? The needle of a magnetic compass points north. This alignment is due to (i) a magnetic force on the needle; (ii) a magnetic torque on the needle; (iii) the magnetic field that the needle itself produces; (iv) both (i) and (ii); (v) all of (i), (ii), and (iii).

27

Magnetic Field and Magnetic Forces

LEARNING OUTCOMES

In this chapter, you'll learn...

27.1 The properties of magnets, and how magnets interact with each other.
27.2 The nature of the force that a moving charged particle experiences in a magnetic field.
27.3 How magnetic field lines are different from electric field lines.
27.4 How to analyze the motion of a charged particle in a magnetic field.
27.5 Some practical applications of magnetic fields in chemistry and physics.
27.6 How to analyze magnetic forces on current-carrying conductors.
27.7 How current loops behave when placed in a magnetic field.
27.8 How direct-current motors work.
27.9 How magnetic forces give rise to the Hall effect.

You'll need to review...

1.10 Vector product of two vectors.
3.4, 5.4 Uniform circular motion.
10.1 Torque.
21.6, 21.7 Electric field lines and electric dipole moment.
22.2, 22.3 Electric flux and Gauss's law.
25.1 Electric current.
26.3 Galvanometers.

Everybody uses magnetic forces. They are at the heart of electric motors, microwave ovens, loudspeakers, computer printers, and disk drives. The most familiar examples of magnetism are permanent magnets, which attract unmagnetized iron objects and can also attract or repel other magnets. A compass needle aligning itself with the earth's magnetism is an example of this interaction. But the *fundamental* nature of magnetism is the interaction of moving electric charges. Unlike electric forces, which act on electric charges whether they are moving or not, magnetic forces act only on *moving* charges.

We saw in Chapter 21 that the electric force arises in two stages: (1) a charge produces an electric field in the space around it, and (2) a second charge responds to this field. Magnetic forces also arise in two stages. First, a *moving* charge or a collection of moving charges (that is, an electric current) produces a *magnetic* field. Next, a second current or moving charge responds to this magnetic field, and so experiences a magnetic force.

In this chapter we study the second stage in the magnetic interaction—that is, how moving charges and currents *respond* to magnetic fields. In particular, we'll see how to calculate magnetic forces and torques, and we'll discover why magnets can pick up iron objects like paper clips. In Chapter 28 we'll complete our picture of the magnetic interaction by examining how moving charges and currents *produce* magnetic fields.

27.1 MAGNETISM

Magnetic phenomena were first observed at least 2500 years ago in fragments of magnetized iron ore found near the ancient city of Magnesia (now Manisa, in western Turkey). These fragments were what are now called **permanent magnets;** you probably have several permanent magnets on your refrigerator door at home. Permanent magnets were found to exert forces on each other as well as on pieces of iron that were not magnetized. It was discovered that when an iron rod is brought in contact with a natural magnet, the rod also becomes magnetized. When such a rod is floated on water or suspended by a string from its center, it tends to line itself up in a north-south direction. The needle of an ordinary compass is just such a piece of magnetized iron.

Before the relationship of magnetic interactions to moving charges was understood, the interactions of permanent magnets and compass needles were described in terms of *magnetic poles*. If a bar-shaped permanent magnet, or *bar magnet,* is free to rotate, one end points north. This end is called a *north pole* or *N pole;* the other end is a *south pole* or *S pole*. Opposite poles attract each other, and like poles repel each other (**Fig. 27.1**). An object that contains iron but is not itself magnetized (that is, it shows no tendency to point north or south) is attracted by *either* pole of a permanent magnet (**Fig. 27.2**). This is the attraction that acts between a magnet and the unmagnetized steel door of a refrigerator. By analogy to electric interactions, we describe the interactions in Figs. 27.1 and 27.2 by saying that a bar magnet sets up a *magnetic field* in the space around it and a second object responds to that field. A compass needle tends to align with the magnetic field at the needle's position.

