? Waves from a broadcasting station produce an alternating current in the circuits of a radio (like the one in this classic car). If a radio is tuned to a station at a frequency of 1000 kHz, it will also detect the transmissions from a station broadcasting at (i) 600 kHz; (ii) 800 kHz; (iii) 1200 kHz; (iv) all of these; (v) none of these.

31

Alternating Current

LEARNING OUTCOMES

In this chapter, you'll learn...

- **31.1** How phasors make it easy to describe sinusoidally varying quantities.
- **31.2** How to use reactance to describe the voltage across a circuit element that carries an alternating current.
- **31.3** How to analyze an *L-R-C* series circuit with a sinusoidal emf.
- **31.4** What determines the amount of power flowing into or out of an alternating-current circuit.
- **31.5** How an *L-R-C* series circuit responds to sinusoidal emfs of different frequencies.
- **31.6** Why transformers are useful, and how they work.

You'll need to review...

- **14.2, 14.8** Simple harmonic motion, resonance.
- **16.5** Resonance and sound.
- **18.3** Root-mean-square (rms) values.
- **25.3** Diodes.
- **26.3** Galvanometers.
- **28.8** Hysteresis in magnetic materials.
- **29.2, 29.6, 29.7** Alternating-current generators; eddy currents; displacement current.
- **30.1, 30.2, 30.5, 30.6** Mutual inductance; voltage across an inductor; *L-C* circuits; *L-R-C* series circuits.

During the 1880s in the United States there was a heated and acrimonious debate between two inventors over the best method of electric-power distribution. Thomas Edison favored direct current (dc)—that is, steady current that does not vary with time. George Westinghouse favored **alternating current (ac),** with sinusoidally varying voltages and currents. He argued that transformers (which we'll study in this chapter) can be used to step the voltage up and down with ac but not with dc; low voltages are safer for consumer use, but high voltages and correspondingly low currents are best for long-distance power transmission to minimize i^2R losses in the cables.

Eventually, Westinghouse prevailed, and most present-day household and industrial power distribution systems operate with alternating current. Any appliance that you plug into a wall outlet uses ac. Circuits in modern communication equipment also make extensive use of ac.

In this chapter we'll learn how resistors, inductors, and capacitors behave in **ac circuits**—that is, circuits with sinusoidally varying voltages and currents. Many of the principles that we found useful in Chapter 30 are applicable, along with several new concepts related to the circuit behavior of inductors and capacitors. A key concept in this discussion is *resonance,* which we studied in Chapter 14 for mechanical systems.

31.1 PHASORS AND ALTERNATING CURRENTS

To supply an alternating current to a circuit, a source of alternating emf or voltage is required. An example of such a source is a coil of wire rotating with constant angular velocity in a magnetic field, which we discussed in Example 29.3 (Section 29.2). This develops a sinusoidal alternating emf and is the prototype of the commercial alternating-current generator or *alternator* (see Fig. 29.8).

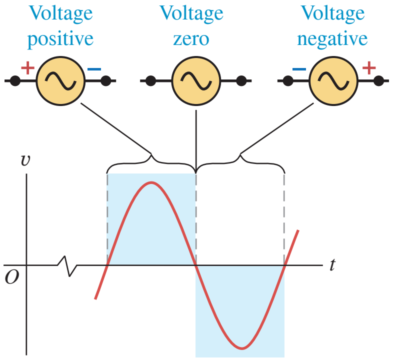
We use the term **ac source** for any device that supplies a sinusoidally varying voltage (potential difference) v or current i. The usual circuit-diagram symbol for an ac source is

A sinusoidal voltage might be described by a function such as

$$v = V\cos\omega t \qquad (31.1)$$

In this expression, (lowercase) v is the *instantaneous* potential difference; (uppercase) V is the maximum potential difference, which we call the **voltage amplitude;** and ω is the *angular frequency,* equal to 2π times the frequency f (**Fig. 31.1**).

Figure **31.1** The voltage across a sinusoidal ac source.



In the United States and Canada, commercial electric-power distribution systems use a frequency $f = 60$ Hz, corresponding to $\omega = (2\pi \text{ rad})(60 \text{ s}^{-1}) = 377$ rad/s; in much of the rest of the world, $f = 50$ Hz ($\omega = 314$ rad/s) is used. Similarly, a sinusoidal current with a maximum value, or **current amplitude,** of I might be described as

Sinusoidal alternating current:

Instantaneous current · Angular frequency

$$i = I\cos\omega t \qquad (31.2)$$

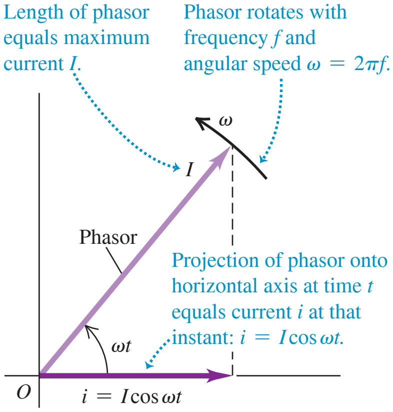
Time

Current amplitude (maximum current)

Phasor Diagrams

To represent sinusoidally varying voltages and currents in ac circuits, we'll use rotating vector diagrams similar to those we used in the study of simple harmonic motion in Section 14.2 (see Figs. 14.5b and 14.6). In these diagrams the instantaneous value of a quantity that varies sinusoidally with time is represented by the *projection* onto a horizontal axis of a vector with a length equal to the amplitude of the quantity. The vector rotates counterclockwise with constant angular speed ω. These rotating vectors are called **phasors,** and diagrams containing them are called **phasor diagrams. Figure 31.2** shows a phasor diagram for the sinusoidal current described by Eq. (31.2). The projection of the phasor onto the horizontal axis at time t is $I\cos\omega t$; this is why we chose to use the cosine function rather than the sine in Eq. (31.2).

Figure **31.2** A phasor diagram.



CAUTION **Just what is a phasor?** A phasor isn't a real physical quantity with a direction in space, such as velocity or electric field. Rather, it's a *geometric* entity that helps us describe physical quantities that vary sinusoidally with time. In Section 14.2 we used a single phasor to represent the position of a particle undergoing simple harmonic motion. Here we'll use phasors to *add* sinusoidal voltages and currents. Combining sinusoidal quantities with phase differences then involves vector addition. We'll use phasors in a similar way in Chapters 35 and 36 in our study of interference effects with light. ❙

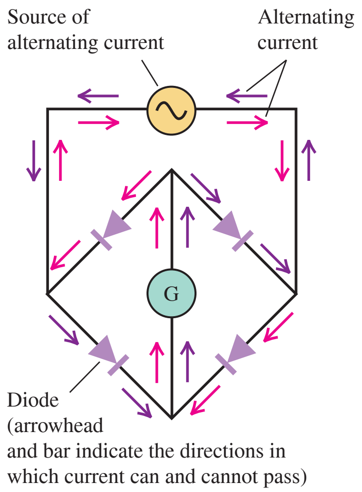
Rectified Alternating Current

How do we measure a sinusoidally varying current? In Section 26.3 we used a d'Arsonval galvanometer to measure steady currents. But if we pass a *sinusoidal* current through a d'Arsonval meter, the torque on the moving coil varies sinusoidally, with one direction half the time and the opposite direction the other half. The needle may wiggle a little if the frequency is low enough, but its average deflection is zero. Hence a d'Arsonval meter by itself isn't very useful for measuring alternating currents.

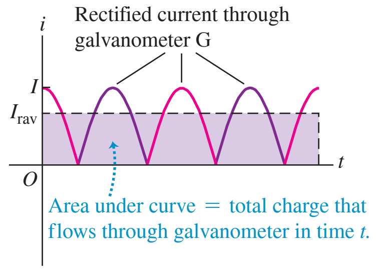
To get a measurable one-way current through the meter, we can use *diodes,* which we described in Section 25.3. A diode is a device that conducts better in one direction than in the other; an ideal diode has zero resistance for one direction of current and infinite resistance for the other. **Figure 31.3a** (next page) shows one possible arrangement, called

Figure **31.3** (a) A full-wave rectifier circuit. (b) Graph of the resulting current through the galvanometer G.

(a) A full-wave rectifier circuit



(b) Graph of the full-wave rectified current and its average value, the rectified average current I_{rav}



a *full-wave rectifier circuit*. The current through the galvanometer G is always upward, regardless of the direction of the current from the ac source (i.e., which part of the cycle the source is in). The graph in Fig. 31.3b shows the current through G: It pulsates but always has the same direction, and the average meter deflection is *not* zero.

The **rectified average current** I_{rav} is defined so that during any whole number of cycles, the total charge that flows is the same as though the current were constant with a value equal to I_{rav}. The notation I_{rav} and the name *rectified average* current emphasize that this is *not* the average of the original sinusoidal current. In Fig. 31.3b the total charge that flows in time t corresponds to the area under the curve of i versus t (recall that $i = dq/dt$, so q is the integral of t); this area must equal the rectangular area with height I_{rav}. We see that I_{rav} is less than the maximum current I; the two are related by

Rectified average value of a sinusoidal current

$$I_{\text{rav}} = \frac{2}{\pi}I = 0.637I \tag{31.3}$$

Current amplitude

(The factor of $2/\pi$ is the average value of $|\cos\omega t|$ or of $|\sin\omega t|$; see Example 29.4 in Section 29.2.) The galvanometer deflection is proportional to I_{rav}. The galvanometer scale can be calibrated to read I, I_{rav}, or, most commonly, I_{rms} (discussed below).

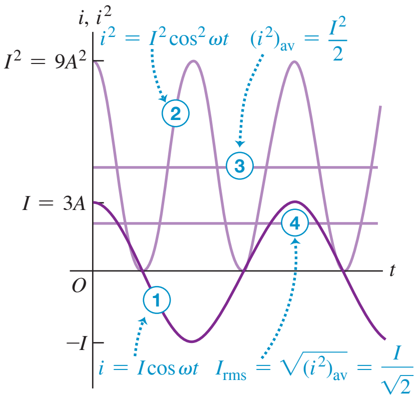
Root-Mean-Square (rms) Values

A more useful way to describe a quantity that can be either positive or negative is the *root-mean-square (rms) value*. We used rms values in Section 18.3 in connection with the speeds of molecules in a gas. We *square* the instantaneous current i, take the *average* (mean) value of i^2, and finally take the *square root* of that average. This procedure defines the **root-mean-square current,** denoted as I_{rms} (**Fig. 31.4**). Even when i is negative, i^2 is always positive, so I_{rms} is never zero (unless i is zero at every instant).

Figure **31.4** Calculating the root-mean-square (rms) value of an alternating current.

Meaning of the rms value of a sinusoidal quantity (here, ac current with $I = 3$ A):

① Graph current i versus time.
② *Square* the instantaneous current i.
③ Take the *average* (mean) value of i^2.
④ Take the *square root* of that average.



Here's how we obtain I_{rms} for a sinusoidal current, like that shown in Fig. 31.4. If the instantaneous current is given by $i = I\cos\omega t$, then

$$i^2 = I^2\cos^2\omega t$$

Using a double-angle formula from trigonometry,

$$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$

we find

$$i^2 = I^2\tfrac{1}{2}(1 + \cos 2\omega t) = \tfrac{1}{2}I^2 + \tfrac{1}{2}I^2\cos 2\omega t$$

The average of $\cos 2\omega t$ is zero because it is positive half the time and negative half the time. Thus the average of i^2 is simply $I^2/2$. The square root of this is I_{rms}:

Root-mean-square (rms) value of a sinusoidal current

$$I_{\text{rms}} = \frac{I}{\sqrt{2}} \tag{31.4}$$

Current amplitude

In the same way, the root-mean-square value of a sinusoidal voltage is

Root-mean-square (rms) value of a sinusoidal voltage

$$V_{\text{rms}} = \frac{V}{\sqrt{2}} \tag{31.5}$$

Voltage amplitude (maximum value)

We can convert a rectifying ammeter into a voltmeter by adding a series resistor, just as for the dc case discussed in Section 26.3. Meters used for ac voltage and current measurements are nearly always calibrated to read rms values, not maximum or rectified average. Voltages and currents in power distribution systems are always described in terms of their rms values. The usual household power supply, "120 volt ac," has an rms voltage of 120 V (**Fig. 31.5**). The voltage amplitude is

$$\begin{aligned} V &= \sqrt{2}\,V_{\mathrm{rms}} \\ &= \sqrt{2}\,(120\ \mathrm{V}) = 170\ \mathrm{V} \end{aligned}$$

Sixty times per second, the instantaneous voltage across a socket's terminals varies from 170 V to −170 V and back again.

Figure **31.5** This wall socket delivers a root-mean-square (rms) voltage of 120 V.

EXAMPLE 31.1 Current in a desktop computer

The plate on the back of a desktop computer says that it draws 2.7 A from a 120 V, 60 Hz line. For this computer, what are (a) the average current, (b) the average of the square of the current, and (c) the current amplitude?

IDENTIFY and SET UP This example is about alternating current. In part (a) we find the average, over a complete cycle, of the alternating current. In part (b) we recognize that the 2.7 A current draw of the computer is the rms value I_{rms}—that is, the *square root* of the *mean* (average) of the *square* of the current, $(i^2)_{\mathrm{av}}$. In part (c) we use Eq. (31.4) to relate I_{rms} to the current amplitude.

EXECUTE (a) The average of *any* sinusoidally varying quantity, over any whole number of cycles, is zero.

(b) We are given $I_{\mathrm{rms}} = 2.7$ A. From the definition of rms value,

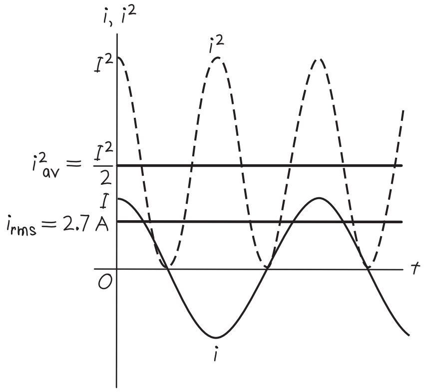
$$I_{\mathrm{rms}} = \sqrt{(i^2)_{\mathrm{av}}} \text{ so } (i^2)_{\mathrm{av}} = (I_{\mathrm{rms}})^2 = (2.7\ \mathrm{A})^2 = 7.3\ \mathrm{A}^2$$

(c) From Eq. (31.4), the current amplitude I is

$$I = \sqrt{2}I_{\mathrm{rms}} = \sqrt{2}(2.7\ \mathrm{A}) = 3.8\ \mathrm{A}$$

Figure 31.6 shows graphs of i and i^2 versus time t.

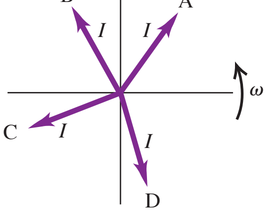
Figure **31.6** Our graphs of the current i and the square of the current i^2 versus time t.



EVALUATE Why would we be interested in the average of the square of the current? Recall that the rate at which energy is dissipated in a resistor R equals i^2R. This rate varies if the current is alternating, so it is best described by its average value $(i^2)_{\mathrm{av}}R = I_{\mathrm{rms}}{}^2R$. We'll use this idea in Section 31.4.

KEYCONCEPT The root-mean-square (rms) value of a time-varying quantity is the square root of the average value of the square of that quantity. To find the rms value of a quantity that varies sinusoidally with time, like the current or voltage in an ac circuit, divide its amplitude by $\sqrt{2}$.

TEST YOUR UNDERSTANDING OF SECTION 31.1 The accompanying figure shows four different current phasors with the same angular frequency ω. At the time shown, which phasor corresponds to (a) a positive current that is becoming more positive; (b) a positive current that is decreasing toward zero; (c) a negative current that is becoming more negative; (d) a negative current that is decreasing in magnitude toward zero?



ANSWER

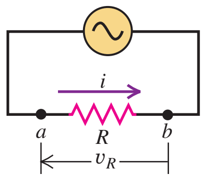
(a) **D**; (b) **A**; (c) **B**; (d) **C** For each phasor, the actual current is represented by the projection of that phasor onto the horizontal axis. The phasors all rotate counterclockwise around the origin with angular speed ω, so at the instant shown the projection of phasor A is positive but trending toward zero; the projection of phasor B is negative and becoming more negative; the projection of phasor C is negative but trending toward zero; and the projection of phasor D is positive and becoming more positive.

31.2 RESISTANCE AND REACTANCE

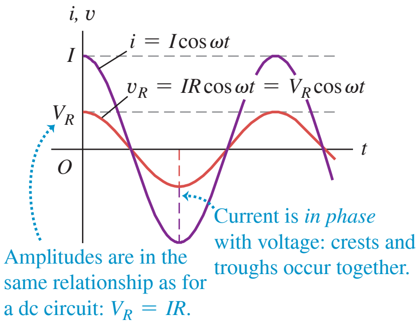
In this section we'll derive voltage–current relationships for individual circuit elements—resistors, inductors, and capacitors—carrying a sinusoidal current.

Figure **31.7** Resistance R connected across an ac source.

(a) Circuit with ac source and resistor



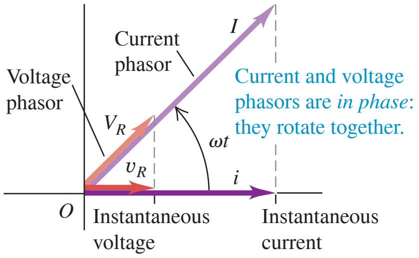
(b) Graphs of current and voltage versus time



Current is *in phase* with voltage: crests and troughs occur together.

Amplitudes are in the same relationship as for a dc circuit: $V_R = IR$.

(c) Phasor diagram



Current and voltage phasors are *in phase*: they rotate together.

Resistor in an ac Circuit

First let's consider a resistor with resistance R through which there is a sinusoidal current given by Eq. (31.2): $i = I\cos\omega t$. The positive direction of current is counterclockwise around the circuit (**Fig. 31.7a**). The current amplitude (maximum current) is I. From Ohm's law the instantaneous potential v_R of point a with respect to point b (that is, the instantaneous voltage across the resistor) is

$$v_R = iR = (IR)\cos\omega t \tag{31.6}$$

The maximum value of the voltage v_R is V_R, the *voltage amplitude:*

Amplitude of voltage across a resistor, ac circuit

Current amplitude

$$V_R = IR \tag{31.7}$$

Resistance

Hence we can also write

$$v_R = V_R\cos\omega t \tag{31.8}$$

Both the current i and the voltage v_R are proportional to $\cos\omega t$, so the current is *in phase* with the voltage. Equation (31.7) shows that the current and voltage amplitudes are related in the same way as in a dc circuit.

Figure 31.7b shows graphs of i and v_R as functions of time. The vertical scales for current and voltage are different, so the relative heights of the two curves are not significant. The corresponding phasor diagram is given in Fig. 31.7c. Because i and v_R are *in phase* and have the same frequency, the current and voltage phasors rotate together; they are parallel at each instant. Their projections on the horizontal axis represent the instantaneous current and voltage, respectively.

