? In one type of welding, electric charge flows between the welding tool and the metal pieces that are to be joined. This produces a glowing arc whose high temperature fuses the pieces together. Why must the tool be held close to the metal pieces? (i) To maximize the potential difference between tool and pieces; (ii) to minimize this potential difference; (iii) to maximize the electric field between tool and pieces; (iv) to minimize this electric field; (v) more than one of these.

23 Electric Potential

LEARNING OUTCOMES

In this chapter, you'll learn...

23.1 How to calculate the electric potential energy of a collection of charges.

23.2 The meaning and significance of electric potential.

23.3 How to calculate the electric potential that a collection of charges produces at a point in space.

23.4 How to use equipotential surfaces to visualize how the electric potential varies in space.

23.5 How to use electric potential to calculate the electric field.

You'll need to review...

7.1–7.4 Conservative forces and potential energy.

21.1–21.6 Electric force and electric fields.

22.4, 22.5 Applications of Gauss's law.

This chapter is about energy associated with electrical interactions. Every time you turn on a light, use a mobile phone, or make toast in a toaster, you are using electrical energy, an indispensable ingredient of our technological society. In Chapters 6 and 7 we introduced the concepts of *work* and *energy* in the context of mechanics; now we'll combine these concepts with what we've learned about electric charge, electric forces, and electric fields. Just as we found for many problems in mechanics, using energy ideas makes it easier to solve a variety of problems in electricity.

When a charged particle moves in an electric field, the field exerts a force that can do *work* on the particle. This work can be expressed in terms of electric potential energy. Just as gravitational potential energy depends on the height of a mass above the earth's surface, electric potential energy depends on the position of the charged particle in the electric field. We'll use a new concept called *electric potential,* or simply *potential* to describe electric potential energy. In circuits, a difference in potential from one point to another is often called *voltage*. The concepts of potential and voltage are crucial to understanding how electric circuits work and have equally important applications to electron beams used in cancer radiotherapy, high-energy particle accelerators, and many other devices.

23.1 ELECTRIC POTENTIAL ENERGY

The concepts of work, potential energy, and conservation of energy proved to be extremely useful in our study of mechanics. In this section we'll show that these concepts are just as useful for understanding and analyzing electrical interactions.

Let's begin by reviewing three essential points from Chapters 6 and 7. First, when a force $\vec{F}$ acts on a particle that moves from point a to point b, the work $W_{a\to b}$ done by the force is given by a *line integral:*

$$W_{a\to b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F\cos\phi\, dl \qquad \text{(work done by a force)} \qquad (23.1)$$

where $d\vec{l}$ is an infinitesimal displacement along the particle's path and ϕ is the angle between $\vec{F}$ and $d\vec{l}$ at each point along the path.

Second, if the force $\vec{F}$ is *conservative*, as we defined the term in Section 7.3, the work done by $\vec{F}$ can always be expressed in terms of a **potential energy** U. When the particle moves from a point where the potential energy is U_a to a point where it is U_b, the change in potential energy is $\Delta U = U_b - U_a$ and

Work done by a conservative force

Potential energy at initial position

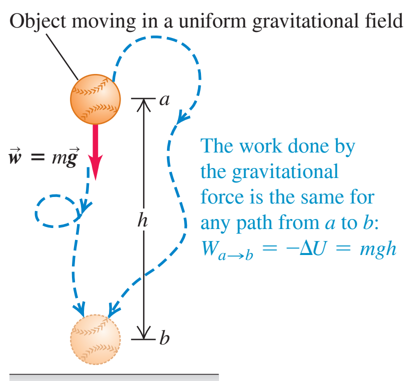
$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U \tag{23.2}$$

Potential energy at final position

Negative of change in potential energy

When $W_{a\to b}$ is positive, U_a is greater than U_b, ΔU is negative, and the potential energy *decreases*. That's what happens when a baseball falls from a high point (a) to a lower point (b) under the influence of the earth's gravity; the force of gravity does positive work, and the gravitational potential energy decreases (**Fig. 23.1**). When a tossed ball is moving upward, the gravitational force does negative work during the ascent, and the potential energy increases.

Figure 23.1 The work done on a baseball moving in a uniform gravitational field.



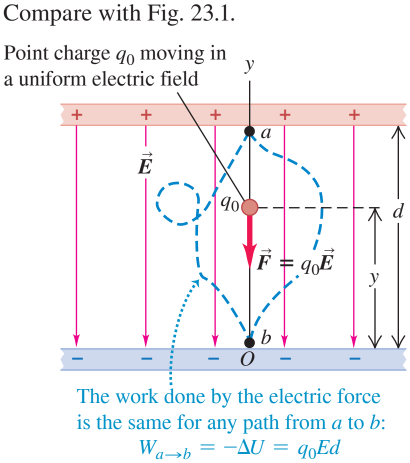
Third, the work–energy theorem says that the change in kinetic energy $\Delta K = K_b - K_a$ during a displacement equals the *total* work done on the particle. If only conservative forces do work, then Eq. (23.2) gives the total work, and $K_b - K_a = -(U_b - U_a)$. We usually write this as

$$K_a + U_a = K_b + U_b \tag{23.3}$$

That is, the total mechanical energy (kinetic plus potential) is *conserved* under these circumstances.

Electric Potential Energy in a Uniform Field

Figure 23.2 The work done on a point charge moving in a uniform electric field. Compare with Fig. 23.1.



Let's look at an electrical example of these concepts. In **Fig. 23.2** a pair of large, charged, parallel metal plates sets up a uniform, downward electric field with magnitude E. The field exerts a downward force with magnitude $F = q_0E$ on a positive test charge q_0. As the charge moves downward a distance d from point a to point b, the force on the test charge is constant and independent of its location. So the work done by the electric field is the product of the force magnitude and the component of displacement in the (downward) direction of the force:

$$W_{a\to b} = Fd = q_0Ed \tag{23.4}$$

This work is positive, since the force is in the same direction as the net displacement of the test charge.

The y-component of the electric force, $F_y = -q_0E$, is constant, and there is no x- or z-component. This is exactly analogous to the gravitational force on a mass m near the earth's surface; for this force, there is a constant y-component $F_y = -mg$ and the x- and z-components are zero. Because of this analogy, we can conclude that the force exerted on q_0 by the uniform electric field in Fig. 23.2 is *conservative,* just as is the gravitational force. This means that the work $W_{a\to b}$ done by the field is independent of the path the particle takes from a to b. We can represent this work with a *potential-energy* function U, just as we did for gravitational potential energy in Section 7.1. The potential energy for the gravitational force $F_y = -mg$ was $U = mgy$; hence the potential energy for the electric force $F_y = -q_0E$ is

$$U = q_0Ey \tag{23.5}$$

When the test charge moves from height y_a to height y_b, the work done on the charge by the field is given by

$$W_{a\to b} = -\Delta U = -(U_b - U_a) = -(q_0Ey_b - q_0Ey_a) = q_0E(y_a - y_b) \tag{23.6}$$

Figure 23.3 A positive charge moving (a) in the direction of the electric field $\vec{E}$ and (b) in the direction opposite $\vec{E}$.

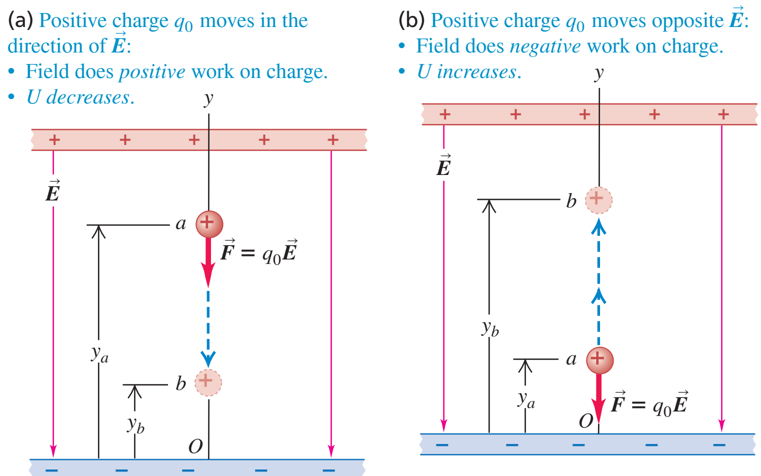
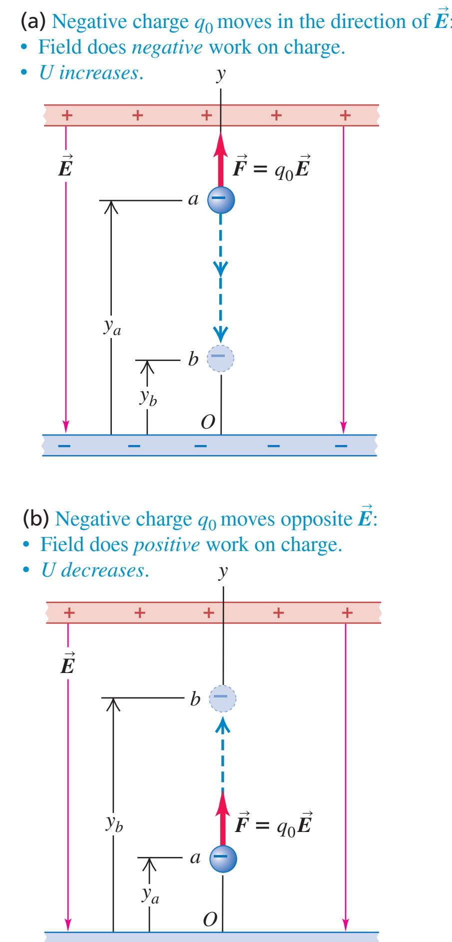


Figure 23.4 A negative charge moving (a) in the direction of the electric field $\vec{E}$ and (b) in the direction opposite $\vec{E}$. Compare with Fig. 23.3.



When y_a is greater than y_b (**Fig. 23.3a**), the positive test charge q_0 moves downward, in the same direction as $\vec{E}$; the displacement is in the same direction as the force $\vec{F} = q_0\vec{E}$, so the field does positive work and U decreases. [In particular, if $y_a - y_b = d$ as in Fig. 23.2, Eq. (23.6) gives $W_{a\to b} = q_0Ed$, in agreement with Eq. (23.4).] When y_a is less than y_b (Fig. 23.3b), the positive test charge q_0 moves upward, in the opposite direction to $\vec{E}$; the displacement is opposite the force, the field does negative work, and U increases.

If the test charge q_0 is *negative,* the potential energy increases when it moves with the field and decreases when it moves against the field (**Fig. 23.4**).

Whether the test charge is positive or negative, the following general rules apply: *U increases* if the test charge q_0 moves in the direction *opposite* the electric force $\vec{F} = q_0\vec{E}$ (Figs. 23.3b and 23.4a); *U decreases* if q_0 moves in the *same* direction as $\vec{F} = q_0\vec{E}$ (Figs. 23.3a and 23.4b). This is the same behavior as for gravitational potential energy, which increases if a mass m moves upward (opposite the direction of the gravitational force) and decreases if m moves downward (in the same direction as the gravitational force).

CAUTION **Electric potential energy** The relationship between electric potential energy change and motion in an electric field is an important one that we'll use often, but that takes some effort to understand. Studying the preceding paragraph as well as Figs. 23.3 and Fig. 23.4 carefully now will help you tremendously later!

Electric Potential Energy of Two Point Charges

The idea of electric potential energy isn't restricted to the special case of a uniform electric field. Indeed, we can apply this concept to a point charge in *any* electric field caused by a static charge distribution. Recall from Chapter 21 that we can represent any charge distribution as a collection of point charges. Therefore it's useful to calculate the work done on a test charge q_0 moving in the electric field caused by a single, stationary point charge q.

We'll consider first a displacement along the *radial* line in **Fig. 23.5**. The force on q_0 is given by Coulomb's law, and its radial component is

$$F_r = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2} \tag{23.7}$$

If q and q_0 have the same sign $(+ \text{ or } -)$, the force is repulsive and F_r is positive; if the two charges have opposite signs, the force is attractive and F_r is negative. The force is *not*

Figure 23.5 Test charge q_0 moves along a straight line extending radially from charge q. As it moves from a to b, the distance varies from r_a to r_b.

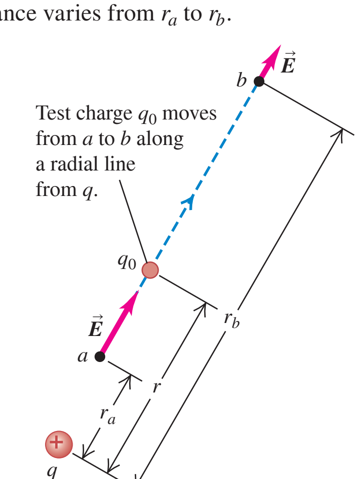
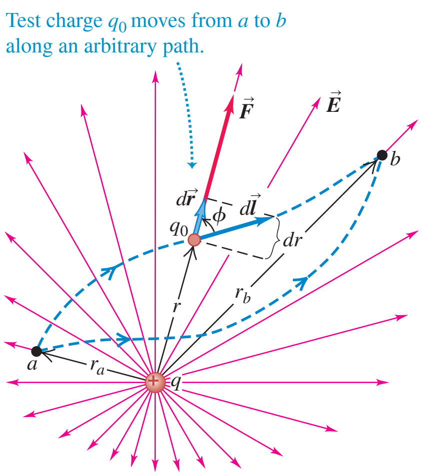


Figure **23.6** The work done on charge q_0 by the electric field of charge q does not depend on the path taken, but only on the distances r_a and r_b.



constant during the displacement, and we must integrate to calculate the work $W_{a\rightarrow b}$ done on q_0 by this force as q_0 moves from a to b:

$$W_{a\rightarrow b} = \int_{r_a}^{r_b} F_r\,dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}dr = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) \qquad (23.8)$$

The work done by the electric force for this path depends on only the endpoints.

Now let's consider a more general displacement (**Fig. 23.6**) in which a and b do *not* lie on the same radial line. From Eq. (23.1) the work done on q_0 during this displacement is given by

$$W_{a\rightarrow b} = \int_{r_a}^{r_b} F\cos\phi\,dl = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\cos\phi\,dl$$

But Fig. 23.6 shows that $\cos\phi\,dl = dr$. That is, the work done during a small displacement $d\vec{l}$ depends only on the change dr in the distance r between the charges, which is the *radial component* of the displacement. Thus Eq. (23.8) is valid even for this more general displacement; the work done on q_0 by the electric field $\vec{E}$ produced by q depends only on r_a and r_b, not on the details of the path. Also, if q_0 returns to its starting point a by a different path, the total work done in the round-trip displacement is zero (the integral in Eq. (23.8) is from r_a back to r_a). These are the needed characteristics for a conservative force, as we defined it in Section 7.3. Thus the force on q_0 is a *conservative* force.

We see that Eqs. (23.2) and (23.8) are consistent if we define the potential energy to be $U_a = qq_0/4\pi\epsilon_0 r_a$ when q_0 is a distance r_a from q, and to be $U_b = qq_0/4\pi\epsilon_0 r_b$ when q_0 is a distance r_b from q. Thus

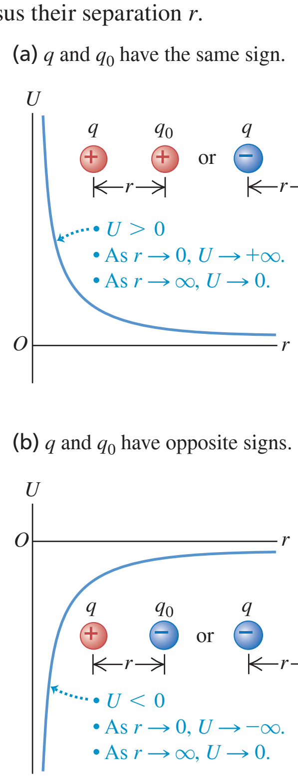
Electric potential energy of two point charges

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \qquad (23.9)$$

Values of two charges
Distance between two charges
Electric constant

Equation (23.9) is valid no matter what the signs of the charges q and q_0. The potential energy is positive if the charges q and q_0 have the same sign (**Fig. 23.7a**) and negative if they have opposite signs (Fig. 23.7b).

Figure **23.7** Graphs of the potential energy U of two point charges q and q_0 versus their separation r.



CAUTION **Electric potential energy vs. electric force** Don't confuse Eq. (23.9) for the potential energy of two point charges with Eq. (23.7) for the radial component of the electric force that one charge exerts on the other. Potential energy U is proportional to $1/r$, while the force component F_r is proportional to $1/r^2$.

Potential energy is always defined relative to some reference point where $U = 0$. In Eq. (23.9), U is zero when q and q_0 are infinitely far apart and $r = \infty$. Therefore U represents the work that would be done on the test charge q_0 by the field of q if q_0 moved from an initial distance r to infinity. If q and q_0 have the same sign, the interaction is repulsive, this work is positive, and U is positive at any finite separation (Fig. 23.7a). If the charges have opposite signs, the interaction is attractive, the work done is negative, and U is negative (Fig. 23.7b).

We emphasize that the potential energy U given by Eq. (23.9) is a *shared* property of the two charges. If the distance between q and q_0 is changed from r_a to r_b, the change in potential energy is the same whether q is held fixed and q_0 is moved or q_0 is held fixed and q is moved. For this reason, we never use the phrase "the electric potential energy *of* a point charge." (Likewise, if a mass m is at a height h above the earth's surface, the gravitational potential energy is a shared property of the mass m and the earth. We emphasized this in Sections 7.1 and 13.3.)

Equation (23.9) also holds if the charge q_0 is outside a spherically symmetric charge *distribution* with total charge q; the distance r is from q_0 to the center of the distribution. That's because Gauss's law tells us that the electric field outside such a distribution is the same as if all of its charge q were concentrated at its center (see Example 22.9 in Section 22.4).

EXAMPLE 23.1 Conservation of energy with electric forces

WITH VARIATION PROBLEMS

A positron (the electron's antiparticle) has mass 9.11×10^{-31} kg and charge $q_0 = +e = +1.60 \times 10^{-19}$ C. Suppose a positron moves in the vicinity of an α (alpha) particle, which has charge $q = +2e = 3.20 \times 10^{-19}$ C and mass 6.64×10^{-27} kg. The α particle's mass is more than 7000 times that of the positron, so we assume that the α particle remains at rest. When the positron is 1.00×10^{-10} m from the α particle, it is moving directly away from the α particle at 3.00×10^6 m/s. (a) What is the positron's speed when the particles are 2.00×10^{-10} m apart? (b) What is the positron's speed when it is very far from the α particle? (c) Suppose the initial conditions are the same but the moving particle is an electron (with the same mass as the positron but charge $q_0 = -e$). Describe the subsequent motion.

IDENTIFY and SET UP The electric force between a positron (or an electron) and an α particle is conservative, so the total mechanical energy (kinetic plus potential) is conserved. Equation (23.9) gives the potential energy U at any separation r: The potential-energy function for parts (a) and (b) looks like that of Fig. 23.7a, and the function for part (c) looks like that of Fig. 23.7b. We are given the positron speed $v_a = 3.00 \times 10^6$ m/s when the separation between the particles is $r_a = 1.00 \times 10^{-10}$ m. In parts (a) and (b) we use Eqs. (23.3) and (23.9) to find the speed for $r = r_b = 2.00 \times 10^{-10}$ m and $r = r_c \rightarrow \infty$, respectively. In part (c) we replace the positron with an electron and reconsider the problem.