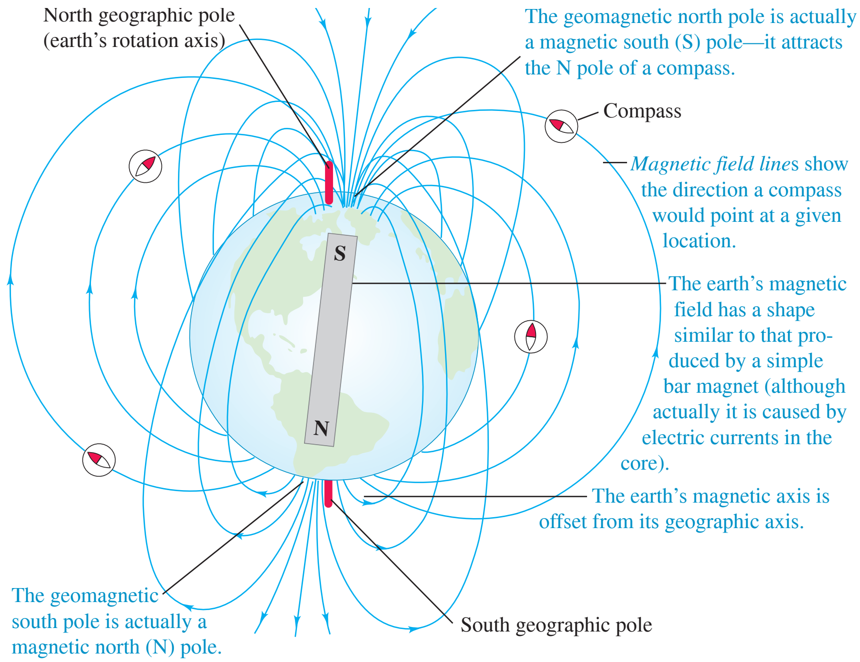
The earth itself is a magnet. Its north geographic pole is close to a magnetic *south* pole, which is why the north pole of a compass needle points north. The earth's magnetic axis is not quite parallel to its geographic axis (the axis of rotation), so a compass reading deviates somewhat from geographic north. This deviation, which varies with location, is called *magnetic declination* or *magnetic variation*. Also, the magnetic field is not horizontal at most points on the earth's surface; its angle up or down is called *magnetic inclination*. At the magnetic poles the magnetic field is vertical.

Figure 27.3 is a sketch of the earth's magnetic field. The lines, called *magnetic field lines,* show the direction that a compass would point at each location; they are discussed in detail in Section 27.3. The direction of the field at any point can be defined as the direction of the force that the field would exert on a magnetic north pole. In Section 27.2 we'll describe a more fundamental way to define the direction and magnitude of a magnetic field.

Magnetic Poles Versus Electric Charge

The concept of magnetic poles may appear similar to that of electric charge, and north and south poles may seem analogous to positive and negative charges. But the analogy can be misleading. While isolated positive and negative charges exist, there is *no* experimental evidence that one isolated magnetic pole exists; poles always appear in pairs. If a bar

Figure **27.3** A sketch of the earth's magnetic field. The field, which is caused by currents in the earth's molten core, changes with time; geologic evidence shows that it reverses direction entirely at irregular intervals of 10^4 to 10^6 years.



The geomagnetic north pole is actually a magnetic south (S) pole—it attracts the N pole of a compass.

Magnetic field lines show the direction a compass would point at a given location.

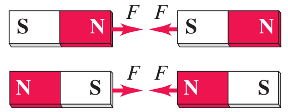
The earth's magnetic field has a shape similar to that produced by a simple bar magnet (although actually it is caused by electric currents in the core).

The earth's magnetic axis is offset from its geographic axis.

The geomagnetic south pole is actually a magnetic north (N) pole.

Figure **27.1** (a) Two bar magnets attract when opposite poles (N and S, or S and N) are next to each other. (b) The bar magnets repel when like poles (N and N, or S and S) are next to each other.

(a) Opposite poles attract.



