30 Inductance

LEARNING OUTCOMES

In this chapter, you'll learn...

30.1 How a time-varying current in one coil can induce an emf in a second, unconnected coil.

30.2 How to relate the induced emf in a circuit to the rate of change of current in the same circuit.

30.3 How to calculate the energy stored in a magnetic field.

30.4 How to analyze circuits that include both a resistor and an inductor (coil).

30.5 Why electrical oscillations occur in circuits that include both an inductor and a capacitor.

30.6 Why oscillations decay in circuits with an inductor, a resistor, and a capacitor.

You'll need to review...

Take a length of copper wire and wrap it around a pencil to form a coil. If you put this coil in a circuit, the coil behaves quite differently than a straight piece of wire. In an ordinary gasoline-powered car, a coil of this kind makes it possible for the 12 volt car battery to provide thousands of volts to the spark plugs in order for the plugs to fire and make the engine run. Other coils are used to keep fluorescent light fixtures shining. Larger coils placed under city streets are used to control the operation of traffic signals. All of these applications, and many others, involve the *induction* effects that we studied in Chapter 29.

A changing current in a coil induces an emf in an adjacent coil. The coupling between the coils is described by their *mutual inductance*. A changing current in a coil also induces an emf in that same coil. Such a coil is called an *inductor,* and the relationship of current to emf is described by the *inductance* (also called *self-inductance*) of the coil. If a coil is initially carrying a current, energy is released when the current decreases; this principle is used in automotive ignition systems. We'll find that this released energy was stored in the magnetic field caused by the current that was initially in the coil, and we'll look at some of the practical applications of magnetic-field energy.

We'll also take a first look at what happens when an inductor is part of a circuit. In Chapter 31 we'll go on to study how inductors behave in alternating-current circuits, and we'll learn why inductors play an essential role in modern electronics.

30.1 MUTUAL INDUCTANCE

In Section 28.4 we considered the magnetic interaction between two wires carrying *steady* currents; the current in one wire causes a magnetic field, which exerts a force on the current in the second wire. But an additional interaction arises between two circuits when there is a *changing* current in one of the circuits. Consider two neighboring coils of wire, as in **Fig. 30.1**. A current flowing in coil 1 produces a magnetic field $\vec{B}$ and hence a magnetic flux through coil 2. If the current in coil 1 changes, the flux through coil 2 changes as well; according to Faraday's law (Section 29.2), this induces an emf in coil 2. In this way, a change in the current in one circuit can induce a current in a second circuit.

Let's analyze the situation shown in Fig. 30.1 in more detail. We'll use lowercase letters to represent quantities that vary with time; for example, a time-varying current is i, often with a subscript to identify the circuit. In Fig. 30.1 a current i_1 in coil 1 sets up a magnetic field $\vec{B}$, and some of the (blue) field lines pass through coil 2. We denote the magnetic flux through *each* turn of coil 2, caused by the current i_1 in coil 1, as Φ_{B2}. (If the flux is different through different turns of the coil, then Φ_{B2} denotes the *average* flux.) The magnetic field is proportional to i_1, so Φ_{B2} is also proportional to i_1. When i_1 changes, Φ_{B2} changes; this changing flux induces an emf $\mathcal{E}_2$ in coil 2, given by

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt} \tag{30.1}$$

We could represent the proportionality of Φ_{B2} and i_1 in the form $\Phi_{B2} = (\text{constant})i_1$, but instead it is more convenient to include the number of turns N_2 in the relationship. Introducing a proportionality constant M_{21}, called the **mutual inductance** of the two coils, we write

$$N_2 \Phi_{B2} = M_{21} i_1 \tag{30.2}$$

where Φ_{B2} is the flux through a *single* turn of coil 2. From this,

$$N_2 \frac{d\Phi_{B2}}{dt} = M_{21} \frac{di_1}{dt}$$

and we can rewrite Eq. (30.1) as

$$\mathcal{E}_2 = -M_{21} \frac{di_1}{dt} \tag{30.3}$$

That is, a change in the current i_1 in coil 1 induces an emf in coil 2 that is directly proportional to the rate of change of i_1 (**Fig. 30.2**).

We may also write the definition of mutual inductance, Eq. (30.2), as

$$M_{21} = \frac{N_2 \Phi_{B2}}{i_1}$$

If the coils are in vacuum, the flux Φ_{B2} through each turn of coil 2 is directly proportional to the current i_1. Then the mutual inductance M_{21} is a constant that depends only on the geometry of the two coils (the size, shape, number of turns, and orientation of each coil and the separation between the coils). If a magnetic material is present, M_{21} also depends on the magnetic properties of the material. If the material has nonlinear magnetic properties—that is, if the relative permeability K_{m} (defined in Section 28.8) is not constant and magnetization is not proportional to magnetic field—then Φ_{B2} is no longer directly proportional to i_1. In that case the mutual inductance also depends on the value of i_1. In this discussion we'll assume that any magnetic material present has constant K_{m} so that flux *is* directly proportional to current and M_{21} depends on geometry only.

We can repeat our discussion for the opposite case in which a changing current i_2 in coil 2 causes a changing flux Φ_{B1} and an emf $\mathcal{E}_1$ in coil 1. It turns out that the corresponding constant M_{12} is *always* equal to M_{21}, even though in general the two coils are not identical and the flux through them is not the same. We call this common value simply the mutual inductance, denoted by the symbol M without subscripts; it characterizes completely the induced-emf interaction of two coils. Then

Mutually induced emfs:

Induced emf in coil 2 | Rate of change of current in coil 1 | Induced emf in coil 1 | Rate of change of current in coil 2

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M \frac{di_2}{dt} \tag{30.4}$$

Mutual inductance of coils 1 and 2

Figure **30.1** A current i_1 in coil 1 gives rise to a magnetic flux through coil 2. (For clarity, only one-half of each coil is shown.)

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.
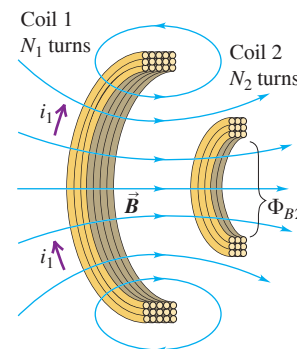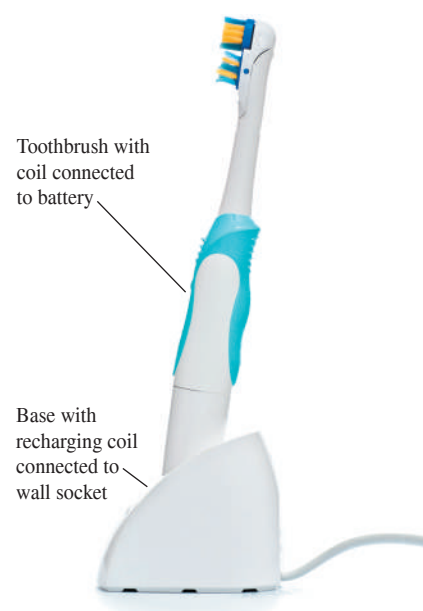
Coil 1
N_1 turns

Coil 2
N_2 turns

i_1

$\vec{B}$

Φ_{B2}

i_1

Figure **30.2** This electric toothbrush makes use of mutual inductance. The base contains a coil that is supplied with alternating current from a wall socket. Even though there is no direct electrical contact between the base and the toothbrush, this varying current induces an emf in a coil within the toothbrush itself, recharging the toothbrush battery.

Toothbrush with coil connected to battery

Base with recharging coil connected to wall socket

The negative signs in Eqs. (30.4) reflect Lenz's law (Section 29.3). The first equation says that a current change in coil 1 causes a flux change through coil 2, inducing an emf in coil 2 that opposes the flux change; in the second equation, the roles of the two coils are interchanged. The mutual inductance M is

Turns in coil 2 · · · ⟶ Magnetic flux through each turn of coil 2 · · · ↓   Turns in coil 1 · ⟶ Magnetic flux through each · · · turn of coil 1

$$\text{Mutual inductance} \cdots\!\!\rightarrow M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2} \qquad (30.5)$$
of coils 1 and 2

Current in coil 1 (causes flux through coil 2)   Current in coil 2 (causes flux through coil 1)

CAUTION **Only a time-varying current induces an emf** Only a *time-varying* current in a coil can induce an emf and hence a current in a second coil. Equations (30.4) show that the induced emf in each coil is directly proportional to the *rate of change* of the current in the other coil, not to the *value* of the current. A steady current in one coil, no matter how strong, cannot induce a current in a neighboring coil. ▮

The SI unit of mutual inductance is called the **henry** (1 H), in honor of the American physicist Joseph Henry (1797–1878), one of the discoverers of electromagnetic induction. From Eq. (30.5), one henry is equal to one weber per ampere. Other equivalent units, obtained by using Eqs. (30.4), are

$$1\ \text{H} = 1\ \text{Wb/A} = 1\ \text{V} \cdot \text{s/A} = 1\ \Omega \cdot \text{s} = 1\ \text{J/A}^2$$

Just as the farad is a rather large unit of capacitance (see Section 24.1), the henry is a rather large unit of mutual inductance. Typical values of mutual inductance can be in the millihenry (mH) or microhenry (μH) range.

Drawbacks and Uses of Mutual Inductance

Mutual inductance can be a nuisance in electric circuits, since variations in current in one circuit can induce unwanted emfs in other nearby circuits. To minimize these effects, multiple-circuit systems must be designed so that M is as small as possible; for example, two coils would be placed far apart.

Happily, mutual inductance also has many useful applications. A *transformer*, used in alternating-current circuits to raise or lower voltages, is fundamentally no different from the two coils shown in Fig. 30.1. A time-varying alternating current in one coil of the transformer produces an alternating emf in the other coil; the value of M, which depends on the geometry of the coils, determines the amplitude of the induced emf in the second coil and hence the amplitude of the output voltage. (We'll describe transformers in more detail in Chapter 31.)

EXAMPLE 30.1 Calculating mutual inductance

WITH √ARIATION PROBLEMS

In one form of Tesla coil (a high-voltage generator popular in science museums), a long solenoid with length l and cross-sectional area A is closely wound with N_1 turns of wire. A coil with N_2 turns surrounds it at its center (**Fig. 30.3**). Find the mutual inductance M.

IDENTIFY and SET UP Mutual inductance occurs here because a current in either coil sets up a magnetic field that causes a flux through the other coil. From Example 28.9 (Section 28.7) we have an expression [Eq. (28.23)] for the field magnitude B_1 at the center of the solenoid (coil 1) in terms of the solenoid current i_1. This allows us to determine the flux through a cross section of the solenoid. Since there is almost no magnetic field outside a very long solenoid, this is also equal to the flux Φ_{B2} through each turn of the *outer* coil (2). We then use Eq. (30.5), in the form $M = N_2 \Phi_{B2}/i_1$, to determine M.

Figure **30.3** A long solenoid with cross-sectional area A and N_1 turns is surrounded at its center by a coil with N_2 turns.

Cross-sectional area A

N_1 turns N_2 turns
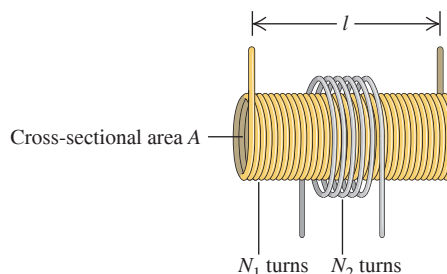
EXECUTE Equation (28.23) is expressed in terms of the number of turns per unit length, which for solenoid (1) is $n_1 = N_1/L$. So

$$B_1 = \mu_0 n_1 i_1 = \frac{\mu_0 N_1 i_1}{l}$$

The flux through a cross section of the solenoid equals $B_1 A$. As we mentioned above, this also equals the flux Φ_{B2} through each turn of the outer coil, independent of its cross-sectional area. From Eq. (30.5), the mutual inductance M is then

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2 B_1 A}{i_1} = \frac{N_2}{i_1} \frac{\mu_0 N_1 i_1}{l} A = \frac{\mu_0 A N_1 N_2}{l}$$

EVALUATE The mutual inductance M of any two coils is proportional to the product $N_1 N_2$ of their numbers of turns. Notice that M depends only on the geometry of the two coils, not on the current.

Here's a numerical example to give you an idea of magnitudes. Suppose $l = 0.50$ m, $A = 10$ cm^2 $= 1.0 \times 10^{-3}$ m^2, $N_1 = 1000$ turns, and $N_2 = 10$ turns. Then

$$M = \frac{(4\pi \times 10^{-7}\ \text{Wb/A} \cdot \text{m})(1.0 \times 10^{-3}\ \text{m}^2)(1000)(10)}{0.50\ \text{m}}$$

$$= 25 \times 10^{-6}\ \text{Wb/A} = 25 \times 10^{-6}\ \text{H} = 25\ \mu\text{H}$$

KEYCONCEPT A current in one coil causes a magnetic flux through an adjacent coil. How much flux is caused in the other coil by a given current in the first coil is determined by the mutual inductance M of the two coils. The value of M doesn't depend on the currents in the coils, only on their geometry.

EXAMPLE 30.2   Emf due to mutual inductance

In Example 30.1, suppose the current i_2 in the outer coil is given by $i_2 = (2.0 \times 10^6\ \text{A/s})t$. (Currents in wires can indeed increase this rapidly for brief periods.) (a) At $t = 3.0\ \mu$s, what is the average magnetic flux through each turn of the solenoid (coil 1) due to the current in the outer coil? (b) What is the induced emf in the solenoid?

IDENTIFY and SET UP In Example 30.1 we found the mutual inductance by relating the current in the solenoid to the flux produced in the outer coil; to do that, we used Eq. (30.5) in the form $M = N_2 \Phi_{B2}/i_1$. Here we are given the current i_2 in the outer coil and want to find the resulting flux Φ_1 in the solenoid. The mutual inductance is the *same* in either case, and we have $M = 25\ \mu$H from Example 30.1. We use Eq. (30.5) in the form $M = N_1 \Phi_{B1}/i_2$ to determine the average flux Φ_{B1} through each turn of the solenoid caused by current i_2 in the outer coil. We then use Eqs. (30.4) to find the emf induced in the solenoid by the time variation of i_2.

EXECUTE (a) At $t = 3.0\ \mu$s $= 3.0 \times 10^{-6}$ s, the current in the outer coil is $i_2 = (2.0 \times 10^6\ \text{A/s})(3.0 \times 10^{-6}\ \text{s}) = 6.0$ A. We solve Eq. (30.5) for the flux Φ_{B1} through each turn of coil 1:

$$\Phi_{B1} = \frac{M i_2}{N_1} = \frac{(25 \times 10^{-6}\ \text{H})(6.0\ \text{A})}{1000} = 1.5 \times 10^{-7}\ \text{Wb}$$

We emphasize that this is an *average* value; the flux can vary considerably between the center and the ends of the solenoid.

(b) We are given $i_2 = (2.0 \times 10^6\ \text{A/s})t$, so $di_2/dt = 2.0 \times 10^6$ A/s; then, from Eqs. (30.4), the induced emf in the solenoid is

$$\mathcal{E}_1 = -M\frac{di_2}{dt} = -(25 \times 10^{-6}\ \text{H})(2.0 \times 10^6\ \text{A/s}) = -50\ \text{V}$$

EVALUATE This is a substantial induced emf in response to a very rapid current change. In an operating Tesla coil, there is a high-frequency alternating current rather than a continuously increasing current as in this example; both di_2/dt and $\mathcal{E}_1$ alternate as well, with amplitudes that can be thousands of times larger than in this example.

KEYCONCEPT When the current in a coil changes, it causes a changing magnetic flux and hence an induced emf in an adjacent coil. The mutual inductance of the two coils determines how much emf is induced in one coil for a given rate of change of the current in the other coil.

TEST YOUR UNDERSTANDING OF SECTION 30.1 Consider the Tesla coil described in Example 30.1. If you make the solenoid out of twice as much wire, so that it has twice as many turns and is twice as long, how much larger is the mutual inductance? (i) M is four times greater; (ii) M is twice as great; (iii) M is unchanged; (iv) M is $\frac{1}{2}$ as great; (v) M is $\frac{1}{4}$ as great.

ANSWER

no effect on this quantity.

duced by the solenoid depends on the number of turns *per unit length*, and the proposed change has

ratio of these quantities, which would remain unchanged. This is because the magnetic field pro-

(N_1) would have *no effect* on the mutual inductance M. Example 30.1 shows that M depends on the

(iii) Doubling both the length of the solenoid (l) and the number of turns of wire in the solenoid

30.2 SELF-INDUCTANCE AND INDUCTORS

In our discussion of mutual inductance we considered two separate, independent circuits: A current in one circuit creates a magnetic field that gives rise to a flux through the second circuit. If the current in the first circuit changes, the flux through the second circuit changes and an emf is induced in the second circuit.

An important related effect occurs in a *single* isolated circuit. A current in a circuit sets up a magnetic field that causes a magnetic flux through the *same* circuit; this flux changes when the current changes. Thus any circuit that carries a varying current has an emf induced in it by the variation in *its own* magnetic field. Such an emf is called a **self-induced emf.** By Lenz's law, a self-induced emf opposes the change in the current that caused the emf and so tends to make it more difficult for variations in current to occur. Hence self-induced emfs can be of great importance whenever there is a varying current.

Self-induced emfs can occur in *any* circuit, since there is always some magnetic flux through the closed loop of a current-carrying circuit. But the effect is greatly enhanced if the circuit includes a coil with N turns of wire (**Fig. 30.4**). As a result of the current i, there is an average magnetic flux Φ_B through each turn of the coil. In analogy to Eq. (30.5) we define the **self-inductance** L of the circuit as

Figure **30.4** The current i in the circuit causes a magnetic field $\vec{B}$ in the coil and hence a flux through the coil.

Self-inductance: If the current i in the coil is changing, the changing flux through the coil induces an emf in the coil.
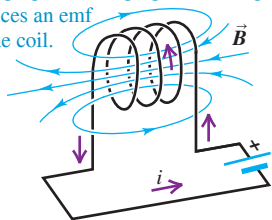
$$\underset{\substack{\text{Self-inductance} \\ \text{(or inductance)} \\ \text{of a coil}}}{} L = \frac{N\Phi_B}{i} \quad \underset{\substack{\text{Number of turns in coil} \\ \text{Flux due to current} \\ \text{through each turn of coil} \\ \text{Current in coil}}}{} \tag{30.6}$$

When there is no danger of confusion with mutual inductance, the self-inductance is called simply the **inductance.** Comparing Eqs. (30.5) and (30.6), we see that the units of self-inductance are the same as those of mutual inductance; the SI unit of self-inductance is the henry.