EXECUTE (a) Both particles have positive charge, so the positron speeds up as it moves away from the α particle. From the energy-conservation equation, Eq. (23.3), the final kinetic energy is

$$K_b = \tfrac{1}{2}mv_b{}^2 = K_a + U_a - U_b$$

In this expression,

$$\begin{aligned}
K_a &= \tfrac{1}{2}mv_a{}^2 = \tfrac{1}{2}(9.11 \times 10^{-31}\text{ kg})(3.00 \times 10^6\text{ m/s})^2 \\
&= 4.10 \times 10^{-18}\text{ J} \\
U_a &= \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r_a} = (9.0 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2) \\
&\quad\times \frac{(3.20 \times 10^{-19}\text{ C})(1.60 \times 10^{-19}\text{ C})}{1.00 \times 10^{-10}\text{ m}} \\
&= 4.61 \times 10^{-18}\text{ J} \\
U_b &= \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r_b} = 2.30 \times 10^{-18}\text{ J}
\end{aligned}$$

Hence the positron kinetic energy and speed at $r = r_b = 2.00 \times 10^{-10}$ m are

$$\begin{aligned}
K_b &= \tfrac{1}{2}mv_b{}^2 \\
&= (4.10 \times 10^{-18}\text{ J}) + (4.61 \times 10^{-18}\text{ J}) - (2.30 \times 10^{-18}\text{ J}) \\
&= 6.41 \times 10^{-18}\text{ J} \\
v_b &= \sqrt{\frac{2K_b}{m}} = \sqrt{\frac{2(6.41 \times 10^{-18}\text{ J})}{9.11 \times 10^{-31}\text{ kg}}} \\
&= 3.8 \times 10^6\text{ m/s}
\end{aligned}$$

(b) When the positron and α particle are very far apart so that $r = r_c \rightarrow \infty$, the final potential energy U_c approaches zero. Again from energy conservation, the final kinetic energy and speed of the positron in this case are

$$\begin{aligned}
K_c &= K_a + U_a - U_c = (4.10 \times 10^{-18}\text{ J}) + (4.61 \times 10^{-18}\text{ J}) - 0 \\
&= 8.71 \times 10^{-18}\text{ J} \\
v_c &= \sqrt{\frac{2K_c}{m}} = \sqrt{\frac{2(8.71 \times 10^{-18}\text{ J})}{9.11 \times 10^{-31}\text{ kg}}} = 4.4 \times 10^6\text{ m/s}
\end{aligned}$$

(c) The electron and α particle have opposite charges, so the force is attractive and the electron slows down as it moves away. Changing the moving particle's sign from $+e$ to $-e$ means that the initial potential energy is now $U_a = -4.61 \times 10^{-18}$ J, which makes the total mechanical energy *negative*:

$$\begin{aligned}
K_a + U_a &= (4.10 \times 10^{-18}\text{ J}) - (4.61 \times 10^{-18}\text{ J}) \\
&= -0.51 \times 10^{-18}\text{ J}
\end{aligned}$$

The total mechanical energy would have to be positive for the electron to move infinitely far away from the α particle. Like a rock thrown upward at low speed from the earth's surface, it will reach a maximum separation $r = r_d$ from the α particle before reversing direction. At this point its speed and its kinetic energy K_d are zero, so at separation r_d we have

$$\begin{aligned}
U_d &= K_a + U_a - K_d = (-0.51 \times 10^{-18}\text{ J}) - 0 \\
U_d &= \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r_d} = -0.51 \times 10^{-18}\text{ J} \\
r_d &= \frac{1}{U_d}\frac{qq_0}{4\pi\epsilon_0} \\
&= \frac{(9.0 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}{-0.51 \times 10^{-18}\text{ J}}(3.20 \times 10^{-19}\text{ C})(-1.60 \times 10^{-19}\text{ C}) \\
&= 9.0 \times 10^{-10}\text{ m}
\end{aligned}$$

For $r_b = 2.00 \times 10^{-10}$ m we have $U_b = -2.30 \times 10^{-18}$ J, so the electron kinetic energy and speed at this point are

$$\begin{aligned}
K_b &= \tfrac{1}{2}mv_b{}^2 \\
&= (4.10 \times 10^{-18}\text{ J}) + (-4.61 \times 10^{-18}\text{ J}) - (-2.30 \times 10^{-18}\text{ J}) \\
&= 1.79 \times 10^{-18}\text{ J} \\
v_b &= \sqrt{\frac{2K_b}{m}} = \sqrt{\frac{2(1.79 \times 10^{-18}\text{ J})}{9.11 \times 10^{-31}\text{ kg}}} = 2.0 \times 10^6\text{ m/s}
\end{aligned}$$

EVALUATE Both particles behave as expected as they move away from the α particle: The positron speeds up, and the electron slows down and eventually turns around. How fast would an electron have to be moving at $r_a = 1.00 \times 10^{-10}$ m to travel infinitely far from the α particle? (*Hint:* See Example 13.5 in Section 13.3.)

KEYCONCEPT The electrostatic forces that two charged particles exert on each other are conservative. The associated electric potential energy U is proportional to the product of the two charges and inversely proportional to the distance r between them. As r approaches infinity, U approaches zero.

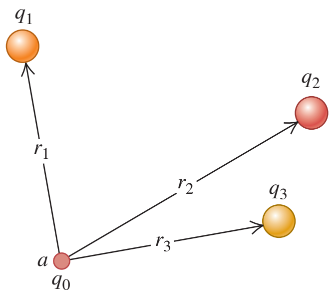


Figure 23.8 The potential energy associated with a charge q_0 at point a depends on the other charges q_1, q_2, and q_3 and on their distances r_1, r_2, and r_3 from point a.

Electric Potential Energy with Several Point Charges

Suppose the electric field $\vec{E}$ in which charge q_0 moves is caused by *several* point charges $q_1, q_2, q_3, \ldots$ at distances $r_1, r_2, r_3, \ldots$ from q_0, as in **Fig. 23.8**. For example, q_0 could be a positive ion moving in the presence of other ions (**Fig. 23.9**). The total electric field at each point is the *vector sum* of the fields due to the individual charges, and the total work done on q_0 during any displacement is the sum of the contributions from the individual charges. From Eq. (23.9) we conclude that the potential energy associated with the test charge q_0 at point a in Fig. 23.8 is the *algebraic* sum (*not* a vector sum):

Electric potential energy of point charge q_0 and collection of charges $q_1, q_2, q_3, \ldots$

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \qquad (23.10)$$

Electric constant

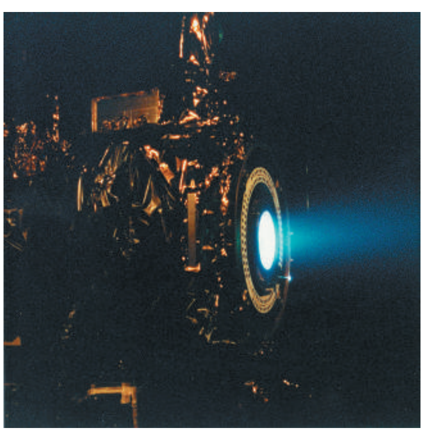
Distances from q_0 to $q_1, q_2, q_3, \ldots$

When q_0 is at a different point b, the potential energy is given by the same expression, but $r_1, r_2, \ldots$ are the distances from $q_1, q_2, \ldots$ to point b. The work done on charge q_0 when it moves from a to b along any path is equal to the difference $U_a - U_b$ between the potential energies when q_0 is at a and at b.

We can represent *any* charge distribution as a collection of point charges, so Eq. (23.10) shows that we can always find a potential-energy function for *any* static electric field. It follows that **for every electric field due to a static charge distribution, the force exerted by that field is conservative.**

Equations (23.9) and (23.10) define U to be zero when distances $r_1, r_2, \ldots$ are infinite—that is, when the test charge q_0 is very far away from all the charges that produce the field. As with any potential-energy function, the point where $U = 0$ is arbitrary; we can always add a constant to make U equal zero at any point we choose. In electrostatics problems it's usually simplest to choose this point to be at infinity. When we analyze electric circuits in Chapters 25 and 26, other choices will be more convenient.

Figure 23.9 This ion engine for spacecraft uses electric forces to eject a stream of positive xenon ions (Xe^+) at speeds in excess of 30 km/s. The thrust produced is very low (about 0.09 newton) but can be maintained continuously for days, in contrast to chemical rockets, which produce a large thrust for a short time (see Fig. 8.34). Such ion engines have been used for maneuvering interplanetary spacecraft.



Equation (23.10) gives the potential energy associated with the presence of the test charge q_0 in the $\vec{E}$ field produced by $q_1, q_2, q_3, \ldots$. But there is also potential energy involved in assembling these charges. If we start with charges $q_1, q_2, q_3, \ldots$ all separated from each other by infinite distances and then bring them together so that the distance between q_i and q_j is r_{ij}, the *total* potential energy U is the sum of the potential energies of interaction for each pair of charges. We can write this as

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} \qquad (23.11)$$

This sum extends over all *pairs* of charges; we don't let $i = j$ (because that would be an interaction of a charge with itself), and we include only terms with $i < j$ to make sure that we count each pair only once. Thus, to account for the interaction between q_3 and q_4, we include a term with $i = 3$ and $j = 4$ but not a term with $i = 4$ and $j = 3$.

Interpreting Electric Potential Energy

As a final comment, here are two viewpoints on electric potential energy. We have defined it in terms of the work done *by the electric field* on a charged particle moving in the field, just as in Chapter 7 we defined potential energy in terms of the work done by gravity or by a spring. When a particle moves from point a to point b, the work done on it by the electric field is $W_{a \to b} = U_a - U_b$. Thus the potential-energy difference $U_a - U_b$ equals *the work that is done by the electric force when the particle moves from a to b*. When U_a is greater than U_b, the field does positive work on the particle as it "falls" from a point of higher potential energy (a) to a point of lower potential energy (b).

An alternative but equivalent viewpoint is to consider how much work we would have to do to "raise" a particle from a point b where the potential energy is U_b to a point a where it has a greater value U_a (pushing two positive charges closer together, for example). To move the particle slowly (so as not to give it any kinetic energy), we need to exert an additional external force $\vec{F}_{\text{ext}}$ that is equal and opposite to the electric-field force and does positive work. The potential-energy difference $U_a - U_b$ is then defined as *the work that must be done by an external force to move the particle slowly from b to a against the electric force*. Because $\vec{F}_{\text{ext}}$ is the negative of the electric-field force and the displacement is in the opposite direction, this definition of the potential difference $U_a - U_b$ is equivalent to that given above. This alternative viewpoint also works if U_a is less than U_b, corresponding to "lowering" the particle; an example is moving two positive charges away from each other. In this case, $U_a - U_b$ is again equal to the work done by the external force, but now this work is negative.

We'll use both of these viewpoints in the next section to interpret what is meant by electric *potential,* or potential energy per unit charge.

EXAMPLE 23.2 A system of point charges

WITH VARIATION PROBLEMS

Two point charges are at fixed positions on the x-axis, $q_1 = -e$ at $x = 0$ and $q_2 = +e$ at $x = a$. (a) Find the work that must be done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$. (b) Find the total potential energy of the system of three charges.

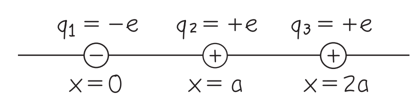
IDENTIFY and SET UP **Figure 23.10** shows the final arrangement of the three charges. In part (a) we need to find the work W that must be done on q_3 by an external force $\vec{F}_{\text{ext}}$ to bring q_3 in from infinity to $x = 2a$. We do this by using Eq. (23.10) to find the potential energy associated with q_3 in the presence of q_1 and q_2. In part (b) we use Eq. (23.11), the expression for the potential energy of a collection of point charges, to find the total potential energy of the system.

EXECUTE (a) The work W equals the difference between (i) the potential energy U associated with q_3 when it is at $x = 2a$ and (ii) the potential energy when it is infinitely far away. The second of these is zero, so the work required is equal to U. The distances between the charges are $r_{13} = 2a$ and $r_{23} = a$, so from Eq. (23.10),

$$W = U = \frac{q_3}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) = \frac{+e}{4\pi\epsilon_0}\left(\frac{-e}{2a} + \frac{+e}{a}\right) = \frac{+e^2}{8\pi\epsilon_0 a}$$

This is positive, just as we should expect. If we bring q_3 in from infinity along the $+x$-axis, it is attracted by q_1 but is repelled more strongly by q_2. Hence we must do positive work to push q_3 to the position at $x = 2a$.

Figure **23.10** Our sketch of the situation after the third charge has been brought in from infinity.



(b) From Eq. (23.11), the total potential energy of the three-charge system is

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{(-e)(e)}{a} + \frac{(-e)(e)}{2a} + \frac{(e)(e)}{a}\right] = \frac{-e^2}{8\pi\epsilon_0 a}$$

EVALUATE Our negative result in part (b) means that the system has lower potential energy than it would if the three charges were infinitely far apart. An external force would have to do *negative* work to bring the three charges from infinity to assemble this entire arrangement and would have to do *positive* work to move the three charges back to infinity.

KEYCONCEPT To find the total potential energy of a system of charges, sum the electric potential energies of all the distinct pairs of charges in the system.

TEST YOUR UNDERSTANDING OF SECTION 23.1 Consider the system of three point charges in Example 21.4 (Section 21.3) and shown in Fig. 21.14. (a) What is the sign of the total potential energy of this system? (i) Positive; (ii) negative; (iii) zero. (b) What is the sign of the total amount of work you would have to do to move these charges infinitely far from each other? (i) Positive; (ii) negative; (iii) zero.

ANSWER

(a) **(i)**, (b) **(ii)** The three charges q_1, q_2, and q_3 are all positive, so all three of the terms in the sum in Eq. (23.11)—q_1q_2/r_{12}, q_1q_3/r_{13}, and q_2q_3/r_{23}—are positive. Hence the total electric potential energy U is positive. This means that it would take positive work to bring the three charges from infinity to the positions shown in Fig. 21.14, and hence *negative* work to move the three charges from these positions back to infinity.

23.2 ELECTRIC POTENTIAL

In Section 23.1 we looked at the potential energy U associated with a test charge q_0 in an electric field. Now we want to describe this potential energy on a "per unit charge" basis, just as electric field describes the force per unit charge on a charged particle in the field. This leads us to the concept of *electric potential,* often called simply *potential*. This concept is very useful in calculations involving energies of charged particles. It also facilitates many electric-field calculations because electric potential is closely related to the electric field $\vec{E}$. When we need to determine an electric field, it is often easier to determine the potential first and then find the field from it.

Potential is *potential energy per unit charge*. We define the potential V at any point in an electric field as the potential energy U *per unit charge* associated with a test charge q_0 at that point:

$$V = \frac{U}{q_0} \quad \text{or} \quad U = q_0 V \qquad (23.12)$$

Potential energy and charge are both scalars, so potential is a scalar. From Eq. (23.12) its units are the units of energy divided by those of charge. The SI unit of potential, called one **volt** (1 V) in honor of the Italian electrical experimenter Alessandro Volta (1745–1827), equals 1 joule per coulomb:

$$1 \text{ V} = 1 \text{ volt} = 1 \text{ J/C} = 1 \text{ joule/coulomb}$$

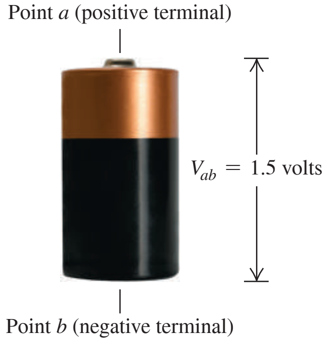
Let's put Eq. (23.2), which equates the work done by the electric force during a displacement from a to b to the quantity $-\Delta U = -(U_b - U_a)$, on a "work per unit charge" basis. We divide this equation by q_0, obtaining

$$\frac{W_{a \to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a) = V_a - V_b \qquad (23.13)$$

where $V_a = U_a/q_0$ is the potential energy per unit charge at point a and similarly for V_b. We call V_a and V_b the *potential at point a* and *potential at point b*, respectively. Thus the work done per unit charge by the electric force when a charged object moves from a to b is equal to the potential at a minus the potential at b.

The difference $V_a - V_b$ is called the *potential of a with respect to b*; we sometimes abbreviate this difference as $V_{ab} = V_a - V_b$ (note the order of the subscripts). This is often called the potential difference between a and b, but that's ambiguous unless we specify which is the reference point. In electric circuits, which we'll analyze in later chapters, the potential difference between two points is often called **voltage** (**Fig. 23.11**). Equation (23.13) then states: **V_{ab}, the potential (in V) of a with respect to b, equals the work (in J) done by the electric force when a UNIT (1 C) charge moves from a to b.**

Figure 23.11 The voltage of this battery equals the difference in potential $V_{ab} = V_a - V_b$ between its positive terminal (point a) and its negative terminal (point b).



Another way to interpret the potential difference V_{ab} in Eq. (23.13) is to use the alternative viewpoint mentioned at the end of Section 23.1. In that viewpoint, $U_a - U_b$ is the amount of work that must be done by an *external* force to move a particle of charge q_0 slowly from b to a against the electric force. The work that must be done *per unit charge* by the external force is then $(U_a - U_b)/q_0 = V_a - V_b = V_{ab}$. In other words: **$V_{ab}$, the potential (in V) of a with respect to b, equals the work (in J) that must be done to move a UNIT (1 C) charge slowly from b to a against the electric force.**

An instrument that measures the difference of potential between two points is called a *voltmeter*. (In Chapter 26 we'll discuss how these devices work.) Voltmeters that can measure a potential difference of 1 μV are common, and sensitivities down to 10^{-12} V can be attained.

Calculating Electric Potential

To find the potential V due to a single point charge q, we divide Eq. (23.9) by q_0:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{23.14}$$

Electric potential due to a point charge; q: Value of point charge; r: Distance from point charge to where potential is measured; ϵ_0: Electric constant

If q is positive, the potential that it produces is positive at all points; if q is negative, it produces a potential that is negative everywhere. In either case, V is equal to zero at $r = \infty$, an infinite distance from the point charge. Note that potential, like electric field, is independent of the test charge q_0 that we use to define it.

Similarly, we divide Eq. (23.10) by q_0 to find the potential due to a collection of point charges:

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \tag{23.15}$$

Electric potential due to a collection of point charges; q_i: Value of ith point charge; r_i: Distance from ith point charge to where potential is measured; ϵ_0: Electric constant

Just as the electric field due to a collection of point charges is the *vector* sum of the fields produced by each charge, the electric potential due to a collection of point charges is the *scalar* sum of the potentials due to each charge. When we have a continuous distribution of charge along a line, over a surface, or through a volume, we divide the charge into elements dq, and the sum in Eq. (23.15) becomes an integral:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \tag{23.16}$$

Electric potential due to a continuous distribution of charge; Integral over charge distribution; dq: Charge element; r: Distance from charge element to where potential is measured; ϵ_0: Electric constant

We'll work out several examples of such cases. The potential defined by Eqs. (23.15) and (23.16) is zero at points that are infinitely far away from *all* the charges. Later we'll encounter cases in which the charge distribution itself extends to infinity. We'll find that in such cases we cannot set $V = 0$ at infinity, and we'll need to exercise care in using and interpreting Eqs. (23.15) and (23.16).

CAUTION **What is electric potential?** Before getting too involved in the details of how to calculate electric potential, remind yourself what potential is. The electric *potential* at a certain point is the potential energy per *unit* charge placed at that point. That's why potential is measured in joules per coulomb, or volts. Keep in mind, too, that there doesn't have to be a charge at a given point for a potential V to exist at that point. (In the same way, an electric field can exist at a given point even if there's no charge there to respond to it.)

Finding Electric Potential from Electric Field

When we are given a collection of point charges, Eq. (23.15) is usually the easiest way to calculate the potential V. But in some problems in which the electric field is known or can be found easily, it is easier to determine V from $\vec{E}$. The force $\vec{F}$ on a test charge q_0 can be written as $\vec{F} = q_0\vec{E}$, so from Eq. (23.1) the work done by the electric force as the test charge moves from a to b is given by

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

BIO APPLICATION Electrocardiography The electrodes used in an electrocardiogram—EKG or ECG for short—measure the potential differences (typically no greater than 1 mV = 10^{-3} V) between different parts of the patient's skin. These are indicative of the potential differences between regions of the heart, and so provide a sensitive way to detect any abnormalities in the electrical activity that drives cardiac function.

If we divide this by q_0 and compare the result with Eq. (23.13), we find

$$\underbrace{V_a - V_b}_{\text{Electric potential difference}} = \overbrace{\int_a^b \underbrace{\vec{E} \cdot d\vec{l}}_{\text{Scalar product of electric field and displacement vector}}}^{\text{Integral along path from } a \text{ to } b} = \int_a^b \underbrace{E}_{\text{Electric-field magnitude}} \cos \underbrace{\phi}_{\text{Angle between } \vec{E} \text{ and } d\vec{l}} \, \underbrace{dl}_{\text{Displacement}} \tag{23.17}$$

The value of $V_a - V_b$ is independent of the path taken from a to b, just as the value of $W_{a \to b}$ is independent of the path. To interpret Eq. (23.17), remember that $\vec{E}$ is the electric force per unit charge on a test charge. If the line integral $\int_a^b \vec{E} \cdot d\vec{l}$ is positive, the electric field does positive work on a positive test charge as it moves from a to b. In this case the electric potential energy decreases as the test charge moves, so the potential energy per unit charge decreases as well; hence V_b is less than V_a and $V_a - V_b$ is positive.

