? A colorful tail-spot wrasse (*Halichoeres melanurus*) is about 10 cm long and can float in the ocean with little effort, while a manta ray (*Manta birostris*) is more than 5 m across and must "flap" its fins continuously to keep from sinking. Which of these best explains the difference? A manta ray has (i) a different shape; (ii) greater mass; (iii) greater volume; (iv) a greater product of mass and volume; (v) a greater ratio of mass to volume.

12 Fluid Mechanics

LEARNING OUTCOMES

In this chapter, you'll learn...

- **12.1** The meaning of the density of a material and the average density of an object.
- **12.2** What is meant by the pressure in a fluid, and how it is measured.
- **12.3** How to calculate the buoyant force that a fluid exerts on an object immersed in it.
- **12.4** The significance of laminar versus turbulent fluid flow, and how the speed of flow in a tube depends on the tube's size.
- **12.5** How to use Bernoulli's equation to relate pressure and flow speed at different points in certain types of flow.
- **12.6** How viscous flow and turbulent flow differ from ideal flow.

You'll need to review...

- **7.1** Mechanical energy change when forces other than gravity do work.
- **11.4** Pressure and its units.

Fluids play a vital role in many aspects of everyday life. We drink them, breathe them, swim in them. They circulate through our bodies and control our weather. The physics of fluids is therefore crucial to our understanding of both nature and technology.

We begin our study with **fluid statics,** the study of fluids at rest in equilibrium situations. Like other equilibrium situations, it is based on Newton's first and third laws. We'll explore the key concepts of density, pressure, and buoyancy. **Fluid dynamics,** the study of fluids in motion, is much more complex; indeed, it is one of the most complex branches of mechanics. Fortunately, we can analyze many important situations by using simple idealized models and familiar principles such as Newton's laws and conservation of energy. Even so, we'll barely scratch the surface of this broad and interesting topic.

12.1 GASES, LIQUIDS, AND DENSITY

A **fluid** is any substance that can flow and change the shape of the volume that it occupies. (By contrast, a solid tends to maintain its shape.) We use the term "fluid" for both gases and liquids. The key difference between them is that a liquid has *cohesion,* while a gas does not. The molecules in a liquid are close to one another, so they can exert attractive forces on each other and thus tend to stay together (that is, to cohere). That's why a quantity of liquid maintains the same volume as it flows: If you pour 500 mL of water into a pan, the water will still occupy a volume of 500 mL. The molecules of a gas, by contrast, are separated on average by distances far larger than the size of a molecule. Hence the forces between molecules are weak, there is little or no cohesion, and a gas can easily change in volume. If you open the valve on a tank of compressed oxygen that has a volume of 500 mL, the oxygen will expand to a far greater volume.

An important property of *any* material, fluid or solid, is its **density,** defined as its mass per unit volume. A homogeneous material such as ice or iron has the same density throughout. We use ρ (the Greek letter rho) for density. For a homogeneous material,

Density of a homogeneous material

$$\rho = \frac{m}{V} \qquad (12.1)$$

m: Mass of material
V: Volume occupied by material

Two objects made of the same material have the same density even though they may have different masses and different volumes. That's because the *ratio* of mass to volume is the same for both objects (**Fig. 12.1**).

The SI unit of density is the kilogram per cubic meter $(1\ \text{kg/m}^3)$. The cgs unit, the gram per cubic centimeter $(1\ \text{g/cm}^3)$, is also widely used:

$$1\ \text{g/cm}^3 = 1000\ \text{kg/m}^3$$

The densities of some common substances at ordinary temperatures are given in **Table 12.1**. Note the wide range of magnitudes. The densest material found on earth is the metal osmium $(\rho = 22{,}500\ \text{kg/m}^3)$, but its density pales by comparison to the densities of exotic astronomical objects, such as white dwarf stars and neutron stars.

The **specific gravity** of a material is the ratio of its density to the density of water at 4.0°C, $1000\ \text{kg/m}^3$; it is a pure number without units. For example, the specific gravity of aluminum is 2.7. "Specific gravity" is a poor term, since it has nothing to do with gravity; "relative density" would have been a better choice.

The density of some materials varies from point to point within the material. One example is the material of the human body, which includes low-density fat (about $940\ \text{kg/m}^3$) and high-density bone (from 1700 to $2500\ \text{kg/m}^3$). Two others are the earth's atmosphere (which is less dense at high altitudes) and oceans (which are denser at greater depths). For these materials, Eq. (12.1) describes the **average density.** In general, the density of a material depends on environmental factors such as temperature and pressure.

Figure **12.1** Two objects with different masses and different volumes but the same density.

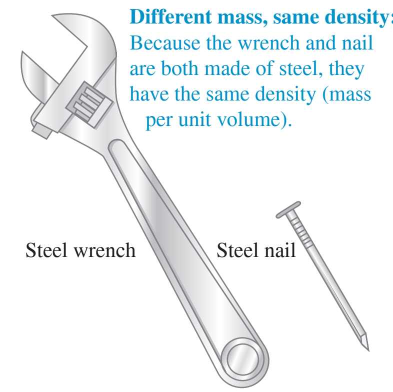


TABLE 12.1 Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerin	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

*To obtain the densities in grams per cubic centimeter, simply divide by 10^3.

BIO APPLICATION Liquid Cohesion in Trees How do trees—some of which grow to heights greater than 100 m—supply water to their highest leaves? The answer lies in the strong cohesive forces between molecules of liquid water. Narrow pipes within the tree extend upward from the roots to the leaves. As water evaporates from the leaves, cohesive forces pull replacement water upward through these pipes.

EXAMPLE 12.1 The weight of a roomful of air

Find the mass and weight of the air at 20°C in a living room with a 4.0 m × 5.0 m floor and a ceiling 3.0 m high, and the mass and weight of an equal volume of water.

IDENTIFY and SET UP We assume that the air density is the same throughout the room. (Air is less dense at high elevations than near sea level, but the density varies negligibly over the room's 3.0 m height; see Section 12.2.) We use Eq. (12.1) to relate the mass m_{air} to the room's volume V (which we'll calculate) and the air density ρ_{air} (given in Table 12.1).

EXECUTE We have $V = (4.0\ \text{m})(5.0\ \text{m})(3.0\ \text{m}) = 60\ \text{m}^3$, so from Eq. (12.1),

$$m_{\text{air}} = \rho_{\text{air}} V = (1.20\ \text{kg/m}^3)(60\ \text{m}^3) = 72\ \text{kg}$$

$$w_{\text{air}} = m_{\text{air}} g = (72\ \text{kg})(9.8\ \text{m/s}^2) = 700\ \text{N} = 160\ \text{lb}$$

The mass and weight of an equal volume of water are

$$m_{\text{water}} = \rho_{\text{water}} V = (1000\ \text{kg/m}^3)(60\ \text{m}^3) = 6.0 \times 10^4\ \text{kg}$$

$$w_{\text{water}} = m_{\text{water}} g = (6.0 \times 10^4\ \text{kg})(9.8\ \text{m/s}^2)$$
$$= 5.9 \times 10^5\ \text{N} = 1.3 \times 10^5\ \text{lb} = 66\ \text{tons}$$

EVALUATE A roomful of air weighs about the same as an average adult. Water is nearly a thousand times denser than air, so its mass and weight are larger by the same factor. The weight of a roomful of water would collapse the floor of an ordinary house.

KEYCONCEPT To find the density of a uniform substance, divide the mass of the substance by the volume that it occupies.

TEST YOUR UNDERSTANDING OF SECTION 12.1 Rank the following objects in order from highest to lowest average density: (i) mass $m = 4.00$ kg, volume $V = 1.60 \times 10^{-3}\ \text{m}^3$; (ii) $m = 8.00$ kg, $V = 1.60 \times 10^{-3}\ \text{m}^3$; (iii) $m = 8.00$ kg, $V = 3.20 \times 10^{-3}\ \text{m}^3$; (iv) $m = 2560$ kg, $V = 0.640\ \text{m}^3$; (v) $m = 2560$ kg, $V = 1.28\ \text{m}^3$.

ANSWER

(ii), (iv), (i) and (iii) (tie), (v) In each case the average density equals the mass divided by the volume:

(i) $\rho = (4.00\ \text{kg})/(1.60 \times 10^{-3}\ \text{m}^3) = 2.50 \times 10^3\ \text{kg/m}^3$;
(ii) $\rho = (8.00\ \text{kg})/(1.60 \times 10^{-3}\ \text{m}^3) = 5.00 \times 10^3\ \text{kg/m}^3$;
(iii) $\rho = (8.00\ \text{kg})/(3.20 \times 10^{-3}\ \text{m}^3) = 2.50 \times 10^3\ \text{kg/m}^3$;
(iv) $\rho = (2560\ \text{kg})/(0.640\ \text{m}^3) = 4.00 \times 10^3\ \text{kg/m}^3$;
(v) $\rho = (2560\ \text{kg})/(1.28\ \text{m}^3) = 2.00 \times 10^3\ \text{kg/m}^3$.

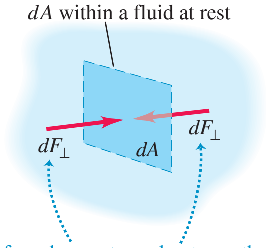
Note that compared to object (i), object (ii) has double the mass but the same volume and so has double the average density. Object (iii) has double the mass and double the volume of object (i), so (i) and (iii) have the same average density. Finally, object (v) has the same mass as object (iv) but double the volume, so (v) has half the average density of (iv).

12.2 PRESSURE IN A FLUID

A fluid exerts a force perpendicular to any surface in contact with it, such as a container wall or an object immersed in the fluid. This is the force that you feel pressing on your legs when you dangle them in a swimming pool. Even when a fluid as a whole is at rest, the molecules that make up the fluid are in motion; the force exerted by the fluid is due to molecules colliding with their surroundings.

Imagine a surface *within* a fluid at rest. For this surface and the fluid to remain at rest, the fluid must exert forces of equal magnitude but opposite direction on the surface's two sides. Consider a small surface of area dA centered on a point in the fluid; the normal force exerted by the fluid on each side is $dF_\perp$ (**Fig. 12.2**). We define the **pressure** p at that point as the normal force per unit area—that is, the ratio of $dF_\perp$ to dA (**Fig. 12.3**):

Figure **12.2** Forces acting on a small surface within a fluid at rest.



$$p = \frac{dF_\perp}{dA} \tag{12.2}$$

Pressure at a point in a fluid (p); Normal force exerted by fluid on a small surface at that point ($dF_\perp$); Area of surface (dA)

If the pressure is the same at all points of a finite plane surface with area A, then

$$p = \frac{F_\perp}{A} \tag{12.3}$$

where $F_\perp$ is the net normal force on one side of the surface. The SI unit of pressure is the **pascal,** where

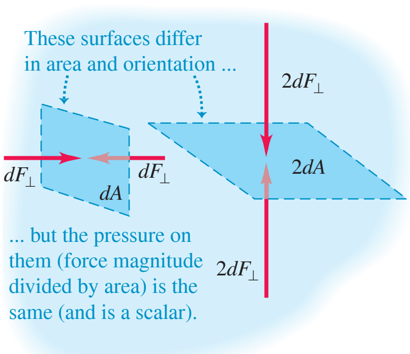
$$1\ \text{pascal} = 1\ \text{Pa} = 1\ \text{N/m}^2$$

We introduced the pascal in Chapter 11. Two related units, used principally in meteorology, are the *bar,* equal to 10^5 Pa, and the *millibar,* equal to 100 Pa.

Atmospheric pressure p_a is the pressure of the earth's atmosphere, the pressure at the bottom of this sea of air in which we live. This pressure varies with weather changes and with elevation. Normal atmospheric pressure at sea level (an average value) is 1 *atmosphere* (atm), defined to be exactly 101,325 Pa. To four significant figures,

$$(p_a)_{av} = 1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}$$
$$= 1.013\ \text{bar} = 1013\ \text{millibar} = 14.70\ \text{lb/in.}^2$$

Figure **12.3** Pressure is a scalar with units of newtons per square meter. By contrast, force is a vector with units of newtons.



CAUTION **Don't confuse pressure and force** In everyday language "pressure" and "force" mean pretty much the same thing. In fluid mechanics, however, these words describe very different quantities. Pressure acts perpendicular to any surface in a fluid, no matter how that surface is oriented (Fig. 12.3). Hence pressure has no direction of its own; it's a scalar. By contrast, force is a vector with a definite direction. Remember, too, that pressure is force per unit area. As Fig. 12.3 shows, a surface with twice the area has twice as much force exerted on it by the fluid, so the pressure is the same.

EXAMPLE 12.2 The force of air

In the room described in Example 12.1, what is the total downward force on the floor due to an air pressure of 1.00 atm?

IDENTIFY and SET UP This example uses the relationship among the pressure p of a fluid (air), the area A subjected to that pressure, and the resulting normal force $F_\perp$ the fluid exerts. The pressure is uniform, so we use Eq. (12.3), $F_\perp = pA$, to determine $F_\perp$. The floor is horizontal, so $F_\perp$ is vertical (downward).

EXECUTE We have $A = (4.0\text{ m})(5.0\text{ m}) = 20\text{ m}^2$, so from Eq. (12.3),

$$F_\perp = pA = (1.013 \times 10^5\text{ N/m}^2)(20\text{ m}^2)$$
$$= 2.0 \times 10^6\text{ N} = 4.6 \times 10^5\text{ lb} = 230\text{ tons}$$

EVALUATE Unlike the water in Example 12.1, $F_\perp$ will not collapse the floor here, because there is an *upward* force of equal magnitude on the floor's underside. If the house has a basement, this upward force is exerted by the air underneath the floor. In this case, if we ignore the thickness of the floor, the *net* force due to air pressure is zero.

KEYCONCEPT To find the force exerted by a fluid perpendicular to a surface, multiply the pressure of the fluid by the surface's area. This relationship comes from the definition of pressure as the normal force per unit area within the fluid.

Pressure, Depth, and Pascal's Law

If the weight of the fluid can be ignored, the pressure in a fluid is the same throughout its volume. We used that approximation in our discussion of bulk stress and strain in Section 11.4. But often the fluid's weight is *not* negligible, and pressure variations are important. Atmospheric pressure is less at high altitude than at sea level, which is why airliner cabins have to be pressurized. When you dive into deep water, you can feel the increased pressure on your ears.

We can derive a relationship between the pressure p at any point in a fluid at rest and the elevation y of the point. We'll assume that the density ρ has the same value throughout the fluid (that is, the density is *uniform*), as does the acceleration due to gravity g. If the fluid is in equilibrium, any thin element of the fluid with thickness dy is also in equilibrium (**Fig. 12.4a**). The bottom and top surfaces each have area A, and they are at elevations y and $y + dy$ above some reference level where $y = 0$. The fluid element has volume $dV = A\,dy$, mass $dm = \rho\,dV = \rho A\,dy$, and weight $dw = dm\,g = \rho g A\,dy$.

What are the other forces on this fluid element (Fig 12.4b)? Let's call the pressure at the bottom surface p; then the total y-component of upward force on this surface is pA. The pressure at the top surface is $p + dp$, and the total y-component of (downward) force on the top surface is $-(p + dp)A$. The fluid element is in equilibrium, so the total y-component of force, including the weight and the forces at the bottom and top surfaces, must be zero:

$$\sum F_y = 0$$

so

$$pA - (p + dp)A - \rho g A\,dy = 0$$

When we divide out the area A and rearrange, we get

$$\frac{dp}{dy} = -\rho g \qquad (12.4)$$

This equation shows that when y increases, p decreases; that is, as we move upward in the fluid, pressure decreases, as we expect. If p_1 and p_2 are the pressures at elevations y_1 and y_2, respectively, and if ρ and g are constant, then

$$\underbrace{p_2 - p_1}_{\text{Pressure difference between two points in a fluid of uniform density}} = -\underbrace{\rho}_{\text{Uniform density of fluid}}\underbrace{g}_{\text{Acceleration due to gravity } (g > 0)}(\underbrace{y_2 - y_1}_{\text{Heights of the two points}}) \qquad (12.5)$$

Figure **12.4** The forces on an element of fluid in equilibrium.

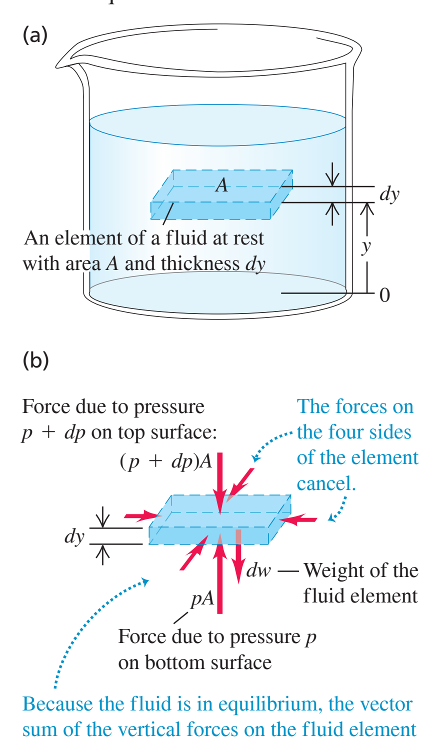
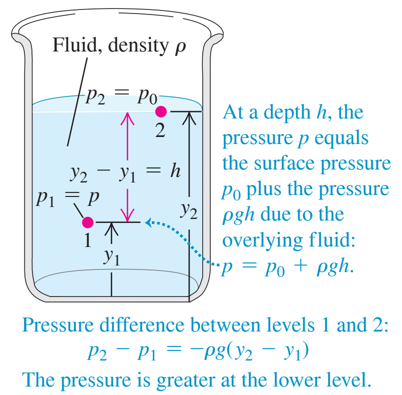


Figure **12.5** How pressure varies with depth in a fluid with uniform density.