If the current i changes, so does the flux Φ_B; after we rearrange Eq. (30.6) and differentiate with respect to time, the rates of change are related by

$$N\frac{d\Phi_B}{dt} = L\frac{di}{dt}$$

From Faraday's law for a coil with N turns, Eq. (29.4), the self-induced emf is $\mathcal{E} = -N\,d\Phi_B/dt$, so it follows that

$$\underset{\substack{\text{Self-induced emf} \\ \text{in a circuit}}}{} \mathcal{E} = -L\frac{di}{dt} \quad \underset{\substack{\text{Inductance of circuit} \\ \text{Rate of change of} \\ \text{current in circuit}}}{} \tag{30.7}$$

The minus sign in Eq. (30.7) is a reflection of Lenz's law; it says that the self-induced emf in a circuit opposes any change in the current in that circuit.

Equation (30.7) states that the self-inductance of a circuit is the magnitude of the self-induced emf per unit rate of change of current. This relationship makes it possible to measure an unknown self-inductance: Change the current at a known rate di/dt, measure the induced emf, and take the ratio to determine L.

Inductors as Circuit Elements

A circuit device that is designed to have a particular inductance is called an **inductor,** or a *choke*. The usual circuit symbol for an inductor is

Like resistors and capacitors, inductors are among the indispensable circuit elements of modern electronics. Their purpose is to oppose any variations in the current through the

APPLICATION Inductors, Power Transmission, and Lightning Strikes
If lightning strikes part of an electrical power transmission system, it causes a sudden spike in voltage that can damage the components of the system as well as anything connected to that system (for example, home appliances). To minimize these effects, large inductors are incorporated into the transmission system. These use the principle that an inductor opposes and suppresses any rapid changes in the current.

circuit. An inductor in a direct-current circuit helps to maintain a steady current despite any fluctuations in the applied emf; in an alternating-current circuit, an inductor tends to suppress variations of the current that are more rapid than desired.

To understand the behavior of circuits containing inductors, we need to develop a general principle analogous to Kirchhoff's loop rule (discussed in Section 26.2). To apply that rule, we go around a conducting loop, measuring potential differences across successive circuit elements as we go. The algebraic sum of these differences around any closed loop must be zero because the electric field produced by charges distributed around the circuit is *conservative*. In Section 29.7 we denoted such a conservative field as $\vec{E}_c$.

When an inductor is included in the circuit, the situation changes. The magnetically induced electric field within the coils of the inductor is *not* conservative; as in Section 29.7, we'll denote it by $\vec{E}_n$. We need to think very carefully about the roles of the various fields. Let's assume we are dealing with an inductor whose coils have negligible resistance. Then a negligibly small electric field is required to make charge move through the coils, so the *total* electric field $\vec{E}_c + \vec{E}_n$ within the coils must be zero, even though neither field is individually zero. Because $\vec{E}_c$ is nonzero, there have to be accumulations of charge on the terminals of the inductor and the surfaces of its conductors to produce this field.

Consider the circuit shown in **Fig. 30.5**; the box contains some combination of batteries and variable resistors that enables us to control the current i in the circuit. According to Faraday's law, Eq. (29.10), the line integral of $\vec{E}_n$ around the circuit is the negative of the rate of change of flux through the circuit, which in turn is given by Eq. (30.7). Combining these two relationships, we get

$$\oint \vec{E}_n \cdot d\vec{l} = -L\frac{di}{dt}$$

where we integrate clockwise around the loop (the direction of the assumed current). But $\vec{E}_n$ is different from zero only within the inductor. Therefore the integral of $\vec{E}_n$ around the whole loop can be replaced by its integral only from a to b through the inductor; that is,

$$\int_a^b \vec{E}_n \cdot d\vec{l} = -L\frac{di}{dt}$$

Next, because $\vec{E}_c + \vec{E}_n = 0$ at each point within the inductor coils, $\vec{E}_n = -\vec{E}_c$. So we can rewrite the above equation as

$$\int_a^b \vec{E}_c \cdot d\vec{l} = L\frac{di}{dt}$$

But this integral is just the potential V_{ab} of point a with respect to point b, so

$$V_{ab} = V_a - V_b = L\frac{di}{dt} \tag{30.8}$$

We conclude that there is a genuine potential difference between the terminals of the inductor, associated with conservative, electrostatic forces, even though the electric field associated with magnetic induction is nonconservative. Thus we are justified in using Kirchhoff's loop rule to analyze circuits that include inductors. Equation (30.8) gives the potential difference across an inductor in a circuit.

> **CAUTION** **Self-induced emf opposes changes in current** Note that the self-induced emf does not oppose the current i itself; rather, it opposes any *change* (di/dt) in the current. Thus the circuit behavior of an inductor is quite different from that of a resistor. **Figure 30.6** compares the behaviors of a resistor and an inductor and summarizes the sign relationships. ∎

Figure **30.5** A circuit containing a source of emf and an ideal inductor. The source is variable, so the current i and its rate of change di/dt can be varied.
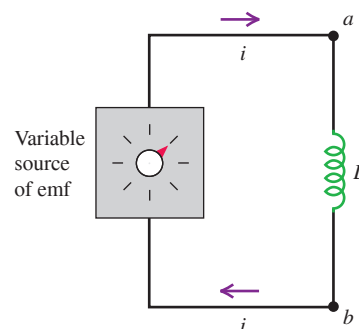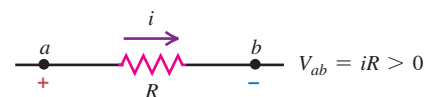

Figure **30.6** (a) The potential difference across a resistor depends on the current, whereas (b), (c), (d) the potential difference across an inductor depends on the rate of change of the current.
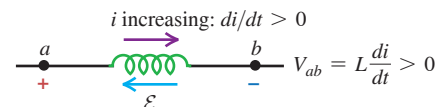
(a) Resistor with current i flowing from a to b: potential drops from a to b.

$$V_{ab} = iR > 0$$

(b) Inductor with *constant* current i flowing from a to b: no potential difference.

i constant: $di/dt = 0$

$$V_{ab} = L\frac{di}{dt} = 0$$
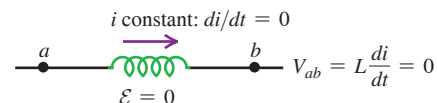
(c) Inductor with *increasing* current i flowing from a to b: potential drops from a to b.

i increasing: $di/dt > 0$

$$V_{ab} = L\frac{di}{dt} > 0$$

(d) Inductor with *decreasing* current i flowing from a to b: potential increases from a to b.

i decreasing: $di/dt < 0$
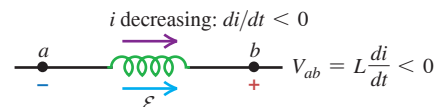

$$V_{ab} = L\frac{di}{dt} < 0$$

Figure **30.7** These fluorescent light tubes are wired in series with an inductor, or ballast, that helps to sustain the current flowing through the tubes.

Applications of Inductors

Because an inductor opposes changes in current, it plays an important role in fluorescent light fixtures (**Fig. 30.7**). In such fixtures, current flows from the wiring into the gas that fills the tube, ionizing the gas and causing it to glow. However, an ionized gas or *plasma* is a highly nonohmic conductor: The greater the current, the more highly ionized the plasma becomes and the lower its resistance. If a sufficiently large voltage is applied to the plasma, the current can grow so much that it damages the circuitry outside the fluorescent tube. To prevent this problem, an inductor or *magnetic ballast* is put in series with the fluorescent tube to keep the current from growing out of bounds.

The ballast also makes it possible for the fluorescent tube to work with the alternating voltage provided by household wiring. This voltage oscillates sinusoidally with a frequency of 60 Hz, so that it goes momentarily to zero 120 times per second. If there were no ballast, the plasma in the fluorescent tube would rapidly deionize when the voltage went to zero and the tube would shut off. With a ballast present, a self-induced emf sustains the current and keeps the tube lit. Magnetic ballasts are also used for this purpose in streetlights (which obtain their light from a glowing mercury or sodium vapor) and in neon lights. (In compact fluorescent lamps, the magnetic ballast is replaced by a more complicated scheme that utilizes transistors, discussed in Chapter 42.)

The self-inductance of a circuit depends on its size, shape, and number of turns. For N turns close together, it is always proportional to N^2. It also depends on the magnetic properties of the material enclosed by the circuit. In the following examples we'll assume that the circuit encloses only vacuum (or air, which from the standpoint of magnetism is essentially vacuum). If, however, the flux is concentrated in a region containing a magnetic material with permeability μ, then in the expression for B we must replace μ_0 (the permeability of vacuum) by $\mu = K_m\mu_0$, as discussed in Section 28.8. If the material is diamagnetic or paramagnetic, this replacement makes very little difference, since K_m is very close to 1. If the material is *ferromagnetic,* however, the difference is of crucial importance. A solenoid wound on a soft iron core having $K_m = 5000$ can have an inductance approximately 5000 times as great as that of the same solenoid with an air core. Ferromagnetic-core inductors are very widely used in a variety of electronic and electric-power applications.

With ferromagnetic materials, the magnetization is in general not a linear function of magnetizing current, especially as saturation is approached. As a result, the inductance is not constant but can depend on current in a fairly complicated way. In our discussion we'll ignore this complication and assume always that the inductance is constant. This is a reasonable assumption even for a ferromagnetic material if the magnetization remains well below the saturation level.

Because automobiles contain steel, a ferromagnetic material, driving an automobile over a coil causes an appreciable increase in the coil's inductance. This effect is **?** used in traffic light sensors, which use a large, current-carrying coil embedded under the road surface near an intersection. The circuitry connected to the coil detects the inductance change as a car drives over. When a preprogrammed number of cars have passed over the coil, the light changes to green to allow the cars through the intersection.

EXAMPLE 30.3 Calculating self-inductance   WITH √ARIATION PROBLEMS

Determine the self-inductance of a toroidal solenoid with cross-sectional area A and mean radius r, closely wound with N turns of wire on a nonmagnetic core (**Fig. 30.8**). Assume that B is uniform across a cross section (that is, neglect the variation of B with distance from the toroid axis).

IDENTIFY and SET UP Our target variable is the self-inductance L of the toroidal solenoid. We can find L by using Eq. (30.6), which requires knowing the flux Φ_B through each turn and the current i in the coil. For this, we use the results of Example 28.10 (Section 28.7), in which we found the magnetic field in the interior of a toroidal solenoid as a function of the current.

Figure **30.8** Determining the self-inductance of a closely wound toroidal solenoid. For clarity, only a few turns of the winding are shown. Part of the toroid has been cut away to show the cross-sectional area A and radius r.

Number of turns = N
(only a few are shown)
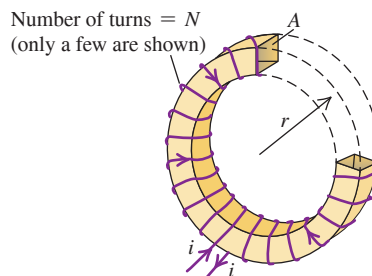
EXECUTE From Eq. (30.6), the self-inductance is $L = N\Phi_B/i$. From Example 28.10, the field magnitude at a distance r from the toroid axis is $B = \mu_0 Ni/2\pi r$. If we assume that the field has this magnitude over the entire cross-sectional area A, then

$$\Phi_B = BA = \frac{\mu_0 NiA}{2\pi r}$$

The flux Φ_B is the same through each turn, and

$$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 A}{2\pi r} \quad \text{(self-inductance of a toroidal solenoid)}$$

EVALUATE Suppose $N = 200$ turns, $A = 5.0 \text{ cm}^2 = 5.0 \times 10^{-4} \text{ m}^2$, and $r = 0.10$ m; then

$$L = \frac{(4\pi \times 10^{-7} \text{ Wb/A} \cdot \text{m})(200)^2(5.0 \times 10^{-4} \text{ m}^2)}{2\pi(0.10 \text{ m})}$$

$$= 40 \times 10^{-6} \text{ H} = 40 \ \mu\text{H}$$

KEYCONCEPT The self-inductance L of a coil (or inductor) tells you how much magnetic flux is produced through that coil by the current that flows in it. Like mutual inductance, the value of L depends only on the coil's geometry, not on how much current it carries.

EXAMPLE 30.4 Calculating self-induced emf

If the current in the toroidal solenoid in Example 30.3 increases uniformly from 0 to 6.0 A in 3.0 μs, find the magnitude and direction of the self-induced emf.

IDENTIFY and SET UP We are given L, the self-inductance, and di/dt, the rate of change of the solenoid current. We find the magnitude of the self-induced emf $\mathcal{E}$ by using Eq. (30.7) and its direction by using Lenz's law.

EXECUTE We have $di/dt = (6.0 \text{ A})/(3.0 \times 10^{-6} \text{ s}) = 2.0 \times 10^6 \text{ A/s}$. From Eq. (30.7), the magnitude of the induced emf is

$$|\mathcal{E}| = L \left| \frac{di}{dt} \right|$$

$$= (40 \times 10^{-6} \text{ H})(2.0 \times 10^6 \text{ A/s}) = 80 \text{ V}$$

The current is increasing, so according to Lenz's law the direction of the emf is opposite to that of the current. This corresponds to the situation in Fig. 30.6c; the emf is in the direction from b to a, like a battery with a as the $+$ terminal and b the $-$ terminal, tending to oppose the current increase from the external circuit.

EVALUATE This example shows that even a small inductance L can give rise to a substantial induced emf if the current changes rapidly.

KEYCONCEPT If the current in a coil changes, the resulting change in magnetic flux through the coil produces an induced emf. The self-inductance of the coil determines how much emf is induced in the coil for a given rate of change of the current in the coil.

TEST YOUR UNDERSTANDING OF SECTION 30.2 Rank the following inductors in order of the potential difference V_{ab}, from most positive to most negative. In each case the inductor has zero resistance and the current flows from point a through the inductor to point b. (i) The current through a 2.0 μH inductor increases from 1.0 A to 2.0 A in 0.50 s; (ii) the current through a 4.0 μH inductor decreases from 3.0 A to 0 in 2.0 s; (iii) the current through a 1.0 μH inductor remains constant at 4.0 A; (iv) the current through a 1.0 μH inductor increases from 0 to 4.0 A in 0.25 s.

ANSWER

❙ **(iv), (i), (iii), (ii)** From Eq. (30.8), the potential difference across the inductor is $V_{ab} = L \, di/dt$. For the four cases we find (i) $V_{ab} = (2.0 \ \mu\text{H})(2.0 \text{ A} - 1.0 \text{ A})/(0.50 \text{ s}) = 4.0 \ \mu\text{V}$; (ii) $V_{ab} = (4.0 \ \mu\text{H})(0 - 3.0 \text{ A})/(2.0 \text{ s}) = -6.0 \ \mu\text{V}$; (iii) $V_{ab} = 0$ because the rate of change of current is zero; and (iv) $V_{ab} = (1.0 \ \mu\text{H})(4.0 \text{ A} - 0)/(0.25 \text{ s}) = 16 \ \mu\text{V}$.

30.3 MAGNETIC-FIELD ENERGY

Establishing a current in an inductor requires an input of energy, and an inductor carrying a current has energy stored in it. Let's see how this comes about. In Fig. 30.5, an increasing current i in the inductor causes an emf $\mathcal{E}$ between its terminals and a corresponding potential difference V_{ab} between the terminals of the source, with point a at higher potential than point b. Thus the source must be adding energy to the inductor, and the instantaneous power P (rate of transfer of energy into the inductor) is $P = V_{ab}i$.

Energy Stored in an Inductor

We can calculate the total energy input U needed to establish a final current I in an inductor with inductance L if the initial current is zero. We assume that the inductor has zero resistance, so no energy is dissipated within the inductor. Let the rate of change of the current i at some instant be di/dt; the current is increasing, so $di/dt > 0$. The voltage between the terminals a and b of the inductor at this instant is $V_{ab} = L\,di/dt$, and the rate P at which energy is being delivered to the inductor (equal to the instantaneous power supplied by the external source) is

$$P = V_{ab}i = Li\frac{di}{dt}$$

The energy dU supplied to the inductor during an infinitesimal time interval dt is $dU = P\,dt$, so

$$dU = Li\,di$$

The total energy U supplied while the current increases from zero to a final value I is

Energy stored ·······→ $U = L \int_0^I i\,di = \frac{1}{2}LI^2$ ←······· Final current (30.9)
in an inductor

Inductance

Integral from initial (zero) value
of instantaneous current to final value

After the current has reached its final steady value I, $di/dt = 0$ and no more energy is input to the inductor. When there is no current, the stored energy U is zero; when the current is I, the energy is $\frac{1}{2}LI^2$.

When the current decreases from I to zero, the inductor acts as a source that supplies a total amount of energy $\frac{1}{2}LI^2$ to the external circuit. If we interrupt the circuit suddenly by opening a switch, the current decreases very rapidly, the induced emf is very large, and the energy may be dissipated in an arc across the switch contacts.

Figure **30.9** A resistor is a device in which energy is irrecoverably dissipated. By contrast, energy stored in a current-carrying inductor can be recovered when the current decreases to zero.

Resistor with current *i*: energy is *dissipated*.
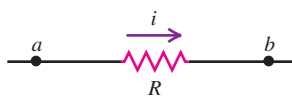
Inductor with current *i*: energy is *stored*.
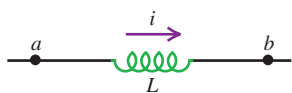
CAUTION **Energy, resistors, and inductors** Don't confuse the behavior of resistors and inductors where energy is concerned (**Fig. 30.9**). Energy flows into a resistor whenever a current passes through it, whether the current is steady or varying; this energy is dissipated in the form of heat. By contrast, energy flows into an ideal, zero-resistance inductor only when the current in the inductor *increases*. This energy is not dissipated; it is stored in the inductor and released when the current *decreases*. When a steady current flows through an inductor, there is no energy flow in or out. ❚

Magnetic Energy Density

The energy in an inductor is actually stored in the magnetic field of the coil, just as the energy of a capacitor is stored in the electric field between its plates. We can develop relationships for magnetic-field energy analogous to those we obtained for electric-field energy in Section 24.3 [Eqs. (24.9) and (24.11)]. We'll concentrate on one simple case, the ideal toroidal solenoid. This system has the advantage that its magnetic field is confined completely to a finite region of space within its core. As in Example 30.3, we assume that the cross-sectional area A is small enough that we can pretend the magnetic field is uniform over the area. The volume V enclosed by the toroidal solenoid is approximately equal to the circumference $2\pi r$ multiplied by the area A: $V = 2\pi rA$. From Example 30.3, the self-inductance of the toroidal solenoid with vacuum within its coils is

$$L = \frac{\mu_0 N^2 A}{2\pi r}$$

From Eq. (30.9), the energy U stored in the toroidal solenoid when the current is I is

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\frac{\mu_0 N^2 A}{2\pi r}I^2$$

The magnetic field and therefore this energy are localized in the volume $V = 2\pi rA$ enclosed by the windings. The energy *per unit volume,* or *magnetic energy density,* is $u = U/V$:

$$u = \frac{U}{2\pi rA} = \tfrac{1}{2}\mu_0 \frac{N^2 I^2}{(2\pi r)^2}$$

We can express this in terms of the magnitude B of the magnetic field inside the toroidal solenoid. From Eq. (28.24) in Example 28.10 (Section 28.7), this is

$$B = \frac{\mu_0 NI}{2\pi r}$$

and so

$$\frac{N^2 I^2}{(2\pi r)^2} = \frac{B^2}{\mu_0^2}$$

When we substitute this into the above equation for u, we finally find the expression for **magnetic energy density** in vacuum:

Magnetic energy density in vacuum $u = \dfrac{B^2 \cdots \text{Magnetic-field magnitude}}{2\mu_0 \cdots \text{Magnetic constant}}$ (30.10)

This is the magnetic analog of the energy per unit volume in an *electric* field in vacuum, $u = \tfrac{1}{2}\epsilon_0 E^2$, which we derived in Section 24.3. As an example, the energy density in the 1.5 T magnetic field of an MRI scanner (see Section 27.7) is $u = B^2/2\mu_0 = (1.5\ \text{T})^2/(2 \times 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}) = 9.0 \times 10^5\ \text{J/m}^3$.