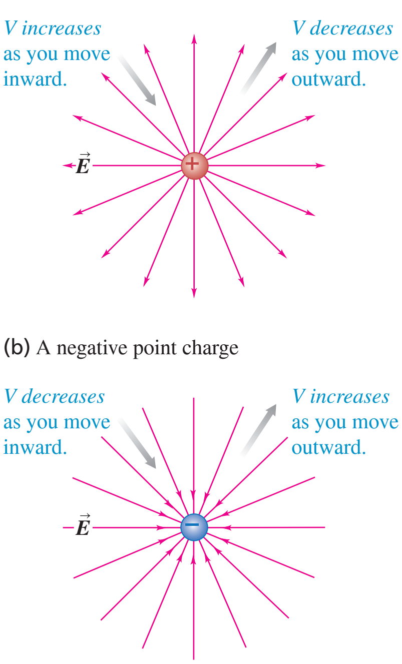
As an illustration, consider a positive point charge (**Fig. 23.12a**). The electric field is directed away from the charge, and $V = q/4\pi\epsilon_0 r$ is positive at any finite distance from the charge. If you move away from the charge, in the direction of $\vec{E}$, you move toward lower values of V; if you move toward the charge, in the direction opposite $\vec{E}$, you move toward greater values of V. For the negative point charge in Fig. 23.12b, $\vec{E}$ is directed toward the charge and $V = q/4\pi\epsilon_0 r$ is negative at any finite distance from the charge. In this case, if you move toward the charge, you are moving in the direction of $\vec{E}$ and in the direction of decreasing (more negative) V. Moving away from the charge, in the direction opposite $\vec{E}$, moves you toward increasing (less negative) values of V. The general rule, valid for *any* electric field, is: Moving *with* the direction of $\vec{E}$ means moving in the direction of *decreasing* V, and moving *against* the direction of $\vec{E}$ means moving in the direction of *increasing* V.

Also, a positive test charge q_0 experiences an electric force in the direction of $\vec{E}$, toward lower values of V; a negative test charge experiences a force opposite $\vec{E}$, toward higher values of V. Thus a positive charge tends to "fall" from a high-potential region to a lower-potential region. The opposite is true for a negative charge.

Figure **23.12** If you move in the direction of $\vec{E}$, electric potential V decreases; if you move in the direction opposite $\vec{E}$, V increases.

(a) A positive point charge

(b) A negative point charge



Notice that Eq. (23.17) can be rewritten as

$$V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{l} \tag{23.18}$$

This has a negative sign compared to the integral in Eq. (23.17), and the limits are reversed; hence Eqs. (23.17) and (23.18) are equivalent. But Eq. (23.18) has a slightly different interpretation. To move a unit charge slowly against the electric force, we must apply an *external* force per unit charge equal to $-\vec{E}$, equal and opposite to the electric force per unit charge $\vec{E}$. Equation (23.18) says that $V_a - V_b = V_{ab}$, the potential of a with respect to b, equals the work done per unit charge by this external force to move a unit charge from b to a. This is the same alternative interpretation we discussed under Eq. (23.13).

Equations (23.17) and (23.18) show that the unit of potential difference (1 V) is equal to the unit of electric field (1 N/C) multiplied by the unit of distance (1 m). Hence the unit of electric field can be expressed as 1 *volt per meter* (1 V/m) as well as 1 N/C:

$$1 \text{ V/m} = 1 \text{ volt/meter} = 1 \text{ N/C} = 1 \text{ newton/coulomb}$$

In practice, the volt per meter is the usual unit of electric-field magnitude.

Electron Volts

The magnitude e of the electron charge can be used to define a unit of energy that is useful in many calculations with atomic and nuclear systems. When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a, the change in the potential energy U is

$$U_a - U_b = q(V_a - V_b) = qV_{ab}$$

If charge q equals the magnitude e of the electron charge, 1.602×10^{-19} C, and the potential difference is $V_{ab} = 1\ \text{V} = 1\ \text{J/C}$, the change in energy is

$$U_a - U_b = (1.602 \times 10^{-19}\ \text{C})(1\ \text{J/C}) = 1.602 \times 10^{-19}\ \text{J}$$

This quantity of energy is defined to be 1 **electron volt** (1 eV):

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

The multiples meV, keV, MeV, GeV, and TeV are often used.

CAUTION Electron volts vs. volts Remember that the electron volt is a unit of energy, *not* a unit of potential or potential difference!

When a particle with charge e moves through a potential difference of 1 volt, the change in potential *energy* is 1 eV. If the charge is some multiple of e—say, Ne—the change in potential energy in electron volts is N times the potential difference in volts. For example, when an alpha particle, which has charge $2e$, moves between two points with a potential difference of 1000 V, the change in potential energy is $2(1000\ \text{eV}) = 2000\ \text{eV}$. To confirm this, we write

$$\begin{aligned} U_a - U_b = qV_{ab} &= (2e)(1000\ \text{V}) = (2)(1.602 \times 10^{-19}\ \text{C})(1000\ \text{V}) \\ &= 3.204 \times 10^{-16}\ \text{J} = 2000\ \text{eV} \end{aligned}$$

Although we defined the electron volt in terms of *potential* energy, we can use it for *any* form of energy, such as the kinetic energy of a moving particle. When we speak of a "one-million-electron-volt proton," we mean a proton with a kinetic energy of one million electron volts (1 MeV), equal to $(10^6)(1.602 \times 10^{-19}\ \text{J}) = 1.602 \times 10^{-13}\ \text{J}$. The Large Hadron Collider near Geneva, Switzerland, is designed to accelerate protons to a kinetic energy of 7 TeV (7×10^{12} eV).

BIO APPLICATION Electron Volts and Cancer Radiotherapy One way to destroy a cancerous tumor is to aim high-energy electrons directly at it. Each electron has a kinetic energy of 4 to 20 million electron volts, or MeV (1 MeV = 10^6 eV), and transfers its energy to the tumor through collisions with the tumor's atoms. Electrons in this energy range can penetrate only a few centimeters into a patient, which makes them useful for treating superficial tumors, such as those on the skin or lips.

EXAMPLE 23.3 Electric force and electric potential

WITH VARIATION PROBLEMS

A proton (charge $+e = 1.602 \times 10^{-19}$ C) moves a distance $d = 0.50$ m in a straight line between points a and b in a linear accelerator. The electric field is uniform along this line, with magnitude $E = 1.5 \times 10^7\ \text{V/m} = 1.5 \times 10^7\ \text{N/C}$ in the direction from a to b. Determine (a) the force on the proton; (b) the work done on it by the field; (c) the potential difference $V_a - V_b$.

IDENTIFY and SET UP This problem uses the relationship between electric field and electric force. It also uses the relationship among force, work, and potential-energy difference. We are given the electric field, so it is straightforward to find the electric force on the proton. Calculating the work is also straightforward because $\vec{E}$ is uniform, so the force on the proton is constant. Once the work is known, we find $V_a - V_b$ from Eq. (23.13).

EXECUTE (a) The force on the proton is in the same direction as the electric field, and its magnitude is

$$\begin{aligned} F = qE &= (1.602 \times 10^{-19}\ \text{C})(1.5 \times 10^7\ \text{N/C}) \\ &= 2.4 \times 10^{-12}\ \text{N} \end{aligned}$$

(b) The force is constant and in the same direction as the displacement, so the work done on the proton is

$$\begin{aligned} W_{a\to b} = Fd &= (2.4 \times 10^{-12}\ \text{N})(0.50\ \text{m}) \\ &= 1.2 \times 10^{-12}\ \text{J} \\ &= (1.2 \times 10^{-12}\ \text{J})\frac{1\ \text{eV}}{1.602 \times 10^{-19}\ \text{J}} \\ &= 7.5 \times 10^6\ \text{eV} = 7.5\ \text{MeV} \end{aligned}$$

(c) From Eq. (23.13) the potential difference is the work per unit charge, which is

$$\begin{aligned} V_a - V_b &= \frac{W_{a\to b}}{q} = \frac{1.2 \times 10^{-12}\ \text{J}}{1.602 \times 10^{-19}\ \text{C}} \\ &= 7.5 \times 10^6\ \text{J/C} = 7.5 \times 10^6\ \text{V} = 7.5\ \text{MV} \end{aligned}$$

We can get this same result even more easily by remembering that 1 electron volt equals 1 volt multiplied by the charge e. The work done is 7.5×10^6 eV and the charge is e, so the potential difference is $(7.5 \times 10^6\ \text{eV})/e = 7.5 \times 10^6$ V.

EVALUATE We can check our result in part (c) by using Eq. (23.17) or (23.18). The angle ϕ between the constant field $\vec{E}$ and the displacement is zero, so Eq. (23.17) becomes

$$V_a - V_b = \int_a^b E\cos\phi\, dl = \int_a^b E\, dl = E\int_a^b dl$$

The integral of dl from a to b is just the distance d, so we again find

$$V_a - V_b = Ed = (1.5 \times 10^7\ \text{V/m})(0.50\ \text{m}) = 7.5 \times 10^6\ \text{V}$$

KEYCONCEPT The potential difference V_{ab} between point a and point b, equal to the difference $V_a - V_b$ of the potentials V at the two points, is the amount of work the electric force does on a unit charge as it moves from a to b. If you move in the direction of the electric field, V decreases (V_{ab} is positive); if you move opposite to the direction of the electric field, V increases (V_{ab} is negative).

EXAMPLE 23.4 Potential due to two point charges

WITH VARIATION PROBLEMS

An electric dipole consists of point charges $q_1 = +12\text{ nC}$ and $q_2 = -12\text{ nC}$ placed 10.0 cm apart (**Fig. 23.13**). Compute the electric potentials at points a, b, and c.

IDENTIFY and SET UP This is the same arrangement as in Example 21.8 (Section 21.5), in which we calculated the electric *field* at each point by doing a *vector* sum. Here our target variable is the electric *potential* V at three points, which we find by doing the *algebraic* sum in Eq. (23.15).

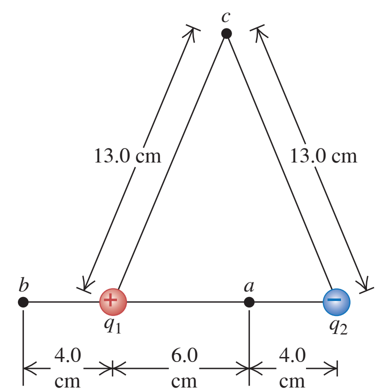
EXECUTE At point a we have $r_1 = 0.060\text{ m}$ and $r_2 = 0.040\text{ m}$, so Eq. (23.15) becomes

$$\begin{aligned} V_a &= \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} \\ &= (9.0 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\text{ C}}{0.060\text{ m}} \\ &\quad + (9.0 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-12 \times 10^{-9}\text{ C})}{0.040\text{ m}} \\ &= 1800\text{ N}\cdot\text{m/C} + (-2700\text{ N}\cdot\text{m/C}) \\ &= 1800\text{ V} + (-2700\text{ V}) = -900\text{ V} \end{aligned}$$

In a similar way you can show that the potential at point b (where $r_1 = 0.040\text{ m}$ and $r_2 = 0.140\text{ m}$) is $V_b = 1930\text{ V}$ and that the potential at point c (where $r_1 = r_2 = 0.130\text{ m}$) is $V_c = 0$.

EVALUATE Let's confirm that these results make sense. Point a is closer to the -12 nC charge than to the $+12\text{ nC}$ charge, so the potential at a is negative. The potential is positive at point b, which is closer to the $+12\text{ nC}$ charge than the -12 nC charge. Finally, point c is equidistant from the $+12\text{ nC}$ charge and the -12 nC charge, so the potential there is zero. (The potential is also equal to zero at a point infinitely far from both charges.)

Figure 23.13 What are the potentials at points a, b, and c due to this electric dipole?



Comparing this example with Example 21.8 shows that it's much easier to calculate electric potential (a scalar) than electric field (a vector). We'll take advantage of this simplification whenever possible.

KEYCONCEPT The electric potential caused at a point P by a single point charge is proportional to the charge and inversely proportional to the distance from the point charge to P. The total electric potential at P due to a system of charges is the sum of the potentials at P due to each individual charge.

EXAMPLE 23.5 Potential and potential energy

WITH VARIATION PROBLEMS

Compute the potential energy associated with a $+4.0\text{ nC}$ point charge if it is placed at points a, b, and c in Fig. 23.13.

IDENTIFY and SET UP The potential energy U associated with a point charge q at a location where the electric potential is V is $U = qV$. We use the values of V from Example 23.4.

EXECUTE At the three points we find

$$\begin{aligned} U_a &= qV_a = (4.0 \times 10^{-9}\text{ C})(-900\text{ J/C}) = -3.6 \times 10^{-6}\text{ J} \\ U_b &= qV_b = (4.0 \times 10^{-9}\text{ C})(1930\text{ J/C}) = 7.7 \times 10^{-6}\text{ J} \\ U_c &= qV_c = 0 \end{aligned}$$

All of these values correspond to U and V being zero at infinity.

EVALUATE Note that *zero* net work is done on the 4.0 nC charge if it moves from point c to infinity *by any path*. In particular, let the path be along the perpendicular bisector of the line joining the other two charges q_1 and q_2 in Fig. 23.13. As shown in Example 21.8 (Section 21.5), at points on the bisector, the direction of $\vec{E}$ is perpendicular to the bisector. Hence the force on the 4.0 nC charge is perpendicular to the path, and no work is done in any displacement along it.

KEYCONCEPT If you place a point charge q at a point P, the associated electric potential energy is equal to q multiplied by the value at P of the electric potential due to all *other* charges.

EXAMPLE 23.6 Finding potential by integration

By integrating the electric field as in Eq. (23.17), find the potential at a distance r from a point charge q.

IDENTIFY and SET UP We let point a in Eq. (23.17) be at distance r and let point b be at infinity (**Fig. 23.14**). As usual, we choose the potential to be zero at an infinite distance from the charge q.

EXECUTE To carry out the integral, we can choose any path we like between points a and b. The most convenient path is a radial line as shown in Fig. 23.14, so that $d\vec{l}$ is in the radial direction and has magnitude dr. Writing $d\vec{l} = \hat{r}dr$, we have from Eq. (23.17)

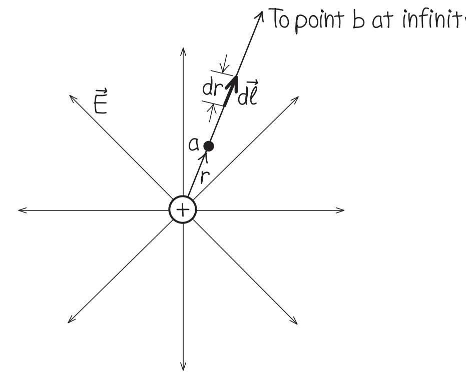
$$V - 0 = V = \int_r^{\infty} \vec{E} \cdot d\vec{l}$$

$$= \int_r^{\infty} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot \hat{r} dr = \int_r^{\infty} \frac{q}{4\pi\epsilon_0 r^2} dr$$

$$= -\frac{q}{4\pi\epsilon_0 r}\bigg|_r^{\infty} = 0 - \left(-\frac{q}{4\pi\epsilon_0 r}\right) = \frac{q}{4\pi\epsilon_0 r}$$

EVALUATE Our result agrees with Eq. (23.14) and is correct for positive or negative q.

Figure 23.14 Calculating the potential by integrating $\vec{E}$ for a single point charge.



KEYCONCEPT The potential difference $V_{ab} = V_a - V_b$ between point a and point b equals the integral of the electric field along a path from a to b. This integral does not depend on the path taken between the two points.

EXAMPLE 23.7 Moving through a potential difference

WITH VARIATION PROBLEMS

In **Fig. 23.15** a dust particle with mass $m = 5.0 \times 10^{-9}$ kg $= 5.0\ \mu$g and charge $q_0 = 2.0$ nC starts from rest and moves in a straight line from point a to point b. What is its speed v at point b?

IDENTIFY and SET UP Only the conservative electric force acts on the particle, so the total mechanical energy is conserved: $K_a + U_a = K_b + U_b$. We get the potential energies U from the corresponding potentials V from Eq. (23.12): $U_a = q_0 V_a$ and $U_b = q_0 V_b$.

EXECUTE We have $K_a = 0$ and $K_b = \frac{1}{2}mv^2$. We substitute these and our expressions for U_a and U_b into the energy-conservation equation, then solve for v. We find

$$0 + q_0 V_a = \tfrac{1}{2}mv^2 + q_0 V_b$$

$$v = \sqrt{\frac{2q_0(V_a - V_b)}{m}}$$

We calculate the potentials from Eq. (23.15), $V = q/4\pi\epsilon_0 r$:

$$V_a = (9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\left(\frac{3.0 \times 10^{-9}\ \text{C}}{0.010\ \text{m}} + \frac{(-3.0 \times 10^{-9}\ \text{C})}{0.020\ \text{m}}\right)$$

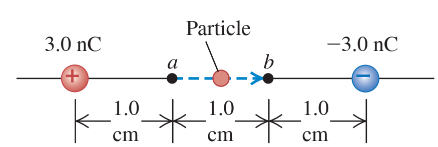
$$= 1350\ \text{V}$$

$$V_b = (9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\left(\frac{3.0 \times 10^{-9}\ \text{C}}{0.020\ \text{m}} + \frac{(-3.0 \times 10^{-9}\ \text{C})}{0.010\ \text{m}}\right)$$

$$= -1350\ \text{V}$$

$$V_a - V_b = (1350\ \text{V}) - (-1350\ \text{V}) = 2700\ \text{V}$$

Figure 23.15 The particle moves from point a to point b; its acceleration is not constant.



Finally,

$$v = \sqrt{\frac{2(2.0 \times 10^{-9}\ \text{C})(2700\ \text{V})}{5.0 \times 10^{-9}\ \text{kg}}} = 46\ \text{m/s}$$

EVALUATE Our result makes sense: The positive dust particle speeds up as it moves away from the $+3.0$ nC charge and toward the -3.0 nC charge. To check unit consistency in the final line of the calculation, note that $1\ \text{V} = 1\ \text{J/C}$, so the numerator under the radical has units of J or kg $\cdot$ m^2/s^2.

KEYCONCEPT When a particle of charge q moves from a point of higher potential to a point of lower potential, its potential energy *decreases* if q is positive; its potential energy *increases* if q is negative.

TEST YOUR UNDERSTANDING OF SECTION 23.2 If the electric *potential* at a certain point is zero, does the electric *field* at that point have to be zero? (*Hint:* Consider point *c* in Examples 23.4 and 21.8.)

ANSWER

no If $V = 0$ at a certain point, $\vec{E}$ does *not* have to be zero at that point. An example is point *c* in Figs. 21.23 and 23.13, for which there is an electric field in the $+x$-direction (see Example 21.9 in Section 21.5) even though $V = 0$ (see Example 23.4). This isn't a surprising result because V and $\vec{E}$ are quite different quantities: V is the net amount of work required to bring a unit charge from infinity to the point in question, whereas $\vec{E}$ is the electric force that acts on a unit charge when it arrives at that point.

23.3 CALCULATING ELECTRIC POTENTIAL

When calculating the potential due to a charge distribution, we usually follow one of two routes. If we know the charge distribution, we can use Eq. (23.15) or (23.16). Or if we know how the electric field depends on position, we can use Eq. (23.17), defining the potential to be zero at some convenient place. Some problems require a combination of these approaches.

As you read through these examples, compare them with the related examples of calculating electric *field* in Section 21.5. You'll see how much easier it is to calculate scalar electric potentials than vector electric fields. The moral is clear: Whenever possible, solve problems by means of an energy approach (using electric potential and electric potential energy) rather than a dynamics approach (using electric fields and electric forces).

PROBLEM-SOLVING STRATEGY 23.1 Calculating Electric Potential

IDENTIFY *the relevant concepts:* Remember that electric potential is *potential energy per unit charge*.

SET UP *the problem* using the following steps:

1. Make a drawing showing the locations and values of the charges (which may be point charges or a continuous distribution of charge) and your choice of coordinate axes.
2. Indicate on your drawing the position of the point at which you want to calculate the electric potential V. Sometimes this position will be an arbitrary one (say, a point a distance r from the center of a charged sphere).

EXECUTE *the solution* as follows:

1. To find the potential due to a collection of point charges, use Eq. (23.15). If you are given a continuous charge distribution, devise a way to divide it into infinitesimal elements and use Eq. (23.16). Carry out the integration, using appropriate limits to include the entire charge distribution.
2. If you are given the electric field, or if you can find it from any of the methods presented in Chapters 21 or 22, it may be easier to find the potential difference between points a and b from Eq. (23.17) or (23.18). When appropriate, make use of your freedom to define V to be zero at some convenient place, and choose this place to be point b. (For point charges, this will usually be at infinity. For other distributions of charge—especially those that themselves extend to infinity—it may be necessary to define V_b to be zero at some finite distance from the charge distribution.) Then the potential at any other point, say a, can by found from Eq. (23.17) or (23.18) with $V_b = 0$.
3. Although potential V is a *scalar* quantity, you may have to use components of the vectors $\vec{E}$ and $d\vec{l}$ when you use Eq. (23.17) or (23.18) to calculate V.

EVALUATE *your answer:* Check whether your answer agrees with your intuition. If your result gives V as a function of position, graph the function to see whether it makes sense. If you know the electric field, you can make a rough check of your result for V by verifying that V decreases if you move in the direction of $\vec{E}$.

EXAMPLE 23.8 A charged conducting sphere

WITH VARIATION PROBLEMS

A solid conducting sphere of radius R has a total charge q. Find the electric potential everywhere, both outside and inside the sphere.

IDENTIFY and SET UP In Example 22.5 (Section 22.4) we used Gauss's law to find the electric *field* at all points for this charge distribution. We can use that result to determine the potential.

