principle, the *principle of conservation of mass and energy*. In some physical phenomena, neither the sum of the rest masses of the particles nor the total energy other than rest energy is separately conserved, but there is a more general conservation principle: In an isolated system, when the sum of the rest masses changes, there is always a change in  $1/c^2$  times the total energy other than the rest energy. This change is equal in magnitude but opposite in sign to the change in the sum of the rest masses.

This more general mass-energy conservation law is the fundamental principle involved in the generation of nuclear power. When a uranium nucleus undergoes fission in a nuclear reactor, the sum of the rest masses of the resulting fragments is *less than* the rest mass of the parent nucleus. An amount of energy is released that equals the mass decrease multiplied by  $c^2$ . Most of this energy can be used to produce steam to operate turbines for electric power generators.

We can also relate the total energy  $E$  of a particle (kinetic energy plus rest energy) directly to its momentum by combining Eq. (37.27) for relativistic momentum and Eq. (37.38) for total energy to eliminate the particle's velocity. The simplest procedure is to rewrite these equations in the following forms:

$$\left(\frac{E}{mc^2}\right)^2 = \frac{1}{1 - v^2/c^2} \quad \text{and} \quad \left(\frac{p}{mc}\right)^2 = \frac{v^2/c^2}{1 - v^2/c^2}$$

Subtracting the second of these from the first and rearranging, we find

$$\begin{array}{l} \text{Total energy,} \\ \text{rest energy,} \\ \text{and momentum:} \end{array} \quad \begin{array}{l} \text{Total} \\ \text{energy} \end{array} \quad \begin{array}{l} \text{Rest} \\ \text{energy} \end{array} \quad \begin{array}{l} \text{Magnitude of} \\ \text{momentum} \end{array} \quad \begin{array}{l} E^2 = (mc^2)^2 + (pc)^2 \\ \text{Rest mass} \quad \text{Speed of light in vacuum} \end{array} \quad (37.39)$$

Again we see that for a particle at rest ( $p = 0$ ),  $E = mc^2$ .

Equation (37.39) also suggests that a particle may have energy and momentum even when it has no rest mass. In such a case,  $m = 0$  and

$$E = pc \quad (\text{zero rest mass}) \quad (37.40)$$

In fact, zero rest mass particles do exist. Such particles always travel at the speed of light in vacuum. One example is the *photon*, the quantum of electromagnetic radiation (to be discussed in Chapter 38). Photons are emitted and absorbed during changes of state of an atomic or nuclear system when the energy and momentum of the system change.

### EXAMPLE 37.10 Energetic electrons

### WITH VARIATION PROBLEMS

(a) Find the rest energy of an electron ( $m = 9.109 \times 10^{-31}$  kg,  $q = -e = -1.602 \times 10^{-19}$  C) in joules and in electron volts. (b) Find the speed of an electron that has been accelerated by an electric field, from rest, through a potential increase of 20.0 kV or of 5.00 MV (typical of a high-voltage x-ray machine).

**IDENTIFY and SET UP** This problem uses the ideas of rest energy, relativistic kinetic energy, and (from Chapter 23) electric potential energy. We use  $E = mc^2$  to find the rest energy and Eqs. (37.7) and (37.38) to find the speed that gives the stated total energy.

**EXECUTE** (a) The rest energy is

$$mc^2 = (9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 8.187 \times 10^{-14} \text{ J}$$

From the definition of the electron volt in Section 23.2,  $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$ . Using this, we find

$$\begin{aligned} mc^2 &= (8.187 \times 10^{-14} \text{ J}) \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \\ &= 5.11 \times 10^5 \text{ eV} = 0.511 \text{ MeV} \end{aligned}$$

*Continued*

### APPLICATION Monitoring Mass-Energy Conversion

Although the control room of a nuclear power plant is very complex, the physical principle on which such a plant operates is a simple one: Part of the rest energy of atomic nuclei is converted to thermal energy, which in turn is used to produce steam to drive electric generators.



(b) In calculations such as this, it is often convenient to work with the quantity  $\gamma = 1/\sqrt{1 - v^2/c^2}$  from Eq. (37.38). Solving this for  $v$ , we find

$$v = c \sqrt{1 - (1/\gamma)^2}$$

The total energy  $E$  of the accelerated electron is the sum of its rest energy  $mc^2$  and the kinetic energy  $eV_{ba}$  that it gains from the work done on it by the electric field in moving from point  $a$  to point  $b$ :

$$E = \gamma mc^2 = mc^2 + eV_{ba} \quad \text{or} \quad \gamma = 1 + \frac{eV_{ba}}{mc^2}$$

An electron accelerated through a potential increase of  $V_{ba} = 20.0$  kV gains 20.0 keV of energy, so for this electron

$$\gamma = 1 + \frac{20.0 \times 10^3 \text{ eV}}{0.511 \times 10^6 \text{ eV}} = 1.039$$

and

$$v = c \sqrt{1 - (1/1.039)^2} = 0.272c = 8.15 \times 10^7 \text{ m/s}$$

Repeating the calculation for  $V_{ba} = 5.00$  MV, we find  $eV_{ba}/mc^2 = 9.78$ ,  $\gamma = 10.78$ , and  $v = 0.996c$ .

**EVALUATE** With  $V_{ba} = 20.0$  kV, the added kinetic energy of 20.0 keV is less than 4% of the rest energy of 0.511 MeV, and the final speed is about one-fourth the speed of light. With  $V_{ba} = 5.00$  MV, the added kinetic energy of 5.00 MeV is much greater than the rest energy and the speed is close to  $c$ .

**CAUTION Three electron energies** All electrons have *rest* energy 0.511 MeV. An electron accelerated from rest through a 5.00 MeV potential increase has *kinetic* energy 5.00 MeV (we call it a “5.00 MeV electron”) and *total* energy 5.51 MeV. Be careful to distinguish these energies from one another. ■

**KEYCONCEPT** The expression for kinetic energy [Eq. (37.36)] is modified in the special theory of relativity. As the speed of a material particle approaches  $c$ , its kinetic energy approaches infinity. The total energy of a particle of rest mass  $m$  is the sum of its kinetic energy and its rest energy  $mc^2$ .

### EXAMPLE 37.11 A relativistic collision

### WITH ✓ VARIATION PROBLEMS

Two protons (each with mass  $m_p = 1.67 \times 10^{-27}$  kg) are initially moving with equal speeds in opposite directions. They continue to exist after a head-on collision that also produces a neutral pion of mass  $m_\pi = 2.40 \times 10^{-28}$  kg (**Fig. 37.22**). If all three particles are at rest after the collision, find the initial speed of the protons. Energy is conserved in the collision.

**IDENTIFY and SET UP** Relativistic total energy is conserved in the collision, so we can equate the (unknown) total energy of the two protons before

the collision to the combined rest energies of the two protons and the pion after the collision. We then use Eq. (37.38) to find the speed of each proton.

**EXECUTE** The total energy of each proton before the collision is  $\gamma m_p c^2$ . By conservation of energy,

$$2(\gamma m_p c^2) = 2(m_p c^2) + m_\pi c^2$$

$$\gamma = 1 + \frac{m_\pi}{2m_p} = 1 + \frac{2.40 \times 10^{-28} \text{ kg}}{2(1.67 \times 10^{-27} \text{ kg})} = 1.072$$

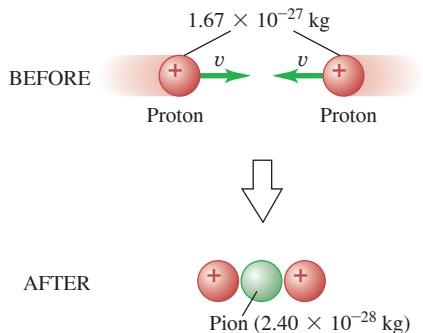
From Eq. (37.38), the initial proton speed is

$$v = c \sqrt{1 - (1/\gamma)^2} = 0.360c$$

**EVALUATE** The proton rest energy is 938 MeV, so the initial kinetic energy of each proton is  $(\gamma - 1)m_p c^2 = 0.072m_p c^2 = (0.072)(938 \text{ MeV}) = 67.5 \text{ MeV}$ . You can verify that the rest energy  $m_\pi c^2$  of the pion is twice this, or 135 MeV. All the kinetic energy “lost” in this completely inelastic collision is transformed into the rest energy of the pion.

**KEYCONCEPT** In a relativistic collision, momentum and total energy are both conserved. Particles can be created or destroyed in such collisions by converting kinetic energy to rest energy, or vice versa.

Figure 37.22 In this collision the kinetic energy of two protons is transformed into the rest energy of a new particle, a pion.



**TEST YOUR UNDERSTANDING OF SECTION 37.8** A proton is accelerated from rest by a constant force that always points in the direction of the particle's motion. Compared to the amount of kinetic energy that the proton gains during the first meter of its travel, how much kinetic energy does the proton gain during one meter of travel while it is moving at 99% of the speed of light? (i) The same amount; (ii) a greater amount; (iii) a smaller amount.