When the material inside the toroid is not vacuum but a material with (constant) magnetic permeability $\mu = K_\text{m}\mu_0$, we replace μ_0 by μ in Eq. (30.10):

Magnetic energy density in a material $u = \dfrac{B^2 \cdots \text{Magnetic-field magnitude}}{2\mu \cdots \text{Permeability of material}}$ (30.11)

Although we have derived Eq. (30.11) for only one special situation, it turns out to be the correct expression for the energy per unit volume associated with *any* magnetic-field configuration in a material with constant permeability. For vacuum, Eq. (30.11) reduces to Eq. (30.10). We'll use the expressions for electric-field and magnetic-field energy in Chapter 32 when we study the energy associated with electromagnetic waves.

Magnetic-field energy plays an important role in the ignition systems of gasoline-powered automobiles. A primary coil of about 250 turns is connected to the car's battery and produces a strong magnetic field. This coil is surrounded by a secondary coil with some 25,000 turns of very fine wire. When it is time for a spark plug to fire (see Fig. 20.5 in Section 20.3), the current to the primary coil is interrupted, the magnetic field quickly drops to zero, and an emf of tens of thousands of volts is induced in the secondary coil. The energy stored in the magnetic field thus goes into a powerful pulse of current that travels through the secondary coil to the spark plug, generating the spark that ignites the fuel–air mixture in the engine's cylinders (**Fig. 30.10**).

APPLICATION **A Magnetic Eruption on the Sun** This composite of two images of the sun shows a coronal mass ejection, a dramatic event in which about 10^{12} kg (a billion tons) of material from the sun's outer atmosphere is ejected into space at speeds of 500 km/s or faster. Such ejections happen at intervals of a few hours to a few days. These immense eruptions are powered by the energy stored in the sun's magnetic field. Unlike the earth's relatively steady magnetic field, the sun's field is constantly changing, and regions of unusually strong field (and hence unusually high magnetic energy density) frequently form. A coronal mass ejection occurs when the energy stored in such a region is suddenly released.
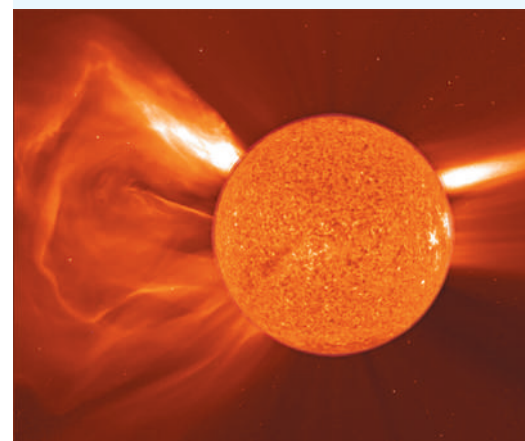

Figure **30.10** The energy required to fire an automobile spark plug is derived from magnetic-field energy stored in the ignition coil.

EXAMPLE 30.5 Storing energy in an inductor

The electric-power industry would like to find efficient ways to store electrical energy generated during low-demand hours to help meet customer requirements during high-demand hours. Could a large inductor be used? What inductance would be needed to store $1.00 \text{ kW} \cdot \text{h}$ of energy in a coil carrying a 200 A current?

IDENTIFY and SET UP We are given the required amount of stored energy U and the current $I = 200$ A. We use Eq. (30.9) to find the self-inductance L.

EXECUTE Here we have $I = 200$ A and $U = 1.00 \text{ kW} \cdot \text{h} = (1.00 \times 10^3 \text{ W})(3600 \text{ s}) = 3.60 \times 10^6$ J. Solving Eq. (30.9) for L, we find

$$L = \frac{2U}{I^2} = \frac{2(3.60 \times 10^6 \text{ J})}{(200 \text{ A})^2} = 180 \text{ H}$$

EVALUATE The required inductance is more than a million times greater than the self-inductance of the toroidal solenoid of Example 30.3. Conventional wires that are to carry 200 A would have to be of large diameter to keep the resistance low and avoid unacceptable energy losses due to I^2R heating. As a result, a 180 H inductor using conventional wire would be very large (room-size). A superconducting inductor can be much smaller, since the resistance of a superconductor is zero and much thinner wires can be used. However, such wires must be kept at low temperature to remain superconducting, and maintaining this temperature itself requires energy. Despite these limitations, superconducting inductors are very efficient for energy storage and are presently in small-scale use by several electric utilities.

KEYCONCEPT Energy is stored in any magnetic field, including the field of a current-carrying inductor. The magnetic energy of an inductor depends on its self-inductance and current.

TEST YOUR UNDERSTANDING OF SECTION 30.3 The current in a solenoid is reversed in direction while keeping the same magnitude. (a) Does this change the magnetic field within the solenoid? (b) Does this change the magnetic energy density in the solenoid?

ANSWER   ❙ (a) **yes**, (b) **no** Reversing the direction of the current has no effect on the magnetic-field magnitude, but it causes the direction of the magnetic field to reverse. It has no effect on the magnetic-field energy density, which is proportional to the square of the *magnitude* of the magnetic field.

30.4 THE *R-L* CIRCUIT

Let's look at some examples of the circuit behavior of an inductor. One thing is clear already; an inductor in a circuit makes it difficult for rapid changes in current to occur, thanks to the effects of self-induced emf. Equation (30.7) shows that the greater the rate of change of current di/dt, the greater the self-induced emf and the greater the potential difference between the inductor terminals. This equation, together with Kirchhoff's rules (see Section 26.2), gives us the principles we need to analyze circuits containing inductors.

PROBLEM-SOLVING STRATEGY 30.1 Inductors in Circuits

IDENTIFY *the relevant concepts:* An inductor is just another circuit element, like a source of emf, a resistor, or a capacitor. One key difference is that when an inductor is included in a circuit, all the voltages, currents, and capacitor charges are in general functions of time, not constants as they have been in most of our previous circuit analysis. But even when the voltages and currents vary with time, Kirchhoff's rules (see Section 26.2) hold at each instant of time.

SET UP *the problem* using the following steps:

1. Follow the procedure given in Problem-Solving Strategy 26.2 (Section 26.2). Draw a circuit diagram and label all quantities, known and unknown. Apply the junction rule immediately to express the currents in terms of as few quantities as possible.

2. Determine which quantities are the target variables.

EXECUTE *the solution* as follows:

1. As in Problem-Solving Strategy 26.2, apply Kirchhoff's loop rule to each loop in the circuit.

2. Review the sign rules given in Problem-Solving Strategy 26.2. To get the correct sign for the potential difference between the terminals of an inductor, apply Lenz's law and the sign rule described in Section 30.2 in connection with Eq. (30.7) and Fig. 30.6. In Kirchhoff's loop rule, when we go through an inductor in the *same* direction as the assumed current, we encounter a voltage *drop* equal to $L\, di/dt$, so the corresponding term in the loop equation is $-L\, di/dt$. When we go through an inductor in the *opposite* direction from the assumed current, the potential difference is reversed and the term to use in the loop equation is $+L\, di/dt$.

3. Solve for the target variables.

EVALUATE *your answer:* Check whether your answer is consistent with the behavior of inductors. By Lenz's law, if the current through an inductor is changing, your result should indicate that the potential difference across the inductor opposes the change.

Current Growth in an *R-L* Circuit

We can learn a great deal about inductor behavior by analyzing the circuit of **Fig. 30.11**. A circuit that includes both a resistor and an inductor, and possibly a source of emf, is called an **R-L circuit.** The inductor helps to prevent rapid changes in current, which can be useful if a steady current is required but the source has a fluctuating emf. The resistor R may be a separate circuit element, or it may be the resistance of the inductor windings; every real-life inductor has some resistance unless it is made of superconducting wire. By closing switch S_1, we can connect the R-L combination to a source with constant emf $\mathcal{E}$. (We assume that the source has zero internal resistance, so the terminal voltage equals $\mathcal{E}$.)

Suppose both switches are open to begin with, and then at some initial time $t = 0$ we close switch S_1. The current cannot change suddenly from zero to some final value, since di/dt and the induced emf in the inductor would both be infinite. Instead, the current begins to grow at a rate that depends on the value of L in the circuit.

Let i be the current at some time t after switch S_1 is closed, and let di/dt be its rate of change at that time. The potential differences v_{ab} (across the resistor) and v_{bc} (across the inductor) are

$$v_{ab} = iR \quad \text{and} \quad v_{bc} = L\frac{di}{dt}$$

Note that if the current is in the direction shown in Fig. 30.11 and is increasing, then both v_{ab} and v_{bc} are positive; a is at a higher potential than b, which in turn is at a higher potential than c. (Compare to Figs. 30.6a and 30.6c.) We apply Kirchhoff's loop rule, starting at the negative terminal and proceeding counterclockwise around the loop:

$$\mathcal{E} - iR - L\frac{di}{dt} = 0 \tag{30.12}$$

Solving this for di/dt, we find that the rate of increase of current is

$$\frac{di}{dt} = \frac{\mathcal{E} - iR}{L} = \frac{\mathcal{E}}{L} - \frac{R}{L}i \tag{30.13}$$

At the instant that switch S_1 is first closed, $i = 0$ and the potential drop across R is zero. The initial rate of change of current is

$$\left(\frac{di}{dt}\right)_{\text{initial}} = \frac{\mathcal{E}}{L}$$

The greater the inductance L, the more slowly the current increases.

As the current increases, the term $(R/L)i$ in Eq. (30.13) also increases, and the *rate* of increase of current given by Eq. (30.13) becomes smaller and smaller. This means that the current is approaching a final, steady-state value I. When the current reaches this value, its rate of increase is zero. Then Eq. (30.13) becomes

$$\left(\frac{di}{dt}\right)_{\text{final}} = 0 = \frac{\mathcal{E}}{L} - \frac{R}{L}I \quad \text{and}$$

$$I = \frac{\mathcal{E}}{R}$$

The *final* current I does not depend on the inductance L; it is the same as it would be if the resistance R alone were connected to the source with emf $\mathcal{E}$.

Figure 30.12 shows the behavior of the current as a function of time. To derive the equation for this curve (that is, an expression for current as a function of time), we proceed just as we did for the charging capacitor in Section 26.4. First we rearrange Eq. (30.13) to the form

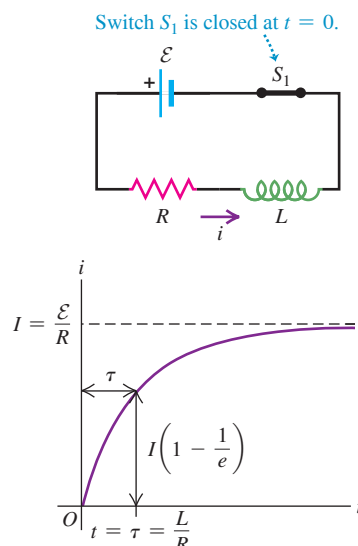$$\frac{di}{i - (\mathcal{E}/R)} = -\frac{R}{L}dt$$

Figure **30.11** An *R-L* circuit.
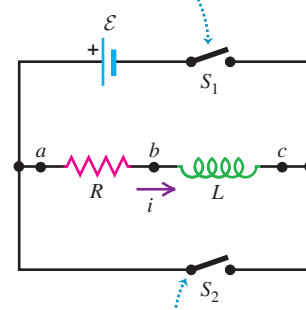
Figure **30.12** Graph of i versus t for growth of current in an *R-L* circuit with an emf in series. The final current is $I = \mathcal{E}/R$; after one time constant τ, the current is $1 - 1/e$ of this value.

Switch S_1 is closed at $t = 0$.

This separates the variables, with i on the left side and t on the right. Then we integrate both sides, renaming the integration variables i' and t' so that we can use i and t as the upper limits. (The lower limit for each integral is zero, corresponding to zero current at the initial time $t = 0$.) We get

$$\int_0^i \frac{di'}{i' - (\mathcal{E}/R)} = -\int_0^t \frac{R}{L}dt'$$

$$\ln\left(\frac{i - (\mathcal{E}/R)}{-\mathcal{E}/R}\right) = -\frac{R}{L}t$$

Now we take exponentials of both sides and solve for i. We leave the details for you to work out; the final result is the equation of the curve in Fig. 30.12:

$$i = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t}) \quad \text{(current in an } R\text{-}L \text{ circuit with emf)} \tag{30.14}$$

Taking the derivative of Eq. (30.14), we find

$$\frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-(R/L)t} \tag{30.15}$$

At time $t = 0$, $i = 0$ and $di/dt = \mathcal{E}/L$. As $t \to \infty$, $i \to \mathcal{E}/R$ and $di/dt \to 0$, as we predicted. As Fig. 30.12 shows, the instantaneous current i first rises rapidly, then increases more slowly and approaches the final value $I = \mathcal{E}/R$ asymptotically. At a time equal to L/R, the current has risen to $(1 - 1/e)$, or about 63%, of its final value. The quantity L/R is therefore a measure of how quickly the current builds toward its final value; this quantity is called the **time constant** for the circuit, denoted by τ:

> **Time constant** ......▸ $\tau = \dfrac{L}{R}$ ◂...... Inductance
> **for an** R-L **circuit**   ◂...... Resistance $\tag{30.16}$

CAUTION **Current and its rate of change in an inductor** Note that the current i cannot change abruptly in a circuit that contains an inductor, so i must be a continuous function of time t. However, the rate of change of the current di/dt *can* change abruptly (for example, when the switch in Fig. 30.12a is closed and the current suddenly starts to increase, as shown at $t = 0$ in Fig. 30.12b). ▮

In a time equal to 2τ, the current reaches 86% of its final value; in 5τ, 99.3%; and in 10τ, 99.995%. (Compare the discussion in Section 26.4 of charging a capacitor of capacitance C that was in series with a resistor of resistance R; the time constant for that situation was the product RC.)

The graphs of i versus t have the same general shape for all values of L. For a given value of R, the time constant τ is greater for greater values of L. When L is small, the current rises rapidly to its final value; when L is large, it rises more slowly. For example, if $R = 100\ \Omega$ and $L = 10$ H,

$$\tau = \frac{L}{R} = \frac{10\text{ H}}{100\ \Omega} = 0.10\text{ s}$$

and the current increases to about 63% of its final value in 0.10 s. (Recall that 1 H $= 1\ \Omega \cdot$ s.) But if $L = 0.010$ H, $\tau = 1.0 \times 10^{-4}$ s $= 0.10$ ms, and the rise is much more rapid.

Energy considerations offer us additional insight into the behavior of an R-L circuit. The instantaneous rate at which the source delivers energy to the circuit is $P = \mathcal{E}i$. The instantaneous rate at which energy is dissipated in the resistor is i^2R, and the rate at which energy is stored in the inductor is $iv_{bc} = Li\,di/dt$ [or, equivalently, $(d/dt)\left(\frac{1}{2}Li^2\right) = Li\,di/dt$]. When we multiply Eq. (30.12) by i and rearrange, we find

$$\mathcal{E}i = i^2R + Li\frac{di}{dt} \tag{30.17}$$

Of the power $\mathcal{E}i$ supplied by the source, part (i^2R) is dissipated in the resistor and part $(Li\,di/dt)$ goes to store energy in the inductor. This discussion is analogous to our power analysis for a charging capacitor, given at the end of Section 26.4.

EXAMPLE 30.6 Analyzing an *R-L* circuit

A sensitive electronic device of resistance $R = 175\ \Omega$ is to be connected to a source of emf (of negligible internal resistance) by a switch. The device is designed to operate with a 36 mA current, but to avoid damage to the device, the current can rise to no more than 4.9 mA in the first 58 μs after the switch is closed. An inductor is therefore connected in series with the device, as in Fig. 30.11; the switch in question is S_1. (a) What is the required source emf $\mathcal{E}$? (b) What is the required inductance L? (c) What is the R-L time constant τ?

IDENTIFY and SET UP This problem concerns current and current growth in an *R-L* circuit, so we can use the ideas of this section. Figure 30.12 shows the current i versus the time t that has elapsed since closing S_1. The graph shows that the final current is $I = \mathcal{E}/R$; we are given $R = 175\ \Omega$, so the emf is determined by the requirement that the final current be $I = 36$ mA. The other requirement is that the current be no more than $i = 4.9$ mA at $t = 58\ \mu$s; to satisfy this, we use Eq. (30.14) for the current as a function of time and solve for the inductance, which is the only unknown quantity. Equation (30.16) then tells us the time constant.

EXECUTE (a) We solve $I = \mathcal{E}/R$ for $\mathcal{E}$:

$$\mathcal{E} = IR = (0.036\ \text{A})(175\ \Omega) = 6.3\ \text{V}$$

(b) To find the required inductance, we solve Eq. (30.14) for L. First we multiply through by $(-R/\mathcal{E})$ and add 1 to both sides:

$$1 - \frac{iR}{\mathcal{E}} = e^{-(R/L)t}$$

Then we take natural logs of both sides, solve for L, and substitute:

$$L = \frac{-Rt}{\ln(1 - iR/\mathcal{E})}$$

$$= \frac{-(175\ \Omega)(58 \times 10^{-6}\ \text{s})}{\ln[1 - (4.9 \times 10^{-3}\ \text{A})(175\ \Omega)/(6.3\ \text{V})]} = 69\ \text{mH}$$

(c) From Eq. (30.16),

$$\tau = \frac{L}{R} = \frac{69 \times 10^{-3}\ \text{H}}{175\ \Omega} = 3.9 \times 10^{-4}\ \text{s} = 390\ \mu\text{s}$$

EVALUATE Note that 58 μs is much less than the time constant. In 58 μs the current builds up from zero to 4.9 mA, a small fraction of its final value of 36 mA; after 390 μs the current equals $(1 - 1/e)$ of its final value, or about $(0.63)(36\ \text{mA}) = 23$ mA.

KEYCONCEPT The presence of an inductor in a circuit that contains a source of emf and a resistor slows the growth of the current when the circuit is completed. The equilibrium value of the current after a very long time is the same as if the inductor were not present.

