? The card reader at your bank's cash machine scans the information that is coded in a magnetic pattern on the back of your card. Why must you remove the card quickly rather than hold it motionless in the card reader's slot? (i) To maximize the magnetic force on the card; (ii) to maximize the magnetic force on the mobile charges in the card reader; (iii) to generate an electric force on the card; (iv) to generate an electric force on the mobile charges in the card reader.

29 Electromagnetic Induction

Almost every modern device or machine, from a computer to a washing machine to a power drill, has electric circuits at its heart. We learned in Chapter 25 that an electromotive force (emf) is required for a current to flow in a circuit; in Chapters 25 and 26 we almost always took the source of emf to be a battery. But for most devices that you plug into a wall socket, the source of emf is *not* a battery but an electric generating station. Such a station produces electrical energy by converting other forms of energy: gravitational potential energy at a hydroelectric plant, chemical energy in a coal-, gas-, or oil-fired plant, nuclear energy at a nuclear plant. But how is this energy conversion done?

The answer is a phenomenon known as *electromagnetic induction:* If the magnetic flux through a circuit changes, an emf and a current are induced in the circuit. In a power-generating station, magnets move relative to coils of wire to produce a changing magnetic flux in the coils and hence an emf.

The central principle of electromagnetic induction is *Faraday's law.* This law relates induced emf to changing magnetic flux in any loop, including a closed circuit. We also discuss Lenz's law, which helps us to predict the directions of induced emfs and currents. These principles will allow us to understand electrical energy-conversion devices such as motors, generators, and transformers.

Electromagnetic induction tells us that a time-varying magnetic field can act as a source of electric field. We'll also see how a time-varying *electric* field can act as a source of *magnetic* field. These remarkable results form part of a neat package of formulas, called *Maxwell's equations,* that describe the behavior of electric and magnetic fields in general. Maxwell's equations pave the way toward an understanding of electromagnetic waves, the topic of Chapter 32.

LEARNING OUTCOMES

In this chapter, you'll learn...

29.1 The experimental evidence that a changing magnetic field induces an emf.

29.2 How Faraday's law relates the induced emf in a loop to the change in magnetic flux through the loop.

29.3 How to determine the direction of an induced emf.

29.4 How to calculate the emf induced in a conductor moving through a magnetic field.

29.5 How a changing magnetic flux generates a circulating electric field.

29.6 How eddy currents arise in a metal that moves in a magnetic field.

29.7 The four fundamental equations that completely describe both electricity and magnetism.

29.8 The remarkable electric and magnetic properties of superconductors.

You'll need to review...

23.1 Conservative electric fields.

25.4 Electromotive force (emf).

27.3, 27.8, 27.9 Magnetic flux; direct-current motors; Hall effect.

28.5–28.7 Magnetic field of a current loop and solenoid; Ampere's law.

29.1 INDUCTION EXPERIMENTS

During the 1830s, several pioneering experiments with magnetically induced emf were carried out in England by Michael Faraday and in the United States by Joseph Henry (1797–1878). **Figure 29.1** shows several examples. In Fig. 29.1a, a coil of wire is connected to a galvanometer. When the nearby magnet is stationary, the meter shows no current. This isn't surprising; there is no source of emf in the circuit. But when we *move* the magnet either toward or away from the coil, the meter shows current in the circuit, but *only* while the magnet is moving (Fig. 29.1b). If we keep the magnet stationary and move the coil, we again detect a current during the motion. We call this an **induced current,** and the corresponding emf required to cause this current is called an **induced emf.**

In Fig. 29.1c we replace the magnet with a second coil connected to a battery. When the second coil is stationary, there is no current in the first coil. However, when we move the second coil toward or away from the first or move the first toward or away from the second, there is current in the first coil, but again *only* while one coil is moving relative to the other.

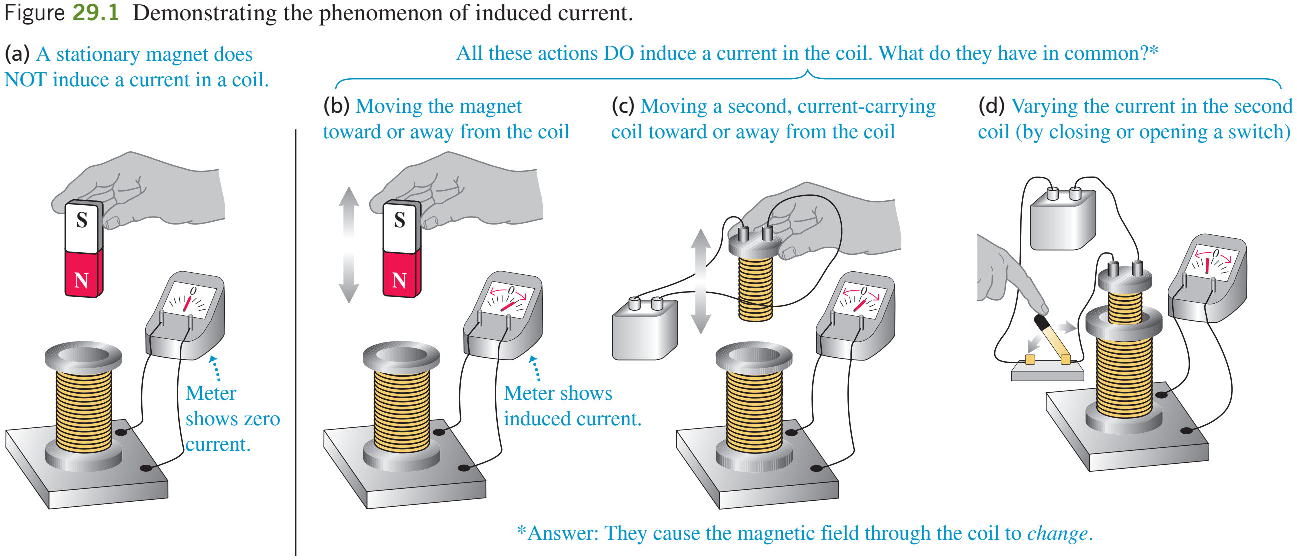
Finally, using the two-coil setup in Fig. 29.1d, we keep both coils stationary and vary the current in the second coil by opening and closing the switch. As we open or close the switch, there is a momentary current pulse in the first coil. The induced current in the first coil is present only while the current in the second coil is changing.

To explore further the common elements in these observations, let's consider a more detailed series of experiments (**Fig. 29.2**). We connect a coil of wire to a galvanometer and then place the coil between the poles of an electromagnet whose magnetic field we can vary. Here's what we observe:

1. When there is no current in the electromagnet, so that $\vec{B} = \mathbf{0}$, the galvanometer shows no current.
2. When the electromagnet is turned on, there is a momentary current through the meter as $\vec{B}$ increases.
3. When $\vec{B}$ levels off at a steady value, the current drops to zero.
4. With the coil in a horizontal plane, we squeeze it so as to decrease the cross-sectional area of the coil. The meter detects current only *during* the deformation, not before or after. When we increase the area to return the coil to its original shape, there is current in the opposite direction, but only while the area of the coil is changing.

BIO APPLICATION Exploring the Brain with Induced emfs Transcranial magnetic stimulation (TMS) is a technique for studying the function of various parts of the brain. A coil held to the subject's head carries a varying electric current, and so produces a varying magnetic field. This field causes an induced emf, and that triggers electric activity in the region of the brain underneath the coil. By observing how the TMS subject responds (for instance, which muscles move as a result of stimulating a certain part of the brain), a physician can test for various neurological conditions.

Figure 29.1 Demonstrating the phenomenon of induced current.



5. If we rotate the coil a few degrees about a horizontal axis, the meter detects current during the rotation, in the same direction as when we decreased the area. When we rotate the coil back, there is a current in the opposite direction during this rotation.
6. If we jerk the coil out of the magnetic field, there is a current during the motion, in the same direction as when we decreased the area.
7. If we decrease the number of turns in the coil by unwinding one or more turns, there is a current during the unwinding, in the same direction as when we decreased the area. If we wind more turns onto the coil, there is a current in the opposite direction during the winding.
8. When the magnet is turned off, there is a momentary current in the direction opposite to the current when it was turned on.
9. The faster we carry out any of these changes, the greater the current.
10. If all these experiments are repeated with a coil that has the same shape but different material and different resistance, the current in each case is inversely proportional to the total circuit resistance. This shows that the induced emfs that are causing the current do not depend on the material of the coil but only on its shape and the magnetic field.

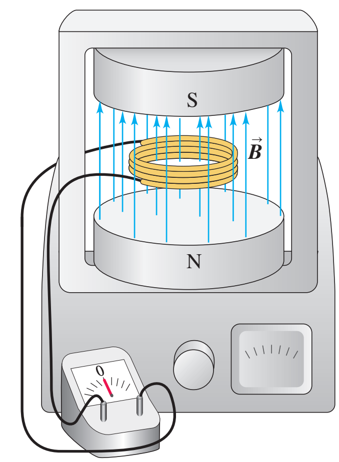


Figure 29.2 A coil in a magnetic field. When the $\vec{B}$ field is constant and the shape, location, and orientation of the coil do not change, no current is induced in the coil. A current is induced when any of these factors change.

The common element in these experiments is changing *magnetic flux* Φ_B through the coil connected to the galvanometer. In each case the flux changes either because the magnetic field changes with time or because the coil is moving through a nonuniform magnetic field. What's more, in each case the induced emf is proportional to the *rate of change* of magnetic flux Φ_B through the coil. The *direction* of the induced emf depends on whether the flux is increasing or decreasing. If the flux is constant, there is no induced emf.

Induced emfs have a tremendous number of practical applications. If you are reading these words indoors, you are making use of induced emfs right now! At the power plant that supplies your neighborhood, an electric generator produces an emf by varying the magnetic flux through coils of wire. (In the next section we'll see how this is done.) This emf supplies the voltage between the terminals of the wall sockets in your home, and this voltage supplies the power to your reading lamp.

Magnetically induced emfs, just like the emfs discussed in Section 25.4, are the result of *nonelectrostatic* forces. We have to distinguish carefully between the electrostatic electric fields produced by charges (according to Coulomb's law) and the nonelectrostatic electric fields produced by changing magnetic fields. We'll return to this distinction later in this chapter and the next.

29.2 FARADAY'S LAW

The common element in all induction effects is changing magnetic flux through a circuit. Before stating the simple physical law that summarizes all of the kinds of experiments described in Section 29.1, let's first review the concept of magnetic flux Φ_B (which we introduced in Section 27.3). For an infinitesimal-area element $d\vec{A}$ in a magnetic field $\vec{B}$ (**Fig. 29.3**), the magnetic flux $d\Phi_B$ through the area is

$$d\Phi_B = \vec{B} \cdot d\vec{A} = B_\perp \, dA = B \, dA \cos\phi$$

where $B_\perp$ is the component of $\vec{B}$ perpendicular to the surface of the area element and ϕ is the angle between $\vec{B}$ and $d\vec{A}$. (As in Chapter 27, be careful to distinguish between two quantities named "phi," ϕ and Φ_B.) The total magnetic flux Φ_B through a finite area is the integral of this expression over the area:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \, dA \cos\phi \qquad (29.1)$$

Figure 29.3 Calculating the magnetic flux through an area element.

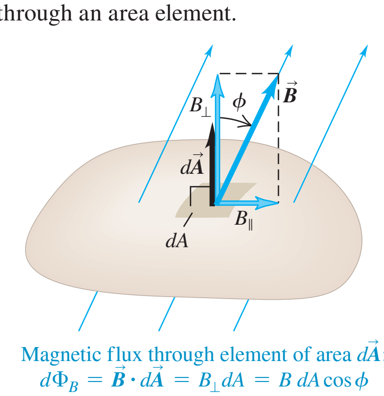
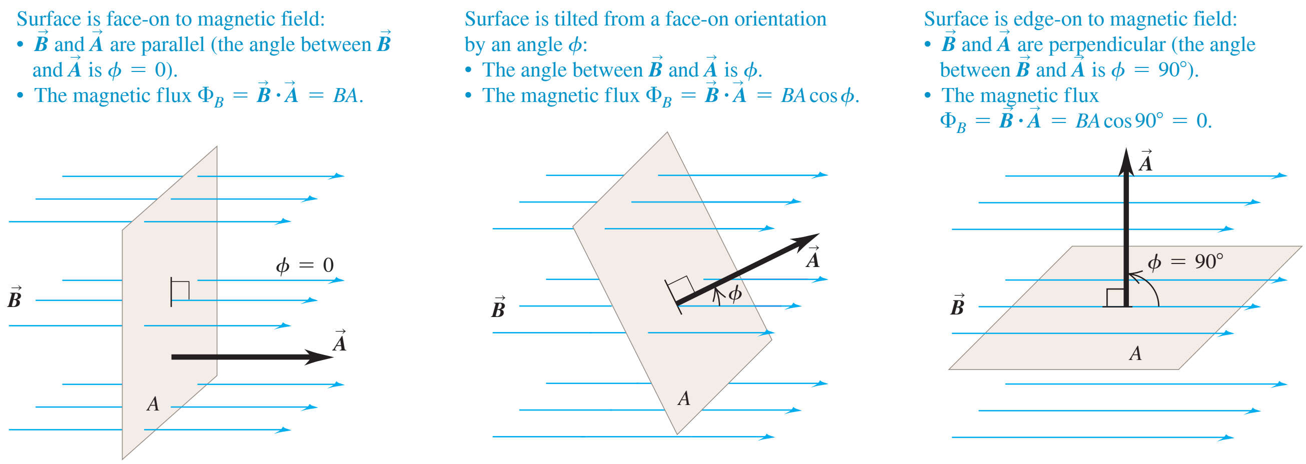


Figure 29.4 Calculating the flux of a uniform magnetic field through a flat area. (Compare to Fig. 22.6, which shows the rules for calculating the flux of a uniform *electric* field.)



If $\vec{B}$ is uniform over a flat area $\vec{A}$, then

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\phi \tag{29.2}$$

Figure 29.4 reviews the rules for using Eq. (29.2).

CAUTION **Choosing the direction of $d\vec{A}$ or $\vec{A}$** In Eqs. (29.1) and (29.2) we must define the direction of the vector area $d\vec{A}$ or $\vec{A}$ unambiguously. There are always two directions perpendicular to any given area, and the sign of the magnetic flux through the area depends on which one we choose. For example, in Fig. 29.3 we chose $d\vec{A}$ to point upward, so ϕ is less than 90° and $\vec{B}\cdot d\vec{A}$ is positive. We could have chosen $d\vec{A}$ to point downward, in which case ϕ would have been greater than 90° and $\vec{B}\cdot d\vec{A}$ would have been negative. Both choices are equally good, but once we make a choice we must stick with it.

Faraday's law of induction states:

Faraday's law: The induced emf in a closed loop ...

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \tag{29.3}$$

... equals the negative of the time rate of change of magnetic flux through the loop.

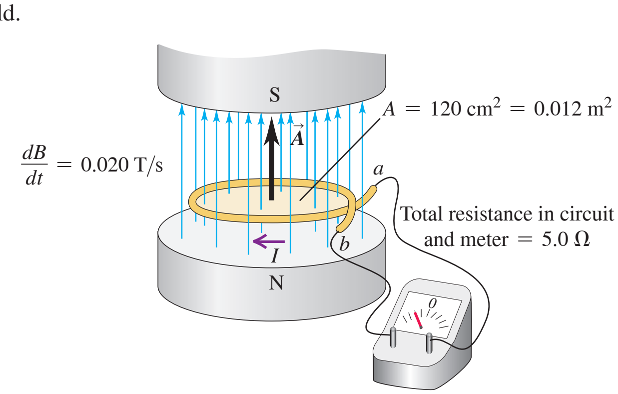
To understand the negative sign, we have to introduce a sign convention for the induced emf $\mathcal{E}$. But first let's look at a simple example of this law in action.

EXAMPLE 29.1 Emf and current induced in a loop

WITH VARIATION PROBLEMS

The magnetic field between the poles of the electromagnet in **Fig. 29.5** is uniform at any time, but its magnitude is increasing at the rate of 0.020 T/s. The area of the conducting loop in the field is 120 cm^2, and the total circuit resistance, including the meter, is 5.0 Ω. (a) Find the induced emf and the induced current in the circuit. (b) If the loop is replaced by one made of an insulator, what effect does this have on the induced emf and induced current?

Figure 29.5 A stationary conducting loop in an increasing magnetic field.



IDENTIFY and SET UP The magnetic flux Φ_B through the loop changes as the magnetic field changes. Hence there will be an induced emf $\mathcal{E}$ and an induced current I in the loop. We calculate Φ_B from Eq. (29.2), then find $\mathcal{E}$ by using Faraday's law. Finally, we calculate I from $\mathcal{E} = IR$, where R is the total resistance of the circuit that includes the loop.

EXECUTE (a) The area vector $\vec{A}$ for the loop is perpendicular to the plane of the loop; we take $\vec{A}$ to be vertically upward. Then $\vec{A}$ and $\vec{B}$ are parallel, and because $\vec{B}$ is uniform the magnetic flux through the loop is $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos 0 = BA$. The area $A = 0.012\ \text{m}^2$ is constant, so the rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{dB}{dt}A = (0.020\ \text{T/s})(0.012\ \text{m}^2)$$
$$= 2.4 \times 10^{-4}\ \text{V} = 0.24\ \text{mV}$$

This, apart from a sign that we haven't discussed yet, is the induced emf $\mathcal{E}$. The corresponding induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{2.4 \times 10^{-4}\ \text{V}}{5.0\ \Omega} = 4.8 \times 10^{-5}\ \text{A} = 0.048\ \text{mA}$$

(b) By changing to an insulating loop, we've made the resistance of the loop very high. Faraday's law, Eq. (29.3), does not involve the resistance of the circuit in any way, so the induced *emf* does not change. But the *current* will be smaller, as given by the equation $I = \mathcal{E}/R$. If the loop is made of a perfect insulator with infinite resistance, the induced current is zero. This situation is analogous to an isolated battery whose terminals aren't connected to anything: An emf is present, but no current flows.

EVALUATE We can verify unit consistency in this calculation by noting that the magnetic-force relationship $\vec{F} = q\vec{v} \times \vec{B}$ implies that the units of $\vec{B}$ are the units of force divided by the units of (charge times velocity): $1\ \text{T} = (1\ \text{N})/(1\ \text{C} \cdot \text{m/s})$. The units of magnetic flux are then $(1\ \text{T})(1\ \text{m}^2) = 1\ \text{N} \cdot \text{s} \cdot \text{m/C}$, and the rate of change of magnetic flux is $1\ \text{N} \cdot \text{m/C} = 1\ \text{J/C} = 1\ \text{V}$. Thus the unit of $d\Phi_B/dt$ is the volt, as required by Eq. (29.3). Also recall from Section 27.3 that the unit of magnetic flux is the weber (Wb): $1\ \text{T} \cdot \text{m}^2 = 1\ \text{Wb}$, so $1\ \text{V} = 1\ \text{Wb/s}$.

KEYCONCEPT When there is a change in the magnetic flux through a curve in space that forms a closed loop, an emf is induced in that loop. If the loop is made of a conductor, an induced current flows in response to the emf.

Direction of Induced emf

We can find the direction of an induced emf or current by using Eq. (29.3) together with some simple sign rules. Here's the procedure:

1. Define a positive direction for the vector area $\vec{A}$.
2. From the directions of $\vec{A}$ and the magnetic field $\vec{B}$, determine the sign of the magnetic flux Φ_B and its rate of change $d\Phi_B/dt$. **Figure 29.6** shows several examples.
3. Determine the sign of the induced emf or current. If the flux is increasing, so $d\Phi_B/dt$ is positive, then the induced emf or current is negative; if the flux is decreasing, $d\Phi_B/dt$ is negative and the induced emf or current is positive.
4. Finally, use your right hand to determine the direction of the induced emf or current. Curl the fingers of your right hand around the $\vec{A}$ vector, with your right thumb in the direction of $\vec{A}$. If the induced emf or current in the circuit is *positive,* it is in the same direction as your curled fingers; if the induced emf or current is *negative,* it is in the opposite direction.

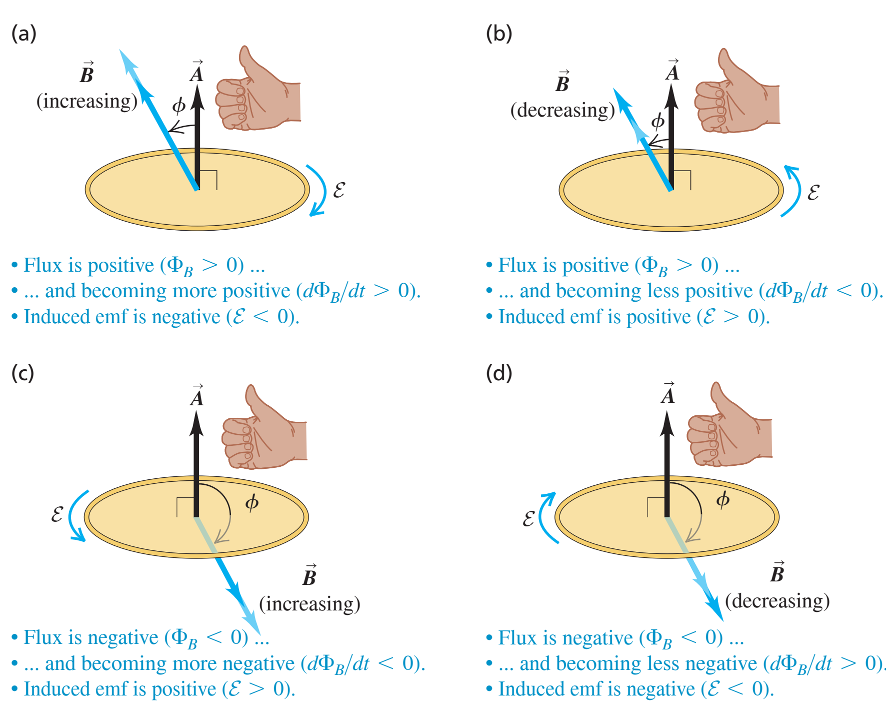


Figure **29.6** The magnetic flux is becoming (a) more positive, (b) less positive, (c) more negative, and (d) less negative. Therefore Φ_B is increasing in (a) and (d) and decreasing in (b) and (c). In (a) and (d) the emfs are negative (they are opposite to the direction of the curled fingers of your right hand when your right thumb points along $\vec{A}$). In (b) and (c) the emfs are positive (in the same direction as the curled fingers).