It's often convenient to express Eq. (12.5) in terms of the *depth* below the surface of a fluid (**Fig. 12.5**). Take point 1 at any level in the fluid and let p represent the pressure at this point. Take point 2 at the *surface* of the fluid, where the pressure is p_0 (subscript zero for zero depth). The depth of point 1 below the surface is $h = y_2 - y_1$, and Eq. (12.5) becomes

$$p_0 - p = -\rho g(y_2 - y_1) = -\rho gh \qquad \text{or}$$

Pressure at depth h in a **fluid of uniform density**

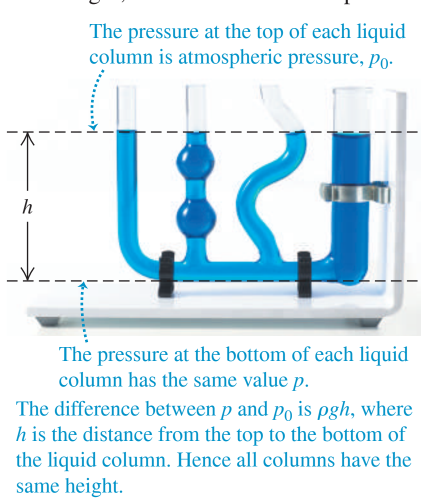
$$p = p_0 + \rho gh \tag{12.6}$$

Pressure at surface of fluid (p_0); Uniform density of fluid (ρ); Acceleration due to gravity ($g > 0$); Depth below surface (h)

The pressure p at a depth h is greater than the pressure p_0 at the surface by an amount ρgh. Note that the pressure is the same at any two points at the same level in the fluid. The *shape* of the container does not matter (**Fig. 12.6**).

Equation (12.6) shows that if we increase the pressure p_0 at the top surface, possibly by using a piston that fits tightly inside the container to push down on the fluid surface, the pressure p at any depth increases by exactly the same amount. This observation is called *Pascal's law.*

Figure **12.6** Each fluid column has the same height, no matter what its shape.



PASCAL'S LAW **Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.**

The hydraulic lift (**Fig. 12.7**) illustrates Pascal's law. A piston with small cross-sectional area A_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure $p = F_1/A_1$ is transmitted through the connecting pipe to a larger piston of area A_2. The applied pressure is the same in both cylinders, so

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{and} \quad F_2 = \frac{A_2}{A_1}F_1 \tag{12.7}$$

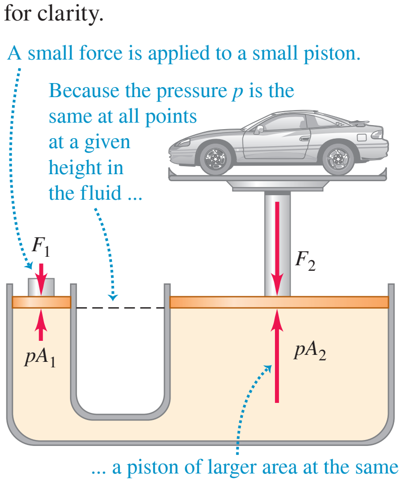
The hydraulic lift is a force-multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lifts and jacks, many elevators, and hydraulic brakes all use this principle.

For gases the assumption that the density ρ is uniform is realistic over only short vertical distances. In a room with a ceiling height of 3.0 m filled with air of uniform density 1.2 kg/m^3, the difference in pressure between floor and ceiling, given by Eq. (12.6), is

$$\rho gh = (1.2\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(3.0\ \text{m}) = 35\ \text{Pa}$$

or about 0.00035 atm, a very small difference. But between sea level and the summit of Mount Everest (8882 m) the density of air changes by nearly a factor of 3, and in this case we cannot use Eq. (12.6). Liquids, by contrast, are nearly incompressible, and it is usually a very good approximation to regard their density as independent of pressure.

Figure **12.7** The hydraulic lift is an application of Pascal's law. The size of the fluid-filled container is exaggerated for clarity.



Absolute Pressure and Gauge Pressure

If the pressure inside a car tire is equal to atmospheric pressure, the tire is flat. The pressure has to be *greater* than atmospheric to support the car, so the significant quantity is the *difference* between the inside and outside pressures. When we say that the pressure in a car tire is "32 pounds" (actually 32 lb/in.^{2}, equal to 220 kPa or 2.2×10^5 Pa), we mean that it is *greater* than atmospheric pressure (14.7 lb/in.^{2} or 1.01×10^5 Pa) by this amount. The *total* pressure in the tire is then 47 lb/in.^{2} or 320 kPa. The excess pressure above atmospheric pressure is usually called **gauge pressure,** and the total pressure is called **absolute pressure.** Engineers use the abbreviations psig and psia for "pounds per square inch gauge" and "pounds per square inch absolute," respectively. If the pressure is *less* than atmospheric, as in a partial vacuum, the gauge pressure is negative.

EXAMPLE 12.3 Finding absolute and gauge pressures

WITH VARIATION PROBLEMS

Water stands 12.0 m deep in a storage tank whose top is open to the atmosphere. What are the absolute and gauge pressures at the bottom of the tank?

IDENTIFY and SET UP Table 11.2 indicates that water is nearly incompressible, so we can treat it as having uniform density. The level of the top of the tank corresponds to point 2 in Fig. 12.5, and the level of the bottom of the tank corresponds to point 1. Our target variable is p in Eq. (12.6). We have $h = 12.0$ m and $p_0 = 1$ atm $= 1.01 \times 10^5$ Pa.

EXECUTE From Eq. (12.6), the pressures are

absolute:
$$\begin{aligned} p &= p_0 + \rho g h \\ &= (1.01 \times 10^5 \text{ Pa}) + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(12.0 \text{ m}) \\ &= 2.19 \times 10^5 \text{ Pa} = 2.16 \text{ atm} = 31.8 \text{ lb/in.}^2 \end{aligned}$$

gauge:
$$\begin{aligned} p - p_0 &= (2.19 - 1.01) \times 10^5 \text{ Pa} \\ &= 1.18 \times 10^5 \text{ Pa} = 1.16 \text{ atm} = 17.1 \text{ lb/in.}^2 \end{aligned}$$

EVALUATE A pressure gauge at the bottom of such a tank would probably be calibrated to read gauge pressure rather than absolute pressure.

KEYCONCEPT Absolute pressure is the total pressure at a given point in a fluid. Gauge pressure is the difference between absolute pressure and atmospheric pressure.

Pressure Gauges

The simplest pressure gauge is the open-tube *manometer* (**Fig. 12.8a**). The U-shaped tube contains a liquid of density ρ, often mercury or water. The left end of the tube is connected to the container where the pressure p is to be measured, and the right end is open to the atmosphere at pressure $p_0 = p_{atm}$. The pressure at the bottom of the tube due to the fluid in the left column is $p + \rho g y_1$, and the pressure at the bottom due to the fluid in the right column is $p_{atm} + \rho g y_2$. These pressures are measured at the same level, so they must be equal:

$$\begin{aligned} p + \rho g y_1 &= p_{atm} + \rho g y_2 \\ p - p_{atm} &= \rho g (y_2 - y_1) = \rho g h \end{aligned} \tag{12.8}$$

In Eq. (12.8), p is the *absolute pressure,* and the difference $p - p_{atm}$ between absolute and atmospheric pressure is the gauge pressure. Thus the gauge pressure is proportional to the difference in height $h = y_2 - y_1$ of the liquid columns.

Another common pressure gauge is the **mercury barometer.** It consists of a long glass tube, closed at one end, that has been filled with mercury and then inverted in a dish of mercury (Fig. 12.8b). The space above the mercury column contains only mercury vapor;

BIO APPLICATION Gauge Pressure of Blood Blood-pressure readings, such as 130/80, give the maximum and minimum gauge pressures in the arteries, measured in mm Hg or torr. Blood pressure varies with vertical position within the body; the standard reference point is the upper arm, level with the heart.

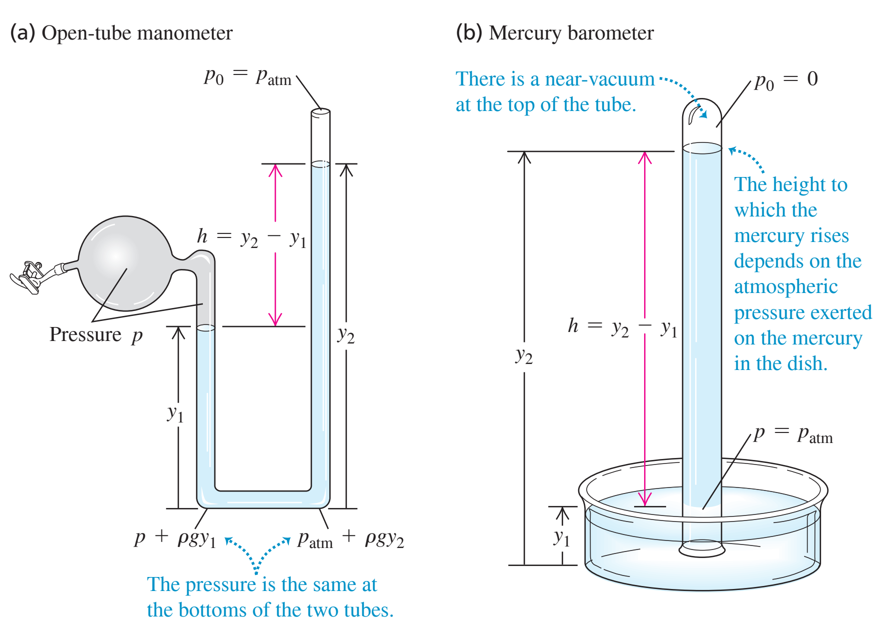
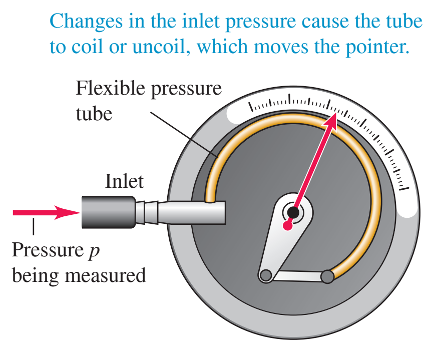


Figure 12.8 Two types of pressure gauge.

Figure **12.9** (a) A Bourdon pressure gauge. When the pressure inside the flexible tube increases, the tube straightens out a little, deflecting the attached pointer. (b) This Bourdon-type pressure gauge is connected to a high-pressure gas line. The gauge pressure shown is just over 5 bars ($1 \text{ bar} = 10^5 \text{ Pa}$).



its pressure is negligibly small, so the pressure p_0 at the top of the mercury column is practically zero. From Eq. (12.6),

$$p_{\text{atm}} = p = 0 + \rho g(y_2 - y_1) = \rho g h \tag{12.9}$$

So the height h of the mercury column indicates the atmospheric pressure p_{atm}.

Pressures are often described in terms of the height of the corresponding mercury column, as so many "inches of mercury" or "millimeters of mercury" (abbreviated mm Hg). A pressure of 1 mm Hg is called *1 torr,* after Evangelista Torricelli, inventor of the mercury barometer. But these units depend on the density of mercury, which varies with temperature, and on the value of g, which varies with location, so the pascal is the preferred unit of pressure.

Many types of pressure gauges use a flexible sealed tube (**Fig. 12.9**). A change in the pressure either inside or outside the tube causes a change in its dimensions. This change is detected optically, electrically, or mechanically.

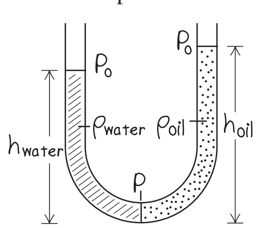
EXAMPLE 12.4 A tale of two fluids

WITH VARIATION PROBLEMS

A manometer tube is partially filled with water. Oil (which does not mix with water) is poured into the left arm of the tube until the oil–water interface is at the midpoint of the tube as shown in **Fig. 12.10**. Both arms of the tube are open to the air. Find a relationship between the heights h_{oil} and h_{water}.

IDENTIFY and SET UP Figure 12.10 shows our sketch. The relationship between pressure and depth given by Eq. (12.6) applies to fluids of uniform density only; we have two fluids of different densities, so we must write a separate pressure–depth relationship for each. Both fluid columns have pressure p at the bottom (where they are in contact and in equilibrium), and both are at atmospheric pressure p_0 at the top (where both are in contact with and in equilibrium with the air).

Figure **12.10** Our sketch for this problem.



EXECUTE Writing Eq. (12.6) for each fluid gives

$$p = p_0 + \rho_{\text{water}} g h_{\text{water}}$$
$$p = p_0 + \rho_{\text{oil}} g h_{\text{oil}}$$

Since the pressure p at the bottom of the tube is the same for both fluids, we set these two expressions equal to each other and solve for h_{oil} in terms of h_{water}:

$$h_{\text{oil}} = \frac{\rho_{\text{water}}}{\rho_{\text{oil}}} h_{\text{water}}$$

EVALUATE Water ($\rho_{\text{water}} = 1000 \text{ kg/m}^3$) is denser than oil ($\rho_{\text{oil}} \approx 850 \text{ kg/m}^3$), so h_{oil} is greater than h_{water} as Fig. 12.10 shows. It takes a greater height of low-density oil to produce the same pressure p at the bottom of the tube.

KEYCONCEPT The pressure is the same at all points *at the same level* in a fluid at rest. This is true even if the fluid contains different substances with different densities.

TEST YOUR UNDERSTANDING OF SECTION 12.2 Mercury is less dense at high temperatures than at low temperatures. Suppose you move a mercury barometer from the cold interior of a tightly sealed refrigerator to outdoors on a hot summer day. You find that the column of mercury remains at the same height in the tube. Compared to the air pressure inside the refrigerator, is the air pressure outdoors (i) higher, (ii) lower, or (iii) the same? (Ignore the very small change in the dimensions of the glass tube due to the temperature change.)

ANSWER

(ii) From Eq. (12.9), the pressure outside the barometer is equal to the product ρgh. When the barometer is taken out of the refrigerator, the density ρ decreases while the height h of the mercury column remains the same. Hence the air pressure must be lower outdoors than inside the refrigerator.

12.3 BUOYANCY

An object immersed in water seems to weigh less than when it is in air. When the object is less dense than the fluid, it floats. The human body usually floats in water, and a helium-filled balloon floats in air. These are examples of **buoyancy,** a phenomenon described by *Archimedes's principle:*

ARCHIMEDES'S PRINCIPLE When an object is completely or partially immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid displaced by the object.

To prove this principle, we consider an arbitrary element of fluid at rest. The dashed curve in **Fig. 12.11a** outlines such an element. The arrows labeled $dF_\perp$ represent the forces exerted on the element's surface by the surrounding fluid.

The entire fluid is in equilibrium, so the sum of all the y-components of force on this element of fluid is zero. Hence the sum of the y-components of the *surface* forces must be an upward force equal in magnitude to the weight mg of the fluid inside the surface. Also, the sum of the torques on the element of fluid must be zero, so the line of action of the resultant y-component of surface force must pass through the center of gravity of this element of fluid.

Now we replace the fluid inside the surface with a solid object that has exactly the same shape (Fig. 12.11b). The pressure at every point is the same as before. So the total upward force exerted on the object by the fluid is also the same, again equal in magnitude to the weight mg of the fluid displaced to make way for the object. We call this upward force the **buoyant force** on the solid object. The line of action of the buoyant force again passes through the center of gravity of the displaced fluid (which doesn't necessarily coincide with the center of gravity of the object).

When a balloon floats in equilibrium in air, its weight (including the gas inside it) must be the same as the weight of the air displaced by the balloon. A fish's flesh is denser than water, yet many fish can float while submerged. These fish have a gas-filled cavity within their bodies, which makes the fish's *average* density the same as water's.

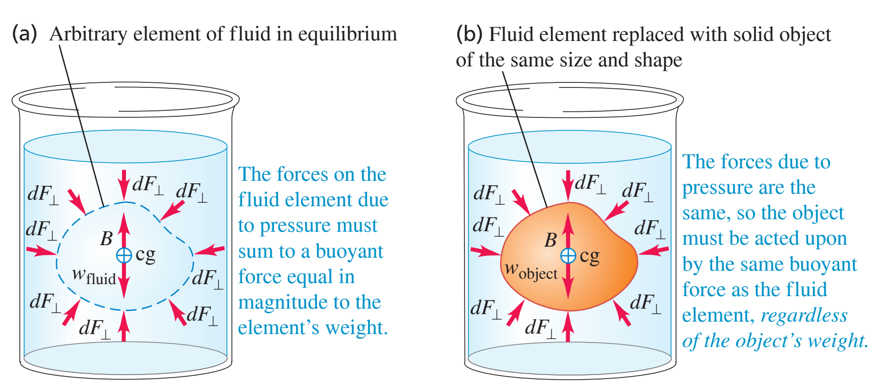
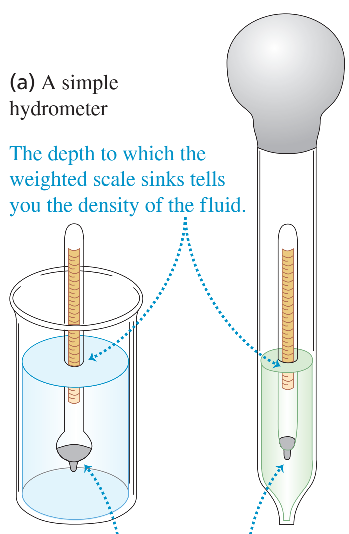


Figure **12.11** Archimedes's principle.

Figure **12.12** Measuring the density of a fluid.



So the net weight of the fish is the same as the weight of the water it displaces. An object whose average density is *less* than that of a liquid can float partially submerged at the free upper surface of the liquid. A ship made of steel (which is much denser than water) can float because the ship is hollow, with air occupying much of its interior volume, so its average density is less than that of water. The greater the density of the liquid, the less of the object is submerged. When you swim in seawater (density 1030 kg/m^3), your body floats higher than in freshwater (1000 kg/m^3).

A practical example of buoyancy is the hydrometer, used to measure the density of liquids (**Fig. 12.12a**). The calibrated float sinks into the fluid until the weight of the fluid it displaces is exactly equal to its own weight. The hydrometer floats *higher* in denser liquids than in less dense liquids, and a scale in the top stem permits direct density readings. Hydrometers like this are used in medical diagnosis to measure the density of urine (which depends on a patient's level of hydration). Figure 12.12b shows a type of hydrometer used to measure the density of battery acid or antifreeze. The bottom of the large tube is immersed in the liquid; the bulb is squeezed to expel air and is then released, like a giant medicine dropper. The liquid rises into the outer tube, and the hydrometer floats in this liquid.

CAUTION **The buoyant force depends on the fluid density** The buoyant force on an object is proportional to the density of the *fluid* in which the object is immersed, *not* the density of the object. If a wooden block and an iron block have the same volume and both are submerged in water, both experience the same buoyant force. The wooden block rises and the iron block sinks because this buoyant force is greater than the weight of the wooden block but less than the weight of the iron block.