Current Decay in an *R-L* Circuit

Now suppose switch S_1 in the circuit of Fig. 30.11 has been closed for a while and the current has reached the value I_0. Resetting our stopwatch to redefine the initial time, we close switch S_2 at time $t = 0$, bypassing the battery. (At the same time we should open S_1 to protect the battery.) The current through R and L does not instantaneously go to zero but decays smoothly, as shown in **Fig. 30.13**. The Kirchhoff's-rule loop equation is obtained from Eq. (30.12) by omitting the $\mathcal{E}$ term. We challenge you to retrace the steps in the above analysis and show that the current i varies with time according to

$$i = I_0 e^{-(R/L)t} \tag{30.18}$$

where I_0 is the initial current at time $t = 0$. The time constant, $\tau = L/R$, is the time for current to decrease to $1/e$, or about 37%, of its original value. In time 2τ it has dropped to 13.5%, in time 5τ to 0.67%, and in 10τ to 0.0045%.

The energy that is needed to maintain the current during this decay is provided by the energy stored in the magnetic field of the inductor. The detailed energy analysis is simpler this time. In place of Eq. (30.17) we have

$$0 = i^2 R + Li\frac{di}{dt} \tag{30.19}$$

Now $Li\ di/dt$ is negative; Eq. (30.19) shows that the energy stored in the inductor *decreases* at the same rate i^2R at which energy is dissipated in the resistor.

This entire discussion should look familiar; the situation is very similar to that of a charging and discharging capacitor, analyzed in Section 26.4. It would be a good idea to compare that section with our discussion of the *R-L* circuit.

Figure **30.13** Graph of i versus t for decay of current in an *R-L* circuit. After one time constant τ, the current is $1/e$ of its initial value.
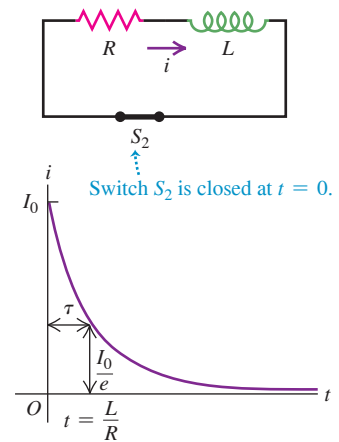
EXAMPLE 30.7 Energy in an *R-L* circuit

When the current in an *R-L* circuit is decaying, what fraction of the original energy stored in the inductor has been dissipated after 2.3 time constants?

IDENTIFY and SET UP This problem concerns current decay in an *R-L* circuit as well as the relationship between the current in an inductor and the amount of stored energy. The current i at any time t is given by Eq. (30.18); the stored energy associated with this current is given by Eq. (30.9), $U = \frac{1}{2}Li^2$.

EXECUTE From Eq. (30.18), the current i at any time t is

$$i = I_0 e^{-(R/L)t}$$

We substitute this into $U = \frac{1}{2}Li^2$ to obtain an expression for the stored energy at any time:

$$U = \frac{1}{2}LI_0^2 e^{-2(R/L)t} = U_0 e^{-2(R/L)t}$$

where $U_0 = \frac{1}{2}LI_0^2$ is the energy at the initial time $t = 0$. When $t = 2.3\tau = 2.3L/R$, we have

$$U = U_0 e^{-2(2.3)} = U_0 e^{-4.6} = 0.010U_0$$

That is, only 0.010 or 1.0% of the energy initially stored in the inductor remains, so 99.0% has been dissipated in the resistor.

EVALUATE To get a sense of what this result means, consider the *R-L* circuit we analyzed in Example 30.6, for which $\tau = 390\ \mu s$. With $L = 69$ mH and $I_0 = 36$ mA, we have

$$U_0 = \frac{1}{2}LI_0^2 = \frac{1}{2}(0.069\ \text{H})(0.036\ \text{A})^2$$
$$= 4.5 \times 10^{-5}\ \text{J}$$

Of this, 99.0% or 4.4×10^{-5} J is dissipated in $2.3(390\ \mu s) = 9.0 \times 10^{-4}$ s $= 0.90$ ms. In other words, this circuit can be almost completely powered off (or powered on) in 0.90 ms, so the minimum time for a complete on–off cycle is 1.8 ms. Even shorter cycle times are required for many purposes, such as in fast switching networks for telecommunications. In such cases a smaller time constant $\tau = L/R$ is needed.

KEYCONCEPT The presence of an inductor in a circuit that contains a resistor slows the decay of the current when the source of emf is removed. As the current decays, the magnetic energy stored in the inductor is dissipated in the resistor.

TEST YOUR UNDERSTANDING OF SECTION 30.4 (a) In Fig. 30.11, what are the algebraic signs of the potential differences v_{ab} and v_{bc} when switch S_1 is closed and switch S_2 is open? (i) $v_{ab} > 0$, $v_{bc} > 0$; (ii) $v_{ab} > 0$, $v_{bc} < 0$; (iii) $v_{ab} < 0$, $v_{bc} > 0$; (iv) $v_{ab} < 0$, $v_{bc} < 0$. (b) What are the signs of v_{ab} and v_{bc} when S_1 is open, S_2 is closed, and current is flowing in the direction shown? (i) $v_{ab} > 0$, $v_{bc} > 0$; (ii) $v_{ab} > 0$, $v_{bc} < 0$; (iii) $v_{ab} < 0$, $v_{bc} > 0$; (iv) $v_{ab} < 0$, $v_{bc} < 0$.

ANSWER

❚ (a) (ii), (b) (ii) Recall that v_{ab} is the potential at a minus the potential at b, and similarly for v_{bc}. For either arrangement of the switches, current flows through the resistor from a to b. The upstream end of the resistor is always at the higher potential, so v_{ab} is positive. With S_1 closed and S_2 open, the current through the inductor flows from b to c and is increasing. The self-induced emf opposes this increase and is therefore directed from c toward b, which means that b is at the higher potential. Hence v_{bc} is positive. With S_1 open and S_2 closed, the inductor current again flows from b to c but is now decreasing. The self-induced emf is directed from b to c in an effort to sustain the decaying current, so c is at the higher potential and v_{bc} is negative.

30.5 THE *L-C* CIRCUIT

A circuit containing an inductor and a capacitor shows an entirely new mode of behavior, characterized by *oscillating* current and charge. This is in sharp contrast to the *exponential* approach to a steady-state situation that we have seen with both *R-C* and *R-L* circuits. In the ***L-C* circuit** in **Fig. 30.14a** we charge the capacitor to a potential difference V_m and initial charge $Q_m = CV_m$ on its left-hand plate and then close the switch. What happens?

The capacitor begins to discharge through the inductor. Because of the induced emf in the inductor, the current cannot change instantaneously; it starts at zero and eventually builds up to a maximum value I_m. During this buildup the capacitor is discharging. At each instant the capacitor potential equals the induced emf, so as the capacitor discharges, the *rate of change* of current decreases. When the capacitor potential becomes zero, the induced emf is also zero, and the current has leveled off at its maximum value I_m.

Figure 30.14 In an oscillating *L-C* circuit, the charge on the capacitor and the current through the inductor both vary sinusoidally with time. Energy is transferred between magnetic energy in the inductor (U_B) and electrical energy in the capacitor (U_E). As in simple harmonic motion, the total energy E remains constant. (Compare Fig. 14.14 in Section 14.3.)
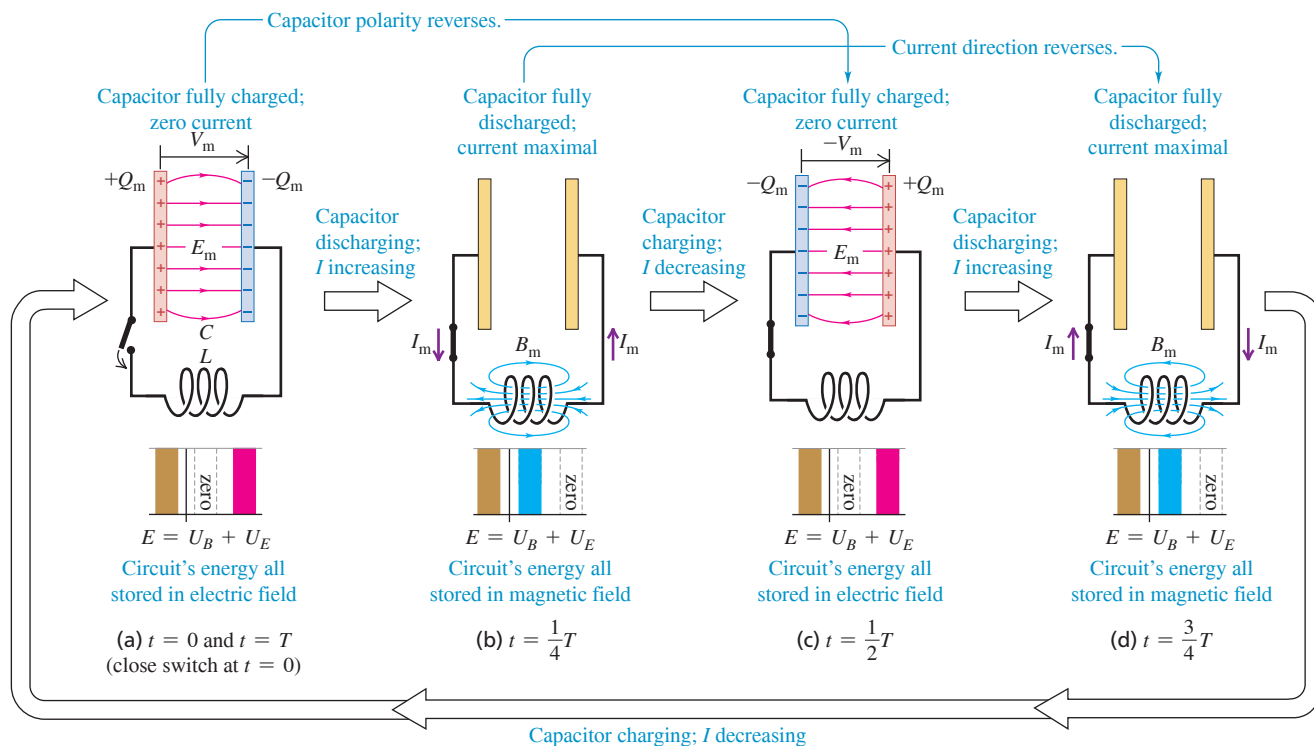

Figure 30.14b shows this situation; the capacitor has completely discharged. The potential difference between its terminals (and those of the inductor) has decreased to zero, and the current has reached its maximum value I_m.

During the discharge of the capacitor, the increasing current in the inductor has established a magnetic field in the space around it, and the energy that was initially stored in the capacitor's electric field is now stored in the inductor's magnetic field.

Although the capacitor is completely discharged in Fig. 30.14b, the current persists (it cannot change instantaneously), and the capacitor begins to charge with polarity opposite to that in the initial state. As the current decreases, the magnetic field also decreases, inducing an emf in the inductor in the *same* direction as the current; this slows down the decrease of the current. Eventually, the current and the magnetic field reach zero, and the capacitor has been charged in the sense *opposite* to its initial polarity (Fig. 30.14c), with potential difference $-V_\mathrm{m}$ and charge $-Q_\mathrm{m}$ on its left-hand plate.

The process now repeats in the reverse direction; a little later, the capacitor has again discharged, and there is a current in the inductor in the opposite direction (Fig. 30.14d). Still later, the capacitor charge returns to its original value (Fig. 30.14a), and the whole process repeats. If there are no energy losses, the charges on the capacitor continue to oscillate back and forth indefinitely. This process is called an **electrical oscillation.** (Before you read further, review the analogous case of *mechanical* oscillation in Sections 14.2 and 14.3.)

From an energy standpoint the oscillations of an electric circuit transfer energy from the capacitor's electric field to the inductor's magnetic field and back. The *total* energy associated with the circuit is constant. This is analogous to the transfer of energy in an oscillating mechanical system from potential energy to kinetic energy and back, with constant total energy (Section 14.3). As we'll see, this analogy goes much further.

Electrical Oscillations in an *L-C* Circuit

Figure **30.15** Applying Kirchhoff's loop rule to the *L-C* circuit. The direction of travel around the loop in the loop equation is shown. Just after the circuit is completed and the capacitor first begins to discharge, as in Fig. 30.14a, the current is negative (opposite to the direction shown).
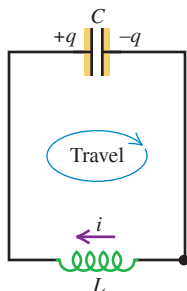
To study the flow of charge in detail, we proceed just as we did for the *R-L* circuit. **Figure 30.15** shows our definitions of q and i.

CAUTION **Positive current in an *L-C* circuit** After you have examined Fig. 30.14, the positive direction for current in Fig. 30.15 may seem backward. In fact, we chose this direction to simplify the relationship between current and capacitor charge. We define the current at each instant to be $i = dq/dt$, the rate of change of the charge on the left-hand capacitor plate. If the capacitor is initially charged and begins to discharge as in Figs. 30.14a and 30.14b, then $dq/dt < 0$ and the initial current i is negative; the current direction is opposite to the (positive) direction shown in Fig. 30.15. ▐

We apply Kirchhoff's loop rule to the circuit in Fig. 30.15. Starting at the lower-right corner of the circuit and adding voltages as we go clockwise around the loop, we obtain

$$-L\frac{di}{dt} - \frac{q}{C} = 0$$

Since $i = dq/dt$, it follows that $di/dt = d^2q/dt^2$. We substitute this expression into the above equation and divide by $-L$ to obtain

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \qquad (L\text{-}C \text{ circuit}) \qquad (30.20)$$

Equation (30.20) has exactly the same form as the equation we derived for simple harmonic motion in Section 14.2, Eq. (14.4): $d^2x/dt^2 = -(k/m)x$, or

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

In an *L-C* circuit the capacitor charge q plays the role of the displacement x, and the current $i = dq/dt$ is analogous to the particle's velocity $v_x = dx/dt$. The inductance L is analogous to the mass m, and the reciprocal of the capacitance, $1/C$, is analogous to the force constant k.

Pursuing this analogy, we recall that the angular frequency $\omega = 2\pi f$ of the harmonic oscillator is equal to $(k/m)^{1/2}$ [Eq. (14.10)], and the position is given as a function of time by Eq. (14.13),

$$x = A\cos(\omega t + \phi)$$

where the amplitude A and the phase angle ϕ depend on the initial conditions. In the analogous electrical situation, the capacitor charge q is given by

$$q = Q\cos(\omega t + \phi) \qquad (30.21)$$

and the angular frequency ω of oscillation is given by

Angular frequency of ┈┈┈➤ $\omega = \sqrt{\dfrac{1}{LC}}$ ┈┈ Capacitance
oscillation in an *L-C* circuit ┈┈┈ Inductance $\qquad (30.22)$

Verify that Eq. (30.21) satisfies the loop equation, Eq. (30.20), when ω has the value given by Eq. (30.22). In doing this, you'll find that the instantaneous current $i = dq/dt$ is

$$i = -\omega Q\sin(\omega t + \phi) \qquad (30.23)$$

Thus the charge and current in an *L-C* circuit oscillate sinusoidally with time, with an angular frequency determined by the values of L and C. The ordinary frequency f, the number of cycles per second, is equal to $\omega/2\pi$. The constants Q and ϕ in Eqs. (30.21) and (30.23) are determined by the initial conditions. If at time $t = 0$ the left-hand capacitor plate in Fig. 30.15 has its maximum charge Q and the current i is zero, then $\phi = 0$. If $q = 0$ at $t = 0$, then $\phi = \pm\pi/2$ rad.

Energy in an *L-C* Circuit

We can also analyze the *L-C* circuit by using an energy approach. The analogy to simple harmonic motion is equally useful here. In the mechanical problem an object with mass m is attached to a spring with force constant k. Suppose we displace the object a distance A from its equilibrium position and release it from rest at time $t = 0$. The kinetic energy of the system at any later time is $\frac{1}{2}mv_x^2$, and its elastic potential energy is $\frac{1}{2}kx^2$. Because the system is conservative, the sum of these energies equals the initial energy of the system, $\frac{1}{2}kA^2$. We find the velocity v_x at any position x just as we did in Section 14.3, Eq. (14.22):

$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2} \qquad (30.24)$$

The *L-C* circuit is also a conservative system. Again let Q be the maximum capacitor charge. The magnetic-field energy $\frac{1}{2}Li^2$ in the inductor at any time corresponds to the kinetic energy $\frac{1}{2}mv^2$ of the oscillating object, and the electric-field energy $q^2/2C$ in the capacitor corresponds to the elastic potential energy $\frac{1}{2}kx^2$ of the spring. The sum of these energies is the total energy $Q^2/2C$ of the system:

$$\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C} \qquad (30.25)$$

The total energy in the *L-C* circuit is *constant;* it oscillates between the magnetic and the electric forms, just as the constant total mechanical energy in simple harmonic motion is constant and oscillates between the kinetic and potential forms.

Solving Eq. (30.25) for i, we find that when the charge on the capacitor is q, the current i is

$$i = \pm\sqrt{\frac{1}{LC}}\sqrt{Q^2 - q^2} \qquad (30.26)$$

Verify this equation by substituting q from Eq. (30.21) and i from Eq. (30.23). Comparing Eqs. (30.24) and (30.26), we see that current $i = dq/dt$ and charge q are related in the same way as are velocity $v_x = dx/dt$ and position x in the mechanical problem.

Table 30.1 summarizes the analogies between simple harmonic motion and *L-C* circuit oscillations. The striking parallels shown there are so close that we can solve complicated mechanical problems by setting up analogous electric circuits and measuring the currents and voltages that correspond to the mechanical quantities to be determined. This is the basic principle of many analog computers. This analogy can be extended to *damped* oscillations, which we consider in the next section. In Chapter 31 we'll extend the analogy further to include *forced* electrical oscillations, which occur in all alternating-current circuits.

TABLE 30.1 Oscillation of a Mass-Spring System Compared with Electrical Oscillation in an *L-C* Circuit

Mass-Spring System

Kinetic energy $= \frac{1}{2}mv_x^2$

Potential energy $= \frac{1}{2}kx^2$

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$

$v_x = dx/dt$

$\omega = \sqrt{\dfrac{k}{m}}$

$x = A\cos(\omega t + \phi)$

Inductor-Capacitor Circuit

Magnetic energy $= \frac{1}{2}Li^2$

Electrical energy $= q^2/2C$

$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$

$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$

$i = dq/dt$

$\omega = \sqrt{\dfrac{1}{LC}}$

$q = Q\cos(\omega t + \phi)$

EXAMPLE 30.8 An oscillating circuit

A 300 V dc power supply is used to charge a 25 μF capacitor. After the capacitor is fully charged, it is disconnected from the power supply and connected across a 10 mH inductor. The resistance in the circuit is negligible. (a) Find the frequency and period of oscillation of the circuit. (b) Find the capacitor charge and the circuit current 1.2 ms after the inductor and capacitor are connected.

IDENTIFY and SET UP Our target variables are the oscillation frequency f and period T, as well as the charge q and current i at a particular time t. We are given the capacitance C and the inductance L, with which we can calculate the frequency and period from Eq. (30.22). We find the charge and current from Eqs. (30.21) and (30.23). Initially the capacitor is fully charged and the current is zero, as in Fig. 30.14a, so the phase angle is $\phi = 0$ [see the discussion that follows Eq. (30.23)].