**ANSWER** (i) As the proton moves a distance  $s$ , the constant force of magnitude  $F$  does work  $W = Fs$  and increases the kinetic energy by an amount  $\Delta K = W = Fs$ . This is true no matter what the speed of the proton before moving this distance. Thus the constant force increases the proton's kinetic energy by the same amount during the first meter of travel as during any subsequent meter of travel. (As the proton approaches the ultimate speed limit of  $c$ , the increase in the proton's speed is less and less with each subsequent meter of travel. That's not what the question asks, however.) ■

## 37.9 NEWTONIAN MECHANICS AND RELATIVITY

The sweeping changes required by the principle of relativity go to the very roots of Newtonian mechanics, including the concepts of length and time, the equations of motion, and the conservation principles. Thus it may appear that we have destroyed the foundations on which Newtonian mechanics is built. In one sense this is true, yet the Newtonian formulation is still accurate whenever speeds are small in comparison with the speed of light in vacuum. In such cases, time dilation, length contraction, and the modifications of the laws of motion are so small that they are unobservable. In fact, every one of the principles of Newtonian mechanics survives as a special case of the more general relativistic formulation.

The laws of Newtonian mechanics are not *wrong*; they are *incomplete*. They are a limiting case of relativistic mechanics. They are *approximately* correct when all speeds are small in comparison to  $c$ , and they become exactly correct in the limit when all speeds approach zero. Thus relativity does not completely destroy the laws of Newtonian mechanics but *generalizes* them. This is a common pattern in the development of physical theory. Whenever a new theory is in partial conflict with an older, established theory, the new must yield the same predictions as the old in areas in which the old theory is supported by experimental evidence. Every new physical theory must pass this test, called the **correspondence principle**.

### The General Theory of Relativity

At this point we may ask whether the special theory of relativity gives the final word on mechanics or whether *further* generalizations are possible or necessary. For example, inertial frames have occupied a privileged position in our discussion. Can the principle of relativity be extended to noninertial frames as well?

Here's an example that illustrates some implications of this question. A student decides to go over Niagara Falls while enclosed in a large wooden box. During her free fall she doesn't fall to the floor of the box because both she and the box are in free fall with a downward acceleration of  $9.8 \text{ m/s}^2$ . But an alternative interpretation, from her point of view, is that she doesn't fall to the floor because her gravitational interaction with the earth has suddenly been turned off. As long as she remains in the box and it remains in free fall, she cannot tell whether she is indeed in free fall or whether the gravitational interaction has vanished.

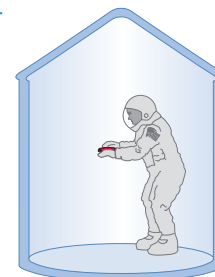
A similar problem occurs in a space station in orbit around the earth. Objects in the space station *seem* to be weightless, but without looking outside the station there is no way to determine whether gravity has been turned off or whether the station and all its contents are accelerating toward the center of the earth. **Figure 37.23** makes a similar point for a spaceship that is not in free fall but may be accelerating relative to an inertial frame or be at rest on the earth's surface.

These considerations form the basis of Einstein's **general theory of relativity**. If we cannot distinguish experimentally between a uniform gravitational field at a particular location and a uniformly accelerated reference frame, then there cannot be any real distinction between the two. Pursuing this concept, we may try to represent *any* gravitational field in terms of special characteristics of the coordinate system. This turns out to require even more sweeping revisions of our spacetime concepts than did the special theory of relativity. In the general theory of relativity the geometric properties of space are affected by the presence of matter (**Fig. 37.24**, next page).

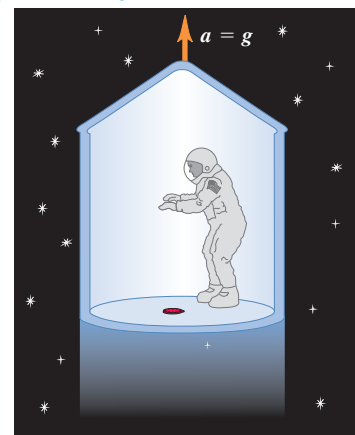
The general theory of relativity has passed several experimental tests, including three proposed by Einstein. One test has to do with understanding the rotation of the axes of the planet Mercury's elliptical orbit, called the *precession of the perihelion*. (The perihelion is the point of closest approach to the sun.) A second test concerns the apparent bending of light rays from distant stars when they pass near the sun. The third test is the *gravitational red shift*, the increase in wavelength of light proceeding outward from a massive source. Some details of the general theory are more difficult to test, but this theory has played a central role in investigations of the formation and evolution of stars, black holes, and studies of the evolution of the universe.

Figure 37.23 Without information from outside the spaceship, the astronaut cannot distinguish situation (b) from situation (c).

(a) An astronaut is about to drop her watch in a spaceship.



(b) In gravity-free space, the floor accelerates upward at  $a = g$  and hits the watch.



(c) On the earth's surface, the watch accelerates downward at  $a = g$  and hits the floor.

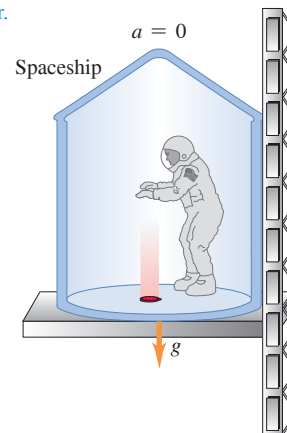


Figure 37.24 A two-dimensional representation of curved space. We imagine the space (a plane) as being distorted as shown by a massive object (the sun). Light from a distant star (solid line) follows the distorted surface on its way to the earth. The dashed line shows the direction from which the light *appears* to be coming. The effect is greatly exaggerated; for the sun, the maximum deviation is only  $0.00048^\circ$ .

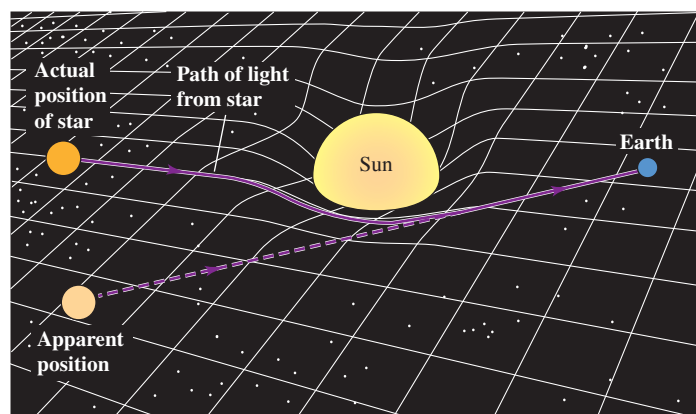


Figure 37.25 A GPS receiver uses radio signals from the orbiting GPS satellites to determine its position. To account for the effects of relativity, the receiver must be tuned to a slightly higher frequency (10.23 MHz) than the frequency emitted by the satellites (10.22999999543 MHz).

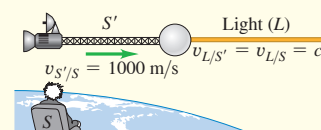


The general theory of relativity may seem to be an exotic bit of knowledge with little practical application. In fact, this theory plays an essential role in the Global Positioning System (GPS), which makes it possible to determine your position on the earth's surface to within a few meters using a handheld receiver (Fig. 37.25). The heart of the GPS system is a collection of more than two dozen satellites in very precise orbits. Each satellite emits carefully timed radio signals, and a GPS receiver simultaneously detects the signals from several satellites. The receiver then calculates the time delay between when each signal was emitted and when it was received, and uses this information to calculate the receiver's position.

To ensure the proper timing of the signals, it's necessary to include corrections due to the special theory of relativity (because the satellites are moving relative to the receiver on earth) as well as the general theory (because the satellites are higher in the earth's gravitational field than the receiver). The corrections due to relativity are small—less than one part in  $10^9$ —but are crucial to the superb precision of the GPS system.

## CHAPTER 37 SUMMARY

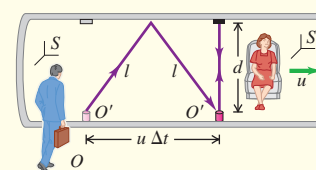
**Invariance of physical laws, simultaneity:** All of the fundamental laws of physics have the same form in all inertial frames of reference. The speed of light in vacuum is the same in all inertial frames and is independent of the motion of the source. Simultaneity is not an absolute concept; events that are simultaneous in one frame are not necessarily simultaneous in a second frame moving relative to the first.



**Time dilation:** If two events occur at the same space point in a particular frame of reference, the time interval  $\Delta t_0$  between the events as measured in that frame is called a proper time interval. If this frame moves with constant velocity  $u$  relative to a second frame, the time interval  $\Delta t$  between the events as observed in the second frame is longer than  $\Delta t_0$ . (See Examples 37.1–37.3.)