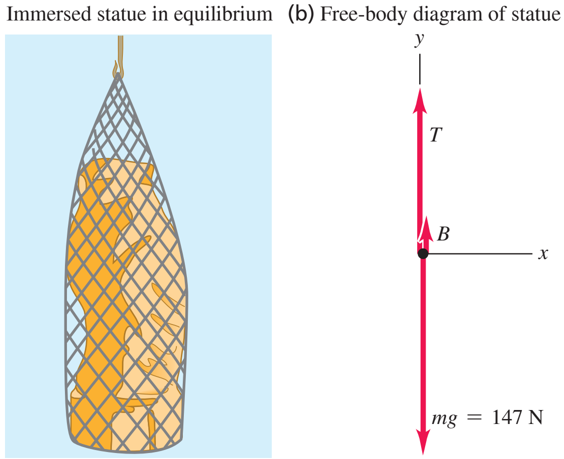
EXAMPLE 12.5 Buoyancy

WITH VARIATION PROBLEMS

A 15.0 kg solid gold statue is raised from the sea bottom (**Fig. 12.13a**). What is the tension in the hoisting cable (assumed massless) when the statue is (a) at rest and completely underwater and (b) at rest and completely out of the water?

IDENTIFY and SET UP In both cases the statue is in equilibrium and experiences three forces: its weight, the cable tension, and a buoyant force equal in magnitude to the weight of the fluid displaced by the statue (seawater in part (a), air in part (b)). Figure 12.13b shows the free-body diagram for the statue. Our target variables are the values of the tension in seawater (T_{sw}) and in air (T_{air}). We are given the mass m_{statue}, and we can calculate the buoyant force in seawater (B_{sw}) and in air (B_{air}) by using Archimedes's principle.

Figure **12.13** What is the tension in the cable hoisting the statue?



EXECUTE (a) To find B_{sw}, we first find the statue's volume V by using the density of gold from Table 12.1:

$$V = \frac{m_{statue}}{\rho_{gold}} = \frac{15.0\text{ kg}}{19.3 \times 10^3\text{ kg/m}^3} = 7.77 \times 10^{-4}\text{ m}^3$$

The buoyant force B_{sw} equals the weight of this same volume of seawater. Using Table 12.1 again:

$$\begin{aligned} B_{sw} &= w_{sw} = m_{sw}g = \rho_{sw}Vg \\ &= (1.03 \times 10^3\text{ kg/m}^3)(7.77 \times 10^{-4}\text{ m}^3)(9.80\text{ m/s}^2) \\ &= 7.84\text{ N} \end{aligned}$$

The statue is at rest, so the net external force acting on it is zero. From Fig. 12.13b,

$$\begin{aligned} \Sigma F_y &= B_{sw} + T_{sw} + (-m_{statue}\,g) = 0 \\ T_{sw} &= m_{statue}\,g - B_{sw} = (15.0\text{ kg})(9.80\text{ m/s}^2) - 7.84\text{ N} \\ &= 147\text{ N} - 7.84\text{ N} = 139\text{ N} \end{aligned}$$

A spring scale attached to the upper end of the cable will indicate a tension 7.84 N less than the statue's actual weight $m_{statue}\,g = 147$ N.

(b) The density of air is about 1.2 kg/m^3, so the buoyant force of air on the statue is

$$\begin{aligned} B_{air} &= \rho_{air}Vg \\ &= (1.2\text{ kg/m}^3)(7.77 \times 10^{-4}\text{ m}^3)(9.80\text{ m/s}^2) \\ &= 9.1 \times 10^{-3}\text{ N} \end{aligned}$$

This is negligible compared to the statue's actual weight $m_{statue}\,g = 147$ N. So within the precision of our data, the tension in the cable with the statue in air is $T_{air} = m_{statue}\,g = 147$ N.

EVALUATE The denser the fluid, the greater the buoyant force and the smaller the cable tension. If the fluid had the same density as the statue, the buoyant force would be equal to the statue's weight and the tension would be zero (the cable would go slack). If the fluid were denser than the statue, the tension would be *negative:* The buoyant force would be greater than the statue's weight, and a downward force would be required to keep the statue from rising upward.

KEYCONCEPT The buoyant force on an object immersed in a fluid is equal to the weight of the fluid that the object displaces (Archimedes's principle). The greater the density of the fluid, the greater the buoyant force that the fluid exerts.

Surface Tension

We've seen that if an object is less dense than water, it will float partially submerged. But a paper clip can rest *atop* a water surface even though its density is several times that of water. This is an example of **surface tension:** The surface of the liquid behaves like a membrane under tension (**Fig. 12.14**). Surface tension arises because the molecules of the liquid exert attractive forces on each other. There is zero net force on a molecule within the interior of the liquid, but a surface molecule is drawn into the interior (**Fig. 12.15**). Thus the liquid tends to minimize its surface area, just as a stretched membrane does.

Surface tension explains why raindrops are spherical (*not* teardrop-shaped): A sphere has a smaller surface area for its volume than any other shape. It also explains why hot, soapy water is used for washing. To wash clothing thoroughly, water must be forced through the tiny spaces between the fibers (**Fig. 12.16**). This requires increasing the surface area of the water, which is difficult to achieve because of surface tension. The job is made easier by increasing the temperature of the water and adding soap, both of which decrease the surface tension.

Surface tension is important for a millimeter-sized water drop, which has a relatively large surface area for its volume. (A sphere of radius r has surface area $4\pi r^2$ and volume $(4\pi/3)r^3$. The ratio of surface area to volume is $3/r$, which increases with decreasing radius.) But for large quantities of liquid, the ratio of surface area to volume is relatively small, and surface tension is negligible compared to pressure forces. For the remainder of this chapter, we'll consider only fluids in bulk and ignore the effects of surface tension.

Figure 12.14 The surface of the water acts like a membrane under tension, allowing this water strider to "walk on water."

Figure 12.15 A molecule at the surface of a liquid is attracted into the bulk liquid, which tends to reduce the liquid's surface area.

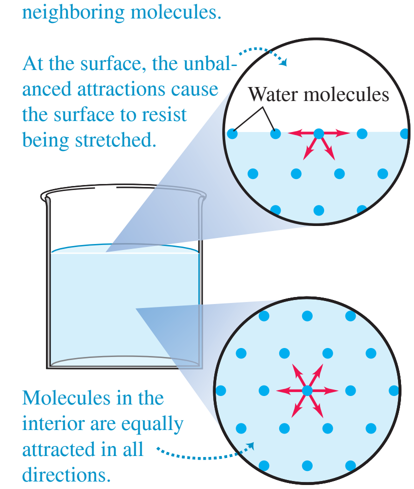
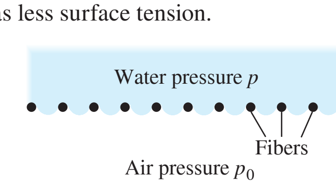


Figure 12.16 Surface tension makes it difficult to force water through small crevices. The required water pressure p can be reduced by using hot, soapy water, which has less surface tension.



TEST YOUR UNDERSTANDING OF SECTION 12.3 You place a container of seawater on a scale and note the reading on the scale. You now suspend the statue of Example 12.5 in the water (**Fig. 12.17**). How does the scale reading change? (i) It increases by 7.84 N; (ii) it decreases by 7.84 N; (iii) it remains the same; (iv) none of these.

Figure 12.17 How does the scale reading change when the statue is immersed in water?

ANSWER

(i) Consider the water, the statue, and the container together as a system; the total weight of the system does not depend on whether the statue is immersed. The total supporting force, including the tension T and the upward force F of the scale on the container (equal to the scale reading), is the same in both cases. But we saw in Example 12.5 that T decreases by 7.84 N when the statue is immersed, so the scale reading F must *increase* by 7.84 N. An alternative viewpoint is that the water exerts an upward buoyant force of 7.84 N on the statue, so the statue must exert an equal downward force on the water, making the scale reading 7.84 N greater than the weight of water and container.

Figure 12.18 A flow tube bounded by flow lines. In steady flow, fluid cannot cross the walls of a flow tube.

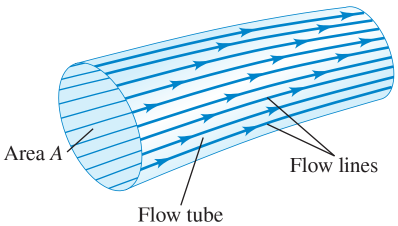


Figure 12.19 Laminar flow around an obstacle.

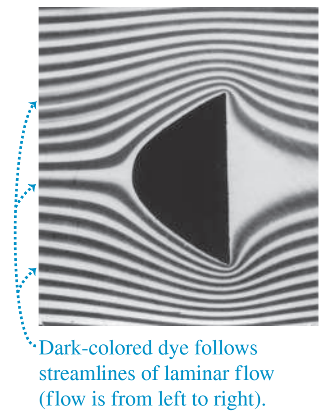
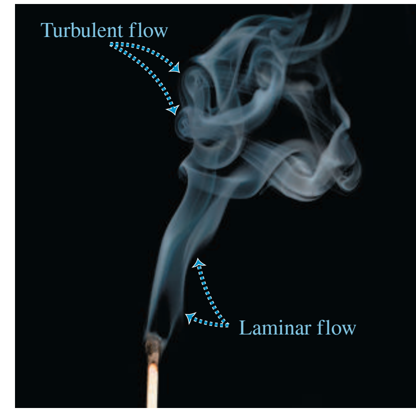


Figure 12.20 The flow of smoke rising from this burnt match is laminar up to a certain point, and then becomes turbulent.



12.4 FLUID FLOW

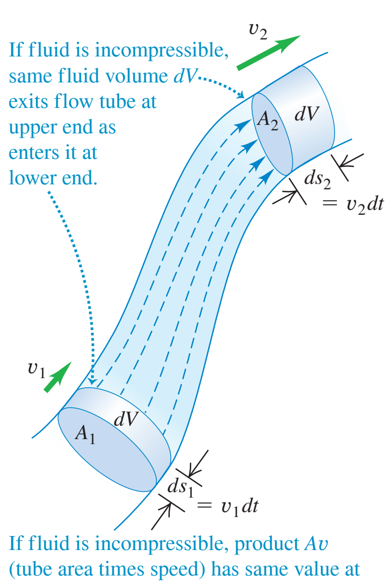
We are now ready to consider *motion* of a fluid. Fluid flow can be extremely complex, as shown by the currents in river rapids or the swirling flames of a campfire. But we can represent some situations by relatively simple idealized models. An **ideal fluid** is a fluid that is *incompressible* (that is, its density cannot change) and has no internal friction (called **viscosity**). Liquids are approximately incompressible in most situations, and we may also treat a gas as incompressible if the pressure differences from one region to another are not too great. Internal friction in a fluid causes shear stresses when two adjacent layers of fluid move relative to each other, as when fluid flows inside a tube or around an obstacle. In some cases we can ignore these shear forces in comparison with forces arising from gravitation and pressure differences.

The path of an individual particle in a moving fluid is called a **flow line.** In **steady flow,** the overall flow pattern does not change with time, so every element passing through a given point follows the same flow line. In this case the "map" of the fluid velocities at various points in space remains constant, although the velocity of a particular particle may change in both magnitude and direction during its motion. A **streamline** is a curve whose tangent at any point is in the direction of the fluid velocity at that point. When the flow pattern changes with time, the streamlines do not coincide with the flow lines. We'll consider only steady-flow situations, for which flow lines and streamlines are identical.

The flow lines passing through the edge of an imaginary element of area, such as area A in **Fig. 12.18**, form a tube called a **flow tube.** From the definition of a flow line, in steady flow no fluid can cross the side walls of a given flow tube.

Figure 12.19 shows the pattern of fluid flow from left to right around an obstacle. The photograph was made by injecting dye into water flowing between two closely spaced glass plates. This pattern is typical of **laminar flow,** in which adjacent layers of fluid slide smoothly past each other and the flow is steady. (A *lamina* is a thin sheet.) At sufficiently high flow rates, or when boundary surfaces cause abrupt changes in velocity, the flow can become irregular and chaotic. This is called **turbulent flow** (**Fig. 12.20**). In turbulent flow there is no steady-state pattern; the flow pattern changes continuously.

Figure 12.21 A flow tube with changing cross-sectional area.



The Continuity Equation

The mass of a moving fluid doesn't change as it flows. This leads to an important relationship called the **continuity equation.** Consider a portion of a flow tube between two stationary cross sections with areas A_1 and A_2 (**Fig. 12.21**). The fluid speeds at these sections are v_1 and v_2, respectively. As we mentioned above, no fluid flows in or out across the side walls of such a tube. During a small time interval dt, the fluid at A_1 moves a distance $ds_1 = v_1\,dt$, so a cylinder of fluid with height $v_1\,dt$ and volume $dV_1 = A_1 v_1\,dt$ flows into the tube across A_1. During this same interval, a cylinder of volume $dV_2 = A_2 v_2\,dt$ flows out of the tube across A_2.

Let's first consider the case of an incompressible fluid so that the density ρ has the same value at all points. The mass dm_1 flowing into the tube across A_1 in time dt is $dm_1 = \rho A_1 v_1\, dt$. Similarly, the mass dm_2 that flows out across A_2 in the same time is $dm_2 = \rho A_2 v_2\, dt$. In steady flow the total mass in the tube is constant, so $dm_1 = dm_2$ and

$$\rho A_1 v_1\, dt = \rho A_2 v_2\, dt \qquad \text{or}$$

Continuity equation for an **incompressible fluid:**

$$A_1 v_1 = A_2 v_2 \tag{12.10}$$

Cross-sectional area of flow tube at two points (see Fig. 12.21)

Speed of flow at the two points

The product Av is the *volume flow rate* dV/dt, the rate at which volume crosses a section of the tube:

Volume flow rate of a fluid

$$\frac{dV}{dt} = Av \tag{12.11}$$

Cross-sectional area of flow tube

Speed of flow

The *mass flow rate* is the mass flow per unit time through a cross section. This is equal to the density ρ times the volume flow rate dV/dt.

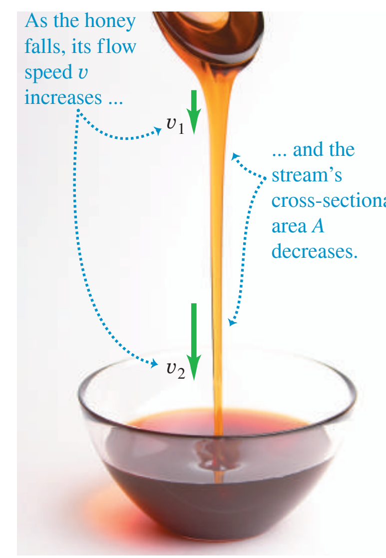
Equation (12.10) shows that the volume flow rate has the same value at all points along any flow tube (**Fig. 12.22**). When the cross section of a flow tube decreases, the speed increases, and vice versa. A broad, deep part of a river has a larger cross section and slower current than a narrow, shallow part, but the volume flow rates are the same in both. This is the essence of the familiar maxim, "Still waters run deep." If a water pipe with 2 cm diameter is connected to a pipe with 1 cm diameter, the flow speed is four times as great in the 1 cm part as in the 2 cm part.

We can generalize Eq. (12.10) for the case in which the fluid is *not* incompressible. If ρ_1 and ρ_2 are the densities at sections 1 and 2, then

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad \text{(continuity equation, compressible fluid)} \tag{12.12}$$

If the fluid is denser at point 2 than at point 1 ($\rho_2 > \rho_1$), the volume flow rate at point 2 will be less than at point 1 ($A_2 v_2 < A_1 v_1$). We leave the details to you. If the fluid is incompressible so that ρ_1 and ρ_2 are always equal, Eq. (12.12) reduces to Eq. (12.10).

Figure **12.22** The continuity equation, Eq. (12.10), helps explain the shape of a stream of honey poured from a spoon.



EXAMPLE 12.6 Flow of an incompressible fluid

Incompressible oil of density 850 kg/m^3 is pumped through a cylindrical pipe at a rate of 9.5 liters per second. (a) The first section of the pipe has a diameter of 8.0 cm. What is the flow speed of the oil? What is the mass flow rate? (b) The second section of the pipe has a diameter of 4.0 cm. What are the flow speed and mass flow rate in that section?

IDENTIFY and SET UP Since the oil is incompressible, the volume flow rate has the *same* value (9.5 L/s) in both sections of pipe. The mass flow rate (the density times the volume flow rate) also has the same value in both sections. (This is just the statement that no fluid is lost or added anywhere along the pipe.) We use the volume flow rate equation, Eq. (12.11), to determine the speed v_1 in the 8.0-cm-diameter section and the continuity equation for incompressible flow, Eq. (12.10), to find the speed v_2 in the 4.0-cm-diameter section.

EXECUTE (a) From Eq. (12.11) the volume flow rate in the first section is $dV/dt = A_1 v_1$, where A_1 is the cross-sectional area of the pipe of diameter 8.0 cm and radius 4.0 cm. Hence

$$v_1 = \frac{dV/dt}{A_1} = \frac{(9.5\ \text{L/s})(10^{-3}\ \text{m}^3/\text{L})}{\pi(4.0 \times 10^{-2}\ \text{m})^2} = 1.9\ \text{m/s}$$

The mass flow rate is $\rho\, dV/dt = (850\ \text{kg/m}^3)(9.5 \times 10^{-3}\ \text{m}^3/\text{s}) = 8.1\ \text{kg/s}$.

(b) From the continuity equation, Eq. (12.10),

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi(4.0 \times 10^{-2}\ \text{m})^2}{\pi(2.0 \times 10^{-2}\ \text{m})^2}(1.9\ \text{m/s}) = 7.6\ \text{m/s} = 4v_1$$

The volume and mass flow rates are the same as in part (a).

EVALUATE The second section of pipe has one-half the diameter and one-fourth the cross-sectional area of the first section. Hence the speed must be four times greater in the second section, which is just what our result shows.

KEYCONCEPT The continuity equation states that as an incompressible fluid moves through a flow tube, the volume flow rate (the flow tube's cross-sectional area times the flow speed) is the same *at all points* along the flow tube. The narrower the flow tube, the faster the flow speed.

TEST YOUR UNDERSTANDING OF SECTION 12.4 A maintenance crew is working on a section of a three-lane highway, leaving only one lane open to traffic. The result is much slower traffic flow (a traffic jam). Do cars on a highway behave like the molecules of (i) an incompressible fluid or (ii) a compressible fluid?