CAUTION Induced emfs are caused by changes in flux It can be tempting to think that *flux* is the cause of induced emf and that an induced emf will appear in a circuit whenever there is a magnetic field in the region bordered by the circuit. But Eq. (29.3) shows that only a *change* in flux through a circuit, not flux itself, can induce an emf in a circuit. If the flux through a circuit has a constant value, whether positive, negative, or zero, there is no induced emf. Note also that an induced emf will appear in a circuit if the flux is changed for any reason, including rotating the circuit relative to the magnetic field or changing the circuit's shape.

In Example 29.1, in which $\vec{A}$ is upward, a positive $\mathcal{E}$ would be directed counterclockwise around the loop, as seen from above. Both $\vec{A}$ and $\vec{B}$ are upward in this example, so Φ_B is positive; the magnitude B is increasing, so $d\Phi_B/dt$ is positive. Hence by Eq. (29.3), $\mathcal{E}$ in Example 29.1 is *negative*—that is, *clockwise* around the loop, as seen from above.

If the loop in Fig. 29.5 is a conductor, the clockwise induced emf causes a clockwise induced current. This induced current produces an additional magnetic field through the loop, and the right-hand rule described in Section 28.5 shows that this field is *opposite* in direction to the increasing field produced by the electromagnet. This is an example of a general rule called *Lenz's law,* which says that any induction effect tends to oppose the change that caused it; in this case the change is the increase in the flux of the electromagnet's field through the loop. (We'll study Lenz's law in detail in the next section.)

You should check out the signs of the induced emfs and currents for the list of experiments in Section 29.1. For example, when the loop in Fig. 29.2 is in a constant field and we tilt it or squeeze it to *decrease* the flux through it, the induced emf and current are counterclockwise, as seen from above.

If a coil has N identical turns and if the flux varies at the same rate through each turn, the *total* rate of change through all turns is N times that for a single turn. If Φ_B is the flux through each turn, the total emf in a coil with N turns is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \tag{29.4}$$

As we discussed in this chapter's introduction, induced emfs play an essential role in the generation of electrical power for commercial use. Several of the following examples explore different methods of producing emfs by changing the flux through a circuit.

PROBLEM-SOLVING STRATEGY 29.1 Faraday's Law

IDENTIFY *the relevant concepts:* Faraday's law applies when a magnetic flux is changing. To use the law, identify an area through which there is a flux of magnetic field. This will usually be the area enclosed by a loop made of a conducting material (though not always—see part (b) of Example 29.1). Identify the target variables.

SET UP *the problem* using the following steps:

1. Faraday's law relates the induced emf to the rate of change of magnetic flux. To calculate this rate of change, you first have to understand what is making the flux change. Is the conductor moving or changing orientation? Is the magnetic field changing?
2. The area vector $\vec{A}$ (or $d\vec{A}$) must be perpendicular to the plane of the area. You always have two choices of its direction; for example, if the area is in a horizontal plane, $\vec{A}$ could point up or down. Choose a direction and use it throughout the problem.

EXECUTE *the solution* as follows:

1. Calculate the magnetic flux from Eq. (29.2) if $\vec{B}$ is uniform over the area of the loop or Eq. (29.1) if it isn't uniform. Remember the direction you chose for the area vector.
2. Calculate the induced emf from Eq. (29.3) or (if your conductor has N turns in a coil) Eq. (29.4). Apply the sign rule (described just after Example 29.1) to determine the positive direction of emf.
3. If the circuit resistance is known, you can calculate the magnitude of the induced current I by using $\mathcal{E} = IR$.

EVALUATE *your answer:* Check your results for the proper units, and double-check that you have properly implemented the sign rules for magnetic flux and induced emf.

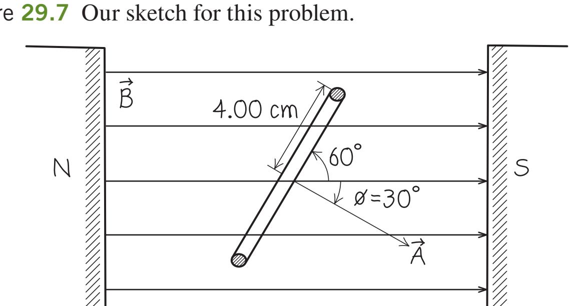
EXAMPLE 29.2 Magnitude and direction of an induced emf

WITH VARIATION PROBLEMS

A 500-loop circular wire coil with radius 4.00 cm is placed between the poles of a large electromagnet. The magnetic field is uniform and makes an angle of 60° with the plane of the coil; it decreases at 0.200 T/s. What are the magnitude and direction of the induced emf?

IDENTIFY and SET UP Our target variable is the emf induced by a varying magnetic flux through the coil. The flux varies because the magnetic field decreases in amplitude. We choose the area vector $\vec{A}$ to be in the direction shown in **Fig. 29.7**. With this choice, the geometry is similar to that of Fig. 29.6b. That figure will help us determine the direction of the induced emf.

Figure 29.7 Our sketch for this problem.



EXECUTE The magnetic field is uniform over the loop, so we can calculate the flux from Eq. (29.2): $\Phi_B = BA\cos\phi$, where $\phi = 30°$. In this expression, the only quantity that changes with time is the magnitude B of the field, so $d\Phi_B/dt = (dB/dt)A\cos\phi$.

CAUTION Remember how ϕ is defined You may have been tempted to say that $\phi = 60°$ in this problem. If so, remember that ϕ is the angle between $\vec{A}$ and $\vec{B}$, *not* the angle between $\vec{B}$ and the plane of the loop.

From Eq. (29.4), the induced emf in the coil of $N = 500$ turns is

$$\begin{aligned}\mathcal{E} &= -N\frac{d\Phi_B}{dt} = -N\frac{dB}{dt}A\cos\phi \\ &= -500(-0.200\ \text{T/s})\pi(0.0400\ \text{m})^2(\cos 30°) = 0.435\ \text{V}\end{aligned}$$

The positive answer means that when you point your right thumb in the direction of area vector $\vec{A}$ (30° below field $\vec{B}$ in Fig. 29.7), the positive direction for $\mathcal{E}$ is in the direction of the curled fingers of your right hand. If you viewed the coil from the left in Fig. 29.7 and looked in the direction of $\vec{A}$, the emf would be clockwise.

EVALUATE If the ends of the wire are connected, the direction of current in the coil is in the same direction as the emf—that is, clockwise as seen from the left side of the coil. A clockwise current increases the magnetic flux through the coil, and therefore tends to oppose the decrease in total flux. This is an example of Lenz's law, which we'll discuss in Section 29.3.

KEYCONCEPT Faraday's law states that the induced emf in a coil is proportional to the rate of change of the magnetic flux through the coil and to the number of turns in the coil. The sign of the induced emf tells you its direction.

EXAMPLE 29.3 Generator I: A simple alternator

WITH VARIATION PROBLEMS

Figure 29.8a shows a simple *alternator,* a device that generates an emf. A rectangular loop is rotated with constant angular speed ω about the axis shown. The magnetic field $\vec{B}$ is uniform and constant. At time $t = 0$, $\phi = 0$. Determine the induced emf.

IDENTIFY and SET UP The magnetic field $\vec{B}$ and the loop area A are constant, but the flux through the loop varies because the loop rotates and so the angle ϕ between $\vec{B}$ and the area vector $\vec{A}$ changes (Fig. 29.8a). Because the angular speed is constant and $\phi = 0$ at $t = 0$, the angle as a function of time is $\phi = \omega t$.

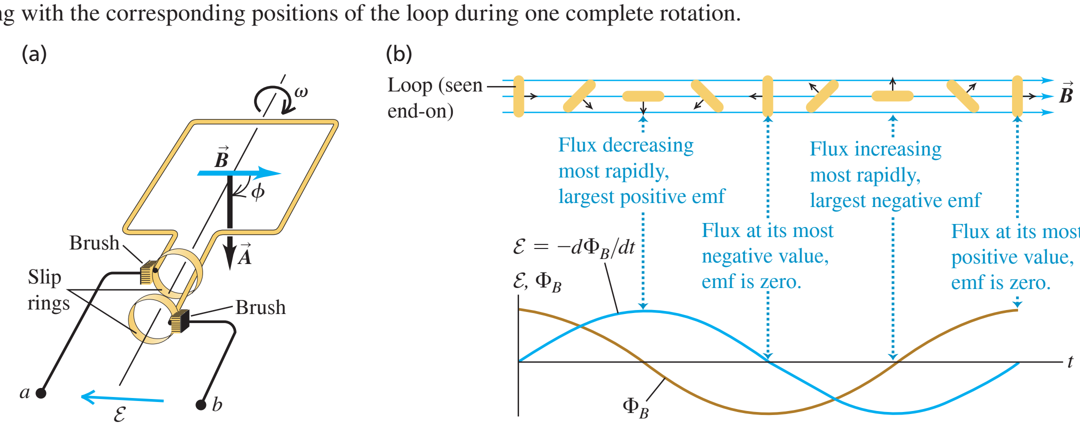
EXECUTE The magnetic field is uniform over the loop, so the magnetic flux is $\Phi_B = BA\cos\phi = BA\cos\omega t$. Hence, by Faraday's law [Eq. (29.3)] the induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA\cos\omega t) = \omega BA\sin\omega t$$

EVALUATE The induced emf $\mathcal{E}$ varies sinusoidally with time (see Fig. 29.8b). When the plane of the loop is perpendicular to $\vec{B}$ ($\phi = 0$ or 180°), Φ_B reaches its maximum and minimum values. At these times, its instantaneous rate of change is zero and $\mathcal{E}$ is zero. Conversely, $\mathcal{E}$ reaches its maximum and minimum values when the plane of the loop is parallel to $\vec{B}$ ($\phi = 90°$ or 270°) and Φ_B is changing most rapidly. We note that the induced emf does not depend on the *shape* of the loop, but only on its area.

We can use the alternator as a source of emf in an external circuit by using two *slip rings* that rotate with the loop, as shown in Fig. 29.8a. The rings slide against stationary contacts called *brushes,* which are connected to the output terminals a and b. Since the emf varies sinusoidally, the current that results in the circuit is an *alternating* current that also

Figure 29.8 (a) Schematic diagram of an alternator. A conducting loop rotates in a magnetic field, producing an emf. Connections from each end of the loop to the external circuit are made by means of that end's slip ring. The system is shown at the time when the angle $\phi = \omega t = 90°$. (b) Graph of the flux through the loop and the resulting emf between terminals a and b, along with the corresponding positions of the loop during one complete rotation.



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varies sinusoidally in magnitude and direction. The amplitude of the emf can be increased by increasing the rotation speed, the field magnitude, or the loop area or by using N loops instead of one, as in Eq. (29.4).

Alternators are used in automobiles to generate the currents in the ignition, the lights, and the entertainment system. The arrangement is a little different than in this example; rather than having a rotating loop in a magnetic field, the loop stays fixed and an electromagnet rotates. (The rotation is provided by a mechanical connection between the alternator and the engine.) But the result is the same; the flux through the loop varies sinusoidally, producing a sinusoidally varying emf. Larger alternators of this same type are used in electric power plants (**Fig. 29.9**).

KEYCONCEPT In an alternator, a coil rotates relative to a magnetic field. The flux through the coil varies sinusoidally with time, as does the emf induced in the coil. The more rapid the rotation, the greater the magnitudes of the induced emf and current.

Figure 29.9 A commercial alternator uses many loops of wire wound around a barrel-like structure called an armature. The armature and wire remain stationary while electromagnets rotate on a shaft (not shown) through the center of the armature. The resulting induced emf is far larger than would be possible with a single loop of wire.

EXAMPLE 29.4 Generator II: A dc generator and back emf in a motor

The alternator in Example 29.3 produces a sinusoidally varying emf and hence an alternating current. **Figure 29.10a** shows a *direct-current* (dc) *generator* that produces an emf that always has the same sign. The arrangement of split rings, called a *commutator,* reverses the connections to the external circuit at angular positions at which the emf reverses. Figure 29.10b shows the resulting emf. Commercial dc generators have a large number of coils and commutator segments, smoothing out the bumps in the emf so that the terminal voltage is not only one-directional but also practically constant. This brush-and-commutator arrangement is the same as that in the direct-current motor discussed in Section 27.8. The motor's *back emf* is just the emf induced by the changing magnetic flux through its rotating coil. Consider a motor with a square, 500-turn coil 10.0 cm on a side. If the magnetic field has magnitude 0.200 T, at what rotation speed is the *average* back emf of the motor equal to 112 V?

IDENTIFY and SET UP As far as the rotating loop is concerned, the situation is the same as in Example 29.3 except that we now have N turns of wire. Without the commutator, the emf would alternate between positive and negative values and have an average value of zero (see Fig. 29.8b). With the commutator, the emf is never negative and its average value is positive (Fig. 29.10b). We'll use our result from Example 29.3 to obtain an expression for this average value and solve it for the rotational speed ω.

EXECUTE Comparison of Figs. 29.8b and 29.10b shows that the back emf of the motor is just N times the absolute value of the emf found for an alternator in Example 29.3, as in Eq. (29.4): $|\mathcal{E}| = N\omega BA|\sin\omega t|$. To find the *average* back emf, we must replace $|\sin\omega t|$ by its average value. We find this by integrating $|\sin\omega t|$ over half a cycle, from $t = 0$ to $t = T/2 = \pi/\omega$, and dividing by the elapsed time π/ω. During this half cycle, the sine function is positive, so $|\sin\omega t| = \sin\omega t$, and we find

$$(|\sin\omega t|)_{\text{av}} = \frac{\int_0^{\pi/\omega} \sin\omega t\, dt}{\pi/\omega} = \frac{2}{\pi}$$

The average back emf is then

$$\mathcal{E}_{\text{av}} = \frac{2N\omega BA}{\pi}$$

Solving for ω, we obtain

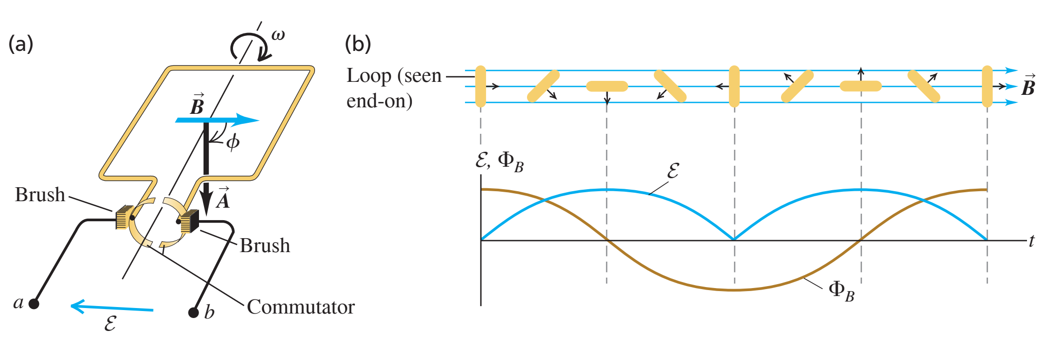
$$\omega = \frac{\pi\mathcal{E}_{\text{av}}}{2NBA} = \frac{\pi(112\text{ V})}{2(500)(0.200\text{ T})(0.100\text{ m})^2} = 176\text{ rad/s}$$

(Recall from Example 29.1 that $1\text{ V} = 1\text{ Wb/s} = 1\text{ T}\cdot\text{m}^2\text{/s}$.)

EVALUATE The average back emf is directly proportional to ω. Hence the slower the rotation speed, the less the back emf and the greater the possibility of burning out the motor, as we described in Example 27.11 (Section 27.8).

KEYCONCEPT The back emf in a motor is a result of Faraday's law. The average value of the back emf is proportional to the rotation speed of the motor.

Figure 29.10 (a) Schematic diagram of a dc generator, using a split-ring commutator. The ring halves are attached to the loop and rotate with it. (b) Graph of the resulting induced emf between terminals a and b. Compare to Fig. 29.8b.



EXAMPLE 29.5 Generator III: The slidewire generator

Figure 29.11 shows a U-shaped conductor in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the figure and directed *into* the page. We lay a metal rod (the "slidewire") with length L across the two arms of the conductor, forming a circuit, and move it to the right with constant velocity $\vec{v}$. This induces an emf and a current, which is why this device is called a *slidewire generator*. Find the magnitude and direction of the resulting induced emf.

IDENTIFY and SET UP The magnetic flux changes because the area of the loop—bounded on the right by the moving rod—is increasing. Our target variable is the emf $\mathcal{E}$ induced in this expanding loop. The magnetic field is uniform over the area of the loop, so we can find the flux from $\Phi_B = BA\cos\phi$. We choose the area vector $\vec{A}$ to point straight into the page, in the same direction as $\vec{B}$. With this choice a positive emf will be one that is directed clockwise around the loop. (You can check this with the right-hand rule: Using your right hand, point your thumb into the page and curl your fingers as in Fig. 29.6.)

EXECUTE Since $\vec{B}$ and $\vec{A}$ point in the same direction, the angle $\phi = 0$ and $\Phi_B = BA$. The magnetic-field magnitude B is constant, so the induced emf is

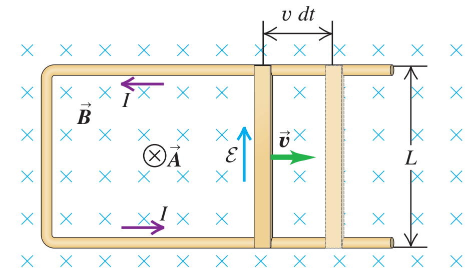
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt}$$

To calculate dA/dt, note that in a time dt the sliding rod moves a distance $v\,dt$ (Fig. 29.11) and the loop area increases by an amount $dA = Lv\,dt$. Hence the induced emf is

$$\mathcal{E} = -B\frac{Lv\,dt}{dt} = -BLv$$

The minus sign tells us that the emf is directed *counterclockwise* around the loop. The induced current is also counterclockwise, as shown in the figure.

Figure **29.11** A slidewire generator. The magnetic field $\vec{B}$ and the vector area $\vec{A}$ are both directed into the figure. The increase in magnetic flux (caused by an increase in area) induces the emf and current.



EVALUATE The emf of a slidewire generator is constant if $\vec{v}$ is constant. Hence the slidewire generator is a *direct-current* generator. It's not a very practical device because the rod eventually moves beyond the U-shaped conductor and loses contact, after which the current stops.

KEYCONCEPT An induced emf can be produced if the area of a circuit in a magnetic field changes. The magnitude of the emf depends on the speed at which the area of the circuit changes.

EXAMPLE 29.6 Work and power in the slidewire generator

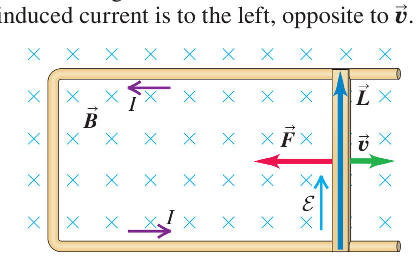
In the slidewire generator of Example 29.5, energy is dissipated in the circuit owing to its resistance. Let the resistance of the circuit (made up of the moving slidewire and the U-shaped conductor that connects the ends of the slidewire) at a given point in the slidewire's motion be R. Find the rate at which energy is dissipated in the circuit and the rate at which work must be done to move the rod through the magnetic field.

IDENTIFY and SET UP Our target variables are the *rates* at which energy is dissipated and at which work is done. Energy is dissipated in the circuit at the rate $P_{\text{dissipated}} = I^2R$. The current I in the circuit equals $|\mathcal{E}|/R$; we found an expression for the induced emf $\mathcal{E}$ in this circuit in Example 29.5. There is a magnetic force $\vec{F} = I\vec{L}\times\vec{B}$ on the rod, where $\vec{L}$ points along the rod in the direction of the current. **Figure 29.12** shows that this force is opposite to the rod velocity $\vec{v}$; to maintain the motion, whoever is pushing the rod must apply a force of equal magnitude in the direction of $\vec{v}$. This force does work at the rate $P_{\text{applied}} = Fv$.

EXECUTE First we'll calculate $P_{\text{dissipated}}$. From Example 29.5, $\mathcal{E} = -BLv$, so the current in the rod is $I = |\mathcal{E}|/R = Blv/R$. Hence

$$P_{\text{dissipated}} = I^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$$

Figure **29.12** The magnetic force $\vec{F} = I\vec{L}\times\vec{B}$ that acts on the rod due to the induced current is to the left, opposite to $\vec{v}$.



To calculate P_{applied}, we first calculate the magnitude of $\vec{F} = I\vec{L}\times\vec{B}$. Since $\vec{L}$ and $\vec{B}$ are perpendicular, this magnitude is

$$F = ILB = \frac{BLv}{R}LB = \frac{B^2L^2v}{R}$$

The applied force has the same magnitude and does work at the rate

$$P_{\text{applied}} = Fv = \frac{B^2L^2v^2}{R}$$

Continued

CAUTION **You can't violate energy conservation** You might think that reversing the direction of $\vec{B}$ or of $\vec{v}$ would allow the magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ to be in the *same* direction as $\vec{v}$. This would be a neat trick. Once the rod was moving, the changing magnetic flux would induce an emf and a current, and the magnetic force on the rod would make it move even faster, increasing the emf and current until the rod was moving at tremendous speed and producing electrical power at a prodigious rate. If this seems too good to be true and a violation of energy conservation, that's because it is. Reversing $\vec{B}$ also reverses the sign of the induced emf and current and hence the direction of $\vec{L}$, so the magnetic force still opposes the motion of the rod; a similar result holds true if we reverse $\vec{v}$.

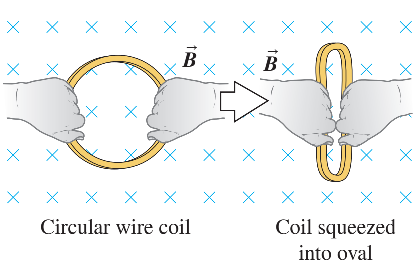
EVALUATE The rate at which work is done is exactly *equal* to the rate at which energy is dissipated in the resistance.

KEYCONCEPT You must do work to move any part of a circuit in a magnetic field. The work that you do is used to make the induced current flow in the presence of the circuit's resistance.

