? The sound from a horn travels more slowly on a cold winter day high in the mountains than on a warm summer day at sea level. This is because at high elevations in winter, the air has lower (i) pressure; (ii) density; (iii) humidity; (iv) temperature; (v) mass per mole.

16 Sound and Hearing

Of all the mechanical waves that occur in nature, the most important in our everyday lives are longitudinal waves in a medium—usually air—called *sound* waves. The reason is that the human ear is tremendously sensitive and can detect sound waves even of very low intensity. The ability to hear an unseen nocturnal predator was essential to the survival of our ancestors, so it is no exaggeration to say that we humans owe our existence to our highly evolved sense of hearing.

In Chapter 15 we described mechanical waves primarily in terms of displacement; however, because the ear is primarily sensitive to changes in pressure, it's often more appropriate to describe sound waves in terms of *pressure* fluctuations. We'll study the relationships among displacement, pressure fluctuation, and intensity and the connections between these quantities and human sound perception.

When a source of sound or a listener moves through the air, the listener may hear a frequency different from the one emitted by the source. This is the Doppler effect, which has important applications in medicine and technology.

16.1 SOUND WAVES

The most general definition of **sound** is a longitudinal wave in a medium. Our main concern is with sound waves in air, but sound can travel through any gas, liquid, or solid. You may be all too familiar with the propagation of sound through a solid if your neighbor's stereo speakers are right next to your wall.

The simplest sound waves are sinusoidal waves, which have definite frequency, amplitude, and wavelength. The human ear is sensitive to waves in the frequency range from about 20 to 20,000 Hz, called the **audible range,** but we also use the term "sound" for similar waves with frequencies above **(ultrasonic)** and below **(infrasonic)** the range of human hearing.

LEARNING OUTCOMES

In this chapter, you'll learn...

16.1 How to describe a sound wave in terms of either particle displacements or pressure fluctuations.

16.2 How to calculate the speed of sound waves in different materials.

16.3 How to calculate the intensity of a sound wave.

16.4 What determines the particular frequencies of sound produced by an organ or a flute.

16.5 How resonance occurs in musical instruments.

16.6 What happens when sound waves from different sources overlap.

16.7 How to describe what happens when two sound waves of slightly different frequencies are combined.

16.8 Why the pitch of a siren changes as it moves past you.

16.9 Why an airplane flying faster than sound produces a shock wave.

You'll need to review...

6.4 Power.

8.1 The impulse–momentum theorem.

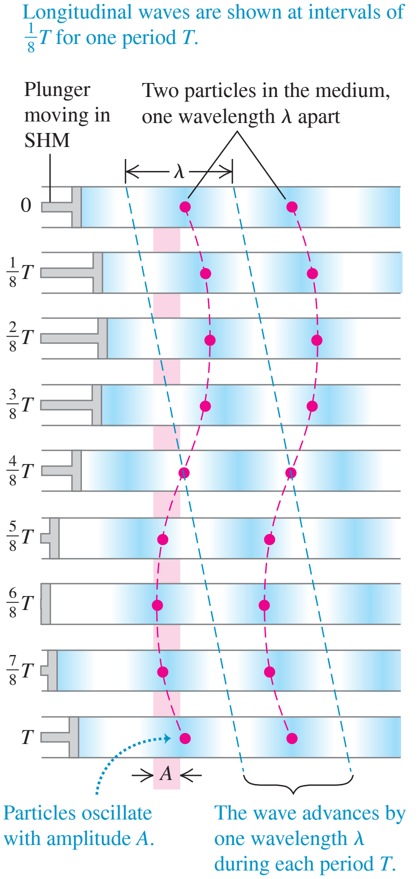
11.4 Bulk modulus and Young's modulus.

12.2 Gauge pressure and absolute pressure.

14.8 Forced oscillations and resonance.

15.1–15.8 Mechanical waves.

Figure **16.1** A sinusoidal longitudinal wave traveling to the right in a fluid. (Compare to Fig. 15.7.)



Sound waves usually travel outward in all directions from the source of sound, with an amplitude that depends on the direction and distance from the source. We'll return to this point in the next section. For now, we concentrate on the idealized case of a sound wave that propagates in the positive x-direction only. As we discussed in Section 15.3, for such a wave, the wave function $y(x, t)$ gives the instantaneous displacement y of a particle in the medium at position x at time t. If the wave is sinusoidal, we can express it by using Eq. (15.7):

$$y(x, t) = A\cos(kx - \omega t) \quad \text{(sound wave propagating in the } +x \text{ direction)} \tag{16.1}$$

In a longitudinal wave the displacements are *parallel* to the direction of travel of the wave, so distances x and y are measured parallel to each other, not perpendicular as in a transverse wave. The amplitude A is the maximum displacement of a particle in the medium from its equilibrium position (**Fig. 16.1**). Hence A is also called the **displacement amplitude.**

Sound Waves as Pressure Fluctuations

We can also describe sound waves in terms of variations of *pressure* at various points. In a sinusoidal sound wave in air, the pressure fluctuates sinusoidally above and below atmospheric pressure p_a with the same frequency as the motions of the air particles. The human ear operates by sensing such pressure variations. A sound wave entering the ear canal exerts a fluctuating pressure on one side of the eardrum; the air on the other side of the eardrum, vented to the outside by the Eustachian tube, is at atmospheric pressure. The pressure difference on the two sides of the eardrum sets it into motion. Microphones and similar devices also usually sense pressure differences, not displacements.

Let $p(x, t)$ be the instantaneous pressure fluctuation in a sound wave at any point x at time t. That is, $p(x, t)$ is the amount by which the pressure *differs* from normal atmospheric pressure p_a. Think of $p(x, t)$ as the *gauge pressure* defined in Section 12.2; it can be either positive or negative. The *absolute* pressure at a point is then $p_a + p(x, t)$.

To see the connection between the pressure fluctuation $p(x, t)$ and the displacement $y(x, t)$ in a sound wave propagating in the $+x$-direction, consider an imaginary cylinder of a wave medium (gas, liquid, or solid) with cross-sectional area S and its axis along the direction of propagation (**Fig. 16.2**). When no sound wave is present, the cylinder has length Δx and volume $V = S\,\Delta x$, as shown by the shaded volume in Fig. 16.2. When a wave is present, at time t the end of the cylinder that is initially at x is displaced by $y_1 = y(x, t)$, and the end that is initially at $x + \Delta x$ is displaced by $y_2 = y(x + \Delta x, t)$; this is shown by the red lines. If $y_2 > y_1$, as shown in Fig. 16.2, the cylinder's volume has increased, which causes a decrease in pressure. If $y_2 < y_1$, the cylinder's volume has decreased and the pressure has increased. If $y_2 = y_1$, the cylinder is simply shifted to the left or right; there is no volume change and no pressure fluctuation. The pressure fluctuation depends on the *difference* between the displacements at neighboring points in the medium.

Quantitatively, the change in volume ΔV of the cylinder is

$$\Delta V = S(y_2 - y_1) = S[y(x + \Delta x, t) - y(x, t)]$$

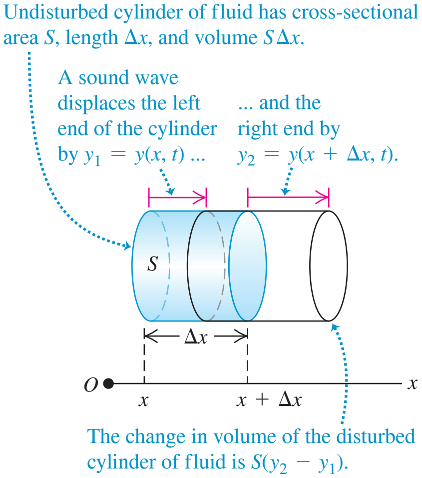
In the limit as $\Delta x \rightarrow 0$, the fractional change in volume dV/V (volume change divided by original volume) is

$$\frac{dV}{V} = \lim_{\Delta x \to 0} \frac{S[y(x + \Delta x, t) - y(x, t)]}{S\,\Delta x} = \frac{\partial y(x, t)}{\partial x} \tag{16.2}$$

The fractional volume change is related to the pressure fluctuation by the bulk modulus B, which by definition [Eq. (11.3)] is $B = -p(x, t)/(dV/V)$ (see Section 11.4). Solving for $p(x, t)$, we have

$$p(x, t) = -B\frac{\partial y(x, t)}{\partial x} \tag{16.3}$$

Figure **16.2** As a sound wave propagates along the x-axis, the left and right ends undergo different displacements y_1 and y_2.



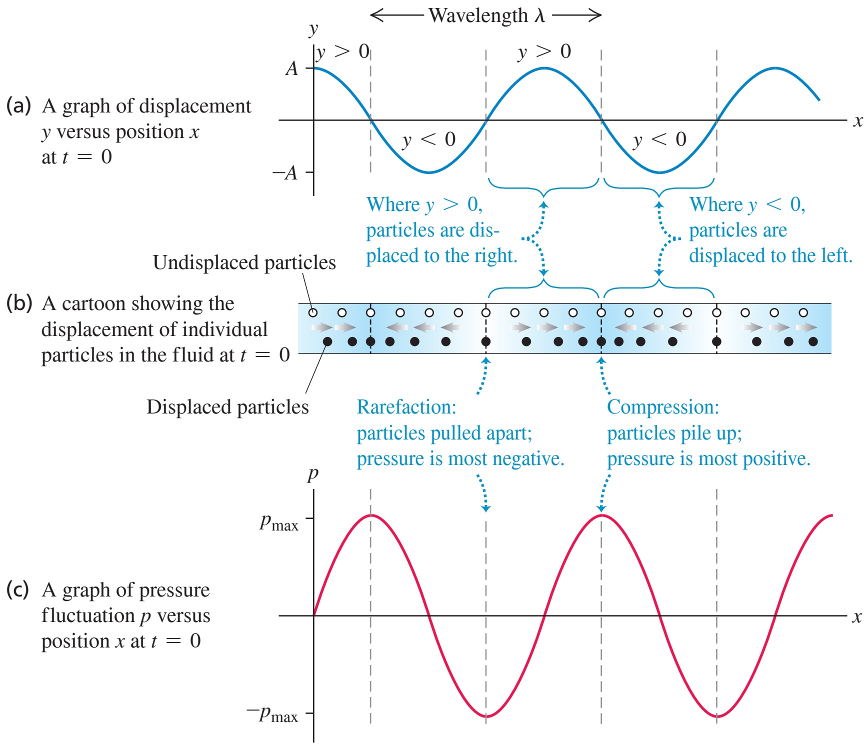


Figure **16.3** Three ways to describe a sound wave.

The negative sign arises because when $\partial y(x, t)/\partial x$ is positive, the displacement is greater at $x + \Delta x$ than at x, corresponding to an increase in volume, a decrease in pressure, and a negative pressure fluctuation.

When we evaluate $\partial y(x, t)/\partial x$ for the sinusoidal wave of Eq. (16.1), we find

$$p(x, t) = BkA\sin(kx - \omega t) \tag{16.4}$$

Figure 16.3 shows $y(x, t)$ and $p(x, t)$ for a sinusoidal sound wave at $t = 0$. It also shows how individual particles of the wave are displaced at this time. While $y(x, t)$ and $p(x, t)$ describe the same wave, these two functions are one-quarter cycle out of phase: At any time, the displacement is greatest where the pressure fluctuation is zero, and vice versa. In particular, note that the compressions (points of greatest pressure and density) and rarefactions (points of lowest pressure and density) are points of *zero* displacement.

Equation (16.4) shows that the quantity BkA represents the maximum pressure fluctuation. We call this the **pressure amplitude,** denoted by $p_{\max}$:

Pressure amplitude, sinusoidal sound wave

Bulk modulus of medium

$$p_{\max} = BkA \tag{16.5}$$

Displacement amplitude

Wave number $= 2\pi/\lambda$

Waves of shorter wavelength λ (larger wave number $k = 2\pi/\lambda$) have greater pressure variations for a given displacement amplitude because the maxima and minima are squeezed closer together. A medium with a large value of bulk modulus B is less compressible and so requires a greater pressure amplitude for a given volume change (that is, a given displacement amplitude).

CAUTION **Graphs of a sound wave** The graphs in Fig. 16.3 show the wave at only *one* instant of time. Because the wave is propagating in the $+x$-direction, as time goes by the wave patterns described by the functions $y(x, t)$ and $p(x, t)$ move to the right at the wave speed $v = \omega/k$. The particles, by contrast, simply oscillate back and forth in simple harmonic motion as shown in Fig. 16.1.

EXAMPLE 16.1 Amplitude of a sound wave in air

In a sinusoidal sound wave of moderate loudness, the maximum pressure variations are about 3.0×10^{-2} Pa above and below atmospheric pressure. Find the corresponding maximum displacement if the frequency is 1000 Hz. In air at normal atmospheric pressure and density, the speed of sound is 344 m/s and the bulk modulus is 1.42×10^5 Pa.

IDENTIFY and SET UP This problem involves the relationship between two ways of describing a sound wave: in terms of displacement and in terms of pressure. The target variable is the displacement amplitude A. We are given the pressure amplitude $p_{\max}$, wave speed v, frequency f, and bulk modulus B. Equation (16.5) relates the target variable A to $p_{\max}$. We use $\omega = vk$ [Eq. (15.6)] to determine the wave number k from v and the angular frequency $\omega = 2\pi f$.

EXECUTE From Eq. (15.6),

$$k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{344 \text{ m/s}} = 18.3 \text{ rad/m}$$

Then from Eq. (16.5), the maximum displacement is

$$A = \frac{p_{\max}}{Bk} = \frac{3.0 \times 10^{-2} \text{ Pa}}{(1.42 \times 10^5 \text{ Pa})(18.3 \text{ rad/m})} = 1.2 \times 10^{-8} \text{ m}$$

EVALUATE This displacement amplitude is only about $\frac{1}{100}$ the size of a human cell. The ear actually senses pressure fluctuations; it detects these minuscule displacements only indirectly.

KEYCONCEPT In a sound wave, the pressure amplitude (maximum pressure fluctuation) and displacement amplitude (maximum displacement of a particle in the medium) are proportional to each other. The proportionality constant depends on the wavelength of the sound and the bulk modulus of the medium.

EXAMPLE 16.2 Amplitude of a sound wave in the inner ear

A sound wave that enters the human ear sets the eardrum into oscillation, which in turn causes oscillation of the *ossicles,* a chain of three tiny bones in the middle ear (**Fig. 16.4**). The ossicles transmit this oscillation to the fluid (mostly water) in the inner ear; there the fluid motion disturbs hair cells that send nerve impulses to the brain with information about the sound. The area of the moving part of the eardrum is about 43 mm^2, and that of the stapes (the smallest of the ossicles) where it connects to the inner ear is about 3.2 mm^2. For the sound in Example 16.1, determine (a) the pressure amplitude and (b) the displacement amplitude of the wave in the fluid of the inner ear, in which the speed of sound is 1500 m/s.

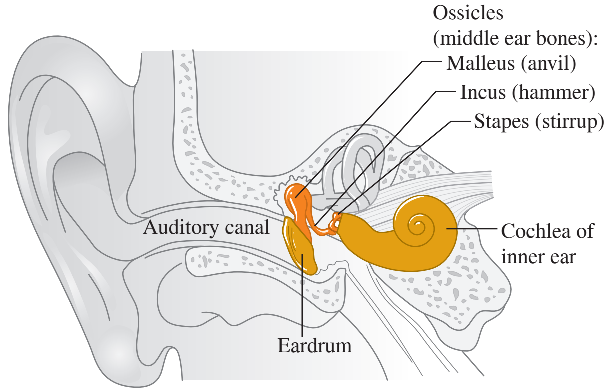
IDENTIFY and SET UP Although the sound wave here travels in liquid rather than air, the same principles and relationships among the properties of the wave apply. We can ignore the mass of the tiny ossicles (about 58 mg $= 5.8 \times 10^{-5}$ kg), so the force they exert on the inner-ear fluid is the same as that exerted on the eardrum and ossicles by the incident sound wave. (In Chapters 4 and 5 we used the same idea to say that the tension is the same at either end of a massless rope.) Hence the pressure amplitude in the inner ear, $p_{\max\,(\text{inner ear})}$, is greater than in the outside air, $p_{\max\,(\text{air})}$, because the same force is exerted on a smaller area (the area of the stapes versus the area of the eardrum). Given $p_{\max\,(\text{inner ear})}$, we find the displacement amplitude $A_{\text{inner ear}}$ from Eq. (16.5).

EXECUTE (a) From the area of the eardrum and the pressure amplitude in air found in Example 16.1, the maximum force exerted by the sound wave in air on the eardrum is $F_{\max} = p_{\max\,(\text{air})} S_{\text{eardrum}}$. Then

$$p_{\max\,(\text{inner ear})} = \frac{F_{\max}}{S_{\text{stapes}}} = p_{\max\,(\text{air})} \frac{S_{\text{eardrum}}}{S_{\text{stapes}}}$$

$$= (3.0 \times 10^{-2} \text{ Pa}) \frac{43 \text{ mm}^2}{3.2 \text{ mm}^2} = 0.40 \text{ Pa}$$

Figure **16.4** The anatomy of the human ear. The middle ear is the size of a small marble; the ossicles (incus, malleus, and stapes) are the smallest bones in the human body.



(b) To find the maximum displacement $A_{\text{inner ear}}$, we use $A = p_{\max}/Bk$ as in Example 16.1. The inner-ear fluid is mostly water, which has a much greater bulk modulus B than air. From Table 11.2 the compressibility of water (unfortunately also called k) is 45.8×10^{-11} Pa^{-1}, so $B_{\text{fluid}} = 1/(45.8 \times 10^{-11} \text{ Pa}^{-1}) = 2.18 \times 10^9$ Pa.

The wave in the inner ear has the same angular frequency ω as the wave in the air because the air, eardrum, ossicles, and inner-ear fluid all oscillate together (see Example 15.8 in Section 15.8). But because the wave speed v is greater in the inner ear than in the air (1500 m/s versus 344 m/s), the wave number $k = \omega/v$ is smaller. Using the value of ω from Example 16.1, we find

$$k_{\text{inner ear}} = \frac{\omega}{v_{\text{inner ear}}} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{1500 \text{ m/s}} = 4.2 \text{ rad/m}$$

Putting everything together, we have

$$A_{\text{inner ear}} = \frac{p_{\max\,(\text{inner ear})}}{B_{\text{fluid}}\,k_{\text{inner ear}}} = \frac{0.40\ \text{Pa}}{(2.18 \times 10^{9}\ \text{Pa})(4.2\ \text{rad/m})}$$

$$= 4.4 \times 10^{-11}\ \text{m}$$

EVALUATE In part (a) we see that the ossicles increase the pressure amplitude by a factor of $(43\ \text{mm}^2)/(3.2\ \text{mm}^2) = 13$. This amplification helps give the human ear its great sensitivity.

The displacement amplitude in the inner ear is even smaller than in the air. But *pressure* variations within the inner-ear fluid are what set the hair cells into motion, so what matters is that the pressure amplitude is larger in the inner ear than in the air.

KEYCONCEPT When a sound wave travels from one medium into a different medium, the wave frequency and angular frequency remain the same. The wave number and wavelength can change, however, as can the pressure amplitude and displacement amplitude.

Perception of Sound Waves

The physical characteristics of a sound wave are directly related to the perception of that sound by a listener. For a given frequency, the greater the pressure amplitude of a sinusoidal sound wave, the greater the perceived **loudness.** The relationship between pressure amplitude and loudness is not a simple one, and it varies from one person to another. One important factor is that the ear is not equally sensitive to all frequencies in the audible range. A sound at one frequency may seem louder than one of equal pressure amplitude at a different frequency. At 1000 Hz the minimum pressure amplitude that can be perceived with normal hearing is about 3×10^{-5} Pa; to produce the same loudness at 200 Hz or 15,000 Hz requires about 3×10^{-4} Pa. Perceived loudness also depends on the health of the ear. Age usually brings a loss of sensitivity at high frequencies.

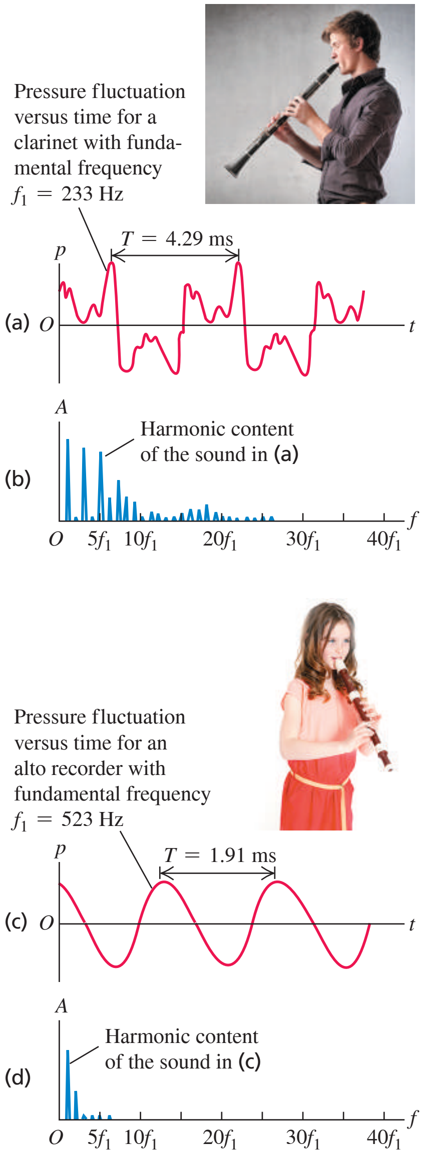
The frequency of a sound wave is the primary factor in determining the **pitch** of a sound, the quality that lets us classify the sound as "high" or "low." The higher the frequency of a sound (within the audible range), the higher the pitch that a listener will perceive. Pressure amplitude also plays a role in determining pitch. When a listener compares two sinusoidal sound waves with the same frequency but different pressure amplitudes, the one with the greater pressure amplitude is usually perceived as louder but also as slightly lower in pitch.

Musical sounds have wave functions that are more complicated than a simple sine function. **Figure 16.5a** shows the pressure fluctuation in the sound wave produced by a clarinet. The pattern is so complex because the column of air in a wind instrument like a clarinet vibrates at a fundamental frequency and at many harmonics at the same time. (In Section 15.8, we described this same behavior for a string that has been plucked, bowed, or struck. We'll examine the physics of wind instruments in Section 16.4.) The sound wave produced in the surrounding air has a similar amount of each harmonic—that is, a similar *harmonic content*. Figure 16.5b shows the harmonic content of the sound of a clarinet. The mathematical process of translating a pressure-time graph like Fig. 16.5a into a graph of harmonic content like Fig. 16.5b is called *Fourier analysis*.

Two tones produced by different instruments might have the same fundamental frequency (and thus the same pitch) but sound different because of different harmonic content. The difference in sound is called **timbre** and is often described in subjective terms such as reedy, mellow, and tinny. A tone that is rich in harmonics, like the clarinet tone in Figs. 16.5a and 16.5b, usually sounds thin and "reedy," while a tone containing mostly a fundamental, like the alto recorder tone in Figs. 16.5c and 16.5d, is more mellow and flute-like. The same principle applies to the human voice, which is another wind instrument; the vowels "a" and "e" sound different because of differences in harmonic content.

Another factor in determining tone quality is the behavior at the beginning (*attack*) and end (*decay*) of a tone. A piano tone begins with a thump and then dies away gradually. A harpsichord tone, in addition to having different harmonic content, begins much more quickly with a click, and the higher harmonics begin before the lower ones. When the key is released, the sound also dies away much more rapidly with a harpsichord than with a piano. Similar effects are present in other musical instruments.

Figure **16.5** Different representations of the sound of (a), (b) a clarinet and (c), (d) an alto recorder. (Graphs adapted from R.E. Berg and D.G. Stork, *The Physics of Sound*, Prentice-Hall, 1982.)



BIO APPLICATION Hearing Loss from Amplified Sound Due to exposure to highly amplified music, many young musicians have suffered permanent ear damage and have hearing typical of persons 65 years of age. Headphones for personal music players used at high volume pose similar threats to hearing. Be careful!

Unlike the tones made by musical instruments, **noise** is a combination of *all* frequencies, not just frequencies that are integer multiples of a fundamental frequency. (An extreme case is "white noise," which contains equal amounts of all frequencies across the audible range.) Examples include the sound of the wind and the hissing sound you make in saying the consonant "s."