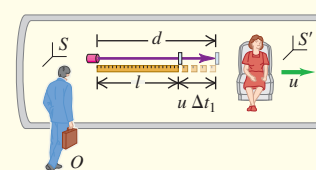
$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \gamma \Delta t_0 \quad (37.6), (37.8)$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \quad (37.7)$$



**Length contraction:** If two points are at rest in a particular frame of reference, the distance  $l_0$  between the points as measured in that frame is called a proper length. If this frame moves with constant velocity  $u$  relative to a second frame and the distances are measured parallel to the motion, the distance  $l$  between the points as measured in the second frame is shorter than  $l_0$ . (See Examples 37.4 and 37.5.)

$$l = l_0 \sqrt{1 - \frac{u^2}{c^2}} = \frac{l_0}{\gamma} \quad (37.16)$$



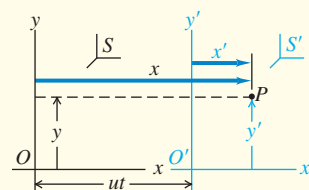


**The Lorentz transformations:** The Lorentz coordinate transformations relate the coordinates and time of an event in an inertial frame  $S$  to the coordinates and time of the same event as observed in a second inertial frame  $S'$  moving at velocity  $u$  relative to the first. For one-dimensional motion, a particle's velocities  $v_x$  in  $S$  and  $v'_x$  in  $S'$  are related by the Lorentz velocity transformation. (See Examples 37.6 and 37.7.)

$$\begin{aligned}x' &= \frac{x - ut}{\sqrt{1 - u^2/c^2}} = \gamma(x - ut) \\y' &= y \quad z' = z\end{aligned}\quad (37.21)$$

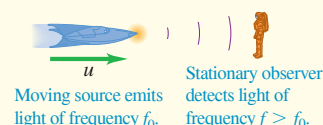
$$\begin{aligned}t' &= \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} = \gamma(t - ux/c^2) \\v'_x &= \frac{v_x - u}{1 - uv_x/c^2}\end{aligned}\quad (37.22)$$

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2}\quad (37.23)$$



**The Doppler effect for electromagnetic waves:** The Doppler effect is the frequency shift in light from a source due to the relative motion of source and observer. For a source moving toward the observer with speed  $u$ , Eq. (37.25) gives the received frequency  $f$  in terms of the emitted frequency  $f_0$ . (See Example 37.8.)

$$f = \sqrt{\frac{c + u}{c - u}} f_0\quad (37.25)$$



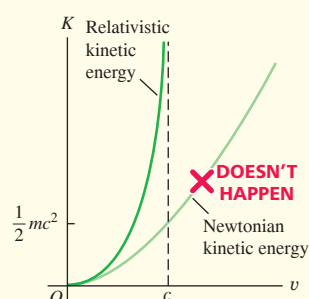
**Relativistic momentum and energy:** For a particle of rest mass  $m$  moving with velocity  $\vec{v}$ , the relativistic momentum  $\vec{p}$  is given by Eq. (37.27) or (37.31) and the relativistic kinetic energy  $K$  is given by Eq. (37.36). The total energy  $E$  is the sum of the kinetic energy and the rest energy  $mc^2$ . The total energy can also be expressed in terms of the magnitude of momentum  $p$  and rest mass  $m$ . (See Examples 37.9–37.11.)

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = \gamma m\vec{v}\quad (37.27), (37.31)$$

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = (\gamma - 1)mc^2\quad (37.36)$$

$$\begin{aligned}E &= K + mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \\&= \gamma mc^2\end{aligned}\quad (37.38)$$

$$E^2 = (mc^2)^2 + (pc)^2\quad (37.39)$$



Chapter 37 Media Assets



## GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

### KEY EXAMPLE VARIATION PROBLEMS

Be sure to review **EXAMPLES 37.1, 37.2, and 37.3** (Section 37.3) and **EXAMPLES 37.4 and 37.5** (Section 37.4) before attempting these problems.

**VP37.5.1** A positive K meson ( $K^+$ ) is an unstable subatomic particle that decays into other particles. At rest, its mean lifetime is  $1.23 \times 10^{-8}$  s. (a) What do you measure the mean lifetime of a  $K^+$  to be if it is moving at  $0.800c$  relative to you? (b) As measured from the reference frame of this  $K^+$ , what is the mean lifetime of a second  $K^+$  that is at rest relative to you?

**VP37.5.2** Alia flies in her spacecraft at  $0.600c$  relative to the planet Arrakis. As she passes Paul, at rest on Arrakis, they both start timers. (a) According to Alia's timer, 20.0 s elapses from when Paul starts his timer to when he stops his timer. What does Paul's timer read when he stops it? (b) Alia stops her timer when it reads 24.0 s. As measured by Paul, how much time elapses from when Alia starts her timer to when she stops it?

**VP37.5.3** At one point in its orbit the earth is  $1.50 \times 10^{11}$  m from the sun. A spacecraft flies along a line from the earth to the sun at  $0.950c$ . (a) As measured by you on the earth, how much time does it take the spacecraft to travel the distance from the earth to the sun? (b) As measured by an astronaut on the spacecraft, what is the distance from the earth to the sun, and how much time does it take her to travel that distance?

**VP37.5.4** A spacecraft flies past the earth in the direction toward the moon, a distance of  $3.84 \times 10^5$  km. As measured by your clock on the earth, it takes 6.00 s for the clock on the spacecraft to tick off 2.00 s. You measure the length of the spacecraft in the direction of its motion to be 24.0 m. As measured by an astronaut at rest on the spacecraft, what are (a) the distance from the earth to the moon and (b) the length of the spacecraft?

Be sure to review **EXAMPLES 37.6 and 37.7** (Section 37.5) before attempting these problems.

**VP37.7.1** Gamora flies her spacecraft in the  $+x$ -direction past the planet Xandar at  $0.750c$  while her sister Nebula is at rest on the surface of the planet. As Gamora passes Nebula, both sisters set their clocks to zero. Each chooses the zero of the  $x$ -axis to be at her position. (a) Nebula sets off fireworks next to her 2.50 s after Gamora passes her. What are the coordinates of this event as measured by Gamora? (b) Gamora detects an explosion in space that occurs  $4.00 \times 10^8$  m in front of her 2.50 s after she passes Nebula. What are the coordinates of this event as measured by Nebula?

**VP37.7.2** Two events occur at different  $x$ -coordinates but at the same time as measured by Kamala. Doreen, who is moving at  $0.800c$  relative to Kamala in the  $+x$ -direction, measures that one event takes place 0.600 s before the other. What is the distance between the two events as measured by (a) Kamala and (b) Doreen?

**VP37.7.3** The spaceship *Nostramo* flies away from the earth in the  $+x$ -direction at  $0.800c$ . It fires a probe in the  $-x$ -direction at  $0.600c$  relative to *Nostramo*. A scoutship is sent from the earth to recover the probe; the scoutship travels in the  $+x$ -direction at  $0.700c$  relative to the earth. Find the velocity (in terms of  $c$ ) of (a) the probe relative to the earth and (b) the scoutship relative to *Nostramo*.

**VP37.7.4** As measured from the earth, the spaceship *Macross* is flying directly toward the earth at  $0.750c$  while on the other side of the earth the spaceship *Yamato* is flying at  $0.650c$  along the same line as *Macross*. Find the speed (in terms of  $c$ ) of *Yamato* relative to *Macross* if *Yamato* is flying (a) toward the earth and (b) away from the earth.

**Be sure to review** **EXAMPLE 37.9** (Section 37.7) and **EXAMPLES 37.10 and 37.11** (Section 37.8) before attempting these problems.

**VP37.11.1** A proton (rest mass  $1.67 \times 10^{-27}$  kg) is moving at  $0.925c$ . Find (a) the momentum of the proton, (b) its acceleration if a force of

$9.00 \times 10^{-14}$  N acts on the proton in the direction of its motion, and (c) its acceleration if a force of  $9.00 \times 10^{-14}$  N acts on the proton perpendicular to the direction of its motion.

**VP37.11.2** A moving electron (rest mass  $9.11 \times 10^{-31}$  kg) has total energy  $4.00 \times 10^{-13}$  J. Find (a) the kinetic energy of the electron, (b) its Lorentz factor  $\gamma$ , and (c) its speed in terms of  $c$ .

**VP37.11.3** A moving electron (rest mass  $9.11 \times 10^{-31}$  kg) has momentum  $2.60 \times 10^{-22}$  kg  $\cdot$  m/s. Find (a) the total energy of the electron, (b) its Lorentz factor  $\gamma$ , and (c) its speed in terms of  $c$ .

**VP37.11.4** The  $\psi(2S)$  meson is an unstable particle with rest energy 3686 MeV. One way in which a  $\psi(2S)$  can decay is into a positive K meson ( $K^+$ ) and a negative K meson ( $K^-$ ), each with rest energy 495 MeV. Assume the  $\psi(2S)$  is at rest before it decays. Find (a) the kinetic energy in MeV, (b) the Lorentz factor  $\gamma$ , and (c) the speed (in terms of  $c$ ) of each K meson.

## BRIDGING PROBLEM Colliding Protons

In an experiment, two protons are shot directly toward each other. Their speeds are such that in the frame of reference of each proton, the other proton is moving at  $0.500c$ . (a) What does an observer in the laboratory measure for the speed of each proton? (b) What is the kinetic energy of each proton as measured by an observer in the laboratory? (c) What is the kinetic energy of each proton as measured by the other proton?

### SOLUTION GUIDE

#### IDENTIFY and SET UP

1. This problem uses the Lorentz velocity transformation, which allows us to relate the velocity  $v_x$  of a proton in one frame to its velocity  $v'_x$  in a different frame. It also uses the idea of relativistic kinetic energy.
2. Draw a sketch of the situation. Take the  $x$ -axis to be the line of motion of the protons, and take the  $+x$ -direction to be to the right. In the frame in which the left-hand proton is at rest, the right-hand proton has velocity  $-0.500c$ . In the laboratory frame the two protons have velocities  $-ac$  and  $+ac$ , where  $a$  (each proton's laboratory speed as a fraction of  $c$ ) is our first

target variable. Given this we can find the laboratory kinetic energy of each proton.

#### EXECUTE

3. Write a Lorentz velocity-transformation equation that relates the velocity of the right-hand proton in the laboratory frame to its velocity in the frame of the left-hand proton. Solve this equation for  $a$ . (Hint: Remember that  $a$  cannot be greater than 1. Why?)
4. Use your result from step 3 to find the laboratory kinetic energy of each proton.
5. Find the kinetic energy of the right-hand proton as measured in the frame of the left-hand proton.