Generators as Energy Converters

Example 29.6 shows that the slidewire generator doesn't produce electrical energy out of nowhere; the energy is supplied by whatever object exerts the force that keeps the rod moving. All that the generator does is *convert* that energy into a different form. The equality between the rate at which *mechanical* energy is supplied to a generator and the rate at which *electrical* energy is generated holds for all types of generators, including the alternator described in Example 29.3. (We are ignoring the effects of friction in the bearings of an alternator or between the rod and the U-shaped conductor of a slidewire generator. The energy lost to friction is not available for conversion to electrical energy, so in real generators the friction is kept to a minimum.)

In Chapter 27 we stated that the magnetic force on moving charges can never do work. You might think, however, that the magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ in Example 29.6 *is* doing (negative) work on the current-carrying rod as it moves, contradicting our earlier statement. In fact, the work done by the magnetic force is zero. The moving charges that make up the current in the rod in Fig. 29.12 have a vertical component of velocity, causing a horizontal component of force on these charges. As a result, there is a horizontal displacement of charge within the rod, the left side acquiring a net positive charge and the right side a net negative charge. The result is a horizontal component of electric field, perpendicular to the length of the rod (analogous to the Hall effect, described in Section 27.9). It is this field, in the direction of motion of the rod, that does work on the mobile charges in the rod and hence indirectly on the atoms making up the rod.



TEST YOUR UNDERSTANDING OF SECTION 29.2 The accompanying figure shows a wire coil being squeezed in a uniform magnetic field. (a) While the coil is being squeezed, is the induced emf in the coil (i) clockwise, (ii) counterclockwise, or (iii) zero? (b) Once the coil has reached its final squeezed shape, is the induced emf in the coil (i) clockwise, (ii) counterclockwise, or (iii) zero?

ANSWER

(a) **(i)**, (b) **(iii)** In (a), initially there is magnetic flux into the plane of the page, which we call positive. While the loop is being squeezed, the flux is becoming less positive ($d\Phi_B/dt < 0$) and so the induced emf is positive as in Fig. 29.6b ($\mathcal{E} = -d\Phi_B/dt > 0$). If you point the thumb of your right hand into the page, your fingers curl clockwise, so this is the direction of positive induced emf. In (b), since the coil's shape is no longer changing, the magnetic flux is not changing and there is no induced emf.

29.3 LENZ'S LAW

Lenz's law is a convenient alternative method for determining the direction of an induced current or emf. Lenz's law, named for the Russian physicist H. F. E. Lenz (1804–1865), is not an independent principle; it can be derived from Faraday's law. It always gives the same results as the sign rules we introduced in connection with Faraday's law, but it is often easier to use. Lenz's law also helps us gain intuitive understanding of various induction effects and of the role of energy conservation.

LENZ'S LAW The direction of any magnetic induction effect is such as to oppose the cause of the effect.

The "cause" may be changing flux through a stationary circuit due to a varying magnetic field, changing flux due to motion of the conductors that make up the circuit, or any combination. If the flux in a stationary circuit changes, as in Examples 29.1 and 29.2, the induced current sets up a magnetic field of its own. Within the area bounded by the circuit, this field is *opposite* to the original field if the original field is *increasing* but is in the *same* direction as the original field if the latter is *decreasing*. That is, the induced current opposes the *change in flux* through the circuit (*not* the flux itself).

If the flux change is due to motion of the conductors, as in Examples 29.3 through 29.6, the direction of the induced current in the moving conductor is such that the direction of the magnetic-field force on the conductor is opposite in direction to its motion. Thus the motion of the conductor, which caused the induced current, is opposed. We saw this explicitly for the slidewire generator in Example 29.6. In all these cases the induced current tries to preserve the *status quo* by opposing motion or a change of flux.

Lenz's law is also directly related to energy conservation. If the induced current in Example 29.6 were in the direction opposite to that given by Lenz's law, the magnetic force on the rod would accelerate it to ever-increasing speed with no external energy source, even though electrical energy is being dissipated in the circuit. This would be a clear violation of energy conservation and doesn't happen in nature.

CONCEPTUAL EXAMPLE 29.7 Lenz's law and the slidewire generator

WITH VARIATION PROBLEMS

In Fig. 29.11, the induced current in the loop causes an additional magnetic field in the area bounded by the loop. The direction of the induced current is counterclockwise, so from the discussion of Section 28.5, this additional magnetic field is directed *out of* the plane of the figure. That direction is opposite that of the original magnetic field, so it tends to cancel the effect of that field. This is just what Lenz's law predicts.

KEYCONCEPT A change in the magnetic flux through a circuit induces a current that produces an additional magnetic field of its own. This induced field always opposes the change in the magnetic flux (Lenz's law).

CONCEPTUAL EXAMPLE 29.8 Lenz's law and the direction of induced current

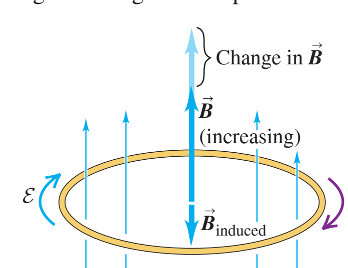
WITH VARIATION PROBLEMS

In **Fig. 29.13** there is a uniform magnetic field $\vec{B}$ through the coil. The magnitude of the field is increasing, so there is an induced emf. Use Lenz's law to determine the direction of the resulting induced current.

SOLUTION This situation is the same as in Example 29.1 (Section 29.2). By Lenz's law the induced current must produce a magnetic field $\vec{B}_{\text{induced}}$ inside the coil that is downward, opposing the change in flux. From the right-hand rule we described in Section 28.5 for the direction of the magnetic field produced by a circular loop, $\vec{B}_{\text{induced}}$ will be in the desired direction if the induced current flows as shown in Fig. 29.13.

Figure 29.14 (next page) shows several applications of Lenz's law to the similar situation of a magnet moving near a conducting loop. In each case, the induced current produces a magnetic field whose direction opposes the change in flux through the loop due to the magnet's motion.

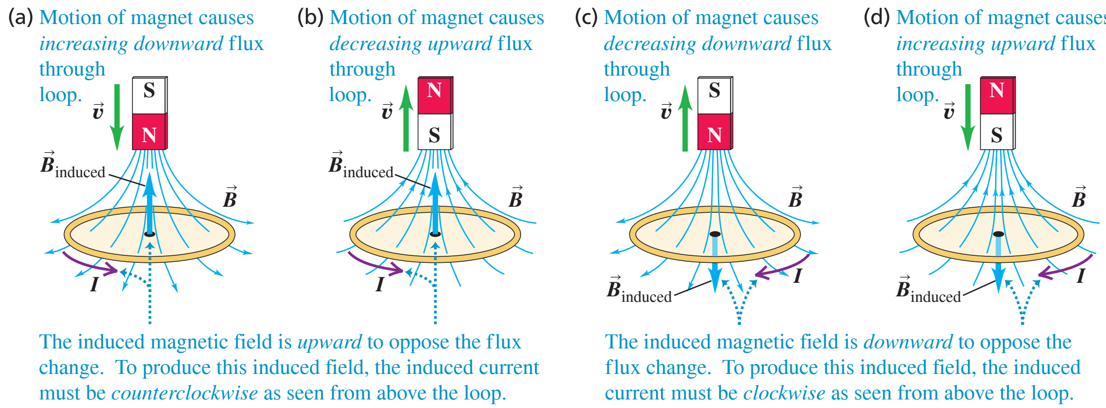
Figure **29.13** The induced current due to the change in $\vec{B}$ is clockwise, as seen from above the loop. The added field $\vec{B}_{\text{induced}}$ that it causes is downward, opposing the change in the upward field $\vec{B}$.



KEYCONCEPT Whenever there is a changing magnetic flux through a circuit, you can use Lenz's law to decide what the induced current direction will be: The current must produce an induced magnetic field that opposes the change in the flux.

Continued

Figure **29.14** Directions of induced currents as a bar magnet moves along the axis of a conducting loop. If the bar magnet is stationary, there is no induced current.



Lenz's Law and the Response to Flux Changes

Since an induced current always opposes any change in magnetic flux through a circuit, how is it possible for the flux to change at all? The answer is that Lenz's law gives only the *direction* of an induced current; the *magnitude* of the current depends on the resistance of the circuit. The greater the circuit resistance, the less the induced current that appears to oppose any change in flux and the easier it is for a flux change to take effect. If the loop in Fig. 29.14 were made out of wood (an insulator), there would be almost no induced current in response to changes in the flux through the loop.

Conversely, the less the circuit resistance, the greater the induced current and the more difficult it is to change the flux through the circuit. If the loop in Fig. 29.14 is a good conductor, an induced current flows as long as the magnet moves relative to the loop. Once the magnet and loop are no longer in relative motion, the induced current very quickly decreases to zero because of the nonzero resistance in the loop.

The extreme case occurs when the resistance of the circuit is *zero*. Then the induced current in Fig. 29.14 will continue to flow even after the induced emf has disappeared—that is, even after the magnet has stopped moving relative to the loop. Thanks to this *persistent current,* it turns out that the flux through the loop is exactly the same as it was before the magnet started to move, so the flux through a loop of zero resistance *never* changes. Exotic materials called *superconductors* do indeed have zero resistance; we discuss these further in Section 29.8.

TEST YOUR UNDERSTANDING OF SECTION 29.3 (a) Suppose the magnet in Fig. 29.14a were stationary and the loop of wire moved upward. Would the induced current in the loop be (i) in the same direction as shown in Fig. 29.14a, (ii) in the direction opposite to that shown in Fig. 29.14a, or (iii) zero? (b) Suppose the magnet and loop of wire in Fig. 29.14a both moved downward at the same velocity. Would the induced current in the loop be (i) in the same direction as shown in Fig. 29.14a, (ii) in the direction opposite to that shown in Fig. 29.14a, or (iii) zero?

ANSWER

(a) **(i)**, (b) **(iii)** In (a), as in the original situation, the magnet and loop are approaching each other and the downward flux through the loop is increasing. Hence the induced emf and induced current are the same. In (b), since the magnet and loop are moving together, the flux through the loop is not changing and no emf is induced.

29.4 MOTIONAL EMF

We've seen several situations in which a conductor moves in a magnetic field, as in the generators discussed in Examples 29.3 through 29.6. We can gain additional insight into the origin of the induced emf in these situations by considering the magnetic forces on mobile charges in the conductor. **Figure 29.15a** shows the same moving rod that we discussed in Example 29.5, separated for the moment from the U-shaped conductor. The magnetic field $\vec{B}$ is uniform and directed into the page, and we move the rod to the right at a constant velocity $\vec{v}$. A charged particle q in the rod then experiences a magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ with magnitude $F = |q|vB$. We'll assume in the following discussion that q is positive; in that case the direction of this force is upward along the rod, from b toward a.

This magnetic force causes the free charges in the rod to move, creating an excess of positive charge at the upper end a and negative charge at the lower end b. This in turn creates an electric field $\vec{E}$ within the rod, in the direction from a toward b (opposite to the magnetic force). Charge continues to accumulate at the ends of the rod until $\vec{E}$ becomes large enough for the downward electric force (with magnitude qE) to cancel exactly the *upward* magnetic force (with magnitude qvB). Then $qE = qvB$ and the charges are in equilibrium.

The magnitude of the potential difference $V_{ab} = V_a - V_b$ is equal to the electric-field magnitude E multiplied by the length L of the rod. From the above discussion, $E = vB$, so

$$V_{ab} = EL = vBL \tag{29.5}$$

with point a at higher potential than point b.

Now suppose the moving rod slides along a stationary U-shaped conductor, forming a complete circuit (Fig. 29.15b). No *magnetic* force acts on the charges in the stationary U-shaped conductor, but the charge that was near points a and b redistributes itself along the stationary conductor, creating an *electric* field within it. This field establishes a current in the direction shown. The moving rod has become a source of emf; within it, charge moves from lower to higher potential, and in the remainder of the circuit, charge moves from higher to lower potential. We call this emf a **motional emf,** denoted by $\mathcal{E}$. From the above discussion, the magnitude of this emf is

Motional emf, conductor length and velocity perpendicular to uniform $\vec{B}$

$$\mathcal{E} = \overset{\text{Conductor speed}}{v}\underset{\text{Magnitude of uniform magnetic field}}{B}\overset{\text{Conductor length}}{L} \tag{29.6}$$

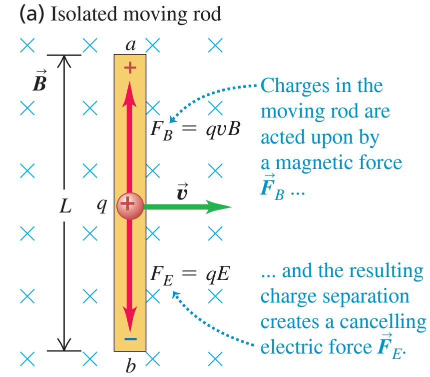
This corresponds to a force per unit charge of magnitude vB acting for a distance L along the moving rod. If the total circuit resistance of the U-shaped conductor and the sliding rod is R, the induced current I in the circuit is given by $vBL = IR$. This is the same result we obtained in Section 29.2 by using Faraday's law, and indeed motional emf is a particular case of Faraday's law. Verify that if we express v in meters per second, B in teslas, and L in meters, then $\mathcal{E}$ is in volts. (Recall that $1\ \text{V} = 1\ \text{J/C} = 1\ \text{T}\cdot\text{m}^2/\text{s}$.)

The emf associated with the moving rod in Fig. 29.15b is analogous to that of a battery with its positive terminal at a and its negative terminal at b, although the origins of the two emfs are quite different. In each case a nonelectrostatic force acts on the charges in the device, in the direction from b to a, and the emf is the work per unit charge done by this force when a charge moves from b to a in the device. When the device is connected to an external circuit, the direction of current is from b to a in the device and from a to b in the external circuit. Note that a motional emf is also present in the isolated moving rod in Fig. 29.15a, in the same way that a battery has an emf even when it's not part of a circuit.

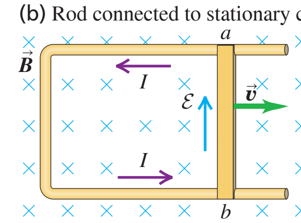
You can determine the direction of the induced emf in Fig. 29.15 by using Lenz's law, even if (as in Fig. 29.15a) the conductor does not form a complete circuit. In this case we can mentally complete the circuit between the ends of the conductor and use Lenz's law to determine the direction of the current. From this we can deduce the polarity of the ends of the open-circuit conductor. The direction from the − end to the + end within the conductor is the direction the current would have if the circuit were complete.

Figure 29.15 A conducting rod moving in a uniform magnetic field. (a) The rod, the velocity, and the field are mutually perpendicular. (b) Direction of induced current in the circuit.

(a) Isolated moving rod

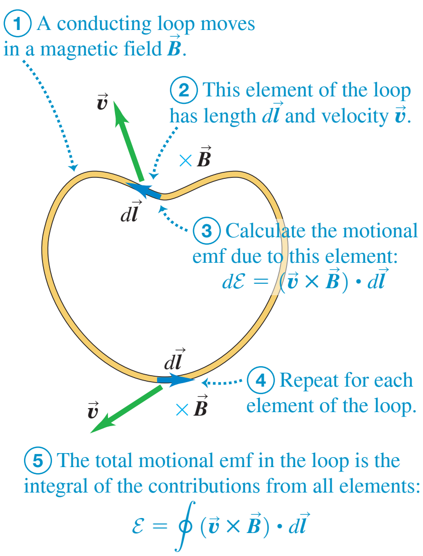


(b) Rod connected to stationary conductor



The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

Figure 29.16 Calculating the motional emf for a moving current loop. The velocity $\vec{v}$ can be different for different elements if the loop is rotating or changing shape. The magnetic field $\vec{B}$ can also have different values at different points around the loop.



Motional emf: General Form

We can generalize the concept of motional emf for a conductor with *any* shape, moving in any magnetic field, uniform or not, if we assume that the magnetic field at each point does not vary with time (**Fig. 29.16**). For an element $d\vec{l}$ of the conductor, the contribution $d\mathcal{E}$ to the emf is the magnitude dl multiplied by the component of $\vec{v} \times \vec{B}$ (the magnetic force per unit charge) parallel to $d\vec{l}$; that is,

$$d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

For any closed conducting loop, the total emf is

Line integral over all elements of **closed conducting loop**

Motional emf, general case

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \qquad (29.7)$$

Length vector of conductor element

Velocity of conductor element

Magnetic field at position of element

This expression looks very different from our original statement of Faraday's law, $\mathcal{E} = -d\Phi_B/dt$ [Eq. (29.3)]. In fact, though, the two statements are equivalent. It can be shown that the rate of change of magnetic flux through a moving conducting loop is always given by the negative of the expression in Eq. (29.7). Thus this equation gives us an alternative formulation of Faraday's law that is often convenient in problems with *moving* conductors. But when we have *stationary* conductors in changing magnetic fields, Eq. (29.7) *cannot* be used; in this case, $\mathcal{E} = -d\Phi_B/dt$ is the only correct way to express Faraday's law.

EXAMPLE 29.9 Motional emf in the slidewire generator

WITH VARIATION PROBLEMS

Suppose the moving rod in Fig. 29.15b is 0.10 m long, the velocity v is 2.5 m/s, the total resistance of the loop is 0.030 Ω, and B is 0.60 T. Find the motional emf, the induced current, and the force acting on the rod.

IDENTIFY and SET UP We'll find the motional emf $\mathcal{E}$ from Eq. (29.6) and the current from the values of $\mathcal{E}$ and the resistance R. The force on the rod is a *magnetic* force, exerted by $\vec{B}$ on the current in the rod; we'll find this force by using $\vec{F} = I\vec{L} \times \vec{B}$.

EXECUTE From Eq. (29.6) the motional emf is

$$\mathcal{E} = vBL = (2.5\ \text{m/s})(0.60\ \text{T})(0.10\ \text{m}) = 0.15\ \text{V}$$

The induced current in the loop is

$$I = \frac{\mathcal{E}}{R} = \frac{0.15\ \text{V}}{0.030\ \Omega} = 5.0\ \text{A}$$

In the expression for the magnetic force, $\vec{F} = I\vec{L} \times \vec{B}$, the vector $\vec{L}$ points in the same direction as the induced current in the rod (from b to a in Fig. 29.15). The right-hand rule for vector products shows that this force is directed *opposite* to the rod's motion. Since $\vec{L}$ and $\vec{B}$ are perpendicular, the force has magnitude

$$F = ILB = (5.0\ \text{A})(0.10\ \text{m})(0.60\ \text{T}) = 0.30\ \text{N}$$

EVALUATE We can check our answer for the direction of $\vec{F}$ by using Lenz's law. If we take the area vector $\vec{A}$ to point into the plane of the loop, the magnetic flux is positive and increasing as the rod moves to the right and increases the area of the loop. Lenz's law tells us that a force appears to oppose this increase in flux. Hence the force on the rod is to the left, opposite its motion.

KEYCONCEPT When a straight conductor moves through a magnetic field, the magnetic forces on the mobile charges give rise to a motional emf. This emf is proportional to the field strength and to the speed and length of the conductor.

EXAMPLE 29.10 The Faraday disk dynamo

Figure 29.17 shows a conducting disk with radius R that lies in the xy-plane and rotates with constant angular velocity ω about the z-axis. The disk is in a uniform, constant $\vec{B}$ field in the z-direction. Find the induced emf between the center and the rim of the disk.

IDENTIFY and SET UP A motional emf arises because the conducting disk moves relative to $\vec{B}$. The complication is that different parts of the disk move at different speeds v, depending on their distance from the rotation axis. We'll address this by considering small segments of the disk and integrating their contributions to determine our target variable, the emf between the center and the rim. Consider the small segment of the disk shown in red in Fig. 29.17 and labeled by its velocity vector $\vec{v}$. The magnetic force per unit charge on this segment is $\vec{v} \times \vec{B}$, which points radially outward from the center of the disk. Hence the induced emf tends to make a current flow radially outward, which tells us that the moving conducting path to think about here is a straight line from the center to the rim. We can find the emf from each small disk segment along this line by using $d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$ and then integrate to find the total emf.

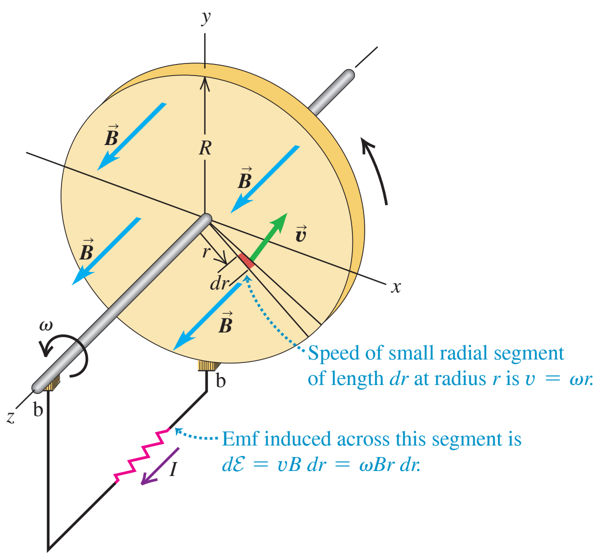


Figure **29.17** A conducting disk with radius R rotating at an angular speed ω in a magnetic field $\vec{B}$. The emf is induced along radial lines of the disk and is applied to an external circuit through the two sliding contacts labeled b.

EXECUTE The length vector $d\vec{l}$ (of length dr) associated with the segment points radially outward, in the same direction as $\vec{v} \times \vec{B}$. The vectors $\vec{v}$ and $\vec{B}$ are perpendicular, and the magnitude of $\vec{v}$ is $v = \omega r$. The emf from the segment is then $d\mathcal{E} = \omega rB\,dr$. The total emf is the integral of $d\mathcal{E}$ from the center $(r = 0)$ to the rim $(r = R)$:

$$\mathcal{E} = \int_0^R \omega Br\,dr = \tfrac{1}{2}\omega BR^2$$

EVALUATE We can use this device as a source of emf in a circuit by completing the circuit through two stationary brushes (labeled b in the figure) that contact the disk and its conducting shaft as shown. Such a disk is called a *Faraday disk dynamo* or a *homopolar generator*. Unlike the alternator in Example 29.3, the Faraday disk dynamo is a direct-current generator; it produces an emf that is constant in time. Can you use Lenz's law to show that for the direction of rotation in Fig. 29.17, the current in the external circuit must be in the direction shown?