TEST YOUR UNDERSTANDING OF SECTION 16.1 You use an electronic signal generator to produce a sinusoidal sound wave in air. You then increase the frequency of the wave from 100 Hz to 400 Hz while keeping the pressure amplitude constant. What effect does this have on the displacement amplitude of the sound wave? (i) It becomes four times greater; (ii) it becomes twice as great; (iii) it is unchanged; (iv) it becomes $\frac{1}{2}$ as great; (v) it becomes $\frac{1}{4}$ as great.

ANSWER

(v) From Eq. (16.5), the displacement amplitude is $A = p_{\max}/Bk$. The pressure amplitude $p_{\max}$ and bulk modulus B remain the same, but the frequency f increases by a factor of 4. Hence the wave number $k = \omega/v = 2\pi f/v$ also increases by a factor of 4. Since A is inversely proportional to k, the displacement amplitude becomes $\frac{1}{4}$ as great. In other words, at higher frequency a smaller maximum displacement is required to produce the same maximum pressure fluctuation.

16.2 SPEED OF SOUND WAVES

We found in Section 15.4 that the speed v of a transverse wave on a string depends on the string tension F and the linear mass density μ: $v = \sqrt{F/\mu}$. What, we may ask, is the corresponding expression for the speed of sound waves in a gas or liquid? On what properties of the medium does the speed depend?

We can make an educated guess about these questions by remembering a claim that we made in Section 15.4: For mechanical waves in general, the expression for the wave speed is of the form

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

A sound wave in a bulk fluid causes compressions and rarefactions of the fluid, so the restoring-force term in the above expression must be related to how difficult it is to compress the fluid. This is precisely what the bulk modulus B of the medium tells us. According to Newton's second law, inertia is related to mass. The "massiveness" of a bulk fluid is described by its density, or mass per unit volume, ρ. Hence we expect that the speed of sound waves should be of the form $v = \sqrt{B/\rho}$.

To check our guess, we'll derive the speed of sound waves in a fluid in a pipe. This is a situation of some importance, since all musical wind instruments are pipes in which a longitudinal wave (sound) propagates in a fluid (air) (**Fig. 16.6**). Human speech works on the same principle; sound waves propagate in your vocal tract, which is an air-filled pipe connected to the lungs at one end (your larynx) and to the outside air at the other end (your mouth). The steps in our derivation are completely parallel to those we used in Section 15.4 to find the speed of transverse waves.

Figure 16.6 When a wind instrument like this French horn is played, sound waves propagate through the air within the instrument's pipes. The properties of the sound that emerges from the large bell depend on the speed of these waves.

Speed of Sound in a Fluid

Figure 16.7 shows a fluid with density ρ in a pipe with cross-sectional area A. In equilibrium (Fig. 16.7a), the fluid is at rest and under a uniform pressure p. We take the x-axis along the length of the pipe. This is also the direction in which we make a longitudinal wave propagate, so the displacement y is also measured along the pipe, just as in Section 16.1 (see Fig. 16.2).

At time $t = 0$ we start the piston at the left end moving toward the right with constant speed v_y. This initiates a wave motion that travels to the right along the length of the pipe, in which successive sections of fluid begin to move and become compressed at successively later times.

Figure 16.7b shows the fluid at time t. All portions of fluid to the left of point P are moving to the right with speed v_y, and all portions to the right of P are still at rest. The boundary between the moving and stationary portions travels to the right with a speed equal to the speed of propagation or wave speed v. At time t the piston has moved a distance $v_y t$, and the boundary has advanced a distance vt. As with a transverse disturbance in a string, we can compute the speed of propagation from the impulse–momentum theorem.

The quantity of fluid set in motion in time t originally occupied a section of the cylinder with length vt, cross-sectional area A, volume vtA, and mass ρvtA. Its longitudinal momentum (that is, momentum along the length of the pipe) is

$$\text{Longitudinal momentum} = (\rho vtA)v_y$$

Next we compute the increase of pressure, Δp, in the moving fluid. The original volume of the moving fluid, Avt, has decreased by an amount $Av_y t$. From the definition of the bulk modulus B, Eq. (11.13) in Section 11.5,

$$B = \frac{-\text{Pressure change}}{\text{Fractional volume change}} = \frac{-\Delta p}{-Av_y t/Avt} \quad \text{and} \quad \Delta p = B\frac{v_y}{v}$$

The pressure in the moving fluid is $p + \Delta p$, and the force exerted on it by the piston is $(p + \Delta p)A$. The net force on the moving fluid (see Fig. 16.7b) is ΔpA, and the longitudinal impulse is

$$\text{Longitudinal impulse} = \Delta pAt = B\frac{v_y}{v}At$$

Because the fluid was at rest at time $t = 0$, the change in momentum up to time t is equal to the momentum at that time. Applying the impulse–momentum theorem (see Section 8.1), we find

$$B\frac{v_y}{v}At = \rho vtAv_y \tag{16.6}$$

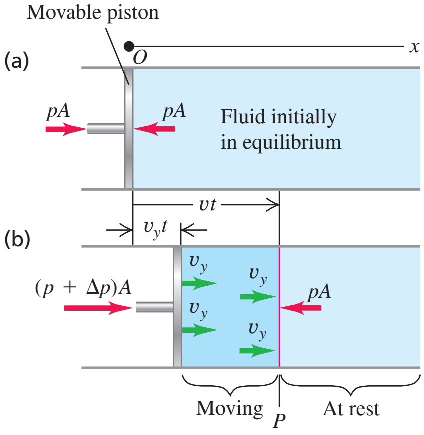
When we solve this expression for v, we get

$$\text{Speed of a longitudinal wave in a fluid} \cdots\rightarrow v = \sqrt{\frac{B \leftarrow\cdots \text{Bulk modulus of fluid}}{\rho \leftarrow\cdots \text{Density of fluid}}} \tag{16.7}$$

which agrees with our educated guess.

While we derived Eq. (16.7) for waves in a pipe, it also applies to longitudinal waves in a bulk fluid, including sound waves traveling in air or water.

Figure **16.7** A sound wave propagating in a fluid confined to a tube. (a) Fluid in equilibrium. (b) A time t after the piston begins moving to the right at speed v_y, the fluid between the piston and point P is in motion. The speed of sound waves is v.



Speed of Sound in a Solid

When a longitudinal wave propagates in a *solid* rod or bar, the situation is somewhat different. The rod expands sideways slightly when it is compressed longitudinally, while a fluid in a pipe with constant cross section cannot move sideways. Using the same kind of reasoning that led us to Eq. (16.7), we can show that the speed of a longitudinal pulse in the rod is given by

$$\text{Speed of a longitudinal wave in a solid rod} \cdots\rightarrow v = \sqrt{\frac{Y \leftarrow\cdots \text{Young's modulus of rod material}}{\rho \leftarrow\cdots \text{Density of rod material}}} \tag{16.8}$$

We defined Young's modulus in Section 11.4.

CAUTION **Solid rods vs. bulk solids** Equation (16.8) applies to only rods whose sides are free to bulge and shrink a little as the wave travels. It does not apply to longitudinal waves in a *bulk* solid because sideways motion in any element of material is prevented by the surrounding material. The speed of longitudinal waves in a bulk solid depends on the density, the bulk modulus, and the *shear* modulus.

Note that Eqs. (16.7) and (16.8) are valid for sinusoidal and other periodic waves, not just for the special case discussed here.

Table 16.1 lists the speed of sound in several bulk materials. Sound waves travel more slowly in lead than in aluminum or steel because lead has a lower bulk modulus and shear modulus and a higher density.

TABLE 16.1 Speed of Sound in Various Bulk Materials

Material	Speed of Sound (m/s)
Gases	
Air (20°C)	344
Helium (20°C)	999
Hydrogen (20°C)	1330
Liquids	
Liquid helium (4 K)	211
Mercury (20°C)	1451
Water (0°C)	1402
Water (20°C)	1482
Water (100°C)	1543
Solids	
Aluminum	6420
Lead	1960
Steel	5941

EXAMPLE 16.3 Wavelength of sonar waves

A ship uses a sonar system (**Fig. 16.8**) to locate underwater objects. Find the speed of sound waves in water using Eq. (16.7), and find the wavelength of a 262 Hz wave.

IDENTIFY and SET UP Our target variables are the speed and wavelength of a sound wave in water. In Eq. (16.7), we use the density of water, $\rho = 1.00 \times 10^3\ \text{kg/m}^3$, and the bulk modulus of water, which we find from the compressibility (see Table 11.2). Given the speed and the frequency $f = 262$ Hz, we find the wavelength from $v = f\lambda$.

EXECUTE In Example 16.2, we used Table 11.2 to find $B = 2.18 \times 10^9$ Pa. Then

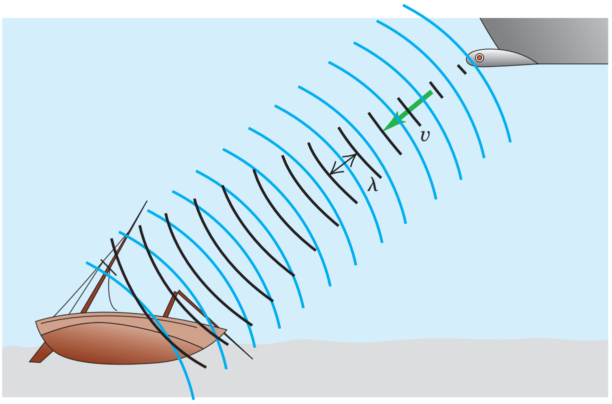
$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.18 \times 10^9\ \text{Pa}}{1.00 \times 10^3\ \text{kg/m}^3}} = 1476\ \text{m/s}$$

and

$$\lambda = \frac{v}{f} = \frac{1476\ \text{m/s}}{262\ \text{s}^{-1}} = 5.64\ \text{m}$$

EVALUATE The calculated value of v agrees well with the value in Table 16.1. Water is denser than air (ρ is larger) but is also much more incompressible (B is much larger), and so the speed $v = \sqrt{B/\rho}$ is greater than the 344 m/s speed of sound in air at ordinary temperatures. The relationship $\lambda = v/f$ then says that a sound wave in water must have a longer wavelength than a wave of the same frequency in air. Indeed, we found in Example 15.1 (Section 15.2) that a 262 Hz sound wave in air has a wavelength of only 1.31 m.

Figure **16.8** A sonar system uses underwater sound waves to detect and locate submerged objects.



KEYCONCEPT The speed of sound waves in a fluid depends on the fluid's bulk modulus and density. A sound wave of a given frequency has a longer wavelength in a medium that has a faster sound speed.

Figure **16.9** This three-dimensional image of a fetus in the womb was made using a sequence of ultrasound scans. Each individual scan reveals a two-dimensional "slice" through the fetus; many such slices were then combined digitally. Ultrasound imaging is also used to study heart valve action and to detect tumors.

Dolphins emit high-frequency sound waves (typically 100,000 Hz) and use the echoes for guidance and for hunting. The corresponding wavelength in water is 1.48 cm. With this high-frequency "sonar" system they can sense objects that are roughly as small as the wavelength (but not much smaller). *Ultrasonic imaging* in medicine uses the same principle; sound waves of very high frequency and very short wavelength, called *ultrasound,* are scanned over the human body, and the "echoes" from interior organs are used to create an image. With ultrasound of frequency 5 MHz $= 5 \times 10^6$ Hz, the wavelength in water (the primary constituent of the body) is 0.3 mm, and features as small as this can be discerned in the image (**Fig. 16.9**). Ultrasound is more sensitive than x rays in distinguishing various kinds of tissues and does not have the radiation hazards associated with x rays.

Speed of Sound in a Gas

Most of the sound waves that we encounter propagate in air. To use Eq. (16.7) to find the speed of sound waves in air, we note that the bulk modulus of a gas depends on pressure: The greater the pressure applied to compress a gas, the more it resists further compression and hence the greater the bulk modulus. (That's why specific values of the bulk modulus for gases are not given in Table 11.1.) The expression for the bulk modulus of a gas for use in Eq. (16.7) is

$$B = \gamma p_0 \tag{16.9}$$

where p_0 is the equilibrium pressure of the gas. The quantity γ (the Greek letter gamma) is called the *ratio of heat capacities.* It is a dimensionless number that characterizes the thermal properties of the gas. (We'll learn more about this quantity in Chapter 19.) As an example, the ratio of heat capacities for air is $\gamma = 1.40$. At normal atmospheric pressure $p_0 = 1.013 \times 10^5$ Pa, so $B = (1.40)(1.013 \times 10^5\ \text{Pa}) = 1.42 \times 10^5$ Pa. This value is minuscule compared to the bulk modulus of a typical solid (see Table 11.1), which is approximately 10^{10} to 10^{11} Pa. This shouldn't be surprising: It's simply a statement that air is far easier to compress than steel.

The density ρ of a gas also depends on the pressure, which in turn depends on the temperature. It turns out that the ratio B/ρ for a given type of ideal gas does *not* depend on the pressure at all, only the temperature. From Eq. (16.7), this means that the speed of sound in a gas is fundamentally a function of temperature T:

Ratio of heat capacities
Gas constant

Speed of sound in an ideal gas

$$v = \sqrt{\frac{\gamma R T}{M}} \qquad (16.10)$$

Absolute temperature
Molar mass

This expression incorporates several quantities that we'll study in Chapters 17, 18, and 19. The temperature T is the *absolute* temperature in kelvins (K), equal to the Celsius temperature plus 273.15; thus 20.00°C corresponds to $T = 293.15$ K. The quantity M is the *molar mass,* or mass per mole of the substance of which the gas is composed. The *gas constant R* has the same value for all gases. The current best numerical value of R is

$$R = 8.3144598(48)\ \text{J/mol} \cdot \text{K}$$

which for practical calculations we can write as 8.314 J/mol · K.

For any particular gas, γ, R, and M are constants, and the wave speed is proportional to the square root of the absolute temperature. We'll see in Chapter 18 that Eq. (16.10) is almost identical to the expression for the average speed of molecules in an ideal gas. This shows that sound speeds and molecular speeds are closely related.

EXAMPLE 16.4 Speed of sound in air

Find the speed of sound in air at $T = 20°\text{C}$, and find the range of wavelengths in air to which the human ear (which can hear frequencies in the range of 20–20,000 Hz) is sensitive. The mean molar mass for air (a mixture of mostly nitrogen and oxygen) is $M = 28.8 \times 10^{-3}$ kg/mol and the ratio of heat capacities is $\gamma = 1.40$.

IDENTIFY and SET UP We use Eq. (16.10) to find the sound speed from γ, T, and M, and we use $v = f\lambda$ to find the wavelengths corresponding to the frequency limits. Note that in Eq. (16.10) temperature T *must* be expressed in kelvins, not Celsius degrees.

EXECUTE At $T = 20°\text{C} = 293$ K, we find

$$v = \sqrt{\frac{\gamma R T}{M}} = \sqrt{\frac{(1.40)(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{28.8 \times 10^{-3}\ \text{kg/mol}}} = 344\ \text{m/s}$$

Using this value of v in $\lambda = v/f$, we find that at 20°C the frequency $f = 20$ Hz corresponds to $\lambda = 17$ m and $f = 20{,}000$ Hz to $\lambda = 1.7$ cm.

EVALUATE Our calculated value of v agrees with the measured sound speed at $T = 20°\text{C}$.

KEYCONCEPT The speed of sound in a gas is determined by the temperature of the gas, its molar mass, and its ratio of heat capacities.

A gas is actually composed of molecules in random motion, separated by distances that are large in comparison with their diameters. The vibrations that constitute a wave in a gas are superposed on the random thermal motion. At atmospheric pressure, a molecule travels an average distance of about 10^{-7} m between collisions, while the displacement amplitude of a faint sound may be only 10^{-9} m. We can think of a gas with a sound wave passing through as being comparable to a swarm of bees; the swarm as a whole oscillates slightly while individual insects move about within the swarm, apparently at random.

TEST YOUR UNDERSTANDING OF SECTION 16.2 Mercury is 13.6 times denser than water. Based on Table 16.1, at 20°C which of these liquids has the greater bulk modulus? (i) Mercury; (ii) water; (iii) both are about the same; (iv) not enough information is given to decide.

ANSWER

(i) From Eq. (16.7), the speed of longitudinal waves (sound) in a fluid is $v = \sqrt{B/\rho}$. We can rewrite this to give an expression for the bulk modulus B in terms of the fluid density ρ and the sound speed v: $B = \rho v^2$. At 20°C the speed of sound in mercury is slightly less than in water (1451 m/s versus 1482 m/s), but the density of mercury is greater than that of water by a large factor (13.6). Hence the bulk modulus of mercury is greater than that of water by a factor of $(13.6)(1451/1482)^2 = 13.0$.

16.3 SOUND INTENSITY

Traveling sound waves, like all other traveling waves, transfer energy from one region of space to another. In Section 15.5 we introduced the *wave intensity* I, equal to the time average rate at which wave energy is transported per unit area across a surface perpendicular to the direction of propagation. Let's see how to express the intensity of a sound wave in a fluid in terms of the displacement amplitude A or pressure amplitude $p_{\max}$.

Let's consider a sound wave propagating in the $+x$-direction so that we can use our expressions from Section 16.1 for the displacement $y(x, t)$ [Eq. (16.1)] and pressure fluctuation $p(x, t)$ [Eq. (16.4)]. In Section 6.4 we saw that power equals the product of force and velocity [see Eq. (6.18)]. So the power per unit area in this sound wave equals the product of $p(x, t)$ (force per unit area) and the *particle* velocity $v_y(x, t)$, which is the velocity at time t of that portion of the wave medium at coordinate x. Using Eqs. (16.1) and (16.4), we find

$$\begin{aligned} v_y(x,t) &= \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t) \\ p(x,t)v_y(x,t) &= [BkA\sin(kx-\omega t)][\omega A \sin(kx - \omega t)] \\ &= B\omega k A^2 \sin^2(kx - \omega t) \end{aligned}$$

CAUTION **Wave velocity vs. particle velocity** Remember that the velocity of the wave as a whole is *not* the same as the particle velocity. While the wave continues to move in the direction of propagation, individual particles in the wave medium merely slosh back and forth, as shown in Fig. 16.1. Furthermore, the maximum speed of a particle of the medium can be very different from the wave speed.

The intensity is the time average value of the power per unit area $p(x, t)v_y(x, t)$. For any value of x the average value of the function $\sin^2(kx - \omega t)$ over one period $T = 2\pi/\omega$ is $\frac{1}{2}$, so

$$I = \tfrac{1}{2}B\omega k A^2 \tag{16.11}$$

Using the relationships $\omega = vk$ and $v = \sqrt{B/\rho}$, we can rewrite Eq. (16.11) as

Intensity of a sinusoidal sound wave in a fluid

$$I = \tfrac{1}{2}\sqrt{\rho B}\,\omega^2 A^2 \tag{16.12}$$

Angular frequency $= 2\pi f$

Displacement amplitude

Density of fluid

Bulk modulus of fluid

It is usually more useful to express I in terms of the pressure amplitude $p_{\max}$. Using Eqs. (16.5) and (16.12) and the relationship $\omega = vk$, we find

$$I = \frac{\omega {p_{\max}}^2}{2Bk} = \frac{v {p_{\max}}^2}{2B} \tag{16.13}$$

By using the wave speed relationship $v = \sqrt{B/\rho}$, we can also write Eq. (16.13) in the alternative forms

Intensity of a sinusoidal sound wave in a fluid

$$I = \frac{{p_{\max}}^2}{2\rho v} = \frac{{p_{\max}}^2}{2\sqrt{\rho B}} \tag{16.14}$$

Pressure amplitude

Wave speed

Bulk modulus of fluid

Density of fluid

You should verify these expressions. Comparison of Eqs. (16.12) and (16.14) shows that sinusoidal sound waves of the same intensity but different frequency have different displacement amplitudes A but the *same* pressure amplitude $p_{\max}$. This is another reason it is usually more convenient to describe a sound wave in terms of pressure fluctuations, not displacement.

The *total* average power carried across a surface by a sound wave equals the product of the intensity at the surface and the surface area, if the intensity over the surface is uniform. The total average sound power emitted by a person speaking in an ordinary conversational tone is about 10^{-5} W, while a loud shout corresponds to about 3×10^{-2} W. If all the residents of New York City were to talk at the same time, the total sound power would be about 100 W, equivalent to the electric power requirement of a medium-sized light bulb. On the other hand, the power required to fill a large auditorium or stadium with loud sound is considerable (see Example 16.7).

If the sound source emits waves in all directions equally, the intensity decreases with increasing distance r from the source according to the inverse-square law (Section 15.5): The intensity is proportional to $1/r^2$. The intensity can be increased by confining the sound waves to travel in the desired direction only (**Fig. 16.10**), although the $1/r^2$ law still applies.

The inverse-square relationship also does not apply indoors because sound energy can reach a listener by reflection from the walls and ceiling. Indeed, part of the architect's job in designing an auditorium is to tailor these reflections so that the intensity is as nearly uniform as possible over the entire auditorium.

Figure **16.10** By cupping your hands like this, you direct the sound waves emerging from your mouth so that they don't propagate to the sides. Hence you can be heard at greater distances.

PROBLEM-SOLVING STRATEGY 16.1 Sound Intensity

IDENTIFY *the relevant concepts:* The relationships between the intensity and amplitude of a sound wave are straightforward. Other quantities are involved in these relationships, however, so it's particularly important to decide which is your target variable.

SET UP *the problem* using the following steps:

1. Sort the physical quantities into categories. Wave properties include the displacement and pressure amplitudes A and $p_{\max}$. The frequency f can be determined from the angular frequency ω, the wave number k, or the wavelength λ. These quantities are related through the wave speed v, which is determined by properties of the medium (B and ρ for a liquid, and γ, T, and M for a gas).
2. List the given quantities and identify the target variables. Find relationships that take you where you want to go.

EXECUTE *the solution:* Use your selected equations to solve for the target variables. Express the temperature in kelvins (Celsius temperature plus 273.15) to calculate the speed of sound in a gas.

EVALUATE *your answer:* If possible, use an alternative relationship to check your results.

EXAMPLE 16.5 Intensity of a sound wave in air

WITH VARIATION PROBLEMS

Find the intensity of the sound wave in Example 16.1, with $p_{\max} = 3.0 \times 10^{-2}$ Pa. Assume the temperature is 20°C so that the density of air is $\rho = 1.20\ \text{kg/m}^3$ and the speed of sound is $v = 344$ m/s.

IDENTIFY and SET UP Our target variable is the intensity I of the sound wave. We are given the pressure amplitude $p_{\max}$ of the wave as well as the density ρ and wave speed v for the medium. We can determine I from $p_{\max}$, ρ, and v from Eq. (16.14).

EXECUTE From Eq. (16.14),

$$I = \frac{{p_{\max}}^2}{2\rho v} = \frac{(3.0 \times 10^{-2}\ \text{Pa})^2}{2(1.20\ \text{kg/m}^3)(344\ \text{m/s})}$$

$$= 1.1 \times 10^{-6}\ \text{J/(s}\cdot\text{m}^2) = 1.1 \times 10^{-6}\ \text{W/m}^2$$

EVALUATE This seems like a very low intensity, but it is well within the range of sound intensities encountered on a daily basis. A very loud sound wave at the threshold of pain has a pressure amplitude of about 30 Pa and an intensity of about 1 W/m^2. The pressure amplitude of the faintest sound wave that can be heard is about 3×10^{-5} Pa, and the corresponding intensity is about $10^{-12}\ \text{W/m}^2$. (Try these values of $p_{\max}$ in Eq. (16.14) to check that the corresponding intensities are as we have stated.)

KEYCONCEPT The intensity (power per unit area) of a sound wave is proportional to the square of the pressure amplitude of the wave. The proportionality constant depends on the density of the medium and the speed of sound in the medium.

EXAMPLE 16.6 Same intensity, different frequencies

WITH VARIATION PROBLEMS

What are the pressure and displacement amplitudes of a 20 Hz sound wave with the same intensity as the 1000 Hz sound wave of Examples 16.1 and 16.5?