Continued

EXECUTE (a) The natural angular frequency is

$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(10 \times 10^{-3} \text{ H})(25 \times 10^{-6} \text{ F})}} = 2.0 \times 10^3 \text{ rad/s}$$

The frequency f and period T are then

$$f = \frac{\omega}{2\pi} = \frac{2.0 \times 10^3 \text{ rad/s}}{2\pi \text{ rad/cycle}} = 320 \text{ Hz}$$

$$T = \frac{1}{f} = \frac{1}{320 \text{ Hz}} = 3.1 \times 10^{-3} \text{ s} = 3.1 \text{ ms}$$

(b) Since the period of the oscillation is $T = 3.1$ ms, $t = 1.2$ ms equals $0.38T$; this corresponds to a situation intermediate between Fig. 30.14b $(t = T/4)$ and Fig. 30.14c $(t = T/2)$. Comparing those figures with Fig. 30.15, we expect the capacitor charge q to be negative (that is, there will be negative charge on the left-hand plate of the capacitor) and the current i to be negative as well (that is, current will flow counterclockwise).

To find q, we use Eq. (30.21), $q = Q \cos(\omega t + \phi)$. The charge is maximum at $t = 0$, so $\phi = 0$ and $Q = C\mathcal{E} = (25 \times 10^{-6} \text{ F})(300 \text{ V}) = 7.5 \times 10^{-3}$ C. Hence Eq. (30.21) becomes

$$q = (7.5 \times 10^{-3} \text{ C}) \cos \omega t$$

At time $t = 1.2 \times 10^{-3}$ s,

$$\omega t = (2.0 \times 10^3 \text{ rad/s})(1.2 \times 10^{-3} \text{ s}) = 2.4 \text{ rad}$$

$$q = (7.5 \times 10^{-3} \text{ C}) \cos(2.4 \text{ rad}) = -5.5 \times 10^{-3} \text{ C}$$

From Eq. (30.23), the current i at any time is $i = -\omega Q \sin \omega t$. At $t = 1.2 \times 10^{-3}$ s,

$$i = -(2.0 \times 10^3 \text{ rad/s})(7.5 \times 10^{-3} \text{ C}) \sin(2.4 \text{ rad}) = -10 \text{ A}$$

EVALUATE The signs of both q and i are negative, as predicted.

KEYCONCEPT An L-C circuit that contains an inductor and a capacitor (and no resistance) is analogous to an ideal mass-spring system. The capacitor charge and the current in the circuit both oscillate sinusoidally, just like the displacement and the velocity for a mass on a spring.

EXAMPLE 30.9 Energy in an oscillating circuit

WITH √ARIATION PROBLEMS

For the L-C circuit of Example 30.8, find the magnetic and electrical energies (a) at $t = 0$ and (b) at $t = 1.2$ ms.

IDENTIFY and SET UP We must calculate the magnetic energy U_B (stored in the inductor) and the electrical energy U_E (stored in the capacitor) at two times during the L-C circuit oscillation. From Example 30.8 we know the values of the capacitor charge q and circuit current i for both times. We use them to calculate $U_B = \frac{1}{2}Li^2$ and $U_E = q^2/2C$.

EXECUTE (a) At $t = 0$ there is no current and $q = Q$. Hence there is no magnetic energy, and all the energy in the circuit is in the form of electrical energy in the capacitor:

$$U_B = \tfrac{1}{2}Li^2 = 0 \qquad U_E = \frac{Q^2}{2C} = \frac{(7.5 \times 10^{-3} \text{ C})^2}{2(25 \times 10^{-6} \text{ F})} = 1.1 \text{ J}$$

(b) From Example 30.8, at $t = 1.2$ ms we have $i = -10$ A and $q = -5.5 \times 10^{-3}$ C. Hence

$$U_B = \tfrac{1}{2}Li^2 = \tfrac{1}{2}(10 \times 10^{-3} \text{ H})(-10 \text{ A})^2 = 0.5 \text{ J}$$

$$U_E = \frac{q^2}{2C} = \frac{(-5.5 \times 10^{-3} \text{ C})^2}{2(25 \times 10^{-6} \text{ F})} = 0.6 \text{ J}$$

EVALUATE The magnetic and electrical energies are the same at $t = 3T/8 = 0.375T$, halfway between the situations in Figs. 30.14b and 30.14c. We saw in Example 30.8 that the time considered in part (b), $t = 1.2$ ms, equals $0.38T$; this is slightly later than $0.375T$, so U_B is slightly less than U_E. At *all* times the *total* energy $E = U_B + U_E$ has the same value, 1.1 J. An L-C circuit without resistance is a conservative system; no energy is dissipated.

KEYCONCEPT In an L-C circuit, the inductor is analogous to the mass in an ideal mass-spring system and the capacitor is analogous to the spring. Magnetic energy in the inductor behaves like the kinetic energy of the mass; electrical energy in the capacitor behaves like the potential energy of the spring.

TEST YOUR UNDERSTANDING OF SECTION 30.5 One way to think about the energy stored in an L-C circuit is to say that the circuit elements do positive or negative work on the charges that move back and forth through the circuit. (a) Between stages (a) and (b) in Fig. 30.14, does the capacitor do positive or negative work on the charges? (b) What kind of force (electric or magnetic) does the capacitor exert on the charges to do this work? (c) During this process, does the inductor do positive or negative work on the charges? (d) What kind of force (electric or magnetic) does the inductor exert on the charges?

ANSWER   ❙ (a) **positive,** (b) **electric,** (c) **negative,** (d) **electric**   The capacitor loses energy between stages (a) and (b), so it does positive work on the charges. It does this by exerting an electric force that pushes current away from the positively charged left-hand capacitor plate and toward the negatively charged right-hand plate. At the same time, the inductor gains energy and does negative work on the moving charges. Although the inductor stores magnetic energy, the force that the inductor exerts is *electric.* This force comes about from the inductor's self-induced emf (see Section 30.2).

30.6 THE *L-R-C* SERIES CIRCUIT

In our discussion of the *L-C* circuit we assumed that there was no *resistance* in the circuit. This is an idealization, of course; every real inductor has resistance in its windings, and there may also be resistance in the connecting wires. Because of resistance, the electro-magnetic energy in the circuit is dissipated and converted to other forms, such as internal energy of the circuit materials. Resistance in an electric circuit is analogous to friction in a mechanical system.

Suppose an inductor with inductance *L* and a resistor of resistance *R* are connected in se-ries across the terminals of a charged capacitor, forming an **L-R-C series circuit.** As before, the capacitor starts to discharge as soon as the circuit is completed. But due to i^2R losses in the resistor, the magnetic-field energy that the inductor acquires when the capacitor is completely discharged is *less* than the original electric-field energy of the capacitor. In the same way, the energy of the capacitor when the magnetic field has decreased to zero is still less and so on.

If the resistance *R* of the resistor is relatively small, the circuit still oscillates, but with **damped harmonic motion** (**Fig. 30.16a**), and we say that the circuit is **underdamped.** If we increase *R*, the oscillations die out more rapidly. When *R* reaches a certain value, the circuit no longer oscillates; it is **critically damped** (Fig. 30.16b). For still larger values of *R*, the circuit is **overdamped** (Fig. 30.16c), and the capacitor charge approaches zero even more slowly. We used these same terms to describe the behavior of the analogous mechanical system, the damped harmonic oscillator, in Section 14.7.

Analyzing an *L-R-C* Series Circuit

To analyze *L-R-C* series circuit behavior in detail, consider the circuit shown in **Fig. 30.17.** It is like the *L-C* circuit of Fig. 30.15 except for the added resistor *R*; we also show the source that charges the capacitor initially. The labeling of the positive senses of *q* and *i* is the same as for the *L-C* circuit.

First we close the switch in the upward position, connecting the capacitor to a source of emf $\mathcal{E}$ for a long enough time to ensure that the capacitor acquires its final charge $Q = C\mathcal{E}$ and any initial oscillations have died out. Then at time $t = 0$ we flip the switch to the downward position, removing the source from the circuit and placing the capacitor in se-ries with the resistor and inductor. Note that the initial current is negative, opposite to the direction of *i* shown in Fig. 30.17.

To find how *q* and *i* vary with time, we apply Kirchhoff's loop rule. Starting at point *a* and going around the loop in the direction *abcda,* we obtain

$$-iR - L\frac{di}{dt} - \frac{q}{C} = 0$$

Replacing *i* with dq/dt and rearranging, we get

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0 \qquad (30.27)$$

Note that when $R = 0$, this reduces to Eq. (30.20) for an *L-C* circuit.

There are general methods for obtaining solutions of Eq. (30.27). The form of the solu-tion is different for the underdamped (small *R*) and overdamped (large *R*) cases. When R^2 is less than $4L/C$, the solution has the form

$$q = Ae^{-(R/2L)t}\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \phi\right) \qquad (30.28)$$

where *A* and ϕ are constants. You can take the first and second derivatives of this function and show by direct substitution that it does satisfy Eq. (30.27).

This solution corresponds to the *underdamped* behavior shown in Fig. 30.16a; the function represents a sinusoidal oscillation with an exponentially decaying amplitude. (Note that the exponential factor $e^{-(R/2L)t}$ is *not* the same as the factor $e^{-(R/L)t}$ that we encountered in describing the *R-L* circuit in Section 30.4.) When $R = 0$, Eq. (30.28) reduces to Eq. (30.21)

Figure 30.16 Graphs of capacitor charge as a function of time in an *L-R-C* series circuit with initial charge *Q*.
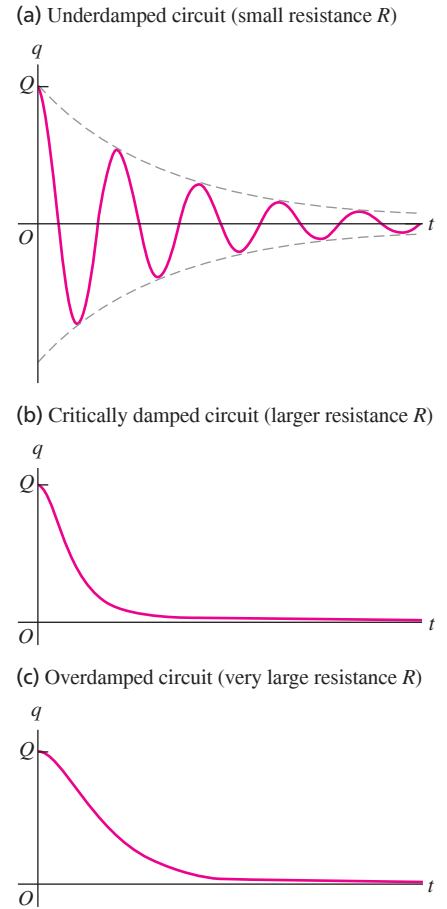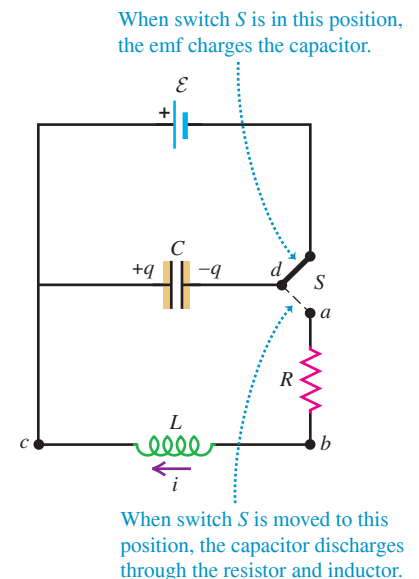
(a) Underdamped circuit (small resistance *R*)

(b) Critically damped circuit (larger resistance *R*)

(c) Overdamped circuit (very large resistance *R*)

Figure 30.17 An *L-R-C* series circuit.

When switch *S* is in this position, the emf charges the capacitor.

When switch *S* is moved to this position, the capacitor discharges through the resistor and inductor.

for the oscillations in an L-C circuit. If R is not zero, the angular frequency of the oscillation is *less* than $1/(LC)^{1/2}$ because of the term containing R. The angular frequency ω' of the damped oscillations is given by

Angular frequency of
underdamped oscillations ·····▸ $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$ ·······Resistance

in an L-R-C series circuit
　　　　　　　　　　　　　　······Inductance

(30.29)

Inductance　　Capacitance

When $R = 0$, this reduces to Eq. (30.22), $\omega = (1/LC)^{1/2}$. As R increases, ω' becomes smaller and smaller. When $R^2 = 4L/C$, the quantity under the radical becomes zero; the system no longer oscillates, and the case of *critical damping* (Fig. 30.16b) has been reached. For still larger values of R the system behaves as in Fig. 30.16c. In this case the circuit is *overdamped,* and q is given as a function of time by the sum of two decreasing exponential functions.

In the *underdamped* case the phase constant ϕ in the cosine function of Eq. (30.28) provides for the possibility of both an initial charge and an initial current at time $t = 0$, analogous to an underdamped harmonic oscillator given both an initial displacement and an initial velocity (see Exercise 30.38).

We emphasize once more that the behavior of the L-R-C series circuit is completely analogous to that of the damped harmonic oscillator (see Section 14.7). We invite you to verify, for example, that if you start with Eq. (14.41) and substitute q for x, L for m, $1/C$ for k, and R for the damping constant b, the result is Eq. (30.27). Similarly, the cross-over point between underdamping and overdamping occurs at $b^2 = 4km$ for the mechanical system and at $R^2 = 4L/C$ for the electrical one. Can you find still other aspects of this analogy?

The practical applications of the L-R-C series circuit emerge when we include a sinusoidally varying source of emf in the circuit. This is analogous to the *forced oscillations* that we discussed in Section 14.7, and there are analogous *resonance* effects. Such a circuit is called an *alternating-current (ac) circuit.* The analysis of ac circuits is the principal topic of the next chapter.

EXAMPLE 30.10　An underdamped L-R-C series circuit　　　　WITH √ARIATION PROBLEMS

What resistance R is required (in terms of L and C) to give an L-R-C series circuit a frequency that is one-half the undamped frequency?

IDENTIFY and SET UP　This problem concerns an underdamped L-R-C series circuit (Fig. 30.16a). We want just enough resistance to reduce the oscillation frequency to one-half of the undamped value. Equation (30.29) gives the angular frequency ω' of an underdamped L-R-C series circuit; Eq. (30.22) gives the angular frequency ω of an undamped L-C circuit. We use these two equations to solve for R.

EXECUTE　From Eqs. (30.29) and (30.22), the requirement $\omega' = \omega/2$ yields

$$\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \tfrac{1}{2}\sqrt{\frac{1}{LC}}$$

We square both sides and solve for R:

$$R = \sqrt{\frac{3L}{C}}$$

For example, adding 35 Ω to the circuit of Example 30.8 ($L = 10$ mH, $C = 25\ \mu$F) would reduce the frequency from 320 Hz to 160 Hz.

EVALUATE　The circuit becomes critically damped with no oscillations when $R = \sqrt{4L/C}$. Our result for R is smaller than that, as it should be; we want the circuit to be *under*damped.

KEYCONCEPT　A series L-R-C circuit is analogous to a mass-spring system with damping. Increasing the resistance reduces the oscillation frequency of the charge and current; if the resistance is too great, the circuit is critically damped or overdamped and there is no oscillation.

TEST YOUR UNDERSTANDING OF SECTION 30.6　An L-R-C series circuit includes a 2.0 Ω resistor. At $t = 0$ the capacitor charge is 2.0 μC. For which of the following values of the inductance and capacitance will the charge on the capacitor *not* oscillate? (i) $L = 3.0\ \mu$H, $C = 6.0\ \mu$F; (ii) $L = 6.0\ \mu$H, $C = 3.0\ \mu$F; (iii) $L = 3.0\ \mu$H, $C = 3.0\ \mu$F.

ANSWER

❚ **(i) and (iii)** There are no oscillations if $R^2 \geq 4L/C$. In each case $R^2 = (2.0\ \Omega)^2 = 4.0\ \Omega^2$. In case (i) $4L/C = 4(3.0\ \mu\text{H})/(6.0\ \mu\text{F}) = 2.0\ \Omega^2$, so there are no oscillations (the system is overdamped); in case (ii) $4L/C = 4(6.0\ \mu\text{H})/(3.0\ \mu\text{F}) = 8.0\ \Omega^2$, so there are oscillations (the system is underdamped); and in case (iii) $4L/C = 4(3.0\ \mu\text{H})/(3.0\ \mu\text{F}) = 4.0\ \Omega^2$, so there are no oscillations (the system is critically damped).

CHAPTER 30 SUMMARY

Mutual inductance: When a changing current i_1 in one circuit causes a changing magnetic flux in a second circuit, an emf $\mathcal{E}_2$ is induced in the second circuit. Likewise, a changing current i_2 in the second circuit induces an emf $\mathcal{E}_1$ in the first circuit. If the circuits are coils of wire with N_1 and N_2 turns, the mutual inductance M can be expressed in terms of the average flux Φ_{B2} through each turn of coil 2 caused by the current i_1 in coil 1, or in terms of the average flux Φ_{B1} through each turn of coil 1 caused by the current i_2 in coil 2. The SI unit of mutual inductance is the henry, abbreviated H. (See Examples 30.1 and 30.2.)

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \quad \text{and}$$

$$\mathcal{E}_1 = -M\frac{di_2}{dt} \tag{30.4}$$

$$M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2} \tag{30.5}$$
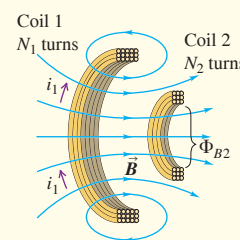
Self-inductance: A changing current i in any circuit causes a self-induced emf $\mathcal{E}$. The inductance (or self-inductance) L depends on the geometry of the circuit and the material surrounding it. The inductance of a coil of N turns is related to the average flux Φ_B through each turn caused by the current i in the coil. An inductor is a circuit device, usually including a coil of wire, intended to have a substantial inductance. (See Examples 30.3 and 30.4.)

$$\mathcal{E} = -L\frac{di}{dt} \tag{30.7}$$
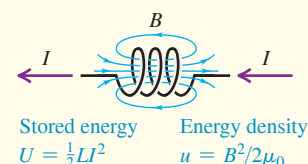
$$L = \frac{N\Phi_B}{i} \tag{30.6}$$
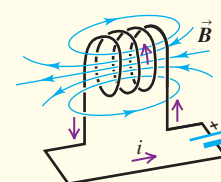
Magnetic-field energy: An inductor with inductance L carrying current I has energy U associated with the inductor's magnetic field. The magnetic energy density u (energy per unit volume) is proportional to the square of the magnetic-field magnitude. (See Example 30.5.)

$$U = \tfrac{1}{2}LI^2 \tag{30.9}$$

$$u = \frac{B^2}{2\mu_0} \quad \text{(in vacuum)} \tag{30.10}$$

$$u = \frac{B^2}{2\mu} \quad \begin{array}{l}\text{(in a material}\\\text{with magnetic}\\\text{permeability } \mu)\end{array} \tag{30.11}$$

Stored energy   Energy density
$U = \tfrac{1}{2}LI^2$ $u = B^2/2\mu_0$

R-L circuits: In a circuit containing a resistor R, an inductor L, and a source of emf, the growth and decay of current are exponential. The time constant τ is the time required for the current to approach within a fraction $1/e$ of its final value. (See Examples 30.6 and 30.7.)

$$\tau = \frac{L}{R} \tag{30.16}$$
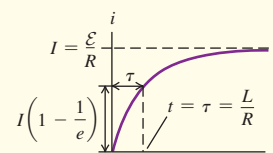
L-C circuits: A circuit that contains inductance L and capacitance C undergoes electrical oscillations with an angular frequency ω that depends on L and C. This is analogous to a mechanical harmonic oscillator, with inductance L analogous to mass m, the reciprocal of capacitance $1/C$ to force constant k, charge q to displacement x, and current i to velocity v_x. (See Examples 30.8 and 30.9.)

$$\omega = \sqrt{\frac{1}{LC}} \tag{30.22}$$
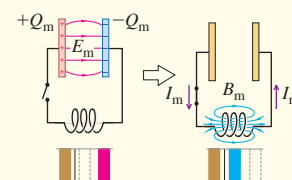
L-R-C series circuits: A circuit that contains inductance, resistance, and capacitance undergoes damped oscillations for sufficiently small resistance. The frequency ω' of damped oscillations depends on the values of L, R, and C. As R increases, the damping increases; if R is greater than a certain value, the behavior becomes overdamped and no longer oscillates. (See Example 30.10.)