EXECUTE From Example 22.5, the field *outside* the sphere is the same as if the sphere were removed and replaced by a point charge q. We take $V = 0$ at infinity, as we did for a point charge. Then the potential at a point outside the sphere at a distance r from its center is the same as that due to a point charge q at the center:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

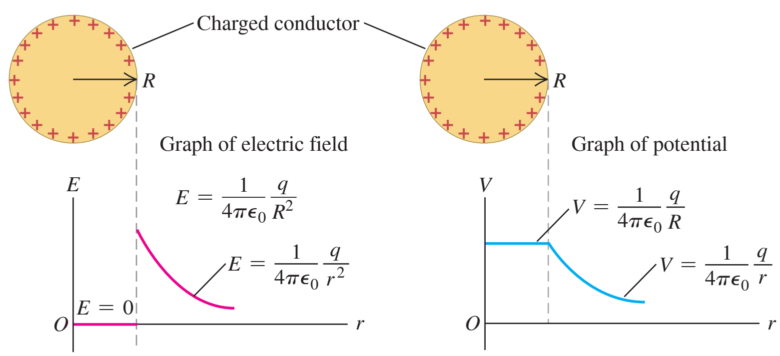
The potential at the surface of the sphere is $V_{\text{surface}} = q/4\pi\epsilon_0 R$.

Inside the sphere, $\vec{E}$ is zero everywhere. Hence no work is done on a test charge that moves from any point to any other point inside the sphere. Thus the potential is the same at every point inside the sphere and is equal to its value $q/4\pi\epsilon_0 R$ at the surface.

EVALUATE **Figure 23.16** shows the field and potential for a positive charge q. In this case the electric field points radially away from the sphere. As you move away from the sphere, in the direction of $\vec{E}$, V decreases (as it should).

KEYCONCEPT If there is excess charge at rest on a solid spherical conductor, all of the charge lies on the surface of the conductor, and the electric field and electric potential outside the sphere are the same as if all the charge were concentrated at the center of the sphere. Everywhere inside the sphere the field is zero and the potential has the same value as at the surface.

Figure 23.16 Electric-field magnitude E and potential V at points inside and outside a positively charged spherical conductor.



Ionization and Corona Discharge

The results of Example 23.8 have numerous practical consequences. One consequence relates to the maximum potential to which a conductor in air can be raised. This potential is limited because air molecules become *ionized,* and air becomes a conductor, at an electric-field magnitude of about 3×10^6 V/m. Assume for the moment that q is positive. When we compare the expressions in Example 23.8 for the potential V_{surface} and field magnitude E_{surface} at the surface of a charged conducting sphere, we note that $V_{\text{surface}} = E_{\text{surface}}R$. Thus, if E_m represents the electric-field magnitude at which air becomes conductive (known as the *dielectric strength* of air), then the maximum potential V_m to which a spherical conductor can be raised is

$$V_m = RE_m$$

For a conducting sphere 1 cm in radius in air, $V_m = (10^{-2}\text{ m})(3 \times 10^6\text{ V/m}) = 30{,}000$ V. No amount of "charging" could raise the potential of a conducting sphere of this size in air higher than about 30,000 V; attempting to raise the potential further by adding extra charge would cause the surrounding air to become ionized and conductive, and the extra added charge would leak into the air.

To attain even higher potentials, high-voltage machines such as Van de Graaff generators use spherical terminals with very large radii (see Fig. 22.26 and the photograph that opens Chapter 22). For example, a terminal of radius $R = 2$ m has a maximum potential $V_m = (2\text{ m})(3 \times 10^6\text{ V/m}) = 6 \times 10^6\text{ V} = 6$ MV.

Figure 23.17 The metal mast at the top of the Empire State Building acts as a lightning rod. It is struck by lightning as many as 500 times each year.

Our result in Example 23.8 also explains what happens with a charged conductor with a very *small* radius of curvature, such as a sharp point or thin wire. Because the maximum potential is proportional to the radius, even relatively small potentials applied to sharp points in air produce sufficiently high fields just outside the point to ionize the surrounding air, making it become a conductor. The resulting current and its associated glow (visible in a dark room) are called *corona discharge*. Laser printers and photocopying machines use corona discharge from fine wires to spray charge on the imaging drum (see Fig. 21.2).

A large-radius conductor is used in situations where it's important to *prevent* corona discharge. An example is the blunt end of a metal lightning rod (**Fig. 23.17**). If there is an excess charge in the atmosphere, as happens during thunderstorms, a substantial charge of the opposite sign can build up on this blunt end. As a result, when the atmospheric charge is discharged through a lightning bolt, it tends to be attracted to the charged lightning rod rather than to other structures that could be damaged. (A conducting wire connecting the lightning rod to the ground then allows the acquired charge to dissipate harmlessly.) A lightning rod with a sharp end would allow less charge buildup and hence be less effective.

EXAMPLE 23.9 Oppositely charged parallel plates

WITH VARIATION PROBLEMS

Find the potential at any height y between the two oppositely charged parallel plates discussed in Section 23.1 (**Fig. 23.18**).

IDENTIFY and SET UP We discussed this situation in Section 23.1. From Eq. (23.5), we know the electric *potential energy* U for a test charge q_0 is $U = q_0Ey$. (We set $y = 0$ and $U = 0$ at the bottom plate.) We use Eq. (23.12), $U = q_0V$, to find the electric *potential* V as a function of y.

EXECUTE The potential $V(y)$ at coordinate y is the potential energy per unit charge:

$$V(y) = \frac{U(y)}{q_0} = \frac{q_0Ey}{q_0} = Ey$$

The potential decreases as we move in the direction of $\vec{E}$ from the upper to the lower plate. At point a, where $y = d$ and $V(y) = V_a$,

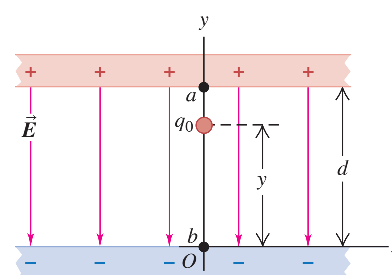
$$V_a - V_b = Ed$$

and

$$E = \frac{V_a - V_b}{d} = \frac{V_{ab}}{d}$$

where V_{ab} is the potential of the positive plate with respect to the negative plate. That is, the electric field equals the potential difference between the plates divided by the distance between them. For a given potential difference V_{ab}, the smaller the distance d between the two plates, the greater the magnitude E of the electric field. (This relationship between E and V_{ab} holds *only* for the planar geometry we have described. It does *not* work for situations such as concentric cylinders or spheres in which the electric field is not uniform.)

Figure 23.18 The charged parallel plates from Fig. 23.2.



EVALUATE Our result shows that $V = 0$ at the bottom plate (at $y = 0$). This is consistent with our choice that $U = q_0V = 0$ for a test charge placed at the bottom plate.

CAUTION "Zero potential" is arbitrary You might think that if a conducting object has zero potential, it must also have zero net charge. But that just isn't so! As an example, the plate at $y = 0$ in Fig. 23.18 has zero potential $(V = 0)$ but has a nonzero charge per unit area $-\sigma$. There's nothing special about the place where potential is zero; we *define* this place to be wherever we want it to be.

KEYCONCEPT In a region of uniform electric field $\vec{E}$, such as between two oppositely charged parallel plates, the electric potential decreases linearly with position as you move in the direction of $\vec{E}$.

EXAMPLE 23.10 An infinite line charge or charged conducting cylinder

WITH VARIATION PROBLEMS

Find the potential at a distance r from a very long line of charge with linear charge density (charge per unit length) λ.

IDENTIFY and SET UP In both Example 21.10 (Section 21.5) and Example 22.6 (Section 22.4) we found that the electric field at a radial distance r from a long straight-line charge (**Fig. 23.19a**) has only a radial component $E_r = \lambda/2\pi\epsilon_0 r$. We use this expression to find the potential by integrating $\vec{E}$ as in Eq. (23.17).

EXECUTE Since the field has only a radial component, we have $\vec{E} \cdot d\vec{l} = E_r dr$. Hence from Eq. (23.17) the potential of any point a with respect to any other point b, at radial distances r_a and r_b from the line of charge, is

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E_r dr = \frac{\lambda}{2\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

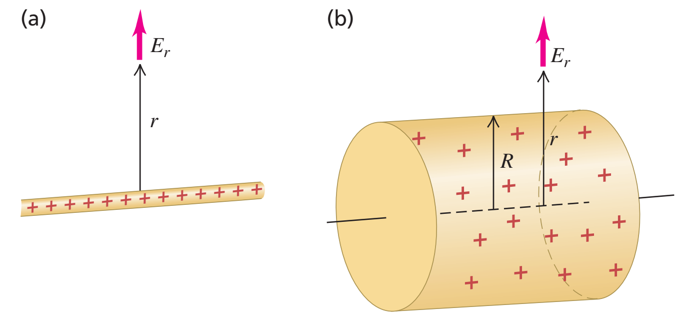
If we take point b at infinity and set $V_b = 0$, we find that V_a is *infinite* for any finite distance r_a from the line charge: $V_a = (\lambda/2\pi\epsilon_0)\ln(\infty/r_a) = \infty$. This is *not* a useful way to define V for this problem! The difficulty is that the charge distribution itself extends to infinity.

Instead, as recommended in Problem-Solving Strategy 23.1, we set $V_b = 0$ at point b at an arbitrary but *finite* radial distance r_0. Then the potential $V = V_a$ at point a at a radial distance r is given by $V - 0 = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$, or

$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_0}{r}$$

EVALUATE According to our result, if λ is positive, then V decreases as r increases. This is as it should be: V decreases as we move in the direction of $\vec{E}$.

Figure **23.19** Electric field outside (a) a long, positively charged wire and (b) a long, positively charged cylinder.



From Example 22.6, the expression for E_r with which we started also applies outside a long, charged conducting cylinder with charge per unit length λ (Fig. 23.19b). Hence our result also gives the potential for such a cylinder, but only for values of r (the distance from the cylinder axis) equal to or greater than the radius R of the cylinder. If we choose r_0 to be the radius R, so that $V = 0$ when $r = R$, then at any point for which $r > R$,

$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{r}$$

Inside the cylinder, $\vec{E} = \mathbf{0}$, and V has the same value (zero) as on the cylinder's surface.

KEYCONCEPT The electric potential outside a very long cylindrical distribution of charge or line of charge decreases logarithmically with the distance from the axis of the distribution or line.

EXAMPLE 23.11 A ring of charge

WITH VARIATION PROBLEMS

Electric charge Q is distributed uniformly around a thin ring of radius a (**Fig. 23.20**). Find the potential at a point P on the ring axis at a distance x from the center of the ring.

IDENTIFY and SET UP We divide the ring into infinitesimal segments and use Eq. (23.16) to find V. All parts of the ring (and therefore all elements of the charge distribution) are at the same distance from P.

EXECUTE Figure 23.20 shows that the distance from each charge element dq to P is $r = \sqrt{x^2 + a^2}$. Hence we can take the factor $1/r$ outside the integral in Eq. (23.16), and

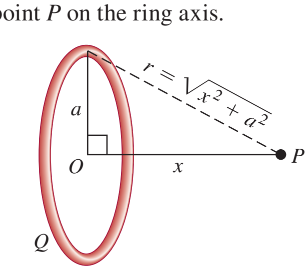
$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2 + a^2}}\int dq$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + a^2}}$$

EVALUATE When x is much larger than a, our expression for V becomes approximately $V = Q/4\pi\epsilon_0 x$, which is the potential at a distance x from a point charge Q. Very far from a charged ring, its electric potential looks like that of a point charge. We drew a similar conclusion about the electric *field* of a ring in Example 21.9 (Section 21.5).

Figure **23.20** All the charge in a ring of charge Q is the same distance r from a point P on the ring axis.



We know the electric field at all points along the x-axis from Example 21.9 (Section 21.5), so we can also find V along this axis by integrating $\vec{E} \cdot d\vec{l}$ as in Eq. (23.17).

KEYCONCEPT To find the electric potential at a point due to a continuous distribution of charge, first divide the distribution into infinitesimally small segments. Then find the potential at the point due to one such segment. Finally, integrate over all segments in the charge distribution.

EXAMPLE 23.12 Potential of a line of charge

WITH VARIATION PROBLEMS

Positive electric charge Q is distributed uniformly along a line of length $2a$ lying along the y-axis between $y = -a$ and $y = +a$ (**Fig. 23.21**). Find the electric potential at a point P on the x-axis at a distance x from the origin.

IDENTIFY and SET UP This is the situation of Example 21.10 (Section 21.5), where we found an expression for the electric field $\vec{E}$ at an arbitrary point on the x-axis. We can find V at point P by using Eq. (23.16) to integrate over the charge distribution. Unlike the situation in Example 23.11, each charge element dQ is a *different* distance from point P, so the integration will take a little more effort.

EXECUTE As in Example 21.10, the element of charge dQ corresponding to an element of length dy on the rod is $dQ = (Q/2a)dy$. The distance from dQ to P is $\sqrt{x^2 + y^2}$, so the contribution dV that the charge element makes to the potential at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{\sqrt{x^2 + y^2}}$$

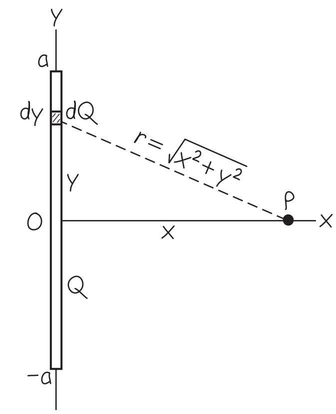
To find the potential at P due to the entire rod, we integrate dV over the length of the rod from $y = -a$ to $y = a$:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{dy}{\sqrt{x^2 + y^2}}$$

You can look up the integral in a table. The final result is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\ln\left(\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right)$$

Figure 23.21 Our sketch for this problem.



EVALUATE We can check our result by letting x approach infinity. In this limit the point P is infinitely far from all of the charge, so we expect V to approach zero; you can verify that it does.

We know the electric field at all points along the x-axis from Example 21.10. We invite you to use this information to find V along this axis by integrating $\vec{E}$ as in Eq. (23.17).

KEYCONCEPT The electric potential due to a symmetrical distribution of charge is most easily calculated at a point of symmetry. Whenever possible, take advantage of the symmetry of the situation to check your results.

TEST YOUR UNDERSTANDING OF SECTION 23.3 If the electric *field* at a certain point is zero, does the electric *potential* at that point have to be zero? (*Hint:* Consider the center of the ring in Examples 23.11 and 21.9.)

ANSWER

no If $\vec{E} = \mathbf{0}$ at a certain point, V does *not* have to be zero at that point. An example is point O at the center of the charged ring in Figs. 21.23 and 23.21. From Example 21.9 (Section 21.5), the electric field is zero at O because the electric-field contributions from different parts of the ring completely cancel. From Example 23.11, however, the potential at O is *not* zero: This point corresponds to $x = 0$, so $V = (1/4\pi\epsilon_0)(Q/a)$. This value of V corresponds to the work that would have to be done to move a unit positive test charge along a path from infinity to point O; it is nonzero because the charged ring repels the test charge, so positive work must be done to move the test charge toward the ring.

23.4 EQUIPOTENTIAL SURFACES

Field lines (see Section 21.6) help us visualize electric fields. In a similar way, the potential at various points in an electric field can be represented graphically by *equipotential surfaces*. These use the same fundamental idea as topographic maps like those used by hikers and mountain climbers (**Fig. 23.22**). On a topographic map, contour lines are drawn through points that are all at the same elevation. Any number of these could be drawn, but typically only a few contour lines are shown at equal spacings of elevation. If a mass m is moved over the terrain along such a contour line, the gravitational potential energy mgy does not change because the elevation y is constant. Thus contour lines on a topographic map are really curves of constant gravitational potential energy. Contour lines are close together where the terrain is steep and there are large changes in elevation over a small horizontal distance; the contour lines are farther apart where the terrain

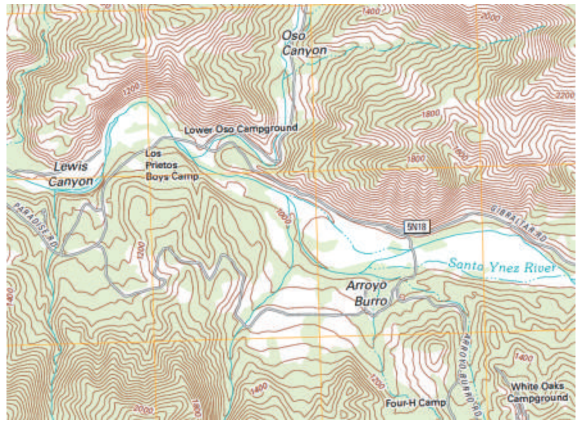


Figure **23.22** Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy.

is gently sloping. A ball allowed to roll downhill will experience the greatest downhill gravitational force where contour lines are closest together.

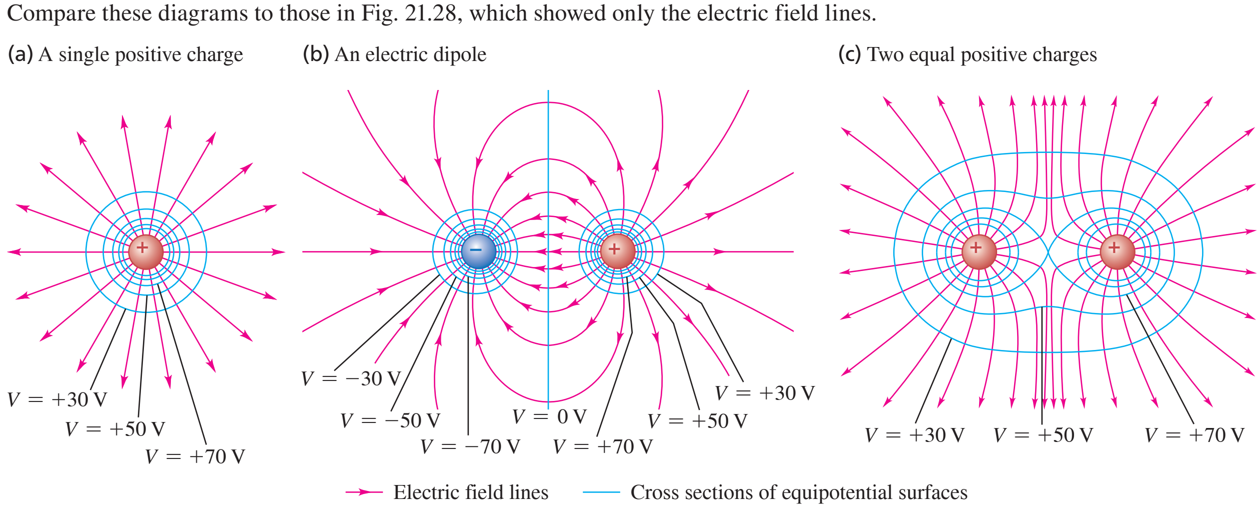
By analogy to contour lines on a topographic map, an **equipotential surface** is a three-dimensional surface on which the *electric potential* V is the same at every point. If a test charge q_0 is moved from point to point on such a surface, the *electric* potential energy q_0V remains constant. In a region where an electric field is present, we can construct an equipotential surface through any point. In diagrams we usually show only a few representative equipotentials, often with equal potential differences between adjacent surfaces. No point can be at two different potentials, so equipotential surfaces for different potentials can never touch or intersect.

Equipotential Surfaces and Field Lines

Because potential energy does not change as a test charge moves over an equipotential surface, the electric field can do no work on such a charge. It follows that $\vec{E}$ must be perpendicular to the surface at every point so that the electric force $q_0\vec{E}$ is always perpendicular to the displacement of a charge moving on the surface. **Field lines and equipotential surfaces are always mutually perpendicular.** In general, field lines are curves, and equipotentials are curved surfaces. For the special case of a *uniform* field, in which the field lines are straight, parallel, and equally spaced, the equipotentials are parallel *planes* perpendicular to the field lines.

Figure 23.23 shows three arrangements of charges. The field lines in the plane of the charges are represented by red lines, and the intersections of the equipotential surfaces with this plane (that is, cross sections of these surfaces) are shown as blue lines. The actual equipotential surfaces are three-dimensional. At each crossing of an equipotential and a field line, the two are perpendicular.

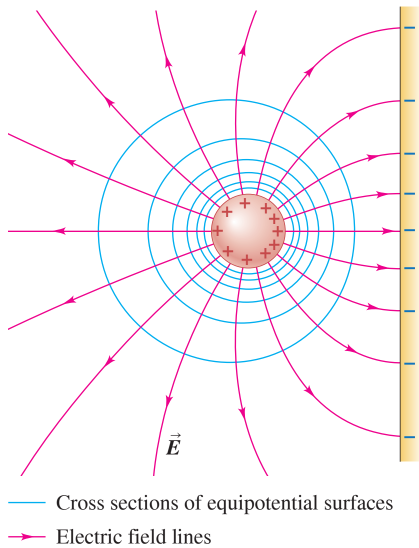
Figure **23.23** Cross sections of equipotential surfaces (blue lines) and electric field lines (red lines) for assemblies of point charges. There are equal potential differences between adjacent surfaces. Compare these diagrams to those in Fig. 21.28, which showed only the electric field lines.