IDENTIFY and SET UP In Examples 16.1 and 16.5 we found that for a 1000 Hz sound wave with $p_{\max} = 3.0 \times 10^{-2}$ Pa, $A = 1.2 \times 10^{-8}$ m and $I = 1.1 \times 10^{-6}\ \text{W/m}^2$. Our target variables are $p_{\max}$ and A for a 20 Hz sound wave of the same intensity I. We can find these using Eqs. (16.14) and (16.12), respectively.

EXECUTE We can rearrange Eqs. (16.14) and (16.12) as $p_{\max}{}^2 = 2I\sqrt{\rho B}$ and $\omega^2 A^2 = 2I/\sqrt{\rho B}$, respectively. These tell us that for a given sound intensity I in a given medium (constant ρ and B), the quantities $p_{\max}$ and ωA (or, equivalently, fA) are *constants* that don't depend on frequency. From the first result we immediately have $p_{\max} = 3.0 \times 10^{-2}$ Pa for $f = 20$ Hz, the same as for $f = 1000$ Hz. If we write the second result as $f_{20}A_{20} = f_{1000}A_{1000}$, we have

$$A_{20} = \left(\frac{f_{1000}}{f_{20}}\right)A_{1000}$$

$$= \left(\frac{1000\ \text{Hz}}{20\ \text{Hz}}\right)(1.2 \times 10^{-8}\ \text{m}) = 6.0 \times 10^{-7}\ \text{m} = 0.60\ \mu\text{m}$$

EVALUATE Our result reinforces the idea that pressure amplitude is a more convenient description of a sound wave and its intensity than displacement amplitude.

KEYCONCEPT If two sound waves in a given medium have the same intensity but different frequencies, the wave with the higher frequency has the greater *displacement* amplitude. The two waves have the same *pressure* amplitude, however.

EXAMPLE 16.7 "Play it loud!"

WITH VARIATION PROBLEMS

For an outdoor concert we want the sound intensity to be 1 W/m^2 at a distance of 20 m from the speaker array. If the sound intensity is uniform in all directions, what is the required average acoustic power output of the array?

IDENTIFY, SET UP, and EXECUTE This example uses the definition of sound intensity as power per unit area. The total power is the target variable; the area in question is a hemisphere centered on the speaker array. We assume that the speakers are on the ground and that none of the acoustic power is directed into the ground, so the acoustic power is uniform over a hemisphere 20 m in radius. The surface area of this hemisphere is $\left(\frac{1}{2}\right)(4\pi)(20\ \text{m})^2$, or about 2500 m^2. The required power is the product of this area and the intensity: $(1\ \text{W/m}^2)(2500\ \text{m}^2) = 2500\ \text{W} = 2.5\ \text{kW}$.

EVALUATE The electrical power input to the speaker would need to be considerably greater than 2.5 kW, because speaker efficiency is not very high (typically a few percent for ordinary speakers, and up to 25% for horn-type speakers).

KEYCONCEPT To find the acoustic power output of a source of sound, multiply the area over which the emitted sound wave is distributed by the average intensity of the sound over that area.

The Decibel Scale

Because the ear is sensitive over a broad range of intensities, a *logarithmic* measure of intensity called **sound intensity level** is often used:

Intensity of sound wave

Sound intensity level

$$\beta = (10\ \text{dB}) \log \frac{I}{I_0} \qquad (16.15)$$

Reference intensity = $10^{-12}\ \text{W/m}^2$

Logarithm to base 10

The chosen reference intensity I_0 in Eq. (16.15) is approximately the threshold of human hearing at 1000 Hz. Sound intensity levels are expressed in **decibels,** abbreviated dB. A decibel is $\frac{1}{10}$ of a *bel,* a unit named for Alexander Graham Bell (the inventor of the telephone). The bel is inconveniently large for most purposes, and the decibel is the usual unit of sound intensity level.

If the intensity of a sound wave equals I_0 or $10^{-12}\ \text{W/m}^2$, its sound intensity level is $\beta = 0$ dB. An intensity of 1 W/m^2 corresponds to 120 dB. **Table 16.2** gives the sound intensity levels of some familiar sounds. You can use Eq. (16.15) to check the value of β given for each intensity in the table.

Because the ear is not equally sensitive to all frequencies in the audible range, some sound-level meters weight the various frequencies unequally. One such scheme leads to the so-called dBA scale; this scale deemphasizes the low and very high frequencies, where the ear is less sensitive.

TABLE 16.2 Sound Intensity Levels from Various Sources (Representative Values)

Source or Description of Sound	Sound Intensity Level, β (dB)	Intensity, I (W/m^2)
Military jet aircraft 30 m away	140	10^2
Threshold of pain	120	1
Riveter	95	3.2×10^{-3}
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Ordinary conversation	65	3.2×10^{-6}
Quiet automobile	50	10^{-7}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing at 1000 Hz	0	10^{-12}

EXAMPLE 16.8 Temporary—or permanent—hearing loss

WITH VARIATION PROBLEMS

A 10 min exposure to 120 dB sound will temporarily shift your threshold of hearing at 1000 Hz from 0 dB up to 28 dB. Ten years of exposure to 92 dB sound will cause a *permanent* shift to 28 dB. What sound intensities correspond to 28 dB and 92 dB?

IDENTIFY and SET UP We are given two sound intensity levels β; our target variables are the corresponding intensities. We can solve Eq. (16.15) to find the intensity I that corresponds to each value of β.

EXECUTE We solve Eq. (16.15) for I by dividing both sides by 10 dB and using the relationship $10^{\log x} = x$:

$$I = I_0 10^{\beta/(10\text{ dB})}$$

For $\beta = 28$ dB and $\beta = 92$ dB, the exponents are $\beta/(10\text{ dB}) = 2.8$ and 9.2, respectively, so that

$$I_{28\text{ dB}} = (10^{-12}\text{ W/m}^2)10^{2.8} = 6.3 \times 10^{-10}\text{ W/m}^2$$
$$I_{92\text{ dB}} = (10^{-12}\text{ W/m}^2)10^{9.2} = 1.6 \times 10^{-3}\text{ W/m}^2$$

EVALUATE If your answers are a factor of 10 too large, you may have entered 10×10^{-12} in your calculator instead of 1×10^{-12}. Be careful!

KEYCONCEPT The sound intensity level (in decibels, or dB) is a logarithmic measure of the intensity of a sound wave. Adding 10 dB to the sound intensity level corresponds to multiplying the intensity by a factor of 10.

EXAMPLE 16.9 A bird sings in a meadow

WITH VARIATION PROBLEMS

Consider an idealized bird (treated as a point source) that emits constant sound power, with intensity obeying the inverse-square law (**Fig. 16.11**). If you move twice the distance from the bird, by how many decibels does the sound intensity level drop?

IDENTIFY and SET UP The decibel scale is logarithmic, so the *difference* between two sound intensity levels (the target variable) corresponds to the *ratio* of the corresponding intensities, which is determined by the inverse-square law. We label the two points P_1 and P_2 (Fig. 16.11). We use Eq. (16.15), the definition of sound intensity level, at each point. We use Eq. (15.26), the inverse-square law, to relate the intensities at the two points.

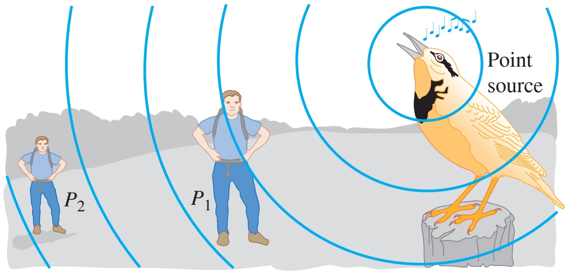
EXECUTE The difference $\beta_2 - \beta_1$ between any two sound intensity levels is related to the corresponding intensities by

$$\beta_2 - \beta_1 = (10\text{ dB})\left(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0}\right)$$
$$= (10\text{ dB})[(\log I_2 - \log I_0) - (\log I_1 - \log I_0)]$$
$$= (10\text{ dB})\log\frac{I_2}{I_1}$$

For this inverse-square-law source, Eq. (15.26) yields $I_2/I_1 = r_1^2/r_2^2 = \frac{1}{4}$, so

$$\beta_2 - \beta_1 = (10\text{ dB})\log\frac{I_2}{I_1} = (10\text{ dB})\log\tfrac{1}{4} = -6.0\text{ dB}$$

Figure **16.11** When you double your distance from a point source of sound, by how much does the sound intensity level decrease?



EVALUATE Our result is negative, which tells us (correctly) that the sound intensity level is less at P_2 than at P_1. The 6 dB difference doesn't depend on the sound intensity level at P_1; *any* doubling of the distance from an inverse-square-law source reduces the sound intensity level by 6 dB.

Note that the perceived *loudness* of a sound is not directly proportional to its intensity. For example, most people interpret an increase of 8 dB to 10 dB in sound intensity level (corresponding to increasing intensity by a factor of 6 to 10) as a doubling of loudness.

KEYCONCEPT The *difference* between the sound intensity levels of two sounds is proportional to the logarithm of the *ratio* of the intensities of those sounds.

TEST YOUR UNDERSTANDING OF SECTION 16.3 You double the intensity of a sound wave in air while leaving the frequency unchanged. (The pressure, density, and temperature of the air remain unchanged as well.) What effect does this have on the displacement amplitude, pressure amplitude, bulk modulus, sound speed, and sound intensity level?

ANSWER

❙ A and p_{max} increase by a factor of $\sqrt{2}$, B and v are unchanged, β increases by 3.0 dB Equations (16.9) and (16.10) show that the bulk modulus B and sound speed v remain the same because the physical properties of the air are unchanged. From Eqs. (16.12) and (16.14), the intensity is proportional to the square of the displacement amplitude or the square of the pressure amplitude. Hence doubling the intensity means that A and p_{max} both increase by a factor of $\sqrt{2}$. Example 16.9 shows that *multiplying* the intensity by a factor of 2 $(I_2/I_1 = 2)$ corresponds to *adding* to the sound intensity level by $(10\text{ dB})\log(I_2/I_1) = (10\text{ dB})\log 2 = 3.0\text{ dB}$.

16.4 STANDING SOUND WAVES AND NORMAL MODES

When longitudinal (sound) waves propagate in a fluid in a pipe, the waves are reflected from the ends in the same way that transverse waves on a string are reflected at its ends. The superposition of the waves traveling in opposite directions again forms a standing wave. Just as for transverse standing waves on a string (see Section 15.7), standing sound waves in a pipe can be used to create sound waves in the surrounding air. This is the principle of the human voice as well as many musical instruments, including woodwinds, brasses, and pipe organs.

Transverse waves on a string, including standing waves, are usually described only in terms of the displacement of the string. But, as we have seen, sound waves in a fluid may be described either in terms of the displacement of the fluid or in terms of the pressure variation in the fluid. To avoid confusion, we'll use the terms **displacement node** and **displacement antinode** to refer to points where particles of the fluid have zero displacement and maximum displacement, respectively.

We can demonstrate standing sound waves in a column of gas using an apparatus called a Kundt's tube (**Fig. 16.12**). A horizontal glass tube a meter or so long is closed at one end and has a flexible diaphragm at the other end that can transmit vibrations. A nearby loudspeaker is driven by an audio oscillator and amplifier; this produces sound waves that force the diaphragm to vibrate sinusoidally with a frequency that we can vary. The sound waves within the tube are reflected at the other, closed end of the tube. We spread a small amount of light powder uniformly along the bottom of the tube. As we vary the frequency of the sound, we pass through frequencies at which the amplitude of the standing waves becomes large enough for the powder to be swept along the tube at those points where the gas is in motion. The powder therefore collects at the displacement nodes (where the gas is not moving). Adjacent nodes are separated by a distance equal to $\lambda/2$.

Figure **16.12** Demonstrating standing sound waves using a Kundt's tube. The blue shading represents the density of the gas at an instant when the gas pressure at the displacement nodes is a maximum or a minimum.

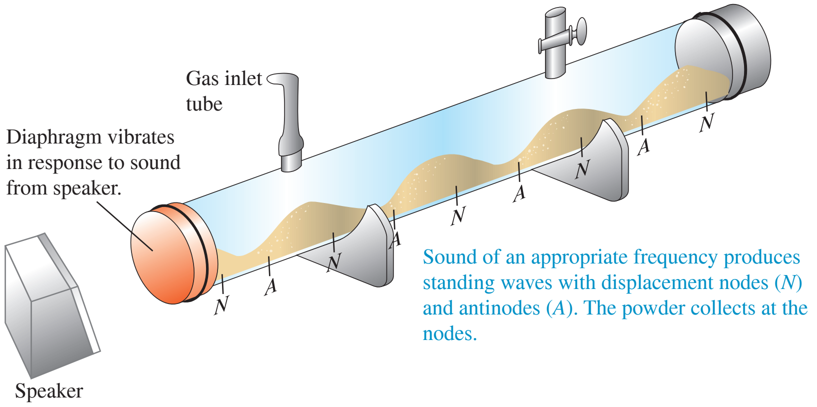


Figure 16.13 shows the motions of nine different particles within a gas-filled tube in which there is a standing sound wave. A particle at a displacement node (N) does not move, while a particle at a displacement antinode (A) oscillates with maximum amplitude. Note that particles on opposite sides of a displacement node vibrate in opposite phase. When these particles approach each other, the gas between them is compressed and the pressure rises; when they recede from each other, there is an expansion and the pressure drops. Hence at a displacement *node* the gas undergoes the maximum amount of compression and expansion, and the variations in pressure and density above and below the average have their maximum value. By contrast, particles on opposite sides of a displacement *antinode* vibrate *in phase;* the distance between the particles is nearly constant, and there is *no* variation in pressure or density at a displacement antinode.

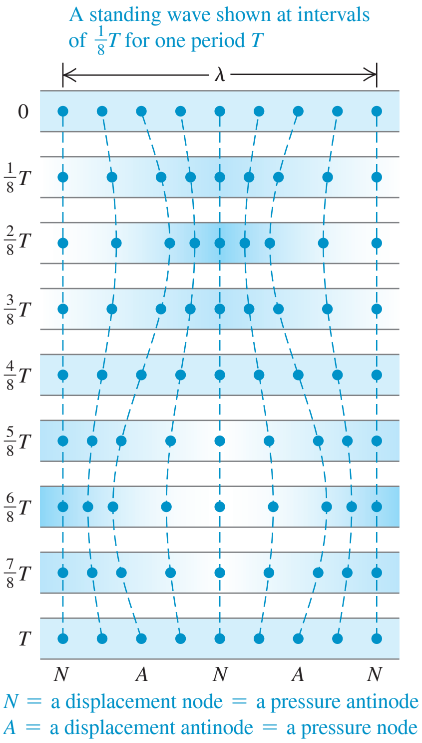
We use the term **pressure node** to describe a point in a standing sound wave at which the pressure and density do not vary and the term **pressure antinode** to describe a point at which the variations in pressure and density are greatest. Using these terms, we can summarize our observations as follows:

A pressure node is always a displacement antinode, and a pressure antinode is always a displacement node.

Figure 16.12 depicts a standing sound wave at an instant at which the pressure variations are greatest; the blue shading shows that the density and pressure of the gas have their maximum and minimum values at the displacement nodes.

When reflection takes place at a *closed* end of a pipe (an end with a rigid barrier or plug), the displacement of the particles at this end must always be zero, analogous to a fixed end of a string. Thus a closed end of a pipe is a displacement node and a pressure antinode; the particles do not move, but the pressure variations are maximum. An *open* end of a pipe is a pressure node because it is open to the atmosphere, where the pressure is constant. Because of this, an open end is always a displacement *antinode,* in analogy to a free end of a string; the particles oscillate with maximum amplitude, but the pressure does not vary. (The pressure node actually occurs somewhat beyond an open end of a pipe. But if the diameter of the pipe is small in comparison to the wavelength, which is true for most musical instruments, this effect can safely be ignored.) Thus longitudinal sound waves are reflected at the closed and open ends of a pipe in the same way that transverse waves in a string are reflected at fixed and free ends, respectively.

Figure **16.13** In a standing sound wave, a displacement node N is a pressure antinode (a point where the pressure fluctuates the most) and a displacement antinode A is a pressure node (a point where the pressure does not fluctuate at all).



CONCEPTUAL EXAMPLE 16.10 The sound of silence

A directional loudspeaker directs a sound wave of wavelength λ at a wall (**Fig. 16.14**). At what distances from the wall could you stand and hear no sound at all?

SOLUTION Your ear detects pressure variations in the air; you'll therefore hear no sound if your ear is at a *pressure node,* which is a displacement antinode. The wall is at a displacement node; the distance from any node to an adjacent antinode is $\lambda/4$, and the distance from one antinode to the next is $\lambda/2$ (Fig. 16.14). Hence the displacement antinodes (pressure nodes), at which no sound will be heard, are at distances $d = \lambda/4$, $d = \lambda/4 + \lambda/2 = 3\lambda/4$, $d = 3\lambda/4 + \lambda/2 = 5\lambda/4, \ldots$ from the wall. If the loudspeaker is not highly directional, this effect is hard to notice because of reflections of sound waves from the floor, ceiling, and other walls.

KEYCONCEPT In a standing sound wave, a pressure node is a displacement antinode, and vice versa. The sound is loudest at a pressure antinode; there is no sound at a pressure node.

Figure **16.14** When a sound wave is directed at a wall, it interferes with the reflected wave to create a standing wave. The N's and A's are *displacement* nodes and antinodes.

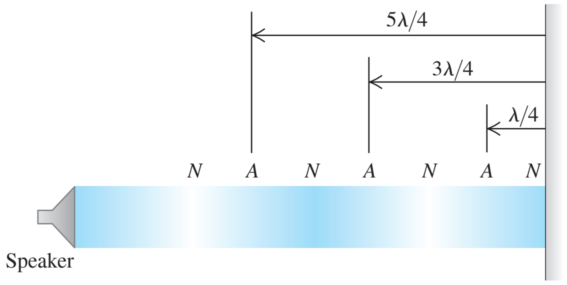


Figure **16.15** Organ pipes of different sizes produce tones with different frequencies.

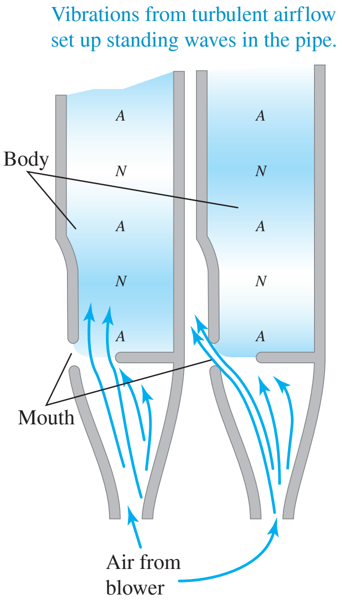
Organ Pipes and Wind Instruments

The most important application of standing sound waves is the production of musical tones. Organ pipes are one of the simplest examples (**Fig. 16.15**). Air is supplied by a blower to the bottom end of the pipe (**Fig. 16.16**). A stream of air emerges from the narrow opening at the edge of the horizontal surface and is directed against the top edge of the opening, which is called the *mouth* of the pipe. The column of air in the pipe is set into vibration, and there is a series of possible normal modes, just as with the stretched string. The mouth acts as an open end; it is a pressure node and a displacement antinode. The other end of the pipe (at the top in Fig. 16.16) may be either open or closed.

In **Fig. 16.17**, both ends of the pipe are open, so both ends are pressure nodes and displacement antinodes. An organ pipe that is open at both ends is called an *open pipe*. The fundamental frequency f_1 corresponds to a standing-wave pattern with a displacement antinode at each end and a displacement node in the middle (Fig. 16.17a). The distance between adjacent antinodes is always equal to one half-wavelength, and in this case that is equal to the length L of the pipe; $\lambda/2 = L$. The corresponding frequency, obtained from the relationship $f = v/\lambda$, is

$$f_1 = \frac{v}{2L} \qquad \text{(open pipe)} \tag{16.16}$$

Figure **16.16** Cross sections of an organ pipe at two instants one half-period apart. The N's and A's are *displacement* nodes and antinodes; as the blue shading shows, these are points of maximum pressure variation and zero pressure variation, respectively.



Figures 16.17b and 16.17c show the second and third harmonics; their vibration patterns have two and three displacement nodes, respectively. For these, a half-wavelength is equal to $L/2$ and $L/3$, respectively, and the frequencies are twice and three times the fundamental, respectively: $f_2 = 2f_1$ and $f_3 = 3f_1$. For *every* normal mode of an open pipe the length L must be an integer number of half-wavelengths, and the possible wavelengths λ_n are given by

$$L = n\frac{\lambda_n}{2} \quad \text{or} \quad \lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots) \quad \text{(open pipe)} \tag{16.17}$$

The corresponding frequencies f_n are given by $f_n = v/\lambda_n$, so all the normal-mode frequencies for a pipe that is open at both ends are given by

Standing waves, open pipe:

Frequency of nth harmonic $(n = 1, 2, 3, \ldots)$

$$f_n = \frac{nv}{2L} \tag{16.18}$$

Speed of sound in pipe

Length of pipe

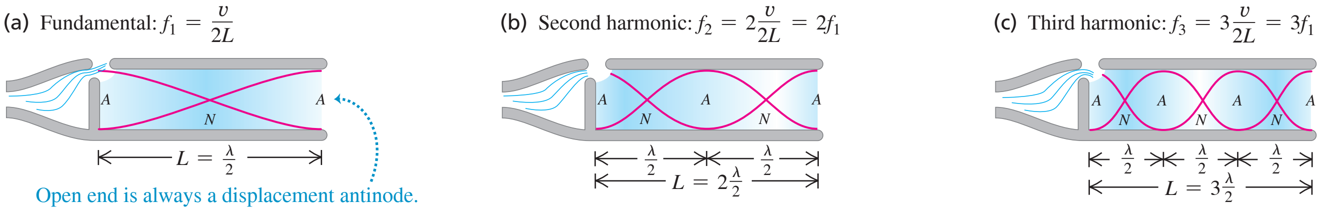
The value $n = 1$ corresponds to the fundamental frequency, $n = 2$ to the second harmonic (or first overtone), and so on. Alternatively, we can say

$$f_n = nf_1 \quad (n = 1, 2, 3, \ldots) \quad \text{(open pipe)} \tag{16.19}$$

with f_1 given by Eq. (16.16).

Figure 16.18 shows a *stopped pipe:* It is open at the left end but closed at the right end. The left (open) end is a displacement antinode (pressure node), but the right (closed) end is a displacement node (pressure antinode). Figure 16.18a shows the lowest-frequency

Figure **16.17** A cross section of an open pipe showing the first three normal modes. The shading indicates the pressure variations. The red curves are graphs of the displacement along the pipe axis at two instants separated in time by one half-period. The N's and A's are the *displacement* nodes and antinodes; interchange these to show the *pressure* nodes and antinodes.

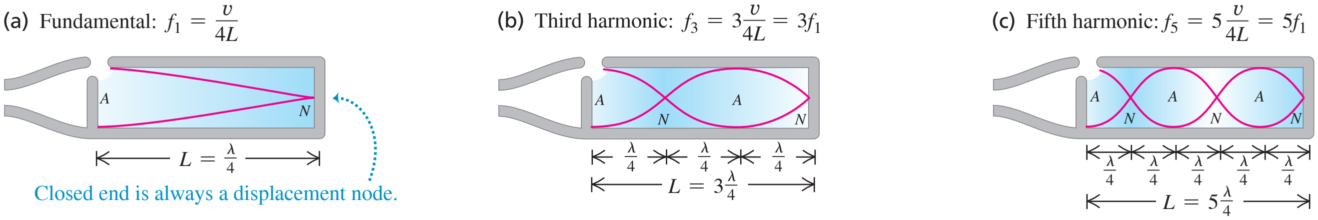


(a) Fundamental: $f_1 = \frac{v}{2L}$

(b) Second harmonic: $f_2 = 2\frac{v}{2L} = 2f_1$

(c) Third harmonic: $f_3 = 3\frac{v}{2L} = 3f_1$

Figure **16.18** A cross section of a stopped pipe showing the first three normal modes as well as the *displacement* nodes and antinodes. Only odd harmonics are possible.