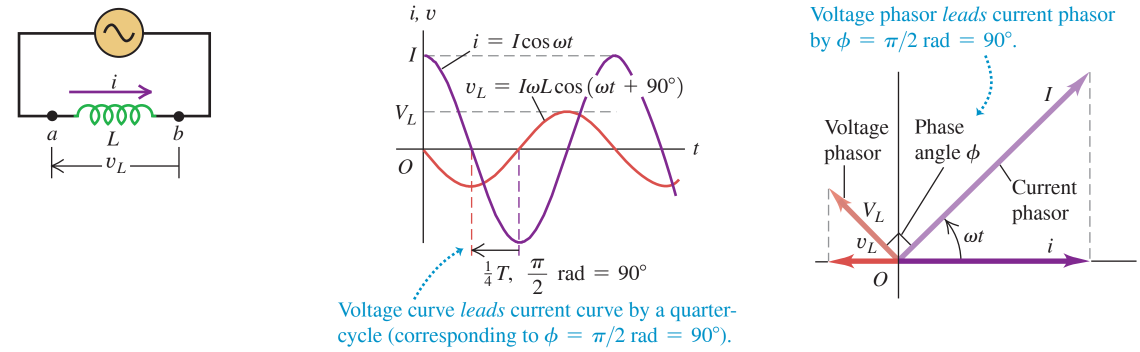
Inductor in an ac Circuit

Now we replace the resistor in Fig. 31.7 with a pure inductor with self-inductance L and zero resistance (**Fig. 31.8a**). Again the current is $i = I\cos\omega t$, and the positive direction of current is counterclockwise around the circuit.

Although there is no resistance, there is a potential difference v_L between the inductor terminals a and b because the current varies with time, giving rise to a self-induced emf. The induced emf in the direction of i is given by Eq. (30.7), $\mathcal{E} = -L\,di/dt$; however, the voltage v_L is *not* simply equal to $\mathcal{E}$. To see why, notice that if the current in the inductor is in the

Figure **31.8** Inductance L connected across an ac source.

(a) Circuit with ac source and inductor (b) Graphs of current and voltage versus time (c) Phasor diagram



Voltage curve *leads* current curve by a quarter-cycle (corresponding to $\phi = \pi/2$ rad $= 90°$).

Voltage phasor *leads* current phasor by $\phi = \pi/2$ rad $= 90°$.

positive (counterclockwise) direction from a to b and is increasing, then di/dt is positive and the induced emf is directed to the left to oppose the increase in current; hence point a is at higher potential than is point b. Thus the potential of point a with respect to point b is positive and is given by $v_L = +L\,di/dt$, the *negative* of the induced emf. (Convince yourself that this expression gives the correct sign of v_L in *all* cases, including i counterclockwise and decreasing, i clockwise and increasing, and i clockwise and decreasing; also review Section 30.2.) So

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}(I\cos\omega t) = -I\omega L\sin\omega t \qquad (31.9)$$

The voltage v_L across the inductor at any instant is proportional to the *rate of change* of the current. The points of maximum voltage on the graph correspond to maximum steepness of the current curve, and the points of zero voltage are the points where the current curve has its maximum and minimum values (Fig. 31.8b). The voltage and current are *out of phase* by a quarter-cycle. Since the voltage peaks occur a quarter-cycle earlier than the current peaks, we say that the voltage *leads* the current by 90°. The phasor diagram in Fig. 31.8c also shows this relationship; the voltage phasor is ahead of the current phasor by 90°.

We can also obtain this phase relationship by rewriting Eq. (31.9) with the identity $\cos(A + 90°) = -\sin A$:

$$v_L = I\omega L\cos(\omega t + 90°) \qquad (31.10)$$

This result shows that the voltage can be viewed as a cosine function with a “head start” of 90° relative to the current.

As we have done in Eq. (31.10), we’ll usually describe the phase of the *voltage* relative to the *current,* not the reverse. Thus if the current i in a circuit is

$$i = I\cos\omega t$$

and the voltage v of one point with respect to another is

$$v = V\cos(\omega t + \phi)$$

we call ϕ the **phase angle;** it gives the phase of the *voltage* relative to the *current*. For a pure resistor, $\phi = 0$, and for a pure inductor, $\phi = 90°$.

From Eq. (31.9) or (31.10) the amplitude V_L of the inductor voltage is

$$V_L = I\omega L \qquad (31.11)$$

We define the **inductive reactance** X_L of an inductor as

$$X_L = \omega L \qquad \text{(inductive reactance)} \qquad (31.12)$$

Using X_L, we can write Eq. (31.11) in a form similar to Eq. (31.7) for a resistor:

$$\underset{\text{Amplitude of voltage across an inductor, ac circuit}}{V_L} = \overset{\text{Current amplitude}}{I}\underset{\text{Inductive reactance}}{X_L} \qquad (31.13)$$

Because X_L is the ratio of a voltage and a current, its SI unit is the ohm, the same as for resistance.

CAUTION Inductor voltage and current are not in phase Equation (31.13) relates the *amplitudes* of the oscillating voltage and current for the inductor in Fig. 31.8a. It does *not* say that the voltage at any instant is equal to the current at that instant multiplied by X_L. As Fig. 31.8b shows, the voltage and current are 90° out of phase. Voltage and current are in phase only for resistors, as in Eq. (31.6).

The Meaning of Inductive Reactance

The inductive reactance X_L is really a description of the self-induced emf that opposes any change in the current through the inductor. From Eq. (31.13), for a given current amplitude I the voltage $v_L = +L\,di/dt$ across the inductor and the self-induced emf $\mathcal{E} = -L\,di/dt$ both have an amplitude V_L that is directly proportional to X_L. According to Eq. (31.12), the inductive reactance and self-induced emf increase with more rapid variation in current (that is, increasing angular frequency ω) and increasing inductance L.

If an oscillating voltage of a given amplitude V_L is applied across the inductor terminals, the resulting current will have a smaller amplitude I for larger values of X_L. Since X_L is proportional to frequency, a high-frequency voltage applied to the inductor gives only a small current, while a lower-frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio-interference filters, to block high frequencies while permitting lower frequencies or dc to pass through. A circuit device that uses an inductor for this purpose is called a *low-pass filter* (see Problem 31.48).

EXAMPLE 31.2 An inductor in an ac circuit

WITH VARIATION PROBLEMS

The current amplitude in a pure inductor in a radio receiver is to be 250 μA when the voltage amplitude is 3.60 V at a frequency of 1.60 MHz (at the upper end of the AM broadcast band). (a) What inductive reactance is needed? What inductance? (b) If the voltage amplitude is kept constant, what will be the current amplitude through this inductor at 16.0 MHz? At 160 kHz?

IDENTIFY and SET UP The circuit may have other elements, but in this example we don't care: All they do is provide the inductor with an oscillating voltage, so the other elements are lumped into the ac source shown in Fig. 31.8a. We are given the current amplitude I and the voltage amplitude V. Our target variables in part (a) are the inductive reactance X_L at 1.60 MHz and the inductance L, which we find from Eqs. (31.13) and (31.12). Knowing L, we use these equations in part (b) to find X_L and I at any frequency.

EXECUTE (a) From Eq. (31.13),

$$X_L = \frac{V_L}{I} = \frac{3.60\ \text{V}}{250 \times 10^{-6}\ \text{A}} = 1.44 \times 10^4\ \Omega = 14.4\ \text{k}\Omega$$

From Eq. (31.12), with $\omega = 2\pi f$,

$$L = \frac{X_L}{2\pi f} = \frac{1.44 \times 10^4\ \Omega}{2\pi(1.60 \times 10^6\ \text{Hz})} = 1.43 \times 10^{-3}\ \text{H} = 1.43\ \text{mH}$$

(b) Combining Eqs. (31.12) and (31.13), we find $I = V_L/X_L = V_L/\omega L = V_L/2\pi f L$. Thus the current amplitude is inversely proportional to the frequency f. Since $I = 250\ \mu\text{A}$ at $f = 1.60$ MHz, the current amplitudes at 16.0 MHz $(10f)$ and 160 kHz = 0.160 MHz $(f/10)$ will be, respectively, one-tenth as great $(25.0\ \mu\text{A})$ and ten times as great $(2500\ \mu\text{A} = 2.50\ \text{mA})$.

EVALUATE In general, the lower the frequency of an oscillating voltage applied across an inductor, the greater the amplitude of the resulting oscillating current.

KEYCONCEPT For any component in an ac circuit, the reactance is the ratio of the amplitude of the voltage across the component to the amplitude of the current through the component. If the component is an inductor, the reactance equals the product of the inductance and the angular frequency of the current.

Capacitor in an ac Circuit

Finally, we connect a capacitor with capacitance C to the source, as in **Fig. 31.9a**, producing a current $i = I\cos\omega t$ through the capacitor. Again, the positive direction of current is counterclockwise around the circuit.

CAUTION Alternating current through a capacitor Charge can't really move through the capacitor because its two plates are insulated from each other. But as the capacitor charges and discharges, there is at each instant a current i into one plate, an equal current out of the other plate, and an equal *displacement* current between the plates. (You should review the discussion of displacement current in Section 29.7.) Thus we often speak about alternating current *through* a capacitor.

To find the instantaneous voltage v_C across the capacitor—that is, the potential of point a with respect to point b —we first let q denote the charge on the left-hand plate of the capacitor in Fig. 31.9a (so $-q$ is the charge on the right-hand plate). The current i is related to q by $i = dq/dt$; with this definition, positive current corresponds to an increasing charge on the left-hand capacitor plate. Then

$$i = \frac{dq}{dt} = I\cos\omega t$$

Integrating this, we get

$$q = \frac{I}{\omega}\sin\omega t \tag{31.14}$$

Also, from Eq. (24.1) the charge q equals the voltage v_C multiplied by the capacitance, $q = Cv_C$. Using this in Eq. (31.14), we find

$$v_C = \frac{I}{\omega C}\sin\omega t \tag{31.15}$$

The instantaneous current i is equal to the rate of change dq/dt of the capacitor charge q; since $q = Cv_C$, i is also proportional to the rate of change of voltage. (Compare to an inductor, for which the situation is reversed and v_L is proportional to the rate of change of i.) Figure 31.9b shows v_C and i as functions of t. Because $i = dq/dt = C\,dv_C/dt$, the current has its greatest magnitude when the v_C curve is rising or falling most steeply and is zero when the v_C curve instantaneously levels off at its maximum and minimum values.

The peaks of capacitor voltage occur a quarter-cycle *after* the corresponding current peaks, and we say that the voltage *lags* the current by 90°. The phasor diagram in Fig. 31.9c shows this relationship; the voltage phasor is behind the current phasor by a quarter-cycle, or 90°.

We can also derive this phase difference by rewriting Eq. (31.15) with the identity $\cos(A - 90°) = \sin A$:

$$v_C = \frac{I}{\omega C}\cos(\omega t - 90°) \qquad (31.16)$$

This corresponds to a phase angle $\phi = -90°$. This cosine function has a "late start" of 90° compared with the current $i = I\cos\omega t$.

Equations (31.15) and (31.16) show that the voltage *amplitude* V_C is

$$V_C = \frac{I}{\omega C} \qquad (31.17)$$

To put this expression in a form similar to Eq. (31.7) for a resistor, $V_R = IR$, we define a quantity X_C, called the **capacitive reactance** of the capacitor, as

$$X_C = \frac{1}{\omega C} \quad \text{(capacitive reactance)} \qquad (31.18)$$

Then

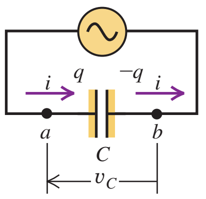
Amplitude of voltage across a capacitor, ac circuit $V_C = IX_C$ **Current amplitude** **Capacitive reactance** (31.19)

The SI unit of X_C is the ohm, the same as for resistance and inductive reactance, because X_C is the ratio of a voltage and a current.

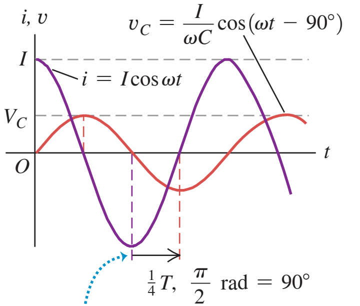
CAUTION **Capacitor voltage and current are not in phase** Remember that Eq. (31.19) for a capacitor, like Eq. (31.13) for an inductor, is *not* a statement about the instantaneous values of voltage and current. The instantaneous values are 90° out of phase, as Fig. 31.9b shows. Rather, Eq. (31.19) relates the *amplitudes* of voltage and current.

Figure **31.9** Capacitor C connected across an ac source.

(a) Circuit with ac source and capacitor

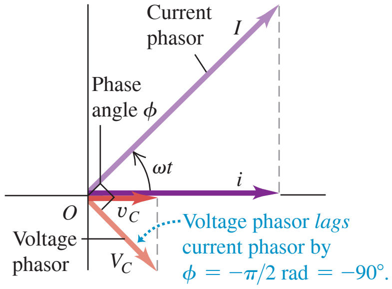


(b) Graphs of current and voltage versus time



Voltage curve *lags* current curve by a quarter-cycle (corresponding to $\phi = -\pi/2$ rad $= -90°$).

(c) Phasor diagram



Voltage phasor *lags* current phasor by $\phi = -\pi/2$ rad $= -90°$.

The Meaning of Capacitive Reactance

The capacitive reactance of a capacitor is inversely proportional both to the capacitance C and to the angular frequency ω; the greater the capacitance and the higher the frequency, the *smaller* the capacitive reactance X_C. Capacitors tend to pass high-frequency current and to block low-frequency currents and dc, just the opposite of inductors. A device that preferentially passes signals of high frequency is called a *high-pass filter* (see Problem 31.47).

EXAMPLE 31.3 A resistor and a capacitor in an ac circuit

WITH VARIATION PROBLEMS

A 200 Ω resistor is connected in series with a 5.0 μF capacitor. The voltage across the resistor is $v_R = (1.20\ \text{V})\cos[(2500\ \text{rad/s})t]$ (**Fig. 31.10**). (a) Derive an expression for the circuit current. (b) Determine the capacitive reactance of the capacitor. (c) Derive an expression for the voltage across the capacitor.

IDENTIFY and SET UP Since this is a series circuit, the current is the same through the capacitor as through the resistor. Our target variables are the current i, the capacitive reactance X_C, and the capacitor voltage v_C. We use Eq. (31.6) to find an expression for i in terms of the angular frequency $\omega = 2500$ rad/s, Eq. (31.18) to find X_C, Eq. (31.19) to find the capacitor voltage amplitude V_C, and Eq. (31.16) to write an expression for v_C.

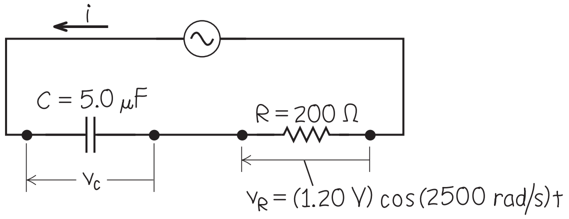
EXECUTE (a) From Eq. (31.6), $v_R = iR$, we find

$$i = \frac{v_R}{R} = \frac{(1.20\ \text{V})\cos[(2500\ \text{rad/s})t]}{200\ \Omega}$$
$$= (6.0 \times 10^{-3}\ \text{A})\cos[(2500\ \text{rad/s})t]$$

(b) From Eq. (31.18), the capacitive reactance at $\omega = 2500$ rad/s is

$$X_C = \frac{1}{\omega C} = \frac{1}{(2500\ \text{rad/s})(5.0 \times 10^{-6}\ \text{F})} = 80\ \Omega$$

Figure **31.10** Our sketch for this problem.



(c) From Eq. (31.19), the capacitor voltage amplitude is

$$V_C = IX_C = (6.0 \times 10^{-3}\ \text{A})(80\ \Omega) = 0.48\ \text{V}$$

(The 80 Ω reactance of the capacitor is 40% of the resistor's 200 Ω resistance, so V_C is 40% of V_R.) The instantaneous capacitor voltage is given by Eq. (31.16):

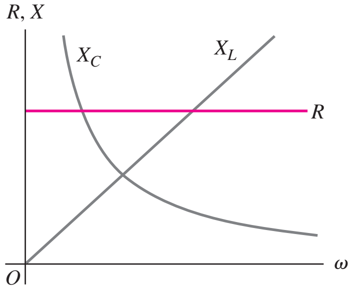
$$v_C = V_C\cos(\omega t - 90°) = (0.48\ \text{V})\cos[(2500\ \text{rad/s})t - \pi/2\ \text{rad}]$$

Continued

EVALUATE Although the same *current* passes through both the capacitor and the resistor, the *voltages* across them are different in both amplitude and phase. Note that in the expression for v_C we converted the 90° to $\pi/2$ rad so that all the angular quantities have the same units. In ac circuit analysis, phase angles are often given in degrees, so be careful to convert to radians when necessary.

KEYCONCEPT The reactance (ratio of voltage amplitude to current amplitude) for a resistor in an ac circuit is equal to its resistance. For a capacitor, the reactance equals the reciprocal of the product of the capacitance and the angular frequency of the current.

Figure **31.11** Graphs of R, X_L, and X_C as functions of angular frequency ω.



Comparing ac Circuit Elements

Table 31.1 summarizes the relationships of voltage and current amplitudes for the three circuit elements we have discussed. Note again that *instantaneous* voltage and current are proportional in a resistor, where there is zero phase difference between v_R and i (see Fig. 31.7b). The instantaneous voltage and current are *not* proportional in an inductor or capacitor, because there is a 90° phase difference in both cases (see Figs. 31.8b and 31.9b).

TABLE 31.1 Circuit Elements with Alternating Current

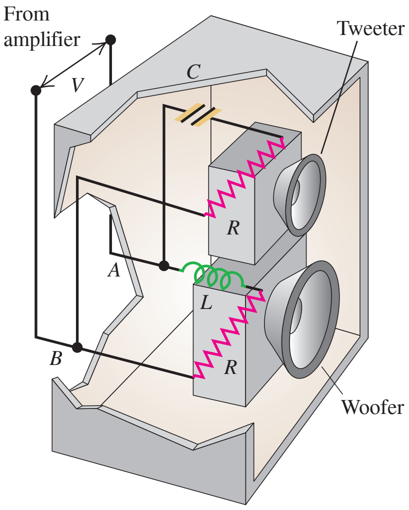
Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	In phase with i
Inductor	$V_L = IX_L$	$X_L = \omega L$	Leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = 1/\omega C$	Lags i by 90°

Figure 31.11 shows how the resistance of a resistor and the reactances of an inductor and a capacitor vary with angular frequency ω. Resistance R is independent of frequency, while the reactances X_L and X_C are not. If $\omega = 0$, corresponding to a dc circuit, there is *no* current through a capacitor because $X_C \rightarrow \infty$, and there is no inductive effect because $X_L = 0$. In the limit $\omega \rightarrow \infty$, X_L also approaches infinity, and the current through an inductor becomes vanishingly small; recall that the self-induced emf opposes rapid changes in current. In this same limit, X_C and the voltage across a capacitor both approach zero; the current changes direction so rapidly that no charge can build up on either plate.

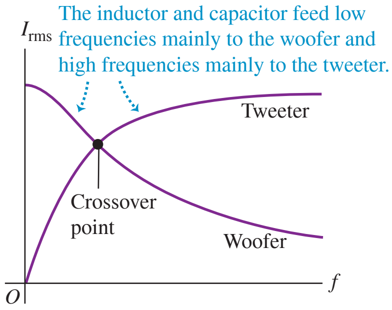
Figure 31.12 shows an application of the above discussion to a loudspeaker system. Low-frequency sounds are produced by the *woofer,* which is a speaker with large diameter; the *tweeter,* a speaker with smaller diameter, produces high-frequency sounds. In order to route signals of different frequency to the appropriate speaker, the woofer and tweeter are connected in parallel across the amplifier output. The capacitor in the tweeter branch blocks the low-frequency components of sound but passes the higher frequencies; the inductor in the woofer branch does the opposite.