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{30.29}$$
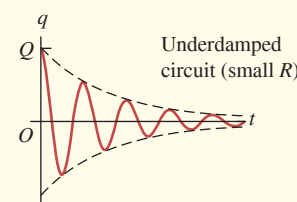
Underdamped circuit (small R)

GUIDED PRACTICE

KEY EXAMPLE √ARIATION PROBLEMS

Be sure to review EXAMPLES 30.1 and 30.2 (Section 30.1) and EXAMPLES 30.3 and 30.4 (Section 30.2) before attempting these problems.

VP30.4.1 A long solenoid (coil 1) is surrounded at its center by a shorter coil (coil 2) as in Fig. 30.3. At an instant when the current in coil 1 is 24.0 A and increasing at a rate of 2.00 A/s, the induced emf in coil 2 is 1.30×10^{-4} V. (a) What is the mutual inductance of the two coils? (b) At this instant, what is the total magnetic flux through all the turns of coil 2 due to the current in coil 1?

VP30.4.2 The mutual inductance of two coils (1 and 2) is 48.0 μH. Find the magnitude of the emf induced in coil 2 if the current in coil 1 is (a) 35.0 A and increasing at 7.00 A/s; (b) 28.0 A and constant; (c) 26.0 A and decreasing at 3.00 A/s.

VP30.4.3 A toroidal solenoid with self-inductance 76.0 μH has 465 turns of wire. Find (a) the magnetic flux through each turn when the current in the solenoid is 12.0 A and (b) the magnitude of the induced emf in the solenoid when the current is 12.0 A and decreasing at 55.0 A/s.

VP30.4.4 When a coil is carrying a current of 25.0 A that is increasing at 145 A/s, the induced emf in the coil has magnitude 3.70 mV. Find (a) the self-inductance of the coil and (b) the magnitude and direction of the induced emf at $t = 2.00$ s if the current is $i(t) = (225 \text{ A/s}^2)t^2$.

Be sure to review EXAMPLES 30.6 and 30.7 (Section 30.4) before attempting these problems.

VP30.7.1 In the circuit shown in Fig. 30.11, the inductance is 4.90 mH and the emf is 24.0 V. Initially both switches are open. You want the current to reach 75.0% of its maximum value 80.0 μs after switch S_1 is closed. Find (a) the required resistance and (b) the current 80.0 μs after switch S_1 is closed.

VP30.7.2 In the circuit shown in Fig. 30.11, $\mathcal{E} = 12.0$ V, $R = 18.0 \ \Omega$, and $L = 37.5$ mH. Initially both switches are open. You close switch S_1

at $t = 0$. At $t = 0.700$ ms, find (a) the current, (b) the rate of change of the current, and (c) the amount of energy stored in the inductor.

VP30.7.3 In the circuit shown in Fig. 30.11, $\mathcal{E} = 12.0$ V, $R = 18.0 \ \Omega$, and $L = 37.5$ mH. Switch S_1 has been closed for a long time and switch S_2 is open. At $t = 0$ you open switch S_1 and close switch S_2. At $t = 0.700$ ms, find (a) the current, (b) the rate of change of the current, and (c) the amount of energy stored in the inductor.

VP30.7.4 For the R-L circuit with an increasing current shown in Fig. 30.12, find (a) the time when the voltage across the inductor has the same magnitude as the voltage across the resistor, (b) the current at this time, and (c) the rate of change of the current at this time.

Be sure to review EXAMPLES 30.8 and 30.9 (Section 30.5) and EXAMPLE 30.10 (Section 30.6) before attempting these problems.

VP30.10.1 The L-C circuit in Fig. 30.15 contains an inductor with $L = 25.0 \ \mu$H. The charge and current in this circuit oscillate with frequency 445 Hz, and the maximum capacitor charge is 5.00 mC. Find (a) the capacitance of the capacitor and (b) the maximum current in the circuit.

VP30.10.2 You construct an L-C circuit that contains a 30.0 μH inductor and a 655 μF capacitor. At $t = 0$ the capacitor has its maximum charge of 0.400 mC. Find (a) the first time after $t = 0$ when the capacitor charge is zero and (b) the magnitude of the current in the circuit at this time.

VP30.10.3 The sum of the electrical and magnetic energies in an L-C circuit is 0.800 J. At a certain instant the energy is exactly half electrical and half magnetic, the capacitor charge is 5.30 mC, and the current is 8.00 A. Find (a) the capacitance, (b) the inductance, and (c) the angular frequency of oscillation.

VP30.10.4 An L-R-C series circuit has inductance 42.0 mH, capacitance C, and resistance R. Without the resistor, the angular frequency of oscillation is 624 rad/s. With the resistor, the angular frequency is 208 rad/s. Find the values of (a) C and (b) R.

BRIDGING PROBLEM Analyzing an *L-C* Circuit

An L-C circuit like that shown in Fig. 30.14 consists of a 60.0 mH inductor and a 250 μF capacitor. The initial charge on the capacitor is 6.00 μC, and the initial current in the inductor is 0.400 mA. (a) What is the maximum energy stored in the inductor? (b) What is the maximum current in the inductor? (c) What is the maximum voltage across the capacitor? (d) When the current in the inductor has half its maximum value, what are the energy stored in the inductor and the voltage across the capacitor?

SOLUTION GUIDE

IDENTIFY and SET UP

1. An L-C circuit is a conservative system—there is no resistance to dissipate energy. The energy oscillates between electrical energy in the capacitor and magnetic energy stored in the inductor.

2. Oscillations in an L-C circuit are analogous to the mechanical oscillations of a particle at the end of an ideal spring (see Table 30.1). Compare this problem to the analogous mechanical problem (see Example 14.3 in Section 14.2 and Example 14.4 in Section 14.3).

3. Which key equations are needed to describe the capacitor? To describe the inductor?

EXECUTE

4. Find the initial total energy in the circuit. Use it to determine the maximum energy stored in the inductor during the oscillation.

5. Use the result of step 4 to find the maximum current in the inductor.

6. Use the result of step 4 to find the maximum energy stored in the capacitor during the oscillation. Then use this to find the maximum capacitor voltage.

7. Find the energy in the inductor and the capacitor charge when the current has half the value that you found in step 5.

EVALUATE

8. Initially, what fraction of the total energy is in the inductor? Is it possible to tell whether this is initially increasing or decreasing?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q30.1 In an electric trolley or bus system, the vehicle's motor draws current from an overhead wire by means of a long arm with an attachment at the end that slides along the overhead wire. A brilliant electric spark is often seen when the attachment crosses a junction in the wires where contact is momentarily lost. Explain this phenomenon.

Q30.2 From Eq. (30.5) 1 H $= 1$ Wb/A, and from Eqs. (30.4) 1 H $=$ 1 $\Omega \cdot$ s. Show that these two definitions are equivalent.

Q30.3 In Fig. 30.1, if coil 2 is turned 90° so that its axis is vertical, does the mutual inductance increase or decrease? Explain.

Q30.4 The tightly wound toroidal solenoid is one of the few configurations for which it is easy to calculate self-inductance. What features of the toroidal solenoid give it this simplicity?

Q30.5 Two identical, closely wound, circular coils, each having self-inductance L, are placed next to each other, so that they are coaxial and almost touching. If they are connected in series, what is the self-inductance of the combination? What if they are connected in parallel? Can they be connected so that the total inductance is zero? Explain.

Q30.6 Two closely wound circular coils have the same number of turns, but one has twice the radius of the other. How are the self-inductances of the two coils related? Explain your reasoning.

Q30.7 You are to make a resistor by winding a wire around a cylindrical form. To make the inductance as small as possible, it is proposed that you wind half the wire in one direction and the other half in the opposite direction. Would this achieve the desired result? Why or why not?

Q30.8 For the same magnetic field strength B, is the energy density greater in vacuum or in a magnetic material? Explain. Does Eq. (30.11) imply that for a long solenoid in which the current is I the energy stored is proportional to $1/\mu$? And does this mean that for the same current less energy is stored when the solenoid is filled with a ferromagnetic material rather than with air? Explain.

Q30.9 In an *R-C* circuit, a resistor, an uncharged capacitor, a dc battery, and an open switch are in series. In an *R-L* circuit, a resistor, an inductor, a dc battery, and an open switch are in series. Compare the behavior of the current in these circuits (a) just after the switch is closed and (b) long after the switch has been closed. In other words, compare the way in which a capacitor and an inductor affect a circuit.

Q30.10 A Differentiating Circuit. The current in a resistanceless inductor is caused to vary with time as shown in the graph of **Fig. Q30.10**. (a) Sketch the pattern that would be observed on the screen of an oscilloscope connected to the terminals of the inductor. (The oscilloscope spot sweeps horizontally across the screen at a constant speed, and its vertical deflection is proportional to the potential difference between the inductor terminals.) (b) Explain why a circuit with an inductor can be described as a "differentiating circuit."

Figure **Q30.10**

Q30.11 In Section 30.5 Kirchhoff's loop rule is applied to an *L-C* circuit where the capacitor is initially fully charged and the equation $-L\,(di/dt) - (q/C) = 0$ is derived. But as the capacitor starts to discharge, the current increases from zero. The equation says $L\,di/dt = -q/C$, so it says $L\,di/dt$ is negative. Explain how $L\,di/dt$ can be negative when the current is increasing.

Q30.12 In Section 30.5 the relationship $i = dq/dt$ is used in deriving Eq. (30.20). But a flow of current corresponds to a decrease in the charge on the capacitor. Explain, therefore, why this is the correct equation to use in the derivation, rather than $i = -dq/dt$.

Q30.13 In the *R-L* circuit shown in Fig. 30.11, when switch S_1 is closed, the potential v_{ac} changes suddenly and discontinuously, but the current does not. Explain why the voltage can change suddenly but the current can't.

Q30.14 In the *R-L* circuit shown in Fig. 30.11, is the current in the resistor always the same as the current in the inductor? How do you know?

Q30.15 Suppose there is a steady current in an inductor. If you attempt to reduce the current to zero instantaneously by quickly opening a switch, an arc can appear at the switch contacts. Why? Is it physically possible to stop the current instantaneously? Explain.

Q30.16 In an *L-R-C* series circuit, what criteria could be used to decide whether the system is overdamped or underdamped? For example, could we compare the maximum energy stored during one cycle to the energy dissipated during one cycle? Explain.

EXERCISES

Section 30.1 Mutual Inductance

30.1 • Two coils have mutual inductance $M = 3.25 \times 10^{-4}$ H. The current i_1 in the first coil increases at a uniform rate of 830 A/s. (a) What is the magnitude of the induced emf in the second coil? Is it constant? (b) Suppose that the current described is in the second coil rather than the first. What is the magnitude of the induced emf in the first coil?

30.2 • Two coils are wound around the same cylindrical form, like the coils in Example 30.1. When the current in the first coil is decreasing at a rate of -0.242 A/s, the induced emf in the second coil has magnitude 1.65×10^{-3} V. (a) What is the mutual inductance of the pair of coils? (b) If the second coil has 25 turns, what is the flux through each turn when the current in the first coil equals 1.20 A? (c) If the current in the second coil increases at a rate of 0.360 A/s, what is the magnitude of the induced emf in the first coil?

30.3 • Two toroidal solenoids are wound around the same form so that the magnetic field of one passes through the turns of the other. Solenoid 1 has 700 turns, and solenoid 2 has 400 turns. When the current in solenoid 1 is 6.52 A, the average flux through each turn of solenoid 2 is 0.0320 Wb. (a) What is the mutual inductance of the pair of solenoids? (b) When the current in solenoid 2 is 2.54 A, what is the average flux through each turn of solenoid 1?

30.4 • A solenoidal coil with 25 turns of wire is wound tightly around another coil with 300 turns (see Example 30.1). The inner solenoid is 25.0 cm long and has a diameter of 2.00 cm. At a certain time, the current in the inner solenoid is 0.120 A and is increasing at a rate of 1.75×10^3 A/s. For this time, calculate: (a) the average magnetic flux through each turn of the inner solenoid; (b) the mutual inductance of the two solenoids; (c) the emf induced in the outer solenoid by the changing current in the inner solenoid.

Section 30.2 Self-Inductance and Inductors

30.5 • A 2.50 mH toroidal solenoid has an average radius of 6.00 cm and a cross-sectional area of 2.00 cm². (a) How many coils does it have? (Make the same assumption as in Example 30.3.) (b) At what rate must the current through it change so that a potential difference of 2.00 V is developed across its ends?

30.6 • A toroidal solenoid has 500 turns, cross-sectional area 6.25 cm², and mean radius 4.00 cm. (a) Calculate the coil's self-inductance. (b) If the current decreases uniformly from 5.00 A to 2.00 A in 3.00 ms, calculate the self-induced emf in the coil. (c) The current is directed from terminal a of the coil to terminal b. Is the direction of the induced emf from a to b or from b to a?

30.7 • At the instant when the current in an inductor is increasing at a rate of 0.0640 A/s, the magnitude of the self-induced emf is 0.0160 V. (a) What is the inductance of the inductor? (b) If the inductor is a solenoid with 400 turns, what is the average magnetic flux through each turn when the current is 0.720 A?

30.8 •• When the current in a toroidal solenoid is changing at a rate of 0.0260 A/s, the magnitude of the induced emf is 12.6 mV. When the current equals 1.40 A, the average flux through each turn of the solenoid is 0.00285 Wb. How many turns does the solenoid have?

30.9 • The inductor in **Fig. E30.9** has inductance 0.260 H and carries a current in the direction shown that is decreasing at a uniform rate, $di/dt = -0.0180$ A/s. (a) Find the self-induced emf. (b) Which end of the inductor, a or b, is at a higher potential?

Figure **E30.9**
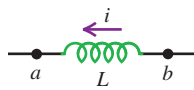
30.10 • The inductor shown in Fig. E30.9 has inductance 0.260 H and carries a current in the direction shown. The current is changing at a constant rate. (a) The potential between points a and b is $V_{ab} = 1.04$ V, with point a at higher potential. Is the current increasing or decreasing? (b) If the current at $t = 0$ is 12.0 A, what is the current at $t = 2.00$ s?

30.11 •• **Inductance of a Solenoid.** (a) A long, straight solenoid has N turns, uniform cross-sectional area A, and length l. Show that the inductance of this solenoid is given by the equation $L = \mu_0 A N^2/l$. Assume that the magnetic field is uniform inside the solenoid and zero outside. (Your answer is approximate because B is actually smaller at the ends than at the center. For this reason, your answer is actually an upper limit on the inductance.) (b) A metallic laboratory spring is typically 5.00 cm long and 0.150 cm in diameter and has 50 coils. If you connect such a spring in an electric circuit, how much self-inductance must you include for it if you model it as an ideal solenoid?

30.12 • A long, straight solenoid has 800 turns. When the current in the solenoid is 2.90 A, the average flux through each turn of the solenoid is 3.25×10^{-3} Wb. What must be the magnitude of the rate of change of the current in order for the self-induced emf to equal 6.20 mV?

Section 30.3 Magnetic-Field Energy

30.13 •• When the current in a long, straight, air-filled solenoid is changing at the rate of 2000 A/s, the voltage across the solenoid is 0.600 V. The solenoid has 1200 turns and uniform cross-sectional area 25.0 mm². Assume that the magnetic field is uniform inside the solenoid and zero outside, so the result $L = \mu_0 A N^2/l$ (see Exercise 30.11) applies. What is the magnitude B of the magnetic field in the interior of the solenoid when the current in the solenoid is 3.00 A?

30.14 • An inductor used in a dc power supply has an inductance of 12.0 H and a resistance of 180 Ω. It carries a current of 0.500 A. (a) What is the energy stored in the magnetic field? (b) At what rate is thermal energy developed in the inductor? (c) Does your answer to part (b) mean that the magnetic-field energy is decreasing with time? Explain.

30.15 • An air-filled toroidal solenoid has a mean radius of 15.0 cm and a cross-sectional area of 5.00 cm². When the current is 12.0 A, the energy stored is 0.390 J. How many turns does the winding have?

30.16 • An air-filled toroidal solenoid has 300 turns of wire, a mean radius of 12.0 cm, and a cross-sectional area of 4.00 cm². If the current is 5.00 A, calculate: (a) the magnetic field in the solenoid; (b) the self-inductance of the solenoid; (c) the energy stored in the magnetic field; (d) the energy density in the magnetic field. (e) Check your answer for part (d) by dividing your answer to part (c) by the volume of the solenoid.

30.17 •• A solenoid 25.0 cm long and with a cross-sectional area of 0.500 cm² contains 400 turns of wire and carries a current of 80.0 A. Calculate: (a) the magnetic field in the solenoid; (b) the energy density in the magnetic field if the solenoid is filled with air; (c) the total energy contained in the coil's magnetic field (assume the field is uniform); (d) the inductance of the solenoid.

30.18 • It has been proposed to use large inductors as energy storage devices. (a) How much electrical energy is converted to light and thermal energy by a 150 W light bulb in one day? (b) If the amount of energy calculated in part (a) is stored in an inductor in which the current is 80.0 A, what is the inductance?

30.19 •• In a proton accelerator used in elementary particle physics experiments, the trajectories of protons are controlled by bending magnets that produce a magnetic field of 4.80 T. What is the magnetic-field energy in a 10.0 cm³ volume of space where $B = 4.80$ T?

30.20 • A region of vacuum contains both a uniform electric field with magnitude E and a uniform magnetic field with magnitude B. (a) What is the ratio E/B if the energy density for the magnetic field equals the energy density for the electric field? (b) If $E = 500$ V/m, what is B, in teslas, if the magnetic-field and electric-field energy densities are equal?