(a) Fundamental: $f_1 = \frac{v}{4L}$

(b) Third harmonic: $f_3 = 3\frac{v}{4L} = 3f_1$

(c) Fifth harmonic: $f_5 = 5\frac{v}{4L} = 5f_1$

mode; the length of the pipe is the distance between a node and the adjacent antinode, or a quarter-wavelength $(L = \lambda_1/4)$. The fundamental frequency is $f_1 = v/\lambda_1$, or

$$f_1 = \frac{v}{4L} \quad \text{(stopped pipe)} \qquad (16.20)$$

This is one-half the fundamental frequency for an *open* pipe of the same length. In musical language, the *pitch* of a closed pipe is one octave lower (a factor of 2 in frequency) than that of an open pipe of the same length. Figure 16.18b shows the next mode, for which the length of the pipe is *three-quarters* of a wavelength, corresponding to a frequency $3f_1$. For Fig. 16.18c, $L = 5\lambda/4$ and the frequency is $5f_1$. The possible wavelengths are given by

$$L = n\frac{\lambda_n}{4} \quad \text{or} \quad \lambda_n = \frac{4L}{n} \quad (n = 1, 3, 5, \ldots) \quad \text{(stopped pipe)} \qquad (16.21)$$

The normal-mode frequencies are given by $f_n = v/\lambda_n$, or

Standing waves, stopped pipe:

$$\overset{\text{Frequency of } n\text{th harmonic } (n = 1, 3, 5, \ldots)}{f_n} = \frac{nv \leftarrow \text{Speed of sound in pipe}}{4L \leftarrow \text{Length of pipe}} \qquad (16.22)$$

or

$$f_n = nf_1 \quad (n = 1, 3, 5, \ldots) \quad \text{(stopped pipe)} \qquad (16.23)$$

with f_1 given by Eq. (16.20). We see that the second, fourth, and all *even* harmonics are missing. In a stopped pipe, the fundamental frequency is $f_1 = v/4L$, and only the odd harmonics in the series $(3f_1, 5f_1, \ldots)$ are possible.

A final possibility is a pipe that is closed at *both* ends, with displacement nodes and pressure antinodes at both ends. This wouldn't be of much use as a musical instrument because the vibrations couldn't get out of the pipe.

EXAMPLE 16.11 A tale of two pipes

WITH ✓ARIATION PROBLEMS

On a day when the speed of sound is 344 m/s, the fundamental frequency of a particular stopped organ pipe is 220 Hz. (a) How long is this pipe? (b) The second *overtone* of this pipe has the same wavelength as the third *harmonic* of an *open* pipe. How long is the open pipe?

IDENTIFY and SET UP This problem uses the relationship between the length and normal-mode frequencies of open pipes (Fig. 16.17) and stopped pipes (Fig. 16.18). In part (a), we determine the length of the stopped pipe from Eq. (16.22). In part (b), we must determine the length of an open pipe, for which Eq. (16.18) gives the frequencies.

EXECUTE (a) For a stopped pipe $f_1 = v/4L$, so

$$L_{\text{stopped}} = \frac{v}{4f_1} = \frac{344 \text{ m/s}}{4(220 \text{ s}^{-1})} = 0.391 \text{ m}$$

(b) The frequency of the second overtone of a stopped pipe (the *third* possible frequency) is $f_5 = 5f_1 = 5(220 \text{ Hz}) = 1100$ Hz. If the wavelengths for the two pipes are the same, the frequencies are also the same. Hence the frequency of the third harmonic of the open pipe, which is at $3f_1 = 3(v/2L)$, equals 1100 Hz. Then

$$1100 \text{ Hz} = 3\left(\frac{344 \text{ m/s}}{2L_{\text{open}}}\right) \quad \text{and} \quad L_{\text{open}} = 0.469 \text{ m}$$

EVALUATE The 0.391 m stopped pipe has a fundamental frequency of 220 Hz; the *longer* (0.469 m) open pipe has a *higher* fundamental frequency, $(1100 \text{ Hz})/3 = 367$ Hz. This is not a contradiction, as you can see if you compare Figs. 16.17a and 16.18a.

KEYCONCEPT For a pipe open at both ends (an "open pipe"), the normal-mode frequencies of a standing sound wave include both even and odd multiples of the pipe's fundamental frequency. For a pipe open at one end and closed at the other (a "stopped pipe"), the only normal-mode frequencies are the odd multiples of the pipe's fundamental frequency. The fundamental frequency of a stopped pipe is half that of an open pipe of the same length.

In an organ pipe in actual use, several modes are always present at once; the motion of the air is a superposition of these modes. This situation is analogous to a string that is struck or plucked, as in Fig. 15.28. Just as for a vibrating string, a complex standing wave in the pipe produces a traveling sound wave in the surrounding air with a harmonic content similar to that of the standing wave. A very narrow pipe produces a sound wave rich in higher harmonics; a fatter pipe produces mostly the fundamental mode, heard as a softer, more flutelike tone. The harmonic content also depends on the shape of the pipe's mouth.

We have talked about organ pipes, but this discussion is also applicable to other wind instruments. The flute and the recorder are directly analogous. The most significant difference is that those instruments have holes along the pipe. Opening and closing the holes with the fingers changes the effective length L of the air column and thus changes the pitch. Any individual organ pipe, by comparison, can play only a single note. The flute and recorder behave as *open* pipes, while the clarinet acts as a *stopped* pipe (closed at the reed end, open at the bell).

Equations (16.18) and (16.22) show that the frequencies of any wind instrument are proportional to the speed of sound v in the air column inside the instrument. As Eq. (16.10) shows, v depends on temperature; it increases when temperature increases. Thus the pitch of all wind instruments rises with increasing temperature. An organ that has some of its pipes at one temperature and others at a different temperature is bound to sound out of tune.

TEST YOUR UNDERSTANDING OF SECTION 16.4 If you connect a hose to one end of a metal pipe and blow compressed air into it, the pipe produces a musical tone. If instead you blow compressed helium into the pipe at the same pressure and temperature, will the pipe produce (i) the same tone, (ii) a higher-pitch tone, or (iii) a lower-pitch tone?

ANSWER **(ii)** Helium is less dense and has a lower molar mass than air, so sound travels faster in helium than in air. The normal-mode frequencies for a pipe are proportional to the sound speed v, so the frequency and hence the pitch increase when the air in the pipe is replaced with helium. ❙

16.5 RESONANCE AND SOUND

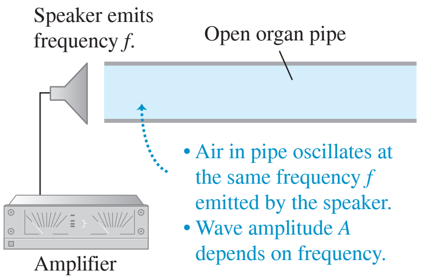
Many mechanical systems have normal modes of oscillation. As we have seen, these include columns of air (as in an organ pipe) and stretched strings (as in a guitar; see Section 15.8). In each mode, every particle of the system oscillates with simple harmonic motion at the same frequency as the mode. Air columns and stretched strings have an infinite series of normal modes, but the basic concept is closely related to the simple harmonic oscillator, discussed in Chapter 14, which has only a single normal mode (that is, only one frequency at which it oscillates after being disturbed).

Suppose we apply a periodically varying force to a system that can oscillate. The system is then forced to oscillate with a frequency equal to the frequency of the applied force (called the *driving frequency*). This motion is called a *forced oscillation*. We talked about forced oscillations of the harmonic oscillator in Section 14.8, including the phenomenon of mechanical **resonance.** A simple example of resonance is pushing Cousin Throckmorton on a swing. The swing is a pendulum; it has only a single normal mode, with a frequency determined by its length. If we push the swing periodically with this frequency, we can build up the amplitude of the motion. But if we push with a very different frequency, the swing hardly moves at all.

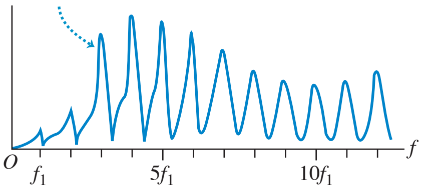
Resonance also occurs when a periodically varying force is applied to a system with many normal modes. In **Fig. 16.19a** an open organ pipe is placed next to a loudspeaker that emits pure sinusoidal sound waves of frequency f, which can be varied by adjusting the amplifier. The air in the pipe is forced to vibrate with the same frequency f as the *driving force* provided by the loudspeaker. In general the amplitude of this motion is relatively small, and the air inside the pipe will not move in any of the normal-mode patterns shown in Fig. 16.17. But if the frequency f of the force is close to one of the normal-mode frequencies, the air in the pipe moves in the normal-mode pattern for that frequency, and the amplitude can become quite large. Figure 16.19b shows the amplitude of oscillation of the air in the pipe as a function of the driving frequency f. This **resonance curve** of the pipe has peaks where f equals the

Figure 16.19 (a) The air in an open pipe is forced to oscillate at the same frequency as the sinusoidal sound waves coming from the loudspeaker. (b) The resonance curve of the open pipe graphs the amplitude of the standing sound wave in the pipe as a function of the driving frequency.

(a)



(b) Resonance curve: graph of amplitude A versus driving frequency f. Peaks occur at normal-mode frequencies of the pipe: f_1, $f_2 = 2f_1$, $f_3 = 3f_1$, … .



normal-mode frequencies of the pipe. The detailed shape of the resonance curve depends on the geometry of the pipe.

If the frequency of the force is precisely *equal* to a normal-mode frequency, the system is in resonance, and the amplitude of the forced oscillation is maximum. If there were no friction or other energy-dissipating mechanism, a driving force at a normal-mode frequency would continue to add energy to the system, the amplitude would increase indefinitely, and the peaks in the resonance curve of Fig. 16.19b would be infinitely high. But in any real system there is always some dissipation of energy, or damping, as we discussed in Section 14.8; the amplitude of oscillation in resonance may be large, but it cannot be infinite.

The "sound of the ocean" you hear when you put your ear next to a large seashell is due to resonance. The noise of the outside air moving past the seashell is a mixture of sound waves of almost all audible frequencies, which forces the air inside the seashell to oscillate. The seashell behaves like an organ pipe, with a set of normal-mode frequencies; hence the inside air oscillates most strongly at those frequencies, producing the seashell's characteristic sound. To hear a similar phenomenon, uncap a full bottle of your favorite beverage and blow across the open top. The noise is provided by your breath blowing across the top, and the "organ pipe" is the column of air inside the bottle above the surface of the liquid. If you take a drink and repeat the experiment, you'll hear a lower tone because the "pipe" is longer and the normal-mode frequencies are lower.

Resonance also occurs when a stretched string is forced to oscillate (see Section 15.8). Suppose that one end of a stretched string is held fixed while the other is given a transverse sinusoidal motion with small amplitude, setting up standing waves. If the frequency of the driving mechanism is *not* equal to one of the normal-mode frequencies of the string, the amplitude at the antinodes is fairly small. However, if the frequency is equal to any one of the normal-mode frequencies, the string is in resonance, and the amplitude at the antinodes is very much larger than that at the driven end. The driven end is not precisely a node, but it lies much closer to a node than to an antinode when the string is in resonance. The photographs of standing waves in Fig. 15.23 were made this way, with the left end of the string fixed and the right end oscillating vertically with small amplitude.

It is easy to demonstrate resonance with a piano. Push down the damper pedal (the right-hand pedal) so that the dampers are lifted and the strings are free to vibrate, and then sing a steady tone into the piano. When you stop singing, the piano seems to continue to sing the same note. The sound waves from your voice excite vibrations in the strings that have natural frequencies close to the frequencies (fundamental and harmonics) present in the note you sang.

A more spectacular example is a singer breaking a wine glass with her amplified voice. A good-quality wine glass has normal-mode frequencies that you can hear by tapping it. If the singer emits a loud note with a frequency corresponding exactly to one of these normal-mode frequencies, large-amplitude oscillations can build up and break the glass (**Fig. 16.20**).

BIO APPLICATION Resonance and the Sensitivity of the Ear The auditory canal of the human ear (see Fig. 16.4) is an air-filled pipe open at one end and closed at the other (eardrum) end. The canal is about 2.5 cm = 0.025 m long, so it has a resonance at its fundamental frequency $f_1 = v/4L = (344\text{ m/s})/[4(0.025\text{ m})] = 3440\text{ Hz}$. The resonance means that a sound at this frequency produces a strong oscillation of the eardrum. That's why your ear is most sensitive to sounds near 3440 Hz.

Figure **16.20** The frequency of the sound from this trumpet exactly matches one of the normal-mode frequencies of the goblet. The resonant vibrations of the goblet have such large amplitude that the goblet tears itself apart.

EXAMPLE 16.12 An organ–guitar duet

WITH VARIATION PROBLEMS

A stopped organ pipe is sounded near a guitar, causing one of the strings to vibrate with large amplitude. We vary the string tension until we find the maximum amplitude. The string is 80% as long as the pipe. If both pipe and string vibrate at their fundamental frequency, calculate the ratio of the wave speed on the string to the speed of sound in air.

IDENTIFY and SET UP The large response of the string is an example of resonance. It occurs because the organ pipe and the guitar string have the same fundamental frequency. If we let the subscripts a and s stand for the air in the pipe and the string, respectively, the condition for resonance is $f_{1\mathrm{a}} = f_{1\mathrm{s}}$. Equation (16.20) gives the fundamental frequency for a stopped pipe, and Eq. (15.32) gives the fundamental frequency for a guitar string held at both ends. These expressions involve the wave speed in air (v_a) and on the string (v_s) and the lengths of the pipe and string. We are given that $L_\mathrm{s} = 0.80L_\mathrm{a}$; our target variable is the ratio $v_\mathrm{s}/v_\mathrm{a}$.

Continued

EXECUTE From Eqs. (16.20) and (15.32), $f_{1a} = v_a/4L_a$ and $f_{1s} = v_s/2L_s$. These frequencies are equal, so

$$\frac{v_a}{4L_a} = \frac{v_s}{2L_s}$$

Substituting $L_s = 0.80L_a$ and rearranging, we get $v_s/v_a = 0.40$.

EVALUATE As an example, if the speed of sound in air is 344 m/s, the wave speed on the string is $(0.40)(344\text{ m/s}) = 138\text{ m/s}$. Note that while the standing waves in the pipe and on the string have the same frequency, they have different *wavelengths* $\lambda = v/f$ because the two media have different wave speeds v. Which standing wave has the greater wavelength?

KEYCONCEPT If you force or drive a mechanical system (such as a guitar string or the air in a pipe) to vibrate at a frequency f, the system will oscillate with maximum amplitude (or *resonate*) if f equals one of the normal-mode frequencies of the system.

TEST YOUR UNDERSTANDING OF SECTION 16.5 A stopped organ pipe of length L has a fundamental frequency of 220 Hz. For which of the following organ pipes will there be a resonance if a tuning fork of frequency 660 Hz is sounded next to the pipe? (There may be more than one correct answer.) (i) A stopped organ pipe of length L; (ii) a stopped organ pipe of length $2L$; (iii) an open organ pipe of length L; (iii) an open organ pipe of length $2L$.

ANSWER

(i) and (iv) There will be a resonance if 660 Hz is one of the pipe's normal-mode frequencies. A stopped organ pipe has normal-mode frequencies that are odd multiples of its fundamental frequency [see Eq. (16.22) and Fig. 16.18]. Hence pipe (i),which has fundamental frequency 220 Hz, also has a normal-mode frequency of $3(220\text{ Hz}) = 660\text{ Hz}$. Pipe (ii) has twice the length of pipe (i); from Eq. (16.20), the fundamental frequency of a stopped pipe is inversely proportional to the length, so pipe (ii) has a fundamental frequency of $\left(\frac{1}{2}\right)(220\text{ Hz}) = 110\text{ Hz}$. Its other normal-mode frequencies are 330 Hz, 550 Hz, 770 Hz, . . . , so a 660 Hz tuning fork will not cause resonance. Pipe (iii) is an open pipe of the same length as pipe (i), so its fundamental frequency is twice as great as for pipe (i) [compare Eqs. (16.16) and (16.20)], or $2(220\text{ Hz}) = 440\text{ Hz}$. Its other normal-mode frequencies are integer multiples of the fundamental frequency [see Eq. (16.19)], or 880 Hz, 1320 Hz, . . . , none of which match the 660 Hz frequency of the tuning fork. Pipe (iv) is also an open pipe but with twice the length of pipe (iii) [see Eq. (16.18)], so its normal-mode frequencies are one-half those of pipe (iii): 220 Hz, 440 Hz, 660 Hz, . . . , so the third harmonic will resonate with the tuning fork.

16.6 INTERFERENCE OF WAVES

Wave phenomena that occur when two or more waves overlap in the same region of space are grouped under the heading *interference*. As we have seen, standing waves are a simple example of an interference effect: Two waves traveling in opposite directions in a medium can combine to produce a standing-wave pattern with nodes and antinodes that do not move.

Figure 16.21 Two speakers driven by the same amplifier. Constructive interference occurs at point P, and destructive interference occurs at point Q.

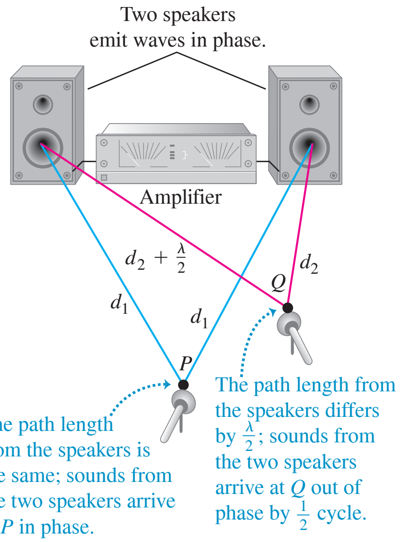


Figure 16.21 shows an example of another type of interference that involves waves that spread out in space. Two speakers, driven in phase by the same amplifier, emit identical sinusoidal sound waves with the same constant frequency. We place a microphone at point P in the figure, equidistant from the speakers. Wave crests emitted from the two speakers at the same time travel equal distances and arrive at point P at the same time; hence the waves arrive in phase, and there is constructive interference. The total wave amplitude that we measure at P is twice the amplitude from each individual wave.

Now let's move the microphone to point Q, where the distances from the two speakers to the microphone differ by a half-wavelength. Then the two waves arrive a half-cycle out of step, or *out of phase;* a positive crest from one speaker arrives at the same time as a negative crest from the other. Destructive interference takes place, and the amplitude measured by the microphone is much *smaller* than when only one speaker is present. If the amplitudes from the two speakers are equal, the two waves cancel each other out completely at point Q, and the total amplitude there is zero.

CAUTION **Interference and traveling waves** The total wave in Fig. 16.21 is a *traveling* wave, not a standing wave. In a standing wave there is no net flow of energy in any direction; by contrast, in Fig. 16.21 there *is* an overall flow of energy from the speakers into the surrounding air, characteristic of a traveling wave. The interference between the waves from the two speakers simply causes the energy flow to be *channeled* into certain directions (for example, toward P) and away from other directions (for example, away from Q). You can see another difference between Fig. 16.21 and a standing wave by considering a point, such as Q, where destructive interference occurs. Such a point is *both* a displacement node *and* a pressure node because there is no wave at all at this point. In a standing wave, a pressure node is a displacement antinode, and vice versa.

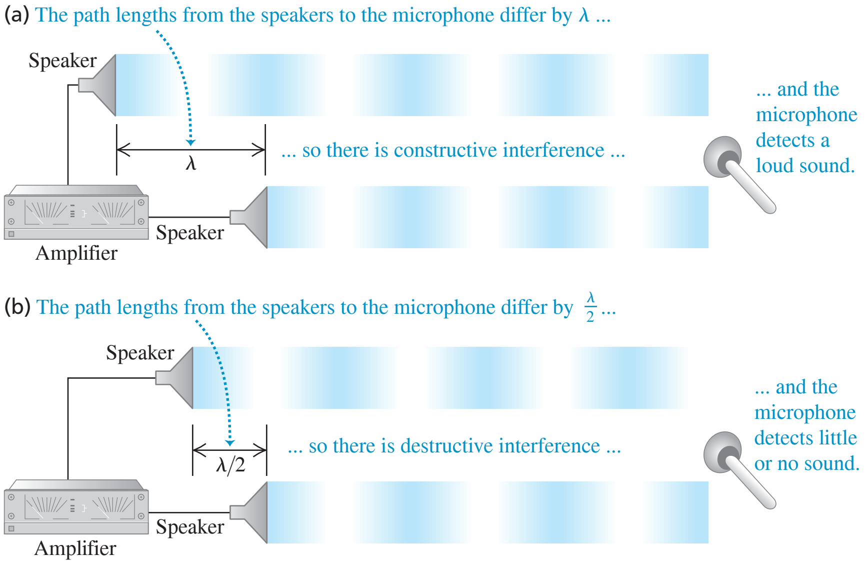


Figure **16.22** Two speakers driven by the same amplifier, emitting waves in phase. Only the waves directed toward the microphone are shown, and they are separated for clarity. (a) Constructive interference occurs when the path difference is $0, \lambda, 2\lambda, 3\lambda, \ldots$. (b) Destructive interference occurs when the path difference is $\lambda/2, 3\lambda/2, 5\lambda/2, \ldots$.

Constructive interference occurs wherever the distances traveled by the two waves differ by a whole number of wavelengths, $0, \lambda, 2\lambda, 3\lambda, \ldots$; then the waves arrive at the microphone in phase (**Fig. 16.22a**). If the distances from the two speakers to the microphone differ by any half-integer number of wavelengths, $\lambda/2, 3\lambda/2, 5\lambda/2, \ldots$, the waves arrive at the microphone out of phase and there will be destructive interference (Fig. 16.22b). In this case, little or no sound energy flows toward the microphone. The energy instead flows in other directions, to where constructive interference occurs.

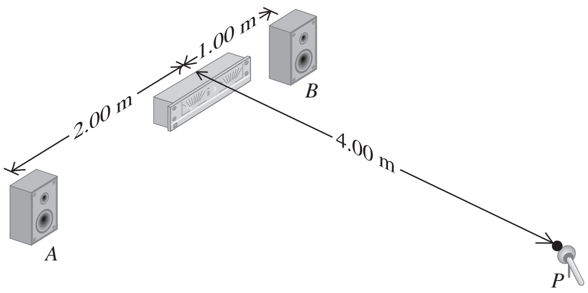
EXAMPLE 16.13 Loudspeaker interference

Two small loudspeakers, A and B (**Fig. 16.23**), are driven by the same amplifier and emit pure sinusoidal waves in phase. (a) For what frequencies does constructive interference occur at point P? (b) For what frequencies does destructive interference occur? The speed of sound is 350 m/s.

IDENTIFY and SET UP The nature of the interference at P depends on the difference d in path lengths from point A to P and from point B to P. We calculate the path lengths using the Pythagorean theorem. Constructive interference occurs when d equals a whole number of wavelengths, while destructive interference occurs when d is a half-integer number of wavelengths. To find the corresponding frequencies, we use $v = f\lambda$.

EXECUTE The A-to-P distance is $[(2.00\text{ m})^2 + (4.00\text{ m})^2]^{1/2} = 4.47\text{ m}$, and the B-to-P distance is $[(1.00\text{ m})^2 + (4.00\text{ m})^2]^{1/2} = 4.12\text{ m}$. The path difference is $d = 4.47\text{ m} - 4.12\text{ m} = 0.35\text{ m}$.

Figure **16.23** What sort of interference occurs at P?



(a) Constructive interference occurs when $d = 0, \lambda, 2\lambda, \ldots$ or $d = 0, v/f, 2v/f, \ldots = nv/f$. So the possible frequencies are

$$f_n = \frac{nv}{d} = n\frac{350\text{ m/s}}{0.35\text{ m}} \qquad (n = 1, 2, 3, \ldots)$$

$$= 1000\text{ Hz}, 2000\text{ Hz}, 3000\text{ Hz}, \ldots$$

(b) Destructive interference occurs when $d = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots$ or $d = v/2f, 3v/2f, 5v/2f, \ldots$. The possible frequencies are

$$f_n = \frac{nv}{2d} = n\frac{350\text{ m/s}}{2(0.35\text{ m})} \qquad (n = 1, 3, 5, \ldots)$$

$$= 500\text{ Hz}, 1500\text{ Hz}, 2500\text{ Hz}, \ldots$$

EVALUATE As we increase the frequency, the sound at point P alternates between large and small (near zero) amplitudes, with maxima and minima at the frequencies given above. This effect may not be strong in an ordinary room because of reflections from the walls, floor, and ceiling.

KEYCONCEPT Two sound waves of the same frequency interfere *constructively* at a certain point if the waves arrive there in phase. If the waves arrive at that point out of phase, they interfere *destructively*.