#### EVALUATE

6. How much total kinetic energy must be imparted to the protons by a scientist in the laboratory? If the experiment were to be repeated with one proton stationary, what kinetic energy would have to be given to the other proton for the collision to be equivalent?

## PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

## DISCUSSION QUESTIONS

**Q37.1** You are standing on a train platform watching a high-speed train pass by. A light inside one of the train cars is turned on and then a little later it is turned off. (a) Who can measure the proper time interval for the duration of the light: you or a passenger on the train? (b) Who can measure the proper length of the train car: you or a passenger on the train? (c) Who can measure the proper length of a sign attached to a post on the train platform: you or a passenger on the train? In each case explain your answer.

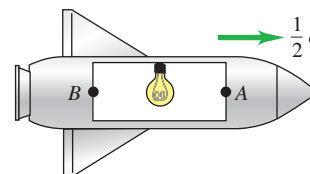
**Q37.2** If simultaneity is not an absolute concept, does that mean that we must discard the concept of causality? If event  $A$  is to *cause* event  $B$ ,  $A$  must occur first. Is it possible that in some frames  $A$  appears to be the cause of  $B$ , and in others  $B$  appears to be the cause of  $A$ ? Explain.

**Q37.3** A rocket is moving to the right at  $\frac{1}{2}$  the speed of light relative to the earth. A light bulb in the center of a room inside the rocket suddenly

turns on. Call the light hitting the front end of the room event  $A$  and the light hitting the back of the room event  $B$  (**Fig. Q37.3**). Which event occurs first,  $A$  or  $B$ , or are they simultaneous, as viewed by (a) an astronaut riding in the rocket and (b) a person at rest on the earth?

**Q37.4** A spaceship is traveling toward the earth from the space colony on Asteroid 1040A. The ship is at the halfway point of the trip, passing Mars at a speed of  $0.9c$  relative to the Mars frame of reference. At the same instant, a passenger on the spaceship receives a radio message from her boyfriend on 1040A and another from her sister on earth. According to the passenger on the ship, were these messages sent simultaneously or at different times? If at different times, which one was sent first? Explain your reasoning.

Figure Q37.3



**Q37.5** The average life span in the United States is about 70 years. Does this mean that it is impossible for an average person to travel a distance greater than 70 light-years away from the earth? (A light-year is the distance light travels in a year.) Explain.

**Q37.6** You are holding an elliptical serving platter. How would you need to travel for the serving platter to appear round to another observer?

**Q37.7** Two events occur at the same space point in a particular inertial frame of reference and are simultaneous in that frame. Is it possible that they may not be simultaneous in a different inertial frame? Explain.

**Q37.8** A high-speed train passes a train platform. Larry is a passenger on the train, Adam is standing on the train platform, and David is riding a bicycle toward the platform in the same direction as the train is traveling. Compare the length of a train car as measured by Larry, Adam, and David.

**Q37.9** The theory of relativity sets an upper limit on the speed that a particle can have. Are there also limits on the energy and momentum of a particle? Explain.

**Q37.10** A student asserts that a material particle must always have a speed slower than that of light, and a massless particle must always move at exactly the speed of light. Is she correct? If so, how do massless particles such as photons and neutrinos acquire this speed? Can't they start from rest and accelerate? Explain.

**Q37.11** The speed of light relative to still water is  $2.25 \times 10^8$  m/s. If the water is moving past us, the speed of light we measure depends on the speed of the water. Do these facts violate Einstein's second postulate? Explain.

**Q37.12** When a monochromatic light source moves toward an observer, its wavelength appears to be shorter than the value measured when the source is at rest. Does this contradict the hypothesis that the speed of light is the same for all observers? Explain.

**Q37.13** In principle, does a hot gas have more mass than the same gas when it is cold? Explain. In practice, would this be a measurable effect? Explain.

**Q37.14** Why do you think the development of Newtonian mechanics preceded the more refined relativistic mechanics by so many years?

**Q37.15** What do you think would be different in everyday life if the speed of light were 10 m/s instead of  $3.00 \times 10^8$  m/s?

## EXERCISES

### Section 37.2 Relativity of Simultaneity

**37.1** • Suppose the two lightning bolts shown in Fig. 37.5a are simultaneous to an observer on the train. Show that they are *not* simultaneous to an observer on the ground. Which lightning strike does the ground observer measure to come first?

### Section 37.3 Relativity of Time Intervals

**37.2** • The positive muon ( $\mu^+$ ), an unstable particle, lives on average  $2.20 \times 10^{-6}$  s (measured in its own frame of reference) before decaying. (a) If such a particle is moving, with respect to the laboratory, with a speed of  $0.900c$ , what average lifetime is measured in the laboratory? (b) What average distance, measured in the laboratory, does the particle move before decaying?

**37.3** • How fast must a rocket travel relative to the earth so that time in the rocket "slows down" to half its rate as measured by earth-based observers? Do present-day jet planes approach such speeds?

**37.4** • A spaceship flies past Mars with a speed of  $0.985c$  relative to the surface of the planet. When the spaceship is directly overhead, a signal light on the Martian surface blinks on and then off. An observer on Mars measures that the signal light was on for  $75.0 \mu\text{s}$ . (a) Does the observer on Mars or the pilot on the spaceship measure the proper time? (b) What is the duration of the light pulse measured by the pilot of the spaceship?

**37.5** • The negative pion ( $\pi^-$ ) is an unstable particle with an average lifetime of  $2.60 \times 10^{-8}$  s (measured in the rest frame of the pion). (a) If the pion is made to travel at very high speed relative to a laboratory, its average lifetime is measured in the laboratory to be  $4.20 \times 10^{-7}$  s. Calculate the speed of the pion expressed as a fraction of  $c$ . (b) What distance, measured in the laboratory, does the pion travel during its average lifetime?

**37.6** • As you pilot your space utility vehicle at a constant speed toward the moon, a race pilot flies past you in her spaceracer at a constant speed of  $0.800c$  relative to you. At the instant the spaceracer passes you, both of you start timers at zero. (a) At the instant when you measure that the spaceracer has traveled  $1.20 \times 10^8$  m past you, what does the race pilot read on her timer? (b) When the race pilot reads the value calculated in part (a) on her timer, what does she measure to be your distance from her? (c) At the instant when the race pilot reads the value calculated in part (a) on her timer, what do you read on yours?

**37.7** • An alien spacecraft is flying overhead at a great distance as you stand in your backyard. You see its searchlight blink on for  $0.150$  s. The first officer on the spacecraft measures that the searchlight is on for  $12.0$  ms. (a) Which of these two measured times is the proper time? (b) What is the speed of the spacecraft relative to the earth, expressed as a fraction of the speed of light  $c$ ?

### Section 37.4 Relativity of Length

**37.8** • At  $x = x' = 0$  and  $t = t' = 0$  a clock ticks aboard an extremely fast spaceship moving past us in the  $+x$ -direction with a Lorentz factor of 100 so  $v \approx c$ . The captain hears the clock tick again  $1.00$  s later. Where and when do we measure the second tick to occur?

**37.9** • A spacecraft of the Trade Federation flies past the planet Coruscant at a speed of  $0.600c$ . A scientist on Coruscant measures the length of the moving spacecraft to be  $74.0$  m. The spacecraft later lands on Coruscant, and the same scientist measures the length of the now stationary spacecraft. What value does she get?

**37.10** • A meter stick moves past you at great speed. Its motion relative to you is parallel to its long axis. If you measure the length of the moving meter stick to be  $1.00$  ft ( $1 \text{ ft} = 0.3048 \text{ m}$ )—for example, by comparing it to a 1 foot ruler that is at rest relative to you—at what speed is the meter stick moving relative to you?

**37.11** • **Why Are We Bombarded by Muons?** Muons are unstable subatomic particles that decay to electrons with a mean lifetime of  $2.2 \mu\text{s}$ . They are produced when cosmic rays bombard the upper atmosphere about  $10$  km above the earth's surface, and they travel very close to the speed of light. The problem we want to address is why we see any of them at the earth's surface. (a) What is the greatest distance a muon could travel during its  $2.2 \mu\text{s}$  lifetime? (b) According to your answer in part (a), it would seem that muons could never make it to the ground. But the  $2.2 \mu\text{s}$  lifetime is measured in the frame of the muon, and muons are moving very fast. At a speed of  $0.999c$ , what is the mean lifetime of a muon as measured by an observer at rest on the earth? How far would the muon travel in this time? Does this result explain why we find muons in cosmic rays? (c) From the point of view of the muon, it still lives for only  $2.2 \mu\text{s}$ , so how does it make it to the ground? What is the thickness of the  $10$  km of atmosphere through which the muon must travel, as measured by the muon? Is it now clear how the muon is able to reach the ground?

**37.12** • A rocket ship flies past the earth at  $91.0\%$  of the speed of light. Inside, an astronaut who is undergoing a physical examination is having his height measured while he is lying down parallel to the direction in which the ship is moving. (a) If his height is measured to be  $2.00$  m by his doctor inside the ship, what height would a person watching this from the earth measure? (b) If the earth-based person had measured  $2.00$  m, what would the doctor in the spaceship have measured for the astronaut's height? Is this a reasonable height? (c) Suppose the astronaut in part (a) gets up after the examination and stands with his body perpendicular to the direction of motion. What would the doctor in the rocket and the observer on earth measure for his height now?

**37.13 •** As measured by an observer on the earth, a spacecraft runway on earth has a length of 3600 m. (a) What is the length of the runway as measured by a pilot of a spacecraft flying past at a speed of  $4.00 \times 10^7$  m/s relative to the earth? (b) An observer on earth measures the time interval from when the spacecraft is directly over one end of the runway until it is directly over the other end. What result does she get? (c) The pilot of the spacecraft measures the time it takes him to travel from one end of the runway to the other end. What value does he get?