(b) Like poles repel.

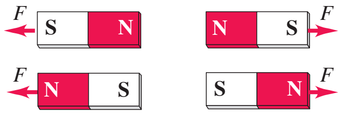
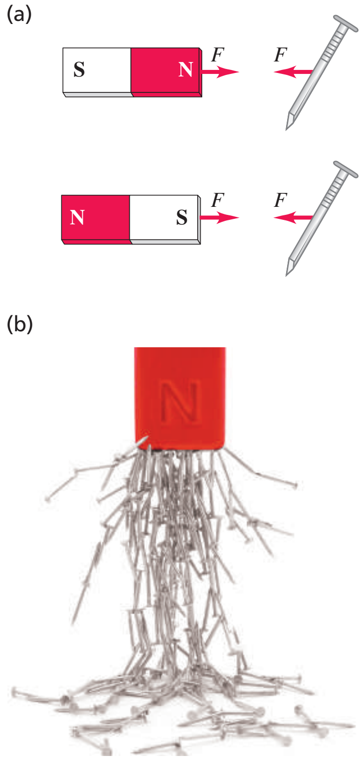


Figure **27.2** (a) Either pole of a bar magnet attracts an unmagnetized object that contains iron, such as a nail. (b) A real-life example of this effect.



magnet is broken in two, each broken end becomes a pole (**Fig. 27.4**). The existence of an isolated magnetic pole, or **magnetic monopole,** would have sweeping implications for theoretical physics. Extensive searches for magnetic monopoles have been carried out, but so far without success.

The first evidence of the relationship of magnetism to moving charges was discovered in 1820 by the Danish scientist Hans Christian Oersted. He found that a compass needle was deflected by a current-carrying wire (**Fig. 27.5**). Similar investigations were carried out in France by André Ampère. A few years later, Michael Faraday in England and Joseph Henry in the United States discovered that moving a magnet near a conducting loop can cause a current in the loop. We now know that the magnetic forces between two objects shown in Figs. 27.1 and 27.2 are fundamentally due to interactions between moving electrons in the atoms of the objects. (There are also *electric* interactions between the two objects, but these are far weaker than the magnetic interactions because the objects are electrically neutral.) Inside a magnetized object such as a permanent magnet, the motion of certain of the atomic electrons is *coordinated;* in an unmagnetized object these motions are not coordinated. (We'll describe these motions further in Section 27.7 and see how the interactions shown in Figs. 27.1 and 27.2 come about.)

Electric and magnetic interactions prove to be intimately connected. Over the next several chapters we'll develop the unifying principles of electromagnetism, culminating in the expression of these principles in *Maxwell's equations*. These equations represent the synthesis of electromagnetism, just as Newton's laws of motion are the synthesis of mechanics, and like Newton's laws they represent a towering achievement of the human intellect.

Figure 27.4 Breaking a bar magnet. Each piece has a north and south pole, even if the pieces are different sizes. (The smaller the piece, the weaker its magnetism.)

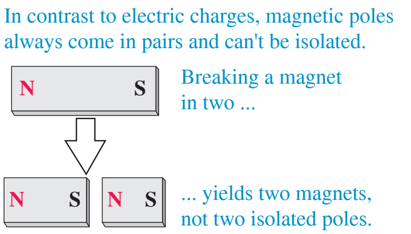
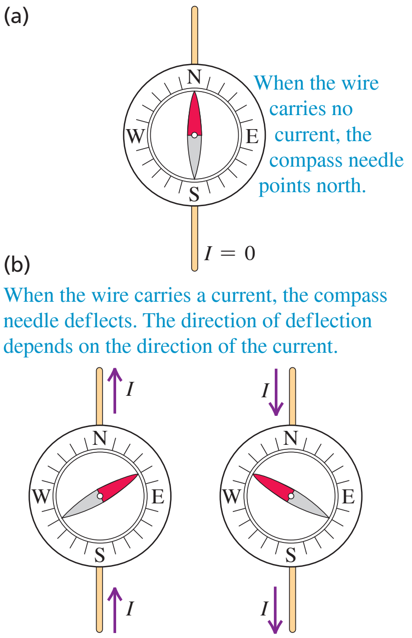


Figure 27.5 In Oersted's experiment, a compass is placed directly over a horizontal wire (here viewed from above).