Figure **16.24** This aviation headset uses destructive interference to minimize the amount of noise from wind and propellers that reaches the wearer's ears.

Interference is the principle behind active noise-reduction headsets, which are used in loud environments such as airplane cockpits (**Fig. 16.24**). A microphone on the headset detects outside noise, and the headset circuitry replays the noise inside the headset shifted in phase by one half-cycle. This phase-shifted sound interferes destructively with the sounds that enter the headset from outside, so the headset wearer experiences very little unwelcome noise.

TEST YOUR UNDERSTANDING OF SECTION 16.6 Suppose that speaker A in Fig. 16.23 emits a sinusoidal sound wave of frequency 500 Hz and speaker B emits a sinusoidal sound wave of frequency 1000 Hz. What sort of interference will there be between these two waves? (i) Constructive interference at various points, including point P, and destructive interference at various other points; (ii) destructive interference at various points, including point P, and constructive interference at various points; (iii) neither (i) nor (ii).

ANSWER

(iii) Constructive and destructive interference between two waves can occur only if the two waves have the same frequency. In this case the frequencies are different, so there are no points where the two waves always reinforce each other (constructive interference) or always cancel each other (destructive interference).

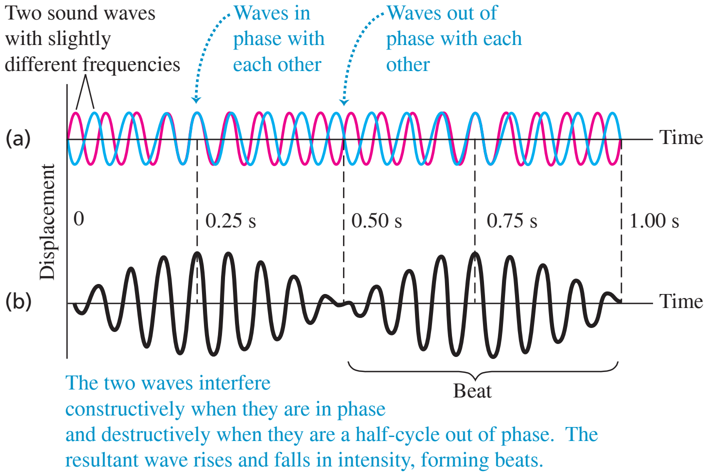
16.7 BEATS

In Section 16.6 we talked about *interference* effects that occur when two different waves with the same frequency overlap in the same region of space. Now let's look at what happens when we have two waves with equal amplitude but slightly different frequencies. This occurs, for example, when two tuning forks with slightly different frequencies are sounded together, or when two organ pipes that are supposed to have exactly the same frequency are slightly "out of tune."

Consider a particular point in space where the two waves overlap. In **Fig. 16.25a** we plot the displacements of the individual waves at this point as functions of time. The total length of the time axis represents 1 second, and the frequencies are 16 Hz (blue graph) and 18 Hz (red graph). Applying the principle of superposition, we add the two displacement functions to find the total displacement function. The result is the graph of Fig. 16.25b. At certain times the two waves are in phase; their maxima coincide and their amplitudes add. But at certain times (like $t = 0.50$ s in Fig. 16.25) the two waves are exactly *out* of phase. The two waves then cancel each other, and the total amplitude is zero.

The resultant wave in Fig. 16.25b looks like a single sinusoidal wave with an amplitude that varies from a maximum to zero and back. In this example the amplitude goes through two maxima and two minima in 1 second, so the frequency of this amplitude variation is 2 Hz. The amplitude variation causes variations of loudness called **beats,** and the frequency with which the loudness varies is called the **beat frequency.** In this example the beat frequency is the *difference* of the two frequencies. If the beat frequency is a few hertz, we hear it as a waver or pulsation in the tone.

Figure **16.25** Beats are fluctuations in amplitude produced by two sound waves of slightly different frequency, here 16 Hz and 18 Hz. (a) Individual waves. (b) Resultant wave formed by superposition of the two waves. The beat frequency is 18 Hz − 16 Hz = 2 Hz.



The two waves interfere constructively when they are in phase and destructively when they are a half-cycle out of phase. The resultant wave rises and falls in intensity, forming beats.

We can prove that the beat frequency is *always* the difference of the two frequencies f_a and f_b. Suppose f_a is larger than f_b; the corresponding periods are T_a and T_b, with $T_a < T_b$. If the two waves start out in phase at time $t = 0$, they are again in phase when the first wave has gone through exactly one more cycle than the second. This happens at a value of t equal to T_{beat}, the *period* of the beat. Let n be the number of cycles of the first wave in time T_{beat}; then the number of cycles of the second wave in the same time is $(n - 1)$, and we have the relationships

$$T_{\text{beat}} = nT_a \qquad \text{and} \qquad T_{\text{beat}} = (n - 1)T_b$$

Eliminating n between these two equations, we find

$$T_{\text{beat}} = \frac{T_a T_b}{T_b - T_a}$$

The reciprocal of the beat period is the beat *frequency*, $f_{\text{beat}} = 1/T_{\text{beat}}$, so

$$f_{\text{beat}} = \frac{T_b - T_a}{T_a T_b} = \frac{1}{T_a} - \frac{1}{T_b}$$

and finally

$$\underset{\textbf{Beat frequency}\text{ for waves } a \text{ and } b}{f_{\text{beat}}} = \overset{\text{Frequency of wave } a}{f_a} - \underset{\text{Frequency of wave } b \text{ (lower than } f_a)}{f_b} \qquad (16.24)$$

As claimed, the beat frequency is the difference of the two frequencies.

CAUTION **Beat frequency tells you only the difference in frequency between two sound waves** The beat frequency for two sound waves always equals the higher frequency minus the lower frequency, so its value alone doesn't tell you which wave frequency is higher. For example, if you hear a beat frequency of 1 Hz and you know one of the sounds has frequency 256 Hz, the frequency of the other sound could be either 257 Hz or 255 Hz.

An alternative way to derive Eq. (16.24) is to write functions to describe the curves in Fig. 16.25a and then add them. Suppose that at a certain position the two waves are given by $y_a(t) = A\sin 2\pi f_a t$ and $y_b(t) = -A\sin 2\pi f_b t$. We use the trigonometric identity

$$\sin a - \sin b = 2\sin\tfrac{1}{2}(a - b)\cos\tfrac{1}{2}(a + b)$$

We can then express the total wave $y(t) = y_a(t) + y_b(t)$ as

$$y_a(t) + y_b(t) = \left[2A\sin\tfrac{1}{2}(2\pi)(f_a - f_b)t\right]\cos\tfrac{1}{2}(2\pi)(f_a + f_b)t$$

The amplitude factor (the quantity in brackets) varies slowly with frequency $\frac{1}{2}(f_a - f_b)$. The cosine factor varies with a frequency equal to the *average* frequency $\frac{1}{2}(f_a + f_b)$. The *square* of the amplitude factor, which is proportional to the intensity that the ear hears, goes through two maxima and two minima per cycle. So the beat frequency f_{beat} that is heard is twice the quantity $\frac{1}{2}(f_a - f_b)$, or just $f_a - f_b$, in agreement with Eq. (16.24).

Beats between two tones can be heard up to a beat frequency of about 6 or 7 Hz. Two piano strings or two organ pipes differing in frequency by 2 or 3 Hz sound wavery and "out of tune," although some organ stops contain two sets of pipes deliberately tuned to beat frequencies of about 1 to 2 Hz for a gently undulating effect. Listening for beats is an important technique in tuning all musical instruments. *Avoiding* beats is part of the task of flying a multiengine propeller airplane (**Fig. 16.26**).

Figure **16.26** If the two propellers on this airplane are not precisely synchronized, the pilots, passengers, and listeners on the ground will hear beats as loud, annoying, throbbing sounds. On some airplanes the propellers are synched electronically; on others the pilot does it by ear, like tuning a piano.

At frequency differences greater than about 6 or 7 Hz, we no longer hear individual beats, and the sensation merges into one of *consonance* or *dissonance,* depending on the frequency ratio of the two tones. In some cases the ear perceives a tone called a *difference tone,* with a pitch equal to the beat frequency of the two tones. For example, if you listen to a whistle that produces sounds at 1800 Hz and 1900 Hz when blown, you'll hear not only these tones but also a much lower 100 Hz tone.

TEST YOUR UNDERSTANDING OF SECTION 16.7 One tuning fork vibrates at 440 Hz, while a second tuning fork vibrates at an unknown frequency. When both tuning forks are sounded simultaneously, you hear a tone that rises and falls in intensity three times per second. What is the frequency of the second tuning fork? (i) 434 Hz; (ii) 437 Hz; (iii) 443 Hz; (iv) 446 Hz; (v) either 434 Hz or 446 Hz; (vi) either 437 Hz or 443 Hz.

ANSWER

(vi) The beat frequency is 3 Hz, so the difference between the two tuning fork frequencies is also 3 Hz. Hence the second tuning fork vibrates at a frequency of either 443 Hz or 437 Hz. You can distinguish between the two possibilities by comparing the pitches of the two tuning forks sounded one at a time: The frequency is 437 Hz if the second tuning fork has a lower pitch and 443 Hz if it has a higher pitch.

16.8 THE DOPPLER EFFECT

When a car approaches you with its horn sounding, the pitch seems to drop as the car passes. This phenomenon, first described by the 19th-century Austrian scientist Christian Doppler, is called the **Doppler effect.** When a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency. A similar effect occurs for light and radio waves; we'll return to this later in this section.

To analyze the Doppler effect for sound, we'll work out a relationship between the frequency shift and the velocities of source and listener relative to the medium (usually air) through which the sound waves propagate. To keep things simple, we consider only the special case in which the velocities of both source and listener lie along the line joining them. Let v_S and v_L be the velocity components along this line for the source and the listener, respectively, relative to the medium. We choose the positive direction for both v_S and v_L to be the direction from the listener L to the source S. The speed of sound relative to the medium, v, is always considered positive.

Moving Listener and Stationary Source

Let's think first about a listener L moving with velocity v_L toward a stationary source S (**Fig. 16.27**). The source emits a sound wave with frequency f_S and wavelength $\lambda = v/f_S$. The figure shows four wave crests, separated by equal distances λ. The wave crests approaching the moving listener have a speed of propagation *relative to the listener* of $(v + v_L)$. So the frequency f_L with which the crests arrive at the listener's position (that is, the frequency the listener hears) is

$$f_L = \frac{v + v_L}{\lambda} = \frac{v + v_L}{v/f_S} \tag{16.25}$$

or

$$f_L = \left(\frac{v + v_L}{v}\right) f_S = \left(1 + \frac{v_L}{v}\right) f_S \quad \text{(moving listener, stationary source)} \tag{16.26}$$

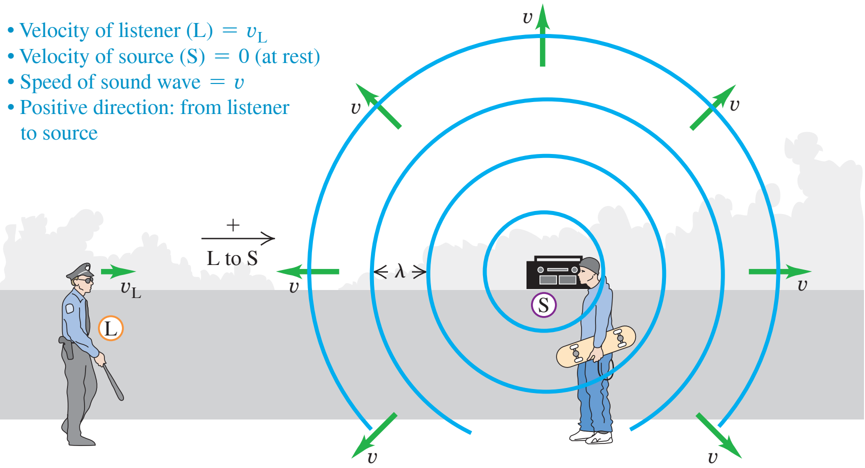


Figure **16.27** A listener moving toward a stationary source hears a frequency that is higher than the source frequency. This is because the relative speed of listener and wave is greater than the wave speed v.

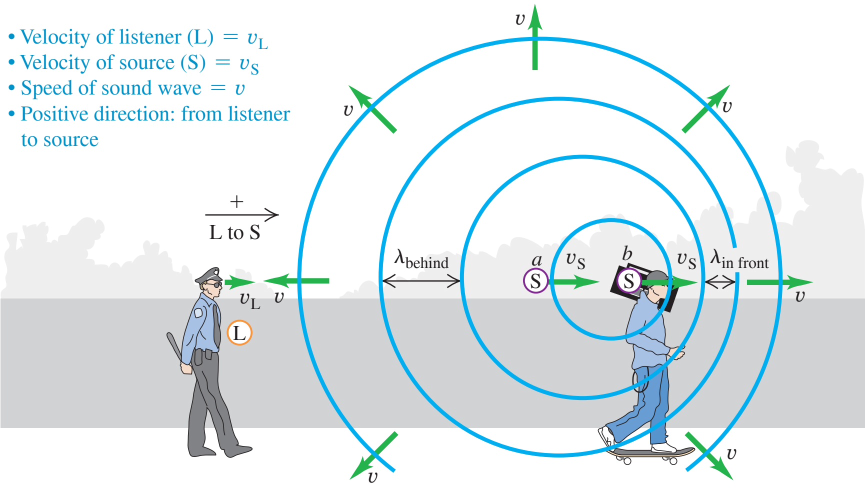


Figure **16.28** Wave crests emitted by a source moving from *a* to *b* are crowded together in front of the source (to the right of this source) and stretched out behind it (to the left of this source).

So a listener moving toward a source ($v_L > 0$), as in Fig. 16.27, hears a higher frequency (higher pitch) than does a stationary listener. A listener moving away from the source ($v_L < 0$) hears a lower frequency (lower pitch).

Moving Source and Moving Listener

Now suppose the source is also moving, with velocity v_S (**Fig. 16.28**). The wave speed relative to the wave medium (air) is still v; it is determined by the properties of the medium and is not changed by the motion of the source. But the wavelength is no longer equal to v/f_S. Here's why. The time for emission of one cycle of the wave is the period $T = 1/f_S$. During this time, the wave travels a distance $vT = v/f_S$ and the source moves a distance $v_S T = v_S/f_S$. The wavelength is the distance between successive wave crests, and this is determined by the *relative* displacement of source and wave. As Fig. 16.28 shows, this is different in front of and behind the source. In the region to the right of the source in Fig. 16.28 (that is, in front of the source), the wavelength is

$$\lambda_{\text{in front}} = \frac{v}{f_S} - \frac{v_S}{f_S} = \frac{v - v_S}{f_S} \quad \text{(wavelength in front of a moving source)} \tag{16.27}$$

In the region to the left of the source (that is, behind the source), it is

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S} \quad \text{(wavelength behind a moving source)} \tag{16.28}$$

The waves in front of and behind the source are compressed and stretched out, respectively, by the motion of the source.

To find the frequency heard by the listener behind the source, we substitute Eq. (16.28) into the first form of Eq. (16.25):

$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} = \frac{v + v_L}{(v + v_S)/f_S}$$

Doppler effect for moving listener L and moving source S:

$$f_L = \frac{v + v_L}{v + v_S} f_S \tag{16.29}$$

Frequency heard by listener

Velocity of listener (+ if from L toward S, − if opposite)

Frequency emitted by source

Speed of sound

Velocity of source (+ if from L toward S, − if opposite)

Figure **16.29** The Doppler effect explains why the siren on a fire engine or ambulance has a high pitch ($f_L > f_S$) when it is approaching you ($v_S < 0$) and a low pitch ($f_L < f_S$) when it is moving away ($v_S > 0$).

Although we derived it for the particular situation shown in Fig. 16.28, Eq. (16.29) includes *all* possibilities for motion of source and listener (relative to the medium) along the line joining them. If the listener happens to be at rest in the medium, v_L is zero. When both source and listener are at rest or have the same velocity relative to the medium, $v_L = v_S$ and $f_L = f_S$. Whenever the direction of the source or listener velocity is opposite to the direction from the listener toward the source (which we have defined as positive), the corresponding velocity to be used in Eq. (16.29) is negative.

As an example, the frequency heard by a listener at rest ($v_L = 0$) is $f_L = [v/(v + v_S)]f_S$. If the source is moving toward the listener (in the negative direction), then $v_S < 0$, $f_L > f_S$, and the listener hears a higher frequency than that emitted by the source. If instead the source is moving away from the listener (in the positive direction), then $v_S > 0$, $f_L < f_S$, and the listener hears a lower frequency. This explains the change in pitch that you hear from the siren of an ambulance as it passes you (**Fig. 16.29**).

PROBLEM-SOLVING STRATEGY 16.2 Doppler Effect

IDENTIFY *the relevant concepts:* The Doppler effect occurs whenever the source of waves, the wave detector (listener), or both are in motion.

SET UP *the problem* using the following steps:

1. Establish a coordinate system, with the positive direction from the listener toward the source. Carefully determine the signs of all relevant velocities. A velocity in the direction from the listener toward the source is positive; a velocity in the opposite direction is negative. All velocities must be measured relative to the air in which the sound travels.
2. Use consistent subscripts to identify the various quantities: S for source and L for listener.
3. Identify which unknown quantities are the target variables.

EXECUTE *the solution* as follows:

1. Use Eq. (16.29) to relate the frequencies at the source and the listener, the sound speed, and the velocities of the source and the listener according to the sign convention of step 1. If the source is moving, you can find the wavelength measured by the listener using Eq. (16.27) or (16.28).
2. When a wave is reflected from a stationary or moving surface, solve the problem in two steps. In the first, the surface is the "listener"; the frequency with which the wave crests arrive at the surface is f_L. In the second, the surface is the "source," emitting waves with this same frequency f_L. Finally, determine the frequency heard by a listener detecting this new wave.

EVALUATE *your answer:* Is the *direction* of the frequency shift reasonable? If the source and the listener are moving toward each other, $f_L > f_S$; if they are moving apart, $f_L < f_S$. If the source and the listener have no relative motion, $f_L = f_S$.

EXAMPLE 16.14 Doppler effect I: Wavelengths

WITH VARIATION PROBLEMS

A police car's siren emits a sinusoidal wave with frequency $f_S = 300$ Hz. The speed of sound is 340 m/s and the air is still. (a) Find the wavelength of the waves if the siren is at rest. (b) Find the wavelengths of the waves in front of and behind the siren if it is moving at 30 m/s.

IDENTIFY and SET UP In part (a) there is no Doppler effect because neither source nor listener is moving with respect to the air; $v = \lambda f$ gives the wavelength. **Figure 16.30** shows the situation in part (b): The source is in motion, so we find the wavelengths using Eqs. (16.27) and (16.28) for the Doppler effect.

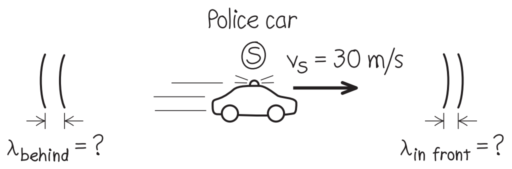
EXECUTE (a) When the source is at rest,

$$\lambda = \frac{v}{f_S} = \frac{340 \text{ m/s}}{300 \text{ Hz}} = 1.13 \text{ m}$$

(b) From Eq. (16.27), in front of the siren

$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S} = \frac{340 \text{ m/s} - 30 \text{ m/s}}{300 \text{ Hz}} = 1.03 \text{ m}$$

Figure **16.30** Our sketch for this problem.



From Eq. (16.28), behind the siren

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S} = \frac{340 \text{ m/s} + 30 \text{ m/s}}{300 \text{ Hz}} = 1.23 \text{ m}$$

EVALUATE The wavelength is shorter in front of the siren and longer behind it, as we expect.

KEYCONCEPT If a source of sound is moving through still air, a listener behind the source hears a sound of increased wavelength. A listener in front of the source hears a sound of decreased wavelength.

EXAMPLE 16.15 Doppler effect II: Frequencies

WITH VARIATION PROBLEMS

If a listener L is at rest and the siren in Example 16.14 is moving away from L at 30 m/s, what frequency does the listener hear?

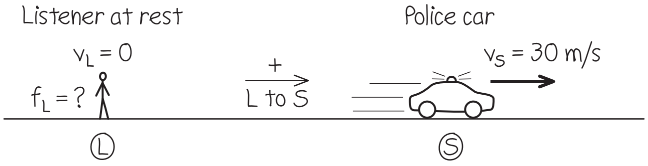
IDENTIFY and SET UP Our target variable is the frequency f_L heard by a listener behind the moving source. **Figure 16.31** shows the situation. We have $v_L = 0$ and $v_S = +30\text{ m/s}$ (positive, since the velocity of the source is in the direction from listener to source).

EXECUTE From Eq. (16.29),

$$f_L = \frac{v}{v + v_S} f_S = \frac{340\text{ m/s}}{340\text{ m/s} + 30\text{ m/s}}(300\text{ Hz}) = 276\text{ Hz}$$

EVALUATE The source and listener are moving apart, so $f_L < f_S$. Here's a check on our numerical result. From Example 16.14, the wavelength behind the source (where the listener in Fig. 16.31 is located) is 1.23 m. The wave speed relative to the stationary listener is $v = 340\text{ m/s}$ even though the source is moving, so

Figure **16.31** Our sketch for this problem.



$$f_L = \frac{v}{\lambda} = \frac{340\text{ m/s}}{1.23\text{ m}} = 276\text{ Hz}$$

KEYCONCEPT If a source of sound is moving through still air, a listener behind the source hears a sound of decreased frequency. A listener in front of the source hears a sound of increased frequency.

EXAMPLE 16.16 Doppler effect III: A moving listener

WITH VARIATION PROBLEMS

If the siren is at rest and the listener is moving away from it at 30 m/s, what frequency does the listener hear?

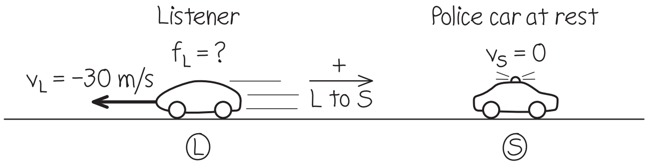
IDENTIFY and SET UP Again our target variable is f_L, but now L is in motion and S is at rest. **Figure 16.32** shows the situation. The velocity of the listener is $v_L = -30\text{ m/s}$ (negative, since the motion is in the direction from source to listener).

EXECUTE From Eq. (16.29),

$$f_L = \frac{v + v_L}{v} f_S = \frac{340\text{ m/s} + (-30\text{ m/s})}{340\text{ m/s}}(300\text{ Hz}) = 274\text{ Hz}$$

EVALUATE Again the source and listener are moving apart, so $f_L < f_S$. Note that the *relative velocity* of source and listener is the same as in Example 16.15, but the Doppler shift is different because v_S and v_L are different.

Figure **16.32** Our sketch for this problem.



KEYCONCEPT If a listener is moving away from a source of sound, the listener hears a sound of decreased frequency. If the listener is moving toward the source, the listener hears a sound of increased frequency.

EXAMPLE 16.17 Doppler effect IV: Moving source, moving listener

WITH VARIATION PROBLEMS

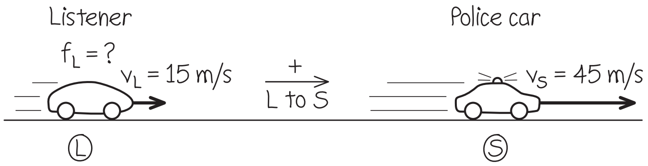
The siren is moving away from the listener with a speed of 45 m/s relative to the air, and the listener is moving toward the siren with a speed of 15 m/s relative to the air. What frequency does the listener hear?

IDENTIFY and SET UP Now *both* L and S are in motion (**Fig. 16.33**). Again our target variable is f_L. Both the source velocity $v_S = +45\text{ m/s}$ and the listener's velocity $v_L = +15\text{ m/s}$ are positive because both velocities are in the direction from listener to source.

EXECUTE From Eq. (16.29),

$$f_L = \frac{v + v_L}{v + v_S} f_S = \frac{340\text{ m/s} + 15\text{ m/s}}{340\text{ m/s} + 45\text{ m/s}}(300\text{ Hz}) = 277\text{ Hz}$$

Figure **16.33** Our sketch for this problem.