ANSWER

(ii) A highway that narrows from three lanes to one is like a pipe whose cross-sectional area narrows to one-third of its value. If cars behaved like the molecules of an incompressible fluid, then as the cars encountered the one-lane section, the spacing between cars (the "density") would stay the same but the cars would triple their speed. This would keep the "volume flow rate" (number of cars per second passing a point on the highway) the same. In real life cars behave like the molecules of a *compressible* fluid: They end up packed closer (the "density" increases) and fewer cars per second pass a point on the highway (the "volume flow rate" decreases).

12.5 BERNOULLI'S EQUATION

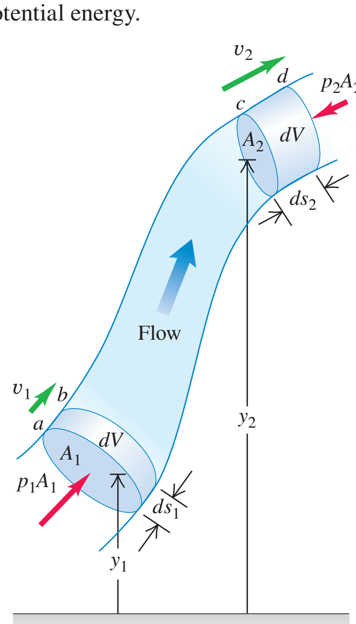
According to the continuity equation, the speed of fluid flow can vary along the paths of the fluid. The pressure can also vary; it depends on height as in the static situation (see Section 12.2), and it also depends on the speed of flow. We can derive an important relationship called *Bernoulli's equation,* which relates the pressure, flow speed, and height for flow of an ideal, incompressible fluid. Bernoulli's equation is useful in analyzing many kinds of fluid flow.

The dependence of pressure on speed follows from the continuity equation, Eq. (12.10). When an incompressible fluid flows along a flow tube with varying cross section, its speed *must* change, and so an element of fluid must have an acceleration. If the tube is horizontal, the force that causes this acceleration has to be applied by the surrounding fluid. This means that the pressure *must* be different in regions of different cross section; if it were the same everywhere, the net force on every fluid element would be zero. When a horizontal flow tube narrows and a fluid element speeds up, it must be moving toward a region of lower pressure in order to have a net forward force to accelerate it. If the elevation also changes, this causes an additional pressure difference.

Deriving Bernoulli's Equation

To derive Bernoulli's equation, we apply the work–energy theorem to the fluid in a section of a flow tube. In **Fig. 12.23** we consider the element of fluid that at some initial time lies between the two cross sections a and c. The speeds at the lower and upper ends are v_1 and v_2. In a small time interval dt, the fluid that is initially at a moves to b, a distance $ds_1 = v_1\,dt$, and the fluid that is initially at c moves to d, a distance $ds_2 = v_2\,dt$. The cross-sectional areas at the two ends are A_1 and A_2, as shown. The fluid is incompressible; hence by the continuity equation, Eq. (12.10), the volume of fluid dV passing *any* cross section during time dt is the same. That is, $dV = A_1\,ds_1 = A_2\,ds_2$.

Figure **12.23** Deriving Bernoulli's equation. The net work done on a fluid element by the pressure of the surrounding fluid equals the change in the kinetic energy plus the change in the gravitational potential energy.



Let's compute the *work* done on this fluid element during dt. If there is negligible internal friction in the fluid (i.e., no viscosity), the only nongravitational forces that do work on the element are due to the pressure of the surrounding fluid. The pressures at the two ends are p_1 and p_2; the force on the cross section at a is p_1A_1, and the force at c is p_2A_2. The net work dW done on the element by the surrounding fluid during this displacement is therefore

$$dW = p_1A_1\,ds_1 - p_2A_2\,ds_2 = (p_1 - p_2)dV \qquad (12.13)$$

The term $p_2A_2\,ds_2$ has a negative sign because the force at c opposes the displacement of the fluid.

The work dW is due to forces other than the conservative force of gravity, so it equals the change in the total mechanical energy (kinetic energy plus gravitational potential energy) associated with the fluid element. The mechanical energy for the fluid between sections b and c does not change. At the beginning of dt the fluid between a and b has volume $A_1\,ds_1$, mass $\rho A_1\,ds_1$, and kinetic energy $\frac{1}{2}\rho(A_1\,ds_1)v_1^2$. At the end of dt the fluid between c and d has kinetic energy $\frac{1}{2}\rho(A_2\,ds_2)v_2^2$. The net change in kinetic energy dK during time dt is

$$dK = \tfrac{1}{2}\rho\,dV(v_2^2 - v_1^2) \qquad (12.14)$$

What about the change in gravitational potential energy? At the beginning of time interval dt, the potential energy for the mass between a and b is $dm\,gy_1 = \rho\,dV\,gy_1$. At the end of dt, the potential energy for the mass between c and d is $dm\,gy_2 = \rho\,dV\,gy_2$. The net change in potential energy dU during dt is

$$dU = \rho\,dV\,g(y_2 - y_1) \tag{12.15}$$

Combining Eqs. (12.13), (12.14), and (12.15) in the energy equation $dW = dK + dU$, we obtain

$$\begin{aligned}(p_1 - p_2)dV &= \tfrac{1}{2}\rho\,dV(v_2^2 - v_1^2) + \rho\,dV\,g(y_2 - y_1)\\ p_1 - p_2 &= \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)\end{aligned} \tag{12.16}$$

This is **Bernoulli's equation.** It states that the work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow. We may also interpret Eq. (12.16) in terms of pressures. The first term on the right is the pressure difference associated with the change of speed of the fluid. The second term on the right is the additional pressure difference caused by the weight of the fluid and the difference in elevation of the two ends.

We can also express Eq. (12.16) in a more convenient form as

$$p_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2 \tag{12.17}$$

Subscripts 1 and 2 refer to *any* two points along the flow tube, so we can write

Bernoulli's equation for an ideal, incompressible fluid:

Pressure Fluid density Value is **same** at all points in flow tube.

$$p + \rho g y + \tfrac{1}{2}\rho v^2 = \text{constant} \tag{12.18}$$

Acceleration due to gravity Elevation Flow speed

Note that when the fluid is *not* moving (so $v_1 = v_2 = 0$), Eq. (12.17) reduces to the pressure relationship we derived for a fluid at rest, Eq. (12.5).

CAUTION Bernoulli's equation applies in certain situations only We stress again that Bernoulli's equation is valid for only incompressible, steady flow of a fluid with no internal friction (no viscosity). It's a simple equation, but don't be tempted to use it in situations in which it doesn't apply!

BIO APPLICATION Why Healthy Giraffes Have High Blood Pressure
Bernoulli's equation suggests that as blood flows upward at roughly constant speed v from the heart to the brain, the pressure p will drop as the blood's height y increases. For blood to reach the brain with the required minimal pressure, the human heart provides a maximum (systolic) gauge pressure of about 120 mm Hg. The vertical distance from heart to brain is much larger for a giraffe, so its heart must produce a much greater maximum gauge pressure (about 280 mm Hg).

PROBLEM-SOLVING STRATEGY 12.1 Bernoulli's Equation

Bernoulli's equation is derived from the work–energy theorem, so much of Problem-Solving Strategy 7.1 (Section 7.1) applies here.

IDENTIFY *the relevant concepts:* Bernoulli's equation is applicable to steady flow of an incompressible fluid that has no internal friction (see Section 12.6). It is generally applicable to flows through large pipes and to flows within bulk fluids (e.g., air flowing around an airplane or water flowing around a fish).

SET UP *the problem* using the following steps:

1. Identify the points 1 and 2 referred to in Bernoulli's equation, Eq. (12.17).
2. Define your coordinate system, particularly the level at which $y = 0$. Take the positive y-direction to be upward.
3. List the unknown and known quantities in Eq. (12.17). Decide which unknowns are the target variables.

EXECUTE *the solution* as follows: Write Bernoulli's equation and solve for the unknowns. You may need the continuity equation, Eq. (12.10), to relate the two speeds in terms of cross-sectional areas of pipes or containers. You may also need Eq. (12.11) to find the volume flow rate.

EVALUATE *your answer:* Verify that the results make physical sense. Check that you have used consistent units: In SI units, pressure is in pascals, density in kilograms per cubic meter, and speed in meters per second. The pressures must be either *all* absolute pressures or *all* gauge pressures.

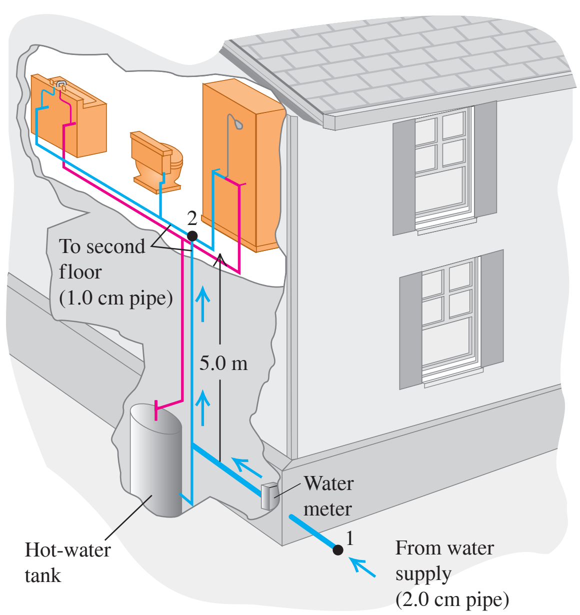
EXAMPLE 12.7 Water pressure in the home

WITH VARIATION PROBLEMS

Water enters a house (**Fig. 12.24**) through a pipe with an inside diameter of 2.0 cm at an absolute pressure of 4.0×10^5 Pa (about 4 atm). A 1.0-cm-diameter pipe leads to the second-floor bathroom 5.0 m above. When the flow speed at the inlet pipe is 1.5 m/s, find the flow speed, pressure, and volume flow rate in the bathroom.

IDENTIFY and SET UP We assume that the water flows at a steady rate. Water is effectively incompressible, so we can use the continuity equation. It's reasonable to ignore internal friction because the pipe has a relatively large diameter, so we can also use Bernoulli's equation. Let points 1 and 2 be at the inlet pipe and at the bathroom, respectively. We are given the pipe diameters at points 1 and 2, from which we calculate the areas A_1 and A_2, as well as the speed $v_1 = 1.5$ m/s and pressure $p_1 = 4.0 \times 10^5$ Pa at the inlet pipe. We take $y_1 = 0$ and $y_2 = 5.0$ m. We find the speed v_2 from the continuity equation and the pressure p_2 from Bernoulli's equation. Knowing v_2, we calculate the volume flow rate v_2A_2.

Figure 12.24 What is the water pressure in the second-story bathroom of this house?



EXECUTE From the continuity equation, Eq. (12.10),

$$v_2 = \frac{A_1}{A_2}v_1 = \frac{\pi(1.0\text{ cm})^2}{\pi(0.50\text{ cm})^2}(1.5\text{ m/s}) = 6.0\text{ m/s}$$

From Bernoulli's equation, Eq. (12.16),

$$\begin{aligned}
p_2 &= p_1 - \tfrac{1}{2}\rho(v_2^2 - v_1^2) - \rho g(y_2 - y_1) \\
&= 4.0 \times 10^5\text{ Pa} - \tfrac{1}{2}(1.0 \times 10^3\text{ kg/m}^3)(36\text{ m}^2/\text{s}^2 - 2.25\text{ m}^2/\text{s}^2) \\
&\quad - (1.0 \times 10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)(5.0\text{ m}) \\
&= 4.0 \times 10^5\text{ Pa} - 0.17 \times 10^5\text{ Pa} - 0.49 \times 10^5\text{ Pa} \\
&= 3.3 \times 10^5\text{ Pa} = 3.3\text{ atm} = 48\text{ lb/in.}^2
\end{aligned}$$

The volume flow rate is

$$\begin{aligned}
\frac{dV}{dt} &= A_2v_2 = \pi(0.50 \times 10^{-2}\text{ m})^2(6.0\text{ m/s}) \\
&= 4.7 \times 10^{-4}\text{ m}^3/\text{s} = 0.47\text{ L/s}
\end{aligned}$$

EVALUATE This is a reasonable flow rate for a bathroom faucet or shower. Note that if the water is turned off, both v_1 and v_2 are zero, the term $\frac{1}{2}\rho(v_2^2 - v_1^2)$ in Bernoulli's equation vanishes, and p_2 rises from 3.3×10^5 Pa to 3.5×10^5 Pa.

KEYCONCEPT Bernoulli's equation allows you to relate the flow speeds at two different points in a fluid to the pressures and heights at those two points.

EXAMPLE 12.8 Speed of efflux

WITH VARIATION PROBLEMS

Figure 12.25 shows a gasoline storage tank with cross-sectional area A_1, filled to a depth h. The space above the gasoline contains air at pressure p_0, and the gasoline flows out the bottom of the tank through a short pipe with cross-sectional area A_2. Derive expressions for the flow speed in the pipe and the volume flow rate.

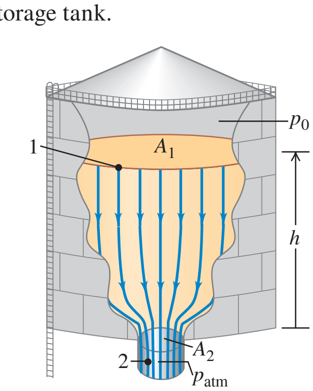
IDENTIFY and SET UP We consider the entire volume of moving liquid as a single flow tube of an incompressible fluid with negligible internal friction. Hence, we can use Bernoulli's equation. Points 1 and 2 are at the surface of the gasoline and at the exit pipe, respectively. At point 1 the pressure is p_0, which we assume to be fixed; at point 2 it is atmospheric pressure p_{atm}. We take $y = 0$ at the exit pipe, so $y_1 = h$ and $y_2 = 0$. Because A_1 is very much larger than A_2, the upper surface of the gasoline will drop very slowly and so v_1 is essentially zero. We find v_2 from Eq. (12.17) and the volume flow rate from Eq. (12.11).

EXECUTE We apply Bernoulli's equation to points 1 and 2:

$$p_0 + \tfrac{1}{2}\rho v_1^2 + \rho gh = p_{atm} + \tfrac{1}{2}\rho v_2^2 + \rho g(0)$$

$$v_2^2 = v_1^2 + 2\left(\frac{p_0 - p_{atm}}{\rho}\right) + 2gh$$

Figure 12.25 Calculating the speed of efflux for gasoline flowing out the bottom of a storage tank.



Using $v_1 = 0$, we find

$$v_2 = \sqrt{2\left(\frac{p_0 - p_{atm}}{\rho}\right) + 2gh}$$

From Eq. (12.11), the volume flow rate is $dV/dt = v_2A_2$.

EVALUATE The speed v_2, sometimes called the *speed of efflux,* depends on both the pressure difference $(p_0 - p_{atm})$ and the height h of the liquid level in the tank. If the top of the tank is vented to the atmosphere, $p_0 = p_{atm}$ and $p_0 - p_{atm} = 0$. Then

$$v_2 = \sqrt{2gh}$$

That is, the speed of efflux from an opening at a distance h below the top surface of the liquid is the *same* as the speed an object would acquire in falling freely through a height h. This result is called *Torricelli's theorem*. It is valid also for a hole in a side wall at a depth h below the surface. If $p_0 = p_{atm}$, the volume flow rate is

$$\frac{dV}{dt} = A_2\sqrt{2gh}$$

KEYCONCEPT When solving problems about the flow of an incompressible fluid with negligible internal friction, you can use both Bernoulli's equation and the continuity equation.

EXAMPLE 12.9 The Venturi meter

WITH VARIATION PROBLEMS

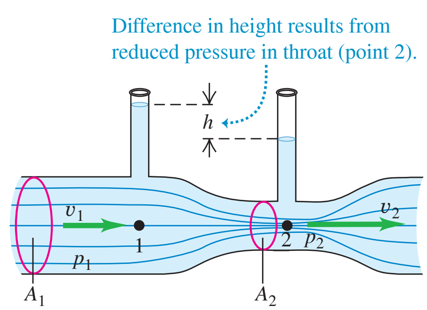
Figure 12.26 shows a *Venturi meter,* used to measure flow speed in a pipe. Derive an expression for the flow speed v_1 in terms of the cross-sectional areas A_1 and A_2.

IDENTIFY and SET UP The flow is steady, and we assume the fluid is incompressible and has negligible internal friction. Hence we can apply Bernoulli's equation to the wide part (point 1) and narrow part (point 2, the *throat*) of the pipe. Equation (12.6) relates h to the pressure difference $p_1 - p_2$.

EXECUTE Points 1 and 2 have the same vertical coordinate $y_1 = y_2$, so Eq. (12.17) says

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2$$

Figure **12.26** The Venturi meter.



From the continuity equation, $v_2 = (A_1/A_2)v_1$. Substituting this and rearranging, we get

$$p_1 - p_2 = \tfrac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

From Eq. (12.6), the pressure difference $p_1 - p_2$ is also equal to ρgh. Substituting this and solving for v_1, we get

$$v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$$

EVALUATE Because A_1 is greater than A_2, v_2 is greater than v_1 and the pressure p_2 in the throat is *less* than p_1. Those pressure differences produce a net force to the right that makes the fluid speed up as it enters the throat, and a net force to the left that slows it as it leaves.

KEYCONCEPT When an incompressible fluid with negligible internal friction flows through a pipe of varying size, the pressure and flow speed both change. Where the cross-sectional area is small, the pressure is low and the speed is high; where the cross-sectional area is large, the pressure is high and the speed is low.

CONCEPTUAL EXAMPLE 12.10 Lift on an airplane wing

Figure 12.27a (next page) shows flow lines around a cross section of an airplane wing. The flow lines crowd together above the wing, corresponding to increased flow speed and reduced pressure, just as in the Venturi throat in Example 12.9. Hence the downward force of the air on the top side of the wing is less than the upward force of the air on the underside of the wing, and there is a net upward force or *lift*. Lift is not simply due to the impulse of air striking the underside of the wing; in fact, the reduced pressure on the upper wing surface makes the greatest contribution to the lift. (This simplified discussion ignores the formation of vortices.)

We can understand the lift force on the basis of momentum changes instead. The vector diagram in Fig. 12.27a shows that there is a net *downward* change in the vertical component of momentum of the air flowing past the wing, corresponding to the downward force the wing exerts on the air. The reaction force *on* the wing is *upward,* as we concluded above.