KEYCONCEPT In general, finding the induced emf in a conductor requires doing an integral. This is because the magnetic force on a mobile charge can be different at different locations in the conductor.

TEST YOUR UNDERSTANDING OF SECTION 29.4 The earth's magnetic field points toward (magnetic) north. For simplicity, assume that the field has no vertical component (as is the case near the earth's equator). (a) If you hold a metal rod in your hand and walk toward the east, how should you orient the rod to get the maximum motional emf between its ends? (i) East-west; (ii) north-south; (iii) up-down; (iv) you get the same motional emf with all of these orientations. (b) How should you hold it to get *zero* emf as you walk toward the east? (i) East-west; (ii) north-south; (iii) up-down; (iv) none of these. (c) In which direction should you travel so that the motional emf across the rod is zero no matter how the rod is oriented? (i) West; (ii) north; (iii) south; (iv) straight up; (v) straight down.

ANSWER

(a) **(iii)**; (b) **(i) or (ii)**; (c) **(ii) or (iii)** You'll get the maximum motional emf if you hold the rod vertically, so that its length is perpendicular to both the magnetic field and the direction of motion. With this orientation, $\vec{L}$ is parallel to $\vec{v} \times \vec{B}$. If you hold the rod in any horizontal orientation, $\vec{L}$ will be perpendicular to $\vec{v} \times \vec{B}$ and no emf will be induced. If you walk due north or south, $\vec{v} \times \vec{B} = \mathbf{0}$ and no emf will be induced for any orientation of the rod.

29.5 INDUCED ELECTRIC FIELDS

When a conductor moves in a magnetic field, we can understand the induced emf on the basis of magnetic forces on charges in the conductor, as described in Section 29.4. But an induced emf also occurs when there is a changing flux through a stationary conductor. What is it that pushes the charges around the circuit in this type of situation?

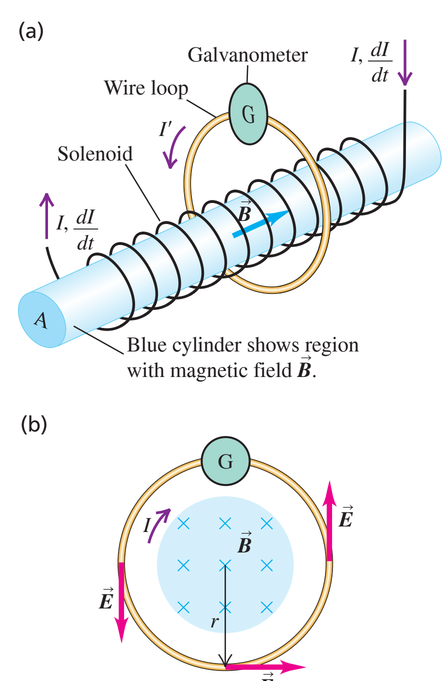
Let's consider the situation shown in **Fig. 29.18**. A long, thin solenoid with cross-sectional area A and n turns per unit length is encircled at its center by a circular conducting loop. The galvanometer G measures the current in the loop. A current I in the winding of the solenoid sets up a magnetic field $\vec{B}$ along the solenoid axis, as shown, with magnitude B as calculated in Section 28.7: $B = \mu_0 nI$, where n is the number of turns per unit length. If we ignore the small field outside the solenoid and take the area vector $\vec{A}$ to point in the same direction as $\vec{B}$, then the magnetic flux Φ_B through the loop is

$$\Phi_B = BA = \mu_0 nIA$$

When the solenoid current I changes with time, the magnetic flux Φ_B also changes, and according to Faraday's law the induced emf in the loop is given by

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 nA\frac{dI}{dt} \tag{29.8}$$

Figure **29.18** (a) The windings of a long solenoid carry a current I that is increasing at a rate dI/dt. The magnetic flux in the solenoid is increasing at a rate $d\Phi_B/dt$, and this changing flux passes through a wire loop. An emf $\mathcal{E} = -d\Phi_B/dt$ is induced in the loop, inducing a current I' that is measured by the galvanometer G. (b) Cross-sectional view.



If the total resistance of the loop is R, the induced current in the loop, which we may call I', is $I' = \mathcal{E}/R$.

But what *force* makes the charges move around the wire loop? It can't be a magnetic force because the loop isn't even *in* a magnetic field. We are forced to conclude that there has to be an **induced electric field** in the conductor *caused by the changing magnetic flux*. Induced electric fields are *very* different from the electric fields caused by charges, which we discussed in Chapter 23. To see this, note that when a charge q goes once around the loop, the total work done on it by the electric field must be equal to q times the emf $\mathcal{E}$. That is, the electric field in the loop *is not conservative,* as we used the term in Section 23.1, because the line integral of $\vec{E}$ around a closed path is not zero. Indeed, this line integral, representing the work done by the induced $\vec{E}$ field per unit charge, is equal to the induced emf $\mathcal{E}$:

$$\oint \vec{E} \cdot d\vec{l} = \mathcal{E} \tag{29.9}$$

From Faraday's law the emf $\mathcal{E}$ is also the negative of the rate of change of magnetic flux through the loop. Thus for this case we can restate Faraday's law as

Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}} \tag{29.10}$$

Note that Faraday's law is *always* true in the form $\mathcal{E} = -d\Phi_B/dt$; the form given in Eq. (29.10) is valid *only* if the path around which we integrate is *stationary*.

Let's apply Eq. (29.10) to the stationary circular loop in Fig. 29.18b, which we take to have radius r. Because of cylindrical symmetry, $\vec{E}$ has the same magnitude at every point on the circle and is tangent to it at each point. (Symmetry would also permit the field to be *radial,* but then Gauss's law would require the presence of a net charge inside the circle, and there is none.) The line integral in Eq. (29.10) becomes simply the magnitude E times the circumference $2\pi r$ of the loop, $\oint \vec{E} \cdot d\vec{l} = 2\pi r E$, and Eq. (29.10) gives

$$E = \frac{1}{2\pi r}\left|\frac{d\Phi_B}{dt}\right| \tag{29.11}$$

The directions of $\vec{E}$ at points on the loop are shown in Fig. 29.18b. We know that $\vec{E}$ has to have the direction shown when $\vec{B}$ in the solenoid is increasing, because $\oint \vec{E} \cdot d\vec{l}$ has to be negative when $d\Phi_B/dt$ is positive. The same approach can be used to find the induced electric field *inside* the solenoid when the solenoid $\vec{B}$ field is changing; we leave the details to you.

Figure **29.19** Applications of induced electric fields. (a) This car is powered by an electric motor. As the car comes to a halt, the spinning wheels run the motor backward so that it acts as a generator. The resulting induced current is used to recharge the car's batteries. (b) The rotating crankshaft of a piston-engine airplane spins a magnet, inducing an emf in an adjacent coil and generating the spark that ignites fuel in the engine cylinders. This keeps the engine running even if the airplane's other electrical systems fail.

Nonelectrostatic Electric Fields

We've learned that Faraday's law, Eq. (29.3), is valid for two rather different situations. In one, an emf is induced by magnetic forces on charges when a conductor moves through a magnetic field. In the other, a time-varying magnetic field induces an electric field and hence an emf; the $\vec{E}$ field is induced even when no conductor is present. This $\vec{E}$ field differs from an electro*static* field in an important way. It is *nonconservative;* the line integral $\oint \vec{E} \cdot d\vec{l}$ around a closed path is not zero, and when a charge moves around a closed path, the field does a nonzero amount of work on it. It follows that for such a field the concept of *potential* has no meaning. We call such a field a **nonelectrostatic field.** In contrast, an *electrostatic* field is *always* conservative, as we discussed in Section 23.1, and always has an associated potential function. Despite this difference, the fundamental effect of *any* electric field is to exert a force $\vec{F} = q\vec{E}$ on a charge q. This relationship is valid whether $\vec{E}$ is conservative and produced by charges or nonconservative and produced by changing magnetic flux.

So a changing magnetic field acts as a source of electric field of a sort that we *cannot* produce with any static charge distribution. What's more, we'll see in Section 29.7 that a changing *electric* field acts as a source of *magnetic* field. We'll explore this symmetry between the two fields in our study of electromagnetic waves in Chapter 32.

If any doubt remains in your mind about the reality of magnetically induced electric fields, consider a few of the many practical applications (**Fig. 29.19**). Pickups in electric guitars use currents induced in stationary pickup coils by the vibration of nearby ferromagnetic strings. Alternators in most cars use rotating magnets to induce currents in stationary coils. Whether we realize it or not, magnetically induced electric fields play an important role in everyday life.

EXAMPLE 29.11 Induced electric fields

WITH VARIATION PROBLEMS

Suppose the long solenoid in Fig. 29.18a has 500 turns per meter and cross-sectional area 4.0 cm^2. The current in its windings is increasing at 100 A/s. (a) Find the magnitude of the induced emf in the wire loop outside the solenoid. (b) Find the magnitude of the induced electric field within the loop if its radius is 2.0 cm.

IDENTIFY and SET UP As in Fig. 29.18b, the increasing magnetic field inside the solenoid causes a change in the magnetic flux through the wire loop and hence induces an electric field $\vec{E}$ around the loop. Our target variables are the induced emf $\mathcal{E}$ and the electric-field magnitude E. We use Eq. (29.8) to determine the emf. The loop and the solenoid share the same central axis. Hence, by symmetry, the electric field is tangent to the loop and has the same magnitude E all the way around its circumference. We can therefore use Eq. (29.9) to find E.

EXECUTE (a) From Eq. (29.8), the induced emf is

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt}\\ &= -(4\pi \times 10^{-7}\ \text{Wb/A}\cdot\text{m})(500\ \text{turns/m})(4.0 \times 10^{-4}\ \text{m}^2)(100\ \text{A/s})\\ &= -25 \times 10^{-6}\ \text{Wb/s} = -25 \times 10^{-6}\ \text{V} = -25\ \mu\text{V}\end{aligned}$$

(b) By symmetry the line integral $\oint \vec{E} \cdot d\vec{l}$ has absolute value $2\pi r E$ no matter which direction we integrate around the loop. This is equal to the absolute value of the emf, so

$$E = \frac{|\mathcal{E}|}{2\pi r} = \frac{25 \times 10^{-6}\ \text{V}}{2\pi(2.0 \times 10^{-2}\ \text{m})} = 2.0 \times 10^{-4}\ \text{V/m}$$

EVALUATE In Fig. 29.18b the magnetic flux *into* the plane of the figure is increasing. According to the right-hand rule for induced emf (Fig. 29.6), a positive emf would be clockwise around the loop; the negative sign of $\mathcal{E}$ shows that the emf is in the counterclockwise direction. Can you also show this by using Lenz's law?

KEYCONCEPT A changing magnetic flux through a closed loop produces an induced electric field $\vec{E}$. The line integral of $\vec{E}$ around the loop is equal to the induced emf in the loop.

TEST YOUR UNDERSTANDING OF SECTION 29.5 If you wiggle a magnet back and forth in your hand, are you generating an electric field? If so, is this field conservative?

ANSWER

yes, no The magnetic field at a fixed position changes as you move the magnet, which induces an electric field. Such induced electric fields are *not* conservative.

29.6 EDDY CURRENTS

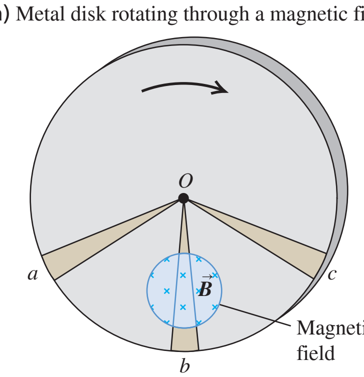
In the examples of induction effects that we have studied, the induced currents have been confined to well-defined paths in conductors and other components forming a circuit. However, many pieces of electrical equipment contain masses of metal moving in magnetic fields or located in changing magnetic fields. In situations like these we can have induced currents that circulate throughout the volume of a material. Because their flow patterns resemble swirling eddies in a river, we call these **eddy currents.**

As an example, consider a metallic disk rotating in a magnetic field perpendicular to the plane of the disk but confined to a limited portion of the disk's area, as shown in **Fig. 29.20a**. Sector Ob is moving across the field and has an emf induced in it. Sectors Oa and Oc are not in the field, but they provide return conducting paths for charges displaced along Ob to return from b to O. The result is a circulation of eddy currents in the disk, somewhat as sketched in Fig. 29.20b.

We can use Lenz's law to decide on the direction of the induced current in the neighborhood of sector Ob. This current must experience a magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ that *opposes* the rotation of the disk, and so this force must be to the right in Fig. 29.20b. Since $\vec{B}$ is directed into the plane of the disk, the current and hence $\vec{L}$ have downward components. The return currents lie outside the field, so they do not experience magnetic forces. The interaction between the eddy currents and the field causes a braking action on the disk. Such effects can be used to stop the rotation of a circular saw quickly when

Figure 29.20 Eddy currents induced in a rotating metal disk.

(a) Metal disk rotating through a magnetic field



(b) Resulting eddy currents and braking force

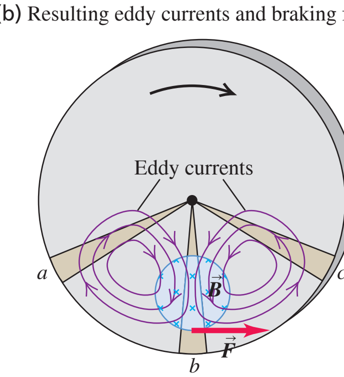
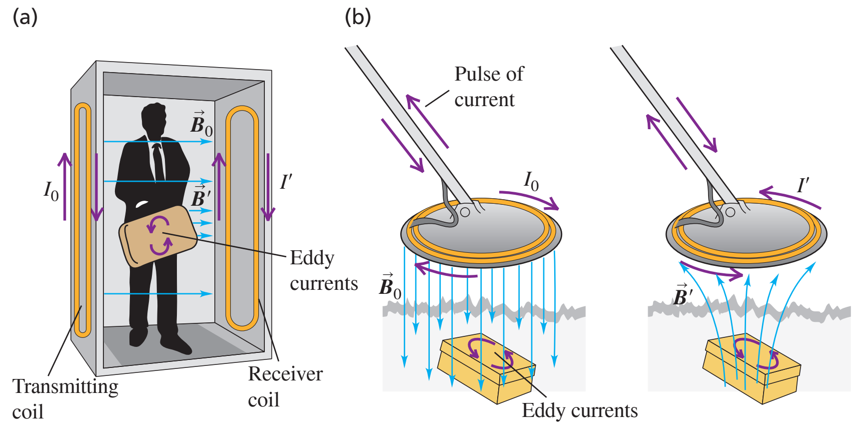
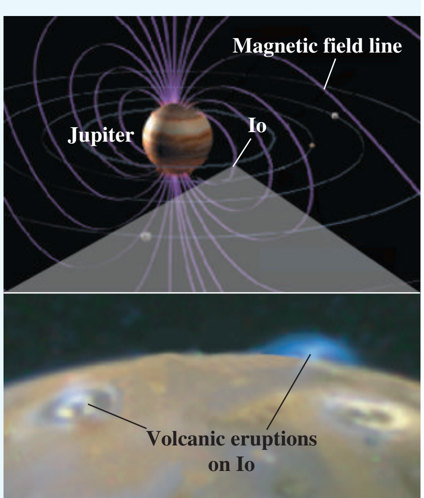


Figure **29.21** (a) A metal detector at an airport security checkpoint generates an alternating magnetic field $\vec{B}_0$. This induces eddy currents in a conducting object carried through the detector. The eddy currents in turn produce an alternating magnetic field $\vec{B}'$, which induces a current in the detector's receiver coil. (b) Portable metal detectors work on the same principle.



APPLICATION Eddy Currents Help Power Io's Volcanoes Jupiter's moon Io is slightly larger than the earth's moon. It moves at more than 60,000 km/h through Jupiter's intense magnetic field (about ten times stronger than the earth's field), which sets up strong eddy currents within Io that dissipate energy at a rate of 10^{12} W. This dissipated energy helps to heat Io's interior and causes volcanic eruptions on its surface, as shown in the lower close-up image. (Gravitational effects from Jupiter cause even more heating.)



the power is turned off. Eddy current braking is used on some electrically powered rapid-transit vehicles. Electromagnets mounted in the cars induce eddy currents in the rails; the resulting magnetic fields cause braking forces on the electromagnets and thus on the cars.

Eddy currents have many other practical uses. In induction furnaces, eddy currents are used to heat materials in completely sealed containers for processes in which it is essential to avoid the slightest contamination of the materials. The metal detectors used at airport security checkpoints (**Fig. 29.21a**) operate by detecting eddy currents induced in metallic objects. Similar devices (Fig. 29.21b) are used to find buried treasure such as bottlecaps and lost pennies.

Eddy currents also have undesirable effects. In an alternating-current transformer, coils wrapped around an iron core carry a sinusoidally varying current. The resulting eddy currents in the core waste energy through I^2R heating and set up an unwanted opposing emf in the coils. To minimize these effects, the core is designed so that the paths for eddy currents are as narrow as possible. We'll describe how this is done when we discuss transformers in Section 31.6.

TEST YOUR UNDERSTANDING OF SECTION 29.6 Suppose that the magnetic field in Fig. 29.20 were directed out of the plane of the figure and the disk were rotating counterclockwise. Compared to the directions of the force $\vec{F}$ and the eddy currents shown in Fig. 29.20b, what would the new directions be? (i) The force $\vec{F}$ and the eddy currents would both be in the same direction; (ii) the force $\vec{F}$ would be in the same direction, but the eddy currents would be in the opposite direction; (iii) the force $\vec{F}$ would be in the opposite direction, but the eddy currents would be in the same direction; (iv) the force $\vec{F}$ and the eddy currents would be in the opposite directions.

ANSWER

(iii) By Lenz's law, the force must oppose the motion of the disk through the magnetic field. Since the disk material is now moving to the right through the field region, the force $\vec{F}$ is to the left—that is, in the opposite direction to that shown in Fig. 29.20b. To produce a leftward magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ on currents moving through a magnetic field $\vec{B}$ directed out of the plane of the figure, the eddy currents must be moving downward in the figure—that is, in the same direction shown in Fig. 29.20b.

29.7 DISPLACEMENT CURRENT AND MAXWELL'S EQUATIONS

We have seen that a varying magnetic field gives rise to an induced electric field. In one of the more remarkable examples of the symmetry of nature, it turns out that a varying *electric* field gives rise to a *magnetic* field. This effect is of tremendous importance, for it turns out to explain the existence of radio waves, gamma rays, and visible light, as well as all other forms of electromagnetic waves.

Generalizing Ampere's Law

To see the origin of the relationship between varying electric fields and magnetic fields, let's return to Ampere's law as given in Section 28.6, Eq. (28.20):

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

The problem with Ampere's law in this form is that it is *incomplete*. To see why, let's consider the process of charging a capacitor (**Fig. 29.22**). Conducting wires lead current i_C into one plate and out of the other; the charge Q increases, and the electric field $\vec{E}$ between the plates increases. The notation i_C indicates *conduction* current to distinguish it from another kind of current we are about to encounter, called *displacement* current i_D. We use lowercase i's and v's to denote instantaneous values of currents and potential differences, respectively, that may vary with time.

Let's apply Ampere's law to the circular path shown. The integral $\oint \vec{B} \cdot d\vec{l}$ around this path equals $\mu_0 I_{\text{encl}}$. For the plane circular area bounded by the circle, I_{encl} is just the current i_C in the left conductor. But the surface that bulges out to the right is bounded by the same circle, and the current through that surface is zero. So $\oint \vec{B} \cdot d\vec{l}$ is equal to $\mu_0 i_C$, and at the same time it is equal to zero! This is a clear contradiction.

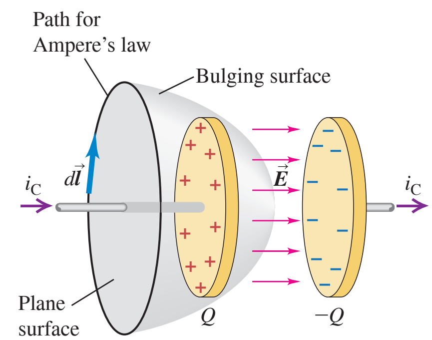


Figure **29.22** Parallel-plate capacitor being charged. The conduction current through the plane surface is i_C, but there is no conduction current through the surface that bulges out to pass between the plates. The two surfaces have a common boundary, so this difference in I_{encl} leads to an apparent contradiction in applying Ampere's law.

However, something else is happening on the bulged-out surface. As the capacitor charges, the electric field $\vec{E}$ and the electric *flux* Φ_E through the surface are increasing. We can determine their rates of change in terms of the charge and current. The instantaneous charge is $q = Cv$, where C is the capacitance and v is the instantaneous potential difference. For a parallel-plate capacitor, $C = \epsilon_0 A/d$, where A is the plate area and d is the spacing. The potential difference v between plates is $v = Ed$, where E is the electric-field magnitude between plates. (We ignore fringing and assume that $\vec{E}$ is uniform in the region between the plates.) If this region is filled with a material with permittivity ϵ, we replace ϵ_0 by ϵ everywhere; we'll use ϵ in the following discussion.

Substituting these expressions for C and v into $q = Cv$, we can express the capacitor charge q in terms of the electric flux $\Phi_E = EA$ through the surface:

$$q = Cv = \frac{\epsilon A}{d}(Ed) = \epsilon EA = \epsilon \Phi_E \tag{29.12}$$

As the capacitor charges, the rate of change of q is the conduction current, $i_C = dq/dt$. Taking the derivative of Eq. (29.12) with respect to time, we get

$$i_C = \frac{dq}{dt} = \epsilon \frac{d\Phi_E}{dt} \tag{29.13}$$

Stretching our imagination a bit, we invent a fictitious **displacement current** i_D in the region between the plates, defined as

Displacement current through an area

$$i_D = \epsilon \frac{d\Phi_E}{dt} \tag{29.14}$$

Time rate of change of electric flux through area

Permittivity of material in area

That is, we imagine that the changing flux through the curved (bulged-out) surface in Fig. 29.22 is equivalent, in Ampere's law, to a conduction current through that surface. We include this fictitious current, along with the real conduction current i_C, in Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_C + i_D)_{\text{encl}} \quad \text{(generalized Ampere's law)} \tag{29.15}$$

Ampere's law in this form is obeyed no matter which surface we use in Fig. 29.22. For the flat surface, i_D is zero; for the curved surface, i_C is zero; and i_C for the flat surface equals i_D for the curved surface. Equation (29.15) remains valid in a magnetic material, provided that the magnetization is proportional to the external field and we replace μ_0 by μ.