Figure **31.12** (a) The two speakers in this loudspeaker system are connected in parallel to the amplifier. (b) Graphs of current amplitude in the tweeter and woofer as functions of frequency for a given amplifier voltage amplitude.

(a) A crossover network in a loudspeaker system



(b) Graphs of rms current as functions of frequency for a given amplifier voltage


The inductor and capacitor feed low frequencies mainly to the woofer and high frequencies mainly to the tweeter.

TEST YOUR UNDERSTANDING OF SECTION 31.2 An oscillating voltage of fixed amplitude is applied across a circuit element. If the frequency of this voltage is increased, will the amplitude of the current through the element (i) increase, (ii) decrease, or (iii) remain the same if it is (a) a resistor, (b) an inductor, or (c) a capacitor?

ANSWER

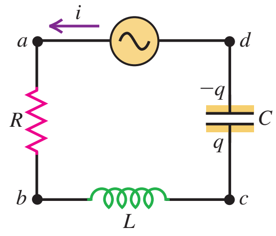
(a) **(iii)**; (b) **(ii)**; (c) **(i)** For a resistor, $V_R = IR$, so $I = V_R/R$. The voltage amplitude V_R and resistance R do not change with frequency, so the current amplitude I remains constant. For an inductor, $V_L = IX_L = I\omega L$, so $I = V_L/\omega L$. The voltage amplitude V_L and inductance L are constant, so the current amplitude I decreases as the frequency increases. For a capacitor, $V_C = IX_C = I/\omega C$, so $I = V_C\omega C$. The voltage amplitude V_C and capacitance C are constant, so the current amplitude I increases as the frequency increases.

31.3 THE *L-R-C* SERIES CIRCUIT

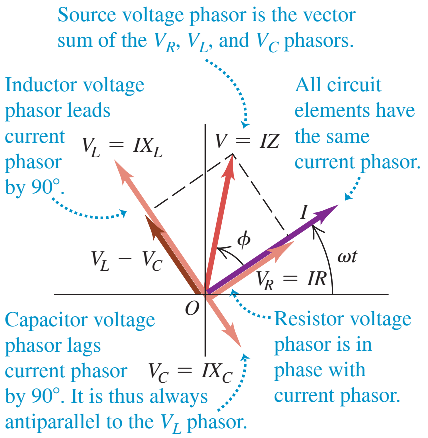
Many ac circuits used in practical electronic systems involve resistance, inductive reactance, and capacitive reactance. **Figure 31.13a** shows a simple example: a series circuit containing a resistor, an inductor, a capacitor, and an ac source. (In Section 30.6 we studied an *L-R-C* series circuit *without* a source.)

Figure **31.13** An *L-R-C* series circuit with an ac source.

(a) *L-R-C* series circuit



(b) Phasor diagram for the case $X_L > X_C$


Source voltage phasor is the vector sum of the V_R, V_L, and V_C phasors.

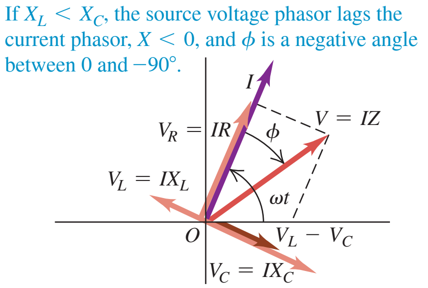
Inductor voltage phasor leads current phasor by 90°.

All circuit elements have the same current phasor.

Capacitor voltage phasor lags current phasor by 90°. It is thus always antiparallel to the V_L phasor.

Resistor voltage phasor is in phase with current phasor.

(c) Phasor diagram for the case $X_L < X_C$


If $X_L < X_C$, the source voltage phasor lags the current phasor, $X < 0$, and ϕ is a negative angle between 0 and −90°.

To analyze this circuit, we'll use a phasor diagram that includes the voltage and current phasors for each of the components. Because of Kirchhoff's loop rule, the instantaneous *total* voltage v_{ad} across all three components is equal to the source voltage at that instant. We'll show that the phasor representing this total voltage is the *vector sum* of the phasors for the individual voltages.

Figures 31.13b and 31.13c show complete phasor diagrams for the circuit of Fig. 31.13a. We assume that the source supplies a current i given by $i = I\cos\omega t$. Because the circuit elements are connected in series, the current at any instant is the same at every point in the circuit. Thus a *single phasor* I, with length proportional to the current amplitude, represents the current in *all* circuit elements.

As in Section 31.2, we use the symbols v_R, v_L, and v_C for the instantaneous voltages across R, L, and C, and the symbols V_R, V_L, and V_C for the maximum voltages. We denote the instantaneous and maximum *source* voltages by v and V. Then, in Fig. 31.13a, $v = v_{ad}$, $v_R = v_{ab}$, $v_L = v_{bc}$, and $v_C = v_{cd}$.

The potential difference between the terminals of a resistor is *in phase* with the current in the resistor. Its maximum value V_R is given by Eq. (31.7):

$$V_R = IR$$

The phasor V_R in Fig. 31.13b, in phase with the current phasor I, represents the voltage across the resistor. Its projection onto the horizontal axis at any instant gives the instantaneous potential difference v_R.

The voltage across an inductor *leads* the current by 90°. Its voltage amplitude is given by Eq. (31.13):

$$V_L = IX_L$$

The phasor V_L in Fig. 31.13b represents the voltage across the inductor, and its projection onto the horizontal axis at any instant equals v_L.

The voltage across a capacitor *lags* the current by 90°. Its voltage amplitude is given by Eq. (31.19):

$$V_C = IX_C$$

The phasor V_C in Fig. 31.13b represents the voltage across the capacitor, and its projection onto the horizontal axis at any instant equals v_C.

The instantaneous potential difference v between terminals a and d is equal at every instant to the (algebraic) sum of the potential differences v_R, v_L, and v_C. That is, it equals the sum of the *projections* of the phasors V_R, V_L, and V_C. But the sum of the projections of these phasors is equal to the *projection* of their *vector sum*. So the vector sum V must be the phasor that represents the source voltage v and the instantaneous total voltage v_{ad} across the series of elements.

To form this vector sum, we first subtract the phasor V_C from the phasor V_L. (These two phasors always lie along the same line, with opposite directions.) This gives the phasor $V_L - V_C$. This is always at right angles to the phasor V_R, so from the Pythagorean theorem the magnitude of the phasor V is

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \tag{31.20}$$

We define the **impedance** Z of an ac circuit as the ratio of the voltage amplitude across the circuit to the current amplitude in the circuit. From Eq. (31.20) the impedance of the *L-R-C* series circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{31.21}$$

so we can rewrite Eq. (31.20) as

$$\underset{\text{Amplitude of voltage across an ac circuit}}{V} = \overset{\text{Current amplitude}}{I}\,\underset{\text{Impedance of circuit}}{Z} \qquad (31.22)$$

While Eq. (31.21) is valid only for an *L-R-C* series circuit, we can use Eq. (31.22) to define the impedance of *any* network of resistors, inductors, and capacitors as the ratio of the amplitude of the voltage across the network to the current amplitude. The SI unit of impedance is the ohm.

The Meaning of Impedance and Phase Angle

Equation (31.22) has a form similar to $V = IR$, with impedance Z in an ac circuit playing the role of resistance R in a dc circuit. Just as direct current tends to follow the path of least resistance, so alternating current tends to follow the path of lowest impedance (**Fig. 31.14**). Note, however, that impedance is actually a function of R, L, and C, as well as of the angular frequency ω. We can see this by substituting Eq. (31.12) for X_L and Eq. (31.18) for X_C into Eq. (31.21), giving the following complete expression for Z for a series circuit:

$$\underset{\text{Impedance of an } L\text{-}R\text{-}C \text{ series circuit}}{Z} = \sqrt{\overset{\text{Resistance}}{R^2} + [\underset{\text{Angular frequency}}{\omega} \overset{\text{Inductance}}{L} - (1/\underset{\text{Angular frequency}}{\omega} \overset{\text{Capacitance}}{C})]^2} \qquad (31.23)$$

Hence for a given amplitude V of the source voltage applied to the circuit, the amplitude $I = V/Z$ of the resulting current will be different at different frequencies. We'll explore this frequency dependence in detail in Section 31.5.

In Fig. 31.13b, the angle ϕ between the voltage and current phasors is the phase angle of the source voltage v with respect to the current i; that is, it is the angle by which the source voltage leads the current. From the diagram,

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \frac{X_L - X_C}{R}$$

$$\underset{\text{Phase angle of voltage with respect to current in an } L\text{-}R\text{-}C \text{ series circuit}}{\tan\phi} = \frac{\overset{\text{Angular frequency}}{\omega} \overset{\text{Inductance}}{L} - 1/\omega \overset{\text{Capacitance}}{C}}{\underset{\text{Resistance}}{R}} \qquad (31.24)$$

If the current is $i = I\cos\omega t$, then the source voltage v is

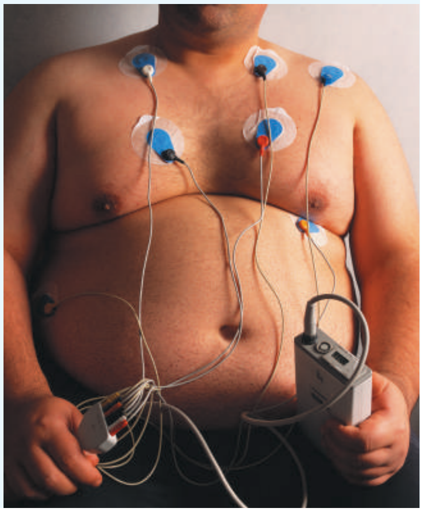
$$v = V\cos(\omega t + \phi) \qquad (31.25)$$

Figure 31.13b shows the behavior of an *L-R-C* series circuit in which $X_L > X_C$. Figure 31.13c shows the behavior when $X_L < X_C$; the voltage phasor V lies on the opposite side of the current phasor I and the voltage *lags* the current. In this case, $X_L - X_C$ is *negative*, $\tan\phi$ is negative, and ϕ is a negative angle between 0° and $-90°$. Since X_L and X_C depend on frequency, the phase angle ϕ depends on frequency as well. We'll examine the consequences of this in Section 31.5.

All of the expressions that we've developed for an *L-R-C* series circuit are still valid if one of the circuit elements is missing. If the resistor is missing, we set $R = 0$; if the inductor is missing, we set $L = 0$. But if the capacitor is missing, we set $C = \infty$, corresponding to the absence of any potential difference ($v_C = q/C = 0$) or any capacitive reactance ($X_C = 1/\omega C = 0$).

Figure **31.14** This gas-filled glass sphere has an alternating voltage between its surface and the electrode at its center. The glowing streamers show the resulting alternating current that passes through the gas. When you touch the outside of the sphere, your fingertips and the inner surface of the sphere act as the plates of a capacitor, and the sphere and your body together form an *L-R-C* series circuit. The current (which is low enough to be harmless) is drawn to your fingers because the path through your body has a low impedance.

BIO APPLICATION Measuring Body Fat by Bioelectric Impedance Analysis The electrodes attached to this overweight patient's chest are applying a small ac voltage of frequency 50 kHz. The attached instrumentation measures the amplitude and phase angle of the resulting current through the patient's body. These depend on the relative amounts of water and fat along the path followed by the current, and so provide a sensitive measure of body composition.



In this entire discussion we have described magnitudes of voltages and currents in terms of their *maximum* values, the voltage and current *amplitudes*. But we remarked at the end of Section 31.1 that these quantities are usually described in terms of rms values, not amplitudes. For any sinusoidally varying quantity, the rms value is always $1/\sqrt{2}$ times the amplitude. All the relationships between voltage and current that we have derived in this and the preceding sections are still valid if we use rms quantities throughout instead of amplitudes. For example, if we divide Eq. (31.22) by $\sqrt{2}$, we get

$$\frac{V}{\sqrt{2}} = \frac{I}{\sqrt{2}}Z$$

which we can rewrite as

$$V_{\text{rms}} = I_{\text{rms}}Z \tag{31.26}$$

We can translate Eqs. (31.7), (31.13), and (31.19) in exactly the same way.

We have considered only ac circuits in which an inductor, a resistor, and a capacitor are in series. You can do a similar analysis for an *L-R-C parallel* circuit; see Problem 31.54.

PROBLEM-SOLVING STRATEGY 31.1 Alternating-Current Circuits

IDENTIFY *the relevant concepts:* In analyzing ac circuits, we can apply all of the concepts used to analyze direct-current circuits, particularly those in Problem-Solving Strategies 26.1 and 26.2. But now we must distinguish between the amplitudes of alternating currents and voltages and their instantaneous values, and among resistance (for resistors), reactance (for inductors or capacitors), and impedance (for composite circuits).

SET UP *the problem* using the following steps:

1. Draw a diagram of the circuit and label all known and unknown quantities.
2. Identify the target variables.

EXECUTE *the solution* as follows:

1. Use the relationships derived in Sections 31.2 and 31.3 to solve for the target variables, using the following hints.
2. It's almost always easiest to work with angular frequency $\omega = 2\pi f$ rather than ordinary frequency f.
3. Keep in mind the following phase relationships: For a resistor, voltage and current are *in phase,* so the corresponding phasors always point in the same direction. For an inductor, the voltage *leads* the current by 90° (i.e., $\phi = +90° = \pi/2$ radians), so the voltage phasor points 90° counterclockwise from the current phasor. For a capacitor, the voltage *lags* the current by 90° (i.e., $\phi = -90° = -\pi/2$ radians), so the voltage phasor points 90° clockwise from the current phasor.
4. Kirchhoff's rules hold *at each instant*. For example, in a series circuit, the instantaneous current is the same in all circuit elements; in a parallel circuit, the instantaneous potential difference is the same across all circuit elements.
5. Inductive reactance, capacitive reactance, and impedance are analogous to resistance; each represents the ratio of voltage amplitude V to current amplitude I in a circuit element or combination of elements. However, phase relationships are crucial. In applying Kirchhoff's loop rule, you must combine the effects of resistance and reactance by *vector* addition of the corresponding voltage phasors, as in Figs. 31.13b and 31.13c. When several circuit elements are in series, for example, you can't *add* all the numerical values of resistance and reactance to get the impedance; that would ignore the phase relationships.

EVALUATE *your answer:* When working with an *L-R-C series* circuit, you can check your results by comparing the values of the inductive and capacitive reactances X_L and X_C. If $X_L > X_C$, then the voltage amplitude across the inductor is greater than that across the capacitor and the phase angle ϕ is positive (between 0° and 90°). If $X_L < X_C$, then the voltage amplitude across the inductor is less than that across the capacitor and the phase angle ϕ is negative (between 0° and −90°).

EXAMPLE 31.4 An *L-R-C* series circuit I

WITH VARIATION PROBLEMS

In the series circuit of Fig. 31.13a, suppose $R = 300\ \Omega$, $L = 60$ mH, $C = 0.50\ \mu$F, $V = 50$ V, and $\omega = 10{,}000$ rad/s. Find the reactances X_L and X_C, the impedance Z, the current amplitude I, the phase angle ϕ, and the voltage amplitude across each circuit element.

IDENTIFY and SET UP This problem uses the ideas developed in Section 31.2 and this section about the behavior of circuit elements in an ac circuit. We use Eqs. (31.12) and (31.18) to determine X_L and X_C, and Eq. (31.23) to find Z. We then use Eq. (31.22) to find the current amplitude and Eq. (31.24) to find the phase angle. The relationships in Table 31.1 then yield the voltage amplitudes.

EXECUTE The inductive and capacitive reactances are

$$X_L = \omega L = (10{,}000\ \text{rad/s})(60\ \text{mH}) = 600\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(10{,}000\ \text{rad/s})(0.50 \times 10^{-6}\ \text{F})} = 200\ \Omega$$

The impedance Z of the circuit is then

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300\ \Omega)^2 + (600\ \Omega - 200\ \Omega)^2}$$
$$= 500\ \Omega$$

Continued

With source voltage amplitude $V = 50$ V, the current amplitude I and phase angle ϕ are

$$I = \frac{V}{Z} = \frac{50\text{ V}}{500\ \Omega} = 0.10\text{ A}$$

$$\phi = \arctan\frac{X_L - X_C}{R} = \arctan\frac{400\ \Omega}{300\ \Omega} = 53^\circ$$

From Table 31.1, the voltage amplitudes V_R, V_L, and V_C across the resistor, inductor, and capacitor, respectively, are

$$V_R = IR = (0.10\text{ A})(300\ \Omega) = 30\text{ V}$$
$$V_L = IX_L = (0.10\text{ A})(600\ \Omega) = 60\text{ V}$$
$$V_C = IX_C = (0.10\text{ A})(200\ \Omega) = 20\text{ V}$$

EVALUATE As in Fig. 31.13b, $X_L > X_C$; hence the voltage amplitude across the inductor is greater than that across the capacitor and ϕ is positive. The value $\phi = 53^\circ$ means that the voltage *leads* the current by 53°.

Note that the source voltage amplitude $V = 50$ V is *not* equal to the sum of the voltage amplitudes across the separate circuit elements: $50\text{ V} \neq 30\text{ V} + 60\text{ V} + 20\text{ V}$. Instead, V is the *vector sum* of the V_R, V_L, and V_C phasors. If you draw the phasor diagram like Fig. 31.13b for this particular situation, you'll see that V_R, $V_L - V_C$, and V constitute a 3-4-5 right triangle.

KEYCONCEPT For an ac circuit that includes an inductor L, resistor R, and capacitor C in series, the impedance—the ratio of the amplitude of the voltage across the L-R-C combination to the amplitude of the current—depends on the values of L, R, C, and the angular frequency of the current.

EXAMPLE 31.5 An *L-R-C* series circuit II

WITH VARIATION PROBLEMS

For the L-R-C series circuit of Example 31.4, find expressions for the time dependence of the instantaneous current i and the instantaneous voltages across the resistor (v_R), inductor (v_L), capacitor (v_C), and ac source (v).

IDENTIFY and SET UP We describe the current by using Eq. (31.2), which assumes that the current is maximum at $t = 0$. The voltages are then given by Eq. (31.8) for the resistor, Eq. (31.10) for the inductor, Eq. (31.16) for the capacitor, and Eq. (31.25) for the source.

EXECUTE The current and the voltages all oscillate with the same angular frequency, $\omega = 10{,}000$ rad/s, and hence with the same period, $2\pi/\omega = 2\pi/(10{,}000\text{ rad/s}) = 6.3 \times 10^{-4}\text{ s} = 0.63$ ms. From Eq. (31.2), the current is

$$i = I\cos\omega t = (0.10\text{ A})\cos[(10{,}000\text{ rad/s})t]$$

The resistor voltage is *in phase* with the current, so

$$v_R = V_R\cos\omega t = (30\text{ V})\cos[(10{,}000\text{ rad/s})t]$$

The inductor voltage *leads* the current by 90°, so

$$v_L = V_L\cos(\omega t + 90^\circ) = -V_L\sin\omega t = -(60\text{ V})\sin[(10{,}000\text{ rad/s})t]$$

The capacitor voltage *lags* the current by 90°, so

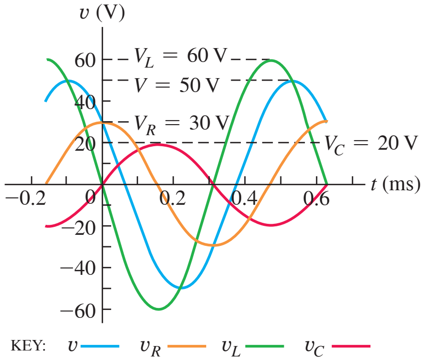
$$v_C = V_C\cos(\omega t - 90^\circ) = V_C\sin\omega t = (20\text{ V})\sin[(10{,}000\text{ rad/s})t]$$

We found in Example 31.4 that the source voltage (equal to the voltage across the entire combination of resistor, inductor, and capacitor) *leads* the current by $\phi = 53^\circ$, so

$$\begin{aligned} v &= V\cos(\omega t + \phi) \\ &= (50\text{ V})\cos\left[(10{,}000\text{ rad/s})t + \left(\frac{2\pi\text{ rad}}{360^\circ}\right)(53^\circ)\right] \\ &= (50\text{ V})\cos[(10{,}000\text{ rad/s})t + 0.93\text{ rad}] \end{aligned}$$

EVALUATE **Figure 31.15** graphs the four voltages versus time. The inductor voltage has a larger amplitude than the capacitor voltage because $X_L > X_C$. The *instantaneous* source voltage v is always equal to the sum of the instantaneous voltages v_R, v_L, and v_C. You should verify this by measuring the values of the voltages shown in the graph at different values of the time t.