Section 30.4 The R-L Circuit

30.21 • An inductor with an inductance of 2.50 H and a resistance of 8.00 Ω is connected to the terminals of a battery with an emf of 6.00 V and negligible internal resistance. Find (a) the initial rate of increase of current in the circuit; (b) the rate of increase of current at the instant when the current is 0.500 A; (c) the current 0.250 s after the circuit is closed; (d) the final steady-state current.

30.22 • In Fig. 30.11, $R = 15.0$ Ω and the battery emf is 6.30 V. With switch S_2 open, switch S_1 is closed. After several minutes, S_1 is opened and S_2 is closed. (a) At 2.00 ms after S_1 is opened, the current has decayed to 0.280 A. Calculate the inductance of the coil. (b) How long after S_1 is opened will the current reach 1.00% of its original value?

30.23 • A 35.0 V battery with negligible internal resistance, a 50.0 Ω resistor, and a 1.25 mH inductor with negligible resistance are all connected in series with an open switch. The switch is suddenly closed. (a) How long after closing the switch will the current through the inductor reach one-half of its maximum value? (b) How long after closing the switch will the energy stored in the inductor reach one-half of its maximum value?

30.24 •• A resistor and an inductor are connected in series to a battery with emf 240 V and negligible internal resistance. The circuit is completed at time $t = 0$. At a later time $t = T$ the current is 5.00 A and is increasing at a rate of 20.0 A/s. After a long time the current in the circuit is 15.0 A. What is the value of T, the time when the current is 5.00 A?

30.25 • A resistor with $R = 30.0$ Ω and an inductor with $L = 0.600$ H are connected in series to a battery that has emf 50.0 V and negligible internal resistance. At time t after the circuit is completed, the energy stored in the inductor is 0.400 J. At this instant, what is the voltage across the inductor?

30.26 • In the circuit shown in Fig. 30.11 switch S_1 has been closed a long time while switch S_2 has been left open. Then S_2 is closed at the same instant when S_1 is opened. Just after S_2 is closed, the current through the resistor is 12.0 A and its rate of decrease is $di/dt = -36.0$ A/s. How long does it take the current to decrease to 6.00 A, one-half its initial value?

30.27 • In Fig. 30.11, suppose that $\mathcal{E} = 60.0$ V, $R = 240$ Ω, and $L = 0.160$ H. With switch S_2 open, switch S_1 is left closed until a constant current is established. Then S_2 is closed and S_1 opened, taking the battery out of the circuit. (a) What is the initial current in the resistor, just after S_2 is closed and S_1 is opened? (b) What is the current in the resistor at $t = 4.00 \times 10^{-4}$ s? (c) What is the potential difference between points b and c at $t = 4.00 \times 10^{-4}$ s? Which point is at a higher potential? (d) How long does it take the current to decrease to half its initial value?

30.28 • In Fig. 30.11, suppose that $\mathcal{E} = 60.0$ V, $R = 240\ \Omega$, and $L = 0.160$ H. Initially there is no current in the circuit. Switch S_2 is left open, and switch S_1 is closed. (a) Just after S_1 is closed, what are the potential differences v_{ab} and v_{bc}? (b) A long time (many time constants) after S_1 is closed, what are v_{ab} and v_{bc}? (c) What are v_{ab} and v_{bc} at an intermediate time when $i = 0.150$ A?

30.29 • In Fig. 30.11 switch S_1 is closed while switch S_2 is kept open. The inductance is $L = 0.380$ H, the resistance is $R = 48.0\ \Omega$, and the emf of the battery is 18.0 V. At time t after S_1 is closed, the current in the circuit is increasing at a rate of $di/dt = 7.20$ A/s. At this instant what is v_{ab}, the voltage across the resistor?

30.30 •• Consider the circuit in Exercise 30.21. (a) Just after the circuit is completed, at what rate is the battery supplying electrical energy to the circuit? (b) When the current has reached its final steady-state value, how much energy is stored in the inductor? What is the rate at which electrical energy is being dissipated in the resistance of the inductor? What is the rate at which the battery is supplying electrical energy to the circuit?

30.31 • Consider the rate P_L at which energy is being stored in the R-L circuit of Fig. 30.12. Answer these questions, in terms of $\mathcal{E}$, R, and L as needed: (a) What is P_L at $t = 0$, just after the circuit is completed? (b) What is P_L at $t \rightarrow \infty$, a long time after the circuit is completed? (c) What is P_L at the instant when $i = \mathcal{E}/2R$, one-half the current's final value?

Section 30.5 The *L-C* Circuit

30.32 •• A 15.0 μF capacitor is charged by a 150.0 V power supply, then disconnected from the power and connected in series with a 0.280 mH inductor. Calculate: (a) the oscillation frequency of the circuit; (b) the energy stored in the capacitor at time $t = 0$ ms (the moment of connection with the inductor); (c) the energy stored in the inductor at $t = 1.30$ ms.

30.33 • In an L-C circuit, $L = 85.0$ mH and $C = 3.20\ \mu$F. During the oscillations the maximum current in the inductor is 0.850 mA. (a) What is the maximum charge on the capacitor? (b) What is the magnitude of the charge on the capacitor at an instant when the current in the inductor has magnitude 0.500 mA?

30.34 • A 7.50 nF capacitor is charged to 12.0 V, then disconnected from the power supply and connected in series through a coil. The period of oscillation of the circuit is then measured to be 8.60×10^{-5} s. Calculate: (a) the inductance of the coil; (b) the maximum charge on the capacitor; (c) the total energy of the circuit; (d) the maximum current in the circuit.

30.35 • *L-C* Oscillations. A capacitor with capacitance 6.00×10^{-5} F is charged by connecting it to a 12.0 V battery. The capacitor is disconnected from the battery and connected across an inductor with $L = 1.50$ H. (a) What are the angular frequency ω of the electrical oscillations and the period of these oscillations (the time for one oscillation)? (b) What is the initial charge on the capacitor? (c) How much energy is initially stored in the capacitor? (d) What is the charge on the capacitor 0.0230 s after the connection to the inductor is made? Interpret the sign of your answer. (e) At the time given in part (d), what is the current in the inductor? Interpret the sign of your answer. (f) At the time given in part (d), how much electrical energy is stored in the capacitor and how much is stored in the inductor?

30.36 • A Radio Tuning Circuit. The minimum capacitance of a variable capacitor in a radio is 4.18 pF. (a) What is the inductance of a coil connected to this capacitor if the oscillation frequency of the L-C circuit is 1600×10^3 Hz, corresponding to one end of the AM radio broadcast band, when the capacitor is set to its minimum capacitance? (b) The frequency at the other end of the broadcast band is 540×10^3 Hz. What is the maximum capacitance of the capacitor if the oscillation frequency is adjustable over the range of the broadcast band?

30.37 •• An *L-C* circuit containing an 80.0 mH inductor and a 1.25 nF capacitor oscillates with a maximum current of 0.750 A. Calculate: (a) the maximum charge on the capacitor and (b) the oscillation frequency of the circuit. (c) Assuming the capacitor had its maximum charge at time $t = 0$, calculate the energy stored in the inductor after 2.50 ms of oscillation.

Section 30.6 The *L-R-C* Series Circuit

30.38 • For the circuit of Fig. 30.17, let $C = 15.0$ nF, $L = 22$ mH, and $R = 75.0\ \Omega$. (a) Calculate the oscillation frequency of the circuit once the capacitor has been charged and the switch has been connected to point a. (b) How long will it take for the amplitude of the oscillation to decay to 10.0% of its original value? (c) What value of R would result in a critically damped circuit?

30.39 • An L-R-C series circuit has $L = 0.450$ H, $C = 2.50 \times 10^{-5}$ F, and resistance R. (a) What is the angular frequency of the circuit when $R = 0$? (b) What value must R have to give a 5.0% decrease in angular frequency compared to the value calculated in part (a)?

30.40 • An L-R-C series circuit has $L = 0.400$ H, $C = 7.00\ \mu$F, and $R = 320\ \Omega$. At $t = 0$ the current is zero and the initial charge on the capacitor is 2.80×10^{-4} C. (a) What are the values of the constants A and ϕ in Eq. (30.28)? (b) How much time does it take for each complete current oscillation after the switch in this circuit is closed? (c) What is the charge on the capacitor after the first complete current oscillation?

PROBLEMS

30.41 •• It is possible to make your own inductor by winding wire around a cylinder, such as a pencil. Assume you have a spool of AWG 20 copper wire, which has a diameter of 0.812 mm. (a) Estimate the diameter of a pencil. (b) Estimate how many times can you tightly wrap AWG 20 copper wire around a pencil to form a solenoid with a length of 4.0 cm. (c) Estimate the inductance of this solenoid by assuming the magnetic field inside is constant. (d) If a current of 1.0 A flows through this solenoid, how much magnetic energy will be stored inside?

30.42 • An inductor is connected to the terminals of a battery that has an emf of 16.0 V and negligible internal resistance. The current is 4.86 mA at 0.940 ms after the connection is completed. After a long time, the current is 6.45 mA. What are (a) the resistance R of the inductor and (b) the inductance L of the inductor?

30.43 •• CP Consider a coil of wire that has radius 3.00 cm and carries a sinusoidal current given by $i(t) = I_0 \sin(2\pi f t)$, where the frequency $f = 60.0$ Hz and the initial current $I_0 = 1.20$ A. (a) Estimate the magnetic flux through this coil as the product of the magnetic field at the center of the coil and the area of the coil. Use this magnetic flux to estimate the self-inductance L of the coil. (b) Use the value of L that you estimated in part (a) to calculate the magnitude of the maximum emf induced in the coil.

30.44 •• CALC A coil has 400 turns and self-inductance 7.50 mH. The current in the coil varies with time according to $i = (680\ \text{mA}) \cos(\pi t/0.0250\ \text{s})$. (a) What is the maximum emf induced in the coil? (b) What is the maximum average flux through each turn of the coil? (c) At $t = 0.0180$ s, what is the magnitude of the induced emf?

30.45 •• Solar Magnetic Energy. Magnetic fields within a sunspot can be as strong as 0.4 T. (By comparison, the earth's magnetic field is about 1/10,000 as strong.) Sunspots can be as large as 25,000 km in radius. The material in a sunspot has a density of about 3×10^{-4} kg/m^3. Assume μ for the sunspot material is μ_0. If 100% of the magnetic-field energy stored in a sunspot could be used to eject the sunspot's material away from the sun's surface, at what speed would that material be ejected? Compare to the sun's escape speed, which is about 6×10^5 m/s. (*Hint:* Calculate the kinetic energy the magnetic field could supply to 1 m^3 of sunspot material.)

30.46 •• CP CALC A Coaxial Cable. A small solid conductor with radius a is supported by insulating, nonmagnetic disks on the axis of a thin-walled tube with inner radius b. The inner and outer conductors carry equal currents i in opposite directions. (a) Use Ampere's law to find the magnetic field at any point in the volume between the conductors. (b) Write the expression for the flux $d\Phi_B$ through a narrow strip of length l parallel to the axis, of width dr, at a distance r from the axis of the cable and lying in a plane containing the axis. (c) Integrate your expression from part (b) over the volume between the two conductors to find the total flux produced by a current i in the central conductor. (d) Show that the inductance of a length l of the cable is

$$ L = l\frac{\mu_0}{2\pi}\ln\!\left(\frac{b}{a}\right) $$

(e) Use Eq. (30.9) to calculate the energy stored in the magnetic field for a length l of the cable.

30.47 • (a) What would have to be the self-inductance of a solenoid for it to store 10.0 J of energy when a 2.00 A current runs through it? (b) If this solenoid's cross-sectional diameter is 4.00 cm, and if you could wrap its coils to a density of 10 coils/mm, how long would the solenoid be? (See Exercise 30.11.) Is this a realistic length for ordinary laboratory use?

30.48 •• CALC Consider the circuit in Fig. 30.11 with both switches open. At $t = 0$ switch S_1 is closed while switch S_2 is left open. (a) Use Eq. (30.14) to derive an equation for the rate P_R at which electrical energy is being consumed in the resistor. In terms of $\mathcal{E}$, R, and L, at what value of t is P_R a maximum? What is that maximum value? (b) Use Eqs. (30.14) and (30.15) to derive an equation for P_L, the rate at which energy is being stored in the inductor. (c) What is P_L at $t = 0$ and as $t \to \infty$? (d) In terms of $\mathcal{E}$, R, and L, at what value of t is P_L a maximum? What is that maximum value? (e) Obtain an expression for $P_{\mathcal{E}}$, the rate at which the battery is supplying electrical energy to the circuit. In terms of $\mathcal{E}$, R, and L, at what value of t is $P_{\mathcal{E}}$ a maximum? What is that maximum value?

30.49 • An Electromagnetic Car Alarm. Your latest invention is a car alarm that produces sound at a particularly annoying frequency of 3500 Hz. To do this, the car-alarm circuitry must produce an alternating electric current of the same frequency. That's why your design includes an inductor and a capacitor in series. The maximum voltage across the capacitor is to be 12.0 V. To produce a sufficiently loud sound, the capacitor must store 0.0160 J of energy. What values of capacitance and inductance should you choose for your car-alarm circuit?
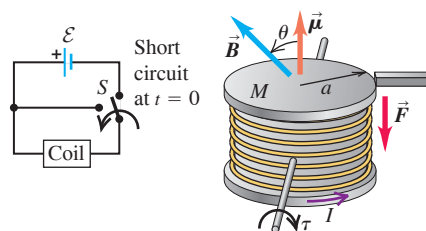
30.50 •• CALC An inductor with inductance $L = 0.300$ H and negligible resistance is connected to a battery, a switch S, and two resistors, $R_1 = 12.0\ \Omega$ and $R_2 = 16.0\ \Omega$ (**Fig. P30.50**). The battery has emf 96.0 V and negligible internal resistance. S is closed at $t = 0$. (a) What are the currents i_1, i_2, and i_3 just after S is closed? (b) What are i_1, i_2, and i_3 after S has been closed a long time? (c) What is the value of t for which i_3 has half of the final value that you calculated in part (b)? (d) When i_3 has half of its final value, what are i_1 and i_2?

Figure **P30.50**
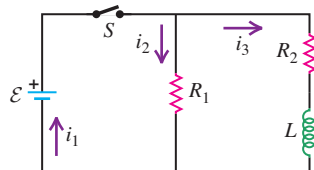
30.51 ••• CP An alternating-current electric motor includes a thin, hollow, cylindrical spool (similar to a ring) with mass $M = 1.11$ kg and radius $a = 5.00$ cm wrapped $N = 500$ times with a copper wire with resistance $R = 5.00\ \Omega$ and inductance $L = 77.0$ mH. Within the spool is a battery that supplies current $I = 1.00$ A, which makes the spool a magnetic dipole with dipole moment $\vec{\mu}$ parallel to the cylinder axis. A constant magnetic field with magnitude $B = 2.00$ T is supplied by an external stator magnet, while the spool turns freely on an axis perpendicular to its own axis. At a certain time, a bar is inserted, stopping the spool's motion (**Fig. P30.51**). At that instant the angle between the spool

axis and the magnetic field is $\theta = 45°$. (a) What is the magnitude of the downward force $\vec{F}$ applied by the bar onto the spool immediately after the bar is inserted? (b) Later, at time $t = 0$ with spool still at rest, the coil is short-circuited and a constant counter-torque $\tau = 0.500$ N·m is applied. The current subsides, and the magnetic torque decreases exponentially. At what time t does the force applied by the bar vanish? (*Hint:* Determine when the magnetic torque balances the counter-torque.) (c) After the spool rotates 180° it becomes stuck on the top side of the bar. The counter-torque is no longer applied, and the switch is returned to its original position. After a long time, the bar is removed. What is the angular acceleration of the spool immediately after the bar is removed? The moment of inertia of the spool for an axis along its diameter is $I = \frac{1}{2}Ma^2$.

Figure **P30.51**

30.52 ••• CALC An inductor with inductance $L = 0.200$ H and negligible resistance is connected to a battery, a switch S, and two resistors, $R_1 = 8.00\ \Omega$ and $R_2 = 6.00\ \Omega$ (**Fig. P30.52**). The battery has emf 48.0 V and negligible internal resistance. S is closed at $t = 0$. (a) What are the currents i_1, i_2, and i_3 just after S is closed? (b) What are i_1, i_2, and i_3 after S has been closed a long time? (c) Apply Kirchhoff's rules to the circuit and obtain a differential equation for $i_3(t)$. Integrate this equation to obtain an equation for i_3 as a function of the time t that has elapsed since S was closed. (d) Use the equation that you derived in part (c) to calculate the value of t for which i_3 has half of the final value that you calculated in part (b). (e) When i_3 has half of its final value, what are i_1 and i_2?

Figure **P30.52**
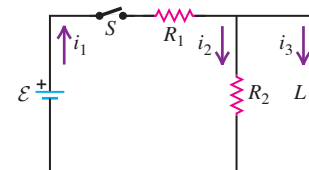
30.53 ••• CP CALC A cylindrical solenoid with radius 1.00 cm and length 10.0 cm consists of 300 windings of AWG 20 copper wire, which has a resistance per length of 0.0333 Ω/m. This solenoid is connected in series with a 10.0 μF capacitor, which is initially uncharged. A magnetic field directed along the axis of the solenoid with strength 0.100 T is switched on abruptly. (a) The solenoid may be considered an inductor and a resistor in series. Use Faraday's law to determine the average emf across the solenoid during the brief switch-on interval, and determine the net charge initially deposited on the capacitor. (See Exercise 29.4.) (b) At time $t = 0$ the capacitor is fully charged and there is no current. How much time does it take for the capacitor to fully discharge the first time? (c) What is the frequency with which the current oscillates? (d) How much energy is stored in the capacitor at $t = 0$? (e) How long does it take for the total energy stored in the circuit to drop to 10% of that value?

30.54 •• A 6.40 nF capacitor is charged to 24.0 V and then disconnected from the battery in the circuit and connected in series with a coil that has $L = 0.0660$ H and negligible resistance. After the circuit has been completed, there are current oscillations. (a) At an instant when the charge of the capacitor is 0.0800 μC, how much energy is stored in the capacitor and in the inductor, and what is the current in the inductor? (b) At the instant when the charge on the capacitor is 0.0800 μC, what are the voltages across the capacitor and across the inductor, and what is the rate at which current in the inductor is changing?

**30.55 •** An *L-C* circuit consists of a 60.0 mH inductor and a 250 μF capacitor. The initial charge on the capacitor is 6.00 μC, and the initial current in the inductor is zero. (a) What is the maximum voltage across the capacitor? (b) What is the maximum current in the inductor? (c) What is the maximum energy stored in the inductor? (d) When the current in the inductor has half its maximum value, what is the charge on the capacitor and what is the energy stored in the inductor?