The fictitious displacement current i_D was invented in 1865 by the Scottish physicist James Clerk Maxwell. There is a corresponding *displacement current density* $j_D = i_D/A$; using $\Phi_E = EA$ and dividing Eq. (29.14) by A, we find

$$j_D = \epsilon \frac{dE}{dt} \tag{29.16}$$

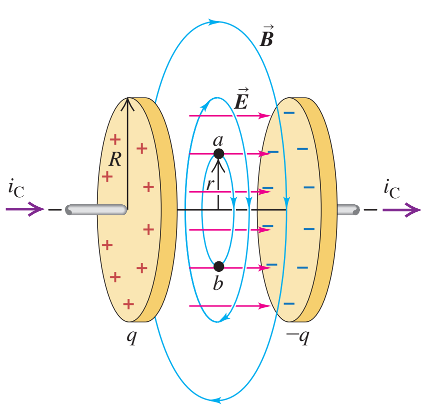
We have pulled the concept out of thin air, as Maxwell did, but we see that it enables us to save Ampere's law in situations such as that in Fig. 29.22.

Another benefit of displacement current is that it lets us generalize Kirchhoff's junction rule, discussed in Section 26.2. Considering the left plate of the capacitor, we have conduction current into it but none out of it. But when we include the displacement current, we have conduction current coming in one side and an equal displacement current coming out the other side. With this generalized meaning of the term "current," we can speak of current going *through* the capacitor.

The Reality of Displacement Current

You might well ask at this point whether displacement current has any real physical significance or whether it is just a ruse to satisfy Ampere's law and Kirchhoff's junction rule. Here's a fundamental experiment that helps to answer that question. We take a plane circular area between the capacitor plates (**Fig. 29.23**). If displacement current really plays the role in Ampere's law that we have claimed, then there ought to be a magnetic field in the region between the plates while the capacitor is charging. We can use our generalized Ampere's law, including displacement current, to predict what this field should be.

Figure 29.23 A capacitor being charged by a current i_C has a displacement current equal to i_C between the plates, with displacement-current density $j_D = \epsilon\, dE/dt$. This can be regarded as the source of the magnetic field between the plates.



To be specific, let's picture round capacitor plates with radius R. To find the magnetic field at a point in the region between the plates at a distance r from the axis, we apply Ampere's law to a circle of radius r passing through the point, with $r < R$. This circle passes through points a and b in Fig. 29.23. The total current enclosed by the circle is j_D times its area, or $(i_D/\pi R^2)(\pi r^2)$. The integral $\oint \vec{B} \cdot d\vec{l}$ in Ampere's law is just B times the circumference $2\pi r$ of the circle, and because $i_D = i_C$ for the charging capacitor, Ampere's law becomes

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 \frac{r^2}{R^2} i_C \qquad \text{or}$$

$$B = \frac{\mu_0}{2\pi} \frac{r}{R^2} i_C \tag{29.17}$$

This result predicts that in the region between the plates $\vec{B}$ is zero at the axis and increases linearly with distance from the axis. A similar calculation shows that *outside* the region between the plates (that is, for $r > R$), $\vec{B}$ is the same as though the wire were continuous and the plates not present at all.

When we *measure* the magnetic field in this region, we find that it really is there and that it behaves just as Eq. (29.17) predicts. This confirms directly the role of displacement current as a source of magnetic field. It is now established beyond reasonable doubt that Maxwell's displacement current, far from being just an artifice, is a fundamental fact of nature.

Maxwell's Equations of Electromagnetism

We are now in a position to wrap up in a single package *all* of the relationships between electric and magnetic fields and their sources. This package consists of four equations, called **Maxwell's equations.** Maxwell did not discover all of these equations single-handedly (though he did develop the concept of displacement current). But he did put them together and recognized their significance, particularly in predicting the existence of electromagnetic waves.

For now we'll state Maxwell's equations in their simplest form, for the case in which we have charges and currents in otherwise empty space. In Chapter 32 we'll discuss how to modify these equations if a dielectric or a magnetic material is present.

Two of Maxwell's equations involve an integral of $\vec{E}$ or $\vec{B}$ over a closed surface. The first is simply Gauss's law for electric fields, Eq. (22.8):

Gauss's law for $\vec{E}$:

$$\overbrace{\oint \vec{E} \cdot d\vec{A}}^{\text{Flux of electric field through a closed surface}} = \frac{Q_{\text{encl}}}{\epsilon_0} \tag{29.18}$$

Charge enclosed by surface (Q_{encl})
Electric constant (ϵ_0)

The second is the analogous relationship for *magnetic* fields, Eq. (27.8):

Gauss's law for $\vec{B}$:

$$\overbrace{\oint \vec{B} \cdot d\vec{A}}^{\text{Flux of magnetic field through any closed surface ...}} = 0 \tag{29.19}$$

... equals zero.

This statement means, among other things, that there are no magnetic monopoles (single magnetic charges) to act as sources of magnetic field.

The third and fourth equations involve a line integral of $\vec{E}$ or $\vec{B}$ around a closed path. Faraday's law states that a changing magnetic flux acts as a source of electric field:

Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}} \tag{29.20}$$

If there is a changing magnetic field, the line integral in Eq. (29.20)—which must be carried out over a *stationary* closed path—is not zero. Thus the $\vec{E}$ field produced by a changing $\vec{B}$ is not conservative.

The fourth and final equation is Ampere's law including displacement current. It states that both a conduction current and a changing electric flux act as sources of magnetic field:

Ampere's law for a stationary integration path:

$$\overbrace{\oint \vec{B} \cdot d\vec{l}}^{\text{Line integral of magnetic field around path}} = \underbrace{\mu_0}_{\text{Magnetic constant}}\Big(\underbrace{i_C}_{\text{Conduction current through path}} + \overbrace{\epsilon_0}^{\text{Electric constant}}\underbrace{\overbrace{\frac{d\Phi_E}{dt}}^{\text{Time rate of change of electric flux through path}}}_{\text{Displacement current through path}}\Big)_{\text{encl}} \tag{29.21}$$

It's worthwhile to look more carefully at the electric field $\vec{E}$ and its role in Maxwell's equations. In general, the total $\vec{E}$ field at a point in space can be the superposition of an electrostatic field $\vec{E}_c$ caused by a distribution of charges at rest and a magnetically induced, nonelectrostatic field $\vec{E}_n$. That is,

$$\vec{E} = \vec{E}_c + \vec{E}_n$$

The electrostatic part $\vec{E}_c$ is *always* conservative, so $\oint \vec{E}_c \cdot d\vec{l} = 0$. This conservative part of the field does not contribute to the integral in Faraday's law, so we can take $\vec{E}$ in Eq. (29.20) to be the *total* electric field $\vec{E}$, including both the part $\vec{E}_c$ due to charges and the magnetically induced part $\vec{E}_n$. Similarly, the nonconservative part $\vec{E}_n$ of the $\vec{E}$ field does not contribute to the integral in Gauss's law, because this part of the field is not caused by static charges. Hence $\oint \vec{E}_n \cdot d\vec{A}$ is always zero. We conclude that in all the Maxwell equations, $\vec{E}$ is the total electric field; these equations don't distinguish between conservative and nonconservative fields.

Figure **29.24** Maxwell's equations in empty space are highly symmetric.

In empty space there are no charges, so the fluxes of $\vec{E}$ and $\vec{B}$ through any closed surface are equal to zero.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In empty space there are no conduction currents, so the line integrals of $\vec{E}$ and $\vec{B}$ around any closed path are related to the rate of change of flux of the other field.

Symmetry in Maxwell's Equations

There is a remarkable symmetry in Maxwell's four equations. In empty space where there is no charge, the first two equations [Eqs. (29.18) and (29.19)] are identical in form, one containing $\vec{E}$ and the other containing $\vec{B}$ (**Fig. 29.24**). When we compare the second two equations, Eq. (29.20) says that a changing magnetic flux creates an electric field, and Eq. (29.21) says that a changing electric flux creates a magnetic field. In empty space, where there is no conduction current, $i_C = 0$ and the two equations have the same form, apart from a numerical constant and a negative sign, with the roles of $\vec{E}$ and $\vec{B}$ exchanged.

We can rewrite Eqs. (29.20) and (29.21) in a different but equivalent form by introducing the definitions of magnetic flux, $\Phi_B = \int \vec{B} \cdot d\vec{A}$, and electric flux, $\Phi_E = \int \vec{E} \cdot d\vec{A}$, respectively. In empty space, where there is no charge or conduction current, $i_C = 0$ and $Q_{encl} = 0$, and we have

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A} \tag{29.22}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d}{dt}\int \vec{E} \cdot d\vec{A} \tag{29.23}$$

Again we notice the symmetry between the roles of $\vec{E}$ and $\vec{B}$ in these expressions.

The most remarkable feature of these equations is that a time-varying field of *either* kind induces a field of the other kind in neighboring regions of space. Maxwell recognized that these relationships predict the existence of electromagnetic disturbances consisting of time-varying electric and magnetic fields that travel or *propagate* from one region of space to another, even if no matter is present in the intervening space. Such disturbances, called *electromagnetic waves,* provide the physical basis for light, radio and television waves, infrared, ultraviolet, and x rays. We'll return to this vitally important topic in Chapter 32.

Although it may not be obvious, *all* the basic relationships between fields and their sources are contained in Maxwell's equations. We can derive Coulomb's law from Gauss's law, we can derive the law of Biot and Savart from Ampere's law, and so on. When we add the equation that defines the $\vec{E}$ and $\vec{B}$ fields in terms of the forces that they exert on a charge q, namely,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{29.24}$$

we have *all* the fundamental relationships of electromagnetism!

Maxwell's equations would have even greater symmetry between the $\vec{E}$ and $\vec{B}$ fields if single magnetic charges (magnetic monopoles) existed. The right side of Eq. (29.19) would contain the total *magnetic* charge enclosed by the surface, and the right side of Eq. (29.20) would include a magnetic monopole current term. However, no magnetic monopoles have yet been found.

In conciseness and generality, Maxwell's equations are in the same league with Newton's laws of motion and the laws of thermodynamics. Indeed, a major goal of science is learning how to express very broad and general relationships in a concise and compact form. Maxwell's synthesis of electromagnetism stands as a towering intellectual achievement, comparable to the Newtonian synthesis we described at the end of Section 13.5 and to the development of relativity and quantum mechanics in the 20th century.

TEST YOUR UNDERSTANDING OF SECTION 29.7 (a) Which of Maxwell's equations explains how a credit card reader works? (b) Which one describes how a wire carrying a steady current generates a magnetic field?

ANSWER

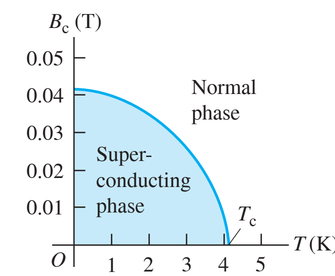
(a) **Faraday's law**, (b) **Ampere's law** A credit card reader works by inducing currents in the reader's coils as the card's magnetized stripe is swiped (see the answer to the chapter opening question). Ampere's law describes how currents of all kinds (both conduction currents and displacement currents) give rise to magnetic fields.

29.8 SUPERCONDUCTIVITY

The most familiar property of a superconductor is the sudden disappearance of all electrical resistance when the material is cooled below a temperature called the *critical temperature,* denoted by T_c. We discussed this behavior and the circumstances of its discovery in Section 25.2. But superconductivity is far more than just the absence of measurable resistance. As we'll see in this section, superconductors also have extraordinary *magnetic* properties.

The first hint of unusual magnetic properties was the discovery that for any superconducting material the critical temperature T_c changes when the material is placed in an externally produced magnetic field $\vec{B}_0$. **Figure 29.25** shows this dependence for mercury, the first element in which superconductivity was observed. As the external field magnitude B_0 increases, the superconducting transition occurs at lower and lower temperature. When B_0 is greater than 0.0412 T, *no* superconducting transition occurs. The minimum magnitude of magnetic field that is needed to eliminate superconductivity at a temperature below T_c is called the *critical field,* denoted by B_c.

Figure 29.25 Phase diagram for pure mercury, showing the critical magnetic field B_c and its dependence on temperature. Superconductivity is impossible above the critical temperature T_c. The curves for other superconducting materials are similar but with different numerical values.



The Meissner Effect

Another aspect of the magnetic behavior of superconductors appears if we place a homogeneous sphere of a superconducting material in a uniform applied magnetic field $\vec{B}_0$ at a temperature T greater than T_c. The material is then in the normal phase, not the superconducting phase (**Fig. 29.26a**). Now we lower the temperature until the superconducting transition occurs. (We assume that the magnitude of $\vec{B}_0$ is not large enough to prevent the phase transition.) What happens to the field?

Measurements of the field outside the sphere show that the field lines become distorted as in Fig. 29.26b. There is no longer any field inside the material, except possibly in a very thin surface layer a hundred or so atoms thick. If a coil is wrapped around the sphere, the emf induced in the coil shows that during the superconducting transition the magnetic flux through the coil decreases from its initial value to zero; this is consistent with the absence of field inside the material. Finally, if the field is now turned off while the material is still in its superconducting phase, no emf is induced in the coil, and measurements show no field outside the sphere (Fig. 29.26c).

We conclude that during a superconducting transition in the presence of the field $\vec{B}_0$, all of the magnetic flux is expelled from the bulk of the sphere, and the magnetic flux Φ_B through the coil becomes zero. This expulsion of magnetic flux is called the *Meissner effect*. As Fig. 29.26b shows, this expulsion crowds the magnetic field lines closer together to the side of the sphere, increasing $\vec{B}$ there.

Superconductor Levitation and Other Applications

The diamagnetic nature of a superconductor has some interesting *mechanical* consequences. A paramagnetic or ferromagnetic material is attracted by a permanent magnet because the magnetic dipoles in the material align with the nonuniform magnetic field of the permanent magnet. (We discussed this in Section 27.7.) For a diamagnetic material

Figure 29.26 A superconducting material (a) above the critical temperature and (b), (c) below the critical temperature.

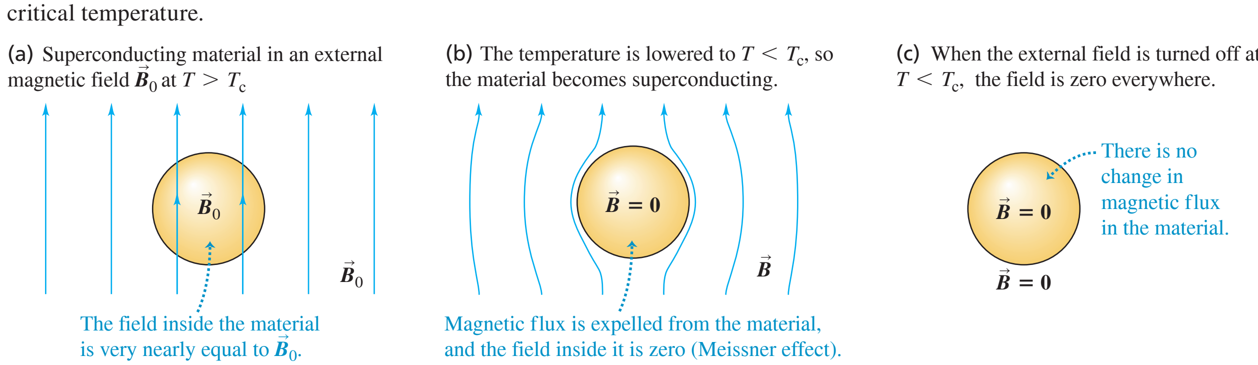
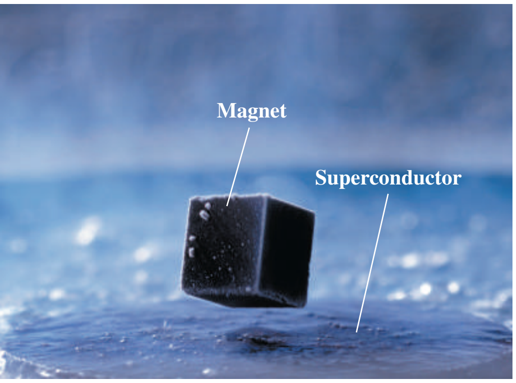


Figure 29.27 A superconductor exerts a repulsive force on a magnet, supporting the magnet in midair.



the magnetization is in the opposite sense, and a diamagnetic material is *repelled* by a permanent magnet. By Newton's third law the magnet is also repelled by the diamagnetic material. **Figure 29.27** shows the repulsion between a specimen of a high-temperature superconductor and a magnet; the magnet is supported ("levitated") by this repulsive magnetic force.

The behavior we have described is characteristic of what are called *type-I superconductors*. There is another class of superconducting materials called *type-II superconductors*. When such a material in the superconducting phase is placed in a magnetic field, the bulk of the material remains superconducting, but thin filaments of material, running parallel to the field, may return to the normal phase. Currents circulate around the boundaries of these filaments, and there *is* magnetic flux inside them. Type-II superconductors are used for electromagnets because they usually have much larger values of B_c than do type-I materials, permitting much larger magnetic fields without destroying the superconducting state. Type-II superconductors have *two* critical magnetic fields: The first, B_{c1}, is the field at which magnetic flux begins to enter the material, forming the filaments just described, and the second, B_{c2}, is the field at which the material becomes normal.

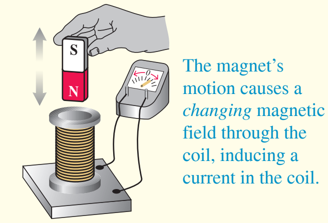
Superconducting electromagnets are in everyday use not only in research laboratories but also in medical MRI (magnetic resonance imaging) scanners. As we described in Section 27.7, scanning a patient through MRI requires a strong magnetic field to align the magnetic dipoles of the patient's atomic nuclei. A steady field of 1.5 T or more is needed, which is very difficult to produce with a conventional electromagnet, since this would require very high currents and hence very high energy losses due to resistance in the electromagnet coils. But with a superconducting electromagnet there is no resistive energy loss, and fields up to 10 T can routinely be attained.

Very sensitive measurements of magnetic fields can be made with superconducting quantum interference devices (SQUIDs), which can detect changes in magnetic flux of less than 10^{-14} Wb; these devices have applications in medicine, geology, and other fields. The number of potential uses for superconductors has increased greatly since the discovery in 1987 of high-temperature superconductors. These materials have critical temperatures that are above the temperature of liquid nitrogen (about 77 K) and so are comparatively easy to attain. Development of practical applications of superconductor science promises to be an exciting chapter in contemporary technology.

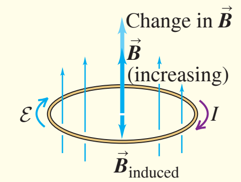
CHAPTER 29 SUMMARY

Faraday's law: Faraday's law states that the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop. This relationship is valid whether the flux change is caused by a changing magnetic field, motion of the loop, or both. (See Examples 29.1–29.6.)

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad (29.3)$$



Lenz's law: Lenz's law states that an induced current or emf always tends to oppose or cancel out the change that caused it. Lenz's law can be derived from Faraday's law and is often easier to use. (See Examples 29.7 and 29.8.)



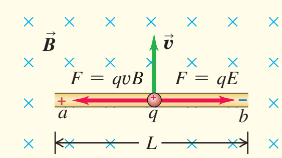
Motional emf: If a conductor moves in a magnetic field, a motional emf is induced. (See Examples 29.9 and 29.10.)

$$\mathcal{E} = vBL \qquad (29.6)$$

(conductor with length L moves in uniform $\vec{B}$ field, $\vec{L}$ and $\vec{v}$ both perpendicular to $\vec{B}$ and to each other)

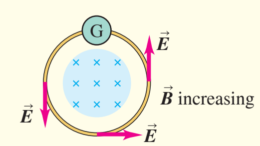
$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \qquad (29.7)$$

(all or part of a closed loop moves in a $\vec{B}$ field)



Induced electric fields: When an emf is induced by a changing magnetic flux through a stationary conductor, there is an induced electric field $\vec{E}$ of nonelectrostatic origin. This field is nonconservative and cannot be associated with a potential. (See Example 29.11.)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad (29.10)$$



Displacement current and Maxwell's equations: A time-varying electric field generates displacement current i_D, which acts as a source of magnetic field in exactly the same way as conduction current. The relationships between electric and magnetic fields and their sources can be stated compactly in four equations, called Maxwell's equations. Together they form a complete basis for the relationship of $\vec{E}$ and $\vec{B}$ fields to their sources.

$$i_D = \epsilon \frac{d\Phi_E}{dt} \qquad (29.14)$$

(displacement current)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad (29.18)$$

(Gauss's law for $\vec{E}$ fields)

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad (29.19)$$

(Gauss's law for $\vec{B}$ fields)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad (29.20)$$

(Faraday's law)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0\left(i_C + \epsilon_0 \frac{d\Phi_E}{dt}\right)_{\text{encl}} \qquad (29.21)$$

(Ampere's law including displacement current)

Chapter 29 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLES 29.1, 29.2, and 29.3 (Section 29.2) and CONCEPTUAL EXAMPLES 29.7 and 29.8 (Section 29.3) before attempting these problems.

VP29.8.1 A single circular loop of wire with radius 2.40 cm lies in the *xy*-plane. There is a uniform magnetic field that changes at a steady rate from 0.140 T in the $+z$-direction at $t = 0$ to 0.110 T in the $-z$-direction at $t = 2.00$ s. Take the area vector for the loop to be in the $+z$-direction. Find (a) the magnetic flux through the loop at $t = 0$ and $t = 2.00$ s, and (b) the magnitude of the induced emf in the loop while the field is changing.

VP29.8.2 A circular coil with 455 turns of wire has radius 5.00 cm and resistance 14.5 Ω. The area vector of the coil points in the $+z$-direction. The coil is in a region of uniform magnetic field that points 20.0° from the $+z$-direction. The magnitude of the field is initially 0.0600 T and decreases at a rate of -3.00×10^{-3} T/s. Find the magnitude of (a) the induced emf and (b) the induced current in the coil while the field is changing.