TEST YOUR UNDERSTANDING OF SECTION 27.1 Suppose you cut off the part of the compass needle shown in Fig. 27.5a that is painted gray. You discard this part, drill a hole in the remaining red part, and place the red part on the pivot at the center of the compass. Will the red part still swing when a current is applied as in Fig. 27.5b?

ANSWER

yes When a magnet is cut apart, each part has a north and south pole (see Fig. 27.4). Hence the small red part behaves much like the original, full-sized compass needle.

BIO APPLICATION Spiny Lobsters and Magnetic Compasses Although the Caribbean spiny lobster (*Panulirus argus*) has a relatively simple nervous system, it is remarkably sensitive to magnetic fields. It has an internal magnetic "compass" that allows it to distinguish north, east, south, and west. This lobster can also sense small differences in the earth's magnetic field from one location to another and may use these differences to help it navigate.

27.2 MAGNETIC FIELD

To introduce the concept of magnetic field properly, let's review our formulation of *electric* interactions in Chapter 21, where we introduced the concept of *electric* field. We represented electric interactions in two steps:

1. A distribution of electric charge creates an electric field $\vec{E}$ in the surrounding space.
2. The electric field exerts a force $\vec{F} = q\vec{E}$ on any other charge q that is present in the field.

We can describe magnetic interactions in a similar way:

1. A moving charge or a current creates a **magnetic field** in the surrounding space (in addition to its *electric* field).
2. The magnetic field exerts a force $\vec{F}$ on any other moving charge or current that is present in the field.

In this chapter we'll concentrate on the *second* aspect of the interaction: Given the presence of a magnetic field, what force does it exert on a moving charge or a current? In Chapter 28 we'll come back to the problem of how magnetic fields are *created* by moving charges and currents.

Like electric field, magnetic field is a *vector field*—that is, a vector quantity associated with each point in space. We'll use the symbol $\vec{B}$ for magnetic field. At any position the direction of $\vec{B}$ is defined as the direction in which the north pole of a compass needle tends to point. The arrows in Fig. 27.3 suggest the direction of the earth's magnetic field; for any magnet, $\vec{B}$ points out of its north pole and into its south pole.

Magnetic Forces on Moving Charges

There are four key characteristics of the magnetic force on a moving charge. First, its magnitude is proportional to the magnitude of the charge. If a 1 μC charge and a 2 μC charge move through a given magnetic field with the same velocity, experiments show that the force on the 2 μC charge is twice as great as the force on the 1 μC charge. Second, the magnitude of the force is also proportional to the magnitude, or "strength," of the field; if we double the magnitude of the field (for example, by using two identical bar magnets instead of one) without changing the charge or its velocity, the force doubles.

A third characteristic is that the magnetic force depends on the particle's velocity. This is quite different from the electric-field force, which is the same whether the charge is moving or not. A charged particle at rest experiences *no* magnetic force. And fourth, we find by experiment that the magnetic force $\vec{F}$ *does not* have the same direction as the magnetic field $\vec{B}$ but instead is always *perpendicular* to both $\vec{B}$ and the velocity $\vec{v}$. The magnitude F of the force is proportional to the component of $\vec{v}$ perpendicular to the field; when that component is zero (that is, when $\vec{v}$ and $\vec{B}$ are parallel or antiparallel), the force is zero.

Figure 27.6 shows these relationships. The direction of $\vec{F}$ is always perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. Its magnitude is given by

$$F = |q|v_{\perp}B = |q|vB\sin\phi \qquad (27.1)$$

where $|q|$ is the magnitude of the charge and ϕ is the angle measured from the direction of $\vec{v}$ to the direction of $\vec{B}$, as shown in the figure.