### Section 37.5 The Lorentz Transformations

**37.14 •** The Lorentz coordinate transformation assumes that  $t = t'$  at  $x = x' = 0$ . At what other values of  $x$  and  $x'$  does  $t = t'$ ?

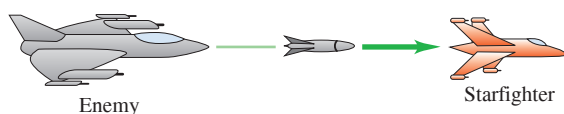
**37.15 •** An observer in frame  $S'$  is moving to the right ( $+x$ -direction) at speed  $u = 0.600c$  away from a stationary observer in frame  $S$ . The observer in  $S'$  measures the speed  $v'$  of a particle moving to the right away from her. What speed  $v$  does the observer in  $S$  measure for the particle if (a)  $v' = 0.400c$ ; (b)  $v' = 0.900c$ ; (c)  $v' = 0.990c$ ?

**37.16 •** Space pilot Mavis zips past Stanley at a constant speed relative to him of  $0.800c$ . Mavis and Stanley start timers at zero when the front of Mavis's ship is directly above Stanley. When Mavis reads 5.00 s on her timer, she turns on a bright light under the front of her spaceship. (a) Use the Lorentz coordinate transformation derived in Example 37.6 to calculate  $x$  and  $t$  as measured by Stanley for the event of turning on the light. (b) Use the time dilation formula, Eq. (37.6), to calculate the time interval between the two events (the front of the spaceship passing overhead and turning on the light) as measured by Stanley. Compare to the value of  $t$  you calculated in part (a). (c) Multiply the time interval by Mavis's speed, both as measured by Stanley, to calculate the distance she has traveled as measured by him when the light turns on. Compare to the value of  $x$  you calculated in part (a).

**37.17 ••** A pursuit spacecraft from the planet Tatooine is attempting to catch up with a Trade Federation cruiser. As measured by an observer on Tatooine, the cruiser is traveling away from the planet with a speed of  $0.600c$ . The pursuit ship is traveling at a speed of  $0.800c$  relative to Tatooine, in the same direction as the cruiser. (a) For the pursuit ship to catch the cruiser, should the velocity of the cruiser relative to the pursuit ship be directed toward or away from the pursuit ship? (b) What is the speed of the cruiser relative to the pursuit ship?

**37.18 ••** An enemy spaceship is moving toward your starfighter with a speed, as measured in your frame, of  $0.400c$ . The enemy ship fires a missile toward you at a speed of  $0.700c$  relative to the enemy ship (**Fig. E37.18**). (a) What is the speed of the missile relative to you? Express your answer in terms of the speed of light. (b) If you measure that the enemy ship is  $8.00 \times 10^6$  km away from you when the missile is fired, how much time, measured in your frame, will it take the missile to reach you?

Figure E37.18



**37.19 ••** Two particles are created in a high-energy accelerator and move off in opposite directions. The speed of one particle, as measured in the laboratory, is  $0.650c$ , and the speed of each particle relative to the other is  $0.950c$ . What is the speed of the second particle, as measured in the laboratory?

**37.20 ••** Two particles in a high-energy accelerator experiment are approaching each other head-on, each with a speed of  $0.9380c$  as measured in the laboratory. What is the magnitude of the velocity of one particle relative to the other?

**37.21 ••** Two particles in a high-energy accelerator experiment approach each other head-on with a relative speed of  $0.890c$ . Both particles travel at the same speed as measured in the laboratory. What is the speed of each particle, as measured in the laboratory?

### Section 37.6 The Doppler Effect for Electromagnetic Waves

**37.22 •** Electromagnetic radiation from a star is observed with an earth-based telescope. The star is moving away from the earth at a speed of  $0.520c$ . If the radiation has a frequency of  $8.64 \times 10^{14}$  Hz in the rest frame of the star, what is the frequency measured by an observer on earth?

**37.23 • Tell It to the Judge.** (a) How fast must you be approaching a red traffic light ( $\lambda = 675$  nm) for it to appear yellow ( $\lambda = 575$  nm)? Express your answer in terms of the speed of light. (b) If you used this as a reason not to get a ticket for running a red light, how much of a fine would you get for speeding? Assume that the fine is \$1.00 for each kilometer per hour that your speed exceeds the posted limit of 90 km/h.

**37.24 •** A source of electromagnetic radiation is moving in a radial direction relative to you. The frequency you measure is 1.25 times the frequency measured in the rest frame of the source. What is the speed of the source relative to you? Is the source moving toward you or away from you?

### Section 37.7 Relativistic Momentum

**37.25 •** A particle zips by us with a Lorentz factor of 1.12. Then another particle zips by us moving at twice the speed of the first particle. (a) What is the Lorentz factor of the second particle? (b) If the particles were moving with a speed much less than  $c$ , the magnitude of the momentum of the second particle would be twice that of the first. However, what is the ratio of the magnitudes of momentum for these relativistic particles?

**37.26 • Relativistic Baseball.** Calculate the magnitude of the force required to give a 0.145 kg baseball an acceleration  $a = 1.00$  m/s<sup>2</sup> in the direction of the baseball's initial velocity when this velocity has a magnitude of (a) 10.0 m/s; (b)  $0.900c$ ; (c)  $0.990c$ . (d) Repeat parts (a), (b), and (c) if the force and acceleration are perpendicular to the velocity.

**37.27 •** A proton has momentum with magnitude  $p_0$  when its speed is  $0.400c$ . In terms of  $p_0$ , what is the magnitude of the proton's momentum when its speed is doubled to  $0.800c$ ?

**37.28 •** A spaceship has length 120 m, diameter 25 m, and mass  $4.0 \times 10^3$  kg as measured by its crew. As the spaceship moves parallel to its cylindrical axis and passes us, we measure its length to be 90 m. (a) What do we measure its diameter to be? (b) What do we measure the magnitude of its momentum to be?

**37.29 •** (a) At what speed is the momentum of a particle twice as great as the result obtained from the nonrelativistic expression  $mv$ ? Express your answer in terms of the speed of light. (b) A force is applied to a particle along its direction of motion. At what speed is the magnitude of force required to produce a given acceleration twice as great as the force required to produce the same acceleration when the particle is at rest? Express your answer in terms of the speed of light.

### Section 37.8 Relativistic Work and Energy

**37.30 •** The sun produces energy by nuclear fusion reactions, in which matter is converted into energy. By measuring the amount of energy we receive from the sun, we know that it is producing energy at a rate of  $3.8 \times 10^{26}$  W. (a) How many kilograms of matter does the sun lose each second? Approximately how many tons of matter is this (1 ton = 2000 lb)? (b) At this rate, how long would it take the sun to use up all its mass?

**37.31 ••** What is the speed of a particle whose kinetic energy is equal to (a) its rest energy and (b) five times its rest energy?

**37.32 •** If a muon is traveling at  $0.999c$ , what are its momentum and kinetic energy? (The mass of such a muon at rest in the laboratory is 207 times the electron mass.)

**37.33 •** A proton (rest mass  $1.67 \times 10^{-27}$  kg) has total energy that is 4.00 times its rest energy. What are (a) the kinetic energy of the proton; (b) the magnitude of the momentum of the proton; (c) the proton's speed?

**37.34 ••** (a) How much work must be done on a particle with mass  $m$  to accelerate it (a) from rest to a speed of  $0.090c$  and (b) from a speed of  $0.900c$  to a speed of  $0.990c$ ? (Express the answers in terms of  $mc^2$ .) (c) How do your answers in parts (a) and (b) compare?



**37.35 •** A particle has rest mass  $6.64 \times 10^{-27}$  kg and momentum  $2.10 \times 10^{-18}$  kg  $\cdot$  m/s. (a) What is the total energy (kinetic plus rest energy) of the particle? (b) What is the kinetic energy of the particle? (c) What is the ratio of the kinetic energy to the rest energy of the particle?

**37.36 ••** Electrons are accelerated through a potential difference of 750 kV, so that their kinetic energy is  $7.50 \times 10^5$  eV. (a) What is the ratio of the speed  $v$  of an electron having this energy to the speed of light,  $c$ ? (b) What would the speed be if it were computed from the principles of classical mechanics?

**37.37 •** Compute the kinetic energy of a proton (mass  $1.67 \times 10^{-27}$  kg) using both the nonrelativistic and relativistic expressions, and compute the ratio of the two results (relativistic divided by nonrelativistic) for speeds of (a)  $8.00 \times 10^7$  m/s and (b)  $2.85 \times 10^8$  m/s.

**37.38 • Creating a Particle.** Two protons (each with rest mass  $M = 1.67 \times 10^{-27}$  kg) are initially moving with equal speeds in opposite directions. The protons continue to exist after a collision that also produces an  $\eta^0$  particle (see Chapter 44). The rest mass of the  $\eta^0$  is  $m = 9.75 \times 10^{-28}$  kg. (a) If the two protons and the  $\eta^0$  are all at rest after the collision, find the initial speed of the protons, expressed as a fraction of the speed of light. (b) What is the kinetic energy of each proton? Express your answer in MeV. (c) What is the rest energy of the  $\eta^0$ , expressed in MeV? (d) Discuss the relationship between the answers to parts (b) and (c).