EVALUATE As in Examples 16.15 and 16.16, the source and listener again move away from each other at 30 m/s, so again $f_L < f_S$. But f_L is different in all three cases because the Doppler effect for sound depends on how the source and listener are moving relative to the *air*, not simply on how they move relative to each other.

KEYCONCEPT When a listener and source of sound are both moving relative to the air, the listener hears a sound of decreased frequency if the listener and source are moving apart. The listener hears a sound of increased frequency if the listener and source are moving closer together.

EXAMPLE 16.18 Doppler effect V: A double Doppler shift

WITH VARIATION PROBLEMS

The police car is moving toward a warehouse at 30 m/s. What frequency does the driver hear reflected from the warehouse?

IDENTIFY This situation has *two* Doppler shifts (**Fig. 16.34**). In the first shift, the warehouse is the stationary "listener." The frequency of sound reaching the warehouse, which we call f_W, is greater than 300 Hz because the source is approaching. In the second shift, the warehouse acts as a source of sound with frequency f_W, and the listener is the driver of the police car; she hears a frequency greater than f_W because she is approaching the source.

SET UP To determine f_W, we use Eq. (16.29) with f_L replaced by f_W. For this part of the problem, $v_L = v_W = 0$ (the warehouse is at rest) and $v_S = -30$ m/s (the siren is moving in the negative direction from source to listener).

To determine the frequency heard by the driver (our target variable), we again use Eq. (16.29) but now with f_S replaced by f_W. For this second part of the problem, $v_S = 0$ because the stationary warehouse is the source and the velocity of the listener (the driver) is $v_L = +30$ m/s. (The listener's velocity is positive because it is in the direction from listener to source.)

EXECUTE The frequency reaching the warehouse is

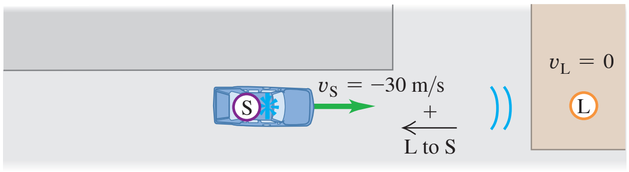
$$\begin{aligned} f_W &= \frac{v}{v + v_S} f_S = \frac{340\text{ m/s}}{340\text{ m/s} + (-30\text{ m/s})}(300\text{ Hz}) \\ &= 329\text{ Hz} \end{aligned}$$

Then the frequency heard by the driver is

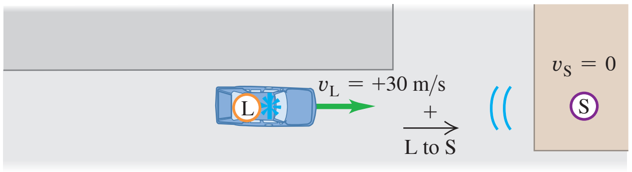
$$f_L = \frac{v + v_L}{v} f_W = \frac{340\text{ m/s} + 30\text{ m/s}}{340\text{ m/s}}(329\text{ Hz}) = 358\text{ Hz}$$

Figure **16.34** Two stages of the sound wave's motion from the police car to the warehouse and back to the police car.

(a) Sound travels from police car's siren (source S) to warehouse ("listener" L).



(b) Reflected sound travels from warehouse (source S) to police car (listener L).



EVALUATE Because there are two Doppler shifts, the reflected sound heard by the driver has an even higher frequency than the sound heard by a stationary listener in the warehouse.

KEYCONCEPT If a source of sound moves relative to a wall (or other reflecting surface), there are two shifts in the frequency of the sound: The frequency received by and reflected from the wall is shifted compared to the sound emitted by the source, and the frequency received back at the source is shifted compared to the sound reflected from the wall. Both shifts increase the frequency if the source is approaching the wall, and both shifts decrease the frequency if the source is moving away from the wall.

Doppler Effect for Electromagnetic Waves

In the Doppler effect for sound, the velocities v_L and v_S are always measured relative to the *air* or whatever medium we are considering. There is also a Doppler effect for *electromagnetic* waves in empty space, such as light waves or radio waves. In this case there is no medium that we can use as a reference to measure velocities, and all that matters is the *relative* velocity of source and receiver. (By contrast, the Doppler effect for sound does not depend simply on this relative velocity, as discussed in Example 16.17.)

To derive the expression for the Doppler frequency shift for light, we have to use the special theory of relativity. We'll discuss this in Chapter 37, but for now we quote the result without derivation. The wave speed is the speed of light, usually denoted by c, and it is the same for both source and receiver. In the frame of reference in which the receiver is at rest, the source is moving away from the receiver with velocity v. (If the source is *approaching* the receiver, v is negative.) The source frequency is again f_S. The frequency f_R measured by the receiver R (the frequency of arrival of the waves at the receiver) is then

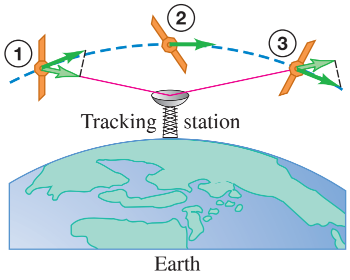
$$f_R = \sqrt{\frac{c - v}{c + v}}\, f_S \qquad \text{(Doppler effect for light)} \qquad (16.30)$$

When v is positive, the source is moving directly *away* from the receiver and f_R is always *less* than f_S; when v is negative, the source is moving directly *toward* the receiver and f_R is *greater* than f_S. The qualitative effect is the same as for sound, but the quantitative relationship is different.

A familiar application of the Doppler effect for radio waves is the radar device mounted on the side window of a police car to check other cars' speeds. The electromagnetic wave emitted by the device is reflected from a moving car, which acts as a moving source, and the wave reflected back to the device is Doppler-shifted in frequency. The transmitted and reflected signals are combined to produce beats, and the speed can be computed from the frequency of the beats. Similar techniques ("Doppler radar") are used to measure wind velocities in the atmosphere.

The Doppler effect is also used to track satellites and other space vehicles. In **Fig. 16.35** a satellite emits a radio signal with constant frequency f_S. As the satellite orbits past, it first approaches and then moves away from the receiver; the frequency f_R of the signal received on earth changes from a value greater than f_S to a value less than f_S as the satellite passes overhead.

Figure **16.35** Change of velocity component along the line of sight of a satellite passing a tracking station. The frequency received at the tracking station changes from high to low as the satellite passes overhead.



TEST YOUR UNDERSTANDING OF SECTION 16.8 You are at an outdoor concert with a wind blowing at 10 m/s from the performers toward you. Is the sound you hear Doppler-shifted? If so, is it shifted to lower or higher frequencies?

ANSWER

no The air (the medium for sound waves) is moving from the source toward the listener. Hence, relative to the air, both the source and the listener are moving in the direction from listener to source. So both velocities are positive and $v_S = v_L = +10$ m/s. The equality of these two velocities means that the numerator and the denominator in Eq. (16.29) are the same, so $f_L = f_S$ and there is *no* Doppler shift.

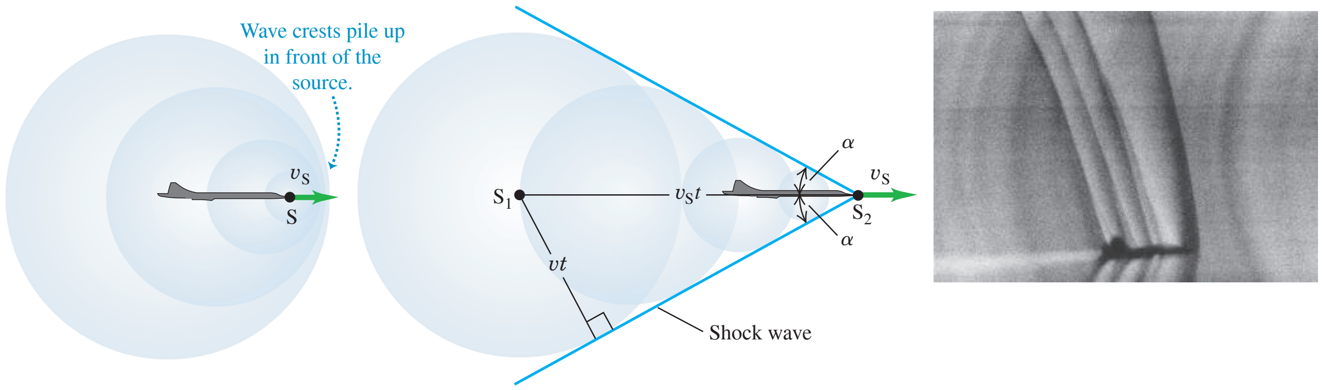
16.9 SHOCK WAVES

You may have experienced "sonic booms" caused by an airplane flying overhead faster than the speed of sound. We can see qualitatively why this happens from **Fig. 16.36**. Let v_S denote the *speed* of the airplane relative to the air, so that it is always positive. The motion of the airplane through the air produces sound; if v_S is less than the speed of sound v, the waves in front of the airplane are crowded together with a wavelength given by Eq. (16.27):

$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S}$$

As the speed v_S of the airplane approaches the speed of sound v, the wavelength approaches zero and the wave crests pile up on each other (Fig. 16.36a). The airplane must exert a large force to compress the air in front of it; by Newton's third law, the air exerts an equally large force back on the airplane. Hence there is a large increase in aerodynamic drag (air resistance) as the airplane approaches the speed of sound, a phenomenon known as the "sound barrier."

Figure **16.36** Wave crests around a sound source S moving (a) slightly slower than the speed of sound v and (b) faster than the sound speed v. (c) This photograph shows a T-38 jet airplane moving at 1.1 times the speed of sound. Separate shock waves are produced by the nose, wings, and tail. The angles of these waves vary because the air speeds up and slows down as it moves around the airplane, so the relative speed v_S of the airplane and air is different for shock waves produced at different points.


(a) Sound source S (airplane) moving at nearly the speed of sound

(b) Sound source moving faster than the speed of sound

(c) Shock waves around a supersonic airplane

When v_S is greater in magnitude than v, the source of sound is **supersonic,** and Eqs. (16.27) and (16.29) for the Doppler effect no longer describe the sound wave in front of the source. Figure 16.36b shows a cross section of what happens. As the airplane moves, it displaces the surrounding air and produces sound. A series of wave crests is emitted from the nose of the airplane; each spreads out in a circle centered at the position of the airplane when it emitted the crest. After a time t the crest emitted from point S_1 has spread to a circle with radius vt, and the airplane has moved a greater distance $v_S t$ to position S_2. You can see that the circular crests interfere constructively at points along the blue line that makes an angle α with the direction of the airplane velocity, leading to a very-large-amplitude wave crest along this line. This large-amplitude crest is called a **shock wave** (Fig. 16.36c).

From the right triangle in Fig. 16.36b we can see that $\sin\alpha = vt/v_S t$, or

Shock wave produced by sound source moving faster than sound:

$$\sin \overset{\text{Angle of shock wave}}{\alpha} = \frac{v \leftarrow \text{Speed of sound}}{v_S \leftarrow \text{Speed of source}} \tag{16.31}$$

Figure 16.37 The first supersonic airplane, the Bell X-1, was shaped much like a 50 caliber bullet—which was known to be able to travel faster than sound.

The ratio v_S/v is called the **Mach number.** It is greater than unity for all supersonic speeds, and $\sin\alpha$ in Eq. (16.31) is the reciprocal of the Mach number. The first person to break the sound barrier was Capt. Chuck Yeager of the U.S. Air Force, flying the Bell X-1 at Mach 1.06 on October 14, 1947 (**Fig. 16.37**).

Shock waves are actually three-dimensional; a shock wave forms a *cone* around the direction of motion of the source. If the source (possibly a supersonic jet airplane or a rifle bullet) moves with constant velocity, the angle α is constant, and the shock-wave cone moves along with the source. It's the arrival of this shock wave that causes the sonic boom you hear after a supersonic airplane has passed by. In front of the shock-wave cone, there is no sound. Inside the cone a stationary listener hears the Doppler-shifted sound of the airplane moving away.

CAUTION **Shock waves** A shock wave is produced *continuously* by any object that moves through the air at supersonic speed, not only at the instant that it "breaks the sound barrier." The sound waves that combine to form the shock wave, as in Fig. 16.36b, are created by the motion of the object itself, not by any sound source that the object may carry. The cracking noises of a bullet and of the tip of a circus whip are due to their supersonic motion. A supersonic jet airplane may have very loud engines, but these do not cause the shock wave. If the pilot were to shut the engines off, the airplane would continue to produce a shock wave as long as its speed remained supersonic.

Shock waves have applications outside of aviation. They are used to break up kidney stones and gallstones without invasive surgery, using a technique with the impressive name *extracorporeal shock-wave lithotripsy.* A shock wave produced outside the body is focused by a reflector or acoustic lens so that as much of it as possible converges on the stone. When the resulting stresses in the stone exceed its tensile strength, it breaks into small pieces and can be eliminated. This technique requires accurate determination of the location of the stone, which may be done using ultrasonic imaging techniques (see Fig. 16.9).

EXAMPLE 16.19 Sonic boom from a supersonic airplane

An airplane is flying at Mach 1.75 at an altitude of 8000 m, where the speed of sound is 320 m/s. How long after the plane passes directly overhead will you hear the sonic boom?

IDENTIFY and SET UP The shock wave forms a cone trailing backward from the airplane, so the problem is really asking for how much time elapses from when the airplane flies overhead to when the shock wave reaches you at point L (**Fig. 16.38**). During the time t (our target variable) since the airplane traveling at speed v_S passed overhead, it has traveled a distance $v_S t$. Equation (16.31) gives the shock cone angle α; we use trigonometry to solve for t.

EXECUTE From Eq. (16.31) the angle α of the shock cone is

$$\alpha = \arcsin\frac{1}{1.75} = 34.8^\circ$$

The speed of the plane is the speed of sound multiplied by the Mach number:

$$v_S = (1.75)(320\ \text{m/s}) = 560\ \text{m/s}$$

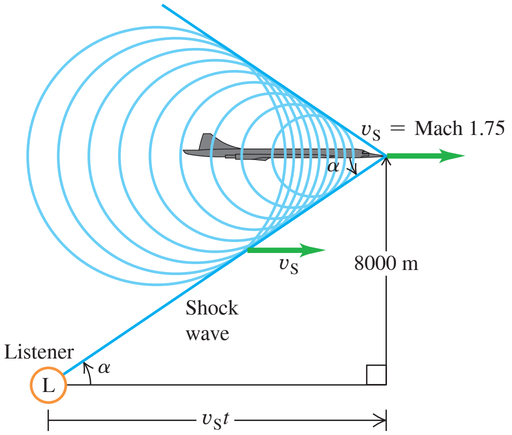


Figure **16.38** You hear a sonic boom when the shock wave reaches you at L (*not* just when the plane breaks the sound barrier). A listener to the right of L has not yet heard the sonic boom but will shortly; a listener to the left of L has already heard the sonic boom.

From Fig. 16.38 we have

$$\tan\alpha = \frac{8000\text{ m}}{v_S t}$$

$$t = \frac{8000\text{ m}}{(560\text{ m/s})(\tan 34.8°)} = 20.5\text{ s}$$

EVALUATE You hear the boom 20.5 s after the airplane passes overhead, at which time it has traveled $(560\text{ m/s})(20.5\text{ s}) = 11.5\text{ km}$ since it passed overhead. We have assumed that the speed of sound is the same at all altitudes, so that $\alpha = \arcsin v/v_S$ is constant and the shock wave forms a perfect cone. In fact, the speed of sound decreases with increasing altitude. How would this affect the value of t?

KEYCONCEPT An object moving through the air faster than the speed of sound continuously produces a cone-shaped shock wave. The angle of the cone depends on the object's Mach number (the ratio of its speed to the speed of sound).

TEST YOUR UNDERSTANDING OF SECTION 16.9 What would you hear if you were directly behind (to the left of) the supersonic airplane in Fig. 16.38? (i) A sonic boom; (ii) the sound of the airplane, Doppler-shifted to higher frequencies; (iii) the sound of the airplane, Doppler-shifted to lower frequencies; (iv) nothing.

ANSWER **(iii)** Figure 16.38 shows that there are sound waves inside the cone of the shock wave. Behind the airplane the wave crests are spread apart, just as they are behind the moving source in Fig. 16.28. Hence the waves that reach you have an increased wavelength and a lower frequency.

CHAPTER 16 SUMMARY

Sound waves: Sound consists of longitudinal waves in a medium. A sinusoidal sound wave is characterized by its frequency f and wavelength λ (or angular frequency ω and wave number k) and by its displacement amplitude A. The pressure amplitude $p_{\max}$ is directly proportional to the displacement amplitude, the wave number, and the bulk modulus B of the wave medium. (See Examples 16.1 and 16.2.)

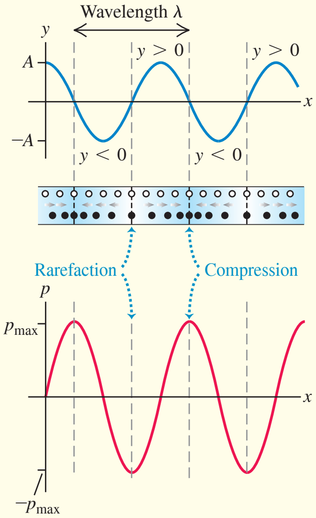
The speed of a sound wave in a fluid depends on the bulk modulus B and density ρ. If the fluid is an ideal gas, the speed can be expressed in terms of the temperature T, molar mass M, and ratio of heat capacities γ of the gas. The speed of longitudinal waves in a solid rod depends on the density and Young's modulus Y. (See Examples 16.3 and 16.4.)

$$p_{\max} = BkA \qquad (16.5)$$
(sinusoidal sound wave)

$$v = \sqrt{\frac{B}{\rho}} \qquad (16.7)$$
(longitudinal wave in a fluid)

$$v = \sqrt{\frac{\gamma RT}{M}} \qquad (16.10)$$
(sound wave in an ideal gas)

$$v = \sqrt{\frac{Y}{\rho}} \qquad (16.8)$$
(longitudinal wave in a solid rod)



Intensity and sound intensity level: The intensity I of a sound wave is the time average rate at which energy is transported by the wave, per unit area. For a sinusoidal wave, the intensity can be expressed in terms of the displacement amplitude A or the pressure amplitude $p_{\max}$. (See Examples 16.5–16.7.)

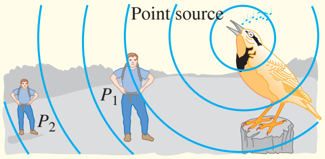
The sound intensity level β of a sound wave is a logarithmic measure of its intensity. It is measured relative to I_0, an arbitrary intensity defined to be $10^{-12}\ \mathrm{W/m^2}$. Sound intensity levels are expressed in decibels (dB). (See Examples 16.8 and 16.9.)

$$I = \tfrac{1}{2}\sqrt{\rho B}\,\omega^2 A^2 = \frac{p_{\max}{}^2}{2\rho v} = \frac{p_{\max}{}^2}{2\sqrt{\rho B}} \qquad (16.12),\ (16.14)$$

(intensity of a sinusoidal sound wave in a fluid)

$$\beta = (10\ \mathrm{dB})\log\frac{I}{I_0} \qquad (16.15)$$

(definition of sound intensity level)



Standing sound waves: Standing sound waves can be set up in a pipe or tube. A closed end is a displacement node and a pressure antinode; an open end is a displacement antinode and a pressure node. For a pipe of length L open at both ends, the normal-mode frequencies are integer multiples of the sound speed divided by $2L$. For a stopped pipe (one that is open at only one end), the normal-mode frequencies are the odd multiples of the sound speed divided by $4L$. (See Examples 16.10 and 16.11.)

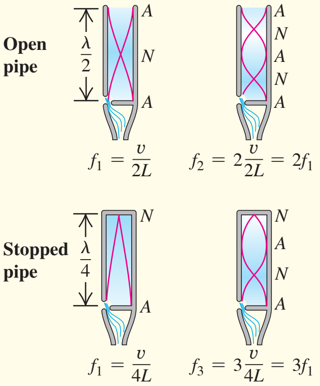
A pipe or other system with normal-mode frequencies can be driven to oscillate at any frequency. A maximum response, or resonance, occurs if the driving frequency is close to one of the normal-mode frequencies of the system. (See Example 16.12.)

$$f_n = \frac{nv}{2L} \qquad (n = 1, 2, 3, \ldots) \qquad (16.18)$$

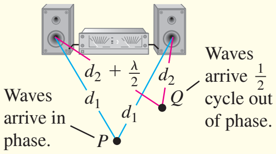
(open pipe)

$$f_n = \frac{nv}{4L} \qquad (n = 1, 3, 5, \ldots) \qquad (16.22)$$

(stopped pipe)



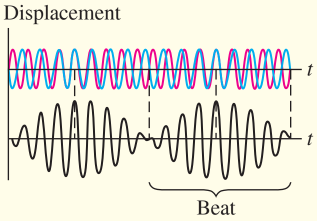
Interference: When two or more waves overlap in the same region of space, the resulting effects are called interference. The resulting amplitude can be either larger or smaller than the amplitude of each individual wave, depending on whether the waves are in phase (constructive interference) or out of phase (destructive interference). (See Example 16.13.)



Beats: Beats are heard when two tones with slightly different frequencies f_a and f_b are sounded together. The beat frequency f_{beat} is the difference between f_a and f_b.

$$f_{\text{beat}} = f_a - f_b \qquad (16.24)$$

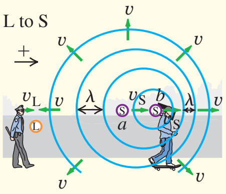
(beat frequency)



Doppler effect: The Doppler effect for sound is the frequency shift that occurs when there is motion of a source of sound, a listener, or both, relative to the medium. The source and listener frequencies f_S and f_L are related by the source and listener velocities v_S and v_L relative to the medium and to the speed of sound v. (See Examples 16.14–16.18.)

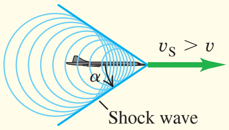
$$f_L = \frac{v + v_L}{v + v_S} f_S \qquad (16.29)$$

(Doppler effect, moving source and moving listener)



Shock waves: A sound source moving with a speed v_S greater than the speed of sound v creates a shock wave. The wave front is a cone with angle α. (See Example 16.19.)

$$\sin\alpha = \frac{v}{v_S} \qquad \text{(shock wave)} \qquad (16.31)$$



Chapter 16 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE ✓ARIATION PROBLEMS

Be sure to review EXAMPLES 16.5, 16.6, 16.7, 16.8, and 16.9 (Section 16.3) before attempting these problems.

VP16.9.1 A 256 Hz sound wave in air (density 1.20 kg/m^3, speed of sound 344 m/s) has intensity $5.50 \times 10^{-8}\ \text{W/m}^2$. (a) What is the wave's pressure amplitude? (b) If the intensity remains the same but the frequency is doubled to 512 Hz, how does this affect the pressure amplitude?

VP16.9.2 At a certain distance from a fire alarm, the sound intensity level is 85.0 dB. (a) What is the intensity of this sound? (b) How many times greater is the intensity of this sound than that of a 67.0 dB sound?

VP16.9.3 A lion can produce a roar with a sound intensity level of 114 dB at a distance of 1.00 m. What is the sound intensity level at a distance of (a) 4.00 m and (b) 15.8 m from the lion? Assume that intensity obeys the inverse-square law.

VP16.9.4 The sound intensity level inside a typical modern airliner in flight is 66.0 dB. The air in the cabin has density 0.920 kg/m^3 (less than in the atmosphere at sea level) and speed of sound 344 m/s. (a) What is the pressure amplitude of this sound? (b) If the pressure amplitude were increased by a factor of 10.0, what would the new sound intensity level be?

Be sure to review EXAMPLES 16.11 and 16.12 (Sections 16.4 and 16.5) before attempting these problems.

VP16.12.1 A particular open organ pipe has a fundamental frequency of 220 Hz (known to musicians as A_3 or "A below middle C") when the speed of sound waves in air is 344 m/s. (a) What is the length of this pipe? (b) The third harmonic of this pipe has the same frequency as the fundamental frequency of a stopped pipe. What is the length of this stopped pipe?