This description does not specify the direction of $\vec{F}$ completely; there are always two directions, opposite to each other, that are both perpendicular to the plane of $\vec{v}$ and $\vec{B}$. To complete the description, we use the same right-hand rule that we used to define the vector product in Section 1.10. (It would be a good idea to review that section before you go on.) Draw the vectors $\vec{v}$ and $\vec{B}$ with their tails together, as in **Fig. 27.7a**. Imagine turning $\vec{v}$ until it points in the direction of $\vec{B}$ (turning through the smaller of the two possible angles). Wrap the fingers of your right hand around the line perpendicular to the plane of $\vec{v}$ and $\vec{B}$ so that they curl around with the sense of rotation from $\vec{v}$ to $\vec{B}$. Your thumb then points in the direction of the force $\vec{F}$ on a *positive* charge.

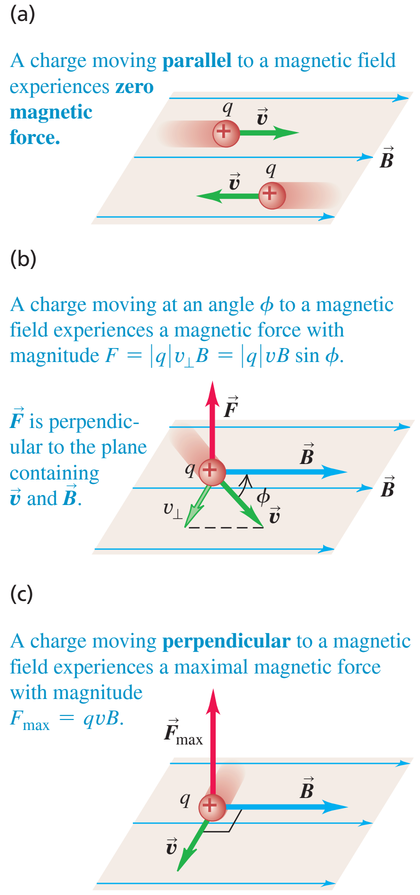
This discussion shows that the force on a charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given, both in magnitude and in direction, by

Magnetic force on a moving charged particle

$$\vec{F} = \overset{\text{Particle's charge}}{q}\underbrace{\vec{v}}_{\text{Particle's velocity}} \times \vec{B} \qquad (27.2)$$

Magnetic field

Figure **27.6** The magnetic force $\vec{F}$ acting on a positive charge q moving with velocity $\vec{v}$ is perpendicular to both $\vec{v}$ and the magnetic field $\vec{B}$. For given values of speed v and magnetic field strength B, the force is greatest when $\vec{v}$ and $\vec{B}$ are perpendicular.



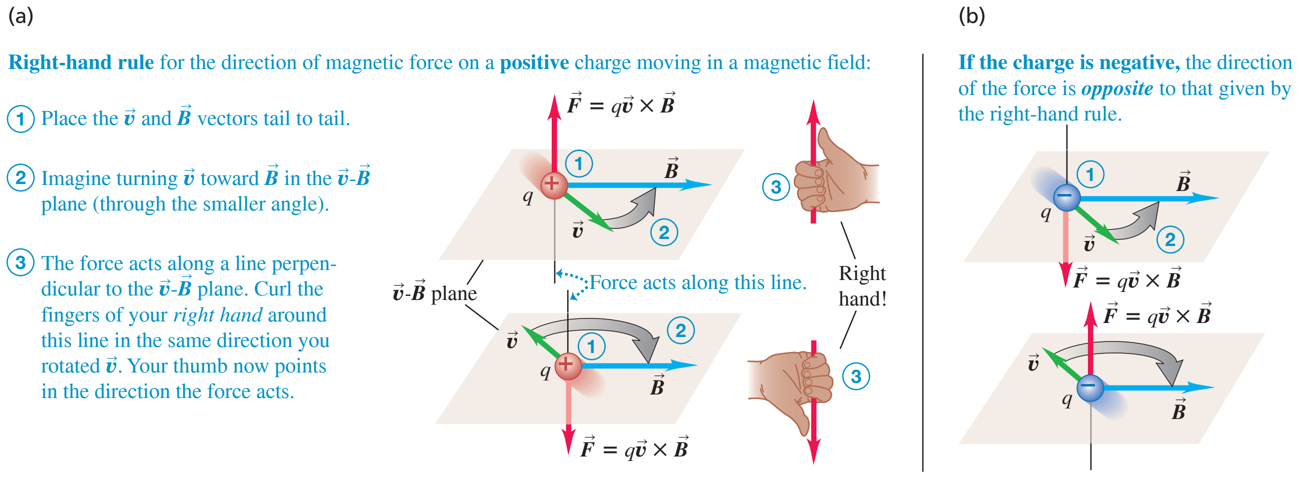
(a) A charge moving **parallel** to a magnetic field experiences **zero magnetic force.**