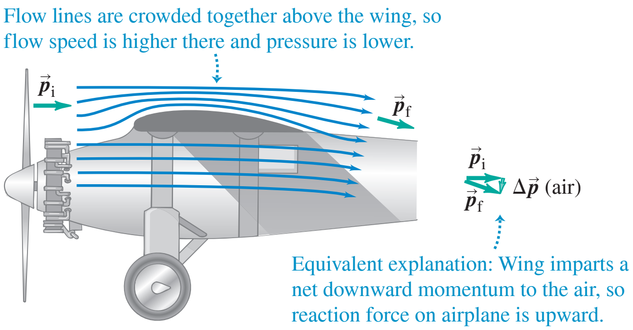
Similar flow patterns and lift forces are found in the vicinity of any humped object in a wind. A moderate wind makes an umbrella "float"; a strong wind can turn it inside out. At high speed, lift can reduce traction on a car's tires; a "spoiler" at the car's tail, shaped like an upside-down wing, provides a compensating downward force.

CAUTION A misconception about wings Some discussions of lift claim that air travels faster over the top of a wing because "it has farther to travel." This claim assumes that air molecules that part company at the front of the wing, one traveling over the wing and one under it, must meet again at the wing's trailing edge. Not so! Figure 12.27b shows a computer simulation of parcels of air flowing around an airplane wing. Parcels that are adjacent at the front of the wing do *not* meet at the trailing edge; the flow over the top of the wing is much faster than if the parcels had to meet. In accordance with Bernoulli's equation, this faster speed means that there is even lower pressure above the wing (and hence greater lift) than the "farther-to-travel" claim would suggest.

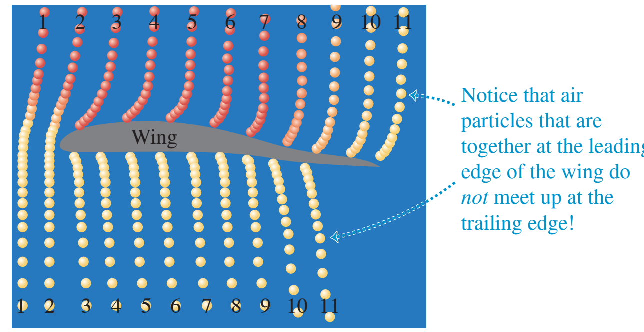
Continued

Figure **12.27** Flow around an airplane wing.

(a) Flow lines around an airplane wing



(b) Computer simulation of air parcels flowing around a wing, showing that air moves much faster over the top than over the bottom



KEYCONCEPT The pressure in a flowing incompressible fluid with negligible internal friction is low at points where the flow lines are crowded together, such as above the upper surface of an airplane wing.

TEST YOUR UNDERSTANDING OF SECTION 12.5 Which is the most accurate statement of Bernoulli's principle? (i) Fast-moving air causes lower pressure; (ii) lower pressure causes fast-moving air; (iii) both (i) and (ii) are equally accurate.

ANSWER

(ii) Newton's second law tells us that an object accelerates (its velocity changes) in response to a net force. In fluid flow, a pressure difference between two points means that fluid particles moving between those two points experience a force, and this force causes the fluid particles to accelerate and change speed.

12.6 VISCOSITY AND TURBULENCE

In our discussion of fluid flow we assumed that the fluid had no internal friction and that the flow was laminar. While these assumptions are often quite valid, in many important physical situations the effects of viscosity (internal friction) and turbulence (nonlaminar flow) are extremely important. Let's take a brief look at some of these situations.

Viscosity

Viscosity is internal friction in a fluid. Viscous forces oppose the motion of one portion of a fluid relative to another. Viscosity is the reason it takes effort to paddle a canoe through calm water, but it is also the reason the paddle works. Viscous effects are important in the flow of fluids in pipes, the flow of blood, the lubrication of engine parts, and many other situations.

Figure **12.28** Lava is an example of a viscous fluid. The viscosity decreases with increasing temperature: The hotter the lava, the more easily it can flow.

Fluids that flow readily, such as water or gasoline, have smaller viscosities than do "thick" liquids such as honey or motor oil. Viscosities of all fluids are strongly temperature dependent, increasing for gases and decreasing for liquids as the temperature increases (**Fig. 12.28**). Oils for engine lubrication must flow equally well in cold and warm conditions, and so are designed to have as *little* temperature variation of viscosity as possible.

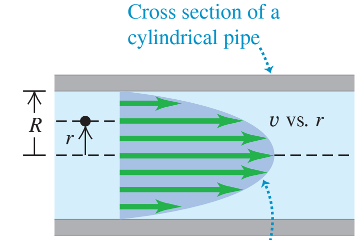
A viscous fluid always tends to cling to a solid surface in contact with it. There is always a thin *boundary layer* of fluid near the surface, in which the fluid is nearly at rest with respect to the surface. That's why dust particles can cling to a fan blade even when it is rotating rapidly, and why you can't get all the dirt off your car by just squirting a hose at it.

Viscosity has important effects on the flow of liquids through pipes, including the flow of blood in the circulatory system. First think about a fluid with zero viscosity so that we can apply Bernoulli's equation, Eq. (12.17). If the two ends of a long cylindrical pipe are at the same height $(y_1 = y_2)$ and the flow speed is the same at both ends

$(v_1 = v_2)$, Bernoulli's equation tells us that the pressure is the same at both ends of the pipe. But this isn't true if we account for viscosity. To see why, consider **Fig. 12.29,** which shows the flow-speed profile for laminar flow of a viscous fluid in a long cylindrical pipe. Due to viscosity, the speed is *zero* at the pipe walls (to which the fluid clings) and is greatest at the center of the pipe. The motion is like a lot of concentric tubes sliding relative to one another, with the central tube moving fastest and the outermost tube at rest. Viscous forces between the tubes oppose this sliding, so to keep the flow going we must apply a greater pressure at the back of the flow than at the front. That's why you have to keep squeezing a tube of toothpaste or a packet of ketchup (both viscous fluids) to keep the fluid coming out of its container. Your fingers provide a pressure at the back of the flow that is far greater than the atmospheric pressure at the front of the flow.

The pressure difference required to sustain a given volume flow rate through a cylindrical pipe of length L and radius R turns out to be proportional to L/R^4. If we decrease R by one-half, the required pressure increases by $2^4 = 16$; decreasing R by a factor of 0.90 (a 10% reduction) increases the required pressure difference by a factor of $(1/0.90)^4 = 1.52$ (a 52% increase). This simple relationship explains the connection between a high-cholesterol diet (which tends to narrow the arteries) and high blood pressure. Due to the R^4 dependence, even a small narrowing of the arteries can result in substantially elevated blood pressure and added strain on the heart muscle.

Figure **12.29** Velocity profile for a viscous fluid in a cylindrical pipe.



Turbulence

When the speed of a flowing fluid exceeds a certain critical value, the flow is no longer laminar. Instead, the flow pattern becomes extremely irregular and complex, and it changes continuously with time; there is no steady-state pattern. This irregular, chaotic flow is called **turbulence.** Figure 12.20 shows the contrast between laminar and turbulent flow for smoke rising in air. Bernoulli's equation is *not* applicable to regions where turbulence occurs because the flow is not steady.

Whether a flow is laminar or turbulent depends in part on the fluid's viscosity. The greater the viscosity, the greater the tendency for the fluid to flow in sheets (laminae) and the more likely the flow is to be laminar. (When we discussed Bernoulli's equation in Section 12.5, we assumed that the flow was laminar and that the fluid had zero viscosity. In fact, a *little* viscosity is needed to ensure that the flow is laminar.)

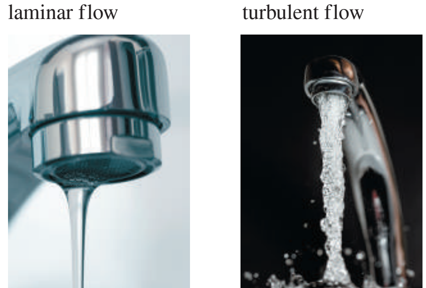
For a fluid of a given viscosity, flow speed is a determining factor for the onset of turbulence. A flow pattern that is stable at low speeds suddenly becomes unstable when a critical speed is reached. Irregularities in the flow pattern can be caused by roughness in the pipe wall, variations in the density of the fluid, and many other factors. At low flow speeds, these disturbances damp out; the flow pattern is *stable* and tends to maintain its laminar nature (**Fig. 12.30a**). When the critical speed is reached, however, the flow pattern becomes unstable. The disturbances no longer damp out but grow until they destroy the entire laminar-flow pattern (Fig. 12.30b).

BIO APPLICATION Listening for Turbulent Flow Normal blood flow in the human aorta is laminar, but a small disturbance such as a heart pathology can cause the flow to become turbulent. Turbulence makes noise, which is why listening to blood flow with a stethoscope is a useful diagnostic technique.

Figure **12.30** The flow of water from a faucet can be (a) laminar or (b) turbulent.

(a) Low speed: laminar flow

(b) High speed: turbulent flow



CONCEPTUAL EXAMPLE 12.11 The curve ball

Does a curve ball *really* curve? Yes, it does, and the reason is turbulence. **Figure 12.31a** shows a nonspinning ball moving through the air from left to right. The flow lines show that to an observer moving with the ball, the air stream appears to move from right to left. Because of the high speeds involved (typically near 35 m/s, or 75 mi/h), there is a region of *turbulent* flow behind the ball.

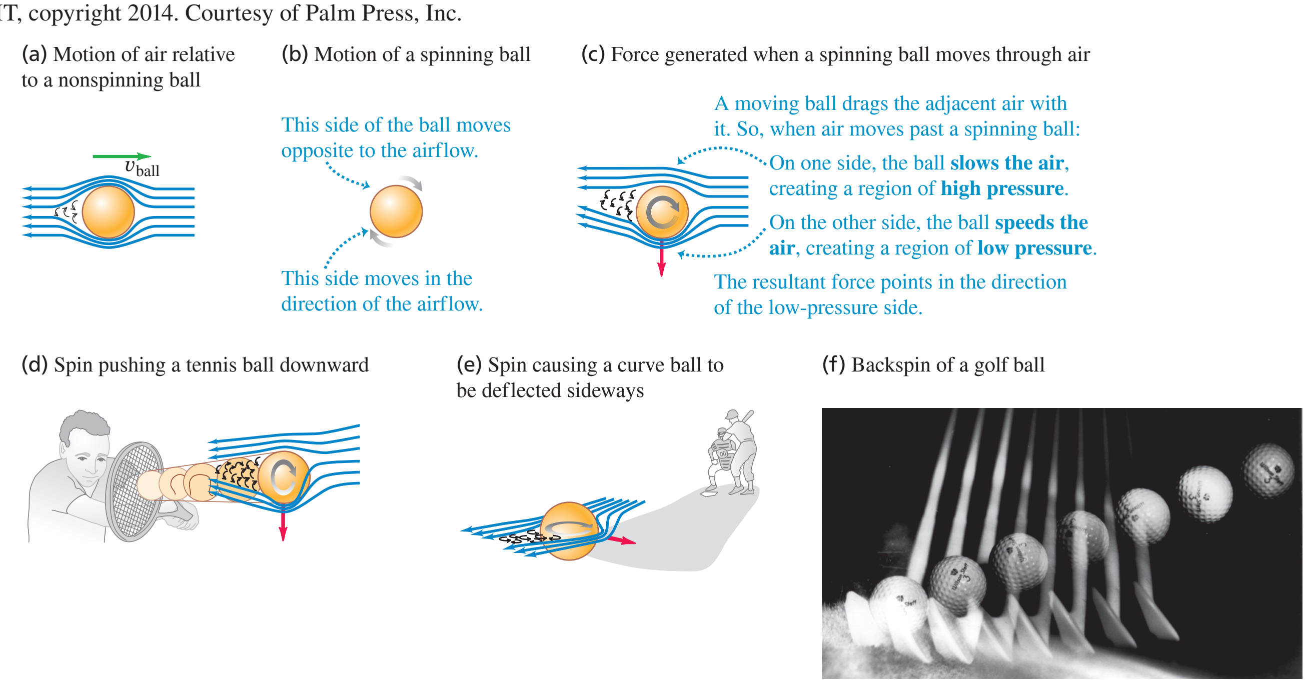
Figure 12.31b shows a *spinning* ball with "top spin." Layers of air near the ball's surface are pulled around in the direction of the spin by friction between the ball and air and by the air's internal friction (viscosity). Hence air moves relative to the ball's surface more slowly at the top of the ball than at the bottom, and turbulence occurs farther forward on the top side than on the bottom. As a result, the average pressure at the top of the ball is now greater than that at the bottom, and the resulting net force deflects the ball downward (Fig. 12.31c). "Top spin" is used in tennis to keep a fast serve in the court (Fig. 12.31d).

In baseball, a curve ball spins about a nearly *vertical* axis and the resulting deflection is sideways. In that case, Fig. 12.31c is a *top* view of the situation. A curve ball thrown by a left-handed pitcher spins as shown in Fig. 12.31e and will curve *toward* a right-handed batter, making it harder to hit.

A similar effect occurs when golf balls acquire "backspin" from impact with the grooved, slanted club face. Figure 12.31f shows the backspin of a golf ball just after impact. The resulting pressure difference between the top and bottom of the ball causes a *lift* force that keeps the ball in the air longer than would be possible without spin. A well-hit drive appears, from the tee, to "float" or even curve *upward* during the initial portion of its flight. This is a real effect, not an illusion. The dimples on the golf ball play an essential role; the viscosity of air gives a dimpled ball a much longer trajectory than an undimpled one with the same initial velocity and spin.

KEYCONCEPT Even in a fluid with low viscosity (that is, little internal friction) such as air, the effects of viscosity can be important for determining how that fluid flows around objects.

Figure **12.31** (a)–(e) Analyzing the motion of a spinning ball through the air. (f) Stroboscopic photograph of a golf ball being struck by a club. The picture was taken at 1000 flashes per second. The ball rotates about once in eight flashes, corresponding to an angular speed of 125 rev/s, or 7500 rpm. Source: Harold Edgerton at MIT, copyright 2014. Courtesy of Palm Press, Inc.



TEST YOUR UNDERSTANDING OF SECTION 12.6 How much more thumb pressure must a nurse use to administer an injection with a hypodermic needle of inside diameter 0.30 mm compared to one with inside diameter 0.60 mm? Assume that the two needles have the same length and that the volume flow rate is the same in both cases. (i) Twice as much; (ii) 4 times as much; (iii) 8 times as much; (iv) 16 times as much; (v) 32 times as much.

ANSWER

❙ **(iv)** The required pressure is proportional to $1/R^4$, where R is the inside radius of the needle (half the inside diameter). With the smaller-diameter needle, the pressure is greater by a factor of $[(0.60\text{ mm})/(0.30\text{ mm})]^4 = 2^4 = 16$.

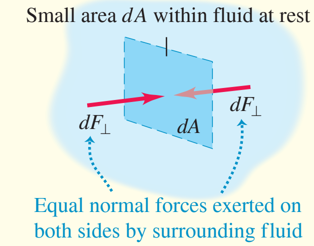
CHAPTER 12 SUMMARY

Density and pressure: Density is mass per unit volume. If a mass m of homogeneous material has volume V, its density ρ is the ratio m/V. Specific gravity is the ratio of the density of a material to the density of water. (See Example 12.1.)

Pressure is normal force per unit area. Pascal's law states that pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid. Absolute pressure is the total pressure in a fluid; gauge pressure is the difference between absolute pressure and atmospheric pressure. The SI unit of pressure is the pascal (Pa): $1\ \text{Pa} = 1\ \text{N/m}^2$. (See Example 12.2.)

$$\rho = \frac{m}{V} \tag{12.1}$$

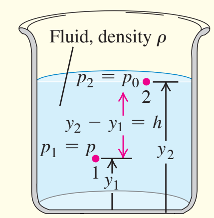
$$p = \frac{dF_\perp}{dA} \tag{12.2}$$



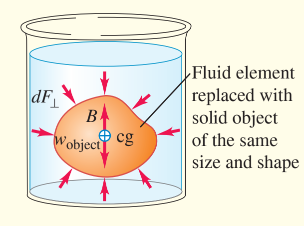
Pressures in a fluid at rest: The pressure difference between points 1 and 2 in a static fluid of uniform density ρ (an incompressible fluid) is proportional to the difference between the elevations y_1 and y_2. If the pressure at the surface of an incompressible liquid at rest is p_0, then the pressure at a depth h is greater by an amount ρgh. (See Examples 12.3 and 12.4.)

$$p_2 - p_1 = -\rho g(y_2 - y_1) \tag{12.5}$$
(pressure in a fluid of uniform density)

$$p = p_0 + \rho gh \tag{12.6}$$
(pressure in a fluid of uniform density)



Buoyancy: Archimedes's principle states that when an object is immersed in a fluid, the fluid exerts an upward buoyant force on the object equal to the weight of the fluid that the object displaces. (See Example 12.5.)



Fluid flow: An ideal fluid is incompressible and has no viscosity (no internal friction). A flow line is the path of a fluid particle; a streamline is a curve tangent at each point to the velocity vector at that point. A flow tube is a tube bounded at its sides by flow lines. In laminar flow, layers of fluid slide smoothly past each other. In turbulent flow, there is great disorder and a constantly changing flow pattern.

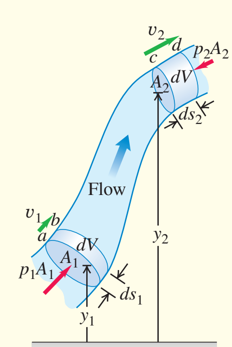
Conservation of mass in an incompressible fluid is expressed by the continuity equation, which relates the flow speeds v_1 and v_2 for two cross sections A_1 and A_2 in a flow tube. The product Av equals the volume flow rate, dV/dt, the rate at which volume crosses a section of the tube. (See Example 12.6.)

Bernoulli's equation states that a quantity involving the pressure p, flow speed v, and elevation y has the same value anywhere in a flow tube, assuming steady flow in an ideal fluid. This equation can be used to relate the properties of the flow at any two points. (See Examples 12.7–12.10.)

$$A_1 v_1 = A_2 v_2 \tag{12.10}$$
(continuity equation, incompressible fluid)

$$\frac{dV}{dt} = Av \tag{12.11}$$
(volume flow rate)

$$p + \rho g y + \tfrac{1}{2}\rho v^2 = \text{constant} \tag{12.18}$$
(Bernoulli's equation)



Chapter 12 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLES 12.3 and 12.4 (Section 12.2) before attempting these problems.