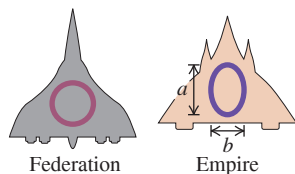
**37.39 •** (a) Through what potential difference does an electron have to be accelerated, starting from rest, to achieve a speed of  $0.980c$ ? (b) What is the kinetic energy of the electron at this speed? Express your answer in joules and in electron volts.

## PROBLEMS

**37.40 •** Inside a spaceship flying past the earth at three-fourths the speed of light, a pendulum is swinging. (a) If each swing takes 1.80 s as measured by an astronaut performing an experiment inside the spaceship, how long will the swing take as measured by a person at mission control (on earth) who is watching the experiment? (b) If each swing takes 1.80 s as measured by a person at mission control, how long will it take as measured by the astronaut in the spaceship?

**37.41 ••** The starships of the Solar Federation are marked with the symbol of the federation, a circle, while starships of the Denebian Empire are marked with the empire's symbol, an ellipse whose major axis is 1.40 times longer than its minor axis ( $a = 1.40b$  in Fig. P37.41). How fast, relative to an observer, does an empire ship have to travel for its marking to be confused with the marking of a federation ship?

Figure P37.41



**37.42 ••** A cube of metal with sides of length  $a$  sits at rest in a frame  $S$  with one edge parallel to the  $x$ -axis. Therefore, in  $S$  the cube has volume  $a^3$ . Frame  $S'$  moves along the  $x$ -axis with a speed  $u$ . As measured by an observer in frame  $S'$ , what is the volume of the metal cube?

**37.43 ••** A space probe is sent to the vicinity of the star Capella, which is 42.2 light-years from the earth. (A light-year is the distance light travels in a year.) The probe travels with a speed of  $0.9930c$ . An astronaut recruit on board is 19 years old when the probe leaves the earth. What is her biological age when the probe reaches Capella?

**37.44 ••** A  $\Sigma^+$  particle has a mean lifetime of 80.2 ps. A physicist measures that mean lifetime to be 403 ps as the particle moves in his lab. The rest mass of the particle is  $2.12 \times 10^{-27}$  kg. (a) How fast is the particle moving? (b) How far does it travel, as measured in the lab frame, over one mean lifetime? (c) What are its rest, kinetic, and total energies in the lab frame of reference? (d) What are its rest, kinetic, and total energies in the particle's frame?

**37.45 ••** Spaceship  $A$  moves past the earth at  $0.80c$  to the west. Spaceship  $B$  approaches  $A$ , moving to the east. Both spaceship crews measure their relative speed of approach to be  $0.98c$ . What mass would the crews of both spaceships measure for the standard kilogram, kept at rest on the earth, (a) according to classical physics and (b) according to the special theory of relativity?

**37.46 ••** One way to strictly enforce a speed limit would be to alter the laws of nature. Suppose the speed of light were 65 mph and your workplace was 30 miles from your home. Assume you travel to work at a typical driving speed of 60 mph. (a) If you drove at that speed for the round trip to and from work, light, how much would your wrist-watch lag your kitchen clock each day? (b) Estimate the length of your car. (c) If you were driving at your estimated driving speed, how long would your car be when viewed from the roadside? (d) What would be the speed relative to you of similar cars traveling toward you in the opposite lane with the same ground speed as you? (e) How long would you measure those cars to be? (f) If the total mass of you and your car was 2000 kg, how much work would be required to get you up to speed? (Note: Your rest mass energy in this world is  $mc^2$ , where  $c = 65$  mph.) (g) How much work would be required in the real world, where the speed of light is  $3.0 \times 10^8$  m/s, to get you up to speed?

**37.47 • The Large Hadron Collider (LHC).** Physicists and engineers from around the world came together to build the largest accelerator in the world, the Large Hadron Collider (LHC) at the CERN Laboratory in Geneva, Switzerland. The machine accelerates protons to high kinetic energies in an underground ring 27 km in circumference. (a) What is the speed  $v$  of a proton in the LHC if the proton's kinetic energy is 7.0 TeV? (Because  $v$  is very close to  $c$ , write  $v = (1 - \Delta)c$  and give your answer in terms of  $\Delta$ .) (b) Find the relativistic mass,  $m_{\text{rel}}$ , of the accelerated proton in terms of its rest mass.

**37.48 ••** The net force  $\vec{F}$  on a particle of mass  $m$  is directed at  $30.0^\circ$  counterclockwise from the  $+x$ -axis. At one instant of time, the particle is traveling in the  $+x$ -direction with a speed (measured relative to the earth) of  $0.700c$ . At this instant, what is the direction of the particle's acceleration?

**37.49 •• Everyday Time Dilation.** Two atomic clocks are carefully synchronized. One remains in New York, and the other is loaded on an airliner that travels at an average speed of 250 m/s and then returns to New York. When the plane returns, the elapsed time on the clock that stayed behind is 4.00 h. By how much will the readings of the two clocks differ, and which clock will show the shorter elapsed time? (Hint: Since  $u \ll c$ , you can simplify  $\sqrt{1 - u^2/c^2}$  by a binomial expansion.)

**37.50 ••** The distance to a particular star, as measured in the earth's frame of reference, is 7.11 light-years (1 light-year is the distance that light travels in 1 y). A spaceship leaves the earth and takes 3.35 y to arrive at the star, as measured by passengers on the ship. (a) How long does the trip take, according to observers on earth? (b) What distance for the trip do passengers on the spacecraft measure?

**37.51 • CP Čerenkov Radiation.** The Russian physicist P. A. Čerenkov discovered that a charged particle traveling in a solid with a speed exceeding the speed of light in that material radiates electromagnetic radiation. (This is analogous to the sonic boom produced by an aircraft moving faster than the speed of sound in air; see Section 16.9. Čerenkov shared the 1958 Nobel Prize for this discovery.) What is the minimum kinetic energy (in electron volts) that an electron must have while traveling inside a slab of crown glass ( $n = 1.52$ ) in order to create this Čerenkov radiation?

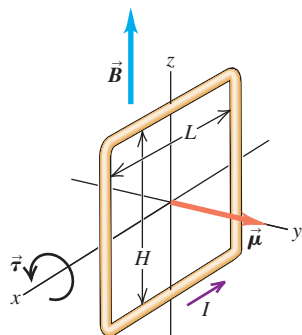
**37.52 ••** Quarks and gluons are fundamental particles that will be discussed in Chapter 44. A proton, which is a bound state of two up quarks and a down quark, has a rest mass of  $m_p = 1.67 \times 10^{-27}$  kg. This is significantly greater than the sum of the rest mass of the up quarks, which is  $m_u = 4.12 \times 10^{-30}$  kg each, and the rest mass of the down quark, which is  $m_d = 8.59 \times 10^{-30}$  kg. Suppose we (incorrectly) model the rest energy of the proton  $m_p c^2$  as derived from the kinetic energy of the three quarks, and we split that energy equally among them. (a) Estimate the Lorentz factor  $\gamma = (1 - v^2/c^2)^{-1/2}$  for each of the up quarks using Eq. (37.36). (b) Similarly estimate the Lorentz factor  $\gamma$  for the down quark. (c) Are the corresponding speeds  $v_u$  and  $v_d$  greater than 99% of the speed of light? (d) More realistically, the quarks are held together by massless gluons, which mediate the strong nuclear interaction. Suppose we model the proton as the three quarks, each with a speed of  $0.90c$ , with the remainder of the proton rest energy supplied by gluons. In this case, estimate the percentage of the proton rest energy associated with gluons. (e) Model a quark as oscillating with an average speed of  $0.90c$  across the diameter of a proton,  $1.7 \times 10^{-15}$  m. Estimate the frequency of that motion.

**37.53 • CP** A nuclear bomb containing 12.0 kg of plutonium explodes. The sum of the rest masses of the products of the explosion is less than the original rest mass by one part in  $10^4$ . (a) How much energy is released in the explosion? (b) If the explosion takes place in  $4.00 \mu\text{s}$ , what is the average power developed by the bomb? (c) What mass of water could the released energy lift to a height of 1.00 km?

**37.54 ••** In the earth's rest frame, two protons are moving away from each other at equal speed. In the frame of each proton, the other proton has a speed of  $0.700c$ . What does an observer in the rest frame of the earth measure for the speed of each proton?

**37.55 •• CP** In a laboratory, a rectangular loop of wire surrounds the origin in the  $xz$ -plane, with extent  $H$  in the  $z$ -direction and extent  $L$  in the  $x$ -direction (Fig. P37.55). The loop carries current  $I$  in the counterclockwise direction as viewed from the positive  $y$ -axis. A magnetic field  $\vec{B} = B\hat{k}$  is present. (a) What is the magnetic dipole moment  $\vec{\mu}$  as seen in the frame  $S$  of the laboratory? (b) What is the torque  $\vec{\tau}$  felt by the current loop? (c) The same loop is viewed from a passing alien spaceship moving with velocity  $\vec{v} = v\hat{i}$  as seen from the laboratory. From the point of view of the aliens, the loop is length contracted in the direction of this motion. What is the magnetic dipole moment  $\vec{\mu}'$  according to the aliens if their coordinate axes are aligned with those of the humans? (d) The torque on the loop is the same when viewed from the laboratory frame and from the spaceship frame. Accordingly, the magnetic field must be frame-dependent. What is the magnetic field  $\vec{B}'$  in the spaceship frame  $S'$ ? (e) If the electromagnetic field is  $(\vec{E}, \vec{B}) = (0, \vec{B})$  in an inertial frame  $S$ , then in another frame  $S'$  moving at velocity  $\vec{v}$  relative to  $S$ , what is the component of the magnetic field  $\vec{B}'_{\perp}$  perpendicular to  $\vec{v}$ ?