Figure **31.15** Graphs of the source voltage v, resistor voltage v_R, inductor voltage v_L, and capacitor voltage v_C as functions of time for the situation of Example 31.4. The current, which is not shown, is in phase with the resistor voltage.



KEYCONCEPT In an L-R-C series circuit that includes an ac source, the instantaneous voltage across the L-R-C combination is the sum of the instantaneous voltages across each of the three components. Because these voltages are not in phase with one another, the amplitude of the voltage across the L-R-C combination is *not* equal to the sum of the individual voltage amplitudes.

TEST YOUR UNDERSTANDING OF SECTION 31.3 Rank the following ac circuits in order of their current amplitude, from highest to lowest value. (i) The circuit in Example 31.4; (ii) the circuit in Example 31.4 with both the capacitor and inductor removed; (iii) the circuit in Example 31.4 with both the resistor and capacitor removed; (iv) the circuit in Example 31.4 with both the resistor and inductor removed.

ANSWER

(iv), (ii), (i), (iii) For the circuit in Example 31.4, $I = V/Z = (50\text{ V})/(500\ \Omega) = 0.10$ A. If the capacitor and inductor are removed so that only the ac source and resistor remain, the circuit is like that shown in Fig. 31.7a; then $I = V/R = (50\text{ V})/(300\ \Omega) = 0.17$ A. If the resistor and capacitor are removed so that only the ac source and inductor remain, the circuit is like that shown in Fig. 31.8a; then $I = V/X_L = (50\text{ V})/(600\ \Omega) = 0.083$ A. Finally, if the resistor and inductor are removed so that only the ac source and capacitor remain, the circuit is like that shown in Fig. 31.9a; then $I = V/X_C = (50\text{ V})/(200\ \Omega) = 0.25$ A.

31.4 POWER IN ALTERNATING-CURRENT CIRCUITS

Alternating currents play a central role in systems for distributing, converting, and using electrical energy, so it's important to look at power relationships in ac circuits. For an ac circuit with instantaneous current i and current amplitude I, we'll consider an element of that circuit across which the instantaneous potential difference is v with voltage amplitude V. The instantaneous power p delivered to this circuit element is

$$p = vi$$

Let's first see what this means for individual circuit elements. We'll assume in each case that $i = I\cos\omega t$.

Power in a Resistor

Suppose first that the circuit element is a *pure resistor* R, as in Fig. 31.7a; then $v = v_R$ and i are *in phase*. We obtain the graph representing p by multiplying the heights of the graphs of v and i in Fig. 31.7b at each instant. The result is the black curve in **Fig. 31.16a**. The product vi is always positive because v and i are always either both positive or both negative. Hence energy is supplied *to* the resistor at every instant for both directions of i, although the power is not constant.

The power curve for a pure resistor is symmetric about a value equal to one-half its maximum value VI, so the *average power* P_{av} is

$$P_{av} = \tfrac{1}{2}VI \qquad \text{(for a pure resistor)} \tag{31.27}$$

An equivalent expression is

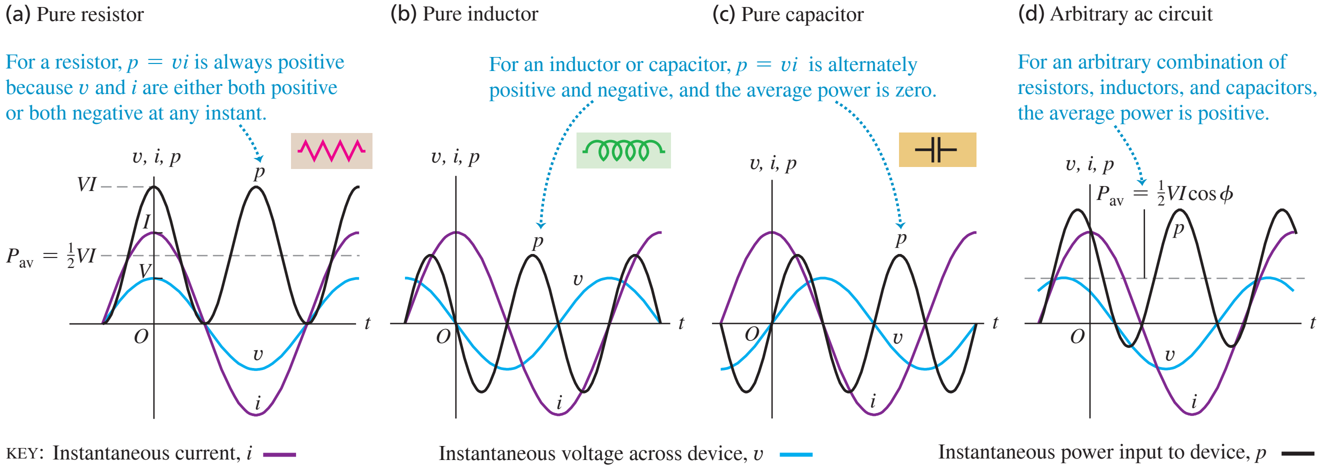
$$P_{av} = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}} = V_{rms}I_{rms} \qquad \text{(for a pure resistor)} \tag{31.28}$$

Also, $V_{rms} = I_{rms}R$, so we can express P_{av} by any of the equivalent forms

$$P_{av} = I_{rms}{}^2R = \frac{V_{rms}{}^2}{R} = V_{rms}I_{rms} \qquad \text{(for a pure resistor)} \tag{31.29}$$

Note that the expressions in Eq. (31.29) have the same form as the corresponding relationships for a dc circuit, Eq. (25.18). Also note that they are valid only for pure resistors, not for more complicated combinations of circuit elements.

Figure **31.16** Graphs of current, voltage, and power as functions of time for (a) a pure resistor, (b) a pure inductor, (c) a pure capacitor, and (d) an arbitrary ac circuit that can have resistance, inductance, and capacitance.



(a) Pure resistor

(b) Pure inductor

(c) Pure capacitor

(d) Arbitrary ac circuit

For a resistor, $p = vi$ is always positive because v and i are either both positive or both negative at any instant.

For an inductor or capacitor, $p = vi$ is alternately positive and negative, and the average power is zero.

For an arbitrary combination of resistors, inductors, and capacitors, the average power is positive.

KEY: Instantaneous current, i — Instantaneous voltage across device, v — Instantaneous power input to device, p —

Power in an Inductor

Next we connect the source to a pure inductor L, as in Fig. 31.8a. The voltage $v = v_L$ leads the current i by 90°. When we multiply the curves of v and i, the product vi is *negative* during the half of the cycle when v and i have *opposite* signs. The power curve, shown in Fig. 31.16b, is symmetric about the horizontal axis; it is positive half the time and negative the other half, and the average power is zero. When p is positive, energy is being supplied to set up the magnetic field in the inductor; when p is negative, the field is collapsing and the inductor is returning energy to the source. The net energy transfer over one cycle is zero.

Power in a Capacitor

Finally, we connect the source to a pure capacitor C, as in Fig. 31.9a. The voltage $v = v_C$ lags the current i by 90°. Figure 31.16c shows the power curve; the average power is again zero. Energy is supplied to charge the capacitor and is returned to the source when the capacitor discharges. The net energy transfer over one cycle is again zero.

Power in a General ac Circuit

In *any* ac circuit, with any combination of resistors, capacitors, and inductors, the voltage v across the entire circuit has some phase angle ϕ with respect to the current i. Then the instantaneous power p is given by

$$p = vi = [V\cos(\omega t + \phi)][I\cos\omega t] \qquad (31.30)$$

The instantaneous power curve has the form shown in Fig. 31.16d. The area between the positive loops and the horizontal axis is greater than the area between the negative loops and the horizontal axis, and the average power is positive.

We can derive from Eq. (31.30) an expression for the *average* power P_{av} by using the identity for the cosine of the sum of two angles:

$$\begin{aligned} p &= [V(\cos\omega t\cos\phi - \sin\omega t\sin\phi)][I\cos\omega t] \\ &= VI\cos\phi\cos^2\omega t - VI\sin\phi\cos\omega t\sin\omega t \end{aligned}$$

From the discussion in Section 31.1 that led to Eq. (31.4), the average value of $\cos^2\omega t$ (over one cycle) is $\frac{1}{2}$. Furthermore, $\cos\omega t\sin\omega t$ is equal to $\frac{1}{2}\sin 2\omega t$, whose average over a cycle is zero. So the average power P_{av} is

Phase angle of voltage with respect to current

Average power into a general ac circuit

$$P_{av} = \frac{1}{2}VI\cos\phi = V_{rms}I_{rms}\cos\phi \qquad (31.31)$$

Voltage amplitude · Current amplitude · rms voltage · rms current

Figure **31.17** Using phasors to calculate the average power for an arbitrary ac circuit.

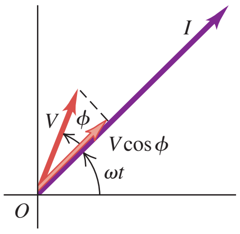

Average power $= \frac{1}{2}I(V\cos\phi)$, where $V\cos\phi$ is the component of V in phase with I.

Figure 31.17 shows the general relationship of the current and voltage phasors. When v and i are in phase, so $\phi = 0$, the average power equals $\frac{1}{2}VI = V_{rms}I_{rms}$; when v and i are 90° out of phase, the average power is zero. In the general case, when v has a phase angle ϕ with respect to i, the average power equals $\frac{1}{2}I$ multiplied by $V\cos\phi$, the component of the voltage phasor that is *in phase* with the current phasor. For the L-R-C series circuit, Figs. 31.13b and 31.13c show that $V\cos\phi$ equals the voltage amplitude V_R for the resistor; hence Eq. (31.31) is the average power dissipated in the resistor. On average there is no energy flow into or out of the inductor or capacitor, so none of P_{av} goes into either of these circuit elements.

The factor $\cos\phi$ is called the **power factor** of the circuit. For a pure resistance, $\phi = 0$, $\cos\phi = 1$, and $P_{av} = V_{rms}I_{rms}$. For a pure inductor or capacitor, $\phi = \pm 90°$, $\cos\phi = 0$, and $P_{av} = 0$. For an L-R-C series circuit the power factor is equal to R/Z; we leave the proof of this statement to you.

A low power factor (large angle ϕ of lag or lead) is usually undesirable in power circuits. The reason is that for a given potential difference, a large current is needed to supply a given amount of power. This results in large i^2R losses in the transmission lines. Many types of ac machinery draw a *lagging* current; that is, the current drawn by the machinery lags the applied voltage. Hence the voltage leads the current, so $\phi > 0$ and $\cos\phi < 1$. The power factor can be corrected toward the ideal value of 1 by connecting a capacitor in parallel with the load. The current drawn by the capacitor *leads* the voltage (that is, the voltage across the capacitor lags the current), which compensates for the lagging current in the other branch of the circuit. The capacitor itself absorbs no net power from the line.

EXAMPLE 31.6 Power in a hair dryer

WITH VARIATION PROBLEMS

An electric hair dryer is rated at 1500 W (the *average* power) at 120 V (the *rms* voltage). Calculate (a) the resistance, (b) the rms current, and (c) the maximum instantaneous power. Assume that the dryer is a pure resistor. (The heating element acts as a resistor.)

IDENTIFY and SET UP We are given $P_{\text{av}} = 1500\text{ W}$ and $V_{\text{rms}} = 120\text{ V}$. Our target variables are the resistance R, the rms current I_{rms}, and the maximum value p_{max} of the instantaneous power p. We solve Eq. (31.29) to find R, Eq. (31.28) to find I_{rms} from V_{rms} and P_{av}, and Eq. (31.30) to find p_{max}.

EXECUTE (a) From Eq. (31.29), the resistance is

$$R = \frac{{V_{\text{rms}}}^2}{P_{\text{av}}} = \frac{(120\text{ V})^2}{1500\text{ W}} = 9.6\ \Omega$$

(b) From Eq. (31.28),

$$I_{\text{rms}} = \frac{P_{\text{av}}}{V_{\text{rms}}} = \frac{1500\text{ W}}{120\text{ V}} = 12.5\text{ A}$$

(c) For a pure resistor, the voltage and current are in phase and the phase angle ϕ is zero. Hence from Eq. (31.30), the instantaneous power is $p = VI\cos^2\omega t$ and the maximum instantaneous power is $p_{\text{max}} = VI$. From Eq. (31.27), this is twice the average power P_{av}, so

$$p_{\text{max}} = VI = 2P_{\text{av}} = 2(1500\text{ W}) = 3000\text{ W}$$

EVALUATE We can use Eq. (31.7) to confirm our result in part (b): $I_{\text{rms}} = V_{\text{rms}}/R = (120\text{ V})/(9.6\ \Omega) = 12.5\text{ A}$. Note that some unscrupulous manufacturers of stereo amplifiers advertise the *peak* power output rather than the lower average value.

KEYCONCEPT The power delivered to a resistor in an ac circuit is not constant, but oscillates between zero and a maximum value. The average power delivered to the resistor is one-half of this maximum, and equals the rms current through the resistor times the rms voltage across the resistor.

EXAMPLE 31.7 Power in an *L-R-C* series circuit

WITH VARIATION PROBLEMS

For the *L-R-C* series circuit of Example 31.4, (a) calculate the power factor and (b) calculate the average power delivered to the entire circuit and to each circuit element.

IDENTIFY and SET UP We can use the results of Example 31.4. The power factor is the cosine of the phase angle ϕ, and we use Eq. (31.31) to find the average power delivered in terms of ϕ and the amplitudes of voltage and current.

EXECUTE (a) The power factor is $\cos\phi = \cos 53° = 0.60$.

(b) From Eq. (31.31),

$$P_{\text{av}} = \tfrac{1}{2}VI\cos\phi = \tfrac{1}{2}(50\text{ V})(0.10\text{ A})(0.60) = 1.5\text{ W}$$

EVALUATE Although P_{av} is the average power delivered to the *L-R-C* combination, all of this power is dissipated in the *resistor*. As Figs. 31.16b and 31.16c show, the average power delivered to a pure inductor or pure capacitor is always zero.

KEYCONCEPT The power factor of an ac circuit equals the cosine of the phase angle ϕ between the oscillating voltage across the circuit and the oscillating current in the circuit. The greater the power factor, the less current is required to supply a given average power for a given voltage amplitude.

TEST YOUR UNDERSTANDING OF SECTION 31.4 Figure 31.16d shows that during part of a cycle of oscillation, the instantaneous power delivered to the circuit is negative. This means that energy is being extracted from the circuit. (a) Where is the energy extracted from? (i) The resistor; (ii) the inductor; (iii) the capacitor; (iv) the ac source; (v) more than one of these. (b) Where does the energy go? (i) The resistor; (ii) the inductor; (iii) the capacitor; (iv) the ac source; (v) more than one of these.

ANSWER

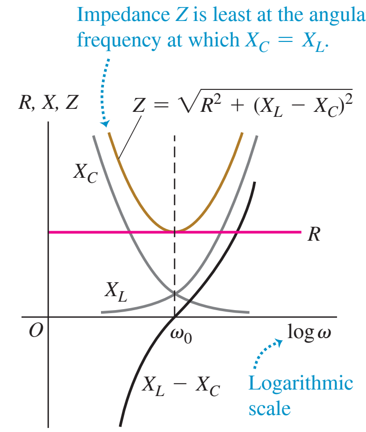
(a) **(v)**; (b) **(iv)** The energy cannot be extracted from the resistor, since energy is dissipated in a resistor and cannot be recovered. Instead, the energy must be extracted from either the inductor (which stores magnetic-field energy) or the capacitor (which stores electric-field energy). Positive power means that energy is being transferred from the ac source to the circuit, so *negative* power implies that energy is being transferred back into the source.

31.5 RESONANCE IN ALTERNATING-CURRENT CIRCUITS

Much of the practical importance of *L-R-C* series circuits arises from the way in which such circuits respond to sources of different angular frequency ω. For example, one type of tuning circuit used in radio receivers is simply an *L-R-C* series circuit. A radio signal of any given frequency produces a current of the same frequency in the receiver circuit, but the amplitude of the current is *greatest* if the signal frequency equals the particular frequency to which the receiver circuit is "tuned." This effect is called *resonance*. The circuit is designed so that signals at other than the tuned frequency produce currents that are too small to make an audible sound come out of the radio's speakers.

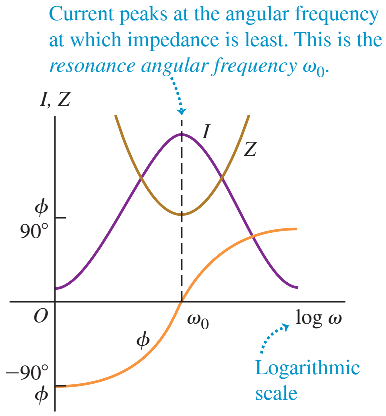
Figure **31.18** How variations in the angular frequency of an ac circuit affect (a) reactance, resistance, and impedance, and (b) impedance, current amplitude, and phase angle.

(a) Reactance, resistance, and impedance as functions of angular frequency



Impedance Z is least at the angular frequency at which $X_C = X_L$.

(b) Impedance, current, and phase angle as functions of angular frequency



Current peaks at the angular frequency at which impedance is least. This is the *resonance angular frequency* ω_0.

To see how an *L-R-C* series circuit can be used in this way, suppose we connect an ac source with constant voltage amplitude V but adjustable angular frequency ω across an *L-R-C* series circuit. The current that appears in the circuit has the same angular frequency as the source and a current amplitude $I = V/Z$, where Z is the impedance of the *L-R-C* series circuit. This impedance depends on the frequency, as Eq. (31.23) shows. **Figure 31.18a** shows graphs of R, X_L, X_C, and Z as functions of ω. We have used a logarithmic angular frequency scale so that we can cover a wide range of frequencies. As the frequency increases, X_L increases and X_C decreases; hence there is always one frequency at which X_L and X_C are equal and $X_L - X_C$ is zero. At this frequency the impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$ has its *smallest* value, equal simply to the resistance R.

Circuit Behavior at Resonance

As we vary the angular frequency ω of the source, the current amplitude $I = V/Z$ varies as shown in Fig. 31.18b; the *maximum* value of I occurs at the frequency at which the impedance Z is *minimum*. This peaking of the current amplitude at a certain frequency is called **resonance.** The angular frequency ω_0 at which the resonance peak occurs is called the **resonance angular frequency.** At $\omega = \omega_0$ the inductive reactance X_L and capacitive reactance X_C are equal, so $\omega_0 L = 1/\omega_0 C$ and

Resonance angular frequency of an *L-R-C* series circuit

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{31.32}$$

Inductance

Capacitance

This is equal to the natural angular frequency of oscillation of an *L-C* circuit, which we derived in Section 30.5, Eq. (30.22). The **resonance frequency** f_0 is $\omega_0/2\pi$. At this frequency, the greatest current appears in the circuit for a given source voltage amplitude; f_0 is the frequency to which the circuit is "tuned."