VP16.12.2 You have two organ pipes, one open and one stopped. Which harmonic (if any) of the stopped pipe has the same frequency as the third harmonic of the open pipe if the stopped pipe length is (a) $\frac{1}{6}$, (b) $\frac{1}{2}$, or (c) $\frac{1}{3}$ that of the open pipe?

VP16.12.3 One of the strings of a bass viol is 0.680 m long and has a fundamental frequency of 165 Hz. (a) What is the speed of waves on this string? (b) When this string vibrates at its fundamental frequency, it causes the air in a nearby stopped organ pipe to vibrate at that pipe's fundamental frequency. The speed of sound in the pipe is 344 m/s. What is the length of this pipe?

VP16.12.4 A stopped pipe 1.00 m in length is filled with helium at 20°C (speed of sound 999 m/s). When the helium in this pipe vibrates at its third harmonic frequency, it causes the air at 20°C (speed of sound 344 m/s) in a nearby open pipe to vibrate at its fifth harmonic frequency. What are the frequency and wavelength of the sound wave (a) in the helium in the stopped pipe and (b) in the air in the open pipe? (c) What is the length of the open pipe?

Be sure to review EXAMPLES 16.14, 16.15, 16.16, 16.17, and 16.18 (Section 16.8) before attempting these problems.

VP16.18.1 The siren on an ambulance emits a sound of frequency 2.80×10^3 Hz. If the ambulance is traveling at 26.0 m/s (93.6 km/h, or 58.2 mi/h), the speed of sound is 340 m/s, and the air is still, what are the frequency and wavelength that you hear if you are standing (a) in front of the ambulance or (b) behind the ambulance?

VP16.18.2 A stationary bagpiper is playing a Highland bagpipe, in which one reed produces a continuous sound of frequency 440 Hz. The air is still and the speed of sound is 340 m/s. (a) What is the wavelength of the sound wave produced by the bagpipe? What are the frequency and wavelength of the sound wave that a bicyclist hears if she is (b) approaching the bagpiper at 10.0 m/s or (c) moving away from the bagpiper at 10.0 m/s?

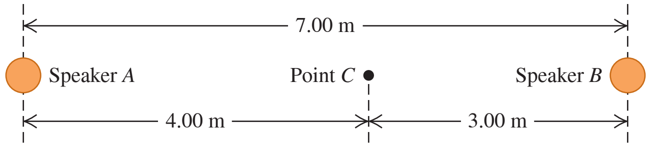
VP16.18.3 A police car moving east at 40.0 m/s is chasing a speeding sports car moving east at 35.0 m/s. The police car's siren has frequency 1.20×10^3 Hz, the speed of sound is 340 m/s, and the air is still. (a) What is the frequency of sound that the driver of the speeding sports car hears? (b) If the speeding sports car were to turn around and drive west at 35.0 m/s toward the approaching police car, what frequency would the driver of the sports car hear?

VP16.18.4 For a scene in an action movie, a car drives at 25.0 m/s directly toward a wall. The car's horn is on continuously and produces a sound of frequency 415 Hz. (a) If the speed of sound is 340 m/s and the air is still, what is the frequency of the sound that the driver of the car hears reflected from the wall? (b) How fast would the car have to move for the reflected sound that the driver hears to have frequency 495 Hz?

BRIDGING PROBLEM Loudspeaker Interference

Loudspeakers A and B are 7.00 m apart and vibrate in phase at 172 Hz. They radiate sound uniformly in all directions. Their acoustic power outputs are 8.00×10^{-4} W and 6.00×10^{-5} W, respectively. The air temperature is 20°C. (a) Determine the difference in phase of the two signals at a point C along the line joining A and B, 3.00 m from B and 4.00 m from A (**Fig. 16.39**). (b) Determine the intensity and sound intensity level at C from speaker A alone (with B turned off) and from speaker B alone (with A turned off). (c) Determine the intensity and sound intensity level at C from both speakers together.

Figure **16.39** The situation for this problem.



SOLUTION GUIDE

IDENTIFY and SET UP

1. Choose the equations that relate power, distance from the source, intensity, pressure amplitude, and sound intensity level.
2. Decide how you'll determine the phase difference in part (a). Once you have found the phase difference, how can you use it to find the amplitude of the combined wave at C due to both sources?
3. List the unknown quantities for each part of the problem and identify your target variables.

Continued

EXECUTE

4. Determine the phase difference at point C.
5. Find the intensity, sound intensity level, and pressure amplitude at C due to each speaker alone.
6. Use your results from steps 4 and 5 to find the pressure amplitude at C due to both loudspeakers together.
7. Use your result from step 6 to find the intensity and sound intensity level at C due to both loudspeakers together.

EVALUATE

8. How do your results from part (c) for intensity and sound intensity level at C compare to those from part (b)? Does this make sense?
9. What result would you have gotten in part (c) if you had (incorrectly) combined the *intensities* from A and B directly, rather than (correctly) combining the *pressure amplitudes* as you did in step 6?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/ or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q16.1 When sound travels from air into water, does the frequency of the wave change? The speed? The wavelength? Explain your reasoning.

Q16.2 The hero of a western movie listens for an oncoming train by putting his ear to the track. Why does this method give an earlier warning of the approach of a train than just listening in the usual way?

Q16.3 Would you expect the pitch (or frequency) of an organ pipe to increase or decrease with increasing temperature? Explain.

Q16.4 In most modern wind instruments the pitch is changed by using keys or valves to change the length of the vibrating air column. The bugle, however, has no valves or keys, yet it can play many notes. How might this be possible? Are there restrictions on what notes a bugle can play?

Q16.5 Symphonic musicians always “warm up” their wind instruments by blowing into them before a performance. What purpose does this serve?

Q16.6 In a popular and amusing science demonstration, a person inhales helium and then his voice becomes high and squeaky. Why does this happen? (*Warning:* Inhaling too much helium can cause unconsciousness or death.)

Q16.7 Lane dividers on highways sometimes have regularly spaced ridges or ripples. When the tires of a moving car roll along such a divider, a musical note is produced. Why? Explain how this phenomenon could be used to measure the car’s speed.

Q16.8 (a) Does a sound level of 0 dB mean that there is no sound? (b) Is there any physical meaning to a sound having a negative intensity level? If so, what is it? (c) Does a sound intensity of zero mean that there is no sound? (d) Is there any physical meaning to a sound having a negative intensity? Why?

Q16.9 Which has a more direct influence on the loudness of a sound wave: the *displacement* amplitude or the *pressure* amplitude? Explain.

Q16.10 If the pressure amplitude of a sound wave is halved, by what factor does the intensity of the wave decrease? By what factor must the pressure amplitude of a sound wave be increased in order to increase the intensity by a factor of 16? Explain.

Q16.11 Does the sound intensity level β obey the inverse-square law? Why?

Q16.12 A small fraction of the energy in a sound wave is absorbed by the air through which the sound passes. How does this modify the inverse-square relationship between intensity and distance from the source? Explain.

Q16.13 A small metal band is slipped onto one of the tines of a tuning fork. As this band is moved closer and closer to the end of the tine, what effect does this have on the wavelength and frequency of the sound the tine produces? Why?

Q16.14 An organist in a cathedral plays a loud chord and then releases the keys. The sound persists for a few seconds and gradually dies away. Why does it persist? What happens to the sound energy when the sound dies away?

Q16.15 Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 860 Hz. Point P is 12.0 m from A and 13.4 m from B. Is the interference at P constructive or destructive? Give the reasoning behind your answer.

Q16.16 Two vibrating tuning forks have identical frequencies, but one is stationary and the other is mounted at the rim of a rotating platform. What does a listener hear? Explain.

Q16.17 A large church has part of the organ in the front of the church and part in the back. A person walking rapidly down the aisle while both segments are playing at once reports that the two segments sound out of tune. Why?

Q16.18 A sound source and a listener are both at rest on the earth, but a strong wind is blowing from the source toward the listener. Is there a Doppler effect? Why or why not?

Q16.19 Can you think of circumstances in which a Doppler effect would be observed for surface waves in water? For elastic waves propagating in a body of water deep below the surface? If so, describe the circumstances and explain your reasoning. If not, explain why not.

Q16.20 Stars other than our sun normally appear featureless when viewed through telescopes. Yet astronomers can readily use the light from these stars to determine that they are rotating and even measure the speed of their surface. How do you think they can do this?

Q16.21 If you wait at a railroad crossing as a train approaches and passes, you hear a Doppler shift in its sound. But if you listen closely, you hear that the change in frequency is continuous; it does not suddenly go from one high frequency to another low frequency. Instead the frequency *smoothly* (but rather quickly) changes from high to low as the train passes. Why does this smooth change occur?

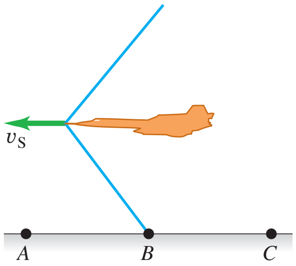
Q16.22 In case 1, a source of sound approaches a stationary observer at speed u. In case 2, the observer moves toward the stationary source at the same speed u. If the source is always producing the same frequency sound, will the observer hear the same frequency in both cases, since the relative speed is the same each time? Why or why not?

Q16.23 Does an aircraft make a sonic boom only at the instant its speed exceeds Mach 1? Explain.

Q16.24 If you are riding in a supersonic aircraft, what do you hear? Explain. In particular, do you hear a continuous sonic boom? Why or why not?

Q16.25 A jet airplane is flying at a constant altitude at a steady speed v_S greater than the speed of sound. Describe what observers at points A, B, and C hear at the instant shown in **Fig. Q16.25**, when the shock wave has just reached point B. Explain.

Figure **Q16.25**



EXERCISES

Unless indicated otherwise, assume the speed of sound in air to be $v = 344$ m/s.

Section 16.1 Sound Waves

16.1 • Example 16.1 (Section 16.1) showed that for sound waves in air with frequency 1000 Hz, a displacement amplitude of 1.2×10^{-8} m produces a pressure amplitude of 3.0×10^{-2} Pa. (a) What is the wavelength of these waves? (b) For 1000 Hz waves in air, what displacement amplitude would be needed for the pressure amplitude to be at the pain threshold, which is 30 Pa? (c) For what wavelength and frequency will waves with a displacement amplitude of 1.2×10^{-8} m produce a pressure amplitude of 1.5×10^{-3} Pa?

16.2 • A loud factory machine produces sound having a displacement amplitude of 1.00 μm, but the frequency of this sound can be adjusted. In order to prevent ear damage to the workers, the maximum pressure amplitude of the sound waves is limited to 10.0 Pa. Under the conditions of this factory, the bulk modulus of air is 1.42×10^5 Pa. What is the highest-frequency sound to which this machine can be adjusted without exceeding the prescribed limit? Is this frequency audible to the workers?

16.3 • Consider a sound wave in air that has displacement amplitude 0.0200 mm. Calculate the pressure amplitude for frequencies of (a) 150 Hz; (b) 1500 Hz; (c) 15,000 Hz. In each case compare the result to the pain threshold, which is 30 Pa.

16.4 • BIO **Ultrasound and Infrasound.** (a) **Whale communication.** Blue whales apparently communicate with each other using sound of frequency 17 Hz, which can be heard nearly 1000 km away in the ocean. What is the wavelength of such a sound in seawater, where the speed of sound is 1531 m/s? (b) **Dolphin clicks.** One type of sound that dolphins emit is a sharp click of wavelength 1.5 cm in the ocean. What is the frequency of such clicks? (c) **Dog whistles.** One brand of dog whistles claims a frequency of 25 kHz for its product. What is the wavelength of this sound? (d) **Bats.** While bats emit a wide variety of sounds, one type emits pulses of sound having a frequency between 39 kHz and 78 kHz. What is the range of wavelengths of this sound? (e) **Sonograms.** Ultrasound is used to view the interior of the body, much as x rays are utilized. For sharp imagery, the wavelength of the sound should be around one-fourth (or less) the size of the objects to be viewed. Approximately what frequency of sound is needed to produce a clear image of a tumor that is 1.0 mm across if the speed of sound in the tissue is 1550 m/s?

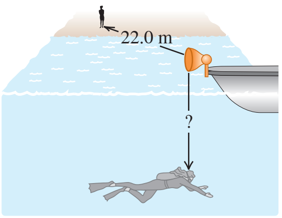
Section 16.2 Speed of Sound Waves

16.5 •• A 60.0-m-long brass rod is struck at one end. A person at the other end hears two sounds as a result of two longitudinal waves, one traveling in the metal rod and the other traveling in air. What is the time interval between the two sounds? (The speed of sound in air is 344 m/s; see Tables 11.1 and 12.1 for relevant information about brass.)

16.6 • (a) In a liquid with density 1300 kg/m^3, longitudinal waves with frequency 400 Hz are found to have wavelength 8.00 m. Calculate the bulk modulus of the liquid. (b) A metal bar with a length of 1.50 m has density 6400 kg/m^3. Longitudinal sound waves take 3.90×10^{-4} s to travel from one end of the bar to the other. What is Young's modulus for this metal?

16.7 • A submerged scuba diver hears the sound of a boat horn directly above her on the surface of the lake. At the same time, a friend on dry land 22.0 m from the boat also hears the horn (**Fig. E16.7**). The horn is 1.2 m above the surface of the water. What is the distance (labeled "?") from the horn to the diver? Both air and water are at 20°C.

Figure **E16.7**



16.8 • At a temperature of 27.0°C, what is the speed of longitudinal waves in (a) hydrogen (molar mass 2.02 g/mol); (b) helium (molar mass 4.00 g/mol); (c) argon (molar mass 39.9 g/mol)? See Table 19.1 for values of γ. (d) Compare your answers for parts (a), (b), and (c) with the speed in air at the same temperature.

16.9 • An oscillator vibrating at 1250 Hz produces a sound wave that travels through an ideal gas at 325 m/s when the gas temperature is 22.0°C. For a certain experiment, you need to have the same oscillator produce sound of wavelength 28.5 cm in this gas. What should the gas temperature be to achieve this wavelength?

16.10 •• CALC (a) Show that the fractional change in the speed of sound (dv/v) due to a very small temperature change dT is given by $dv/v = \frac{1}{2}dT/T$. [*Hint:* Start with Eq. (16.10).] (b) The speed of sound in air at 20°C is found to be 344 m/s. Use the result in part (a) to find the change in the speed of sound for a 1.0°C change in air temperature.

Section 16.3 Sound Intensity

16.11 •• BIO **Energy Delivered to the Ear.** Sound is detected when a sound wave causes the tympanic membrane (the eardrum) to vibrate. Typically, the diameter of this membrane is about 8.4 mm in humans. (a) How much energy is delivered to the eardrum each second when someone whispers (20 dB) a secret in your ear? (b) To comprehend how sensitive the ear is to very small amounts of energy, calculate how fast a typical 2.0 mg mosquito would have to fly (in mm/s) to have this amount of kinetic energy.

16.12 • (a) By what factor must the sound intensity be increased to raise the sound intensity level by 13.0 dB? (b) Explain why you don't need to know the original sound intensity.

16.13 •• **Eavesdropping!** You are trying to overhear a juicy conversation, but from your distance of 15.0 m, it sounds like only an average whisper of 20.0 dB. How close should you move to the chatterboxes for the sound level to be 60.0 dB?

16.14 • A small source of sound waves emits uniformly in all directions. The total power output of the source is P. By what factor must P increase if the sound intensity level at a distance of 20.0 m from the source is to increase 5.00 dB?

16.15 • A sound wave in air at 20°C has a frequency of 320 Hz and a displacement amplitude of 5.00×10^{-3} mm. For this sound wave calculate the (a) pressure amplitude (in Pa); (b) intensity (in W/m^2); (c) sound intensity level (in decibels).

16.16 •• You live on a busy street, but as a music lover, you want to reduce the traffic noise. (a) If you install special sound-reflecting windows that reduce the sound intensity level (in dB) by 30 dB, by what fraction have you lowered the sound intensity (in W/m^2)? (b) If, instead, you reduce the intensity by half, what change (in dB) do you make in the sound intensity level?

16.17 • BIO For a person with normal hearing, the faintest sound that can be heard at a frequency of 400 Hz has a pressure amplitude of about 6.0×10^{-5} Pa. Calculate the (a) intensity; (b) sound intensity level; (c) displacement amplitude of this sound wave at 20°C.

16.18 •• The intensity due to a number of independent sound sources is the sum of the individual intensities. (a) When four quadruplets cry simultaneously, how many decibels greater is the sound intensity level than when a single one cries? (b) To increase the sound intensity level again by the same number of decibels as in part (a), how many more crying babies are required?

16.19 • **CP** A baby's mouth is 30 cm from her father's ear and 1.50 m from her mother's ear. What is the difference between the sound intensity levels heard by the father and by the mother?

16.20 •• (a) If two sounds differ by 5.00 dB, find the ratio of the intensity of the louder sound to that of the softer one. (b) If one sound is 100 times as intense as another, by how much do they differ in sound intensity level (in decibels)? (c) If you increase the volume of your stereo so that the intensity doubles, by how much does the sound intensity level increase?

16.21 •• **CP** At point *A*, 3.0 m from a small source of sound that is emitting uniformly in all directions, the sound intensity level is 53 dB. (a) What is the intensity of the sound at *A*? (b) How far from the source must you go so that the intensity is one-fourth of what it was at *A*? (c) How far must you go so that the sound intensity level is one-fourth of what it was at *A*? (d) Does intensity obey the inverse-square law? What about sound intensity level?

16.22 •• The pattern of displacement nodes *N* and antinodes *A* in a pipe is *ANANANANANA* when the standing-wave frequency is 1710 Hz. The pipe contains air at 20°C (See Table 16.1.) (a) Is it an open or a closed (stopped) pipe? (b) Which harmonic is this? (c) What is the length of the pipe? (d) What is the fundamental frequency? (e) What would be the fundamental frequency of the pipe if it contained helium at 20°C ?

Section 16.4 Standing Sound Waves and Normal Modes

16.23 • Standing sound waves are produced in a pipe that is 1.20 m long. For the fundamental and first two overtones, determine the locations along the pipe (measured from the left end) of the displacement nodes and the pressure nodes if (a) the pipe is open at both ends and (b) the pipe is closed at the left end and open at the right end.

16.24 • The fundamental frequency of a pipe that is open at both ends is 524 Hz. (a) How long is this pipe? If one end is now closed, find (b) the wavelength and (c) the frequency of the new fundamental.

16.25 • **BIO The Human Voice.** The human vocal tract is a pipe that extends about 17 cm from the lips to the vocal folds (also called "vocal cords") near the middle of your throat. The vocal folds behave rather like the reed of a clarinet, and the vocal tract acts like a stopped pipe. Estimate the first three standing-wave frequencies of the vocal tract. Use $v = 344$ m/s. (The answers are only an estimate, since the position of lips and tongue affects the motion of air in the vocal tract.)

16.26 •• **BIO The Vocal Tract.** Many professional singers have a range of $2\frac{1}{2}$ octaves or even greater. Suppose a soprano's range extends from A below middle C (frequency 220 Hz) up to E-flat above high C (frequency 1244 Hz). Although the vocal tract is complicated, we can model it as a resonating air column, like an organ pipe, that is open at the top and closed at the bottom. The column extends from the mouth down to the diaphragm in the chest cavity. Assume that the lowest note is the fundamental. How long is this column of air if $v = 354$ m/s? Does your result seem reasonable, on the basis of observations of your body?

16.27 • The longest pipe found in most medium-size pipe organs is 4.88 m (16 ft) long. What is the frequency of the note corresponding to the fundamental mode if the pipe is (a) open at both ends, (b) open at one end and closed at the other?

16.28 • **Singing in the Shower.** A pipe closed at both ends can have standing waves inside of it, but you normally don't hear them because little of the sound can get out. But you *can* hear them if you are *inside* the pipe, such as someone singing in the shower. (a) Show that the wavelengths of standing waves in a pipe of length L that is closed at both ends are $\lambda_n = 2L/n$ and the frequencies are given by $f_n = nv/2L = nf_1$, where $n = 1, 2, 3, \dots$. (b) Modeling it as a pipe, find the frequency of the fundamental and the first two overtones for a shower 2.50 m tall. Are these frequencies audible?

16.29 •• The pattern of displacement nodes *N* and antinodes *A* in a pipe is *NANANANANA* when the standing-wave frequency is 1710 Hz. The pipe contains air at 20°C. (See Table 16.1.) (a) Is it an open or a closed (stopped) pipe? (b) Which harmonic is this? (c) What is the length of the pipe? (d) What is the fundamental frequency?

Section 16.5 Resonance and Sound

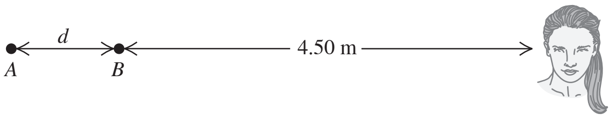
16.30 •• **CP** You have a stopped pipe of adjustable length close to a taut 62.0 cm, 7.25 g wire under a tension of 4110 N. You want to adjust the length of the pipe so that, when it produces sound at its fundamental frequency, this sound causes the wire to vibrate in its second *overtone* with very large amplitude. How long should the pipe be?

16.31 • You blow across the open mouth of an empty test tube and produce the fundamental standing wave in the 14.0-cm-long air column in the test tube, which acts as a stopped pipe. (a) What is the frequency of this standing wave? (b) What is the frequency of the fundamental standing wave in the air column if the test tube is half filled with water?

Section 16.6 Interference of Waves

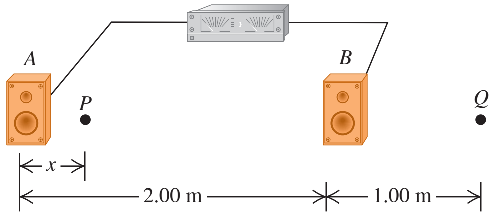
16.32 • Small speakers *A* and *B* are driven in phase at 725 Hz by the same audio oscillator. Both speakers start out 4.50 m from the listener, but speaker *A* is slowly moved away (**Fig. E16.32**). (a) At what distance *d* will the sound from the speakers first produce destructive interference at the listener's location? (b) If *A* is moved even farther away than in part (a), at what distance *d* will the speakers next produce destructive interference at the listener's location? (c) After *A* starts moving away from its original spot, at what distance *d* will the speakers first produce constructive interference at the listener's location?

Figure **E16.32**



16.33 • Two loudspeakers, *A* and *B* (**Fig. E16.33**), are driven by the same amplifier and emit sinusoidal waves in phase. Speaker *B* is 2.00 m to the right of speaker *A*. Consider point *Q* along the extension of the line connecting the speakers, 1.00 m to the right of speaker *B*. Both speakers emit sound waves that travel directly from the speaker to point *Q*. What is the lowest frequency for which (a) *constructive* interference occurs at point *Q*; (b) *destructive* interference occurs at point *Q*?

Figure **E16.33**



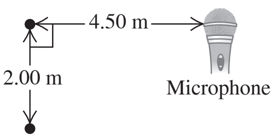
16.34 •• Two loudspeakers, *A* and *B* (see Fig. E16.33), are driven by the same amplifier and emit sinusoidal waves in phase. Speaker *B* is 2.00 m to the right of speaker *A*. The frequency of the sound waves produced by the loudspeakers is 206 Hz. Consider a point *P* between the speakers and along the line connecting them, a distance *x* to the right of *A*. Both speakers emit sound waves that travel directly from the speaker to point *P*. For what values of *x* will (a) *destructive* interference occur at *P*; (b) *constructive* interference occur at *P*? (c) Interference effects like those in parts (a) and (b) are almost never a factor in listening to home stereo equipment. Why not?

16.35 •• Two loudspeakers, *A* and *B*, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker *B* is 12.0 m to the right of speaker *A*. The frequency of the waves emitted by each speaker is 688 Hz. You are standing between the speakers, along the line connecting them, and are at a point of constructive interference. How far must you walk toward speaker *B* to move to a point of destructive interference?

16.36 • Two loudspeakers, *A* and *B*, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 172 Hz. You are 8.00 m from *A*. What is the closest you can be to *B* and be at a point of destructive interference?

16.37 •• Two small stereo speakers are driven in step by the same variable-frequency oscillator. Their sound is picked up by a microphone arranged as shown in **Fig. E16.37**. For what frequencies does their sound at the speakers produce (a) constructive interference and (b) destructive interference?