VP29.8.3 A square single loop of wire 4.00 cm on a side with 875 turns lies in the *xy*-plane. The loop is in a uniform magnetic field that changes at a steady rate from $\vec{B} = (0.200\text{ T})\hat{\imath} + (0.150\text{ T})\hat{k}$ at $t = 0$ to $\vec{B} = (0.300\text{ T})\hat{\imath} + (-0.200\text{ T})\hat{k}$ at $t = 3.00$ s. At the time $t = 2.00$ s, find (a) the magnitude of the induced emf and (b) the direction of the induced current in the coil as seen from a point on the $+z$-axis.

VP29.8.4 A coil with area A and N turns of wire lies in the *xy*-plane. Its area vector points in the $+z$-direction. It is exposed to a uniform time-varying magnetic field $\vec{B} = (B_0 \sin \omega t)\hat{k}$, where ω is a positive constant. Find (a) the induced emf in the coil and (b) the direction of the induced current in the coil as seen from a point on the $+z$-axis at $t = 0$ and at $t = \pi/\omega$.

Be sure to review EXAMPLE 29.9 (Section 29.4) before attempting these problems.

VP29.9.1 An isolated conducting rod of length 8.00 cm is oriented parallel to the *x*-axis. It moves in the $+y$-direction at 3.90 m/s in the presence of a uniform magnetic field of magnitude 0.600 T that points in the $-z$-direction. Find (a) the magnitude of the motional emf in the rod and (b) the magnitude and direction of the electric field produced in the rod.

VP29.9.2 In the slidewire generator of Fig. 29.15b, you reverse the rod's motion so that it moves to the left rather than to the right. The moving rod is 5.00 cm long and moves at 2.40 m/s, and the uniform magnetic field has magnitude 0.150 T. If the resistance of the circuit at a given instant is 0.0200 Ω, find the current in the circuit and its direction around the circuit.

VP29.9.3 An isolated conducting rod of length 8.00 cm is oriented parallel to the *x*-axis. At $t = 0$, end a is at $x = y = z = 0$ and end b is at $x = 8.00$ cm, $y = z = 0$. The rod moves with constant velocity $\vec{v} = (1.50\text{ m/s})\hat{\imath} + (2.00\text{ m/s})\hat{\jmath}$ in a uniform magnetic field $\vec{B} = (-0.120\text{ T})\hat{\imath} + (0.150\text{ T})\hat{k}$. (a) Find the magnitude of the induced emf in the rod. (b) Which end of the rod, a or b, is at higher potential?

VP29.9.4 The magnetic force on the moving rod in a slidewire generator (Fig. 29.15b) has magnitude 0.0600 N at an instant when the resistance of the circuit is 0.750 Ω. The rod is 0.190 m long and the magnetic field (which is perpendicular to the plane of the generator) has magnitude 0.550 T. Find the speed of the rod.

Be sure to review EXAMPLE 29.11 (Section 29.5) before attempting these problems.

VP29.11.1 A long solenoid has a cross-sectional area of 3.00 cm^2. The current through the windings is decreasing at a rate of 22.0 A/s. A wire loop of radius 3.10 cm is around the solenoid, parallel with its coils, and centered on the axis of the solenoid. The magnitude of the induced emf is 15.0 μV. Find (a) the number of turns per meter in the solenoid and (b) the magnitude of the induced electric field within the loop.

VP29.11.2 A circular wire loop of radius 0.360 cm lies in the *xz*-plane. There is a uniform magnetic field in the *y*-direction that decreases at 0.0150 T/s. Find the magnitude of the induced electric field in the wire.

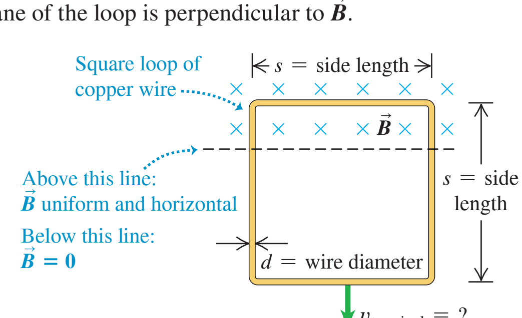
VP29.11.3 A long solenoid with cross-sectional area 4.00 cm^2 and 965 turns per meter is oriented with its axis along the *z*-axis. The field inside the solenoid points in the $+z$-direction. A wire loop of radius 5.00 cm is around the solenoid, parallel with its coils, centered on the axis of the solenoid, and lying in the *xy*-plane. Find the rate of change of the current in the solenoid if the electric field in the loop at the point $x = 5.00$ cm, $y = 0$, $z = 0$ is (a) $\vec{E} = (+1.20 \times 10^{-5}\text{ V/m})\hat{\jmath}$; (b) $\vec{E} = (-1.80 \times 10^{-5}\text{ V/m})\hat{\jmath}$.

VP29.11.4 A long solenoid has 585 turns per meter and a cross-sectional area of 2.70 cm^2. The current through the windings as a function of time is $(0.600\text{ A/s}^2)t^2$. A wire loop of resistance 0.600 Ω is around the solenoid, parallel with its coils, and centered on the axis of the solenoid. What is the current induced in the loop at $t = 13.9$ s?

BRIDGING PROBLEM A Falling Square Loop

A vertically oriented square loop of copper wire falls from rest in a region in which the field $\vec{B}$ is horizontal, uniform, and perpendicular to the plane of the loop, into a field-free region (**Fig. 29.28**). The side length of the loop is s and the wire diameter is d. The resistivity of copper is ρ_R and the density of copper is ρ_m. If the loop reaches its terminal speed while its upper segment is still in the magnetic-field region, find an expression for the terminal speed.

Figure 29.28 A wire loop falling in a horizontal magnetic field $\vec{B}$. The plane of the loop is perpendicular to $\vec{B}$.



SOLUTION GUIDE

IDENTIFY and SET UP

1. The motion of the loop through the magnetic field induces an emf and a current in the loop. The field then gives rise to a magnetic

force on this current that opposes the downward force of gravity. The loop reaches terminal speed (it no longer accelerates) when the upward magnetic force balances the downward force of gravity.

2. Consider the case in which the entire loop is in the magnetic-field region. Is there an induced emf in this case? If so, what is its direction?
3. Consider the case in which only the upper segment of the loop is in the magnetic-field region. Is there an induced emf in this case? If so, what is its direction?
4. For the case in which there is an induced emf and hence an induced current, what is the direction of the magnetic force on each of the four sides of the loop? What is the direction of the *net* magnetic force on the loop?

EXECUTE

5. For the case in which the loop is falling at speed v and there is an induced emf, find (i) the emf, (ii) the induced current, and (iii) the magnetic force on the loop in terms of its resistance R.
6. Find R and the mass of the loop in terms of the given information about the loop.
7. Use your results from steps 5 and 6 to find an expression for the terminal speed.

EVALUATE

8. How does the terminal speed depend on the magnetic-field magnitude B? Explain why this makes sense.

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q29.1 A sheet of copper is placed between the poles of an electromagnet with the magnetic field perpendicular to the sheet. When the sheet is pulled out, a considerable force is required, and the force required increases with speed. Explain. Is a force required also when the sheet is inserted between the poles? Explain.

Q29.2 In Fig. 29.8, if the angular speed ω of the loop is doubled, then the frequency with which the induced current changes direction doubles, and the maximum emf also doubles. Why? Does the torque required to turn the loop change? Explain.

Q29.3 Two circular loops lie side by side in the same plane. One is connected to a source that supplies an increasing current; the other is a simple closed ring. Is the induced current in the ring in the same direction as the current in the loop connected to the source, or opposite? What if the current in the first loop is decreasing? Explain.

Q29.4 For Eq. (29.6), show that if v is in meters per second, B in teslas, and L in meters, then the units of the right-hand side of the equation are joules per coulomb or volts (the correct SI units for $\mathcal{E}$).

Q29.5 A long, straight conductor passes through the center of a metal ring, perpendicular to its plane. If the current in the conductor increases, is a current induced in the ring? Explain.

Q29.6 A student asserted that if a permanent magnet is dropped down a vertical copper pipe, it eventually reaches a terminal velocity even if there is no air resistance. Why should this be? Or should it?

Q29.7 An airplane is in level flight over Antarctica, where the magnetic field of the earth is mostly directed upward away from the ground. As viewed by a passenger facing toward the front of the plane, is the left or the right wingtip at higher potential? Does your answer depend on the direction the plane is flying?

Q29.8 Consider the situation in Exercise 29.21. In part (a), find the direction of the force that the large circuit exerts on the small one. Explain how this result is consistent with Lenz's law.

Q29.9 A metal rectangle is close to a long, straight, current-carrying wire, with two of its sides parallel to the wire. If the current in the long wire is decreasing, is the rectangle repelled by or attracted to the wire? Explain why this result is consistent with Lenz's law.

Q29.10 A square conducting loop is in a region of uniform, constant magnetic field. Can the loop be rotated about an axis along one side and no emf be induced in the loop? Discuss, in terms of the orientation of the rotation axis relative to the magnetic-field direction.

Q29.11 Example 29.6 discusses the external force that must be applied to the slidewire to move it at constant speed. If there were a break in the left-hand end of the U-shaped conductor, how much force would be needed to move the slidewire at constant speed? As in the example, you can ignore friction.

Q29.12 In the situation shown in Fig. 29.18, would it be appropriate to ask how much *energy* an electron gains during a complete trip around the wire loop with current I'? Would it be appropriate to ask what *potential difference* the electron moves through during such a complete trip? Explain your answers.

Q29.13 A metal ring is oriented with the plane of its area perpendicular to a spatially uniform magnetic field that increases at a steady rate. If the radius of the ring is doubled, by what factor do (a) the emf induced in the ring and (b) the electric field induced in the ring change?

Q29.14 Small one-cylinder gasoline engines sometimes use a device called a *magneto* to supply current to the spark plug. A permanent magnet is attached to the flywheel, and a stationary coil is mounted adjacent to it. Explain how this device is able to generate current. What happens when the magnet passes the coil?

Q29.15 Does Lenz's law say that the induced current in a metal loop always flows to oppose the magnetic flux through that loop? Explain.

Q29.16 Does Faraday's law say that a large magnetic flux induces a large emf in a coil? Explain.

Q29.17 Can one have a displacement current as well as a conduction current within a conductor? Explain.

Q29.18 Your physics study partner asks you to consider a parallel-plate capacitor that has a dielectric completely filling the volume between the plates. He then claims that Eqs. (29.13) and (29.14) show that the conduction current in the dielectric equals the displacement current in the dielectric. Do you agree? Explain.

Q29.19 Match the mathematical statements of Maxwell's equations as given in Section 29.7 to these verbal statements. (a) Closed electric field lines are evidently produced only by changing magnetic flux. (b) Closed magnetic field lines are produced both by the motion of electric charge and by changing electric flux. (c) Electric field lines can start on positive charges and end on negative charges. (d) Evidently there are no magnetic monopoles on which to start and end magnetic field lines.

Q29.20 If magnetic monopoles existed, the right-hand side of Eq. (29.20) would include a term proportional to the current of magnetic monopoles. Suppose a steady monopole current is moving in a long straight wire. Sketch the *electric* field lines that such a current would produce.

Q29.21 A type-II superconductor in an external field between B_{c1} and B_{c2} has regions that contain magnetic flux and have resistance, and also has superconducting regions. What is the resistance of a long, thin cylinder of such material?

EXERCISES

Section 29.2 Faraday's Law

29.1 • A single loop of wire with an area of 0.0900 m^2 is in a uniform magnetic field that has an initial value of 3.80 T, is perpendicular to the plane of the loop, and is decreasing at a constant rate of 0.190 T/s. (a) What emf is induced in this loop? (b) If the loop has a resistance of 0.600 Ω, find the current induced in the loop.

29.2 •• In a physics laboratory experiment, a coil with 200 turns enclosing an area of 12 cm^2 is rotated in 0.040 s from a position where its plane is perpendicular to the earth's magnetic field to a position where its plane is parallel to the field. The earth's magnetic field at the lab location is 6.0×10^{-5} T. (a) What is the magnetic flux through each turn of the coil before it is rotated? After it is rotated? (b) What is the average emf induced in the coil?

29.3 • The magnetic flux through a coil is given by $\Phi_B = \alpha t - \beta t^3$, where α and β are constants. (a) What are the units of α and β? (b) If the induced emf is zero at $t = 0.500$ s, how is α related to β? (c) If the emf at $t = 0$ is -1.60 V, what is the emf at $t = 0.250$ s?

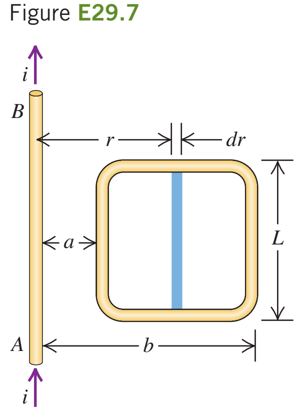
29.4 •• A small, closely wound coil has N turns, area A, and resistance R. The coil is initially in a uniform magnetic field that has magnitude B and a direction perpendicular to the plane of the loop. The coil is then rapidly pulled out of the field so that the flux through the coil is reduced to zero in time Δt. (a) What are the magnitude of the average emf $\mathcal{E}_{av}$ and average current I_{av} induced in the coil? (b) The total charge Q that flows through the coil is given by $Q = I_{av}\Delta t$. Derive an expression for Q in terms of N, A, B, and R. Note that Q does not depend on Δt. (c) What is Q if $N = 150$ turns, $A = 4.50$ cm^2, $R = 30.0$ Ω, and $B = 0.200$ T?

29.5 • A circular loop of wire with a radius of 12.0 cm and oriented in the horizontal xy-plane is located in a region of uniform magnetic field. A field of 1.5 T is directed along the positive z-direction, which is upward. (a) If the loop is removed from the field region in a time interval of 2.0 ms, find the average emf that will be induced in the wire loop during the extraction process. (b) If the coil is viewed looking down on it from above, is the induced current in the loop clockwise or counterclockwise?

29.6 • CALC A coil 4.00 cm in radius, containing 500 turns, is placed in a uniform magnetic field that varies with time according to $B = (0.0120\ \text{T/s})t + (3.00 \times 10^{-5}\ \text{T/s}^4)t^4$. The coil is connected to a 600 Ω resistor, and its plane is perpendicular to the magnetic field. You can ignore the resistance of the coil. (a) Find the magnitude of the induced emf in the coil as a function of time. (b) What is the current in the resistor at time $t = 5.00$ s?

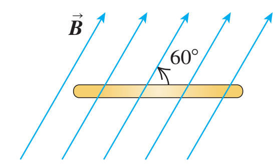
29.7 • CALC The current in the long, straight wire AB shown in **Fig. E29.7** is upward and is increasing steadily at a rate di/dt. (a) At an instant when the current is i, what are the magnitude and direction of the field $\vec{B}$ at a distance r to the right of the wire? (b) What is the flux $d\Phi_B$ through the narrow, shaded strip? (c) What is the total flux through the loop? (d) What is the induced emf in the loop? (e) Evaluate the numerical value of the induced emf if $a = 12.0$ cm, $b = 36.0$ cm, $L = 24.0$ cm, and $di/dt = 9.60$ A/s.

Figure E29.7



29.8 • CALC A flat, circular, steel loop of radius 75 cm is at rest in a uniform magnetic field, as shown in an edge-on view in **Fig. E29.8**. The field is changing with time, according to $B(t) = (1.4\ \text{T})e^{-(0.057\ \text{s}^{-1})t}$. (a) Find the emf induced in the loop as a function of time. (b) When is the induced emf equal to $\frac{1}{10}$ of its initial value? (c) Find the direction of the current induced in the loop, as viewed from above the loop.

Figure E29.8



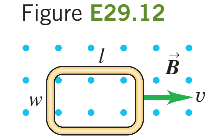
29.9 • **Shrinking Loop.** A circular loop of flexible iron wire has an initial circumference of 165.0 cm, but its circumference is decreasing at a constant rate of 12.0 cm/s due to a tangential pull on the wire. The loop is in a constant, uniform magnetic field oriented perpendicular to the plane of the loop and with magnitude 0.500 T. (a) Find the emf induced in the loop at the instant when 9.0 s have passed. (b) Find the direction of the induced current in the loop as viewed looking along the direction of the magnetic field.

29.10 • A closely wound rectangular coil of 80 turns has dimensions of 25.0 cm by 40.0 cm. The plane of the coil is rotated from a position where it makes an angle of 37.0° with a magnetic field of 1.70 T to a position perpendicular to the field. The rotation takes 0.0600 s. What is the average emf induced in the coil?

29.11 •• CALC A circular loop of wire with radius 2.00 cm and resistance 0.600 Ω is in a region of a spatially uniform magnetic field $\vec{B}$ that is perpendicular to the plane of the loop. At $t = 0$ the magnetic field has magnitude $B_0 = 3.00$ T. The magnetic field then decreases according to the equation $B(t) = B_0 e^{-t/\tau}$, where $\tau = 0.500$ s. (a) What is the maximum magnitude of the current I induced in the loop? (b) What is the induced current I when $t = 1.50$ s?

29.12 • A flat, rectangular coil of dimensions l and w is pulled with uniform speed v through a uniform magnetic field B with the plane of its area perpendicular to the field (**Fig. E29.12**). (a) Find the emf induced in this coil. (b) If the speed and magnetic field are both tripled, what is the induced emf?

Figure E29.12

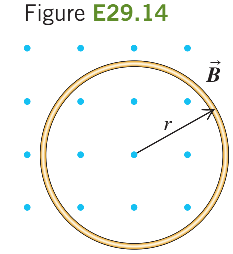


29.13 •• The armature of a small generator consists of a flat, square coil with 120 turns and sides with a length of 1.60 cm. The coil rotates in a magnetic field of 0.0750 T. What is the angular speed of the coil if the maximum emf produced is 24.0 mV?

Section 29.3 Lenz's Law

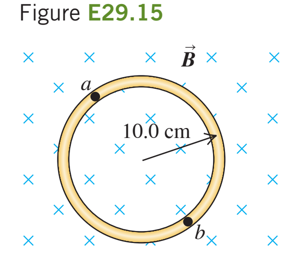
29.14 • A circular loop of wire with radius $r = 0.0480$ m and resistance $R = 0.160$ Ω is in a region of spatially uniform magnetic field, as shown in **Fig. E29.14**. The magnetic field is directed out of the plane of the figure. The magnetic field has an initial value of 8.00 T and is decreasing at a rate of $dB/dt = -0.680$ T/s. (a) Is the induced current in the loop clockwise or counterclockwise? (b) What is the rate at which electrical energy is being dissipated by the resistance of the loop?

Figure E29.14



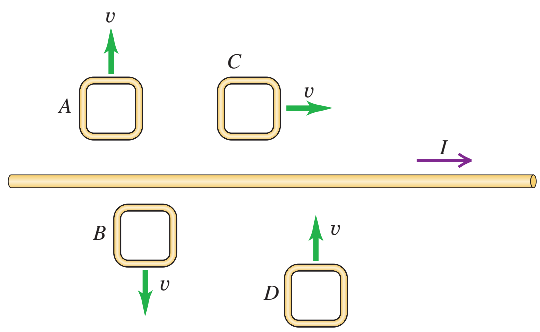
29.15 • A circular loop of wire is in a region of spatially uniform magnetic field, as shown in **Fig. E29.15**. The magnetic field is directed into the plane of the figure. Determine the direction (clockwise or counterclockwise) of the induced current in the loop when (a) B is increasing; (b) B is decreasing; (c) B is constant with value B_0. Explain your reasoning.

Figure E29.15



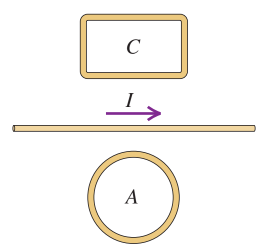
29.16 • The current I in a long, straight wire is constant and is directed toward the right as in **Fig. E29.16**. Conducting loops A, B, C, and D are moving, in the directions shown, near the wire. (a) For each loop, is the direction of the induced current clockwise or counterclockwise, or is the induced current zero? (b) For each loop, what is the direction of the net force that the wire exerts on the loop? Give your reasoning for each answer.

Figure **E29.16**



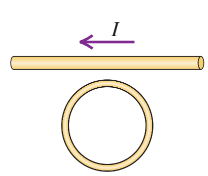
29.17 • Two closed loops A and C are close to a long wire carrying a current I (**Fig. E29.17**). (a) Find the direction (clockwise or counterclockwise) of the current induced in each loop if I is steadily decreasing. (b) While I is decreasing, what is the direction of the net force that the wire exerts on each loop? Explain how you obtain your answer.

Figure **E29.17**



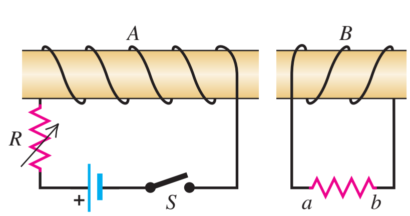
29.18 • The current in **Fig. E29.18** obeys the equation $I(t) = I_0 e^{-bt}$, where $b > 0$. Find the direction (clockwise or counterclockwise) of the current induced in the round coil for $t > 0$.

Figure **E29.18**



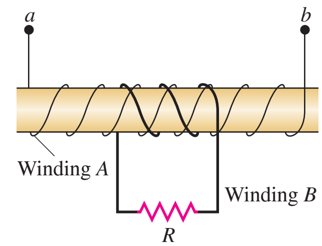
29.19 • Using Lenz's law, determine the direction of the current in resistor ab of **Fig. E29.19** when (a) switch S is opened after having been closed for several minutes; (b) coil B is brought closer to coil A with the switch closed; (c) the resistance of R is decreased while the switch remains closed.

Figure **E29.19**



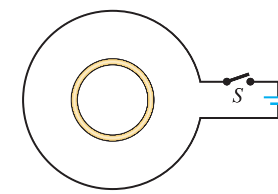
29.20 • A cardboard tube is wrapped with two windings of insulated wire wound in opposite directions, as shown in **Fig. E29.20**. Terminals a and b of winding A may be connected to a battery through a reversing switch. State whether the induced current in the resistor R is from left to right or from right to left in the following circumstances: (a) the current in winding A is from a to b and is increasing; (b) the current in winding A is from b to a and is decreasing; (c) the current in winding A is from b to a and is increasing.

Figure **E29.20**



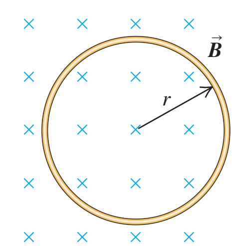
29.21 • A small, circular ring is inside a larger loop that is connected to a battery and a switch (**Fig. E29.21**). Use Lenz's law to find the direction of the current induced in the small ring (a) just after switch S is closed; (b) after S has been closed a long time; (c) just after S has been reopened after it was closed for a long time.