30.56 •• A charged capacitor with $C = 590$ μF is connected in series to an inductor that has $L = 0.330$ H and negligible resistance. At an instant when the current in the inductor is $i = 2.50$ A, the current is increasing at a rate of $di/dt = 73.0$ A/s. During the current oscillations, what is the maximum voltage across the capacitor?

30.57 ••• CP In the circuit shown in Fig. P30.57, the switch *S* has been open for a long time and is suddenly closed. Neither the battery nor the inductors have any appreciable resistance. What do the ammeter and the voltmeter read (a) just after *S* is closed; (b) after *S* has been closed a very long time; (c) 0.115 ms after *S* is closed?

Figure **P30.57**
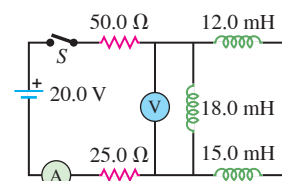
30.58 ••• CP In the circuit shown in **Fig. P30.58**, find the reading in each ammeter and voltmeter (a) just after switch *S* is closed and (b) after *S* has been closed a very long time.

Figure **P30.58**
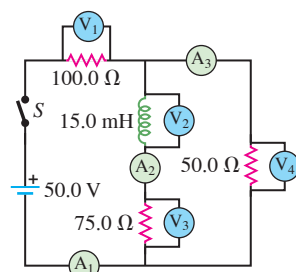
30.59 •• DATA To investigate the properties of a large industrial solenoid, you connect the solenoid and a resistor in series with a battery. Switches allow the battery to be replaced by a short circuit across the solenoid and resistor. Therefore Fig. 30.11 applies, with $R = R_{ext} + R_L$, where R_L is the resistance of the solenoid and R_{ext} is the resistance of the series resistor. With switch S_2 open, you close switch S_1 and keep it closed until the current *i* in the solenoid is constant (Fig. 30.11). Then you close S_2 and open S_1 simultaneously, using a rapid-response switching mechanism. With high-speed electronics you measure the time t_{half} that it takes for the current to decrease to half of its initial value. You repeat this measurement for several values of R_{ext} and obtain these results:

R_{ext} (Ω)	3.0	4.0	5.0	6.0	7.0	8.0	10.0	12.0
t_{half} (s)	0.735	0.654	0.589	0.536	0.491	0.453	0.393	0.347

(a) Graph your data in the form of $1/t_{half}$ versus R_{ext}. Explain why the data points plotted this way fall close to a straight line. (b) Use your graph from part (a) to calculate the resistance R_L and inductance *L* of the solenoid. (c) If the current in the solenoid is 20.0 A, how much energy is stored there? At what rate is electrical energy being dissipated in the resistance of the solenoid?
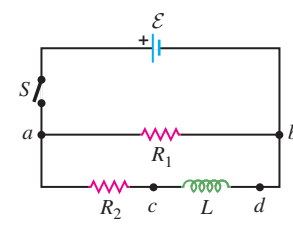
30.60 •• In the circuit shown in **Fig. P30.60**, switch S_1 has been closed for a long enough time so that the current reads a steady 3.50 A. Suddenly, switch S_2 is closed and S_1 is opened at the same instant. (a) What is the maximum charge that the capacitor will receive? (b) What is the current in the inductor at this time?

Figure **P30.60**
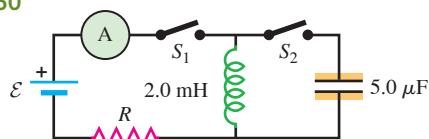
30.61 •• CP In the circuit shown in Fig. **P30.61**, $\mathcal{E} = 60.0$ V, $R_1 = 40.0$ Ω, $R_2 = 25.0$ Ω, and $L = 0.300$ H. Switch *S* is closed at $t = 0$. Just after the switch is closed, (a) what is the potential difference v_{ab} across the resistor R_1; (b) which point, *a* or *b*, is at a higher potential; (c) what is the potential difference v_{cd} across the inductor 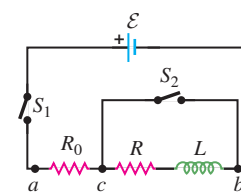*L*; (d) which point, *c* or *d*, is at a higher potential? The switch is left closed a long time and then opened. Just after the switch is opened, (e) what is the potential difference v_{ab} across the resistor R_1; (f) which point, *a* or *b*, is at a higher potential; (g) what is the potential difference v_{cd} across the inductor *L*; (h) which point, *c* or *d*, is at a higher potential?

Figure **P30.61**

30.62 •• CP In the circuit shown in Fig. P30.61, $\mathcal{E} = 60.0$ V, $R_1 = 40.0$ Ω, $R_2 = 25.0$ Ω, and $L = 0.300$ H. (a) Switch *S* is closed. At some time *t* afterward, the current in the inductor is increasing at a rate of $di/dt = 50.0$ A/s. At this instant, what are the current i_1 through R_1 and the current i_2 through R_2? (*Hint:* Analyze two separate loops: one containing $\mathcal{E}$ and R_1 and the other containing $\mathcal{E}$, R_2, and *L*.) (b) After the switch has been closed a long time, it is opened again. Just after it is opened, what is the current through R_1?

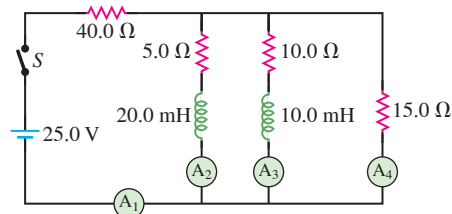**30.63 •• CALC** Consider the circuit shown in **Fig. P30.63**. Let $\mathcal{E} = 36.0$ V, $R_0 = 50.0$ Ω, $R = 150$ Ω, and $L = 4.00$ H. (a) Switch S_1 is closed and switch S_2 is left open. Just after S_1 is closed, what are the current i_0 through R_0 and the potential differences v_{ac} and v_{cb}? (b) After S_1 has been closed a long time (S_2 is still open) so that the current has reached its final, steady value, what are i_0, v_{ac}, and v_{cb}? (c) Find the expressions for i_0, v_{ac}, and v_{cb} as functions of the time *t* since S_1 was closed. Your results should agree with part (a) when $t = 0$ and with part (b) when $t \to \infty$. Graph i_0, v_{ac}, and v_{cb} versus time.

Figure **P30.63**

30.64 •• After the current in the circuit of Fig. P30.63 has reached its final, steady value with switch S_1 closed and S_2 open, switch S_2 is closed, thus short-circuiting the inductor. (Switch S_1 remains closed. See Problem 30.63 for numerical values of the circuit elements.) (a) Just after S_2 is closed, what are v_{ac} and v_{cb}, and what are the currents through R_0, *R*, and S_2? (b) A long time after S_2 is closed, what are v_{ac} and v_{cb}, and what are the currents through R_0, *R*, and S_2? (c) Derive expressions for the currents through R_0, *R*, and S_2 as functions of the time *t* that has elapsed since S_2 was closed. Your results should agree with part (a) when $t = 0$ and with part (b) when $t \to \infty$. Graph these three currents versus time.

30.65 •• CP In the circuit shown in **Fig. P30.65**, switch *S* is closed at time $t = 0$. (a) Find the reading of each meter just after *S* is closed. (b) What does each meter read long after *S* is closed?

Figure **P30.65**

30.66 ••• **CP** In the circuit shown in **Fig. P30.66**, neither the battery nor the inductors have any appreciable resistance, the capacitors are initially uncharged, and the switch S has been in position 1 for a very long time. (a) What is the current in the circuit? (b) The switch is now suddenly flipped to position 2. Find the maximum charge that each capacitor will receive, and how much time after the switch is flipped it will take them to acquire this charge.

Figure **P30.66**
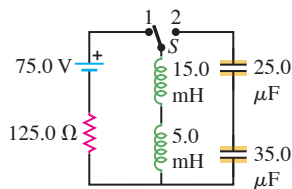
30.67 •• **DATA** During a summer internship as an electronics technician, you are asked to measure the self-inductance L of a solenoid. You connect the solenoid in series with a $10.0 \ \Omega$ resistor, a battery that has negligible internal resistance, and a switch. Using an ideal voltmeter, you measure and digitally record the voltage v_L across the solenoid as a function of the time t that has elapsed since the switch is closed. Your measured values are shown in **Fig. P30.67**, where v_L is plotted versus t. In addition, you measure that $v_L = 50.0$ V just after the switch is closed and $v_L = 20.0$ V a long time after it is closed. (a) Apply the loop rule to the circuit and obtain an equation for v_L as a function of t. [*Hint*: Use an analysis similar to that used to derive Eq. (30.15).] (b) What is the emf $\mathcal{E}$ of the battery? (c) According to your measurements, what is the voltage amplitude across the $10.0 \ \Omega$ resistor as $t \to \infty$? Use this result to calculate the current in the circuit as $t \to \infty$. (d) What is the resistance R_L of the solenoid? (e) Use the theoretical equation from part (a), Fig. P30.67, and the values of $\mathcal{E}$ and R_L from parts (b) and (d) to calculate L. (*Hint:* According to the equation, what is v_L when $t = \tau$, one time constant? Use Fig. P30.67 to estimate the value of $t = \tau$.)

Figure **P30.67**
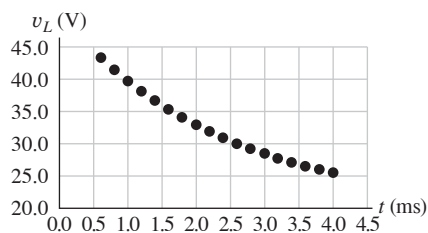
30.68 •• **DATA** You are studying a solenoid of unknown resistance and inductance. You connect it in series with a $50.0 \ \Omega$ resistor, a 25.0 V battery that has negligible internal resistance, and a switch. Using an ideal voltmeter, you measure and digitally record the voltage v_R across the resistor as a function of the time t that has elapsed after the switch is closed. Your measured values are shown in **Fig. P30.68**, where v_R is plotted versus t. In addition, you measure that $v_R = 0$ just after the switch is closed and $v_R = 25.0$ V a long time after it is closed. (a) What is the resistance R_L of the solenoid? (b) Apply the loop rule to the circuit and obtain an equation for v_R as a function of t. (c) According to the equation that you derived in part (b), what is v_R when $t = \tau$, one time constant? Use Fig. P30.68 to estimate the value of $t = \tau$. What is the inductance of the solenoid? (d) How much energy is stored in the inductor a long time after the switch is closed?

Figure **P30.68**
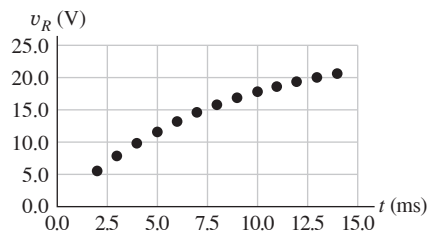
CHALLENGE PROBLEMS

30.69 ••• A long solenoid with N_1 windings and radius b surrounds a coaxial, narrower solenoid with N_2 windings and radius $a < b$, as shown in **Fig. P30.69**. At the far end, the outer solenoid is attached to the inner solenoid by a wire, so that current flows down the outer coil and then back through the inner coil as shown. (a) The two leads are attached to a supply circuit that includes a battery, supplying current I as indicated. What is the magnetic flux through each turn of the inner coil, taking rightward as the positive direction? (b) What is the magnetic flux through each turn of the outer coil? (c) What is the inductance as seen by the two leads? (d) What would be the inductance if the sense of the inner coil were reversed? (e) In the original configuration, what would be the inductance if $\lambda = 20.0$ cm, $a = 1.00$ cm, $b = 2.00$ cm, $N_1 = 1200$, and $N_2 = 750$? (f) Using the values from part (e), what would be the inductance in the configuration of part (d)?

Figure **P30.69**
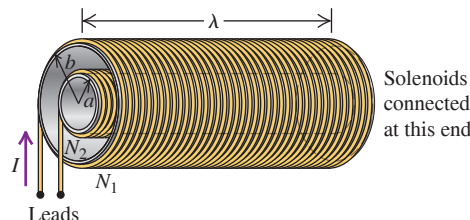
30.70 ••• **CP A Volume Gauge.** A tank containing a liquid has turns of wire wrapped around it, causing it to act like an inductor. The liquid content of the tank can be measured by using its inductance to determine the height of the liquid in the tank. The inductance of the tank changes from a value of L_0 corresponding to a relative permeability of 1 when the tank is empty to a value of L_f corresponding to a relative permeability of K_m (the relative permeability of the liquid) when the tank is full. The appropriate electronic circuitry can determine the inductance to five significant figures and thus the effective relative permeability of the combined air and liquid within the rectangular cavity of the tank. The four sides of the tank each have width W and height D (**Fig. P30.70**). The height of the liquid in the tank is d. You can ignore any fringing effects and assume that the relative permeability of the material of which the tank is made can be ignored. (a) Derive an expression for d as a function of L, the inductance corresponding to a certain fluid height, L_0, L_f, and D. (b) What is the inductance (to five significant figures) for a tank $\frac{1}{4}$ full, $\frac{1}{2}$ full, $\frac{3}{4}$ full, and completely full if the tank contains liquid oxygen? Take $L_0 = 0.63000$ H. The magnetic susceptibility of liquid oxygen is $\chi_m = 1.52 \times 10^{-3}$. (c) Repeat part (b) for mercury. The magnetic susceptibility of mercury is given in Table 28.1. (d) For which material is this volume gauge more practical?

Figure **P30.70**
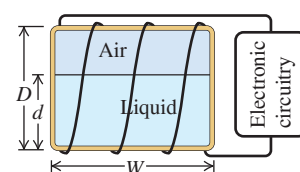
30.71 ••• **CP CALC** Consider the circuit shown in **Fig. P30.71**. Switch S is closed at time $t = 0$, causing a current i_1 through the inductive branch and a current i_2 through the capacitive branch. The initial charge on the capacitor is zero, and the charge at time t is q_2. (a) Derive expressions for i_1, i_2, and q_2 as functions of time. Express your answers in terms of $\mathcal{E}$, L, C, R_1, R_2, and t. For the remainder of the problem let the circuit elements have the following values: $\mathcal{E} = 48$ V, $L = 8.0$ H, $C = 20$ μF, $R_1 = 25$ Ω, and $R_2 = 5000$ Ω. (b) What is the initial current through the inductive branch? What is the initial current through the capacitive branch? (c) What are the currents through the inductive and capacitive branches a long time after the switch has been closed? How long is a "long time"? Explain. (d) At what time t_1 (accurate to two significant figures) will the currents i_1 and i_2 be equal? (*Hint:* You might consider using series expansions for the exponentials.) (e) For the conditions given in part (d), determine i_1. (f) The total current through the battery is $i = i_1 + i_2$. At what time t_2 (accurate to two significant figures) will i equal one-half of its final value? (*Hint:* The numerical work is greatly simplified if one makes suitable approximations. A sketch of i_1 and i_2 versus t may help you decide what approximations are valid.)

Figure **P30.71**
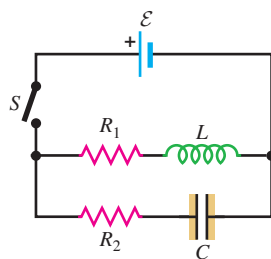
MCAT-STYLE PASSAGE PROBLEMS

BIO Quenching an MRI Magnet. Magnets carrying very large currents are used to produce the uniform, large-magnitude magnetic fields that are required for *magnetic resonance imaging* (MRI). A typical MRI magnet may be a solenoid that is 2.0 m long and 1.0 m in diameter, has a self-inductance of 4.4 H, and carries a current of 750 A. A normal wire carrying that much current would dissipate a great deal of electrical power as heat, so most MRI magnets are made with coils of superconducting wire cooled by liquid helium at a temperature just under its boiling point (4.2 K). After a current is established in the wire, the power supply is disconnected and the magnet leads are shorted together through a piece of superconductor so that the current flows without resistance as long as the liquid helium keeps the magnet cold.

Under rare circumstances, a small segment of the magnet's wire may lose its superconducting properties and develop resistance. In this segment, electrical energy is converted to thermal energy, which can boil off some of the liquid helium. More of the wire then warms up and loses its superconducting properties, thus dissipating even more energy as heat. Because the latent heat of vaporization of liquid helium is quite low (20.9 kJ/kg), once the wire begins to warm up, all of the liquid helium may boil off rapidly. This event, called a *quench,* can damage the magnet. Also, a large volume of helium gas is generated as the liquid helium boils off, causing an asphyxiation hazard, and the resulting rapid pressure buildup can lead to an explosion. You can see how important it is to keep the wire resistance in an MRI magnet at zero and to have devices that detect a quench and shut down the current immediately.

30.72 How many turns does this typical MRI magnet have? (a) 1100; (b) 3000; (c) 4000; (d) 22,000.

30.73 If a small part of this magnet loses its superconducting properties and the resistance of the magnet wire suddenly rises from 0 to a constant 0.005 Ω, how much time will it take for the current to decrease to half of its initial value? (a) 4.7 min; (b) 10 min; (c) 15 min; (d) 30 min.

30.74 If part of the magnet develops resistance and liquid helium boils away, rendering more and more of the magnet nonsuperconducting, how will this quench affect the time for the current to drop to half of its initial value? (a) The time will be shorter because the resistance will increase; (b) the time will be longer because the resistance will increase; (c) the time will be the same; (d) not enough information is given.

30.75 If all of the magnetic energy stored in this MRI magnet is converted to thermal energy, how much liquid helium will boil off? (a) 27 kg; (b) 38 kg; (c) 60 kg; (d) 110 kg.

ANSWERS

Chapter Opening Question ?

(iii) As explained in Section 30.2, traffic light sensors work by measuring the change in inductance of a coil embedded under the road surface when a car (which contains ferromagnetic material) drives over it.

Key Example √ARIATION Problems

VP30.4.1 (a) 6.50×10^{-5} H (b) 1.56×10^{-3} T$\cdot$m^2
VP30.4.2 (a) 3.36×10^{-4} V (b) zero (c) 1.44×10^{-4} V
VP30.4.3 (a) 1.96×10^{-6} T$\cdot$m^2 (b) 4.18×10^{-3} V
VP30.4.4 (a) 25.5 μH (b) 23.0 mV, opposite to the current
VP30.7.1 (a) 84.9 Ω (b) 0.212 A
VP30.7.2 (a) 0.190 A (b) $+229$ A/s (c) 6.79×10^{-4} J
VP30.7.3 (a) 0.476 A (b) -229 A/s (c) 4.26×10^{-3} J

VP30.7.4 (a) $(L/R)\ln 2 = 0.693(L/R)$ (b) $\mathcal{E}/2R$ (c) $\mathcal{E}/2L$
VP30.10.1 (a) 5.12×10^{-3} F (b) 14.0 A
VP30.10.2 (a) 2.20×10^{-4} s (b) 2.85 A
VP30.10.3 (a) 35.1 μF (b) 0.0125 H (c) 1.51×10^{3} rad/s
VP30.10.4 (a) 61.1 μF (b) 49.4 Ω

Bridging Problem

(a) 7.68×10^{-8} J
(b) 1.60 mA
(c) 24.8 mV
(d) 1.92×10^{-8} J, 21.5 mV