In Fig. 23.23 we have drawn equipotentials so that there are equal potential differences between adjacent surfaces. In regions where the magnitude of $\vec{E}$ is large, the equipotential surfaces are close together because the field does a relatively large amount of work on a test charge in a relatively small displacement. This is the case near the point charge in Fig. 23.23a or between the two point charges in Fig. 23.23b; note that in these regions the field lines are also closer together. This is directly analogous to the downhill force of gravity being greatest in regions on a topographic map where contour lines are close together. Conversely, in regions where the field is weaker, the equipotential surfaces are farther apart; this happens at larger radii in Fig. 23.23a, to the left of the negative charge or the right of the positive charge in Fig. 23.23b, and at greater distances from both charges in Fig. 23.23c. (It may appear that two equipotential surfaces intersect at the center of Fig. 23.23c, in violation of the rule that this can never happen. In fact this is a single figure-8–shaped equipotential surface.)

CAUTION ***E* need not be constant over an equipotential surface** On a given equipotential surface, the potential V has the same value at every point. In general, however, the electric-field magnitude E is *not* the same at all points on an equipotential surface. For instance, on equipotential surface "$V = -30$ V" in Fig. 23.23b, E is less to the left of the negative charge than it is between the two charges. On the figure-8–shaped equipotential surface in Fig. 23.23c, $E = 0$ at the middle point halfway between the two charges; at any other point on this surface, E is nonzero.

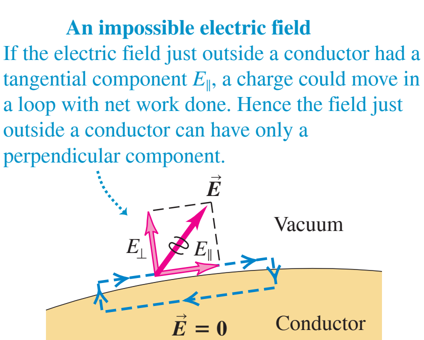
Figure **23.24** When charges are at rest, a conducting surface is always an equipotential surface. Field lines are perpendicular to a conducting surface.



Equipotentials and Conductors

Here's an important statement about equipotential surfaces: **When all charges are at rest, the surface of a conductor is always an equipotential surface.** Since the electric field $\vec{E}$ is always perpendicular to an equipotential surface, we can prove this statement by proving that **when all charges are at rest, the electric field just outside a conductor must be perpendicular to the surface at every point** (**Fig. 23.24**). We know that $\vec{E} = \mathbf{0}$ everywhere inside the conductor; otherwise, charges would move. In particular, at any point just inside the surface the component of $\vec{E}$ tangent to the surface is zero. It follows that the tangential component of $\vec{E}$ is also zero just *outside* the surface. If it were not, a charge could move around a rectangular path partly inside and partly outside (**Fig. 23.25**) and return to its starting point with a net amount of work having been done on it. This would violate the conservative nature of electrostatic fields, so the tangential component of $\vec{E}$ just outside the surface must be zero at every point on the surface. Thus $\vec{E}$ is perpendicular to the surface at each point, proving our statement.

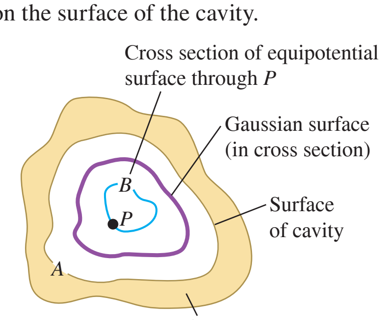
Figure **23.25** At all points on a conductor's surface, the electric field must be perpendicular to the surface. If $\vec{E}$ had a tangential component, a net amount of work would be done on a test charge by moving it around a loop as shown—which is impossible because the electric force is conservative.



It also follows that **when all charges are at rest, the entire solid volume of a conductor is at the same potential.** Equation (23.17) states that the potential difference between two points a and b within the conductor's solid volume, $V_a - V_b$, is equal to the line integral $\int_a^b \vec{E} \cdot d\vec{l}$ of the electric field from a to b. Since $\vec{E} = \mathbf{0}$ everywhere inside the conductor, the integral is guaranteed to be zero for any two such points a and b. Hence the potential is the same for any two points within the solid volume of the conductor. We describe this by saying that the solid volume of the conductor is an *equipotential volume.*

We can now prove a theorem that we quoted without proof in Section 22.5. The theorem is as follows: In an electrostatic situation, if a conductor contains a cavity and if no charge is present inside the cavity, then there can be no net charge *anywhere* on the surface of the cavity. This means that if you're inside a charged conducting box, you can safely touch any point on the inside walls of the box without being shocked. To prove this theorem, we first prove that *every point in the cavity is at the same potential.* In **Fig. 23.26** the conducting surface A of the cavity is an equipotential surface, as we have just proved. Suppose point P in the cavity is at a different potential; then we can construct a different equipotential surface B including point P.

Figure **23.26** A cavity in a conductor. If the cavity contains no charge, every point in the cavity is at the same potential, the electric field is zero everywhere in the cavity, and there is no charge anywhere on the surface of the cavity.



Now consider a Gaussian surface, shown in Fig. 23.26, between the two equipotential surfaces. Because of the relationship between $\vec{E}$ and the equipotentials, we know that the field at every point between the equipotentials is from A toward B, or else at every point it is from B toward A, depending on which equipotential surface is at higher potential.

In either case the flux through this Gaussian surface is certainly not zero. But then Gauss's law says that the charge enclosed by the Gaussian surface cannot be zero. This contradicts our initial assumption that there is *no* charge in the cavity. So the potential at P *cannot* be different from that at the cavity wall.

The entire region of the cavity must therefore be at the same potential. But for this to be true, *the electric field inside the cavity must be zero everywhere*. Finally, Gauss's law shows that the electric field at any point on the surface of a conductor is proportional to the surface charge density σ at that point. We conclude that *the surface charge density on the wall of the cavity is zero at every point*. This chain of reasoning may seem tortuous, but it is worth careful study.

CAUTION **Equipotential surfaces vs. Gaussian surfaces** Don't confuse equipotential surfaces with the Gaussian surfaces we encountered in Chapter 22. Gaussian surfaces have relevance only when we are using Gauss's law, and we can choose *any* Gaussian surface that's convenient. We *cannot* choose equipotential surfaces; the shape is determined by the charge distribution.

TEST YOUR UNDERSTANDING OF SECTION 23.4 Would the shapes of the equipotential surfaces in Fig. 23.23 change if the sign of each charge were reversed?

ANSWER

no If the positive charges in Fig. 23.23 were replaced by negative charges, and vice versa, the equipotential surfaces would be the same but the sign of the potential would be reversed. For example, the surfaces in Fig. 23.23b with potential $V = +30\text{ V}$ and $V = -50\text{ V}$ would have potential $V = -30\text{ V}$ and $V = +50\text{ V}$, respectively.

23.5 POTENTIAL GRADIENT

Electric field and potential are closely related. Equation (23.17), restated here, expresses one aspect of that relationship:

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

If we know $\vec{E}$ at various points, we can use this equation to calculate potential differences. In this section we show how to turn this around; if we know the potential V at various points, we can use it to determine $\vec{E}$. Regarding V as a function of the coordinates (x, y, z) of a point in space, we'll show that the components of $\vec{E}$ are related to the *partial derivatives* of V with respect to x, y, and z.

In Eq. (23.17), $V_a - V_b$ is the potential of a with respect to b—that is, the change of potential encountered on a trip from b to a. We can write this as

$$V_a - V_b = \int_b^a dV = -\int_a^b dV$$

where dV is the infinitesimal change of potential accompanying an infinitesimal element $d\vec{l}$ of the path from b to a. Comparing to Eq. (23.17), we have

$$-\int_a^b dV = \int_a^b \vec{E} \cdot d\vec{l}$$

These two integrals must be equal for *any* pair of limits a and b, and for this to be true the *integrands* must be equal. Thus, for *any* infinitesimal displacement $d\vec{l}$,

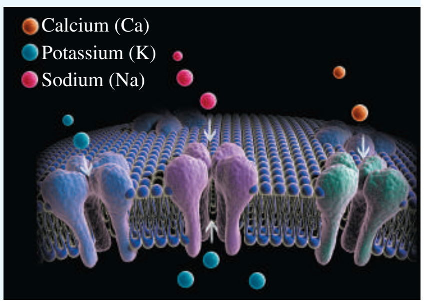
$$-dV = \vec{E} \cdot d\vec{l}$$

To interpret this expression, we write $\vec{E}$ and $d\vec{l}$ in terms of their components: $\vec{E} = \hat{\imath}\, E_x + \hat{\jmath}\, E_y + \hat{k}\, E_z$ and $d\vec{l} = \hat{\imath}\, dx + \hat{\jmath}\, dy + \hat{k}\, dz$. Then

$$-dV = E_x dx + E_y dy + E_z dz$$

Suppose the displacement is parallel to the x-axis, so $dy = dz = 0$. Then $-dV = E_x dx$ or $E_x = -(dV/dx)_{y,\,z\text{ constant}}$, where the subscript reminds us that only x varies in the

BIO APPLICATION Potential Gradient Across a Cell Membrane The interior of a human cell is at a lower electric potential V than the exterior. (The potential difference when the cell is inactive is about -70 mV in neurons and about -95 mV in skeletal muscle cells.) Hence there is a potential gradient $\vec{\nabla} V$ that points from the *interior* to the *exterior* of the cell membrane, and an electric field $\vec{E} = -\vec{\nabla} V$ that points from the *exterior* to the *interior*. This field affects how ions flow into or out of the cell through special channels in the membrane.



derivative; recall that V is in general a function of x, y, and z. But this is just what is meant by the partial derivative $\partial V/\partial x$. The y- and z-components of $\vec{E}$ are related to the corresponding derivatives of V in the same way, so

Electric field components found from potential:

Each electric field component ...

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z} \tag{23.19}$$

... equals the negative of the corresponding partial derivative of electric potential function V.

This is consistent with the units of electric field being V/m. In terms of unit vectors we can write

Electric field vector found from potential:

Electric field

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \tag{23.20}$$

Partial derivatives of electric potential function V

The following operation is called the **gradient** of the function f:

$$\vec{\nabla} f = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) f \tag{23.21}$$

The operator denoted by $\vec{\nabla}$ is called "grad" or "del." Thus in vector notation,

$$\vec{E} = -\vec{\nabla} V \tag{23.22}$$

This is read "$\vec{E}$ is the negative of the gradient of V" or "$\vec{E}$ equals negative grad V." The quantity $\vec{\nabla} V$ is called the *potential gradient*.

At each point, the potential gradient $\vec{\nabla} V$ points in the direction in which V *increases* most rapidly with a change in position. Hence at each point the direction of $\vec{E} = -\vec{\nabla} V$ is the direction in which V *decreases* most rapidly and is always perpendicular to the equipotential surface through the point. This agrees with our observation in Section 23.2 that moving in the direction of the electric field means moving in the direction of decreasing potential.

Equation (23.22) doesn't depend on the particular choice of the zero point for V. If we were to change the zero point, the effect would be to change V at every point by the same amount; the derivatives of V would be the same.

If $\vec{E}$ has a radial component E_r with respect to a point or an axis and r is the distance from the point or axis, the relationship corresponding to Eqs. (23.19) is

$$E_r = -\frac{\partial V}{\partial r} \qquad \text{(radial electric field component)} \tag{23.23}$$

Often we can compute the electric field caused by a charge distribution in either of two ways: directly, by adding the $\vec{E}$ fields of point charges, or by first calculating the potential and then taking its gradient to find the field. The second method is often easier because potential is a *scalar* quantity, requiring at worst the integration of a scalar function. Electric field is a *vector* quantity, requiring computation of components for each element of charge and a separate integration for each component. Thus, quite apart from its fundamental significance, potential offers a very useful computational technique in field calculations. In the next two examples, a knowledge of V is used to find the electric field.

We stress once more that if we know $\vec{E}$ as a function of position, we can calculate V from Eq. (23.17) or (23.18), and if we know V as a function of position, we can calculate $\vec{E}$ from Eq. (23.19), (23.20), or (23.23). Deriving V from $\vec{E}$ requires integration, and deriving $\vec{E}$ from V requires differentiation.

EXAMPLE 23.13 Potential and field of a point charge

From Eq. (23.14) the potential at a radial distance r from a point charge q is $V = q/4\pi\epsilon_0 r$. Find the vector electric field from this expression for V.

IDENTIFY and SET UP This problem uses the general relationship between the electric potential as a function of position and the electric-field vector. By symmetry, the electric field here has only a radial component E_r. We use Eq. (23.23) to find this component.

EXECUTE From Eq. (23.23),

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

so the vector electric field is

$$\vec{E} = \hat{r}E_r = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

EVALUATE Our result agrees with Eq. (21.7), as it must.

An alternative approach is to ignore the radial symmetry, write the radial distance as $r = \sqrt{x^2 + y^2 + z^2}$, and take the derivatives of V with respect to x, y, and z as in Eq. (23.20). We find

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{x^2 + y^2 + z^2}}\right) = -\frac{1}{4\pi\epsilon_0}\frac{qx}{(x^2 + y^2 + z^2)^{3/2}}$$
$$= -\frac{qx}{4\pi\epsilon_0 r^3}$$

and similarly

$$\frac{\partial V}{\partial y} = -\frac{qy}{4\pi\epsilon_0 r^3} \qquad \frac{\partial V}{\partial z} = -\frac{qz}{4\pi\epsilon_0 r^3}$$

Then from Eq. (23.20),

$$\vec{E} = -\left[\hat{\imath}\left(-\frac{qx}{4\pi\epsilon_0 r^3}\right) + \hat{\jmath}\left(-\frac{qy}{4\pi\epsilon_0 r^3}\right) + \hat{k}\left(-\frac{qz}{4\pi\epsilon_0 r^3}\right)\right]$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\left(\frac{x\hat{\imath} + y\hat{\jmath} + z\hat{k}}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

This approach gives us the same answer, but with more effort. Clearly it's best to exploit the symmetry of the charge distribution whenever possible.

KEYCONCEPT If you know the electric potential as a function of position in a region of space, you can find the electric field in that region by calculating the negative gradient of the potential.

EXAMPLE 23.14 Potential and field of a ring of charge

In Example 23.11 (Section 23.3) we found that for a ring of charge with radius a and total charge Q, the potential at a point P on the ring's symmetry axis a distance x from the center (see Fig. 23.20) is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + a^2}}$$

Find the electric field at P.

IDENTIFY and SET UP We are given V as a function of x along the x-axis, and we wish to find the electric field at a point on this axis. From the symmetry of the charge distribution, the electric field along the symmetry (x-) axis of the ring can have only an x-component. We find it by using the first of Eqs. (23.19).

EXECUTE The x-component of the electric field is

$$E_x = -\frac{\partial V}{\partial x} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{3/2}}$$

EVALUATE This agrees with our result in Example 21.9.

CAUTION Don't use expressions where they don't apply In this example, V is not a function of y or z on the ring axis, so $\partial V/\partial y = \partial V/\partial z = 0$ and $E_y = E_z = 0$. But that does not mean that it's true *everywhere;* our expressions for V and E_x are valid *on the ring axis only*. If we had an expression for V valid at *all* points in space, we could use it to find the components of $\vec{E}$ at any point by using Eqs. (23.19).

KEYCONCEPT If you know the electric potential as a function of position only along a line, you can find the component of the electric field parallel to that line by calculating the negative partial derivative of the potential with respect to position on that line. You cannot find the electric field at points off the line in this way.

TEST YOUR UNDERSTANDING OF SECTION 23.5 In a certain region of space the potential is given by $V = A + Bx + Cy^3 + Dxy$, where A, B, C, and D are positive constants. Which of these statements about the electric field $\vec{E}$ in this region of space is correct? (There may be more than one correct answer.) (i) Increasing the value of A will increase the value of $\vec{E}$ at all points; (ii) increasing the value of A will decrease the value of $\vec{E}$ at all points; (iii) $\vec{E}$ has no z-component; (iv) the electric field is zero at the origin ($x = 0$, $y = 0$, $z = 0$).

ANSWER

(iii) From Eqs. (23.19), the components of the electric field are $E_x = -\partial V/\partial x = -(B + Dy)$, $E_y = -\partial V/\partial y = -(3Cy^2 + Dx)$, and $E_z = -\partial V/\partial z = 0$. The value of A has no effect, which means that we can add a constant to the electric potential at all points without changing $\vec{E}$ or the potential difference between two points. The potential does not depend on z, so the z-component of $\vec{E}$ is zero. Note that at the origin the electric field is not zero because it has a nonzero x-component: $E_x = -B$, $E_y = 0$, $E_z = 0$.

CHAPTER 23 SUMMARY

Electric potential energy: The electric force caused by any collection of charges at rest is a conservative force. The work W done by the electric force on a charged particle moving in an electric field can be represented by the change in a potential-energy function U.

The electric potential energy for two point charges q and q_0 depends on their separation r. The electric potential energy for a charge q_0 in the presence of a collection of charges q_1, q_2, q_3 depends on the distance from q_0 to each of these other charges. (See Examples 23.1 and 23.2.)

$$W_{a \to b} = U_a - U_b \tag{23.2}$$

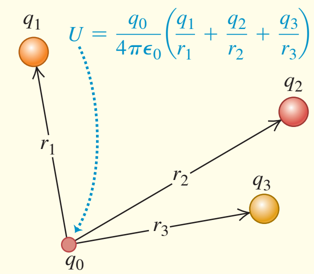
$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \tag{23.9}$$

(two point charges)

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right)$$

$$= \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \tag{23.10}$$

(q_0 in presence of other point charges)



Electric potential: Potential, denoted by V, is potential energy per unit charge. The potential difference between two points equals the amount of work per charge that would be required to move a positive test charge between those points. The potential V due to a quantity of charge can be calculated by summing (if the charge is a collection of point charges) or by integrating (if the charge is a distribution). (See Examples 23.3, 23.4, 23.5, 23.7, 23.11, and 23.12.)

The potential difference between two points a and b, also called the potential of a with respect to b, is given by the line integral of $\vec{E}$. The potential at a given point can be found by first finding $\vec{E}$ and then carrying out this integral. (See Examples 23.6, 23.8, 23.9, and 23.10.)

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{23.14}$$

(due to a point charge)

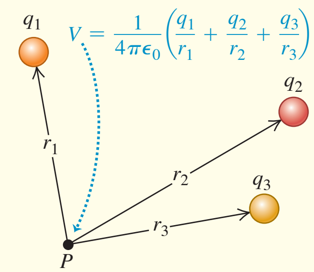
$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \tag{23.15}$$

(due to a collection of point charges)

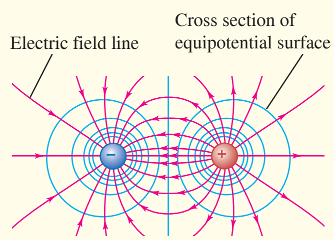
$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \tag{23.16}$$

(due to a charge distribution)

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E\cos\phi \, dl \tag{23.17}$$



Equipotential surfaces: An equipotential surface is a surface on which the potential has the same value at every point. At a point where a field line crosses an equipotential surface, the two are perpendicular. When all charges are at rest, the surface of a conductor is always an equipotential surface and all points in the interior of a conductor are at the same potential. When a cavity within a conductor contains no charge, the entire cavity is an equipotential region and there is no surface charge anywhere on the surface of the cavity.



Finding electric field from electric potential: If the potential V is known as a function of the coordinates x, y, and z, the components of electric field $\vec{E}$ at any point are given by partial derivatives of V. (See Examples 23.13 and 23.14.)

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \tag{23.19}$$

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \tag{23.20}$$

(vector form)

Chapter 23 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLES 23.1 and 23.2 (Section 23.1) before attempting these problems.

VP23.2.1 An electron (charge $-e = -1.60 \times 10^{-19}$ C) is at rest at a distance 1.00×10^{-10} m from the center of a nucleus of lead (charge $+82e = +1.31 \times 10^{-17}$ C). You may treat both objects as point charges. (a) How much work would you have to do to move the electron to a distance 5.00×10^{-10} m from the center of the lead nucleus? (b) If you release the electron from the position in (a), what will be its kinetic energy when it is a distance 8.00×10^{-12} m from the center of the lead nucleus? (You can assume that the lead nucleus remains at rest, since it is much more massive than the electron.)

VP23.2.2 In a nuclear physics experiment, a proton (mass 1.67×10^{-27} kg, charge $+e = +1.60 \times 10^{-19}$ C) is fired directly at a target nucleus of unknown charge. (You can treat both objects as point charges, and assume that the nucleus remains at rest.) When it is far from its target, the proton has speed 2.50×10^{6} m/s. The proton comes momentarily to rest at a distance 5.29×10^{-13} m from the center of the target nucleus, then flies back in the direction from which it came. (a) What is the electric potential energy of the proton and nucleus when they are 5.29×10^{-13} m apart? (b) What is the charge of the target nucleus?