Figure **E29.21**



29.22 • **CALC** A circular loop of wire with radius $r = 0.0250$ m and resistance $R = 0.390\ \Omega$ is in a region of spatially uniform magnetic field, as shown in **Fig. E29.22**. The magnetic field is directed into the plane of the figure. At $t = 0$, $B = 0$. The magnetic field then begins increasing, with $B(t) = (0.380\ \text{T/s}^3)t^3$. What is the current in the loop (magnitude and direction) at the instant when $B = 1.33$ T?

Figure **E29.22**



Section 29.4 Motional Electromotive Force

29.23 • A magnetic field of 0.080 T is in the y-direction. The velocity of wire segment S has a magnitude of 78 m/s and components of 18 m/s in the x-direction, 24 m/s in the y-direction, and 72 m/s in the z-direction. The segment has length 0.50 m and is parallel to the z-axis as it moves. (a) Find the motional emf induced between the ends of the segment. (b) What would the motional emf be if the wire segment was parallel to the y-axis?

29.24 • A rectangular loop of wire with dimensions 1.50 cm by 8.00 cm and resistance $R = 0.600\ \Omega$ is being pulled to the right out of a region of uniform magnetic field. The magnetic field has magnitude $B = 2.40$ T and is directed into the plane of **Fig. E29.24**. At the instant when the speed of the loop is 3.00 m/s and it is still partially in the field region, what force (magnitude and direction) does the magnetic field exert on the loop?

29.25 • In **Fig. E29.25** a conducting rod of length $L = 30.0$ cm moves in a magnetic field $\vec{B}$ of magnitude 0.450 T directed into the plane of the figure. The rod moves with speed $v = 5.00$ m/s in the direction shown. (a) What is the potential difference between the ends of the rod? (b) Which point, a or b, is at higher potential? (c) When the charges in the rod are in equilibrium, what are the magnitude and direction of the electric field within the rod? (d) When the charges in the rod are in equilibrium, which point, a or b, has an excess of positive charge? (e) What is the potential difference across the rod if it moves (i) parallel to ab and (ii) directly out of the page?

Figure **E29.24**

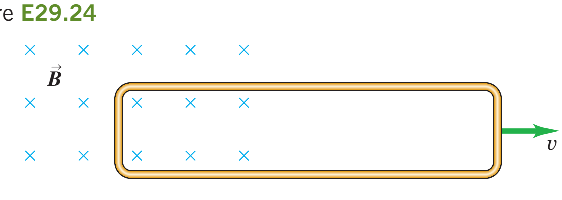
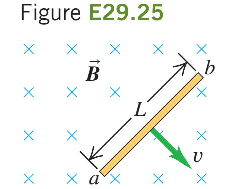
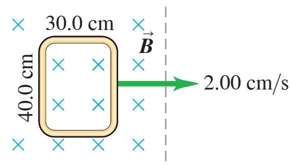


Figure **E29.25**



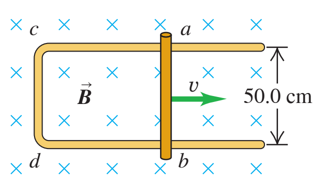
29.26 • A rectangle measuring 30.0 cm by 40.0 cm is located inside a region of a spatially uniform magnetic field of 1.25 T, with the field perpendicular to the plane of the coil (**Fig. E29.26**). The coil is pulled out at a steady rate of 2.00 cm/s traveling perpendicular to the field lines. The region of the field ends abruptly as shown. Find the emf induced in this coil when it is (a) all inside the field; (b) partly inside the field; (c) all outside the field.

Figure **E29.26**



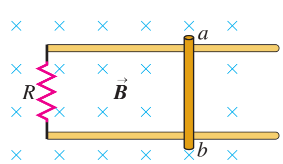
29.27 • The conducting rod *ab* shown in **Fig. E29.27** makes contact with metal rails *ca* and *db*. The apparatus is in a uniform magnetic field of 0.800 T, perpendicular to the plane of the figure. (a) Find the magnitude of the emf induced in the rod when it is moving toward the right with a speed 7.50 m/s. (b) In what direction does the current flow in the rod? (c) If the resistance of the circuit *abdc* is 1.50 Ω (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.50 m/s. You can ignore friction. (d) Compare the rate at which mechanical work is done by the force (Fv) with the rate at which thermal energy is developed in the circuit (I^2R).

Figure **E29.27**



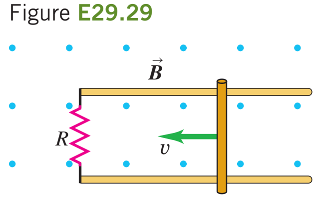
29.28 • A 0.650-m-long metal bar is pulled to the right at a steady 5.0 m/s perpendicular to a uniform, 0.750 T magnetic field. The bar rides on parallel metal rails connected through a 25.0 Ω resistor (**Fig. E29.28**), so the apparatus makes a complete circuit. Ignore the resistance of the bar and the rails. (a) Calculate the magnitude of the emf induced in the circuit. (b) Find the direction of the current induced in the circuit by using (i) the magnetic force on the charges in the moving bar; (ii) Faraday's law; (iii) Lenz's law. (c) Calculate the current through the resistor.

Figure **E29.28**



29.29 • A 0.360-m-long metal bar is pulled to the left by an applied force *F*. The bar rides on parallel metal rails connected through a 45.0 Ω resistor, as shown in **Fig. E29.29**, so the apparatus makes a complete circuit. You can ignore the resistance of the bar and rails. The circuit is in a uniform 0.650 T magnetic field that is directed out of the plane of the figure. At the instant when the bar is moving to the left at 5.90 m/s, (a) is the induced current in the circuit clockwise or counterclockwise and (b) what is the rate at which the applied force is doing work on the bar?

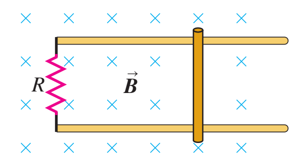
Figure **E29.29**



29.30 • Consider the circuit shown in Fig. E29.29, but with the bar moving to the right with speed v. As in Exercise 29.29, the bar has length 0.360 m, $R = 45.0\ \Omega$, and $B = 0.650$ T. (a) Is the induced current in the circuit clockwise or counterclockwise? (b) At an instant when the 45.0 Ω resistor is dissipating electrical energy at a rate of 0.840 J/s, what is the speed of the bar?

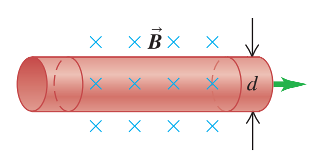
29.31 • A 0.250-m-long bar moves on parallel rails that are connected through a 6.00 Ω resistor, as shown in **Fig. E29.31**, so the apparatus makes a complete circuit. You can ignore the resistance of the bar and rails. The circuit is in a uniform magnetic field $B = 1.20$ T that is directed into the plane of the figure. At an instant when the induced current in the circuit is counterclockwise and equal to 1.75 A, what is the velocity of the bar (magnitude and direction)?

Figure **E29.31**



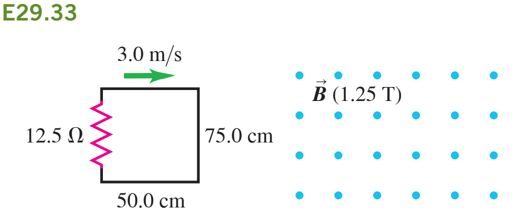
29.32 •• **BIO Measuring Blood Flow.** Blood contains positive and negative ions and thus is a conductor. A blood vessel, therefore, can be viewed as an electrical wire. We can even picture the flowing blood as a series of parallel conducting slabs whose thickness is the diameter d of the vessel moving with speed v. (See **Fig. E29.32**.) (a) If the blood vessel is placed in a magnetic field B perpendicular to the vessel, as in the figure, show that the motional potential difference induced across it is $\mathcal{E} = vBd$. (b) If you expect that the blood will be flowing at 15 cm/s for a vessel 5.0 mm in diameter, what strength of magnetic field will you need to produce a potential difference of 1.0 mV? (c) Show that the volume rate of flow (R) of the blood is equal to $R = \pi\mathcal{E}d/4B$. (*Note:* Although the method developed here is useful in measuring the rate of blood flow in a vessel, it is limited to use in surgery because measurement of the potential $\mathcal{E}$ must be made directly across the vessel.)

Figure **E29.32**



29.33 •• A rectangular circuit is moved at a constant velocity of 3.0 m/s into, through, and then out of a uniform 1.25 T magnetic field, as shown in **Fig. E29.33**. The magnetic-field region is considerably wider than 50.0 cm. Find the magnitude and direction (clockwise or counterclockwise) of the current induced in the circuit as it is (a) going into the magnetic field; (b) totally within the magnetic field, but still moving; and (c) moving out of the field. (d) Sketch a graph of the current in this circuit as a function of time, including the preceding three cases.

Figure **E29.33**



Section 29.5 Induced Electric Fields

29.34 • A metal ring 4.50 cm in diameter is placed between the north and south poles of large magnets with the plane of its area perpendicular to the magnetic field. These magnets produce an initial uniform field of 1.12 T between them but are gradually pulled apart, causing this field to remain uniform but decrease steadily at 0.250 T/s. (a) What is the magnitude of the electric field induced in the ring? (b) In which direction (clockwise or counterclockwise) does the current flow as viewed by someone on the south pole of the magnet?

29.35 •• A long, thin solenoid has 400 turns per meter and radius 1.10 cm. The current in the solenoid is increasing at a uniform rate di/dt. The induced electric field at a point near the center of the solenoid and 3.50 cm from its axis is 8.00×10^{-6} V/m. Calculate di/dt.

29.36 •• A long, thin solenoid has 900 turns per meter and radius 2.50 cm. The current in the solenoid is increasing at a uniform rate of 36.0 A/s. What is the magnitude of the induced electric field at a point near the center of the solenoid and (a) 0.500 cm from the axis of the solenoid; (b) 1.00 cm from the axis of the solenoid?

29.37 • A long, straight solenoid with a cross-sectional area of 8.00 cm^2 is wound with 90 turns of wire per centimeter, and the windings carry a current of 0.350 A. A second winding of 12 turns encircles the solenoid at its center. The current in the solenoid is turned off such that the magnetic field of the solenoid becomes zero in 0.0400 s. What is the average induced emf in the second winding?

Section 29.7 Displacement Current and Maxwell's Equations

29.38 • A parallel-plate, air-filled capacitor is being charged as in Fig. 29.23. The circular plates have radius 4.00 cm, and at a particular instant the conduction current in the wires is 0.520 A. (a) What is the displacement current density j_D in the air space between the plates? (b) What is the rate at which the electric field between the plates is changing? (c) What is the induced magnetic field between the plates at a distance of 2.00 cm from the axis? (d) At 1.00 cm from the axis?

29.39 • **Displacement Current in a Dielectric.** Suppose that the parallel plates in Fig. 29.23 have an area of 3.00 cm^2 and are separated by a 2.50-mm-thick sheet of dielectric that completely fills the volume between the plates. The dielectric has dielectric constant 4.70. (You can ignore fringing effects.) At a certain instant, the potential difference between the plates is 120 V and the conduction current i_C equals 6.00 mA. At this instant, what are (a) the charge q on each plate; (b) the rate of change of charge on the plates; (c) the displacement current in the dielectric?

29.40 • CALC In Fig. 29.23 the capacitor plates have area 5.00 cm^2 and separation 2.00 mm. The plates are in vacuum. The charging current i_C has a *constant* value of 1.80 mA. At $t = 0$ the charge on the plates is zero. (a) Calculate the charge on the plates, the electric field between the plates, and the potential difference between the plates when $t = 0.500\ \mu$s. (b) Calculate dE/dt, the time rate of change of the electric field between the plates. Does dE/dt vary in time? (c) Calculate the displacement current density j_D between the plates, and from this the total displacement current i_D. How do i_C and i_D compare?

29.41 • CALC The electric flux is $(4.0\ \text{V} \cdot \text{m/s}^5)t^5$ through a certain area of a dielectric that has dielectric constant 2.5. (a) Find the displacement current through that area at $t = 1.5$ s. (b) At what time was the displacement current $\frac{1}{6}$ as much?

Section 29.8 Superconductivity

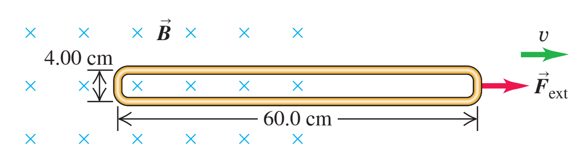
29.42 • At temperatures near absolute zero, B_c approaches 0.142 T for vanadium, a type-I superconductor. The normal phase of vanadium has a magnetic susceptibility close to zero. Consider a long, thin vanadium cylinder with its axis parallel to an external magnetic field $\vec{B}_0$ in the $+x$-direction. At points far from the ends of the cylinder, by symmetry, all the magnetic vectors are parallel to the x-axis. At temperatures near absolute zero, what are the resultant magnetic field $\vec{B}$ and the magnetization $\vec{M}$ inside and outside the cylinder (far from the ends) for (a) $\vec{B}_0 = (0.130\ \text{T})\hat{\imath}$ and (b) $\vec{B}_0 = (0.260\ \text{T})\hat{\imath}$?

PROBLEMS

29.43 •• CP A motor vehicle generates electrical power using an alternator, which employs electromagnetic induction to convert mechanical energy to electrical energy. The alternator acts as a dc generator (Example 29.4). The alternator maintains and replenishes charge on the car's battery and operates headlights, radiator fans, windshield wipers, power windows, computer systems, sensors, sound systems, and other components. (a) A typical car battery provides 70 amp-hours of charge. How many coulombs is that? (b) If headlights each draw 20 A of current, a radiator fan draws 10 A, and windshield wipers each draw 5 A, estimate the peak current needed for a car to operate on a rainy night. (c) A car's alternator supplies an average emf of 14 V as emf induced in a sequence of stator coils in the presence of a magnetic field created by rotor coil electromagnets turned by a pulley system. A stator coil may have 42 windings and a cross-sectional diameter of 5.0 cm, and it rotates at 400 Hz. Estimate the strength of the magnetic field generated by a rotor coil.

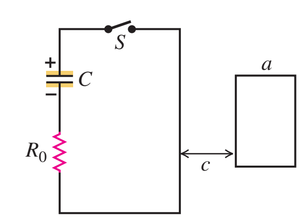
29.44 •• A very long, rectangular loop of wire can slide without friction on a horizontal surface. Initially the loop has part of its area in a region of uniform magnetic field that has magnitude $B = 2.90$ T and is perpendicular to the plane of the loop. The loop has dimensions 4.00 cm by 60.0 cm, mass 24.0 g, and resistance $R = 5.00 \times 10^{-3}\ \Omega$. The loop is initially at rest; then a constant force $F_{ext} = 0.180$ N is applied to the loop to pull it out of the field (**Fig. P29.44**). (a) What is the acceleration of the loop when $v = 3.00$ cm/s? (b) What are the loop's terminal speed and acceleration when the loop is moving at that terminal speed? (c) What is the acceleration of the loop when it is completely out of the magnetic field?

Figure **P29.44**



29.45 •• CP CALC In the circuit shown in **Fig. P29.45**, the capacitor has capacitance $C = 20\ \mu$F and is initially charged to 100 V with the polarity shown. The resistor R_0 has resistance 10 Ω. At time $t = 0$ the switch S is closed. The small circuit is not connected in any way to the large one. The wire of the small circuit has a resistance of 1.0 Ω/m and contains 25 loops. The large circuit is a rectangle 2.0 m by 4.0 m, while the small one has dimensions $a = 10.0$ cm and $b = 20.0$ cm. The distance c is 5.0 cm. (The figure is not drawn to scale.) Both circuits are held stationary. Assume that only the wire nearest the small circuit produces an appreciable magnetic field through it. (a) Find the current in the large circuit 200 μs after S is closed. (b) Find the current in the small circuit 200 μs after S is closed. (*Hint:* See Exercise 29.7.) (c) Find the direction of the current in the small circuit. (d) Justify why we can ignore the magnetic field from all the wires of the large circuit except for the wire closest to the small circuit.

Figure **P29.45**

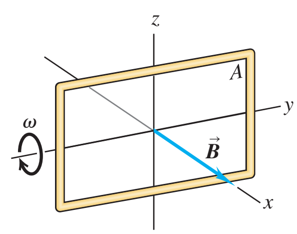


29.46 •• CP CALC In the circuit in Fig. P29.45, an emf of 90.0 V is added in series with the capacitor and the resistor, and the capacitor is initially uncharged. The emf is placed between the capacitor and switch S, with the positive terminal of the emf adjacent to the capacitor. Otherwise, the two circuits are the same as in Problem 29.45. The switch is closed at $t = 0$. When the current in the large circuit is 5.00 A, what are the magnitude and direction of the induced current in the small circuit?

29.47 •• **CALC** A very long, straight solenoid with a cross-sectional area of $2.00\ \text{cm}^2$ is wound with 90.0 turns of wire per centimeter. Starting at $t = 0$, the current in the solenoid is increasing according to $i(t) = (0.160\ \text{A/s}^2)t^2$. A secondary winding of 5 turns encircles the solenoid at its center, such that the secondary winding has the same cross-sectional area as the solenoid. What is the magnitude of the emf induced in the secondary winding at the instant that the current in the solenoid is 3.20 A?

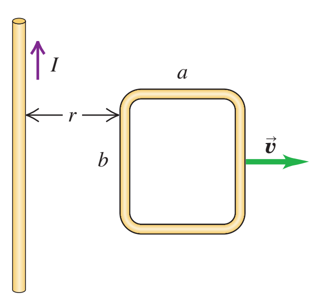
29.48 • Suppose the loop in **Fig. P29.48** is (a) rotated about the y-axis; (b) rotated about the x-axis; (c) rotated about an edge parallel to the z-axis. What is the maximum induced emf in each case if $A = 600\ \text{cm}^2$, $\omega = 35.0\ \text{rad/s}$, and $B = 0.320\ \text{T}$?

Figure **P29.48**



29.49 • In **Fig. P29.49** the loop is being pulled to the right at constant speed v. A constant current I flows in the long wire, in the direction shown. (a) Calculate the magnitude of the net emf $\mathcal{E}$ induced in the loop. Do this two ways: (i) by using Faraday's law of induction (*Hint:* See Exercise 29.7) and (ii) by looking at the emf induced in each segment of the loop due to its motion. (b) Find the direction (clockwise or counterclockwise) of the current induced in the loop. Do this two ways: (i) using Lenz's law and (ii) using the magnetic force on charges in the loop. (c) Check your answer for the emf in part (a) in the following special cases to see whether it is physically reasonable: (i) The loop is stationary; (ii) the loop is very thin, so $a \rightarrow 0$; (iii) the loop gets very far from the wire.

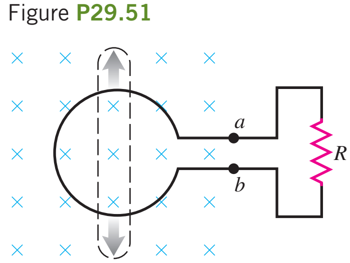
Figure **P29.49**



29.50 •• If you secure a refrigerator magnet about 2 mm from the metallic surface of a refrigerator door and then move the magnet sideways, you can feel a resistive force, indicating the presence of eddy currents in the surface. (a) Estimate the magnetic field strength B of the magnet to be 5 mT (Problem 28.53) and assume the magnet is rectangular with dimensions 4 cm wide by 2 cm high, so its area A is $8\ \text{cm}^2$. Now estimate the magnetic flux Φ_B into the refrigerator door behind the magnet. (b) If you move the magnet sideways at a speed of 2 cm/s, what is a corresponding estimate of the time rate at which the magnetic flux through an area A fixed on the refrigerator is changing as the magnet passes over? Use this estimate to estimate the emf induced under the rectangle on the door's surface.

29.51 • A flexible circular loop 6.50 cm in diameter lies in a magnetic field with magnitude 1.35 T, directed into the plane of the page as shown in **Fig. P29.51**. The loop is pulled at the points indicated by the arrows, forming a loop of zero area in 0.250 s. (a) Find the average induced emf in the circuit. (b) What is the direction of the current in R: from a to b or from b to a? Explain your reasoning.

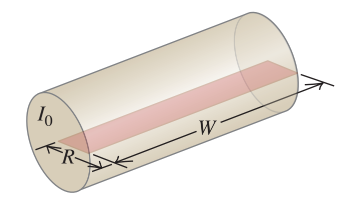
Figure **P29.51**



29.52 ••• **CALC** A conducting rod with length $L = 0.200$ m, mass $m = 0.120$ kg, and resistance $R = 80.0\ \Omega$ moves without friction on metal rails as shown in Fig. 29.11. A uniform magnetic field with magnitude $B = 1.50$ T is directed into the plane of the figure. The rod is initially at rest, and then a constant force with magnitude $F = 1.90$ N and directed to the right is applied to the rod. How many seconds after the force is applied does the rod reach a speed of 25.0 m/s?

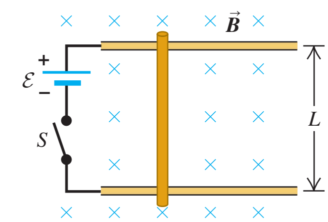
29.53 •• **CALC** A very long, cylindrical wire of radius R carries a current I_0 uniformly distributed across the cross section of the wire. Calculate the magnetic flux through a rectangle that has one side of length W running down the center of the wire and another side of length R, as shown in **Fig. P29.53** (see Exercise 29.7).

Figure **P29.53**



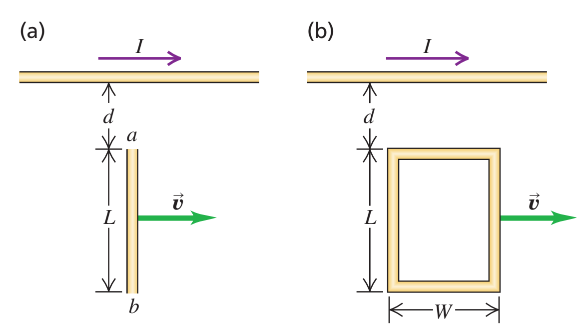
29.54 •• **CP CALC Terminal Speed.** A bar of length $L = 0.36$ m is free to slide without friction on horizontal rails as shown in **Fig. P29.54**. A uniform magnetic field $B = 2.4$ T is directed into the plane of the figure. At one end of the rails there is a battery with emf $\mathcal{E} = 12$ V and a switch S. The bar has mass 0.90 kg and resistance 5.0 Ω; ignore all other resistance in the circuit. The switch is closed at time $t = 0$. (a) Sketch the bar's speed as a function of time. (b) Just after the switch is closed, what is the acceleration of the bar? (c) What is the acceleration of the bar when its speed is 2.0 m/s? (d) What is the bar's terminal speed?