Figure P37.55



**37.56 •• CP** A small sphere with charge  $Q$  is resting motionless on an insulated pedestal in a laboratory. In the frame of reference  $S$  of the laboratory, the  $z$ -axis points upward, there is a magnetic field  $\vec{B} = -B\hat{j}$ , and there is no electric field. (a) What is the net force on the sphere? (b) The same sphere is viewed from a passing spacecraft moving with velocity  $\vec{v} = v\hat{i}$  as seen from the laboratory. Using the result of Problem 37.55, what is the magnetic field  $\vec{B}'$  seen by observers in the spacecraft if their coordinate axes are aligned with those of the laboratory? (c) From the perspective of the spacecraft, the sphere has a nonzero velocity and therefore feels a nonzero magnetic force. What is that

magnetic force? (d) The net electromagnetic force on the sphere is necessarily the same when viewed from the laboratory frame and from the spaceship frame. This can be true only if there is a nonzero electric field  $\vec{E}'$  in the spaceship frame. Determine  $\vec{E}'$ . (e) Generalize your conclusion: If the electromagnetic field is  $(\vec{E}, \vec{B}) = (0, \vec{B})$  in inertial frame  $S$ , then in another frame  $S'$  moving at velocity  $\vec{v}$  relative to  $S$ , what is the component of the electric field  $\vec{E}'_{\perp}$  perpendicular to  $\vec{v}$ ?

**37.57 •** One of the wavelengths of light emitted by hydrogen atoms under normal laboratory conditions is  $\lambda = 656.3$  nm, in the red portion of the electromagnetic spectrum. In the light emitted from a distant galaxy this same spectral line is observed to be Doppler-shifted to  $\lambda = 953.4$  nm, in the infrared portion of the spectrum. How fast are the emitting atoms moving relative to the earth? Are they approaching the earth or receding from it?

**37.58 ••** Two events are observed in a frame of reference  $S$  to occur at the same space point, the second occurring 1.80 s after the first. In a frame  $S'$  moving relative to  $S$ , the second event is observed to occur 2.15 s after the first. What is the difference between the positions of the two events as measured in  $S'$ ?

**37.59 •• Measuring Speed by Radar.** A baseball coach uses a radar device to measure the speed of an approaching pitched baseball. This device sends out electromagnetic waves with frequency  $f_0$  and then measures the shift in frequency  $\Delta f$  of the waves reflected from the moving baseball. If the fractional frequency shift produced by a baseball is  $\Delta f/f_0 = 2.86 \times 10^{-7}$ , what is the baseball's speed in km/h? (Hint: Are the waves Doppler-shifted a second time when reflected off the ball?)

**37.60 •• Albert in Wonderland.** Einstein and Lorentz, avid tennis players, play a fast-paced game on a court where they stand 20.0 m from each other. They play without a net. The tennis ball has mass 0.0580 kg. Ignore gravity and assume that the ball travels parallel to the ground as it travels between the two players. Unless otherwise specified, all measurements are made by the two men. (a) Lorentz serves the ball at  $80.0$  m/s. What is the ball's kinetic energy? (b) Einstein slams a return at  $1.80 \times 10^8$  m/s. What is the ball's kinetic energy? (c) During Einstein's return of the ball in part (a), a white rabbit runs beside the court in the direction from Einstein to Lorentz. The rabbit has a speed of  $2.20 \times 10^8$  m/s relative to the two men. What is the speed of the rabbit relative to the ball? (d) What does the rabbit measure as the distance from Einstein to Lorentz? (e) How much time does it take for the rabbit to run 20.0 m, according to the players? (f) The white rabbit uses his pocket watch to measure the time (as he sees it) for the distance from Einstein to Lorentz to pass by under him. What time does he measure?

**37.61 • CP** In a particle accelerator a proton moves at constant speed  $0.750c$  in a circle of radius 628 m. What is the net force on the proton?

**37.62 ••** A spaceship moving at constant speed  $u$  relative to us broadcasts a radio signal at constant frequency  $f_0$ . As the spaceship approaches us, we receive a higher frequency  $f$ ; after it has passed, we receive a lower frequency. (a) As the spaceship passes by, so it is instantaneously moving neither toward nor away from us, show that the frequency we receive is not  $f_0$ , and derive an expression for the frequency we do receive. Is the frequency we receive higher or lower than  $f_0$ ? (Hint: In this case, successive wave crests move the same distance to the observer and so they have the same transit time. Thus  $f$  equals  $1/T$ . Use the time dilation formula to relate the periods in the stationary and moving frames.) (b) A spaceship emits electromagnetic waves of frequency  $f_0 = 345$  MHz as measured in a frame moving with the ship. The spaceship is moving at a constant speed  $0.758c$  relative to us. What frequency  $f$  do we receive when the spaceship is approaching us? When it is moving away? In each case what is the shift in frequency,  $f - f_0$ ? (c) Use the result of part (a) to calculate the frequency  $f$  and the frequency shift  $(f - f_0)$  we receive at the instant that the ship passes by us. How does the shift in frequency calculated here compare to the shifts calculated in part (b)?

**37.63 •• DATA** As a research scientist at a linear accelerator, you are studying an unstable particle. You measure its mean lifetime  $\Delta t$  as a function of the particle's speed relative to your laboratory equipment. You record the speed of the particle  $u$  as a fraction of the speed of light in vacuum  $c$ . The table gives the results of your measurements.

$u/c$	0.70	0.80	0.85	0.88	0.90	0.92	0.94
$\Delta t$ ( $10^{-8}$ s)	3.57	4.41	5.02	5.47	6.05	6.58	7.62

(a) Your team leader suggests that if you plot your data as  $(\Delta t)^2$  versus  $(1 - u^2/c^2)^{-1}$ , the data points will be fit well by a straight line. Construct this graph and verify the team leader's prediction. Use the best-fit straight line to your data to calculate the mean lifetime of the particle in its rest frame. (b) What is the speed of the particle relative to your lab equipment (expressed as  $u/c$ ) if the lifetime that you measure is four times its rest-frame lifetime?

**37.64 •• CP** The French physicist Armand Fizeau was the first to measure the speed of light accurately. He also found experimentally that the speed, relative to the lab frame, of light traveling in a tank of water that is itself moving at a speed  $V$  relative to the lab frame is

$$v = \frac{c}{n} + kV$$

where  $n = 1.333$  is the index of refraction of water. Fizeau called  $k$  the dragging coefficient and obtained an experimental value of  $k = 0.44$ . What value of  $k$  do you calculate from relativistic transformations?

**37.65 •• DATA** You are a scientist studying small aerosol particles that are contained in a vacuum chamber. The particles carry a net charge, and you use a uniform electric field to exert a constant force of  $8.00 \times 10^{-14}$  N on one of them. That particle moves in the direction of the exerted force. Your instruments measure the acceleration of the particle as a function of its speed  $v$ . The table gives the results of your measurements for this particular particle.

$v/c$	0.60	0.65	0.70	0.75	0.80	0.85
$a$ ( $10^3$ m/s <sup>2</sup> )	20.3	17.9	14.8	11.2	8.5	5.9

(a) Graph your data so that the data points are well fit by a straight line. Use the slope of this line to calculate the mass  $m$  of the particle. (b) What magnitude of acceleration does the exerted force produce if the speed of the particle is 100 m/s?

**37.66 •• DATA** You are an astronomer investigating four astronomical sources of infrared radiation. You have identified the nature of each source, so you know the frequency  $f_0$  of each when it is at rest relative to you. Your detector, which is at rest relative to the earth, measures the frequency  $f$  of the moving source. Your results are given in the table.

Source	A	B	C	D
$f$ (THz)	7.1	5.4	6.1	8.1
$f_0$ (THz)	9.2	8.6	7.9	8.9

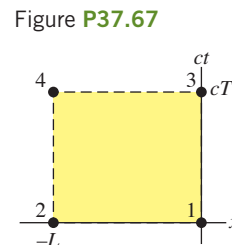
(a) Which source is moving at the highest speed relative to your detector? What is its speed? Is that source moving toward or away from the detector? (b) Which source is moving at the lowest speed relative to your detector? What is its speed? Is that source moving toward or away from the detector? (c) For source B, what frequency would your detector measure if the source were moving at the same speed relative to the detector but toward it rather than away from it?

## CHALLENGE PROBLEMS

**37.67 •••** A fluorescent tube with length  $L$  is fixed horizontally above a ticket window in a train station. At time  $t = 0$  the tube lights up,

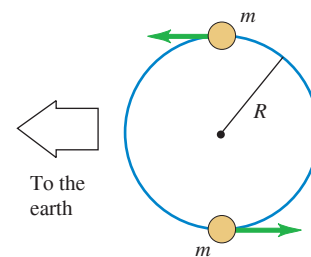
changing color from white to bright yellow. After a duration  $T$  the tube turns off, reverting to its white color. The tube lies along the  $x$ -axis of frame  $S$  fixed with respect to the station, with its right end at the origin. We can represent the history of the tube using a “spacetime” diagram, as shown in

**Fig. P37.67**. Events 1 and 2 correspond, respectively, to the right and left ends of the tube at time  $t = 0$ , while events 3 and 4 correspond to the right and left ends of the tube at time  $t = T$ . The totality of events corresponding to positions on the tube when it is lit correspond to the yellow region on the diagram. The tube is observed from the window of a rocket train passing the station from left to right at speed  $v$ . The frame  $S'$  is fixed with respect to the train such that the  $x'$ -axis coincides with the  $x$ -axis of frame  $S$ , and such that event 1 occurs at the origin  $(x', t') = (0, 0)$ . (a) Use the Lorentz transformations to determine the spacetime coordinates  $(x', t')$  for events 1, 2, 3, and 4 in the train frame  $S'$ . (b) Construct a spacetime diagram analogous to Figure P37.67 showing the tube in the train frame  $S'$ . Carefully label relevant points, using the economical definition  $\gamma = (1 - v^2/c^2)^{-1/2}$  when needed. (c) We use the product  $ct$  rather than  $t$  on the vertical axis of our figure so that both axes have the dimension of distance. Thus the yellow region has a well-defined area. What is the area of the yellow region in the  $S$  frame plot? (d) Compute the area of the yellow region in the  $S'$  frame plot you constructed in part (b). (Hint: The area of a parallelogram is given by the magnitude of the vector product of two vectors describing adjacent edges.) (e) Is the area of the yellow region the same when we change the frame of reference from  $S$  to  $S'$ ?