VP23.2.3 Two point charges are at fixed positions on the y-axis: $q_1 = +e$ at $y = 0$ and $q_2 = -e$ at $y = a$. Find (a) the work you must do to bring a third charge $q_3 = -e$ from infinity to being at rest at $y = -2a$ and (b) the total potential energy of the system of three charges.

VP23.2.4 A point charge $q_1 = +5.00$ nC is at the fixed position $x = 0$, $y = 0$, $z = 0$. You find that you must do 8.10×10^{-6} J of work to bring a second point charge from infinity to the position $x = +4.00$ cm, $y = 0$, $z = 0$. (a) What is the value of the second charge? (b) Once the second charge is in place, how much additional work would you have to do to bring a third point charge $q_3 = +2.00$ nC from infinity to the position $x = 0$, $y = +3.00$ cm, $z = 0$?

Be sure to review EXAMPLES 23.3, 23.4, 23.5, and 23.7 (Section 23.2) before attempting these problems.

VP23.7.1 In a certain region of space the electric field is uniform and given by $\vec{E} = (5.00 \times 10^{2}\ \text{V/m})\hat{\imath}$. If the electric potential at the point $x = 0$, $y = 0$, $z = 0$ is equal to V_0, find the potential difference $V_0 - V_P$ for each of the following points P: (a) $x = +5.00$ cm, $y = 0$, $z = 0$; (b) $x = +3.00$ cm, $y = +4.00$ cm, $z = 0$; (c) $x = 0$, $y = +5.00$ cm, $z = 0$; (d) $x = -5.00$ cm, $y = 0$, $z = 0$.

VP23.7.2 You have two identical point charges of +6.0 nC each, one at the position $x = 3.0$ cm, $y = 0$, $z = 0$ and the other at $x = -3.0$ cm, $y = 0$, $z = 0$. Find the electric potential due to these charges at (a) $x = 0$, $y = 0$, $z = 0$; (b) $x = 0$, $y = 4.0$ cm, $z = 0$; (c) $x = 8.0$ cm, $y = 0$, $z = 0$. Take $V = 0$ at infinity. (d) At which (if any) of these points is the electric *field* equal to zero? If the field is zero at a certain point, is the potential necessarily zero there as well?

VP23.7.3 One point charge of +5.90 nC is at the position $x = 3.00$ cm, $y = 0$, $z = 0$, and one point charge of −5.90 nC is at $x = -3.00$ cm, $y = 0$, $z = 0$. Find the electric potential due to these charges at (a) $x = 3.00$ cm, $y = 8.00$ cm, $z = 0$; (b) $x = -3.00$ cm, $y = 8.00$ cm, $z = 0$. Take $V = 0$ at infinity. (c) A proton (mass 1.67×10^{-27} kg, charge $+e = +1.60 \times 10^{-19}$ C) follows a path from point (b) to point (a). If its speed at (b) is 3.00×10^{5} m/s, what is its speed at (a)?

VP23.7.4 In Example 22.9 (Section 22.4) we found that the electric field *inside* a uniformly charged sphere with positive charge Q and radius R points radially outward from the center of the sphere and has magnitude $E = Qr/4\pi\epsilon_0 R^3$ at a distance r from the center. (a) By integrating the electric field, find the potential difference between the center of the sphere and its surface. (b) Which is at higher potential: the center or the surface?

Be sure to review EXAMPLES 23.8, 23.9, 23.10, 23.11, and 23.12 (Section 23.3) before attempting these problems.

VP23.12.1 A conducting sphere of radius R carries positive charge q. Calculate the amount of work that would be required to move a small positive test charge q_0 slowly (a) from $r = 5R$ to $r = 3R$; (b) from $r = 2R$ to $r = 7R$; (c) from $r = 4R$ to $r = R/2$. In each case assume that the presence of q_0 has no effect on how the charge q is distributed over the sphere.

VP23.12.2 For the two oppositely charged parallel plates in Fig. 23.18, the potential difference between the plates is $V_{ab} = 24.0$ V, $V = 0$ at the bottom plate, and the distance between the plates is $d = 4.50$ mm. (a) What is the electric potential at a point 3.00 mm above the lower plate? (b) What is the potential energy for a dust particle of mass 5.00×10^{-9} kg and charge +2.00 nC at the position in part (a)? (c) If the particle in part (b) is released from rest, will it move toward the upper plate or the lower plate? What will be its speed when it hits that plate?

VP23.12.3 The uniformly charged ring shown in Fig. 23.20 has charge +5.00 nC and radius 2.50 cm. A point charge $q = +3.00$ nC of mass 4.00×10^{-9} kg is on the ring axis 8.00 cm from the center of the ring and is moving toward the center at 60.0 m/s. Take $V = 0$ at infinity. What is the potential energy of the point charge (a) at 8.00 cm from the ring's center and (b) at the ring's center? (c) What are the kinetic energy and speed of the point charge when it reaches the ring's center?

VP23.12.4 Positive charge Q is distributed uniformly along a rod of length L that lies along the x-axis from $x = L$ to $x = 2L$. (a) How much charge is contained within a segment of the rod of length dx? (b) Integrate to find the total electric potential at the origin ($x = 0$) due to the rod.

BRIDGING PROBLEM A Point Charge and a Line of Charge

Positive electric charge Q is distributed uniformly along a thin rod of length $2a$. The rod lies along the x-axis between $x = -a$ and $x = +a$ (**Fig. 23.27**). Calculate how much work you must do to bring a positive point charge q from infinity to the point $x = +L$ on the x-axis, where $L > a$.

SOLUTION GUIDE

IDENTIFY and SET UP

1. In this problem you must first calculate the potential V at $x = +L$ due to the charged rod. You can then find the change in potential energy involved in bringing the point charge q from infinity (where $V = 0$) to $x = +L$.
2. To find V, divide the rod into infinitesimal segments of length dx'. How much charge is on such a segment? Consider one such segment located at $x = x'$, where $-a \le x' \le a$. What is the potential dV at $x = +L$ due to this segment?
3. The total potential at $x = +L$ is the integral of dV, including contributions from all of the segments for x' from $-a$ to $+a$. Set up this integral.

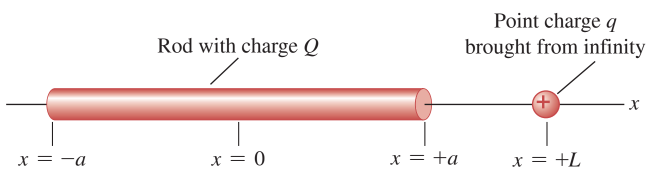
EXECUTE

4. Integrate your expression from step 3 to find the potential V at $x = +L$. A simple, standard substitution will do the trick; use a table of integrals only as a last resort.
5. Use your result from step 4 to find the potential energy for a point charge q placed at $x = +L$.
6. Use your result from step 5 to find the work you must do to bring the point charge from infinity to $x = +L$.

EVALUATE

7. What does your result from step 5 become in the limit $a \rightarrow 0$? Does this make sense?
8. Suppose the point charge q were negative rather than positive. How would this affect your result in step 4? In step 5?

Figure 23.27 How much work must you do to bring point charge q in from infinity?



PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q23.1 A student asked, "Since electrical potential is always proportional to potential energy, why bother with the concept of potential at all?" How would you respond?

Q23.2 The potential (relative to a point at infinity) midway between two charges of equal magnitude and opposite sign is zero. Is it possible to bring a test charge from infinity to this midpoint in such a way that no work is done in any part of the displacement? If so, describe how it can be done. If it is not possible, explain why.

Q23.3 Is it possible to have an arrangement of two point charges separated by a finite distance such that the electric potential energy of the arrangement is the same as if the two charges were infinitely far apart? Why or why not? What if there are three charges? Explain.

Q23.4 Since potential can have any value you want depending on the choice of the reference level of zero potential, how does a voltmeter know what to read when you connect it between two points?

Q23.5 If $\vec{E}$ is zero everywhere along a certain path that leads from point A to point B, what is the potential difference between those two points? Does this mean that $\vec{E}$ is zero everywhere along *any* path from A to B? Explain.

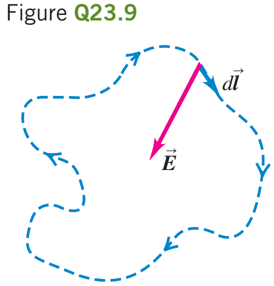
Q23.6 If $\vec{E}$ is zero throughout a certain region of space, is the potential necessarily also zero in this region? Why or why not? If not, what *can* be said about the potential?

Q23.7 Which way do electric field lines point, from high to low potential or from low to high? Explain.

Q23.8 (a) Take $V = 0$ at infinity. If the potential is zero at a point, is the electric field necessarily zero at that point? (b) If the electric field is zero at a point, is the potential necessarily zero there? Prove your answers, using simple examples.

Q23.9 If you carry out the integral of the electric field $\int \vec{E} \cdot d\vec{l}$ for a *closed* path like that shown in **Fig. Q23.9**, the integral will *always* be equal to zero, independent of the shape of the path and independent of where charges may be located relative to the path. Explain why.

Figure Q23.9



Q23.10 The potential difference between the two terminals of an AA battery (used in flashlights and portable stereos) is 1.5 V. If two AA batteries are placed end to end with the positive terminal of one battery touching the negative terminal of the other, what is the potential difference between the terminals at the exposed ends of the combination? What if the two positive terminals are touching each other? Explain your reasoning.

Q23.11 It is easy to produce a potential difference of several thousand volts between your body and the floor by scuffing your shoes across a nylon carpet. When you touch a metal doorknob, you get a mild shock. Yet contact with a power line of comparable voltage would probably be fatal. Why is there a difference?

Q23.12 If the electric potential at a single point is known, can $\vec{E}$ at that point be determined? If so, how? If not, why not?

Q23.13 Because electric field lines and equipotential surfaces are always perpendicular, two equipotential surfaces can never cross; if they did, the direction of $\vec{E}$ would be ambiguous at the crossing points. Yet two equipotential surfaces appear to cross at the center of Fig. 23.23c. Explain why there is no ambiguity about the direction of $\vec{E}$ in this particular case.

Q23.14 A uniform electric field is directed due east. Point B is 2.00 m west of point A, point C is 2.00 m east of point A, and point D is 2.00 m south of A. For each point, B, C, and D, is the potential at that point larger, smaller, or the same as at point A? Give the reasoning behind your answers.

Q23.15 We often say that if point A is at a higher potential than point B, A is at positive potential and B is at negative potential. Does it necessarily follow that a point at positive potential is positively charged, or that a point at negative potential is negatively charged? Illustrate your answers with clear, simple examples.

Q23.16 A conducting sphere is to be charged by bringing in positive charge a little at a time until the total charge is Q. The total work required for this process is alleged to be proportional to Q^2. Is this correct? Why or why not?

Q23.17 In electronics it is customary to define the potential of ground (thinking of the earth as a large conductor) as zero. Is this consistent with the fact that the earth has a net electric charge that is not zero?

Q23.18 A conducting sphere is placed between two charged parallel plates such as those shown in Fig. 23.2. Does the electric field inside the sphere depend on precisely where between the plates the sphere is placed? What about the electric potential inside the sphere? Do the answers to these questions depend on whether or not there is a net charge on the sphere? Explain your reasoning.

Q23.19 A conductor that carries a net charge Q has a hollow, empty cavity in its interior. Does the potential vary from point to point within the material of the conductor? What about within the cavity? How does the potential inside the cavity compare to the potential within the material of the conductor?

Q23.20 A high-voltage dc power line falls on a car, so the entire metal body of the car is at a potential of 10,000 V with respect to the ground. What happens to the occupants (a) when they are sitting in the car and (b) when they step out of the car? Explain your reasoning.

Q23.21 When a thunderstorm is approaching, sailors at sea sometimes observe a phenomenon called "St. Elmo's fire," a bluish flickering light at the tips of masts. What causes this? Why does it occur at the tips of masts? Why is the effect most pronounced when the masts are wet? (*Hint:* Seawater is a good conductor of electricity.)

Q23.22 A positive point charge is placed near a very large conducting plane. A professor of physics asserted that the field caused by this configuration is the same as would be obtained by removing the plane and placing a negative point charge of equal magnitude in the mirror-image position behind the initial position of the plane. Is this correct? Why or why not? (*Hint:* Inspect Fig. 23.23b.)

EXERCISES

Section 23.1 Electric Potential Energy

23.1 •• A point charge $q_1 = +2.40\ \mu\text{C}$ is held stationary at the origin. A second point charge $q_2 = -4.30\ \mu\text{C}$ moves from the point $x = 0.150$ m, $y = 0$ to the point $x = 0.250$ m, $y = 0.250$ m. How much work is done by the electric force on q_2?

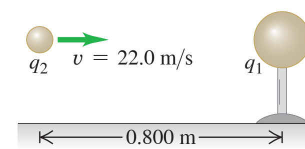
23.2 • A point charge q_1 is held stationary at the origin. A second charge q_2 is placed at point a, and the electric potential energy of the pair of charges is $+5.4 \times 10^{-8}$ J. When the second charge is moved to point b, the electric force on the charge does -1.9×10^{-8} J of work. What is the electric potential energy of the pair of charges when the second charge is at point b?

23.3 •• **Energy of the Nucleus.** How much work is needed to assemble an atomic nucleus containing three protons (such as Li) if we model it as an equilateral triangle of side 2.00×10^{-15} m with a proton at each vertex? Assume the protons started from very far away.

23.4 •• (a) How much work would it take to push two protons very slowly from a separation of 2.00×10^{-10} m (a typical atomic distance) to 3.00×10^{-15} m (a typical nuclear distance)? (b) If the protons are both released from rest at the closer distance in part (a), how fast are they moving when they reach their original separation?

23.5 •• A small metal sphere, carrying a net charge of $q_1 = -2.80\ \mu\text{C}$, is held in a stationary position by insulating supports. A second small metal sphere, with a net charge of $q_2 = -7.80\ \mu\text{C}$ and mass 1.50 g, is projected toward q_1. When the two spheres are 0.800 m apart, q_2 is moving toward q_1 with speed 22.0 m/s (**Fig. E23.5**). Assume that the two spheres can be treated as point charges. You can ignore the force of gravity. (a) What is the speed of q_2 when the spheres are 0.400 m apart? (b) How close does q_2 get to q_1?

Figure **E23.5**



23.6 •• **BIO Energy of DNA Base Pairing.** (See Exercise 21.18.) (a) Calculate the electric potential energy of the adenine–thymine bond, using the same combinations of molecules (O–H–N and N–H–N) as in Exercise 21.18. (b) Compare this energy with the potential energy of the proton–electron pair in the hydrogen atom.

23.7 •• Two protons, starting several meters apart, are aimed directly at each other with speeds of 2.00×10^5 m/s, measured relative to the earth. Find the maximum electric force that these protons will exert on each other.

23.8 •• Three equal $1.20\ \mu\text{C}$ point charges are placed at the corners of an equilateral triangle with sides 0.400 m long. What is the potential energy of the system? (Take as zero the potential energy of the three charges when they are infinitely far apart.)

23.9 •• Two protons are released from rest when they are 0.750 nm apart. (a) What is the maximum speed they will reach? When does this speed occur? (b) What is the maximum acceleration they will achieve? When does this acceleration occur?

23.10 •• Four electrons are located at the corners of a square 10.0 nm on a side, with an alpha particle at its midpoint. How much work is needed to move the alpha particle to the midpoint of one of the sides of the square?

Section 23.2 Electric Potential

23.11 •• **CALC** Points a and b lie in a region where the y-component of the electric field is $E_y = \alpha + \beta/y^2$. The constants in this expression have the values $\alpha = 600$ N/C and $\beta = 5.00\ \text{N} \cdot \text{m}^2/\text{C}$. Points a and b are on the $+y$-axis. Point a is at $y = 2.00$ cm and point b is at $y = 3.00$ cm. What is the potential difference $V_a - V_b$ between these two points and which point, a or b, is at higher potential?

23.12 • An object with charge $q = -6.00 \times 10^{-9}$ C is placed in a region of uniform electric field and is released from rest at point A. After the charge has moved to point B, 0.500 m to the right, it has kinetic energy 3.00×10^{-7} J. (a) If the electric potential at point A is $+30.0$ V, what is the electric potential at point B? (b) What are the magnitude and direction of the electric field?

23.13 • A small particle has charge $-5.00\ \mu\text{C}$ and mass 2.00×10^{-4} kg. It moves from point A, where the electric potential is $V_A = +200$ V, to point B, where the electric potential is $V_B = +800$ V. The electric force is the only force acting on the particle. The particle has speed 5.00 m/s at point A. What is its speed at point B? Is it moving faster or slower at B than at A? Explain.

23.14 • A particle with charge $+4.20$ nC is in a uniform electric field $\vec{E}$ directed to the left. The charge is released from rest and moves to the left; after it has moved 6.00 cm, its kinetic energy is $+2.20 \times 10^{-6}$ J. What are (a) the work done by the electric force, (b) the potential of the starting point with respect to the end point, and (c) the magnitude of $\vec{E}$?

23.15 • A charge of 28.0 nC is placed in a uniform electric field that is directed vertically upward and has a magnitude of 4.00×10^4 V/m. What work is done by the electric force when the charge moves (a) 0.450 m to the right; (b) 0.670 m upward; (c) 2.60 m at an angle of 45.0° downward from the horizontal?

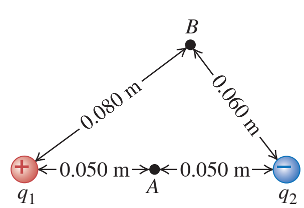
23.16 • Two stationary point charges +3.00 nC and +2.00 nC are separated by a distance of 50.0 cm. An electron is released from rest at a point midway between the two charges and moves along the line connecting the two charges. What is the speed of the electron when it is 10.0 cm from the +3.00 nC charge?

23.17 •• Point charges $q_1 = +2.00\ \mu C$ and $q_2 = -2.00\ \mu C$ are placed at adjacent corners of a square for which the length of each side is 3.00 cm. Point a is at the center of the square, and point b is at the empty corner closest to q_2. Take the electric potential to be zero at a distance far from both charges. (a) What is the electric potential at point a due to q_1 and q_2? (b) What is the electric potential at point b? (c) A point charge $q_3 = -5.00\ \mu C$ moves from point a to point b. How much work is done on q_3 by the electric forces exerted by q_1 and q_2? Is this work positive or negative?

23.18 • Two point charges of equal magnitude Q are held a distance d apart. Consider only points on the line passing through both charges; take $V = 0$ at infinity. (a) If the two charges have the same sign, find the location of all points (if there are any) at which (i) the potential is zero (is the electric field zero at these points?), and (ii) the electric field is zero (is the potential zero at these points?). (b) Repeat part (a) for two point charges having opposite signs.

23.19 • Two point charges $q_1 = +2.40$ nC and $q_2 = -6.50$ nC are 0.100 m apart. Point A is midway between them; point B is 0.080 m from q_1 and 0.060 m from q_2 (**Fig. E23.19**). Take the electric potential to be zero at infinity. Find (a) the potential at point A; (b) the potential at point B; (c) the work done by the electric field on a charge of 2.50 nC that travels from point B to point A.

Figure **E23.19**



23.20 •• (a) An electron is to be accelerated from 3.00×10^6 m/s to 8.00×10^6 m/s. Through what potential difference must the electron pass to accomplish this? (b) Through what potential difference must the electron pass if it is to be slowed from 8.00×10^6 m/s to a halt?

23.21 •• Points A and B lie within a region of space where there is a uniform electric field that has no x- or z-component; only the y-component E_y is nonzero. Point A is at $y = 8.00$ cm and point B is at $y = 15.0$ cm. The potential difference between B and A is $V_B - V_A = +12.0$ V, so point B is at higher potential than point A. (a) Is E_y positive or negative? (b) What is the magnitude of the electric field? (c) Point C has coordinates $x = 5.00$ cm, $y = 5.00$ cm. What is the potential difference between points B and C?

23.22 • At a certain distance from a point charge, the potential and electric-field magnitude due to that charge are 4.98 V and 16.2 V/m, respectively. (Take $V = 0$ at infinity.) (a) What is the distance to the point charge? (b) What is the magnitude of the charge? (c) Is the electric field directed toward or away from the point charge?

23.23 • A uniform electric field has magnitude E and is directed in the negative x-direction. The potential difference between point a (at $x = 0.60$ m) and point b (at $x = 0.90$ m) is 240 V. (a) Which point, a or b, is at the higher potential? (b) Calculate the value of E. (c) A negative point charge $q = -0.200\ \mu C$ is moved from b to a. Calculate the work done on the point charge by the electric field.