Figure **P29.54**



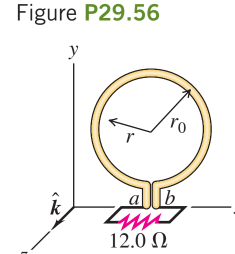
29.55 • **CALC** The long, straight wire shown in **Fig. P29.55a** carries constant current I. A metal bar with length L is moving at constant velocity $\vec{v}$, as shown in the figure. Point a is a distance d from the wire. (a) Calculate the emf induced in the bar. (b) Which point, a or b, is at higher potential? (c) If the bar is replaced by a rectangular wire loop of resistance R (Fig. P29.55b), what is the magnitude of the current induced in the loop?

Figure **P29.55**



29.56 • **CALC** A circular conducting ring with radius $r_0 = 0.0420$ m lies in the xy-plane in a region of uniform magnetic field $\vec{B} = B_0[1 - 3(t/t_0)^2 + 2(t/t_0)^3]\hat{k}$. In this expression, $t_0 = 0.0100$ s and is constant, t is time, $\hat{k}$ is the unit vector in the $+z$-direction, and $B_0 = 0.0800$ T and is constant. At points a and b (**Fig. P29.56**) there is a small gap in the ring with wires leading to an external circuit of resistance $R = 12.0\ \Omega$. There is no magnetic field at the location of the external circuit. (a) Derive an expression, as a function of time, for the total magnetic flux Φ_B through the ring. (b) Determine the emf induced in the ring at time $t = 5.00 \times 10^{-3}$ s. What is the polarity of the emf? (c) Because of the internal resistance of the ring, the current through R at the time given in part (b) is only 3.00 mA. Determine the internal resistance of the ring. (d) Determine the emf in the ring at a time $t = 1.21 \times 10^{-2}$ s. What is the polarity of the emf? (e) Determine the time at which the current through R reverses its direction.

Figure **P29.56**

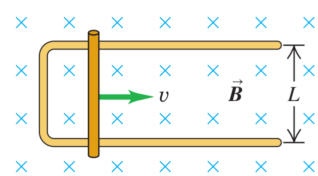


29.57 • CALC A slender rod, 0.240 m long, rotates with an angular speed of 8.80 rad/s about an axis through one end and perpendicular to the rod. The plane of rotation of the rod is perpendicular to a uniform magnetic field with a magnitude of 0.650 T. (a) What is the induced emf in the rod? (b) What is the potential difference between its ends? (c) Suppose instead the rod rotates at 8.80 rad/s about an axis through its center and perpendicular to the rod. In this case, what is the potential difference between the ends of the rod? Between the center of the rod and one end?

29.58 •• A 25.0-cm-long metal rod lies in the xy-plane and makes an angle of 36.9° with the positive x-axis and an angle of 53.1° with the positive y-axis. The rod is moving in the $+x$-direction with a speed of 6.80 m/s. The rod is in a uniform magnetic field $\vec{B} = (0.120\text{ T})\hat{\imath} - (0.220\text{ T})\hat{\jmath} - (0.0900\text{ T})\hat{k}$. (a) What is the magnitude of the emf induced in the rod? (b) Indicate in a sketch which end of the rod is at higher potential.

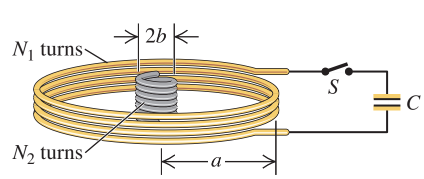
29.59 •• CP CALC A rectangular loop with width L and a slidewire with mass m are as shown in **Fig. P29.59**. A uniform magnetic field $\vec{B}$ is directed perpendicular to the plane of the loop into the plane of the figure. The slidewire is given an initial speed of v_0 and then released. There is no friction between the slidewire and the loop, and the resistance of the loop is negligible in comparison to the resistance R of the slidewire. (a) Obtain an expression for F, the magnitude of the force exerted on the wire while it is moving at speed v. (b) Show that the distance x that the wire moves before coming to rest is $x = mv_0R/L^2B^2$.

Figure **P29.59**



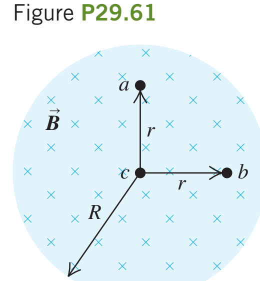
29.60 ••• CP A circular coil with $N_1 = 5000$ turns is made of a conducting material with resistance 0.0100 Ω/m and radius $a = 40.0$ cm. The coil is attached to a $C = 10.00\ \mu$F capacitor as shown in **Fig. P29.60**. A second coil with radius $b = 4.00$ cm, made of the same wire, with $N_2 = 100$ turns, is concentric with the first coil and parallel to it. The capacitor has a charge of $+100\ \mu$C on its upper plate, and the switch S is open. At time $t = 0$ the switch is closed. (a) What is the magnitude of the current in the larger coil immediately after the switch is closed? (b) What is the magnetic flux through each turn of the smaller coil immediately after the switch is closed? (Since $b << a$, we may treat the magnetic field in the smaller coil due to the larger coil as uniform.) (c) What is the direction of the current in the smaller coil immediately after the switch is closed? (d) What is the direction of the current in the smaller coil at $t = 1.26$ ms? (e) What is the magnitude of the current in the smaller coil at $t = 1.26$ ms?

Figure **P29.60**



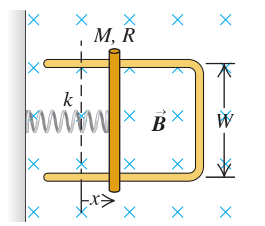
29.61 • The magnetic field $\vec{B}$, at all points within a circular region of radius R, is uniform in space and directed into the plane of the page as shown in **Fig. P29.61**. (The region could be a cross section inside the windings of a long, straight solenoid.) If the magnetic field is increasing at a rate dB/dt, what are the magnitude and direction of the force on a stationary positive point charge q located at points a, b, and c? (Point a is a distance r above the center of the region, point b is a distance r to the right of the center, and point c is at the center of the region.)

Figure **P29.61**



29.62 ••• CP A bar with mass $M = 1.20$ kg and resistance $R = 0.500\ \Omega$ slides without friction on a horizontal U-shaped rail with width $W = 40.0$ cm and negligible resistance. The bar is attached to a spring with spring constant $k = 90.0$ N/m, as shown in **Fig. P29.62**. A constant magnetic field with magnitude 1.00 T points into the plane everywhere in the vicinity. At time $t = 0$ the bar is stretched beyond its equilibrium position by an amount $x = 10.0$ cm and released from rest. (a) This system behaves like a damped oscillator, described by Eq. (14.41). What is the damping coefficient b? (b) With what frequency does the bar oscillate around its equilibrium position? (c) What is the amplitude of the motion at time $t = 5.00$ s? (d) What is the magnitude of the current in the bar when it passes the equilibrium position for the first time? (e) What is the direction of that current?

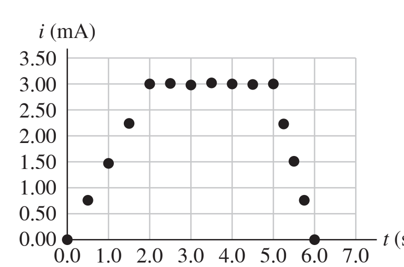
Figure **P29.62**



29.63 ••• CALC A dielectric of permittivity 3.5×10^{-11} F/m completely fills the volume between two capacitor plates. For $t > 0$ the electric flux through the dielectric is $(8.0 \times 10^3\ \text{V}\cdot\text{m/s}^3)t^3$. The dielectric is ideal and nonmagnetic; the conduction current in the dielectric is zero. At what time does the displacement current in the dielectric equal 21 μA?

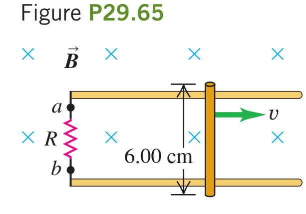
29.64 •• DATA You are evaluating the performance of a large electromagnet. The magnetic field of the electromagnet is zero at $t = 0$ and increases as the current through the windings of the electromagnet is increased. You determine the magnetic field as a function of time by measuring the time dependence of the current induced in a small coil that you insert between the poles of the electromagnet, with the plane of the coil parallel to the pole faces as in Fig. 29.5. The coil has 4 turns, a radius of 0.800 cm, and a resistance of 0.250 Ω. You measure the current i in the coil as a function of time t. Your results are shown in **Fig. P29.64**. Throughout your measurements, the current induced in the coil remains in the same direction. Calculate the magnetic field at the location of the coil for (a) $t = 2.00$ s, (b) $t = 5.00$ s, and (c) $t = 6.00$ s.

Figure **P29.64**



29.65 •• DATA You are conducting an experiment in which a metal bar of length 6.00 cm and mass 0.200 kg slides without friction on two parallel metal rails (**Fig. P29.65**). A resistor with resistance $R = 0.800\ \Omega$ is connected across one end of the rails so that the bar, rails, and resistor form a complete conducting path. The resistances of the rails and of the bar are much less than R and can be ignored. The entire apparatus is in a uniform magnetic field $\vec{B}$ that is directed into the plane of the figure. You give the bar an initial velocity $v = 20.0$ cm/s to the right and then release it, so that the only force on the bar then is the force exerted by the magnetic field. Using high-speed photography, you measure the magnitude of the acceleration of the bar as a function of its speed. Your results are given in the table (next page):

Figure **P29.65**

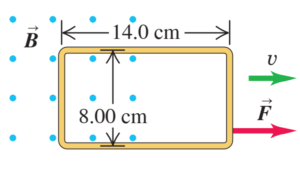


v (cm/s)	20.0	16.0	14.0	12.0	10.0	8.0
a (cm/s^2)	6.2	4.9	4.3	3.7	3.1	2.5

(a) Plot the data as a graph of a versus v. Explain why the data points plotted this way lie close to a straight line, and determine the slope of the best-fit straight line for the data. (b) Use your graph from part (a) to calculate the magnitude B of the magnetic field. (c) While the bar is moving, which end of the resistor, a or b, is at higher potential? (d) How many seconds does it take the speed of the bar to decrease from 20.0 cm/s to 10.0 cm/s?

29.66 ••• DATA You measure the magnitude of the external force $\vec{F}$ that must be applied to a rectangular conducting loop to pull it at constant speed v out of a region of uniform magnetic field $\vec{B}$ that is directed out of the plane of **Fig. P29.66**. The loop has dimensions 14.0 cm by 8.00 cm and resistance $4.00 \times 10^{-3}\ \Omega$; it does not change shape as it moves. The measurements you collect are listed in the table.

Figure **P29.66**



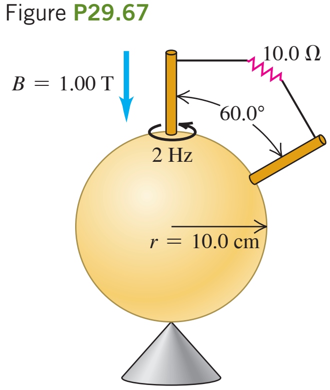
F (N)	0.10	0.21	0.31	0.41	0.52
v (cm/s)	2.0	4.0	6.0	8.0	10.0

(a) Plot the data as a graph of F versus v. Explain why the data points plotted this way lie close to a straight line, and determine the slope of the best-fit straight line for the data. (b) Use your graph from part (a) to calculate the magnitude B of the uniform magnetic field. (c) In Fig. P29.66, is the current induced in the loop clockwise or counterclockwise? (d) At what rate is electrical energy being dissipated in the loop when the speed of the loop is 5.00 cm/s?

CHALLENGE PROBLEMS

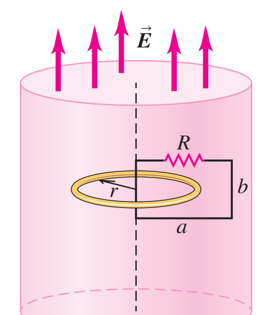
29.67 ••• CP A conducting spherical shell with radius 10.0 cm spins about a vertical axis twice every second in the presence of a constant magnetic field $\vec{B}$ with magnitude 1.00 T that points downward. Two conducting rods supported by a frame contact the sphere with conducting brushes and extend away from the sphere radially. One rod extends from the top of the sphere and the other forms a 60.0° angle with vertical, as shown in **Fig. P29.67**. The outer ends of the rods are connected to each other by a conducting wire that includes a 10.0 Ω resistor. (a) Construct a Cartesian coordinate system with the origin at the center of the sphere, the z-axis pointing upward, and the y-axis pointing rightward so that both rods lie in the yz-plane. What is the velocity $\vec{v}$ of a point on the sphere in the yz-plane at angle θ measured from the positive z-axis? (b) What is the vector product $\vec{v} \times \vec{B}$? (c) What is the line element $d\vec{l}$ along the shortest path on the sphere from the upper rod to the angled rod at angle θ? (d) What is the magnitude of the current in the wire? (e) What direction does the current flow in the vertical rod?

Figure **P29.67**



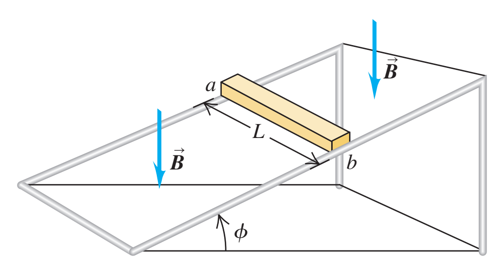
29.68 ••• CP A uniform electric field is directed axially in a cylindrical region that includes a rectangular loop of wire with total resistance R. This loop has radially oriented width a and axially oriented length b, and sits tight against the cylinder axis, as shown in **Fig. P29.68**. The electric field is zero at time $t = 0$ and then increases in time according to $\vec{E} = \eta t^2 \hat{k}$, where η is a constant with units of $\mathrm{V/(m \cdot s^2)}$. (a) What is the magnitude of the displacement current through a circular loop centered on the cylinder axis with radius $r \le a$, at time t? (b) Use Ampere's law to determine the magnitude of the magnetic field a distance $r \le a$ from the cylinder axis at time t. (c) What is the magnetic flux at time t through the rectangular wire loop? (d) What magnitude of current flows in the wire? (e) Does the current flow clockwise or counterclockwise from the perspective shown in the figure?

Figure **P29.68**



29.69 ••• A metal bar with length L, mass m, and resistance R is placed on frictionless metal rails that are inclined at an angle ϕ above the horizontal. The rails have negligible resistance. A uniform magnetic field of magnitude B is directed downward as shown in **Fig. P29.69**. The bar is released from rest and slides down the rails. (a) Is the direction of the current induced in the bar from a to b or from b to a? (b) What is the terminal speed of the bar? (c) What is the induced current in the bar when the terminal speed has been reached? (d) After the terminal speed has been reached, at what rate is electrical energy being converted to thermal energy in the resistance of the bar? (e) After the terminal speed has been reached, at what rate is work being done on the bar by gravity? Compare your answer to that in part (d).

Figure **P29.69**

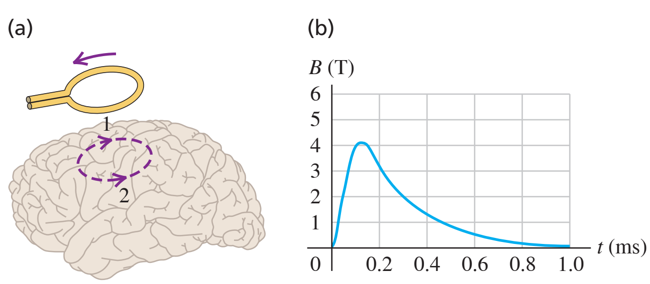


29.70 ••• CP CALC A square, conducting, wire loop of side L, total mass m, and total resistance R initially lies in the horizontal xy-plane, with corners at $(x, y, z) = (0, 0, 0)$, $(0, L, 0)$, $(L, 0, 0)$, and $(L, L, 0)$. There is a uniform, upward magnetic field $\vec{B} = B\hat{k}$ in the space within and around the loop. The side of the loop that extends from $(0, 0, 0)$ to $(L, 0, 0)$ is held in place on the x-axis; the rest of the loop is free to pivot around this axis. When the loop is released, it begins to rotate due to the gravitational torque. (a) Find the *net* torque (magnitude and direction) that acts on the loop when it has rotated through an angle ϕ from its original orientation and is rotating downward at an angular speed ω. (b) Find the angular acceleration of the loop at the instant described in part (a). (c) Compared to the case with zero magnetic field, does it take the loop a longer or shorter time to rotate through 90°? Explain. (d) Is mechanical energy conserved as the loop rotates downward? Explain.

MCAT-STYLE PASSAGE PROBLEMS

BIO Stimulating the Brain. Communication in the nervous system is based on the propagation of electrical signals called *action potentials* along axons, which are extensions of nerve cells (see the MCAT-style Passage Problems in Chapter 26). Action potentials are generated when the electric potential difference across the membrane of the nerve cell changes: Specifically, the inside of the cell becomes more positive. Researchers in clinical medicine and neurobiology cannot stimulate nerves (even noninvasively) at specific locations in conscious human subjects. Using electrodes to apply current to the skin is painful and requires large currents, which could be dangerous.

Anthony Barker and colleagues at the University of Sheffield in England developed a technique called *transcranial magnetic stimulation* (TMS). In this widely used procedure, a coil positioned near the skull produces a time-varying magnetic field that induces in the conductive tissue of the brain (see part (a) of the figure) electric currents that are sufficient to cause action potentials in nerve cells. For example, if the coil is placed near the motor cortex (the region of the brain that controls voluntary movement), scientists can monitor muscle contraction and assess the connections between the brain and the muscles. Part (b) of the figure is a graph of the typical dependence on time t of the magnetic field B produced by the coil.

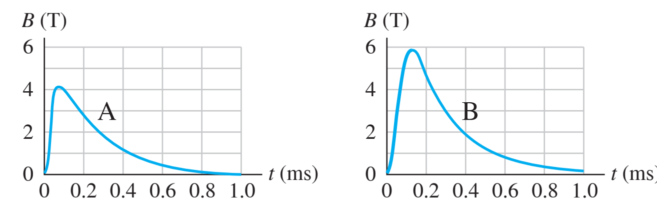


29.71 In part (a) of the figure, a current pulse increases to a peak and then decreases to zero in the direction shown in the stimulating coil. What will be the direction of the induced current (dashed line) in the brain tissue? (a) 1; (b) 2; (c) 1 while the current increases in the stimulating coil, 2 while the current decreases; (d) 2 while the current increases in the stimulating coil, 1 while the current decreases.

29.72 Consider the brain tissue at the level of the dashed line to be a series of concentric circles, each behaving independently of the others. Where will the induced emf be the greatest? (a) At the center of the dashed line; (b) at the periphery of the dashed line; (c) nowhere—it will be the same in all concentric circles; (d) at the center while the stimulating current increases, at the periphery while the current decreases.

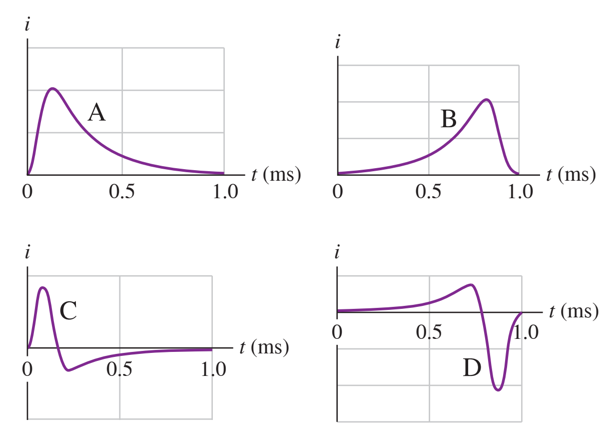
29.73 It may be desirable to increase the maximum induced current in the brain tissue. In **Fig. P29.73**, which time-dependent graph of the magnetic field B in the coil achieves that goal? Assume that everything else remains constant. (a) A; (b) B; (c) either A or B; (d) neither A nor B.

Figure P29.73



29.74 Which graph in **Fig. P29.74** best represents the time t dependence of the current i induced in the brain tissue, assuming that this tissue can be modeled as a resistive circuit? (The units of i are arbitrary.) (a) A; (b) B; (c) C; (d) D.

Figure P29.74



ANSWERS

Chapter Opening Question ?

(iv) As the magnetic stripe moves through the card reader, the coded pattern of magnetization in the stripe causes a varying magnetic flux. An electric field is induced, which causes a current in the reader's circuits. If the card does not move, there is no induced current and none of the credit card's information is read.

Key Example VARIATION Problems

VP29.8.1 (a) $+2.53 \times 10^{-4}\ \mathrm{T \cdot m^2}$ at $t = 0$, $-1.99 \times 10^{-4}\ \mathrm{T \cdot m^2}$ at $t = 2.00$ s (b) 2.26×10^{-4} V
VP29.8.2 (a) 1.01×10^{-2} V (b) 6.95×10^{-4} A
VP29.8.3 (a) 0.163 V (b) counterclockwise
VP29.8.4 (a) $-\omega N B_0 A \cos \omega t$ (b) clockwise at $t = 0$, counterclockwise at $t = \pi/\omega$
VP29.9.1 (a) 0.187 V (b) 2.34 V/m, $+x$-direction
VP29.9.2 0.900 A, clockwise
VP29.9.3 (a) 2.40×10^{-2} V (b) end b
VP29.9.4 4.12 m/s
VP29.11.1 (a) 1.81×10^3 turns/meter (b) 7.70×10^{-5} V/m
VP29.11.2 2.70×10^{-5} V/m
VP29.11.3 (a) -7.77 A/s (b) $+11.7$ A/s
VP29.11.4 5.52×10^{-6} A

Bridging Problem

$v_{\mathrm{terminal}} = 16\rho_m \rho_R g / B^2$