**37.68 ••• CP Determining the Masses of Stars.** Many of the stars in the sky are actually *binary stars*, in which two stars orbit about their common center of mass. If the orbital speeds of the stars are high enough, the motion of the stars can be detected by the Doppler shifts of the light they emit. Stars for which this is the case are called *spectroscopic binary stars*.

Figure P37.68

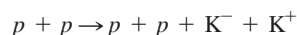


**Figure P37.68** shows the simplest case of a spectroscopic binary star:

two identical stars, each with mass  $m$ , orbiting their center of mass in a circle of radius  $R$ . The plane of the stars' orbits is edge-on to the line of sight of an observer on the earth. (a) The light produced by heated hydrogen gas in a laboratory on the earth has a frequency of  $4.568110 \times 10^{14}$  Hz. In the light received from the stars by a telescope on the earth, hydrogen light is observed to vary in frequency between  $4.567710 \times 10^{14}$  Hz and  $4.568910 \times 10^{14}$  Hz. Determine whether the binary star system as a whole is moving toward or away from the earth, the speed of this motion, and the orbital speeds of the stars. (Hint: The speeds involved are much less than  $c$ , so you may use the approximate result  $\Delta f/f = u/c$  given in Section 37.6.) (b) The light from each star in the binary system varies from its maximum frequency to its minimum frequency and back again in 11.0 days. Determine the orbital radius  $R$  and the mass  $m$  of each star. Give your answer for  $m$  in kilograms and as a multiple of the mass of the sun,  $1.99 \times 10^{30}$  kg. Compare the value of  $R$  to the distance from the earth to the sun,  $1.50 \times 10^{11}$  m. (This technique is actually used in astronomy to determine the masses of stars. In practice, the problem is more complicated because the two stars in a binary system are usually not identical, the orbits are usually not circular, and the plane of the orbits is usually tilted with respect to the line of sight from the earth.)



**37.69 ••• CP Kaon Production.** In high-energy physics, new particles can be created by collisions of fast-moving projectile particles with stationary particles. Some of the kinetic energy of the incident particle is used to create the mass of the new particle. A proton–proton collision can result in the creation of a negative kaon ( $K^-$ ) and a positive kaon ( $K^+$ ):



(a) Calculate the minimum kinetic energy of the incident proton that will allow this reaction to occur if the second (target) proton is initially at rest. The rest energy of each kaon is 493.7 MeV, and the rest energy of each proton is 938.3 MeV. (*Hint:* It is useful here to work in the frame in which the total momentum is zero. But note that the Lorentz transformation must be used to relate the velocities in the laboratory frame to those in the zero-total-momentum frame.) (b) How does this calculated minimum kinetic energy compare with the total rest mass energy of the created kaons? (c) Suppose that instead the two protons are both in motion with velocities of equal magnitude and opposite direction. Find the minimum combined kinetic energy of the two protons that will allow the reaction to occur. How does this calculated minimum kinetic energy compare with the total rest mass energy of the created kaons? (This example shows that when colliding beams of particles are used instead of a stationary target, the energy requirements for producing new particles are reduced substantially.)

**37.70 ••• CP CALC Relativity and the Wave Equation.** (a) Consider the Galilean transformation along the  $x$ -direction:  $x' = x - vt$  and  $t' = t$ . In frame  $S$  the wave equation for electromagnetic waves in a vacuum is

$$\frac{\partial^2 E(x, t)}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E(x, t)}{\partial t^2} = 0$$

where  $E$  represents the electric field in the wave. Show that by using the Galilean transformation the wave equation in frame  $S'$  is found to be

$$\left(1 - \frac{v^2}{c^2}\right) \frac{\partial^2 E(x', t')}{\partial x'^2} + \frac{2v}{c^2} \frac{\partial^2 E(x', t')}{\partial x' \partial t'} - \frac{1}{c^2} \frac{\partial^2 E(x', t')}{\partial t'^2} = 0$$

This has a different form than the wave equation in  $S$ . Hence the Galilean transformation *violates* the first relativity postulate that all physical laws have the same form in all inertial reference frames. (*Hint:* Express the derivatives  $\partial/\partial x$  and  $\partial/\partial t$  in terms of  $\partial/\partial x'$  and  $\partial/\partial t'$  by use of the chain rule.) (b) Repeat the analysis of part (a), but use the Lorentz coordinate transformations, Eqs. (37.21), and show that in frame  $S'$  the wave equation has the same form as in frame  $S$ :

$$\frac{\partial^2 E(x', t')}{\partial x'^2} - \frac{1}{c^2} \frac{\partial^2 E(x', t')}{\partial t'^2} = 0$$

Explain why this shows that the speed of light in vacuum is  $c$  in both frames  $S$  and  $S'$ .

## MCAT-STYLE PASSAGE PROBLEMS

**Speed of Light.** Our universe has properties that are determined by the values of the fundamental physical constants, and it would be a much different place if the charge of the electron, the mass of the proton, or the speed of light was substantially different from its actual value. For instance, the speed of light is so great that the effects of relativity usually go unnoticed in everyday events. Let's imagine an alternate universe where the speed of light is 1,000,000 times less than it is in our universe to see what would happen.

**37.71** An airplane has a length of 60 m when measured at rest. When the airplane is moving at 180 m/s (400 mph) in the alternate universe, how long would the plane appear to be to a stationary observer? (a) 24 m; (b) 36 m; (c) 48 m; (d) 60 m; (e) 75 m.

**37.72** If the airplane of Passage Problem 37.71 has a rest mass of 20,000 kg, what is its relativistic mass when the plane is moving at 180 m/s? (a) 8000 kg; (b) 12,000 kg; (c) 16,000 kg; (d) 25,000 kg; (e) 33,300 kg.

**37.73** In our universe, the rest energy of an electron is approximately  $8.2 \times 10^{-14}$  J. What would it be in the alternate universe? (a)  $8.2 \times 10^{-8}$  J; (b)  $8.2 \times 10^{-26}$  J; (c)  $8.2 \times 10^{-2}$  J; (d) 0.82 J.

**37.74** In the alternate universe, how fast must an object be moving for it to have a kinetic energy equal to its rest mass? (a) 225 m/s; (b) 260 m/s; (c) 300 m/s; (d) The kinetic energy could not be equal to the rest mass.

## ANSWERS

### Chapter Opening Question ?

(v) From Eq. (37.36), the relativistic expression for the kinetic energy of a particle of mass  $m$  moving at speed  $v$  is  $K = (\gamma - 1)mc^2$ , where  $\gamma = 1/\sqrt{1 - v^2/c^2}$ . If  $v = 0.99000c$ ,  $\gamma - 1 = 6.08881$ ; if  $v = 0.99995c$ ,  $\gamma - 1 = 99.001$ , which is 16.260 times greater than the value at  $v = 0.99000c$ . As the speed approaches  $c$ , a relatively small increase in  $v$  corresponds to a large increase in kinetic energy (see Fig. 37.21).

### Key Example VARIATION Problems

**VP37.5.1** (a)  $2.05 \times 10^{-8}$  s (b)  $2.05 \times 10^{-8}$  s

**VP37.5.2** (a) 16.0 s (b) 30.0 s

**VP37.5.3** (a) 526 s (b)  $4.68 \times 10^{10}$  m, 164 s

**VP37.5.4** (a)  $1.28 \times 10^5$  km (b) 72.0 m

**VP37.7.1** (a)  $x' = -8.50 \times 10^8$  m,  $t' = 3.78$  s

(b)  $x = 1.46 \times 10^9$  m,  $t = 5.29$  s

**VP37.7.2** (a)  $1.35 \times 10^8$  m (b)  $2.25 \times 10^8$  m

**VP37.7.3** (a)  $+0.385c$  (b)  $-0.227c$

**VP37.7.4** (a)  $0.941c$  (b)  $0.195c$

**VP37.11.1** (a)  $1.22 \times 10^{-18}$  kg  $\cdot$  m/s (b)  $2.96 \times 10^{12}$  m/s<sup>2</sup>

(c)  $2.05 \times 10^{13}$  m/s<sup>2</sup>

**VP37.11.2** (a)  $3.18 \times 10^{-13}$  J (b) 4.88 (c)  $0.979c$

**VP37.11.3** (a)  $1.13 \times 10^{-13}$  J (b) 1.38 (c)  $0.689c$

**VP37.11.4** (a) 1348 MeV (b) 3.72 (c)  $0.963c$

### Bridging Problem

(a)  $0.268c$

(b) 35.6 MeV

(c) 145 MeV