VP12.4.1 Liquefied natural gas (LNG) in a vertical storage tank has a density of 455 kg/m^3. The depth of LNG in the tank is 2.00 m, and the absolute pressure of the air inside the tank above the upper surface of the LNG is 1.22×10^5 Pa. What is the absolute pressure at the bottom of the tank?

VP12.4.2 You fill a vertical glass tube to a depth of 15.0 cm with freshwater. You then pour on top of the water an additional 15.0 cm of gasoline (density $7.40 \times 10^2\ kg/m^3$), which does not mix with water. The upper surface of the gasoline is exposed to the air. Find the gauge pressure (a) at the interface between the gasoline and water and (b) at the bottom of the tube.

VP12.4.3 In an open-tube manometer (see Fig. 12.8a), the absolute pressure in the container of gas on the left is 2.10×10^5 Pa. If atmospheric pressure is 1.01×10^5 Pa and the liquid in the manometer is mercury (density $1.36 \times 10^4\ kg/m^3$), what will be the difference in the heights of the left and right columns of liquid?

VP12.4.4 In the manometer tube shown in Fig. 12.10, the oil in the right-hand arm is olive oil of density 916 kg/m^3. (a) If the top of the oil is 25.0 cm above the bottom of the tube, what is the height of the top of the water above the bottom of the tube? (b) What is the gauge pressure 15.0 cm beneath the surface of the water? (c) At what depth below the surface of the oil is the gauge pressure the same as in part (b)?

Be sure to review EXAMPLE 12.5 (Section 12.3) before attempting these problems.

VP12.5.1 An object of volume $7.50 \times 10^{-4}\ m^3$ and density $1.15 \times 10^3\ kg/m^3$ is completely submerged in a fluid. (a) Calculate the weight of this object. (b) Calculate the buoyant force on this object if the fluid is (i) air (density 1.20 kg/m^3), (ii) water (density $1.00 \times 10^3\ kg/m^3$), and (iii) glycerin (density $1.26 \times 10^3\ kg/m^3$). In each case state whether the object will rise or sink if released while submerged in the fluid.

VP12.5.2 A sphere of volume $1.20 \times 10^{-3}\ m^3$ hangs from a cable. When the sphere is completely submerged in water, the tension in the cable is 29.4 N. (a) What is the buoyant force on the submerged sphere? (b) What is the weight of the sphere? (c) What is the density of the sphere?

VP12.5.3 A cube of volume $5.50 \times 10^{-3}\ m^3$ and density $7.50 \times 10^3\ kg/m^3$ hangs from a cable. When the cube has the lower half of its volume submerged in an unknown liquid, the tension in the cable is 375 N. What is the density of the liquid? (Ignore the small buoyant force exerted by the air on the upper half of the cube.)

VP12.5.4 A wooden cylinder of length L and cross-sectional area A is partially submerged in a liquid with the axis of the cylinder oriented straight up and down. The density of the liquid is ρ_L. (a) If the length of the cylinder that is below the surface of the liquid is d, what is the buoyant force that the liquid exerts on the cylinder? (b) If the cylinder floats in the position described in part (a), what is the density of the cylinder? (Ignore the small buoyant force exerted by the air on the part of the cylinder above the surface of the liquid.)

Be sure to review EXAMPLES 12.7, 12.8, and 12.9 (Section 12.5) before attempting these problems.

VP12.9.1 A pipe leads from a storage tank on the roof of a building to the ground floor. The absolute pressure of the water in the storage tank where it connects to the pipe is 3.0×10^5 Pa, the pipe has a radius of 1.0 cm where it connects to the storage tank, and the speed of flow in this pipe is 1.6 m/s. The pipe on the ground floor has a radius of 0.50 cm and is 9.0 m below the storage tank. Find (a) the speed of flow and (b) the pressure in the pipe on the ground floor.

VP12.9.2 The storage tank in Fig. 12.25 contains ethanol (density $8.1 \times 10^2\ kg/m^3$). The tank is 4.0 m in radius, the short pipe at the bottom of the tank is 1.0 cm in radius, and the height of ethanol in the tank is 3.2 m. The volume flow rate of ethanol from the short pipe is $4.4 \times 10^{-3}\ m^3/s$. (a) What is the speed at which ethanol flows out of the short pipe? (b) What is the gauge pressure of the air inside the tank in the space above the ethanol?

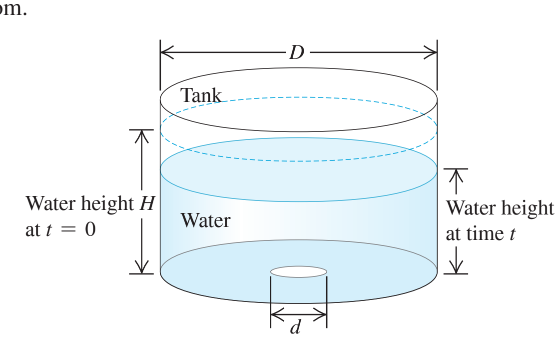
VP12.9.3 In a Venturi meter (see Fig. 12.26) that uses freshwater, the pressure difference between points 1 and 2 is 8.1×10^2 Pa, and the wide and narrow parts of the pipe have radii 2.5 cm and 1.2 cm, respectively. Find (a) the difference in the heights of the liquid levels in the two vertical tubes and (b) the volume flow rate through the horizontal pipe.

VP12.9.4 In the storage tank shown in Fig. 12.25, suppose area A_1 is *not* very much larger than area A_2. In this case we cannot treat the speed v_1 of the upper surface of the gasoline as zero. Derive an expression for the flow speed in the pipe in this situation in terms of p_0, p_{atm} g, h, A_1, A_2 and ρ.

BRIDGING PROBLEM How Long to Drain?

A large cylindrical tank with diameter D is open to the air at the top. The tank contains water to a height H. A small circular hole with diameter d, where $d \ll D$, is then opened at the bottom of the tank (**Fig. 12.32**). Ignore any effects of viscosity. (a) Find y, the height of water in the tank a time t after the hole is opened, as a function of t. (b) How long does it take to drain the tank completely? (c) If you double height H, by what factor does the time to drain the tank increase?

Figure 12.32 A water tank that is open at the top and has a hole at the bottom.



SOLUTION GUIDE

IDENTIFY and SET UP

1. Draw a sketch of the situation that shows all of the relevant dimensions.
2. List the unknown quantities, and decide which of these are the target variables.
3. At what speed does water flow out of the bottom of the tank? How is this related to the volume flow rate of water out of the tank? How is the volume flow rate related to the rate of change of y?

EXECUTE

4. Use your results from step 3 to write an equation for dy/dt.
5. Your result from step 4 is a relatively simple differential equation. With your knowledge of calculus, you can integrate it to find y as a function of t. (*Hint:* Once you've done the integration, you'll still have to do a little algebra.)
6. Use your result from step 5 to find the time when the tank is empty. How does your result depend on the initial height H?

EVALUATE

7. Check whether your answers are reasonable. A good check is to draw a graph of y versus t. According to your graph, what is the algebraic sign of dy/dt at different times? Does this make sense?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q12.1 A cube of oak wood with very smooth faces normally floats in water. Suppose you submerge it completely and press one face flat against the bottom of a tank so that no water is under that face. Will the block float to the surface? Is there a buoyant force on it? Explain.

Q12.2 A rubber hose is attached to a funnel, and the free end is bent around to point upward. When water is poured into the funnel, it rises in the hose to the same level as in the funnel, even though the funnel has a lot more water in it than the hose does. Why? What supports the extra weight of the water in the funnel?

Q12.3 Comparing Example 12.1 (Section 12.1) and Example 12.2 (Section 12.2), it seems that 700 N of air is exerting a downward force of 2.0×10^6 N on the floor. How is this possible?

Q12.4 Equation (12.7) shows that an area ratio of 100 to 1 can give 100 times more output force than input force. Doesn't this violate conservation of energy? Explain.

Q12.5 You have probably noticed that the lower the tire pressure, the larger the contact area between the tire and the road. Why?

Q12.6 In hot-air ballooning, a large balloon is filled with air heated by a gas burner at the bottom. Why must the air be heated? How does the balloonist control ascent and descent?

Q12.7 In describing the size of a large ship, one uses such expressions as "it displaces 20,000 tons." What does this mean? Can the weight of the ship be obtained from this information?

Q12.8 You drop a solid sphere of aluminum in a bucket of water that sits on the ground. The buoyant force equals the weight of water displaced; this is less than the weight of the sphere, so the sphere sinks to the bottom. If you take the bucket with you on an elevator that accelerates upward, the apparent weight of the water increases and the buoyant force on the sphere increases. Could the acceleration of the elevator be great enough to make the sphere pop up out of the water? Explain.

Q12.9 A rigid, lighter-than-air dirigible filled with helium cannot continue to rise indefinitely. Why? What determines the maximum height it can attain?

Q12.10 Which has a greater buoyant force on it: a 25 cm^3 piece of wood floating with part of its volume above water or a 25 cm^3 piece of submerged iron? Or, must you know their masses before you can answer? Explain.

Q12.11 The purity of gold can be tested by weighing it in air and in water. How? Do you think you could get away with making a fake gold brick by gold-plating some cheaper material?

Q12.12 During the Great Mississippi Flood of 1993, the levees in St. Louis tended to rupture first at the bottom. Why?

Q12.13 A cargo ship travels from the Atlantic Ocean (salt water) to Lake Ontario (freshwater) via the St. Lawrence River. The ship rides several centimeters lower in the water in Lake Ontario than it did in the ocean. Explain.

Q12.14 You push a piece of wood under the surface of a swimming pool. After it is completely submerged, you keep pushing it deeper and deeper. As you do this, what will happen to the buoyant force on it? Will the force keep increasing, stay the same, or decrease? Why?

Q12.15 An old question is "Which weighs more, a pound of feathers or a pound of lead?" If the weight in pounds is the gravitational force, will a pound of feathers balance a pound of lead on opposite pans of an equal-arm balance? Explain, taking into account buoyant forces.

Q12.16 Suppose the door that opens into a room makes an airtight but frictionless fit in its frame. Do you think you could open the door if the air pressure outside the room were standard atmospheric pressure and the air pressure inside the room was 1% greater? Explain.

Q12.17 At a certain depth in an incompressible liquid, the absolute pressure is p. At twice this depth, will the absolute pressure be equal to $2p$, greater than $2p$, or less than $2p$? Justify your answer.

Q12.18 A piece of iron is glued to the top of a block of wood. When the block is placed in a bucket of water with the iron on top, the block floats. The block is now turned over so that the iron is submerged beneath the wood. Does the block float or sink? Does the water level in the bucket rise, drop, or stay the same? Explain.

Q12.19 You take an empty glass jar and push it into a tank of water with the open mouth of the jar downward, so that the air inside the jar is trapped and cannot get out. If you push the jar deeper into the water, does the buoyant force on the jar stay the same? If not, does it increase or decrease? Explain.

Q12.20 You are floating in a canoe in the middle of a swimming pool. Your friend is at the edge of the pool, carefully noting the level of the water on the side of the pool. You have a bowling ball with you in the canoe. If you carefully drop the bowling ball over the side of the canoe and it sinks to the bottom of the pool, does the water level in the pool rise or fall?

Q12.21 You are floating in a canoe in the middle of a swimming pool. A large bird flies up and lights on your shoulder. Does the water level in the pool rise or fall?

Q12.22 Two identical buckets are filled to the brim with water, but one of them has a piece of wood floating in it. Which bucket of water weighs more? Explain.

Q12.23 An ice cube floats in a glass of water. When the ice melts, will the water level in the glass rise, fall, or remain unchanged? Explain.

Q12.24 A helium-filled balloon is tied to a light string inside a car at rest. The other end of the string is attached to the floor of the car, so the balloon pulls the string vertical. The car's windows are closed. Now the car accelerates forward. Does the balloon move? If so, does it move forward or backward? Justify your reasoning with reference to buoyancy. (If you have a chance, try this experiment yourself—but with someone else driving!)

Q12.25 If the velocity at each point in space in steady-state fluid flow is constant, how can a fluid particle accelerate?

Q12.26 In a store-window vacuum cleaner display, a table-tennis ball is suspended in midair in a jet of air blown from the outlet hose of a tank-type vacuum cleaner. The ball bounces around a little but always moves back toward the center of the jet, even if the jet is tilted from the vertical. How does this behavior illustrate Bernoulli's equation?

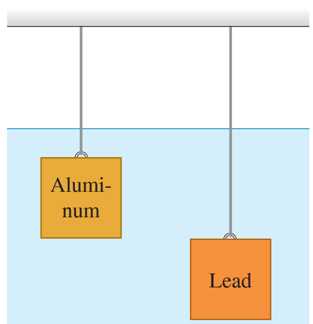
Q12.27 A tornado consists of a rapidly whirling air vortex. Why is the pressure always much lower in the center than at the outside? How does this condition account for the destructive power of a tornado?

Q12.28 Airports at high elevations have longer runways for takeoffs and landings than do airports at sea level. One reason is that aircraft engines develop less power in the thin air well above sea level. What is another reason?

Q12.29 When a smooth-flowing stream of water comes out of a faucet, it narrows as it falls. Explain.

Q12.30 Identical-size lead and aluminum cubes are suspended at different depths by two wires in a large vat of water (**Fig. Q12.30**). (a) Which cube experiences a greater buoyant force? (b) For which cube is the tension in the wire greater? (c) Which cube experiences a greater force on its lower face? (d) For which cube is the difference in pressure between the upper and lower faces greater?

Figure **Q12.30**



EXERCISES

Section 12.1 Gases, Liquids, and Density

12.1 •• On a part-time job, you are asked to bring a cylindrical iron rod of length 85.8 cm and diameter 2.85 cm from a storage room to a machinist. Will you need a cart? (To answer, calculate the weight of the rod.)

12.2 •• A cube 5.0 cm on each side is made of a metal alloy. After you drill a cylindrical hole 2.0 cm in diameter all the way through and perpendicular to one face, you find that the cube weighs 6.30 N. (a) What is the density of this metal? (b) What did the cube weigh before you drilled the hole in it?

12.3 • You purchase a rectangular piece of metal that has dimensions $5.0 \times 15.0 \times 30.0$ mm and mass 0.0158 kg. The seller tells you that the metal is gold. To check this, you compute the average density of the piece. What value do you get? Were you cheated?

12.4 • (a) What is the average density of the sun? (b) What is the average density of a neutron star that has the same mass as the sun but a radius of only 20.0 km?

12.5 •• A uniform lead sphere and a uniform aluminum sphere have the same mass. What is the ratio of the radius of the aluminum sphere to the radius of the lead sphere?

12.6 •• A hollow cylindrical copper pipe is 1.50 m long and has an outside diameter of 3.50 cm and an inside diameter of 2.50 cm. How much does it weigh?

Section 12.2 Pressure in a Fluid

12.7 •• **Oceans on Mars.** Scientists have found evidence that Mars may once have had an ocean 0.500 km deep. The acceleration due to gravity on Mars is 3.71 m/s^2. (a) What would be the gauge pressure at the bottom of such an ocean, assuming it was freshwater? (b) To what depth would you need to go in the earth's ocean to experience the same gauge pressure?

12.8 •• **BIO** (a) Calculate the difference in blood pressure between the feet and top of the head for a person who is 1.65 m tall. (b) Consider a cylindrical segment of a blood vessel 2.00 cm long and 1.50 mm in diameter. What *additional* outward force would such a vessel need to withstand in the person's feet compared to a similar vessel in her head?

12.9 • **BIO** In intravenous feeding, a needle is inserted in a vein in the patient's arm and a tube leads from the needle to a reservoir of fluid (density 1050 kg/m^3) located at height h above the arm. The top of the reservoir is open to the air. If the gauge pressure inside the vein is 5980 Pa, what is the minimum value of h that allows fluid to enter the vein? Assume the needle diameter is large enough that you can ignore the viscosity (see Section 12.6) of the fluid.

12.10 • A barrel contains a 0.120 m layer of oil floating on water that is 0.250 m deep. The density of the oil is 600 kg/m^3. (a) What is the gauge pressure at the oil–water interface? (b) What is the gauge pressure at the bottom of the barrel?

12.11 • A U-shaped tube with both arms open to the air has a 35.0 cm column of liquid of unknown density in its right arm. Beneath this liquid and not mixing with it is glycerin that extends into the left arm of the tube. The surface of the glycerin in the left arm is 12.0 cm below the surface of the unknown liquid in the right arm. What is the density of the unknown liquid?

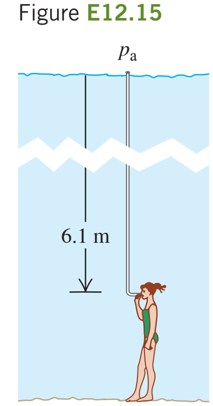
12.12 •• You are designing a diving bell to withstand the pressure of seawater at a depth of 250 m. (a) What is the gauge pressure at this depth? (You can ignore changes in the density of the water with depth.) (b) At this depth, what is the net force due to the water outside and the air inside the bell on a circular glass window 30.0 cm in diameter if the pressure inside the diving bell equals the pressure at the surface of the water? (Ignore the small variation of pressure over the surface of the window.)

12.13 •• **BIO Ear Damage from Diving.** If the force on the tympanic membrane (eardrum) increases by about 1.5 N above the force from atmospheric pressure, the membrane can be damaged. When you go scuba diving in the ocean, below what depth could damage to your eardrum start to occur? The eardrum is typically 8.2 mm in diameter. (Consult Table 12.1.)

12.14 •• The liquid in the open-tube manometer in Fig. 12.8a is mercury, $y_1 = 3.00$ cm, and $y_2 = 7.00$ cm. Atmospheric pressure is 980 millibars. What are (a) the absolute pressure at the bottom of the U-shaped tube; (b) the absolute pressure in the open tube at a depth of 4.00 cm below the free surface; (c) the absolute pressure of the gas in the container; (d) the gauge pressure of the gas in pascals?

12.15 • **BIO** There is a maximum depth at which a diver can breathe through a snorkel tube (**Fig. E12.15**) because as the depth increases, so does the pressure difference, which tends to collapse the diver's lungs. Since the snorkel connects the air in the lungs to the atmosphere at the surface, the pressure inside the lungs is atmospheric pressure. What is the external–internal pressure difference when the diver's lungs are at a depth of 6.1 m (about 20 ft)? Assume that the diver is in freshwater. (A scuba diver breathing from compressed air tanks can operate at greater depths than can a snorkeler, since the pressure of the air inside the scuba diver's lungs increases to match the external pressure of the water.)