It's instructive to look at what happens to the *voltages* in an *L-R-C* series circuit at resonance. The current at any instant is the same in L and C. The voltage across an inductor always *leads* the current by 90°, or $\frac{1}{4}$ cycle, and the voltage across a capacitor always *lags* the current by 90°. Therefore the instantaneous voltages across L and C always differ in phase by 180°, or $\frac{1}{2}$ cycle; they have opposite signs at each instant. At the resonance frequency, and *only* at the resonance frequency, $X_L = X_C$ and the voltage amplitudes $V_L = IX_L$ and $V_C = IX_C$ are *equal;* then the instantaneous voltages across L and C add to zero at each instant, and the *total* voltage v_{bd} across the *L-C* combination in Fig. 31.13a is exactly zero. The voltage across the resistor is then equal to the source voltage. So at the resonance frequency the circuit behaves as if the inductor and capacitor weren't there at all!

The *phase* of the voltage relative to the current is given by Eq. (31.24). At frequencies below resonance, X_C is greater than X_L; the capacitive reactance dominates, the voltage *lags* the current, and the phase angle ϕ is between 0° and −90°. Above resonance, the inductive reactance dominates, the voltage *leads* the current, and the phase angle ϕ is between 0° and +90°. Figure 31.18b shows this variation of ϕ with angular frequency.

Tailoring an ac Circuit

If we can vary the inductance L or the capacitance C of a circuit, we can also vary the resonance frequency. This is exactly how a radio is "tuned" to receive a particular station. In the early days of radio this was accomplished by the use of capacitors with movable metal plates whose overlap could be varied to change C. (This is what is being done with the radio tuning knob shown in the photograph that opens this chapter.) Another approach is to vary L by using a coil with a ferrite core that slides in or out.

In an L-R-C series circuit the impedance is a minimum and the current is a maximum at the resonance frequency. The middle curve in **Fig. 31.19** is a graph of current as a function of frequency for such a circuit, with source voltage amplitude $V = 100\ \text{V}$, $L = 2.0\ \text{H}$, $C = 0.50\ \mu\text{F}$, and $R = 500\ \Omega$. This curve, called a *response curve* or *resonance curve,* has a peak at the resonance angular frequency $\omega_0 = \sqrt{LC} = 1000\ \text{rad/s}$.

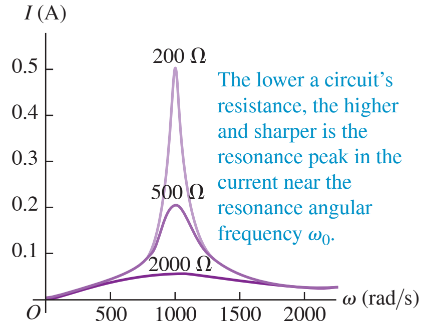
The resonance frequency is determined by L and C; what happens when we change R? Figure 31.19 also shows graphs of I as a function of ω for $R = 200\ \Omega$ and for $R = 2000\ \Omega$. The curves are similar for frequencies far away from resonance, where the impedance is dominated by X_L or X_C. But near resonance, where X_L and X_C nearly cancel each other, the curve is higher and more sharply peaked for small values of R and broader and flatter for large values of R. At resonance, $Z = R$ and $I = V/R$, so the maximum height of the curve is inversely proportional to R.

The shape of the response curve is important in the design of radio receiving circuits. The sharply peaked curve is what makes it possible to discriminate between two stations broadcasting on adjacent frequency bands. But if the peak is *too* sharp, some of the information in the received signal is lost, such as the high-frequency sounds in music. The shape of the resonance curve is also related to the overdamped and underdamped oscillations that we described in Section 30.6. A sharply peaked resonance curve corresponds to a small value of R and a lightly damped oscillating system; a broad, flat curve goes with a large value of R and a heavily damped system.

In this section we have discussed resonance in an L-R-C *series* circuit. Resonance can also occur in an ac circuit in which the inductor, resistor, and capacitor are connected in *parallel*. We leave the details to you (see Problems 31.54 and 31.55).

Resonance phenomena occur not just in ac circuits, but in all areas of physics. We discussed examples of resonance in *mechanical* systems in Sections 14.8 and 16.5. The amplitude of a mechanical oscillation peaks when the driving-force frequency is close to a natural frequency of the system; this is analogous to the peaking of the current in an L-R-C series circuit.

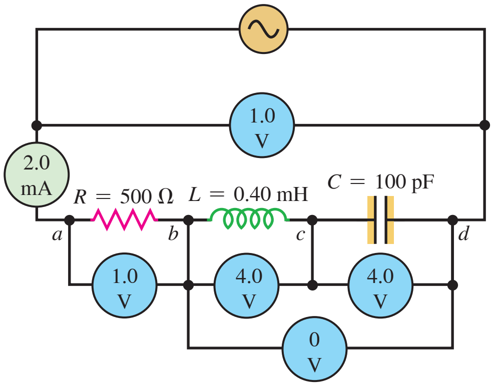
Figure **31.19** Graph of current amplitude I as a function of angular frequency ω for an L-R-C series circuit with $V = 100\ \text{V}$, $L = 2.0\ \text{H}$, $C = 0.50\ \mu\text{F}$, and three different values of the resistance R.



EXAMPLE 31.8 Tuning a radio

The series circuit in **Fig. 31.20** is similar to some radio tuning circuits. It is connected to a variable-frequency ac source with an rms terminal voltage of 1.0 V. (a) Find the resonance frequency. At the resonance frequency, find (b) the inductive reactance X_L, the capacitive reactance X_C, and the impedance Z; (c) the rms current I_{rms}; (d) the rms voltage across each circuit element.

Figure **31.20** A radio tuning circuit at resonance. The circles denote rms current and voltages.



IDENTIFY and SET UP Figure 31.20 shows an L-R-C series circuit, with ideal meters inserted to measure the rms current and voltages, our target variables. Equation (31.32) gives the formula for the resonance angular frequency ω_0, from which we find the resonance frequency f_0. We use Eqs. (31.12) and (31.18) to find X_L and X_C, which are equal at resonance; at resonance, from Eq. (31.23), we have $Z = R$. We use Eqs. (31.7), (31.13), and (31.19) to find the voltages across the circuit elements.

EXECUTE (a) The values of ω_0 and f_0 are

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.40 \times 10^{-3}\ \text{H})(100 \times 10^{-12}\ \text{F})}}\\ &= 5.0 \times 10^6\ \text{rad/s}\\ f_0 &= 8.0 \times 10^5\ \text{Hz} = 800\ \text{kHz}\end{aligned}$$

This frequency is in the lower part of the AM radio band.

(b) At this frequency,

$$X_L = \omega L = (5.0 \times 10^6\ \text{rad/s})(0.40 \times 10^{-3}\ \text{H}) = 2000\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(5.0 \times 10^6\ \text{rad/s})(100 \times 10^{-12}\ \text{F})} = 2000\ \Omega$$

Since $X_L = X_C$ at resonance as stated above, $Z = R = 500\ \Omega$.

Continued

(c) From Eq. (31.26) the rms current at resonance is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} = \frac{1.0\ \text{V}}{500\ \Omega} = 0.0020\ \text{A} = 2.0\ \text{mA}$$

(d) The rms potential difference across the resistor is

$$V_{R\text{-rms}} = I_{\text{rms}}R = (0.0020\ \text{A})(500\ \Omega) = 1.0\ \text{V}$$

The rms potential differences across the inductor and capacitor are

$$V_{L\text{-rms}} = I_{\text{rms}}X_L = (0.0020\ \text{A})(2000\ \Omega) = 4.0\ \text{V}$$

$$V_{C\text{-rms}} = I_{\text{rms}}X_C = (0.0020\ \text{A})(2000\ \Omega) = 4.0\ \text{V}$$

EVALUATE The potential differences across the inductor and the capacitor have equal rms values and amplitudes, but are 180° out of phase and so add to zero at each instant. Note also that at resonance, $V_{R\text{-rms}}$ is equal to the source voltage V_{rms}, while in this example, $V_{L\text{-rms}}$ and $V_{C\text{-rms}}$ are both considerably *larger* than V_{rms}.

KEYCONCEPT In an ac circuit, the current amplitude is maximum for a given source voltage amplitude when the source frequency equals the circuit's resonance frequency. For a series *L-R-C* circuit the resonance frequency is the same as the circuit's natural oscillation frequency without the resistor or the source.

TEST YOUR UNDERSTANDING OF SECTION 31.5 How does the resonance frequency of an *L-R-C* series circuit change if the plates of the capacitor are brought closer together? (i) It increases; (ii) it decreases; (iii) it is unaffected.

ANSWER

❙ **(ii)** The capacitance C increases if the plate spacing is decreased (see Section 24.1). Hence the resonance frequency $f_0 = \omega_0/2\pi = 1/2\pi\sqrt{LC}$ decreases.

31.6 TRANSFORMERS

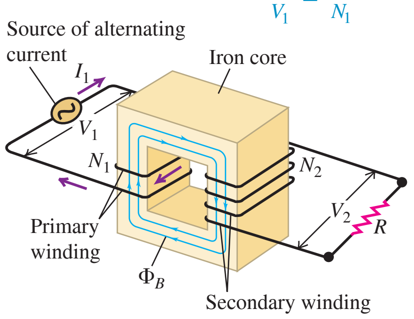
One advantage of ac over dc for electric-power distribution is that it is much easier to step voltage levels up and down with ac than with dc. For long-distance power transmission it is desirable to use as high a voltage and as small a current as possible; this reduces i^2R losses in the transmission lines, and smaller wires can be used, saving on material costs. Present-day transmission lines operate at rms voltages of the order of 500 kV. However, safety considerations dictate relatively low voltages in generating equipment and in household and industrial power distribution. The standard voltage for household wiring is 120 V in North and Central America, and 220 V to 240 V in most of the rest of the world. The necessary voltage conversion is accomplished by using **transformers.**

BIO APPLICATION Dangers of ac Versus dc Voltages Alternating current at high voltage (above 500 V) is more dangerous than direct current at the same voltage. When a person touches a high-voltage dc source, it usually causes a single muscle contraction that can be strong enough to push the person away from the source. By contrast, touching a high-voltage ac source can cause a continuing muscle contraction that prevents the victim from letting go of the source. Lowering the ac voltage with a transformer reduces the risk of injury.

How Transformers Work

Figure 31.21 shows an idealized transformer. Its key components are two coils or *windings*, electrically insulated from each other but wound on the same core. The core is typically made of a material, such as iron, with a very large relative permeability K_m. This keeps the magnetic field lines due to a current in one winding almost completely within the core. Hence almost all of these field lines pass through the other winding, maximizing the *mutual inductance* of the two windings (see Section 30.1). The winding to which power is supplied is called the **primary;** the winding from which power is delivered is called the **secondary.** The circuit symbol for a transformer with an iron core is

Figure 31.21 Schematic diagram of an idealized step-up transformer. The primary is connected to an ac source; the secondary is connected to a device with resistance R.



The induced emf *per turn* is the same in both coils, so we adjust the ratio of terminal voltages by adjusting the ratio of turns: $\frac{V_2}{V_1} = \frac{N_2}{N_1}$

Here’s how a transformer works. The ac source causes an alternating current in the primary, which sets up an alternating flux in the core; this induces an emf in each winding, in accordance with Faraday’s law. The induced emf in the secondary gives rise to an alternating current in the secondary, and this delivers energy to the device to which the secondary is connected. All currents and emfs have the same frequency as the ac source.

Let’s see how the voltage across the secondary can be made larger or smaller in amplitude than the voltage across the primary. We ignore the resistance of the windings and assume that all the magnetic field lines are confined to the iron core, so at any instant the magnetic flux Φ_B is the same in each turn of the primary and secondary windings. The primary winding has N_1 turns and the secondary winding has N_2 turns. When the magnetic flux changes because of changing currents in the two coils, the resulting induced emfs are

$$\mathcal{E}_1 = -N_1\frac{d\Phi_B}{dt} \quad \text{and} \quad \mathcal{E}_2 = -N_2\frac{d\Phi_B}{dt} \tag{31.33}$$

The flux *per turn* Φ_B is the same in both the primary and the secondary, so Eqs. (31.33) show that the induced emf *per turn* is the same in each. The ratio of the secondary emf $\mathcal{E}_2$ to the primary emf $\mathcal{E}_1$ is therefore equal at any instant to the ratio of secondary to primary turns:

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \tag{31.34}$$

Since $\mathcal{E}_1$ and $\mathcal{E}_2$ both oscillate with the same frequency as the ac source, Eq. (31.34) also gives the ratio of the amplitudes or of the rms values of the induced emfs. If the windings have zero resistance, the induced emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ are equal to the terminal voltages across the primary and the secondary, respectively; hence

Terminal voltages in a transformer:

Secondary voltage amplitude or rms value

Number of turns in secondary

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \tag{31.35}$$

Primary voltage amplitude or rms value

Number of turns in primary

By choosing the appropriate turns ratio N_2/N_1, we may obtain any desired secondary voltage from a given primary voltage. If $N_2 > N_1$, as in Fig. 31.21, then $V_2 > V_1$ and we have a *step-up* transformer; if $N_2 < N_1$, then $V_2 < V_1$ and we have a *step-down* transformer. At a power-generating station, step-up transformers are used; the primary is connected to the power source and the secondary is connected to the transmission lines, giving the desired high voltage for transmission. Near the consumer, step-down transformers lower the voltage to a value suitable for use in home or industry (**Fig. 31.22**).

Even the relatively low voltage provided by a household wall socket is too high for many electronic devices, so a further step-down transformer is necessary. This is the role of an “ac adapter” such as those used to recharge a mobile phone or laptop computer from line voltage (**Fig. 31.23**).

Energy Considerations for Transformers

If the secondary circuit is completed by a resistance R, then the amplitude or rms value of the current in the secondary circuit is $I_2 = V_2/R$. From energy considerations, the power delivered to the primary equals that taken out of the secondary (since there is no resistance in the windings), so

Terminal voltages and currents in a transformer:

Primary voltage amplitude or rms value

Secondary voltage amplitude or rms value

$$V_1I_1 = V_2I_2 \tag{31.36}$$

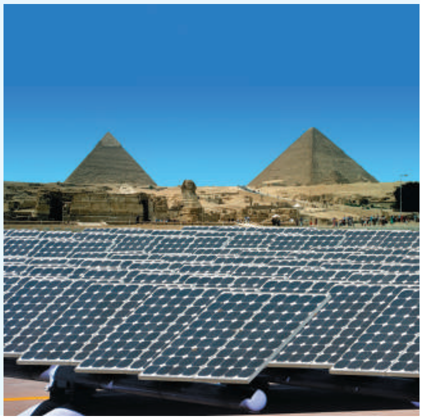
Current in primary

Current in secondary

Figure **31.22** The cylindrical can near the top of this power pole is a step-down transformer. It converts the high-voltage ac in the power lines to low-voltage (120 V) ac, which is then distributed to the surrounding homes and businesses.

Figure **31.23** An ac adapter like this one converts household ac into low-voltage dc for use in electronic devices. It contains a step-down transformer to change the line voltage to a lower value, typically 3 to 12 V, as well as diodes to convert alternating current to the direct current that small electronic devices require (see Fig. 31.3).

APPLICATION When dc Power Transmission Is Better Than ac In the cables used for transmitting ac power, eddy currents oppose and cancel the current near the central axis of the cable. Hence current flows only in a reduced area near the cable's surface, increasing the cable's effective resistance and causing the loss of more power. For this and other reasons, high-voltage dc can be more efficient than high-voltage ac for transmitting power over long distances, such as in a planned application to take solar power generated in North Africa to Europe through dc lines at the bottom of the Mediterranean.



We can combine Eqs. (31.35) and (31.36) and the relationship $I_2 = V_2/R$ to eliminate V_2 and I_2; we obtain

$$\frac{V_1}{I_1} = \frac{R}{(N_2/N_1)^2} \tag{31.37}$$

This shows that when the secondary circuit is completed through a resistance R, the result is the same as if the *source* had been connected directly to a resistance equal to R divided by the square of the turns ratio, $(N_2/N_1)^2$. In other words, the transformer "transforms" not only voltages and currents, but resistances as well. More generally, we can regard a transformer as "transforming" the *impedance* of the network to which the secondary circuit is completed.

Equation (31.37) has many practical consequences. The power supplied by a source to a resistor depends on the resistances of both the resistor and the source. It can be shown that the power transfer is greatest when the two resistances are *equal*. The same principle applies in both dc and ac circuits. When a high-impedance ac source must be connected to a low-impedance circuit, such as an audio amplifier connected to a loudspeaker, the source impedance can be *matched* to that of the circuit by the use of a transformer with an appropriate turns ratio N_2/N_1.

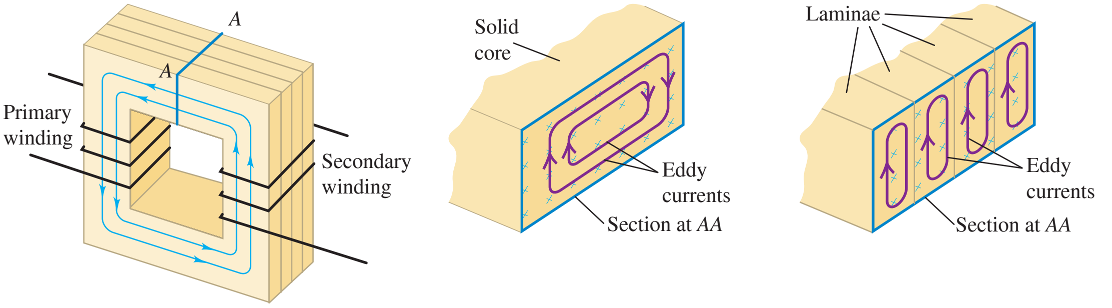
Real transformers always have some energy losses. (That's why an ac adapter like the one shown in Fig. 31.23 feels warm to the touch after it's been in use for a while; the transformer is heated by the dissipated energy.) The windings have some resistance, leading to i^2R losses. There are also energy losses through hysteresis in the core (see Section 28.8). Hysteresis losses are minimized by the use of soft iron with a narrow hysteresis loop.

Eddy currents (Section 29.6) also cause energy loss in transformers. Consider a section AA through an iron transformer core (**Fig. 31.24a**). Since iron is a conductor, any such section can be pictured as several conducting circuits, one within the other (Fig. 31.24b). The flux through each of these circuits is continually changing, so eddy currents circulate in the entire volume of the core, with lines of flow that form planes perpendicular to the flux. These eddy currents waste energy through i^2R heating and themselves set up an opposing flux.

The effects of eddy currents can be minimized by the use of a *laminated* core—that is, one built up of thin sheets, or laminae. The large electrical surface resistance of each lamina, due either to a natural coating of oxide or to an insulating varnish, effectively confines the eddy currents to individual laminae (Fig. 31.24c). The possible eddy-current paths are narrower, the induced emf in each path is smaller, and the eddy currents are greatly reduced. The alternating magnetic field exerts forces on the current-carrying laminae that cause them to vibrate back and forth; this vibration causes the characteristic "hum" of an operating transformer. You can hear this same "hum" from the magnetic ballast of a fluorescent light fixture (see Section 30.2).

Thanks to the use of soft iron cores and lamination, transformer efficiencies are usually well over 90%; in large installations they may reach 99%.