23.24 •• A small sphere with charge $q = -5.00\ \mu C$ is moving in a uniform electric field that has no y- or z-component. The only force on the sphere is the force exerted by the electric field. Point A is on the x-axis at $x = -0.400$ m, and point B is at the origin. At point A the sphere has kinetic energy $K_A = 8.00 \times 10^{-4}$ J, and at point B its kinetic energy is $K_B = 3.00 \times 10^{-4}$ J. (a) What is the potential difference $V_{AB} = V_A - V_B$? Which point, A or B, is at higher potential? (b) What are the magnitude and direction of the electric field?

23.25 •• Identical point charges q_1 and q_2 each have positive charge $+6.00\ \mu C$. Charge q_1 is held fixed on the x-axis at $x = +0.400$ m, and q_2 is held fixed on the x-axis at $x = -0.400$ m. A small sphere has charge $Q = -0.200\ \mu C$ and mass 12.0 g. The sphere is initially very far from the origin. It is released from rest and moves along the y-axis toward the origin. (a) As the sphere moves from very large y to $y = 0$, how much work is done on it by the resultant force exerted by q_1 and q_2? (b) If the only force acting on the sphere is the force exerted by the point charges, what is its speed when it reaches the origin?

Section 23.3 Calculating Electric Potential

23.26 •• A solid conducting sphere of radius 5.00 cm carries a net charge. To find the value of the charge, you measure the potential difference $V_{AB} = V_A - V_B$ between point A, which is 8.00 cm from the center of the sphere, and point B, which is a distance r from the center of the sphere. You repeat these measurements for several values of $r > 8.00$ cm. When you plot your data as V_{AB} versus $1/r$, the values lie close to a straight line with slope $-18.0\ \text{V} \cdot \text{m}$. What does your data give for the net charge on the sphere? Is the net charge positive or negative?

23.27 •• A thin spherical shell with radius $R_1 = 3.00$ cm is concentric with a larger thin spherical shell with radius $R_2 = 5.00$ cm. Both shells are made of insulating material. The smaller shell has charge $q_1 = +6.00$ nC distributed uniformly over its surface, and the larger shell has charge $q_2 = -9.00$ nC distributed uniformly over its surface. Take the electric potential to be zero at an infinite distance from both shells. (a) What is the electric potential due to the two shells at the following distance from their common center: (i) $r = 0$; (ii) $r = 4.00$ cm; (iii) $r = 6.00$ cm? (b) What is the magnitude of the potential difference between the surfaces of the two shells? Which shell is at higher potential: the inner shell or the outer shell?

23.28 • A total electric charge of 3.50 nC is distributed uniformly over the surface of a metal sphere with a radius of 24.0 cm. If the potential is zero at a point at infinity, find the value of the potential at the following distances from the center of the sphere: (a) 48.0 cm; (b) 24.0 cm; (c) 12.0 cm.

23.29 •• A uniformly charged, thin ring has radius 15.0 cm and total charge +24.0 nC. An electron is placed on the ring's axis a distance 30.0 cm from the center of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest. (a) Describe the subsequent motion of the electron. (b) Find the speed of the electron when it reaches the center of the ring.

23.30 • A solid conducting sphere has net positive charge and radius $R = 0.400$ m. At a point 1.20 m from the center of the sphere, the electric potential due to the charge on the sphere is 24.0 V. Assume that $V = 0$ at an infinite distance from the sphere. What is the electric potential at the center of the sphere?

23.31 • Charge $Q = 5.00\ \mu C$ is distributed uniformly over the volume of an insulating sphere that has radius $R = 12.0$ cm. A small sphere with charge $q = +3.00\ \mu C$ and mass 6.00×10^{-5} kg is projected toward the center of the large sphere from an initial large distance. The large sphere is held at a fixed position and the small sphere can be treated as a point charge. What minimum speed must the small sphere have in order to come within 8.00 cm of the surface of the large sphere?

23.32 •• An infinitely long line of charge has linear charge density 5.00×10^{-12} C/m. A proton (mass 1.67×10^{-27} kg, charge $+1.60 \times 10^{-19}$ C) is 18.0 cm from the line and moving directly toward the line at 3.50×10^{3} m/s. (a) Calculate the proton's initial kinetic energy. (b) How close does the proton get to the line of charge?

23.33 •• A very long insulating cylindrical shell of radius 6.00 cm carries charge of linear density 8.50 μC/m spread uniformly over its outer surface. What would a voltmeter read if it were connected between (a) the surface of the cylinder and a point 4.00 cm above the surface, and (b) the surface and a point 1.00 cm from the central axis of the cylinder?

23.34 •• A very long insulating cylinder of charge of radius 2.50 cm carries a uniform linear density of 15.0 nC/m. If you put one probe of a voltmeter at the surface, how far from the surface must the other probe be placed so that the voltmeter reads 175 V?

23.35 •• A very small sphere with positive charge $q = +8.00\ \mu$C is released from rest at a point 1.50 cm from a very long line of uniform linear charge density $\lambda = +3.00\ \mu$C/m. What is the kinetic energy of the sphere when it is 4.50 cm from the line of charge if the only force on it is the force exerted by the line of charge?

23.36 • CP Two large, parallel conducting plates carrying opposite charges of equal magnitude are separated by 2.20 cm. (a) If the surface charge density for each plate has magnitude 47.0 nC/m^2, what is the magnitude of $\vec{E}$ in the region between the plates? (b) What is the potential difference between the two plates? (c) If the separation between the plates is doubled while the surface charge density is kept constant at the value in part (a), what happens to the magnitude of the electric field and to the potential difference?

23.37 • Two large, parallel, metal plates carry opposite charges of equal magnitude. They are separated by 45.0 mm, and the potential difference between them is 360 V. (a) What is the magnitude of the electric field (assumed to be uniform) in the region between the plates? (b) What is the magnitude of the force this field exerts on a particle with charge +2.40 nC? (c) Use the results of part (b) to compute the work done by the field on the particle as it moves from the higher-potential plate to the lower. (d) Compare the result of part (c) to the change of potential energy of the same charge, computed from the electric potential.

23.38 • BIO **Electrical Sensitivity of Sharks.** Certain sharks can detect an electric field as weak as 1.0 μV/m. To grasp how weak this field is, if you wanted to produce it between two parallel metal plates by connecting an ordinary 1.5 V AA battery across these plates, how far apart would the plates have to be?

23.39 • The electric field at the surface of a charged, solid, copper sphere with radius 0.200 m is 3800 N/C, directed toward the center of the sphere. What is the potential at the center of the sphere, if we take the potential to be zero infinitely far from the sphere?

23.40 •• (a) How much excess charge must be placed on a copper sphere 25.0 cm in diameter so that the potential of its center is 3.75 kV? Take the point where $V = 0$ to be infinitely far from the sphere. (b) What is the potential of the sphere's surface?

Section 23.4 Equipotential Surfaces and Section 23.5 Potential Gradient

23.41 •• CALC A metal sphere with radius r_a is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius r_b. There is charge $+q$ on the inner sphere and charge $-q$ on the outer spherical shell. (a) Calculate the potential $V(r)$ for (i) $r < r_a$; (ii) $r_a < r < r_b$; (iii) $r > r_b$. (*Hint:* The net potential is the sum of the potentials due to the individual spheres.) Take V to be zero when r is infinite. (b) Show that the potential of the inner sphere with respect to the outer is

$$V_{ab} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(c) Use Eq. (23.23) and the result from part (a) to show that the electric field at any point between the spheres has magnitude

$$E(r) = \frac{V_{ab}}{(1/r_a - 1/r_b)}\frac{1}{r^2}$$

(d) Use Eq. (23.23) and the result from part (a) to find the electric field at a point outside the larger sphere at a distance r from the center, where $r > r_b$. (e) Suppose the charge on the outer sphere is not $-q$ but a negative charge of different magnitude, say $-Q$. Show that the answers for parts (b) and (c) are the same as before but the answer for part (d) is different.

23.42 • A very large plastic sheet carries a uniform charge density of -6.00 nC/m^2 on one face. (a) As you move away from the sheet along a line perpendicular to it, does the potential increase or decrease? How do you know, without doing any calculations? Does your answer depend on where you choose the reference point for potential? (b) Find the spacing between equipotential surfaces that differ from each other by 1.00 V. What type of surfaces are these?

23.43 • CALC In a certain region of space, the electric potential is $V(x, y, z) = Axy - Bx^2 + Cy$, where A, B, and C are positive constants. (a) Calculate the x-, y-, and z-components of the electric field. (b) At which points is the electric field equal to zero?

23.44 • CALC In a certain region of space the electric potential is given by $V = +Ax^2y - Bxy^2$, where $A = 5.00$ V/m^3 and $B = 8.00$ V/m^3. Calculate the magnitude and direction of the electric field at the point in the region that has coordinates $x = 2.00$ m, $y = 0.400$ m, and $z = 0$.

23.45 • A metal sphere with radius $r_a = 1.20$ cm is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius $r_b = 9.60$ cm. Charge $+q$ is put on the inner sphere and charge $-q$ on the outer spherical shell. The magnitude of q is chosen to make the potential difference between the spheres 500 V, with the inner sphere at higher potential. (a) Use the result of Exercise 23.41(b) to calculate q. (b) With the help of the result of Exercise 23.41(a), sketch the equipotential surfaces that correspond to 500, 400, 300, 200, 100, and 0 V. (c) In your sketch, show the electric field lines. Are the electric field lines and equipotential surfaces mutually perpendicular? Are the equipotential surfaces closer together when the magnitude of $\vec{E}$ is largest?

PROBLEMS

23.46 • CP A point charge $q_1 = +5.00\ \mu$C is held fixed in space. From a horizontal distance of 6.00 cm, a small sphere with mass 4.00×10^{-3} kg and charge $q_2 = +2.00\ \mu$C is fired toward the fixed charge with an initial speed of 40.0 m/s. Gravity can be neglected. What is the acceleration of the sphere at the instant when its speed is 25.0 m/s?

23.47 ••• A point charge $q_1 = 4.00$ nC is placed at the origin, and a second point charge $q_2 = -3.00$ nC is placed on the x-axis at $x = +20.0$ cm. A third point charge $q_3 = 2.00$ nC is to be placed on the x-axis between q_1 and q_2. (Take as zero the potential energy of the three charges when they are infinitely far apart.) (a) What is the potential energy of the system of the three charges if q_3 is placed at $x = +10.0$ cm? (b) Where should q_3 be placed to make the potential energy of the system equal to zero?

23.48 •• A point charge +8.00 nC is on the $-x$-axis at $x = -0.200$ m, and a point charge -4.00 nC is on the $+x$-axis at $x = 0.200$ m. (a) In addition to $x = \pm\infty$, at what point on the x-axis is the resultant field of the two charges equal to zero? (b) Let $V = 0$ at $x = \pm\infty$. At what two other points on the x-axis is the total electric potential due to the two charges equal to zero? (c) Is $E = 0$ at either of the points in part (b) where $V = 0$? Explain.

23.49 •• A very long uniform line of charge with charge per unit length $\lambda = +5.00\ \mu C/m$ lies along the x-axis, with its midpoint at the origin. A very large uniform sheet of charge is parallel to the xy-plane; the center of the sheet is at $z = +0.600$ m. The sheet has charge per unit area $\sigma = +8.00\ \mu C/m^2$, and the center of the sheet is at $x = 0$, $y = 0$. Point A is on the z-axis at $z = +0.300$ m, and point B is on the z-axis at $z = -0.200$ m. What is the potential difference $V_{AB} = V_A - V_B$ between points A and B? Which point, A or B, is at higher potential?

23.50 ••• A small sphere with mass 5.00×10^{-7} kg and charge $+7.00\ \mu C$ is released from rest a distance of 0.400 m above a large horizontal insulating sheet of charge that has uniform surface charge density $\sigma = +8.00\ pC/m^2$. Using energy methods, calculate the speed of the sphere when it is 0.100 m above the sheet.

23.51 •• A gold nucleus has a radius of 7.3×10^{-15} m and a charge of $+79e$. Through what voltage must an alpha particle, with charge $+2e$, be accelerated so that it has just enough energy to reach a distance of 2.0×10^{-14} m from the surface of a gold nucleus? (Assume that the gold nucleus remains stationary and can be treated as a point charge.)

23.52 •• CP A proton and an alpha particle are released from rest when they are 0.225 nm apart. The alpha particle (a helium nucleus) has essentially four times the mass and two times the charge of a proton. Find the maximum *speed* and maximum *acceleration* of each of these particles. When do these maxima occur: just following the release of the particles or after a very long time?

23.53 • A particle with charge +7.60 nC is in a uniform electric field directed to the left. Another force, in addition to the electric force, acts on the particle so that when it is released from rest, it moves to the right. After it has moved 8.00 cm, the additional force has done 6.50×10^{-5} J of work and the particle has 4.35×10^{-5} J of kinetic energy. (a) What work was done by the electric force? (b) What is the potential of the starting point with respect to the end point? (c) What is the magnitude of the electric field?

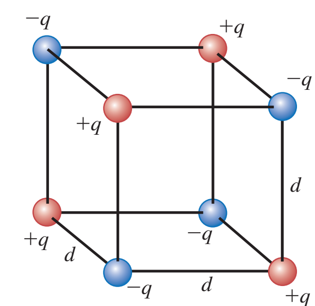
23.54 •• Identical charges $q = +5.00\ \mu C$ are placed at opposite corners of a square that has sides of length 8.00 cm. Point A is at one of the empty corners, and point B is at the center of the square. A charge $q_0 = -3.00\ \mu C$ is placed at point A and moves along the diagonal of the square to point B. (a) What is the magnitude of the net electric force on q_0 when it is at point A? Sketch the placement of the charges and the direction of the net force. (b) What is the magnitude of the net electric force on q_0 when it is at point B? (c) How much work does the electric force do on q_0 during its motion from A to B? Is this work positive or negative? When it goes from A to B, does q_0 move to higher potential or to lower potential?

23.55 •• CALC A vacuum tube diode consists of concentric cylindrical electrodes, the negative cathode and the positive anode. Because of the accumulation of charge near the cathode, the electric potential between the electrodes is given by $V(x) = Cx^{4/3}$ where x is the distance from the cathode and C is a constant, characteristic of a particular diode and operating conditions. Assume that the distance between the cathode and anode is 13.0 mm and the potential difference between electrodes is 240 V. (a) Determine the value of C. (b) Obtain a formula for the electric field between the electrodes as a function of x. (c) Determine the force on an electron when the electron is halfway between the electrodes.

23.56 •• When you scuff your feet on a carpet, you gain electrons and become negatively charged. If you then place your finger near a metallic surface, such as a doorknob, an electric field develops between your finger and the doorknob. As your finger gets closer to the surface, the magnitude of the electric field increases. When it exceeds a threshold of 3 MV/m, the air breaks down, creating a small "lightning" strike, which you feel as a shock. (a) Estimate the distance between your finger and a doorknob at the point you feel the shock. (b) Using this estimate and the threshold field strength of 3 MV/m, find the potential between your finger and the doorknob. (c) At this (small) distance, we can treat the tip of your finger and the end of the doorknob as infinite planar surfaces with opposite charge density. Estimate that density. (d) Estimate the effective area of your fingertip as presented to the doorknob. (e) Use these results to estimate the magnitude of charge on your finger. (f) Assuming that your net excess charge has built up on your finger, attracted there by the doorknob, estimate the number of electrons that you lost while scuffing your feet.

23.57 •• An Ionic Crystal. Figure P23.57 shows eight point charges arranged at the corners of a cube with sides of length d. The values of the charges are $+q$ and $-q$, as shown. This is a model of one cell of a cubic ionic crystal. In sodium chloride (NaCl), for instance, the positive ions are Na^+ and the negative ions are Cl^-. (a) Calculate the potential energy U of this arrangement. (Take as zero the potential energy of the eight charges when they are infinitely far apart.) (b) In part (a), you should have found that $U < 0$. Explain the relationship between this result and the observation that such ionic crystals exist in nature.

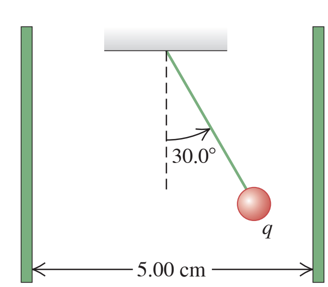
Figure P23.57



23.58 •• Electrical power is transmitted to our homes by overhead wires strung between poles. A typical residential utility line has a maximum potential of 22 kV relative to ground. We can treat this potential as constant and model it as generated by a net charge distributed on the wire. (a) Estimate the height of an electrical transmission line. (b) Treat the electrical wire as a long conducting cylinder with a radius of 2 cm. If the potential between the surface of the wire and a position directly beneath the wire is 22 kV, what is the linear charge density on the wire? (c) Use your estimate of the linear charge density to estimate the strength of the electric field on the ground beneath the wire.

23.59 •• CP A small sphere with mass 1.50 g hangs by a thread between two very large parallel vertical plates 5.00 cm apart (**Fig. P23.59**). The plates are insulating and have uniform surface charge densities $+\sigma$ and $-\sigma$. The charge on the sphere is $q = 8.90 \times 10^{-6}$ C. What potential difference between the plates will cause the thread to assume an angle of 30.0° with the vertical?

Figure P23.59



23.60 •• Two spherical shells have a common center. The inner shell has radius $R_1 = 5.00$ cm and charge $q_1 = +3.00 \times 10^{-6}$ C; the outer shell has radius $R_2 = 15.0$ cm and charge $q_2 = -5.00 \times 10^{-6}$ C. Both charges are spread uniformly over the shell surface. What is the electric potential due to the two shells at the following distances from their common center: (a) $r = 2.50$ cm; (b) $r = 10.0$ cm; (c) $r = 20.0$ cm? Take $V = 0$ at a large distance from the shells.

23.61 • CALC Coaxial Cylinders. A long metal cylinder with radius a is supported on an insulating stand on the axis of a long, hollow, metal tube with radius b. The positive charge per unit length on the inner cylinder is λ, and there is an equal negative charge per unit length on the outer cylinder. (a) Calculate the potential $V(r)$ for (i) $r < a$; (ii) $a < r < b$; (iii) $r > b$. (*Hint:* The net potential is the sum of the potentials due to the individual conductors.) Take $V = 0$ at $r = b$. (b) Show that the potential of the inner cylinder with respect to the outer is

$$V_{ab} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a}$$

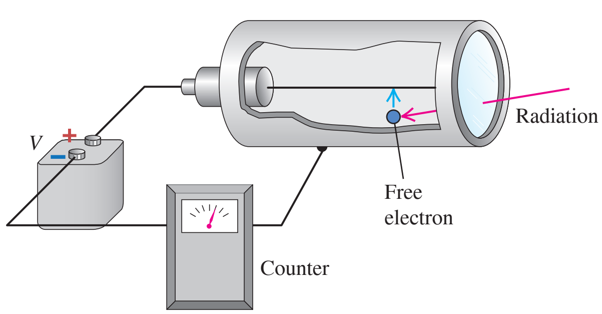
(c) Use Eq. (23.23) and the result from part (a) to show that the electric field at any point between the cylinders has magnitude

$$E(r) = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}$$

(d) What is the potential difference between the two cylinders if the outer cylinder has no net charge?

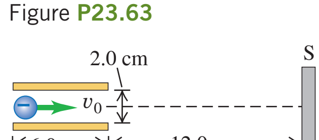
23.62 •• A *Geiger counter* detects radiation such as alpha particles by using the fact that the radiation ionizes the air along its path. A thin wire lies on the axis of a hollow metal cylinder and is insulated from it (**Fig. P23.62**). A large potential difference is established between the wire and the outer cylinder, with the wire at higher potential; this sets up a strong electric field directed radially outward. When ionizing radiation enters the device, it ionizes a few air molecules. The free electrons produced are accelerated by the electric field toward the wire and, on the way there, ionize many more air molecules. Thus a current pulse is produced that can be detected by appropriate electronic circuitry and converted to an audible "click." Suppose the radius of the central wire is 145 μm and the radius of the hollow cylinder is 1.80 cm. What potential difference between the wire and the cylinder produces an electric field of 2.00×10^4 V/m at a distance of 1.20 cm from the axis of the wire? (The wire and cylinder are both very long in comparison to their radii, so the results of Problem 23.61 apply.)

Figure **P23.62**



23.63 • CP Deflection in a CRT. Cathode-ray tubes (CRTs) were often found in oscilloscopes and computer monitors. In **Fig. P23.63** an electron with an initial speed of 6.50×10^6 m/s is projected along the axis midway between the deflection plates of a cathode-ray tube. The potential difference between the two plates is 22.0 V and the lower plate is the one at higher potential. (a) What is the force (magnitude and direction) on the electron when it is between the plates? (b) What is the acceleration of the electron (magnitude and direction) when acted on by the force in part (a)? (c) How far below the axis has the electron moved when it reaches the end of the plates? (d) At what angle with the axis is it moving as it leaves the plates? (e) How far below the axis will it strike the fluorescent screen S?