Figure **E16.37**



Section 16.7 Beats

16.38 •• Two guitarists attempt to play the same note of wavelength 64.8 cm at the same time, but one of the instruments is slightly out of tune and plays a note of wavelength 65.2 cm instead. What is the frequency of the beats these musicians hear when they play together?

16.39 •• **Tuning a Violin.** A violinist is tuning her instrument to concert A (440 Hz). She plays the note while listening to an electronically generated tone of exactly that frequency and hears a beat frequency of 3 Hz, which increases to 4 Hz when she tightens her violin string slightly. (a) What was the frequency of the note played by her violin when she heard the 3 Hz beats? (b) To get her violin perfectly tuned to concert A, should she tighten or loosen her string from what it was when she heard the 3 Hz beats?

16.40 •• Two organ pipes, open at one end but closed at the other, are each 1.14 m long. One is now lengthened by 2.00 cm. Find the beat frequency that they produce when playing together in their fundamentals.

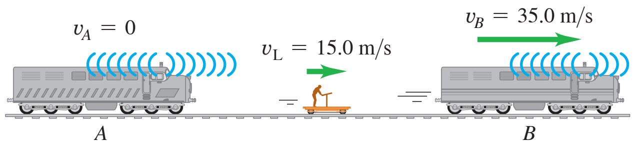
Section 16.8 The Doppler Effect

16.41 •• On the planet Arrakis a male ornithoid is flying toward his mate at 25.0 m/s while singing at a frequency of 1200 Hz. If the stationary female hears a tone of 1240 Hz, what is the speed of sound in the atmosphere of Arrakis?

16.42 • A railroad train is traveling at 25.0 m/s in still air. The frequency of the note emitted by the locomotive whistle is 400 Hz. What is the wavelength of the sound waves (a) in front of the locomotive and (b) behind the locomotive? What is the frequency of the sound heard by a stationary listener (c) in front of the locomotive and (d) behind the locomotive?

16.43 • Two train whistles, *A* and *B*, each have a frequency of 392 Hz. *A* is stationary and *B* is moving toward the right (away from *A*) at a speed of 35.0 m/s. A listener is between the two whistles and is moving toward the right with a speed of 15.0 m/s (**Fig. E16.43**). No wind is blowing. (a) What is the frequency from *A* as heard by the listener? (b) What is the frequency from *B* as heard by the listener? (c) What is the beat frequency detected by the listener?

Figure **E16.43**



16.44 • **Moving Source vs. Moving Listener.** (a) A sound source producing 1.00 kHz waves moves toward a stationary listener at one-half the speed of sound. What frequency will the listener hear? (b) Suppose instead that the source is stationary and the listener moves toward the source at one-half the speed of sound. What frequency does the listener hear? How does your answer compare to that in part (a)? Explain on physical grounds why the two answers differ.

16.45 • A swimming duck paddles the water with its feet once every 1.6 s, producing surface waves with this period. The duck is moving at constant speed in a pond where the speed of surface waves is 0.32 m/s, and the crests of the waves ahead of the duck are spaced 0.12 m apart. (a) What is the duck's speed? (b) How far apart are the crests behind the duck?

16.46 • A railroad train is traveling at 30.0 m/s in still air. The frequency of the note emitted by the train whistle is 352 Hz. What frequency is heard by a passenger on a train moving in the opposite direction to the first at 18.0 m/s and (a) approaching the first and (b) receding from the first?

16.47 • A car alarm is emitting sound waves of frequency 520 Hz. You are on a motorcycle, traveling directly away from the parked car. How fast must you be traveling if you detect a frequency of 490 Hz?

16.48 •• While sitting in your car by the side of a country road, you are approached by your friend, who happens to be in an identical car. You blow your car's horn, which has a frequency of 260 Hz. Your friend blows his car's horn, which is identical to yours, and you hear a beat frequency of 6.0 Hz. How fast is your friend approaching you?

16.49 • A police car is traveling due east at a speed of 15.0 m/s relative to the earth. You are in a convertible following behind the police car. Your car is also moving due east at 15.0 m/s relative to the earth, so the speed of the police car relative to you is zero. The siren of the police car is emitting sound of frequency 500 Hz. The speed of sound in the still air is 340 m/s. (a) What is the speed of the sound waves relative to you? (b) What is the wavelength of the sound waves at your location? (c) What frequency do you detect?

16.50 •• The siren of a fire engine that is driving northward at 30.0 m/s emits a sound of frequency 2000 Hz. A truck in front of this fire engine is moving northward at 20.0 m/s. (a) What is the frequency of the siren's sound that the fire engine's driver hears reflected from the back of the truck? (b) What wavelength would this driver measure for these reflected sound waves?

16.51 •• A stationary police car emits a sound of frequency 1200 Hz that bounces off a car on the highway and returns with a frequency of 1250 Hz. The police car is right next to the highway, so the moving car is traveling directly toward or away from it. (a) How fast was the moving car going? Was it moving toward or away from the police car? (b) What frequency would the police car have received if it had been traveling toward the other car at 20.0 m/s?

16.52 •• A stationary source emits sound waves of frequency f_s. There is no wind blowing. A device for detecting sound waves and measuring their observed frequency moves toward the source with speed v_L, and the observed frequency of the sound waves is f_L. The measurement is repeated for different values of v_L. You plot the results as f_L versus v_L and find that your data lie close to a straight line that has slope 1.75 m^{-1} and *y*-intercept 600.0 Hz. What are your experimental results for the speed of sound in the still air and for the frequency f_s of the source?

Section 16.9 Shock Waves

16.53 •• A jet plane flies overhead at Mach 1.70 and at a constant altitude of 1250 m. (a) What is the angle α of the shock-wave cone? (b) How much time after the plane passes directly overhead do you hear the sonic boom? Neglect the variation of the speed of sound with altitude.

16.54 • The shock-wave cone created by a space shuttle at one instant during its reentry into the atmosphere makes an angle of 58.0° with its direction of motion. The speed of sound at this altitude is 331 m/s. (a) What is the Mach number of the shuttle at this instant, and (b) how fast (in m/s and in mi/h) is it traveling relative to the atmosphere? (c) What would be its Mach number and the angle of its shock-wave cone if it flew at the same speed but at low altitude where the speed of sound is 344 m/s?

PROBLEMS

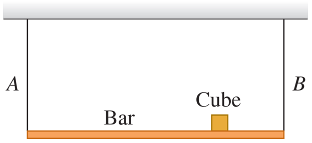
16.55 •• Use Eq. (16.10) and the information given in Example 16.4 to show that the speed of sound in air at 0°C is 332 m/s. (a) Using the first two terms of the power series expansion of $(1 + x)^n$ (see Appendix B), show that the speed of sound in air at Celsius temperature T_C is given approximately by $v = (332\text{ m/s})(1 + T_C/546)$. (b) Use the result in part (a) to calculate v at 20°C. Compare your result to the value given in Table 16.1. Is the expression in part (a) accurate at 20°C? (c) Do you expect the expression in part (a) to be accurate at 120°C? Explain. (*Hint:* Compare the second and third terms in the power series expansion.)

16.56 •• **CP** The sound from a trumpet radiates uniformly in all directions in 20°C air. At a distance of 5.00 m from the trumpet the sound intensity level is 52.0 dB. The frequency is 587 Hz. (a) What is the pressure amplitude at this distance? (b) What is the displacement amplitude? (c) At what distance is the sound intensity level 30.0 dB?

16.57 ••• **CALC** The air temperature over a lake decreases linearly with height after sunset, since air cools faster than water. (a) If the temperature at the surface is 25.00°C and the temperature at a height of 300.0 m is 5.000°C, how long does it take sound to rise 300.0 m directly upward? [*Hint:* Use Eq. (16.10) and integrate.] (b) At a height of 300.0 m, how far does sound travel horizontally in this same time interval? The change in wave speed with altitude due to the nocturnal temperature inversion over the lake gives rise to a change in direction of the sound waves. This phenomenon is called refraction and will be discussed in detail for electromagnetic waves in Chapter 33.

16.58 •• **CP** A uniform 165 N bar is supported horizontally by two identical wires A and B (**Fig. P16.58**). A small 185 N cube of lead is placed three-fourths of the way from A to B. The wires are each 75.0 cm long and have a mass of 5.50 g. If both of them are simultaneously plucked at the center, what is the frequency of the beats that they will produce when vibrating in their fundamental?

Figure **P16.58**

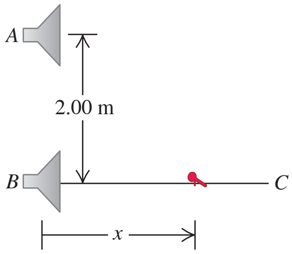


16.59 • An organ pipe has two successive harmonics with frequencies 1372 and 1764 Hz. (a) Is this an open or a stopped pipe? Explain. (b) What two harmonics are these? (c) What is the length of the pipe?

16.60 •• A Kundt's tube is filled with helium gas. The speed of sound for helium at 20°C is given in Table 16.1. A tuning fork that produces sound waves with frequency 1200 Hz is used to set up standing waves inside the tube. You measure the node-to-node distance to be 47.0 cm. What is the temperature of the helium gas in the tube?

16.61 •• Two identical loudspeakers are located at points A and B, 2.00 m apart. The loudspeakers are driven by the same amplifier and produce sound waves with a frequency of 784 Hz. Take the speed of sound in air to be 344 m/s. A small microphone is moved out from point B along a line perpendicular to the line connecting A and B (line BC in **Fig. P16.61**). (a) At what distances from B will there be *destructive* interference? (b) At what distances from B will there be *constructive* interference? (c) If the frequency is made low enough, there will be no positions along the line BC at which destructive interference occurs. How low must the frequency be for this to be the case?

Figure **P16.61**



16.62 •• A bat flies toward a wall, emitting a steady sound of frequency 1.70 kHz. This bat hears its own sound plus the sound reflected by the wall. How fast should the bat fly in order to hear a beat frequency of 8.00 Hz?

16.63 •• The sound source of a ship's sonar system operates at a frequency of 18.0 kHz. The speed of sound in water (assumed to be at a uniform 20°C) is 1482 m/s. (a) What is the wavelength of the waves emitted by the source? (b) What is the difference in frequency between the directly radiated waves and the waves reflected from a whale traveling directly toward the ship at 4.95 m/s? The ship is at rest in the water.

16.64 •• Consider a thunderstorm with a flash of lightning followed by a crash of thunder. (a) Estimate the time delay between the lightning flash and the sound of the thunder. (b) Determine the distance sound travels in that time, which provides an estimate of the distance to the lightning flash. (c) By comparing to the sounds listed in Table 16.2, estimate for your location the average intensity level of the sound in decibels and calculate the intensity in W/m^2. (d) Assume the sound was generated at the site of the lightning flash and assume the sound was transmitted uniformly in all directions. Use the estimated distance to estimate the average sound power generated by the thunder. (e) Estimate the duration of the thunderclap. Multiply your estimate by the average power to determine the sound energy released by a lightning strike.

16.65 ••• **CP** Suppose that you are at a bowling alley. (a) By comparing to the sounds listed in Table 16.2, for your location estimate the sound intensity level in decibels and the intensity in W/m^2 of the crashing pins after a well-executed strike. (b) Using 18 m as the length of a bowling alley, determine the average power associated with that sound. (c) Estimate the duration of the crashing sound. Multiply your estimate by the average power to obtain the sound energy released in the strike. (d) Estimate the time it took the bowling ball to travel down the alley prior to the strike. Use that time to estimate the speed of the ball. (e) Assume the ball has a mass of 6.4 kg. Account for both translational and rotational motions to estimate the kinetic energy of the bowling ball immediately prior to the strike. (f) What fraction of the ball's energy was converted to sound?

16.66 ••• **BIO** **Ultrasound in Medicine.** A 2.00 MHz sound wave travels through a pregnant woman's abdomen and is reflected from the fetal heart wall of her unborn baby. The heart wall is moving toward the sound receiver as the heart beats. The reflected sound is then mixed with the transmitted sound, and 72 beats per second are detected. The speed of sound in body tissue is 1500 m/s. Calculate the speed of the fetal heart wall at the instant this measurement is made.

16.67 ••• **BIO** Horseshoe bats (genus *Rhinolophus*) emit sounds from their nostrils and then listen to the frequency of the sound reflected from their prey to determine the prey's speed. (The "horseshoe" that gives the bat its name is a depression around the nostrils that acts like a focusing mirror, so that the bat emits sound in a narrow beam like a flashlight.) A *Rhinolophus* flying at speed v_{bat} emits sound of frequency f_{bat}; the sound it hears reflected from an insect flying toward it has a higher frequency f_{refl}. (a) Show that the speed of the insect is

$$v_{\text{insect}} = v\left[\frac{f_{\text{refl}}(v - v_{\text{bat}}) - f_{\text{bat}}(v + v_{\text{bat}})}{f_{\text{refl}}(v - v_{\text{bat}}) + f_{\text{bat}}(v + v_{\text{bat}})}\right]$$

where v is the speed of sound. (b) If $f_{\text{bat}} = 80.7$ kHz, $f_{\text{refl}} = 83.5$ kHz, and $v_{\text{bat}} = 3.9$ m/s, calculate the speed of the insect.

16.68 • **CP** A police siren of frequency f_{siren} is attached to a vibrating platform. The platform and siren oscillate up and down in simple harmonic motion with amplitude A_p and frequency f_p. (a) Find the maximum and minimum sound frequencies that you would hear at a position directly above the siren. (b) At what point in the motion of the platform is the maximum frequency heard? The minimum frequency? Explain.

16.69 •• **CP** A turntable 1.50 m in diameter rotates at 75 rpm. Two speakers, each giving off sound of wavelength 31.3 cm, are attached to the rim of the table at opposite ends of a diameter. A listener stands in front of the turntable. (a) What is the greatest beat frequency the listener will receive from this system? (b) Will the listener be able to distinguish individual beats?

16.70 •• **DATA** A long, closed cylindrical tank contains a diatomic gas that is maintained at a uniform temperature that can be varied. When you measure the speed of sound v in the gas as a function of the temperature T of the gas, you obtain these results:

***T* (°C)**	−20.0	0.0	20.0	40.0	60.0	80.0
***v* (m/s)**	324	337	349	361	372	383

(a) Explain how you can plot these results so that the graph will be well fit by a straight line. Construct this graph and verify that the plotted points do lie close to a straight line. (b) Because the gas is diatomic, $\gamma = 1.40$. Use the slope of the line in part (a) to calculate M, the molar mass of the gas. Express M in grams/imole. What type of gas is in the tank?

16.71 •• **DATA** A long tube contains air at a pressure of 1.00 atm and a temperature of 77.0°C. The tube is open at one end and closed at the other by a movable piston. A tuning fork that vibrates with a frequency of 500 Hz is placed near the open end. Resonance is produced when the piston is at distances 18.0 cm, 55.5 cm, and 93.0 cm from the open end. (a) From these values, what is the speed of sound in air at 77.0°C? (b) From the result of part (a), what is the value of γ? (c) These results show that a displacement antinode is slightly outside the open end of the tube. How far outside is it?

16.72 ••• **DATA** **Supernova!** (a) Equation (16.30) can be written as

$$f_R = f_S\left(1 - \frac{v}{c}\right)^{1/2}\left(1 + \frac{v}{c}\right)^{-1/2}$$

where c is the speed of light in vacuum, 3.00×10^8 m/s. Most objects move much slower than this (v/c is very small), so calculations made with Eq. (16.30) must be done carefully to avoid rounding errors. Use the binomial theorem to show that if $v \ll c$, Eq. (16.30) approximately reduces to $f_R = f_S[1 - (v/c)]$. (b) The gas cloud known as the Crab Nebula can be seen with even a small telescope. It is the remnant of a *supernova,* a cataclysmic explosion of a star. (The explosion was seen on the earth on July 4, 1054 C.E.) Its streamers glow with the characteristic red color of heated hydrogen gas. In a laboratory on the earth, heated hydrogen produces red light with frequency 4.568×10^{14} Hz; the red light received from streamers in the Crab Nebula that are pointed toward the earth has frequency 4.586×10^{14} Hz. Estimate the speed with which the outer edges of the Crab Nebula are expanding. Assume that the speed of the center of the nebula relative to the earth is negligible. (c) Assuming that the expansion speed of the Crab Nebula has been constant since the supernova that produced it, estimate the diameter of the Crab Nebula. Give your answer in meters and in light-years. (d) The angular diameter of the Crab Nebula as seen from the earth is about 5 arc-minutes $\left(1 \text{ arc-minute} = \frac{1}{60} \text{ degree}\right)$. Estimate the distance (in light-years) to the Crab Nebula, and estimate the year in which the supernova actually took place.

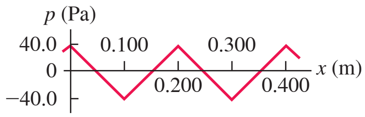
16.73 ••• **CP** A one-string Aeolian harp is constructed by attaching the lower end of a 1.56-m-long rigid and uniform pole with a mass of 8.00 kg to a tree at a pivot and then using a light rope to hang a 39.0-cm-long hollow steel tube with a mass of 4.00 kg from its upper end. A horizontal wire between the pole and the tree is attached to the pole at a height h above the pivot and holds the pole at a 45.0° angle with the vertical. The wire is uniform and has linear mass density $\mu = 1.40$ g/m. When the wind blows, the wire resonates at its fundamental frequency. The height h is chosen such that the sound emitted by the wire stimulates the fundamental standing wave in the tube, which harmonizes with the resonating wire. (a) What is the frequency of the note produced by this instrument? (b) At what height h should the wire be placed? (*Hint:* The pole is in static equilibrium, so the net torque on it is zero and this determines the tension in the wire as a function of h.)

16.74 ••• Two powerful speakers, separated by 15.00 m, stand on the floor in front of the stage in a large amphitheater. An aisle perpendicular to the stage is directly in front of one of the speakers and extends 50.00 m to an exit door at the back of the amphitheater. (a) If the speakers produce in-phase, coherent 440 Hz tones, at how many points along the aisle is the sound minimal? (b) What is the distance between the farthest such point and the door at the back of the aisle? (c) Suppose the coherent sound emitted from both speakers is a linear superposition of a 440 Hz tone and another tone with frequency f. What is the smallest value of f so that minimal sound is heard at any point where the 440 Hz sound is minimal? (d) At how many additional points in the aisle is the 440 Hz tone present but the second tone is minimal? (e) What is the distance from the closest of these points to the speaker at the front of the aisle?

CHALLENGE PROBLEMS

16.75 ••• **CALC** **Figure P16.75** shows the pressure fluctuation p of a nonsinusoidal sound wave as a function of x for $t = 0$. The wave is traveling in the $+x$-direction. (a) Graph the pressure fluctuation p as a function of t for $x = 0$. Show at least two cycles of oscillation. (b) Graph the displacement y in this sound wave as a function of x at $t = 0$. At $x = 0$, the displacement at $t = 0$ is zero. Show at least two wavelengths of the wave. (c) Graph the displacement y as a function of t for $x = 0$. Show at least two cycles of oscillation. (d) Calculate the maximum velocity and the maximum acceleration of an element of the air through which this sound wave is traveling. (e) Describe how the cone of a loudspeaker must move as a function of time to produce the sound wave in this problem.

Figure **P16.75**



16.76 ••• CP Longitudinal Waves on a Spring. A long spring such as a Slinky™ is often used to demonstrate longitudinal waves. (a) Show that if a spring that obeys Hooke's law has mass m, length L, and force constant k', the speed of longitudinal waves on the spring is $v = L\sqrt{k'/m}$ (see Section 16.2). (b) Evaluate v for a spring with $m = 0.250$ kg, $L = 2.00$ m, and $k' = 1.50$ N/m.

MCAT-STYLE PASSAGE PROBLEMS

BIO Ultrasound Imaging. A typical ultrasound transducer used for medical diagnosis produces a beam of ultrasound with a frequency of 1.0 MHz. The beam travels from the transducer through tissue and partially reflects when it encounters different structures in the tissue. The same transducer that produces the ultrasound also detects the reflections. The transducer emits a short pulse of ultrasound and waits to receive the reflected echoes before emitting the next pulse. By measuring the time between the initial pulse and the arrival of the reflected signal, we can use the speed of ultrasound in tissue, 1540 m/s, to determine the distance from the transducer to the structure that produced the reflection.

As the ultrasound beam passes through tissue, the beam is attenuated through absorption. Thus deeper structures return weaker echoes. A typical attenuation in tissue is −100 dB/m · MHz; in bone it is −500 dB/m · MHz. In determining attenuation, we take the reference intensity to be the intensity produced by the transducer.

16.77 If the deepest structure you wish to image is 10.0 cm from the transducer, what is the maximum number of pulses per second that can be emitted? (a) 3850; (b) 7700; (c) 15,400; (d) 1,000,000.

16.78 After a beam passes through 10 cm of tissue, what is the beam's intensity as a fraction of its initial intensity from the transducer? (a) 1×10^{-11}; (b) 0.001; (c) 0.01; (d) 0.1.

16.79 Because the speed of ultrasound in bone is about twice the speed in soft tissue, the distance to a structure that lies beyond a bone can be measured incorrectly. If a beam passes through 4 cm of tissue, then 2 cm of bone, and then another 1 cm of tissue before echoing off a cyst and returning to the transducer, what is the difference between the true distance to the cyst and the distance that is measured by assuming the speed is always 1540 m/s? Compared with the measured distance, the structure is actually (a) 1 cm farther; (b) 2 cm farther; (c) 1 cm closer; (d) 2 cm closer.

16.80 In some applications of ultrasound, such as its use on cranial tissues, large reflections from the surrounding bones can produce standing waves. This is of concern because the large pressure amplitude in an antinode can damage tissues. For a frequency of 1.0 MHz, what is the distance between antinodes in tissue? (a) 0.38 mm; (b) 0.75 mm; (c) 1.5 mm; (d) 3.0 mm.

16.81 For cranial ultrasound, why is it advantageous to use frequencies in the kHZ range rather than the MHz range? (a) The antinodes of the standing waves will be closer together at the lower frequencies than at the higher frequencies; (b) there will be no standing waves at the lower frequencies; (c) cranial bones will attenuate the ultrasound more at the lower frequencies than at the higher frequencies; (d) cranial bones will attenuate the ultrasound less at the lower frequencies than at the higher frequencies.

ANSWERS

Chapter Opening Question ?

(iv) Equation (16.10) in Section 16.2 says that the speed of sound in a gas depends on the temperature and on the kind of gas (through the ratio of heat capacities and the molar mass). Winter air in the mountains has a lower temperature than summer air at sea level, but they have essentially the same composition. Hence the lower temperature alone explains the slower speed of sound in winter in the mountains.

Key Example √ARIATION Problems

VP16.9.1 (a) $p_{\max} = 6.74 \times 10^{-3}$ Pa (b) $p_{\max}$ is unchanged
VP16.9.2 (a) 3.16×10^{-4} W/m^2 (b) 63.1
VP16.9.3 (a) 102 dB (b) 90 dB
VP16.9.4 (a) 5.02×10^{-2} Pa (b) 86.0 dB
VP16.12.1 (a) 0.782 m (b) 0.130 m
VP16.12.2 (a) $n = 1$ (the fundamental frequency) (b) $n = 3$ (the third harmonic) (c) none
VP16.12.3 (a) 224 m/s (b) 0.521 m
VP16.12.4 (a) $f = 749$ Hz, $\lambda = 1.33$ m (b) $f = 749$ Hz, $\lambda = 0.459$ m (c) 1.15 m
VP16.18.1 (a) $f = 3.03 \times 10^3$ Hz, $\lambda = 0.112$ m (b) $f = 2.60 \times 10^3$ Hz, $\lambda = 0.131$ m
VP16.18.2 (a) $\lambda = 0.773$ m (b) $f = 453$ Hz, $\lambda = 0.773$ m (c) $f = 427$ Hz, $\lambda = 0.773$ m
VP16.18.3 (a) 1.22×10^3 Hz (b) 1.50×10^3 Hz
VP16.18.4 (a) 481 Hz (b) 29.9 m/s

Bridging Problem

(a) $180° = \pi$ rad
(b) A alone: $I = 3.98 \times 10^{-6}$ W/m^2, $\beta = 66.0$ dB;
B alone: $I = 5.31 \times 10^{-7}$ W/m^2, $\beta = 57.2$ dB
(c) $I = 1.60 \times 10^{-6}$ W/m^2, $\beta = 62.1$ dB