Figure 31.24 (a) Primary and secondary windings in a transformer. (b) Eddy currents in the iron core, shown in the cross section at AA. (c) Using a laminated core reduces the eddy currents.



(a) Schematic transformer

(b) Large eddy currents in solid core

(c) Smaller eddy currents in laminated core

EXAMPLE 31.9 “Wake up and smell the (transformer)!”

A friend returns to the United States from Europe with a 960 W coffeemaker, designed to operate from a 240 V line. (a) What can she do to operate it at the USA-standard 120 V? (b) What current will the coffeemaker draw from the 120 V line? (c) What is the resistance of the coffeemaker? (The voltages are rms values.)

IDENTIFY and SET UP Our friend needs a step-up transformer to convert 120 V ac to the 240 V ac that the coffeemaker requires. We use Eq. (31.35) to determine the transformer turns ratio N_2/N_1, $P_{\mathrm{av}} = V_{\mathrm{rms}}I_{\mathrm{rms}}$ for a resistor to find the current draw, and Eq. (31.37) to calculate the resistance.

EXECUTE (a) To get $V_2 = 240\ \mathrm{V}$ from $V_1 = 120\ \mathrm{V}$, the required turns ratio is $N_2/N_1 = V_2/V_1 = (240\ \mathrm{V})/(120\ \mathrm{V}) = 2$. That is, the secondary coil (connected to the coffeemaker) should have twice as many turns as the primary coil (connected to the 120 V line).

(b) We find the rms current I_1 in the 120 V primary by using $P_{\mathrm{av}} = V_1 I_1$, where P_{av} is the average power drawn by the coffeemaker and hence the power supplied by the 120 V line. (We’re assuming that no energy is lost in the transformer.) Hence $I_1 = P_{\mathrm{av}}/V_1 = (960\ \mathrm{W})/(120\ \mathrm{V}) = 8.0\ \mathrm{A}$. The secondary current is then $I_2 = P_{\mathrm{av}}/V_2 = (960\ \mathrm{W})/(240\ \mathrm{V}) = 4.0\ \mathrm{A}$.

(c) We have $V_1 = 120\ \mathrm{V}$, $I_1 = 8.0\ \mathrm{A}$, and $N_2/N_1 = 2$, so

$$\frac{V_1}{I_1} = \frac{120\ \mathrm{V}}{8.0\ \mathrm{A}} = 15\ \Omega$$

From Eq. (31.37),

$$R = 2^2(15\ \Omega) = 60\ \Omega$$

EVALUATE As a check, $V_2/R = (240\ \mathrm{V})/(60\ \Omega) = 4.0\ \mathrm{A} = I_2$, the same value obtained previously. You can also check this result for R by using the expression $P_{\mathrm{av}} = V_2^2/R$ for the power drawn by the coffeemaker.

KEYCONCEPT A transformer can be used to raise or lower the voltage amplitude of an alternating current. If there are more turns in the secondary coil than in the primary coil (step-up transformer), the transformer increases the voltage amplitude and decreases the current amplitude; the reverse is true if there are fewer turns in the secondary than in the primary (step-down transformer).

TEST YOUR UNDERSTANDING OF SECTION 31.6 Each of the following four transformers has 1000 turns in its primary. Rank the transformers from largest to smallest number of turns in the secondary. (i) Converts 120 V ac into 6.0 V ac; (ii) converts 120 V ac into 240 V ac; (iii) converts 240 V ac into 6.0 V ac; (iv) converts 240 V ac into 120 V ac.

ANSWER

(ii), (iv), (i), (iii) From Eq. (31.35) the turns ratio is $N_2/N_1 = V_2/V_1$, so the number of turns in the secondary is $N_2 = N_1V_2/V_1$. Hence for the four cases we have (i) $N_2 = (1000)(6.0\ \mathrm{V})/(120\ \mathrm{V}) = 50$ turns; (ii) $N_2 = (1000)(240\ \mathrm{V})/(120\ \mathrm{V}) = 2000$ turns; (iii) $N_2 = (1000)(6.0\ \mathrm{V})/(240\ \mathrm{V}) = 25$ turns; and (iv) $N_2 = (1000)(120\ \mathrm{V})/(240\ \mathrm{V}) = 500$ turns. Note that (i), (iii), and (iv) are step-down transformers with fewer turns in the secondary than in the primary, while (ii) is a step-up transformer with more turns in the secondary than in the primary.

CHAPTER 31 SUMMARY

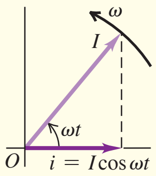
Phasors and alternating current: An alternator or ac source produces an emf that varies sinusoidally with time. A sinusoidal voltage or current can be represented by a phasor, a vector that rotates counterclockwise with constant angular velocity ω equal to the angular frequency of the sinusoidal quantity. Its projection on the horizontal axis at any instant represents the instantaneous value of the quantity.

For a sinusoidal current, the rectified average and rms (root-mean-square) currents are proportional to the current amplitude I. Similarly, the rms value of a sinusoidal voltage is proportional to the voltage amplitude V. (See Example 31.1.)

$$I_{\text{rav}} = \frac{2}{\pi}I = 0.637I \qquad (31.3)$$

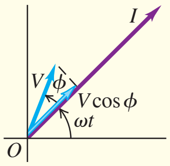
$$I_{\text{rms}} = \frac{I}{\sqrt{2}} \qquad (31.4)$$

$$V_{\text{rms}} = \frac{V}{\sqrt{2}} \qquad (31.5)$$



Voltage, current, and phase angle: In general, the instantaneous voltage $v = V\cos(\omega t + \phi)$ between two points in an ac circuit is not in phase with the instantaneous current passing through those points. The quantity ϕ is called the phase angle of the voltage relative to the current.

$$i = I\cos\omega t \qquad (31.2)$$

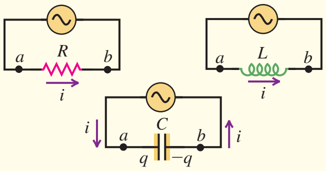


Resistance and reactance: The voltage across a resistor R is in phase with the current. The voltage across an inductor L leads the current by 90° ($\phi = +90°$), while the voltage across a capacitor C lags the current by 90° ($\phi = -90°$). The voltage amplitude across each type of device is proportional to the current amplitude I. An inductor has inductive reactance $X_L = \omega L$, and a capacitor has capacitive reactance $X_C = 1/\omega C$. (See Examples 31.2 and 31.3.)

$$V_R = IR \qquad (31.7)$$

$$V_L = IX_L \qquad (31.13)$$

$$V_C = IX_C \qquad (31.19)$$

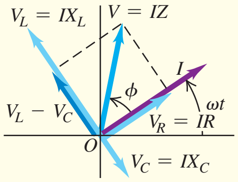


Impedance and the *L-R-C* series circuit: In a general ac circuit, the voltage and current amplitudes are related by the circuit impedance Z. In an L-R-C series circuit, the values of L, R, C, and the angular frequency ω determine the impedance and the phase angle ϕ of the voltage relative to the current. (See Examples 31.4 and 31.5.)

$$V = IZ \qquad (31.22)$$

$$Z = \sqrt{R^2 + [\omega L - (1/\omega C)]^2} \qquad (31.23)$$

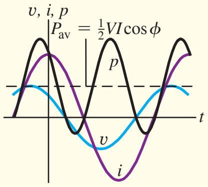
$$\tan\phi = \frac{\omega L - 1/\omega C}{R} \qquad (31.24)$$



Power in ac circuits: The average power input P_{av} to an ac circuit depends on the voltage and current amplitudes (or, equivalently, their rms values) and the phase angle ϕ of the voltage relative to the current. The quantity $\cos\phi$ is called the power factor. (See Examples 31.6 and 31.7.)

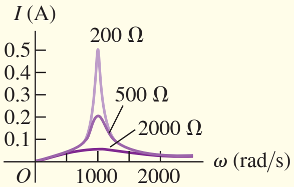
$$P_{\text{av}} = \tfrac{1}{2}VI\cos\phi \qquad (31.31)$$

$$= V_{\text{rms}}I_{\text{rms}}\cos\phi$$



Resonance in ac circuits: In an L-R-C series circuit, the current becomes maximum and the impedance becomes minimum at an angular frequency called the resonance angular frequency. This phenomenon is called resonance. At resonance the voltage and current are in phase, and the impedance Z is equal to the resistance R. (See Example 31.8.)

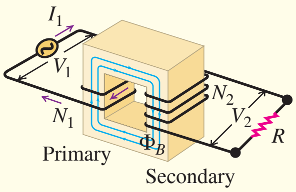
$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (31.32)$$



Transformers: A transformer is used to transform the voltage and current levels in an ac circuit. In an ideal transformer with no energy losses, if the primary winding has N_1 turns and the secondary winding has N_2 turns, the amplitudes (or rms values) of the two voltages are related by Eq. (31.35). The amplitudes (or rms values) of the primary and secondary voltages and currents are related by Eq. (31.36). (See Example 31.9.)

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad (31.35)$$

$$V_1I_1 = V_2I_2 \qquad (31.36)$$



Chapter 31 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLES 31.2 and 31.3 (Section 31.2) before attempting these problems.

VP31.3.1 An inductor with $L = 2.50$ mH is part of the circuit for an aviation radio beacon that operates at 108 MHz. Find (a) the inductive reactance of the inductor and (b) the amplitude of the current in the inductor if the amplitude of the voltage across the inductor is 4.20 kV.

VP31.3.2 An ac circuit includes a resistor with $R = 125\ \Omega$ connected in series with a capacitor with $C = 7.00\ \mu\text{F}$. The current through the resistor and capacitor is $i = (2.40 \times 10^{-3}\ \text{A})\cos[(1.75 \times 10^{3}\ \text{rad/s})t]$. Find (a) the voltage across the resistor as a function of time, (b) the capacitive reactance of the capacitor, and (c) the voltage across the capacitor as a function of time.

VP31.3.3 An ac circuit includes a 155 Ω resistor in series with a 8.00 μF capacitor. The current in the circuit has amplitude 4.00×10^{-3} A. (a) Find the frequency for which the capacitive reactance equals the resistance. (b) At this frequency, what are the amplitudes of the voltages across the resistor and capacitor? (c) At this frequency, when the voltage across the resistor is maximum, what is the voltage across the capacitor? When the voltage across the capacitor is maximum, what is the voltage across the resistor?

VP31.3.4 A 116 Ω resistor and an unknown capacitor are connected in series to an ac source with angular frequency 5.10×10^{3} rad/s. The amplitude of the resistor voltage is 2.45 V, and the current in the circuit has its maximum positive value at $t = 0$. Find (a) the current amplitude, (b) the current at $t = 3.50$ ms, (c) the maximum voltage across the capacitor, and (d) the capacitance.

Be sure to review EXAMPLES 31.4 and 31.5 (Section 31.3) before attempting these problems.

VP31.5.1 In the series circuit of Fig. 31.13a the resistance is 255 Ω, the inductance is 45.0 mH, the capacitance is 0.400 μF, and the source has voltage amplitude 55.0 V. The inductive reactance is equal to the resistance. Find (a) the frequency of the source, (b) the impedance, (c) the current amplitude, (d) the phase angle, and (e) whether the voltage leads or lags the current.

VP31.5.2 In the series circuit of Fig. 31.13a the source has voltage amplitude 20.0 V and angular frequency 5.40×10^{3} rad/s, the inductance is 6.50 mH, the capacitance is 0.600 μF, and the impedance is 474 Ω. Find (a) the resistance, (b) the current amplitude, (c) the resistor voltage amplitude, (d) the inductor voltage amplitude, and (e) the capacitor voltage amplitude.

VP31.5.3 The source in an *L-R-C* series circuit has angular frequency 1.20×10^{4} rad/s. The capacitance is 0.965 μF and the resistance is 65.0 Ω. If you want the source voltage to lead the current by 15.0°, find (a) the required value of $X_L - X_C$ and (b) the required inductance.

VP31.5.4 The current in an *L-R-C* series circuit has amplitude 0.120 A and angular frequency 8.00×10^{3} rad/s, and it has its maximum positive value at $t = 0$. The resistance is 95.0 Ω, the inductance is 6.50 mH, and the capacitance is 0.440 μF. For the resistor, inductor, and capacitor, find (a) the voltage amplitudes and (b) the instantaneous voltages at $t = 0.305$ ms.

Be sure to review EXAMPLES 31.6 and 31.7 (Section 31.4) before attempting these problems.

VP31.7.1 In operation an electric toaster has an rms current of 3.95 A when plugged into a wall socket with rms voltage 1.20×10^{2} V. Find the toaster's (a) average power, (b) maximum instantaneous power, and (c) resistance.

VP31.7.2 An *L-R-C* series circuit has a source with voltage amplitude 35.0 V and angular frequency 1.30×10^{3} rad/s. The resistance is 275 Ω, the inductance is 82.3 mH, and the capacitance is 1.10 μF. Find (a) the inductive and capacitive reactances, (b) the phase angle, and (c) the power factor.

VP31.7.3 For the circuit in the preceding problem, find (a) the impedance, (b) the current amplitude, and (c) the average power delivered to the resistor.

VP31.7.4 The power factor of an *L-R-C* series circuit is 0.800. The angular frequency is 1.30×10^{4} rad/s, the inductance is 7.50 mH, and the capacitance is 0.440 μF. Find (a) the inductive and capacitive reactances, (b) the phase angle, and (c) the resistance.

BRIDGING PROBLEM An Alternating-Current Circuit

A series circuit like the circuit in Fig. 31.13a consists of a 1.50 mH inductor, a 125 Ω resistor, and a 25.0 nF capacitor connected across an ac source having an rms voltage of 35.0 V and variable frequency. (a) At what angular frequencies will the current amplitude be equal to $\frac{1}{3}$ of its maximum possible value? (b) At the frequencies in part (a), what are the current amplitude and the voltage amplitude across each circuit element (including the ac source)?

SOLUTION GUIDE

IDENTIFY and SET UP

1. The maximum current amplitude occurs at the resonance angular frequency. This problem concerns the angular frequencies at which the current amplitude is one-third of that maximum.
2. Choose the equation that will allow you to find the angular frequencies in question, and choose the equations that you'll then use to find the current and voltage amplitudes at each angular frequency.

EXECUTE

3. Find the impedance at the angular frequencies in part (a); then solve for the values of angular frequency.
4. Find the voltage amplitude across the source and the current amplitude for each of the angular frequencies in part (a). (*Hint:* Be careful to distinguish between *amplitude* and *rms value.*)
5. Use the results of steps 3 and 4 to find the reactances at each angular frequency. Then calculate the voltage amplitudes for the resistor, inductor, and capacitor.

EVALUATE

6. Are any voltage amplitudes greater than the voltage amplitude of the source? If so, does this mean your results are in error?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q31.1 Household electric power in most of western Europe is supplied at 240 V, rather than the 120 V that is standard in the United States and Canada. What are the advantages and disadvantages of each system?

Q31.2 The current in an ac power line changes direction 120 times per second, and its average value is zero. Explain how it is possible for power to be transmitted in such a system.

Q31.3 In an ac circuit, why is the average power for an inductor and a capacitor zero, but not for a resistor?

Q31.4 Equation (31.14) was derived by using the relationship $i = dq/dt$ between the current and the charge on the capacitor. In Fig. 31.9a the positive counterclockwise current increases the charge on the capacitor. When the charge on the left plate is positive but decreasing in time, is $i = dq/dt$ still correct or should it be $i = -dq/dt$? Is $i = dq/dt$ still correct when the right-hand plate has positive charge that is increasing or decreasing in magnitude? Explain.

Q31.5 Fluorescent lights often use an inductor, called a ballast, to limit the current through the tubes. Why is it better to use an inductor rather than a resistor for this purpose?

Q31.6 Equation (31.9) says that $v_{ab} = L\,di/dt$ (see Fig. 31.8a). Using Faraday's law, explain why point a is at higher potential than point b when i is in the direction shown in Fig. 31.8a and is increasing in magnitude. When i is counterclockwise and decreasing in magnitude, is $v_{ab} = L\,di/dt$ still correct, or should it be $v_{ab} = -L\,di/dt$? Is $v_{ab} = L\,di/dt$ still correct when i is clockwise and increasing or decreasing in magnitude? Explain.

Q31.7 Is it possible for the power factor of an *L-R-C* series ac circuit to be zero? Justify your answer on *physical* grounds.

Q31.8 In an *L-R-C* series circuit, can the instantaneous voltage across the capacitor exceed the source voltage at that same instant? Can this be true for the instantaneous voltage across the inductor? Across the resistor? Explain.

Q31.9 In an *L-R-C* series circuit, what are the phase angle ϕ and power factor $\cos\phi$ when the resistance is much smaller than the inductive or capacitive reactance and the circuit is operated far from resonance? Explain.

Q31.10 When an *L-R-C* series circuit is connected across a 120 V ac line, the voltage rating of the capacitor may be exceeded even if it is rated at 200 or 400 V. How can this be?

Q31.11 In Example 31.6 (Section 31.4), a hair dryer is treated as a pure resistor. But because there are coils in the heating element and in the motor that drives the blower fan, a hair dryer also has inductance. Qualitatively, does including an inductance increase or decrease the values of R, I_{rms}, and P?

Q31.12 A light bulb and a parallel-plate capacitor with air between the plates are connected in series to an ac source. What happens to the brightness of the bulb when a dielectric is inserted between the plates of the capacitor? Explain.

Q31.13 A coil of wire wrapped on a hollow tube and a light bulb are connected in series to an ac source. What happens to the brightness of the bulb when an iron rod is inserted in the tube?

Q31.14 A circuit consists of a light bulb, a capacitor, and an inductor connected in series to an ac source. What happens to the brightness of the bulb when the inductor is omitted? When the inductor is left in the circuit but the capacitor is omitted? Explain.

Q31.15 A circuit consists of a light bulb, a capacitor, and an inductor connected in series to an ac source. Is it possible for both the capacitor and the inductor to be removed and the brightness of the bulb to remain the same? Explain.

Q31.16 Can a transformer be used with dc? Explain. What happens if a transformer designed for 120 V ac is connected to a 120 V dc line?

Q31.17 An ideal transformer has N_1 windings in the primary and N_2 windings in its secondary. If you double only the number of secondary windings, by what factor does (a) the voltage amplitude in the secondary change, and (b) the effective resistance of the secondary circuit change?

Q31.18 An inductor, a capacitor, and a resistor are all connected in series across an ac source. If the resistance, inductance, and capacitance are all doubled, by what factor does each of the following quantities change? Indicate whether they increase or decrease: (a) the resonance angular frequency; (b) the inductive reactance; (c) the capacitive reactance. (d) Does the impedance double?

Q31.19 You want to double the resonance angular frequency of an *L-R-C* series circuit by changing only the *pertinent* circuit elements all by the same factor. (a) Which ones should you change? (b) By what factor should you change them?

EXERCISES

Section 31.1 Phasors and Alternating Currents

31.1 • You have a special light bulb with a *very* delicate wire filament. The wire will break if the current in it ever exceeds 1.50 A, even for an instant. What is the largest root-mean-square current you can run through this bulb?

31.2 • The voltage across the terminals of an ac power supply varies with time according to Eq. (31.1). The voltage amplitude is $V = 45.0$ V. What are (a) the root-mean-square potential difference V_{rms} and (b) the average potential difference V_{av} between the two terminals of the power supply?

Section 31.2 Resistance and Reactance

31.3 • An inductor with $L = 9.50$ mH is connected across an ac source that has voltage amplitude 45.0 V. (a) What is the phase angle ϕ for the source voltage relative to the current? Does the source voltage lag or lead the current? (b) What value for the frequency of the source results in a current amplitude of 3.90 A?