Figure **E12.15**



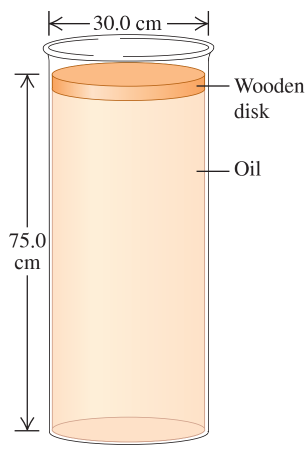
12.16 •• **BIO** The lower end of a long plastic straw is immersed below the surface of the water in a plastic cup. An average person sucking on the upper end of the straw can pull water into the straw to a vertical height of 1.1 m above the surface of the water in the cup. (a) What is the lowest gauge pressure that the average person can achieve inside his lungs? (b) Explain why your answer in part (a) is negative.

12.17 •• An electrical short cuts off all power to a submersible diving vehicle when it is 30 m below the surface of the ocean. The crew must push out a hatch of area 0.75 m^2 and weight 300 N on the bottom to escape. If the pressure inside is 1.0 atm, what downward force must the crew exert on the hatch to open it?

12.18 •• A tall cylinder with a cross-sectional area 12.0 cm^2 is partially filled with mercury; the surface of the mercury is 8.00 cm above the bottom of the cylinder. Water is slowly poured in on top of the mercury, and the two fluids don't mix. What volume of water must be added to double the gauge pressure at the bottom of the cylinder?

12.19 •• A cylindrical disk of wood weighing 45.0 N and having a diameter of 30.0 cm floats on a cylinder of oil of density 0.850 g/cm^3 (**Fig. E12.19**). The cylinder of oil is 75.0 cm deep and has a diameter the same as that of the wood. (a) What is the gauge pressure at the top of the oil column? (b) Suppose now that someone puts a weight of 83.0 N on top of the wood, but no oil seeps around the edge of the wood. What is the *change* in pressure at (i) the bottom of the oil and (ii) halfway down in the oil?

Figure **E12.19**



12.20 • You are doing experiments from a research ship in the Atlantic Ocean. On a day when the atmospheric pressure at the surface of the water is 1.03×10^5 Pa, at what depth below the surface of the water is the absolute pressure (a) twice the pressure at the surface and (b) four times the pressure at the surface?

12.21 •• **Hydraulic Lift I.** For the hydraulic lift shown in Fig. 12.7, what must be the ratio of the diameter of the vessel at the car to the diameter of the vessel where the force F_1 is applied so that a 1520 kg car can be lifted with a force F_1 of just 125 N?

12.22 • **Hydraulic Lift II.** The piston of a hydraulic automobile lift is 0.30 m in diameter. What gauge pressure, in pascals, is required to lift a car with a mass of 1200 kg? Also express this pressure in atmospheres.

12.23 • Early barometers contained wine instead of mercury. On a day when the height h of the liquid in the barometer tube is 750 mm for a mercury barometer, what is h for a barometer that uses wine with density 990 kg/m^3?

Section 12.3 Buoyancy

12.24 • Estimate the fraction of your body's total volume that is above the surface of the water when you float in seawater (density 1030 kg/m^3). (a) Use this estimate and your weight to calculate the total volume of your body. (b) What is your average density? How does your average density compare to the density of seawater?

12.25 •• A 900 N athlete in very good condition does not float in a freshwater pool. To keep him from sinking to the bottom, an upward force of 20 N must be applied to him. What are his volume and his average density?

12.26 •• A rock has mass 1.80 kg. When the rock is suspended from the lower end of a string and totally immersed in water, the tension in the string is 12.8 N. What is the smallest density of a liquid in which the rock will float?

12.27 • A 950 kg cylindrical buoy floats vertically in seawater. The diameter of the buoy is 0.900 m. Calculate the additional distance the buoy will sink when an 80.0 kg man stands on top of it.

12.28 •• A slab of ice floats on a freshwater lake. What minimum volume must the slab have for a 65.0 kg woman to be able to stand on it without getting her feet wet?

12.29 •• An ore sample weighs 17.50 N in air. When the sample is suspended by a light cord and totally immersed in water, the tension in the cord is 11.20 N. Find the total volume and the density of the sample.

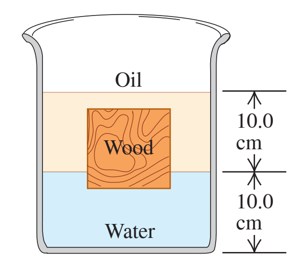
12.30 •• You are preparing some apparatus for a visit to a newly discovered planet Caasi having oceans of glycerin and a surface acceleration due to gravity of 5.40 m/s^2. If your apparatus floats in the oceans on earth with 25.0% of its volume submerged, what percentage will be submerged in the glycerin oceans of Caasi?

12.31 •• A rock with density 1200 kg/m^3 is suspended from the lower end of a light string. When the rock is in air, the tension in the string is 28.0 N. What is the tension in the string when the rock is totally immersed in a liquid with density 750 kg/m^3?

12.32 • A hollow plastic sphere is held below the surface of a freshwater lake by a cord anchored to the bottom of the lake. The sphere has a volume of 0.650 m^3 and the tension in the cord is 1120 N. (a) Calculate the buoyant force exerted by the water on the sphere. (b) What is the mass of the sphere? (c) The cord breaks and the sphere rises to the surface. When the sphere comes to rest, what fraction of its volume will be submerged?

12.33 •• A cubical block of wood, 10.0 cm on a side, floats at the interface between oil and water with its lower surface 1.50 cm below the interface (**Fig. E12.33**). The density of the oil is 790 kg/m^3. (a) What is the gauge pressure at the upper face of the block? (b) What is the gauge pressure at the lower face of the block? (c) What are the mass and density of the block?

Figure **E12.33**



12.34 • A solid aluminum ingot weighs 89 N in air. (a) What is its volume? (b) The ingot is suspended from a rope and totally immersed in water. What is the tension in the rope (the *apparent* weight of the ingot in water)?

12.35 •• A rock is suspended by a light string. When the rock is in air, the tension in the string is 39.2 N. When the rock is totally immersed in water, the tension is 28.4 N. When the rock is totally immersed in an unknown liquid, the tension is 21.5 N. What is the density of the unknown liquid?

12.36 • A uniform plastic block floats in water with 30.0% of its volume above the surface of the water. The block is placed in a second liquid and floats with 20.0% of its volume above the surface of the liquid. What is the density of the second liquid?

12.37 •• A large plastic cylinder with mass 30.0 kg and density 370 kg/m^3 is in the water of a lake. A light vertical cable runs between the bottom of the cylinder and the bottom of the lake and holds the cylinder so that 30.0% of its volume is above the surface of the water. What is the tension in the cable?

Section 12.4 Fluid Flow

12.38 •• Water runs into a fountain, filling all the pipes, at a steady rate of 0.750 m^3/s. (a) How fast will it shoot out of a hole 4.50 cm in diameter? (b) At what speed will it shoot out if the diameter of the hole is three times as large?

12.39 •• A shower head has 20 circular openings, each with radius 1.0 mm. The shower head is connected to a pipe with radius 0.80 cm. If the speed of water in the pipe is 3.0 m/s, what is its speed as it exits the shower-head openings?

12.40 • Water is flowing in a pipe with a varying cross-sectional area, and at all points the water completely fills the pipe. At point 1 the cross-sectional area of the pipe is 0.070 m^2, and the magnitude of the fluid velocity is 3.50 m/s. (a) What is the fluid speed at points in the pipe where the cross-sectional area is (a) 0.105 m^2 and (b) 0.047 m^2? (c) Calculate the volume of water discharged from the open end of the pipe in 1.00 hour.

12.41 • Water is flowing in a pipe with a circular cross section but with varying cross-sectional area, and at all points the water completely fills the pipe. (a) At one point in the pipe the radius is 0.150 m. What is the speed of the water at this point if water is flowing into this pipe at a steady rate of $1.20\ m^3/s$? (b) At a second point in the pipe the water speed is 3.80 m/s. What is the radius of the pipe at this point?

12.42 •• On another planet that you are exploring, a large tank is open to the atmosphere and contains ethanol. A horizontal pipe of cross-sectional area $9.0 \times 10^{-4}\ m^2$ has one end inserted into the tank just above the bottom of the tank. The other end of the pipe is open to the atmosphere. The viscosity of the ethanol can be neglected. You measure the volume flow rate of the ethanol from the tank as a function of the depth h of the ethanol in the tank. If you graph the volume flow rate squared as a function of h, your data lie close to a straight line that has slope $1.94 \times 10^{-5}\ m^5/s^2$. What is the value of g, the acceleration of a free-falling object at the surface of the planet?

12.43 •• A large, cylindrical water tank with diameter 3.00 m is on a platform 2.00 m above the ground. The vertical tank is open to the air and the depth of the water in the tank is 2.00 m. There is a hole with diameter 0.500 cm in the side of the tank just above the bottom of the tank. The hole is plugged with a cork. You remove the cork and collect in a bucket the water that flows out the hole. (a) When 1.00 gal of water flows out of the tank, what is the change in the height of the water in the tank? (b) How long does it take you to collect 1.00 gal of water in the bucket? Based on your answer in part (a), is it reasonable to ignore the change in the depth of the water in the tank as 1.00 gal of water flows out?

Section 12.5 Bernoulli's Equation

12.44 • A soft drink (mostly water) flows in a pipe at a beverage plant with a mass flow rate that would fill 220 0.355-L cans per minute. At point 2 in the pipe, the gauge pressure is 152 kPa and the cross-sectional area is $8.00\ cm^2$. At point 1, 1.35 m above point 2, the cross-sectional area is $2.00\ cm^2$. Find the (a) mass flow rate; (b) volume flow rate; (c) flow speeds at points 1 and 2; (d) gauge pressure at point 1.

12.45 •• A sealed tank containing seawater to a height of 11.0 m also contains air above the water at a gauge pressure of 3.00 atm. Water flows out from the bottom through a small hole. How fast is this water moving?

12.46 •• BIO **Artery Blockage.** A medical technician is trying to determine what percentage of a patient's artery is blocked by plaque. To do this, she measures the blood pressure just before the region of blockage and finds that it is 1.20×10^4 Pa, while in the region of blockage it is 1.15×10^4 Pa. Furthermore, she knows that blood flowing through the normal artery just before the point of blockage is traveling at 30.0 cm/s, and the specific gravity of this patient's blood is 1.06. What percentage of the cross-sectional area of the patient's artery is blocked by the plaque?

12.47 • What gauge pressure is required in the city water mains for a stream from a fire hose connected to the mains to reach a vertical height of 15.0 m? (Assume that the mains have a much larger diameter than the fire hose.)

12.48 • A small circular hole 6.00 mm in diameter is cut in the side of a large water tank, 14.0 m below the water level in the tank. The top of the tank is open to the air. Find (a) the speed of efflux of the water and (b) the volume discharged per second.

12.49 • At a certain point in a horizontal pipeline, the water's speed is 2.50 m/s and the gauge pressure is 1.80×10^4 Pa. Find the gauge pressure at a second point in the line if the cross-sectional area at the second point is twice that at the first.

12.50 •• At one point in a pipeline the water's speed is 3.00 m/s and the gauge pressure is 5.00×10^4 Pa. Find the gauge pressure at a second point in the line, 11.0 m lower than the first, if the pipe diameter at the second point is twice that at the first.

12.51 •• A golf course sprinkler system discharges water from a horizontal pipe at the rate of $7200\ cm^3/s$. At one point in the pipe, where the radius is 4.00 cm, the water's absolute pressure is 2.40×10^5 Pa. At a second point in the pipe, the water passes through a constriction where the radius is 2.00 cm. What is the water's absolute pressure as it flows through this constriction?

Section 12.6 Viscosity and Turbulence

12.52 • A pressure difference of 6.00×10^4 Pa is required to maintain a volume flow rate of $0.800\ m^3/s$ for a viscous fluid flowing through a section of cylindrical pipe that has radius 0.210 m. What pressure difference is required to maintain the same volume flow rate if the radius of the pipe is decreased to 0.0700 m?

12.53 •• BIO **Clogged Artery.** Viscous blood is flowing through an artery partially clogged by cholesterol. A surgeon wants to remove enough of the cholesterol to double the flow rate of blood through this artery. If the original diameter of the artery is D, what should be the new diameter (in terms of D) to accomplish this for the same pressure gradient?

PROBLEMS

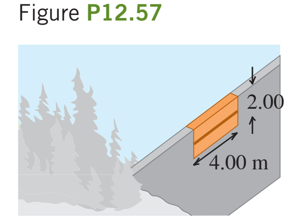
12.54 •• CP The deepest point known in any of the earth's oceans is in the Marianas Trench, 10.92 km deep. (a) Assuming water is incompressible, what is the pressure at this depth? Use the density of seawater. (b) The actual pressure is 1.16×10^8 Pa; your calculated value will be less because the density actually varies with depth. Using the compressibility of water and the actual pressure, find the density of the water at the bottom of the Marianas Trench. What is the percent change in the density of the water?

12.55 ••• CALC A swimming pool is 5.0 m long, 4.0 m wide, and 3.0 m deep. Compute the force exerted by the water against (a) the bottom and (b) either end. (*Hint:* Calculate the force on a thin, horizontal strip at a depth h, and integrate this over the end of the pool.) Do not include the force due to air pressure.

12.56 •• A rock is suspended from the lower end of a light string. When the rock is totally immersed in water, the tension in the string is 1.20 N. When the rock is totally immersed in ethanol, the tension in the string is 1.60 N. Use the densities in Table 12.1 to calculate the weight of the rock.

12.57 ••• CP CALC The upper edge of a gate in a dam runs along the water surface. The gate is 2.00 m high and 4.00 m wide and is hinged along a horizontal line through its center (**Fig. P12.57**). Calculate the torque about the hinge arising from the force due to the water. (*Hint:* Use a procedure similar to that used in Problem 12.55; calculate the torque on a thin, horizontal strip at a depth h and integrate this over the gate.)

Figure **P12.57**



12.58 ••• A large tank has a 40.0-cm-deep layer of water floating on top of a 60.0-cm-deep layer of glycerin. A dense wooden cube has a side length of 2.00 cm and a mass of 9.20×10^{-3} kg. When the cube floats in the tank, what fraction of its volume is below the surface of the glycerin?

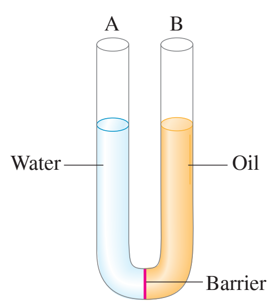
12.59 •• A block of plastic in the shape of a rectangular solid that has height 8.00 cm and area A for its top and bottom surfaces is floating in water. You place coins on the top surface of the block (at the center, so the top surface of the block remains horizontal). By measuring the height of the block above the surface of the water, you can determine the height h below the surface. You measure h for various values of the total mass m of the coins that you have placed on the block. You plot h versus m and find that your data lie close to a straight line that has slope 0.0390 m/kg and y-intercept 0.0312 m. What is the mass of the block?

12.60 •• Ballooning on Mars. It has been proposed that we could explore Mars using inflated balloons to hover just above the surface. The buoyancy of the atmosphere would keep the balloon aloft. The density of the Martian atmosphere is $0.0154\ \text{kg/m}^3$ (although this varies with temperature). Suppose we construct these balloons of a thin but tough plastic having a density such that each square meter has a mass of 5.00 g. We inflate them with a very light gas whose mass we can ignore. (a) What should be the radius and mass of these balloons so they just hover above the surface of Mars? (b) If we released one of the balloons from part (a) on earth, where the atmospheric density is $1.20\ \text{kg/m}^3$, what would be its initial acceleration assuming it was the same size as on Mars? Would it go up or down? (c) If on Mars these balloons have five times the radius found in part (a), how heavy an instrument package could they carry?

12.61 •• A 0.180 kg cube of ice (frozen water) is floating in glycerin. The gylcerin is in a tall cylinder that has inside radius 3.50 cm. The level of the glycerin is well below the top of the cylinder. If the ice completely melts, by what distance does the height of liquid in the cylinder change? Does the level of liquid rise or fall? That is, is the surface of the water above or below the original level of the glycerin before the ice melted?

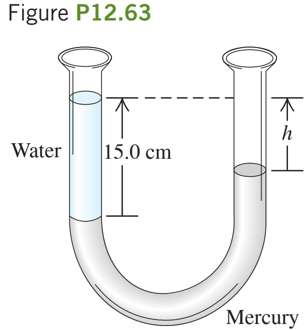
12.62 •• A narrow, U-shaped glass tube with open ends is filled with 25.0 cm of oil (of specific gravity 0.80) and 25.0 cm of water on opposite sides, with a barrier separating the liquids (**Fig. P12.62**). (a) Assume that the two liquids do not mix, and find the final heights of the columns of liquid in each side of the tube after the barrier is removed. (b) For the following cases, arrive at your answer by simple physical reasoning, not by calculations: (i) What would be the height on each side if the oil and water had equal densities? (ii) What would the heights be if the oil's density were much less than that of water?

Figure **P12.62**



12.63 • A U-shaped tube open to the air at both ends contains some mercury. A quantity of water is carefully poured into the left arm of the U-shaped tube until the vertical height of the water column is 15.0 cm (**Fig. P12.63**). (a) What is the gauge pressure at the water–mercury interface? (b) Calculate the vertical distance h from the top of the mercury in the right-hand arm of the tube to the top of the water in the left-hand arm.

Figure **P12.63**



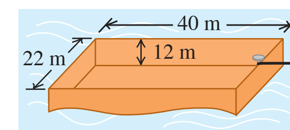
12.64 •• CALC The Great Molasses Flood. On the afternoon of January 15, 1919, an unusually warm day in Boston, a 17.7-m-high, 27.4-m-diameter cylindrical metal tank used for storing molasses ruptured. Molasses flooded into the streets in a 5-m-deep stream, killing pedestrians and horses and knocking down buildings. The molasses had a density of $1600\ \text{kg/m}^3$. If the tank was full before the accident, what was the total outward force the molasses exerted on its sides? (*Hint:* Consider the outward force on a circular ring of the tank wall of width dy and at a depth y below the surface. Integrate to find the total outward force. Assume that before the tank ruptured, the pressure at the surface of the molasses was equal to the air pressure outside the tank.)