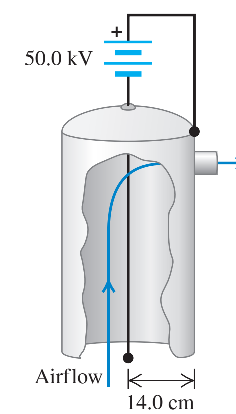
Figure **P23.63**



23.64 •• CP Deflecting Plates of an Oscilloscope. The vertical deflecting plates of a typical classroom oscilloscope are a pair of parallel square metal plates carrying equal but opposite charges. Typical dimensions are about 3.0 cm on a side, with a separation of about 5.0 mm. The potential difference between the plates is 25.0 V. The plates are close enough that we can ignore fringing at the ends. Under these conditions: (a) how much charge is on each plate, and (b) how strong is the electric field between the plates? (c) If an electron is ejected at rest from the negative plate, how fast is it moving when it reaches the positive plate?

23.65 •• *Electrostatic precipitators* use electric forces to remove pollutant particles from smoke, in particular in the smokestacks of coal-burning power plants. One form of precipitator consists of a vertical, hollow, metal cylinder with a thin wire, insulated from the cylinder, running along its axis (**Fig. P23.65**). A large potential difference is established between the wire and the outer cylinder, with the wire at lower potential. This sets up a strong radial electric field directed inward. The field produces a region of ionized air near the wire. Smoke enters the precipitator at the bottom, ash and dust in it pick up electrons, and the charged pollutants are accelerated toward the outer cylinder wall by the electric field. Suppose the radius of the central wire is 90.0 μm, the radius of the cylinder is 14.0 cm, and a potential difference of 50.0 kV is established between the wire and the cylinder. Also assume that the wire and cylinder are both very long in comparison to the cylinder radius, so the results of Problem 23.61 apply. (a) What is the magnitude of the electric field midway between the wire and the cylinder wall? (b) What magnitude of charge must a 30.0 μg ash particle have if the electric field computed in part (a) is to exert a force ten times the weight of the particle?

Figure **P23.65**



23.66 •• CALC A disk with radius R has uniform surface charge density σ. (a) By regarding the disk as a series of thin concentric rings, calculate the electric potential V at a point on the disk's axis a distance x from the center of the disk. Assume that the potential is zero at infinity. (*Hint:* Use the result of Example 23.11 in Section 23.3.) (b) Calculate $-\partial V/\partial x$. Show that the result agrees with the expression for E_x calculated in Example 21.11 (Section 21.5).

23.67 ••• CALC Self-Energy of a Sphere of Charge. A solid sphere of radius R contains a total charge Q distributed uniformly throughout its volume. Find the energy needed to assemble this charge by bringing infinitesimal charges from far away. This energy is called the "self-energy" of the charge distribution. (*Hint:* After you have assembled a charge q in a sphere of radius r, how much energy would it take to add a spherical shell of thickness dr having charge dq? Then integrate to get the total energy.)

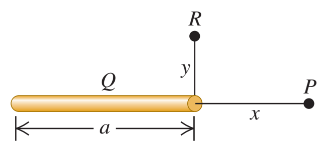
23.68 • CALC A thin insulating rod is bent into a semicircular arc of radius a, and a total electric charge Q is distributed uniformly along the rod. Calculate the potential at the center of curvature of the arc if the potential is assumed to be zero at infinity.

23.69 •• Charge $Q = +4.00\ \mu$C is distributed uniformly over the volume of an insulating sphere that has radius $R = 5.00$ cm. What is the potential difference between the center of the sphere and the surface of the sphere?

23.70 • (a) If a spherical raindrop of radius 0.650 mm carries a charge of -3.60 pC uniformly distributed over its volume, what is the potential at its surface? (Take the potential to be zero at an infinite distance from the raindrop.) (b) Two identical raindrops, each with radius and charge specified in part (a), collide and merge into one larger raindrop. What is the radius of this larger drop, and what is the potential at its surface, if its charge is uniformly distributed over its volume?

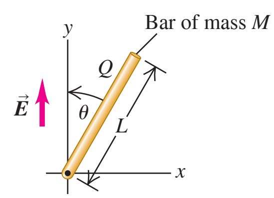
23.71 • CALC Electric charge is distributed uniformly along a thin rod of length a, with total charge Q. Take the potential to be zero at infinity. Find the potential at the following points (**Fig. P23.71**): (a) point P, a distance x to the right of the rod, and (b) point R, a distance y above the right-hand end of the rod. (c) In parts (a) and (b), what does your result reduce to as x or y becomes much larger than a?

Figure **P23.71**



23.72 ••• CP CALC A rigid bar with mass M, length L, and a uniformly distributed positive charge Q is free to pivot about the origin in the presence of a spatially uniform electric field $\vec{E} = E\hat{j}$, as shown in **Fig. P23.72**. Assume that the electric forces are large enough for gravity to be neglected. (a) Write the potential $V(y)$ due to the electric field as a function of y, using the convention $V(0) = V_0$, where V_0 is a constant to be determined. (b) Determine the electric potential energy U of the system as a function of θ and V_0. (*Hint:* $U = \int V\,dq$.) (c) For what value of V_0 does the potential energy vanish when $\theta = 0$? (d) If the bar is released from rest at the position $\theta = 90°$, what is the angular speed of the bar as it passes the position $\theta = 0$? (e) If the bar starts from rest at a small value of θ, with what frequency will the bar oscillate around the position $\theta = 0$?

Figure **P23.72**



23.73 ••• CP A helium nucleus, also known as an α (alpha) particle, consists of two protons and two neutrons and has a diameter of 10^{-15} m = 1 fm. The protons, with a charge of $+e$, are subject to a repulsive Coulomb force. Since the neutrons have zero charge, there must be an attractive force that counteracts the electric repulsion and keeps the protons from flying apart. This so-called strong force plays a central role in particle physics. (a) As a crude model, assume that an α particle consists of two pointlike protons attracted by a Hooke's-law spring with spring constant k, and ignore the neutrons. Assume further that in the absence of other forces, the spring has an equilibrium separation of zero. Write an expression for the potential energy when the protons are separated by distance d. (b) Minimize this potential to find the equilibrium separation d_0 in terms of e and k. (c) If $d_0 = 1.00$ fm, what is the value of k? (d) How much energy is stored in this system, in terms of electron volts? (e) A proton has a mass of 1.67×10^{-27} kg. If the spring were to break, the α particle would disintegrate and the protons would fly off in opposite directions. What would be their ultimate speed?

23.74 • A metal sphere with radius R_1 has a charge Q_1. Take the electric potential to be zero at an infinite distance from the sphere. (a) What are the electric field and electric potential at the surface of the sphere? This sphere is now connected by a long, thin conducting wire to another sphere of radius R_2 that is several meters from the first sphere. Before the connection is made, this second sphere is uncharged. After electrostatic equilibrium has been reached, what are (b) the total charge on each sphere; (c) the electric potential at the surface of each sphere; (d) the electric field at the surface of each sphere? Assume that the amount of charge on the wire is much less than the charge on each sphere.

23.75 • An alpha particle with kinetic energy 9.50 MeV (when far away) collides head-on with a lead nucleus at rest. What is the distance of closest approach of the two particles? (Assume that the lead nucleus remains stationary and may be treated as a point charge. The atomic number of lead is 82. The alpha particle is a helium nucleus, with atomic number 2.)

23.76 • CALC The electric potential V in a region of space is given by

$$V(x, y, z) = A(x^2 - 3y^2 + z^2)$$

where A is a constant. (a) Derive an expression for the electric field $\vec{E}$ at any point in this region. (b) The work done by the field when a 1.50 μC test charge moves from the point $(x, y, z) = (0, 0, 0.250 \text{ m})$ to the origin is measured to be 6.00×10^{-5} J. Determine A. (c) Determine the electric field at the point (0, 0, 0.250 m). (d) Show that in every plane parallel to the xz-plane the equipotential contours are circles. (e) What is the radius of the equipotential contour corresponding to $V = 1280$ V and $y = 2.00$ m?

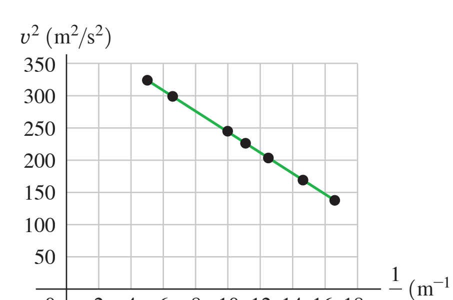
23.77 •• DATA The electric potential in a region that is within 2.00 m of the origin of a rectangular coordinate system is given by $V = Ax^l + By^m + Cz^n + D$, where $A, B, C, D, l, m,$ and n are constants. The units of $A, B, C,$ and D are such that if $x, y,$ and z are in meters, then V is in volts. You measure V and each component of the electric field at four points and obtain these results:

Point	(x, y, z) (m)	V (V)	E_x (V/m)	E_y (V/m)	E_z (V/m)
1	(0, 0, 0)	10.0	0	0	0
2	(1.00, 0, 0)	4.0	12.0	0	0
3	(0, 1.00, 0)	6.0	0	12.0	0
4	(0, 0, 1.00)	8.0	0	0	12.0

(a) Use the data in the table to calculate $A, B, C, D, l, m,$ and n. (b) What are V and the magnitude of E at the points (0, 0, 0), (0.50 m, 0.50 m, 0.50 m), and (1.00 m, 1.00 m, 1.00 m)?

23.78 •• DATA A small, stationary sphere carries a net charge Q. You perform the following experiment to measure Q: From a large distance you fire a small particle with mass $m = 4.00 \times 10^{-4}$ kg and charge $q = 5.00 \times 10^{-8}$ C directly at the center of the sphere. The apparatus you are using measures the particle's speed v as a function of the distance x from the sphere. The sphere's mass is much greater than the mass of the projectile particle, so you assume that the sphere remains at rest. All of the measured values of x are much larger than the radius of either object, so you treat both objects as point particles. You plot your data on a graph of v^2 versus $(1/x)$ (**Fig. P23.78**). The straight line $v^2 = 400 \text{ m}^2/\text{s}^2 - [(15.75 \text{ m}^3/\text{s}^2)/x]$ gives a good fit to the data points. (a) Explain why the graph is a straight line. (b) What is the initial speed v_0 of the particle when it is very far from the sphere? (c) What is Q? (d) How close does the particle get to the sphere? Assume that this distance is much larger than the radii of the particle and sphere, so continue to treat them as point particles and to assume that the sphere remains at rest.

Figure **P23.78**



23.79 ••• DATA The Millikan Oil-Drop Experiment. The charge of an electron was first measured by the American physicist Robert Millikan during 1909–1913. In his experiment, oil was sprayed in very fine drops (about 10^{-4} mm in diameter) into the space between two parallel horizontal plates separated by a distance d. A potential difference V_{AB} was maintained between the plates, causing a downward electric field between them. Some of the oil drops acquired a negative charge because of frictional effects or because of ionization of the surrounding air by x rays or radioactivity. The drops were observed through a microscope. (a) Show that an oil drop of radius r at rest between the plates remained at rest if the magnitude of its charge was

$$q = \frac{4\pi}{3}\frac{\rho r^3 gd}{V_{AB}}$$

where ρ is oil's density. (Ignore the buoyant force of the air.) By adjusting V_{AB} to keep a given drop at rest, Millikan determined the charge on that drop, provided its radius r was known. (b) Millikan's oil drops were much too small to measure their radii directly. Instead, Millikan determined r by cutting off the electric field and measuring the *terminal speed* v_t of the drop as it fell. (We discussed terminal speed in Section 5.3.) The viscous force F on a sphere of radius r moving at speed v through a fluid with viscosity η is given by Stokes's law: $F = 6\pi\eta r v$. When a drop fell at v_t, the viscous force just balanced the drop's weight $w = mg$. Show that the magnitude of the charge on the drop was

$$q = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}$$

(c) You repeat the Millikan oil-drop experiment. Four of your measured values of V_{AB} and v_t are listed in the table:

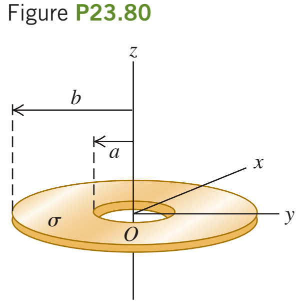
Drop	**1**	**2**	**3**	**4**
V_{AB} (V)	9.16	4.57	12.32	6.28
v_t (10^{-5} m/s)	2.54	0.767	4.39	1.52

In your apparatus, the separation d between the horizontal plates is 1.00 mm. The density of the oil you use is 824 kg/m^3. For the viscosity η of air, use the value 1.81×10^{-5} N·s/m^2. Assume that $g = 9.80$ m/s^2. Calculate the charge q of each drop. (d) If electric charge is *quantized* (that is, exists in multiples of the magnitude of the charge of an electron), then the charge on each drop is $-ne$, where n is the number of excess electrons on each drop. (All four drops in your table have negative charge.) Drop 2 has the smallest magnitude of charge observed in the experiment, for all 300 drops on which measurements were made, so assume that its charge is due to an excess charge of one electron. Determine the number of excess electrons n for each of the other three drops. (e) Use $q = -ne$ to calculate e from the data for each of the four drops, and average these four values to get your best experimental value of e.

CHALLENGE PROBLEMS

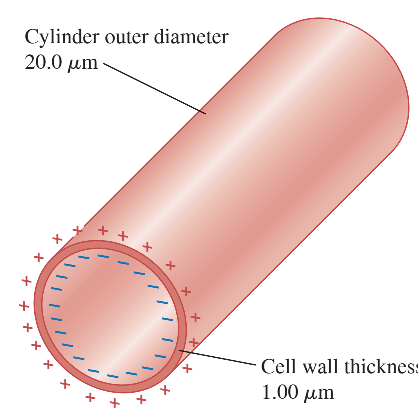
23.80 ••• CALC An annulus with an inner radius of a and an outer radius of b has charge density σ and lies in the xy-plane with its center at the origin, as shown in **Fig. P23.80**. (a) Using the convention that the potential vanishes at infinity, determine the potential at all points on the z-axis. (b) Determine the electric field at all points on the z-axis by differentiating the potential. (c) Show that in the limit $a \to 0$, $b \to \infty$ the electric field reproduces the result obtained in Example 22.7 for an infinite plane sheet of charge. (d) If $a = 5.00$ cm, $b = 10.0$ cm and the total charge on the annulus is 1.00 μC, what is the potential at the origin? (e) If a particle with mass 1.00 g (much less than the mass of the annulus) and charge 1.00 μC is placed at the origin and given the slightest nudge, it will be projected along the z-axis. In this case, what will be its ultimate speed?

Figure **P23.80**



23.81 ••• BIO A heart cell can be modeled as a cylindrical shell that is 100 μm long, with an outer diameter of 20.0 μm and a cell wall thickness of 1.00 μm, as shown in **Fig. P23.81**. Potassium ions move across the cell wall, depositing positive charge on the outer surface and leaving a net negative charge on the inner surface. During the so-called resting phase, the inside of the cell has a potential that is 90.0 mV lower than the potential on its outer surface. (a) If the net charge of the cell is zero, what is the magnitude of the total charge on either cell wall membrane? Ignore edge effects and treat the cell as a very long cylinder. (b) What is the magnitude of the electric field just inside the cell wall? (c) In a subsequent depolarization event, sodium ions move through channels in the cell wall, so that the inner membrane becomes positively charged. At the end of this event, the inside of the cell has a potential that is 20.0 mV higher than the potential outside the cell. If we model this event by charge moving from the outer membrane to the inner membrane, what magnitude of charge moves across the cell wall during this event? (d) If this were done entirely by the motion of sodium ions, Na^+, how many ions have moved?

Figure **P23.81**



23.82 ••• CALC A hollow, thin-walled insulating cylinder of radius R and length L (like the cardboard tube in a roll of toilet paper) has charge Q uniformly distributed over its surface. (a) Calculate the electric potential at all points along the axis of the tube. Take the origin to be at the center of the tube, and take the potential to be zero at infinity. (b) Show that if $L \ll R$, the result of part (a) reduces to the potential on the axis of a ring of charge of radius R. (See Example 23.11 in Section 23.3.) (c) Use the result of part (a) to find the electric field at all points along the axis of the tube.

23.83 ••• CP In experiments in which atomic nuclei collide, head-on collisions like that described in Problem 23.75 do happen, but "near misses" are more common. Suppose the alpha particle in that problem is not "aimed" at the center of the lead nucleus but has an initial nonzero angular momentum (with respect to the stationary lead nucleus) of magnitude $L = p_0 b$, where p_0 is the magnitude of the particle's initial momentum and $b = 1.00 \times 10^{-12}$ m. What is the distance of closest approach? Repeat for $b = 1.00 \times 10^{-13}$ m and $b = 1.00 \times 10^{-14}$ m.

MCAT-STYLE PASSAGE PROBLEMS

Materials Analysis with Ions. *Rutherford backscattering spectrometry* (RBS) is a technique used to determine the structure and composition of materials. A beam of ions (typically helium ions) is accelerated to high energy and aimed at a sample. By analyzing the distribution and energy of the ions that are scattered from (that is, deflected by collisions with) the atoms in the sample, researchers can determine the sample's composition. To accelerate the ions to high energies, a *tandem electrostatic accelerator* may be used. In this device, negative ions (He^-) start at a potential $V = 0$ and are accelerated by a high positive voltage at the midpoint of the accelerator. The high voltage produces a constant electric field in the acceleration tube through which the ions move. When accelerated ions reach the midpoint, the electrons are stripped off, turning the negative ions into doubly positively charged ions (He^{++}). These positive ions are then repelled from the midpoint by the high positive voltage there and continue to accelerate to the far end of the accelerator, where again $V = 0$.

23.84 For a particular experiment, helium ions are to be given a kinetic energy of 3.0 MeV. What should the voltage at the center of the accelerator be, assuming that the ions start essentially at rest? (a) -3.0 MV; (b) $+3.0$ MV; (c) $+1.5$ MV; (d) $+1.0$ MV.

23.85 A helium ion (He^{++}) that comes within about 10 fm of the center of the nucleus of an atom in the sample may induce a nuclear reaction instead of simply scattering. Imagine a helium ion with a kinetic energy of 3.0 MeV heading straight toward an atom at rest in the sample. Assume that the atom stays fixed. What minimum charge can the nucleus of the atom have such that the helium ion gets no closer than 10 fm from the center of the atomic nucleus? (1 fm $= 1 \times 10^{-15}$ m, and e is the magnitude of the charge of an electron or a proton.) (a) $2e$; (b) $11e$; (c) $20e$; (d) $22e$.

23.86 The maximum voltage at the center of a typical tandem electrostatic accelerator is 6.0 MV. If the distance from one end of the acceleration tube to the midpoint is 12 m, what is the magnitude of the average electric field in the tube under these conditions? (a) 41,000 V/m; (b) 250,000 V/m; (c) 500,000 V/m; (d) 6,000,000 V/m.

ANSWERS

Chapter Opening Question ?

(iii) A large, constant potential difference V_{ab} is maintained between the welding tool (a) and the metal pieces to be welded (b). For a given potential difference between two conductors a and b, the smaller the distance d separating the conductors, the greater is the magnitude E of the field between them. Hence d must be small in order for E to be large enough to ionize the gas between the conductors (see Section 23.3) and produce an arc through this gas.

Key Example VARIATION Problems

VP23.2.1 (a) 1.51×10^{-16} J (b) 2.32×10^{-15} J
VP23.2.2 (a) 5.22×10^{-15} J (b) 1.92×10^{-18} C
VP23.2.3 (a) $-e^2/24\pi\epsilon_0 a$ (b) $-7e^2/24\pi\epsilon_0 a$
VP23.2.4 (a) 7.21×10^{-9} C (b) 5.59×10^{-6} J
VP23.7.1 (a) $+25.0$ V (b) $+15.0$ V (c) 0 (d) -25.0 V
VP23.7.2 (a) 3.6×10^3 V (b) 2.2×10^3 V (c) 1.6×10^3 V (d) point (a); no
VP23.7.3 (a) $+133$ V (b) -133 V (c) 1.98×10^5 m/s
VP23.7.4 (a) $Q/8\pi\epsilon_0 R$ (b) the center
VP23.12.1 (a) $qq_0/30\pi\epsilon_0 R$ (b) $-5qq_0/56\pi\epsilon_0 R$ (c) $3qq_0/16\pi\epsilon_0 R$
VP23.12.2 (a) $+16.0$ V (b) 3.20×10^{-8} J (c) lower, 3.58 m/s
VP23.12.3 (a) 1.61×10^{-6} J (b) 5.39×10^{-6} J (c) $K = 3.42 \times 10^{-6}$ J, $v = 41.3$ m/s
VP23.12.4 (a) $dq = (Q/L)dx$ (b) $(Q/4\pi\epsilon_0 L)\ln 2$

Bridging Problem

$$\frac{qQ}{8\pi\epsilon_0 a}\ln\left(\frac{L+a}{L-a}\right)$$