31.4 • A capacitor is connected across an ac source that has voltage amplitude 60.0 V and frequency 80.0 Hz. (a) What is the phase angle ϕ for the source voltage relative to the current? Does the source voltage lag or lead the current? (b) What is the capacitance C of the capacitor if the current amplitude is 5.30 A?

31.5 • (a) What is the reactance of a 3.00 H inductor at a frequency of 80.0 Hz? (b) What is the inductance of an inductor whose reactance is 120 Ω at 80.0 Hz? (c) What is the reactance of a 4.00 μF capacitor at a frequency of 80.0 Hz? (d) What is the capacitance of a capacitor whose reactance is 120 Ω at 80.0 Hz?

31.6 • A capacitance C and an inductance L are operated at the same angular frequency. (a) At what angular frequency will they have the same reactance? (b) If $L = 5.00$ mH and $C = 3.50\ \mu$F, what is the numerical value of the angular frequency in part (a), and what is the reactance of each element?

31.7 • (a) Compute the reactance of a 0.450 H inductor at frequencies of 60.0 Hz and 600 Hz. (b) Compute the reactance of a 2.50 μF capacitor at the same frequencies. (c) At what frequency is the reactance of a 0.450 H inductor equal to that of a 2.50 μF capacitor?

31.8 • **A Radio Inductor.** You want the current amplitude through a 0.450 mH inductor (part of the circuitry for a radio receiver) to be 1.80 mA when a sinusoidal voltage with amplitude 12.0 V is applied across the inductor. What frequency is required?

31.9 •• A 0.180 H inductor is connected in series with a 90.0 Ω resistor and an ac source. The voltage across the inductor is $v_L = -(12.0\text{ V})\sin[(480\text{ rad/s})t]$. (a) Derive an expression for the voltage v_R across the resistor. (b) What is v_R at $t = 2.00$ ms?

31.10 •• A 250 Ω resistor is connected in series with a 4.80 μF capacitor and an ac source. The voltage across the capacitor is $v_C = (7.60\text{ V})\sin[(120\text{ rad/s})t]$. (a) Determine the capacitive reactance of the capacitor. (b) Derive an expression for the voltage v_R across the resistor.

31.11 •• A 150 Ω resistor is connected in series with a 0.250 H inductor and an ac source. The voltage across the resistor is $v_R = (3.80\text{ V})\cos[(720\text{ rad/s})t]$. (a) Derive an expression for the circuit current. (b) Determine the inductive reactance of the inductor. (c) Derive an expression for the voltage v_L across the inductor.

Section 31.3 The *L-R-C* Series Circuit

31.12 • You have a 200 Ω resistor, a 0.400 H inductor, and a 6.00 μF capacitor. Suppose you take the resistor and inductor and make a series circuit with a voltage source that has voltage amplitude 30.0 V and an angular frequency of 250 rad/s. (a) What is the impedance of the circuit? (b) What is the current amplitude? (c) What are the voltage amplitudes across the resistor and across the inductor? (d) What is the phase angle ϕ of the source voltage with respect to the current? Does the source voltage lag or lead the current? (e) Construct the phasor diagram.

31.13 • The resistor, inductor, capacitor, and voltage source described in Exercise 31.12 are connected to form an *L-R-C* series circuit. (a) What is the impedance of the circuit? (b) What is the current amplitude? (c) What is the phase angle of the source voltage with respect to the current? Does the source voltage lag or lead the current? (d) What are the voltage amplitudes across the resistor, inductor, and capacitor? (e) Explain how it is possible for the voltage amplitude across the capacitor to be greater than the voltage amplitude across the source.

31.14 •• A 200 Ω resistor, 0.900 H inductor, and 6.00 μF capacitor are connected in series across a voltage source that has voltage amplitude 30.0 V and an angular frequency of 250 rad/s. (a) What are v, v_R, v_L, and v_C at $t = 20.0$ ms? Compare $v_R + v_L + v_C$ to v at this instant. (b) What are V_R, V_L, and V_C? Compare V to $V_R + V_L + V_C$. Explain why these two quantities are not equal.

31.15 • In an *L-R-C* series circuit, the rms voltage across the resistor is 30.0 V, across the capacitor it is 90.0 V, and across the inductor it is 50.0 V. What is the rms voltage of the source?

31.16 •• An *L-R-C* series circuit has voltage amplitudes $V_L = 180$ V, $V_C = 120$ V, and $V_R = 160$ V. At time t the instantaneous voltage across the inductor is 80.0 V. At this instant, what is the voltage across the capacitor and across the resistor?

31.17 • An *L-R-C* series circuit has source voltage amplitude $V = 240$ V, and the voltage amplitudes for the inductor and capacitor are $V_L = 310$ V and $V_C = 180$ V. What is the phase angle ϕ?

Section 31.4 Power in Alternating-Current Circuits

31.18 •• A resistor with $R = 300$ Ω and an inductor are connected in series across an ac source that has voltage amplitude 500 V. The rate at which electrical energy is dissipated in the resistor is 286 W. What is (a) the impedance Z of the circuit; (b) the amplitude of the voltage across the inductor; (c) the power factor?

31.19 • The power of a certain CD player operating at 120 V rms is 20.0 W. Assuming that the CD player behaves like a pure resistor, find (a) the maximum instantaneous power; (b) the rms current; (c) the resistance of this player.

31.20 •• In an *L-R-C* series circuit, the components have the following values: $L = 20.0$ mH, $C = 140$ nF, and $R = 350$ Ω. The generator has an rms voltage of 120 V and a frequency of 1.25 kHz. Determine (a) the power supplied by the generator and (b) the power dissipated in the resistor.

31.21 •• (a) Use Figs. 31.13b and 31.13c to show that for a series *L-R-C* circuit, $V\cos\phi = IR$. Use this result in Eq. (31.31) to show that the average power delivered by the source is $P_{\text{av}} = \frac{1}{2}I^2R$. Therefore the average power delivered by the source is equal to the average power consumed by the resistor. This means the average power for the capacitor and for the inductor is zero. These circuit elements consume electrical energy during the part of the current oscillations when they are storing energy, but all the stored energy in each is then released during another part of the current oscillations. (b) In an *L-R-C* series circuit the amplitude of the source voltage is 120 V, the source voltage leads the current by 53.1°, and the average power supplied by the source is 80.0 W. What is the resistance R of the resistor in the circuit?

31.22 •• A circuit has an ac voltage source and a resistor and capacitor connected in series. There is no inductor. The ac voltage source has voltage amplitude 900 V and angular frequency $\omega = 20.0$ rad/s. The voltage amplitude across the capacitor is 500 V. The resistor has resistance $R = 300$ Ω. (a) What is the voltage amplitude across the resistor? (b) What is the capacitance C of the capacitor? (c) Does the source voltage lag or lead the current? (d) What is the average rate at which the ac source supplies electrical energy to the circuit?

31.23 • An *L-R-C* series circuit with $L = 0.120$ H, $R = 240$ Ω, and $C = 7.30$ μF carries an rms current of 0.450 A with a frequency of 400 Hz. (a) What are the phase angle and power factor for this circuit? (b) What is the impedance of the circuit? (c) What is the rms voltage of the source? (d) What average power is delivered by the source? (e) What is the average rate at which electrical energy is converted to thermal energy in the resistor? (f) What is the average rate at which electrical energy is dissipated (converted to other forms) in the capacitor? (g) In the inductor?

31.24 •• An *L-R-C* series circuit is connected to a 120 Hz ac source that has $V_{\text{rms}} = 80.0$ V. The circuit has a resistance of 75.0 Ω and an impedance at this frequency of 105 Ω. What average power is delivered to the circuit by the source?

31.25 •• A series ac circuit contains a 250 Ω resistor, a 15 mH inductor, a 3.5 μF capacitor, and an ac power source of voltage amplitude 45 V operating at an angular frequency of 360 rad/s. (a) What is the power factor of this circuit? (b) Find the average power delivered to the entire circuit. (c) What is the average power delivered to the resistor, to the capacitor, and to the inductor?

Section 31.5 Resonance in Alternating-Current Circuits

31.26 •• In an *L-R-C* series circuit the source is operated at its resonant angular frequency. At this frequency, the reactance X_C of the capacitor is 200 Ω and the voltage amplitude across the capacitor is 600 V. The circuit has $R = 300$ Ω. What is the voltage amplitude of the source?

31.27 • **Analyzing an *L-R-C* Circuit.** You have a 200 Ω resistor, a 0.400 H inductor, a 5.00 μF capacitor, and a variable-frequency ac source with an amplitude of 3.00 V. You connect all four elements together to form a series circuit. (a) At what frequency will the current in the circuit be greatest? What will be the current amplitude at this frequency? (b) What will be the current amplitude at an angular frequency of 400 rad/s? At this frequency, will the source voltage lead or lag the current?

31.28 • An *L-R-C* series circuit is constructed using a 175 Ω resistor, a 12.5 μF capacitor, and an 8.00 mH inductor, all connected across an ac source having a variable frequency and a voltage amplitude of 25.0 V. (a) At what angular frequency will the impedance be smallest, and what is the impedance at this frequency? (b) At the angular frequency in part (a), what is the maximum current through the inductor? (c) At the angular frequency in part (a), find the potential difference across the ac source, the resistor, the capacitor, and the inductor at the instant that the current is equal to one-half its greatest positive value. (d) In part (c), how are the potential differences across the resistor, inductor, and capacitor related to the potential difference across the ac source?

31.29 • In an L-R-C series circuit, $R = 300\ \Omega$, $L = 0.400$ H, and $C = 6.00 \times 10^{-8}$ F. When the ac source operates at the resonance frequency of the circuit, the current amplitude is 0.500 A. (a) What is the voltage amplitude of the source? (b) What is the amplitude of the voltage across the resistor, across the inductor, and across the capacitor? (c) What is the average power supplied by the source?

31.30 • An L-R-C series circuit consists of a source with voltage amplitude 120 V and angular frequency 50.0 rad/s, a resistor with $R = 400\ \Omega$, an inductor with $L = 3.00$ H, and a capacitor with capacitance C. (a) For what value of C will the current amplitude in the circuit be a maximum? (b) When C has the value calculated in part (a), what is the amplitude of the voltage across the inductor?

31.31 • In an L-R-C series circuit, $R = 150\ \Omega$, $L = 0.750$ H, and $C = 0.0180\ \mu$F. The source has voltage amplitude $V = 150$ V and a frequency equal to the resonance frequency of the circuit. (a) What is the power factor? (b) What is the average power delivered by the source? (c) The capacitor is replaced by one with $C = 0.0360\ \mu$F and the source frequency is adjusted to the new resonance value. Then what is the average power delivered by the source?

31.32 • An L-R-C series circuit has $R = 400\ \Omega$, $L = 0.600$ H, and $C = 5.00 \times 10^{-8}$ F. The voltage amplitude of the source is 80.0 V. When the ac source operates at the resonance frequency of the circuit, what is the average power delivered by the source?

31.33 •• In an L-R-C series circuit, $L = 0.280$ H and $C = 4.00\ \mu$F. The voltage amplitude of the source is 120 V. (a) What is the resonance angular frequency of the circuit? (b) When the source operates at the resonance angular frequency, the current amplitude in the circuit is 1.70 A. What is the resistance R of the resistor? (c) At the resonance angular frequency, what are the peak voltages across the inductor, the capacitor, and the resistor?

Section 31.6 Transformers

31.34 • **Off to Europe!** You plan to take your hair dryer to Europe, where the electrical outlets put out 240 V instead of the 120 V seen in the United States. The dryer puts out 1600 W at 120 V. (a) What could you do to operate your dryer via the 240 V line in Europe? (b) What current will your dryer draw from a European outlet? (c) What resistance will your dryer appear to have when operated at 240 V?

31.35 • **A Step-Down Transformer.** A transformer connected to a 120 V (rms) ac line is to supply 12.0 V (rms) to a portable electronic device. The load resistance in the secondary is 5.00 Ω. (a) What should the ratio of primary to secondary turns of the transformer be? (b) What rms current must the secondary supply? (c) What average power is delivered to the load? (d) What resistance connected directly across the 120 V line would draw the same power as the transformer? Show that this is equal to 5.00 Ω times the square of the ratio of primary to secondary turns.

31.36 • **A Step-Up Transformer.** A transformer connected to a 120 V (rms) ac line is to supply 13,000 V (rms) for a neon sign. To reduce shock hazard, a fuse is to be inserted in the primary circuit; the fuse is to blow when the rms current in the secondary circuit exceeds 8.50 mA. (a) What is the ratio of secondary to primary turns of the transformer? (b) What power must be supplied to the transformer when the rms secondary current is 8.50 mA? (c) What current rating should the fuse in the primary circuit have?

PROBLEMS

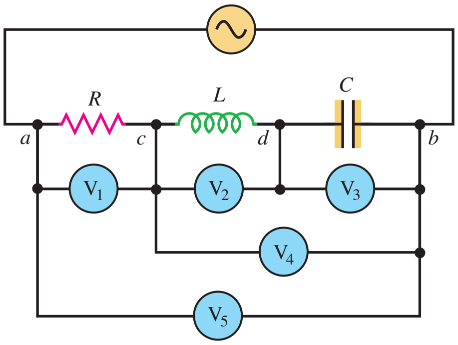
31.37 • A coil has a resistance of 48.0 Ω. At a frequency of 80.0 Hz the voltage across the coil leads the current in it by 52.3°. Determine the inductance of the coil.

31.38 •• In an L-R-C series circuit the phase angle is 53.0° and the source voltage lags the current. The resistance of the resistor is 300 Ω and the reactance of the capacitor is 500 Ω. The average power delivered by the source is 80.0 W. (a) What is the reactance of the inductor? (b) What is the current amplitude in the circuit? (c) What is the voltage amplitude of the source?

31.39 •• An L-R-C series circuit has $C = 4.80\ \mu$F, $L = 0.520$ H, and source voltage amplitude $V = 56.0$ V. The source is operated at the resonance frequency of the circuit. If the voltage across the capacitor has amplitude 80.0 V, what is the value of R for the resistor in the circuit?

31.40 •• Five infinite-impedance voltmeters, calibrated to read rms values, are connected as shown in **Fig. P31.40**. Let $R = 200\ \Omega$, $L = 0.400$ H, $C = 6.00\ \mu$F, and $V = 30.0$ V. What is the reading of each voltmeter if (a) $\omega = 200$ rad/s and (b) $\omega = 1000$ rad/s?

Figure **P31.40**

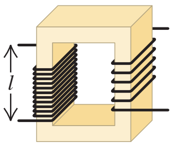


31.41 •• **CP** A parallel-plate capacitor having square plates 4.50 cm on each side and 8.00 mm apart is placed in series with the following: an ac source of angular frequency 650 rad/s and voltage amplitude 22.5 V; a 75.0 Ω resistor; and an ideal solenoid that is 9.00 cm long, has a circular cross section 0.500 cm in diameter, and carries 125 coils per centimeter. What is the resonance angular frequency of this circuit? (See Exercise 30.11.)

31.42 •• **CP** A toroidal solenoid has 2900 closely wound turns, cross-sectional area 0.450 cm^2, mean radius 9.00 cm, and resistance $R = 2.80\ \Omega$. Ignore the variation of the magnetic field across the cross section of the solenoid. What is the amplitude of the current in the solenoid if it is connected to an ac source that has voltage amplitude 24.0 V and frequency 495 Hz?

31.43 •• A series circuit has an impedance of 60.0 Ω and a power factor of 0.720 at 50.0 Hz. The source voltage lags the current. (a) What circuit element, an inductor or a capacitor, should be placed in series with the circuit to raise its power factor? (b) What size element will raise the power factor to unity?

Figure **P31.44**



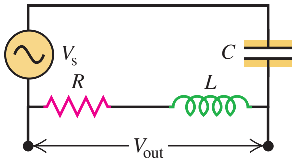
31.44 •• Consider the ac adapter that bisects the power cord to your laptop computer, cell phone charger, or another electronic device. (a) What dc output voltage V and current I are listed on your device? (b) What power P is listed on your device? Does $P = IV$? (c) Your ac adapter is most likely not a simple transformer; however, it is instructive to analyze the device as if it were: Assume your device is a transformer with a 120 V rms voltage on its primary coil and a full-wave rectifier following its secondary coil. If an R-C circuit with a sufficient time constant followed, the ultimate output would maintain the dc value of the voltage amplitude on the rectified secondary coil. If the primary coil has 200 turns, how many turns are needed on the secondary coil to produce the stated dc output according to this scheme? (Give the closest integer value.) (d) Estimate the current amplitude in the primary coil. (e) Based on the size of your device and assuming the transformer geometry shown in **Fig. P31.44**, estimate the length l of the primary coil. (f) Use Ampere's law to estimate the average strength of the magnetic field inside the core, assuming a relative permeability of 5000.

31.45 •• In an L-R-C series circuit, $R = 300\ \Omega$, $X_C = 300\ \Omega$, and $X_L = 500\ \Omega$. The average electrical power consumed in the resistor is 60.0 W. (a) What is the power factor of the circuit? (b) What is the rms voltage of the source?

31.46 • At a frequency ω_1 the reactance of a certain capacitor equals that of a certain inductor. (a) If the frequency is changed to $\omega_2 = 2\omega_1$, what is the ratio of the reactance of the inductor to that of the capacitor? Which reactance is larger? (b) If the frequency is changed to $\omega_3 = \omega_1/3$, what is the ratio of the reactance of the inductor to that of the capacitor? Which reactance is larger? (c) If the capacitor and inductor are placed in series with a resistor of resistance R to form an L-R-C series circuit, what will be the resonance angular frequency of the circuit?

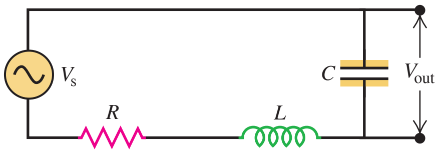
31.47 •• **A High-Pass Filter.** One application of L-R-C series circuits is to high-pass or low-pass filters, which filter out either the low- or high-frequency components of a signal. A high-pass filter is shown in **Fig. P31.47**, where the output voltage is taken across the L-R combination. (The L-R combination represents an inductive coil that also has resistance due to the large length of wire in the coil.) Derive an expression for $V_{\text{out}}/V_{\text{s}}$, the ratio of the output and source voltage amplitudes, as a function of the angular frequency ω of the source. Show that when ω is small, this ratio is proportional to ω and thus is small, and show that the ratio approaches unity in the limit of large frequency.

Figure **P31.47**



31.48 •• **A Low-Pass Filter. Figure P31.48** shows a low-pass filter (see Problem 31.47); the output voltage is taken across the capacitor in an L-R-C series circuit. Derive an expression for $V_{\text{out}}/V_{\text{s}}$, the ratio of the output and source voltage amplitudes, as a function of the angular frequency ω of the source. Show that when ω is large, this ratio is proportional to ω^{-2} and thus is very small, and show that the ratio approaches unity in the limit of small frequency.

Figure **P31.48**



31.49 ••• An L-R-C series circuit is connected to an ac source of constant voltage amplitude V and variable angular frequency ω. (a) Show that the current amplitude, as a function of ω, is

$$I = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

(b) Show that the average power dissipated in the resistor is

$$P = \frac{V^2 R/2}{R^2 + (\omega L - 1/\omega C)^2}$$

(c) Show that I and P are *both* maximum when $\omega = 1/\sqrt{LC}$, the resonance frequency of the circuit. (d) Graph P as a function of ω for $V = 100$ V, $R = 200\ \Omega$, $L = 2.0$ H, and $C = 0.50\ \mu$F. Compare to the light purple curve in Fig. 31.19. Discuss the behavior of I and P in the limits $\omega = 0$ and $\omega \rightarrow \infty$.