12.65 •• A large, 40.0 kg cubical block of wood with uniform density is floating in a freshwater lake with 20.0% of its volume above the surface of the water. You want to load bricks onto the floating block and then push it horizontally through the water to an island where you are building an outdoor grill. (a) What is the volume of the block? (b) What is the maximum mass of bricks that you can place on the block without causing it to sink below the water surface?

12.66 ••• A hot-air balloon has a volume of $2200\ \text{m}^3$. The balloon fabric (the envelope) weighs 900 N. The basket with gear and full propane tanks weighs 1700 N. If the balloon can barely lift an additional 3200 N of passengers, breakfast, and champagne when the outside air density is $1.23\ \text{kg/m}^3$, what is the average density of the heated gases in the envelope?

12.67 • An open barge has the dimensions shown in **Fig. P12.67.** If the barge is made out of 4.0-cm-thick steel plate on each of its four sides and its bottom, what mass of coal can the barge carry in freshwater without sinking? Is there enough room in the barge to hold this amount of coal? (The density of coal is about $1500\ \text{kg/m}^3$.)

Figure **P12.67**



12.68 ••• A piece of wood is 0.600 m long, 0.250 m wide, and 0.080 m thick. Its density is $700\ \text{kg/m}^3$. What volume of lead must be fastened underneath it to sink the wood in calm water so that its top is just even with the water level? What is the mass of this volume of lead?

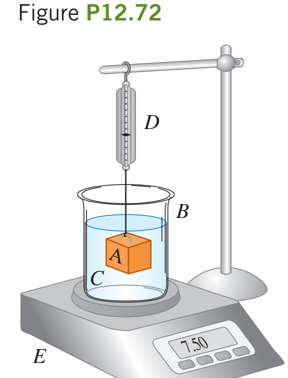
12.69 •• A gallon of milk in a full plastic jug is sitting on the edge of your kitchen table. Estimate the vertical distance between the top surface of the milk and the bottom of the jug. Also estimate the distance from the tabletop to the floor. You punch a small hole in the side of the jug just above the bottom of the jug, and milk flows out the hole. When the milk first starts to flow out the hole, what horizontal distance does it travel before reaching the floor? Assume the milk is in free fall after it has passed through the hole, and neglect the viscosity of the milk.

12.70 •• Ethanol is flowing in a pipe and at all points completely fills the pipe. At point A in the pipe the gauge pressure is p and the cross-sectional area of the pipe is $0.0500\ \text{m}^2$. The other end of the pipe (point B) is open to the air, has cross-sectional area $0.0200\ \text{m}^2$, and is at a vertical height of 3.00 m above point A. What must the gauge pressure p be at A if the volume flow rate out of the pipe at point B is $0.0800\ \text{m}^3/\text{s}$?

12.71 •• CP A firehose must be able to shoot water to the top of a building 28.0 m tall when aimed straight up. Water enters this hose at a steady rate of $0.500\ \text{m}^3/\text{s}$ and shoots out of a round nozzle. Neglect air resistance. (a) What is the maximum diameter this nozzle can have? (b) If the only nozzle available has a diameter twice as great, what is the highest point the water can reach?

12.72 •• Block A in **Fig. P12.72** hangs by a cord from spring balance D and is submerged in a liquid C contained in beaker B. The mass of the beaker is 1.00 kg; the mass of the liquid is 1.80 kg. Balance D reads 3.50 kg, and balance E reads 7.50 kg. The volume of block A is $3.80 \times 10^{-3}\ \text{m}^3$. (a) What is the density of the liquid? (b) What will each balance read if block A is pulled up out of the liquid?

Figure **P12.72**



12.73 ••• CALC A closed and elevated vertical cylindrical tank with diameter 2.00 m contains water to a depth of 0.800 m. A worker accidently pokes a circular hole with diameter 0.0200 m in the bottom of the tank. As the water drains from the tank, compressed air above the water in the tank maintains a gauge pressure of 5.00×10^3 Pa at the surface of the water. Ignore any effects of viscosity. (a) Just after the hole is made, what is the speed of the water as it emerges from the hole? What is the ratio of this speed to the efflux speed if the top of the tank is open to the air? (b) How much time does it take for all the water to drain from the tank? What is the ratio of this time to the time it takes for the tank to drain if the top of the tank is open to the air?

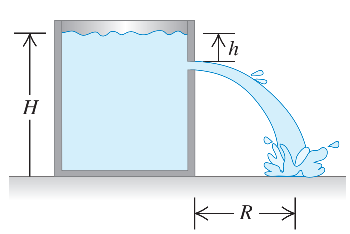
12.74 •• A plastic ball has radius 12.0 cm and floats in water with 24.0% of its volume submerged. (a) What force must you apply to the ball to hold it at rest totally below the surface of the water? (b) If you let go of the ball, what is its acceleration the instant you release it?

12.75 ••• A cubical block of density ρ_B and with sides of length L floats in a liquid of greater density ρ_L. (a) What fraction of the block's volume is above the surface of the liquid? (b) The liquid is denser than water (density ρ_W) and does not mix with it. If water is poured on the surface of that liquid, how deep must the water layer be so that the water surface just rises to the top of the block? Express your answer in terms of L, ρ_B, ρ_L, and ρ_W. (c) Find the depth of the water layer in part (b) if the liquid is mercury, the block is made of iron, and $L = 10.0$ cm.

12.76 •• A barge is in a rectangular lock on a freshwater river. The lock is 60.0 m long and 20.0 m wide, and the steel doors on each end are closed. With the barge floating in the lock, a 2.50×10^6 N load of scrap metal is put onto the barge. The metal has density 7200 kg/m^3. (a) When the load of scrap metal, initially on the bank, is placed onto the barge, what vertical distance does the water in the lock rise? (b) The scrap metal is now pushed overboard into the water. Does the water level in the lock rise, fall, or remain the same? If it rises or falls, by what vertical distance does it change?

12.77 • CP Water stands at a depth H in a large, open tank whose side walls are vertical (**Fig. P12.77**). A hole is made in one of the walls at a depth h below the water surface. (a) At what distance R from the foot of the wall does the emerging stream strike the floor? (b) How far above the bottom of the tank could a second hole be cut so that the stream emerging from it could have the same range as for the first hole?

Figure **P12.77**



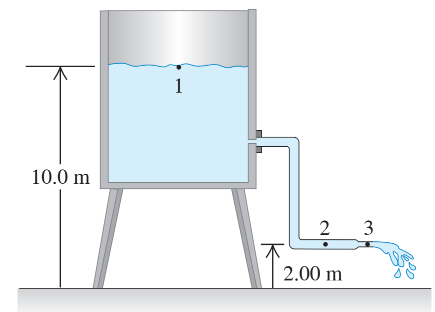
12.78 •• Your uncle is in the below-deck galley of his boat while you are spear fishing in the water nearby. An errant spear makes a small hole in the boat's hull, and water starts to leak into the galley. (a) If the hole is 0.900 m below the water surface and has area 1.20 cm^2, how long does it take 10.0 L of water to leak into the boat? (b) Do you need to take into consideration the fact that the boat sinks lower into the water as water leaks in?

12.79 •• CP You hold a hose at waist height and spray water horizontally with it. The hose nozzle has a diameter of 1.80 cm, and the water splashes on the ground a distance of 0.950 m horizontally from the nozzle. If you constrict the nozzle to a diameter of 0.750 cm, how far from the nozzle, horizontally, will the water travel before it hits the ground? (Ignore air resistance.)

12.80 ••• A cylindrical bucket, open at the top, is 25.0 cm high and 10.0 cm in diameter. A circular hole with a cross-sectional area 1.50 cm^2 is cut in the center of the bottom of the bucket. Water flows into the bucket from a tube above it at the rate of 2.40×10^{-4} m^3/s. How high will the water in the bucket rise?

12.81 • Water flows steadily from an open tank as in **Fig. P12.81.** The elevation of point 1 is 10.0 m, and the elevation of points 2 and 3 is 2.00 m. The cross-sectional area at point 2 is 0.0480 m^2; at point 3 it is 0.0160 m^2. The area of the tank is very large compared with the cross-sectional area of the pipe. Assuming that Bernoulli's equation applies, compute (a) the discharge rate in cubic meters per second and (b) the gauge pressure at point 2.

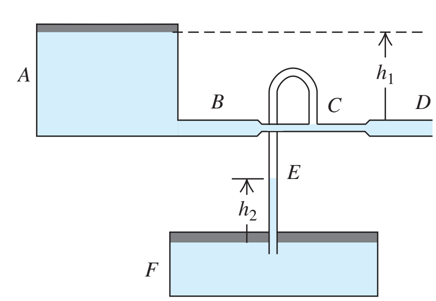
Figure **P12.81**



12.82 •• CP In 1993 the radius of Hurricane Emily was about 350 km. The wind speed near the center ("eye") of the hurricane, whose radius was about 30 km, reached about 200 km/h. As air swirled in from the rim of the hurricane toward the eye, its angular momentum remained roughly constant. Estimate (a) the wind speed at the rim of the hurricane; (b) the pressure difference at the earth's surface between the eye and the rim. (*Hint:* See Table 12.1.) Where is the pressure greater? (c) If the kinetic energy of the swirling air in the eye could be converted completely to gravitational potential energy, how high would the air go? (d) In fact, the air in the eye is lifted to heights of several kilometers. How can you reconcile this with your answer to part (c)?

12.83 •• Two very large open tanks A and F (**Fig. P12.83**) contain the same liquid. A horizontal pipe BCD, having a constriction at C and open to the air at D, leads out of the bottom of tank A, and a vertical pipe E opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross-sectional area at C is one-half the area at D and if D is a distance h_1 below the level of the liquid in A, to what height h_2 will liquid rise in pipe E? Express your answer in terms of h_1.

Figure **P12.83**

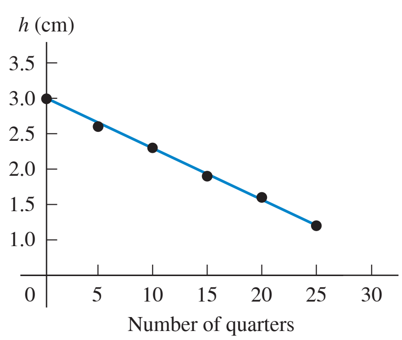


12.84 • A liquid flowing from a vertical pipe has a definite shape as it flows from the pipe. To get the equation for this shape, assume that the liquid is in free fall once it leaves the pipe. Just as it leaves the pipe, the liquid has speed v_0 and the radius of the stream of liquid is r_0. (a) Find an equation for the speed of the liquid as a function of the distance y it has fallen. Combining this with the equation of continuity, find an expression for the radius of the stream as a function of y. (b) If water flows out of a vertical pipe at a speed of 1.20 m/s, how far below the outlet will the radius be one-half the original radius of the stream?

12.85 •• DATA The density values in Table 12.1 are listed in increasing order. A chemistry student notices that the first four chemical elements that are included are also listed in order of increasing atomic mass. (a) See whether there is a simple relationship between density and atomic mass by plotting a graph of density (in g/cm^3) versus atomic mass for all eight elements in that table. (See Appendix D for their atomic masses in grams per mole.) (b) Can you draw a straight line or simple curve through the points to find a "simple" relationship? (c) Explain why "More massive atoms result in more dense solids" does not tell the whole story.

12.86 •• DATA You have a bucket containing an unknown liquid. You also have a cube-shaped wooden block that you measure to be 8.0 cm on a side, but you don't know the mass or density of the block. To find the density of the liquid, you perform an experiment. First you place the wooden block in the liquid and measure the height of the top of the floating block above the liquid surface. Then you stack various numbers of U.S. quarter-dollar coins onto the block and measure the new value of h. The straight line that gives the best fit to the data you have collected is shown in **Fig. P12.86.** Find the mass of one quarter (see www.usmint.gov for quarters dated 2012). Use this information and the slope and intercept of the straight-line fit to your data to calculate (a) the density of the liquid (in kg/m^3) and (b) the mass of the block (in kg).

Figure **P12.86**



12.87 ••• DATA The Environmental Protection Agency is investigating an abandoned chemical plant. A large, closed cylindrical tank contains an unknown liquid. You must determine the liquid's density and the height of the liquid in the tank (the vertical distance from the surface of the liquid to the bottom of the tank). To maintain various values of the gauge pressure in the air that is above the liquid in the tank, you can use compressed air. You make a small hole at the bottom of the side of the tank, which is on a concrete platform—so the hole is 50.0 cm above the ground. The table gives your measurements of the horizontal distance R that the initially horizontal stream of liquid pouring out of the tank travels before it strikes the ground and the gauge pressure p_g of the air in the tank.

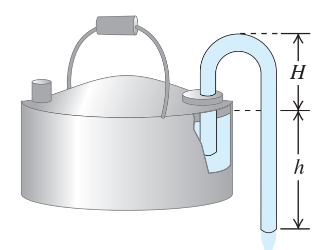
p_g (atm)	0.50	1.00	2.00	3.00	4.00
R (m)	5.4	6.5	8.2	9.7	10.9

(a) Graph R^2 as a function of p_g. Explain why the data points fall close to a straight line. Find the slope and intercept of that line. (b) Use the slope and intercept found in part (a) to calculate the height h (in meters) of the liquid in the tank and the density of the liquid (in kg/m^3). Use $g = 9.80\ m/s^2$. Assume that the liquid is nonviscous and that the hole is small enough compared to the tank's diameter so that the change in h during the measurements is very small.

CHALLENGE PROBLEM

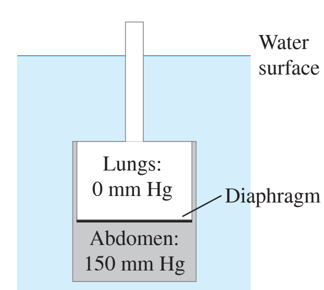
12.88 ••• A *siphon* (**Fig. P12.88**) is a convenient device for removing liquids from containers. To establish the flow, the tube must be initially filled with fluid. Let the fluid have density ρ, and let the atmospheric pressure be p_{atm}. Assume that the cross-sectional area of the tube is the same at all points along it. (a) If the lower end of the siphon is at a distance h below the surface of the liquid in the container, what is the speed of the fluid as it flows out the lower end of the siphon? (Assume that the container has a very large diameter, and ignore any effects of viscosity.) (b) A curious feature of a siphon is that the fluid initially flows "uphill." What is the greatest height H that the high point of the tube can have if flow is still to occur?

Figure **P12.88**



MCAT-STYLE PASSAGE PROBLEMS

BIO Elephants Under Pressure. An elephant can swim or walk with its chest several meters underwater while the animal breathes through its trunk, which remains above the water surface and acts like a snorkel. The elephant's tissues are at an increased pressure due to the surrounding water, but the lungs are at atmospheric pressure because they are connected to the air through the trunk. The figure shows the gauge pressures in an elephant's lungs and abdomen when the elephant's chest is submerged to a particular depth in a lake. In this situation, the elephant's diaphragm, which separates the lungs from the abdomen, must sustain the difference in pressure between the lungs and the abdomen. The diaphragm of an elephant is typically 3.0 cm thick and 120 cm in diameter. (See "Why Doesn't the Elephant Have a Pleural Space?" by John B. West, *Physiology*, Vol. 17:47–50, April 1, 2002.)



12.89 For the situation shown, the tissues in the elephant's abdomen are at a gauge pressure of 150 mm Hg. This pressure corresponds to what distance below the surface of a lake? (a) 1.5 m; (b) 2.0 m; (c) 3.0 m; (d) 15 m.

12.90 The maximum force the muscles of the diaphragm can exert is 24,000 N. What maximum pressure difference can the diaphragm withstand? (a) 160 mm Hg; (b) 760 mm Hg; (c) 920 mm Hg; (d) 5000 mm Hg.

12.91 How does the force the diaphragm experiences due to the difference in pressure between the lungs and abdomen depend on the abdomen's distance below the water surface? The force (a) increases linearly with distance; (b) increases as distance squared; (c) increases as distance cubed; (d) increases exponentially with distance.

12.92 If the elephant were to snorkel in salt water, which is more dense than freshwater, would the maximum depth at which it could snorkel be different from that in freshwater? (a) Yes—that depth would increase, because the pressure would be lower at a given depth in salt water than in freshwater; (b) yes—that depth would decrease, because the pressure would be higher at a given depth in salt water than in freshwater; (c) no, because pressure differences within the submerged elephant depend on only the density of air, not the density of the water; (d) no, because the buoyant force on the elephant would be the same in both cases.

ANSWERS

Chapter Opening Question ?

(v) The ratio of mass to volume is density. The flesh of both the wrasse and the ray is denser than seawater, but a wrasse has a gas-filled body cavity called a swimbladder. Hence the *average* density of the wrasse's body is the same as that of seawater, and the fish neither sinks nor rises. Rays have no such cavity, so they must swim continuously to avoid sinking: Their fins provide lift, much like the wings of a bird or airplane (see Section 12.5).

Key Example VARIATION Problems

VP12.4.1 1.31×10^5 Pa
VP12.4.2 (a) 1.09×10^3 Pa (b) 2.56×10^3 Pa
VP12.4.3 81.8 cm
VP12.4.4 (a) 22.9 cm (b) 1.47×10^3 Pa (c) 16.4 cm
VP12.5.1 (a) 8.45 N (b) (i) 8.82×10^{-3} N, sink; (ii) 7.35 N, sink; (iii) 9.26 N, rise
VP12.5.2 (a) 11.8 N (b) 41.2 N (c) 3.50×10^3 kg/m^3
VP12.5.3 1.1×10^3 kg/m^3
VP12.5.4 (a) $B = \rho_L A d g$ (b) $\rho_{\text{cylinder}} = \rho_L(d/L)$
VP12.9.1 (a) 6.4 m/s (b) 3.7×10^5 Pa
VP12.9.2 (a) 14 m/s (b) 5.4×10^4 Pa
VP12.9.3 (a) 8.3 cm (b) 5.9×10^{-4} m^3/s
VP12.9.4 $v_2 = \sqrt{\left[2\left(\dfrac{p_0 - p_{\text{atm}}}{\rho}\right) + 2gh\right] \Big/ \left[1 - \left(\dfrac{A_2}{A_1}\right)^2\right]}$

Bridging Problem

(a) $y = H - \left(\dfrac{d}{D}\right)^2 \sqrt{2gH}\, t + \left(\dfrac{d}{D}\right)^4 \dfrac{gt^2}{2}$

(b) $T = \sqrt{\dfrac{2H}{g}}\left(\dfrac{D}{d}\right)^2$

(c) $\sqrt{2}$