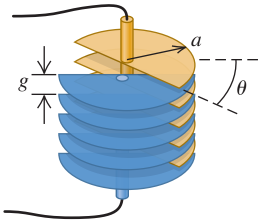
31.50 ••• An L-R-C series circuit is connected to an ac source of constant voltage amplitude V and variable angular frequency ω. Using the results of Problem 31.49, find an expression for (a) the amplitude V_L of the voltage across the inductor as a function of ω and (b) the amplitude V_C of the voltage across the capacitor as a function of ω. (c) Graph V_L and V_C as functions of ω for $V = 100$ V, $R = 200\ \Omega$, $L = 2.0$ H, and $C = 0.50\ \mu$F. (d) Discuss the behavior of V_L and V_C in the limits $\omega = 0$ and $\omega \rightarrow \infty$. For what value of ω is $V_L = V_C$? What is the significance of this value of ω?

31.51 •• In an L-R-C series circuit the magnitude of the phase angle is 54.0°, with the source voltage lagging the current. The reactance of the capacitor is 350 Ω, and the resistor resistance is 180 Ω. The average power delivered by the source is 140 W. Find (a) the reactance of the inductor; (b) the rms current; (c) the rms voltage of the source.

31.52 •• **CP** Cell phones that use 4G technology receive signals broadcast between 2 GHz and 8 GHz. (a) If you want to create a simple L-R-C series circuit to detect a 4.0 GHz cell phone signal, what is the relevant value of the product LC, where L is the inductance and C is the capacitance? (b) If you choose a capacitor that has $C = 1.0 \times 10^{-15}$ F, what inductance do you need? (c) Suppose you want to wind your own toroidal inductor and fit it inside a box as thin as your cell phone. Based on the size of your phone, estimate the largest cross-sectional area possible for this. (d) Assume the largest allowable radius of the toroid is 1.0 cm and estimate the lowest number of windings needed to create your inductor, assuming the material inside has a relative permeability of 1.

31.53 ••• **CP** Five semicircular conducting plates, each with radius a, are attached to a conducting rod along their common axis such that the plates are parallel with a separation gap g between adjacent plates. A similar arrangement involving five additional semicircular conducting plates is attached to a second conducting rod symmetrically intertwined, upside down, with the first arrangement such that the plates can rotate freely, as shown in **Fig. P31.53**. The two five-plate constructions, with an adjacent plate separation of $g/2$, are insulated from each other, and wire leads are attached to the separate axles. This is a common type of variable capacitor, with a capacitance proportional to the overlap angle θ. (a) Determine a formula for the capacitance C in terms of a, g, and θ, where θ is in radians. (b) The plate radius is $a = 6.00$ cm while the gap distance is $g = 2.00$ mm. This variable capacitor is placed in series with a 100 μH inductor and a 10.0 Ω resistor. A voltage source is supplied by an amplified signal from an antenna, and the output voltage is taken across the capacitor. The circuit is used as a radio receiver that receives a signal frequency equal to the resonance frequency of the L-R-C circuit. To receive an AM radio signal at 1180 kHz, what angle θ, in degrees, should be selected? (c) If the input signal is a sinusoidal voltage with amplitude 100.0 mV and frequency 1180 kHz, what is the amplitude of the output voltage? (d) If the knob on the capacitor is turned so that $\theta = 120°$, to what frequency is the circuit tuned? (e) If the input signal has the frequency resonant with this setting and voltage amplitude 100 mV, what is the voltage amplitude of the output signal?

Figure **P31.53**



31.54 •• **The L-R-C Parallel Circuit.** A resistor, an inductor, and a capacitor are connected in parallel to an ac source with voltage amplitude V and angular frequency ω. Let the source voltage be given by $v = V\cos\omega t$. (a) Show that each of the instantaneous voltages v_R, v_L, and v_C at any instant is equal to v and that $i = i_R + i_L + i_C$, where i is the current through the source and i_R, i_L, and i_C are the currents through the resistor, inductor, and capacitor, respectively. (b) What are the phases of i_R, i_L, and i_C with respect to v? Use current phasors to represent i, i_R, i_L, and i_C. In a phasor diagram, show the phases of these four currents with respect to v. (c) Use the phasor diagram of part (b) to show that the current amplitude I for the current i through the source is $I = \sqrt{I_R^2 + (I_C - I_L)^2}$. (d) Show that the result of part (c) can be written as $I = V/Z$, with $1/Z = \sqrt{(1/R^2) + [\omega C - (1/\omega L)]^2}$.

31.55 •• The impedance of an L-R-C parallel circuit was derived in Problem 31.54. (a) Show that at the resonance angular frequency $\omega_0 = 1/\sqrt{LC}$, the impedance Z is a maximum and therefore the current through the ac source is a minimum. (b) A 100 Ω resistor, a 0.100 μF capacitor, and a 0.300 H inductor are connected in parallel to a voltage source with amplitude 240 V. What is the resonance angular frequency? For this circuit, what is (c) the maximum current through the source at the resonance frequency; (d) the maximum current in the resistor at resonance; (e) the maximum current in the inductor at resonance; (f) the maximum current in the branch containing the capacitor at resonance?

31.56 •• A 400 Ω resistor and a 6.00 μF capacitor are connected in parallel to an ac generator that supplies an rms voltage of 180 V at an angular frequency of 360 rad/s. Use the results of Problem 31.54. Note that since there is no inductor in this circuit, the $1/\omega L$ term is not present in the expression for $1/Z$. Find (a) the current amplitude in the resistor; (b) the current amplitude in the capacitor; (c) the phase angle of the source current with respect to the source voltage; (d) the amplitude of the current through the generator. (e) Does the source current lag or lead the source voltage?

31.57 ••• An L-R-C series circuit consists of a 2.50 μF capacitor, a 5.00 mH inductor, and a 75.0 Ω resistor connected across an ac source of voltage amplitude 15.0 V having variable frequency. (a) Under what circumstances is the average power delivered to the circuit equal to $\frac{1}{2}V_{\text{rms}}I_{\text{rms}}$? (b) Under the conditions of part (a), what is the average power delivered to each circuit element and what is the maximum current through the capacitor?

31.58 •• An L-R-C series circuit has $R = 60.0\ \Omega$, $L = 0.800$ H, and $C = 3.00 \times 10^{-4}$ F. The ac source has voltage amplitude 90.0 V and angular frequency 120 rad/s. (a) What is the maximum energy stored in the inductor? (b) When the energy stored in the inductor is a maximum, how much energy is stored in the capacitor? (c) What is the maximum energy stored in the capacitor?

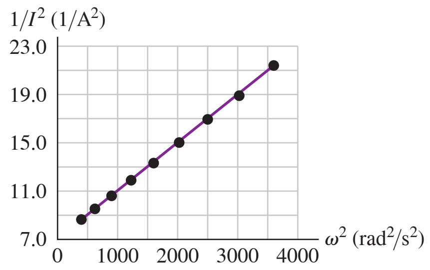
31.59 • In an L-R-C series circuit, the source has a voltage amplitude of 120 V, $R = 80.0\ \Omega$, and the reactance of the capacitor is 480 Ω. The voltage amplitude across the capacitor is 360 V. (a) What is the current amplitude in the circuit? (b) What is the impedance? (c) What two values can the reactance of the inductor have? (d) For which of the two values found in part (c) is the angular frequency less than the resonance angular frequency? Explain.

31.60 •• In an L-R-C series ac circuit, the source has a voltage amplitude of 240 V, $R = 90.0\ \Omega$, and the reactance of the inductor is 320 Ω. The voltage amplitude across the resistor is 135 V. (a) What is the current amplitude in the circuit? (b) What is the voltage amplitude across the inductor? (c) What two values can the reactance of the capacitor have? (d) For which of the two values found in part (c) is the angular frequency less than the resonance angular frequency? Explain.

31.61 • A resistance R, capacitance C, and inductance L are connected in series to a voltage source with amplitude V and variable angular frequency ω. If $\omega = \omega_0$, the resonance angular frequency, find (a) the maximum current in the resistor; (b) the maximum voltage across the capacitor; (c) the maximum voltage across the inductor; (d) the maximum energy stored in the capacitor; (e) the maximum energy stored in the inductor. Give your answers in terms of R, C, L, and V.

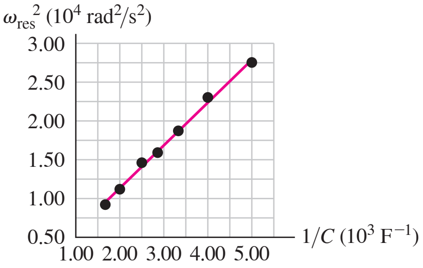
31.62 •• **DATA** A coworker of yours was making measurements of a large solenoid that is connected to an ac voltage source. Unfortunately, she left for vacation before she completed the analysis, and your boss has asked you to finish it. You are given a graph of $1/I^2$ versus ω^2 (**Fig. P31.62**), where I is the current in the circuit and ω is the angular frequency of the source. A note attached to the graph says that the voltage amplitude of the source was kept constant at 12.0 V. Calculate the resistance and inductance of the solenoid.

Figure **P31.62**



31.63 •• **DATA** You are analyzing an ac circuit that contains a solenoid and a capacitor in series with an ac source that has voltage amplitude 90.0 V and angular frequency ω. For different capacitors in the circuit, each with known capacitance, you measure the value of the frequency ω_{res} for which the current in the circuit is a maximum. You plot your measured values on a graph of $\omega_{\text{res}}{}^2$ versus $1/C$ (**Fig. P31.63**). The maximum current for each value of C is the same, you note, and equal to 4.50 A. Calculate the resistance and inductance of the solenoid.

Figure **P31.63**



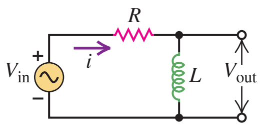
31.64 •• **DATA** You are given this table of data recorded for a circuit that has a resistor, an inductor with negligible resistance, and a capacitor, all in series with an ac voltage source:

f **(Hz)**	80	160
Z **(Ω)**	15	13
ϕ **(deg)**	−71	67

Here f is the frequency of the voltage source, Z is the impedance of the circuit, and ϕ is the phase angle. (a) Use the data at both frequencies to calculate the resistance of the resistor. Calculate the average of these two values of the resistance, and use the result as the value of R in the rest of the analysis. (b) Use the data at 80 Hz and 160 Hz to calculate the inductance L and capacitance C of the circuit. (c) What is the resonance frequency for the circuit, and what are the impedance and phase angle at the resonance frequency?

31.65 ••• **CP** The frequency response of a filter circuit is commonly specified, in decibels, as the attenuation $G = (20\ \text{dB})\log_{10}(V_{\text{out}}/V_{\text{in}})$, where V_{in} is the input voltage amplitude and V_{out} is the output voltage amplitude. This is analogous to the definition of decibels given in Eq. (16.15). For the R-L circuit shown in **Fig. P31.65**, the input signal is $v_{\text{in}}(t) = V_{\text{in}}\cos(\omega t)$. (a) The current is $i(t) = I\cos(\omega t - \phi)$. What is the current amplitude I? (b) What is the phase angle ϕ? (c) What is the ratio $V_{\text{out}}/V_{\text{in}}$? (d) For what frequency $f = f_{\text{3db}}$ does this circuit have an attenuation of −3.0 dB? (The frequency f_{3db} is commonly cited as the nominal boundary demarcating the "pass band.") (e) If $R = 100\ \Omega$, what inductance L will "pass" frequencies higher than 10.0 kHz and block frequencies lower than 10.0 kHz?

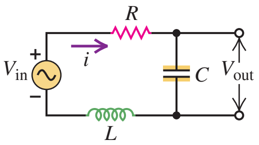
Figure **P31.65**



CHALLENGE PROBLEMS

31.66 ••• CALC The *L-R-C* series circuit shown in **Fig. P31.66** has an ac voltage source $v_{\text{in}}(t) = V_{\text{in}}\cos(\omega t)$, where V_{in} is the input voltage amplitude and ω is its angular frequency. The values of the resistance, inductance, and capacitance of the components are *R, L,* and *C*. The capacitance is variable. The output voltage is $v_{\text{out}}(t) = V_{\text{out}}\cos(\omega t + \theta)$. (a) What is the output voltage amplitude V_{out}? (b) What is the phase angle θ of the output voltage? (c) What is the value of θ at resonance? (d) If $R = 100\ \Omega$, $L = 1.00\ \mu\text{H}$, and $V_{\text{in}} = 10.0$ V, what value of C would result in a resonance frequency of 100 kHz? (e) In that case what would be the output voltage amplitude at resonance?

Figure **P31.66**



31.67 •• CALC In an *L-R-C* series circuit the current is given by $i = I\cos\omega t$. The voltage amplitudes for the resistor, inductor, and capacitor are V_R, V_L, and V_C. (a) Show that the instantaneous power into the resistor is $p_R = V_R I\cos^2\omega t = \frac{1}{2}V_R I(1 + \cos 2\omega t)$. What does this expression give for the average power into the resistor? (b) Show that the instantaneous power into the inductor is $p_L = -V_L I\sin\omega t\cos\omega t = -\frac{1}{2}V_L I\sin 2\omega t$. What does this expression give for the average power into the inductor? (c) Show that the instantaneous power into the capacitor is $p_C = V_C I\sin\omega t\cos\omega t = \frac{1}{2}V_C I\sin 2\omega t$. What does this expression give for the average power into the capacitor? (d) The instantaneous power delivered by the source is shown in Section 31.4 to be $p = VI\cos\omega t(\cos\phi\cos\omega t - \sin\phi\sin\omega t)$. Show that $p_R + p_L + p_C$ equals p at each instant of time.

31.68 ••• CALC (a) At what angular frequency is the voltage amplitude across the *resistor* in an *L-R-C* series circuit at maximum value? (b) At what angular frequency is the voltage amplitude across the *inductor* at maximum value? (c) At what angular frequency is the voltage amplitude across the *capacitor* at maximum value? (You may want to refer to the results of Problem 31.49.)

MCAT-STYLE PASSAGE PROBLEMS

BIO Converting dc to ac. An individual cell such as an egg cell (an ovum, produced in the ovaries) is commonly organized spatially, as manifested in part by asymmetries in the cell membrane. These asymmetries include nonuniform distributions of ion transport mechanisms, which result in a net electric current entering one region of the membrane and leaving another. These steady cellular currents may regulate cell polarity, leading (in the case of eggs) to embryonic polarity; therefore scientists are interested in measuring them.

These cellular currents move in loops through extracellular fluid. Ohm's law requires that there be voltage differences between any two points in this current-carrying fluid surrounding cells. Although the currents may be significant, the extracellular voltage differences are tiny—on the order of nanovolts. If we can map the voltage differences in the fluid outside a cell, we can calculate the current density by using Ohm's law, assuming that the resistivity of the fluid is known. We cannot measure these voltage differences by spacing two electrodes 10 or 20 μm apart, because the dc impedance (the resistance) of such electrodes is high and the inherent noise in signals detected at the electrodes far exceeds the cellular voltages.

One successful method of measurement uses an electrode with a ball-shaped end made of platinum that is moved sinusoidally between two points in the fluid outside a cell. The electric potential that the electrode measures, with respect to a distant reference electrode, also varies sinusoidally. The dc potential difference between the two extremes (the two points in the fluid) is then converted to a sine-wave ac potential difference. The platinum electrode behaves as a capacitor in series with the resistance of the extracellular fluid. This resistance, called the *access resistance* (R_{A}), has a value of about $\rho/10a$, where ρ is the resistivity of the fluid (usually expressed in $\Omega \cdot$cm) and a is the radius of the ball electrode. The platinum ball typically has a diameter of 20 μm and a capacitance of 10 nF; the resistivity of many biological fluids is 100 $\Omega \cdot$cm.

31.69 What is the dc impedance of the electrode, assuming that it behaves as an ideal capacitor? (a) 0; (b) infinite; (c) $\sqrt{2} \times 10^4\ \Omega$; (d) $\sqrt{2} \times 10^6\ \Omega$.

31.70 If the electrode oscillates between two points 20 μm apart at a frequency of $(5000/\pi)$ Hz, what is the electrode's impedance? (a) 0; (b) infinite; (c) $\sqrt{2} \times 10^4\ \Omega$; (d) $\sqrt{2} \times 10^6\ \Omega$.

31.71 The signal from the oscillating electrode is fed into an amplifier, which reports the measured voltage as an rms value, 1.5 nV. What is the potential difference between the two extremes? (a) 1.5 nV; (b) 3.0 nV; (c) 2.1 nV; (d) 4.2 nV.

31.72 If the frequency at which the electrode is oscillated is increased to a very large value, the electrode's impedance (a) approaches infinity; (b) approaches zero; (c) approaches a constant but nonzero value; (d) does not change.

ANSWERS

Chapter Opening Question ?

(iv) A radio simultaneously detects transmissions at *all* frequencies. However, a radio is an *L-R-C* series circuit, and at any given time it is tuned to have a resonance at just one frequency. Hence the response of the radio to that frequency is much greater than its response to any other frequency, which is why you hear only one broadcasting station through the radio's speaker. (You can sometimes hear a second station if its frequency is sufficiently close to the tuned frequency.)

Key Example ✓ARIATION Problems

VP31.3.1 (a) $1.70 \times 10^6\ \Omega$ (b) 2.48×10^{-3} A

VP31.3.2 (a) $(0.300\text{ V})\cos[(1.75 \times 10^3\text{ rad/s})t]$ (b) 81.6 Ω (c) $(0.196\text{ V})\cos[(1.75 \times 10^3\text{ rad/s})t - \pi/2\text{ rad}]$

VP31.3.3 (a) 128 Hz (b) both are 0.620 V (c) zero; zero

VP31.3.4 (a) 0.0215 A (b) 0.0116 A (c) 2.45 V (d) 1.72 μF

VP31.5.1 (a) 902 Hz (b) 316 Ω (c) 0.174 A (d) –36.1° (e) lags

VP31.5.2 (a) 387 Ω (b) 0.0422 A (c) 16.3 V (d) 1.48 V (e) 13.0 V

VP31.5.3 (a) 17.4 Ω (b) 8.65 mH

VP31.5.4 (a) $V_R = 11.4$ V, $V_L = 6.24$ V, $V_C = 34.1$ V (b) $v_R = -8.71$ V, $v_L = -4.03$ V, $v_C = +22.0$ V

VP31.7.1 (a) 474 W (b) 948 W (c) 30.4 Ω

VP31.7.2 (a) $X_L = 107\ \Omega$, $X_C = 699\ \Omega$ (b) –65.1° (c) 0.421

VP31.7.3 (a) 653 Ω (b) 0.0536 A (c) 0.395 W

VP31.7.4 (a) $X_L = 97.5\ \Omega$, $X_C = 85.5\ \Omega$ (b) 36.9° (c) 16.0 Ω

Bridging Problem

(a) 8.35×10^4 rad/s and 3.19×10^5 rad/s

(b) At 8.35×10^4 rad/s: $V_{\text{source}} = 49.5$ V, $I = 0.132$ A, $V_R = 16.5$ V, $V_L = 16.5$ V, $V_C = 63.2$ V

At 3.19×10^5 rad/s: $V_{\text{source}} = 49.5$ V, $I = 0.132$ A, $V_R = 16.5$ V, $V_L = 63.2$ V, $V_C = 16.5$ V