(b) A charge moving at an angle ϕ to a magnetic field experiences a magnetic force with magnitude $F = |q|v_{\perp}B = |q|vB\sin\phi$.

$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

(c) A charge moving **perpendicular** to a magnetic field experiences a maximal magnetic force with magnitude $F_{\max} = qvB$.

Figure **27.7** Finding the direction of the magnetic force on a moving charged particle.



(a) **Right-hand rule** for the direction of magnetic force on a **positive** charge moving in a magnetic field:

① Place the $\vec{v}$ and $\vec{B}$ vectors tail to tail.

② Imagine turning $\vec{v}$ toward $\vec{B}$ in the $\vec{v}$-$\vec{B}$ plane (through the smaller angle).

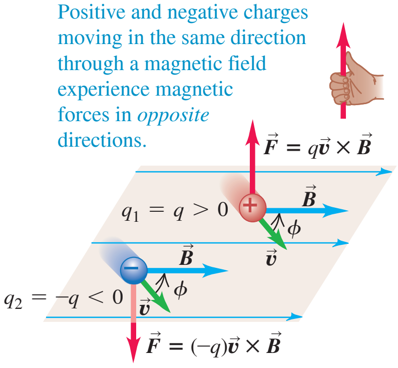
③ The force acts along a line perpendicular to the $\vec{v}$-$\vec{B}$ plane. Curl the fingers of your *right hand* around this line in the same direction you rotated $\vec{v}$. Your thumb now points in the direction the force acts.

Force acts along this line.

Right hand!

(b) **If the charge is negative,** the direction of the force is ***opposite*** to that given by the right-hand rule.

Figure **27.8** Reversing the sign of a moving charge reverses the direction of the magnetic force that acts on it.



Positive and negative charges moving in the same direction through a magnetic field experience magnetic forces in *opposite* directions.

This is the first of several vector products we'll encounter in our study of magnetic-field relationships. It's important to note that Eq. (27.2) was *not* deduced theoretically; it is an observation based on *experiment*.

Equation (27.2) is valid for both positive and negative charges. When q is negative, the direction of the force $\vec{F}$ is opposite to that of $\vec{v} \times \vec{B}$ (Fig. 27.7b). If two charges with equal magnitude and opposite sign move in the same $\vec{B}$ field with the same velocity (**Fig. 27.8**), the forces have equal magnitude and opposite direction. Figures 27.6, 27.7, and 27.8 show several examples of the relationships of the directions of $\vec{F}$, $\vec{v}$, and $\vec{B}$ for both positive and negative charges. Be sure you understand the relationships shown in these figures.

Equation (27.1) gives the magnitude of the magnetic force $\vec{F}$ in Eq. (27.2). Since ϕ is the angle between the directions of vectors $\vec{v}$ and $\vec{B}$, we may interpret $B \sin\phi$ as the component of $\vec{B}$ perpendicular to $\vec{v}$—that is, $B_\perp$. With this notation the force magnitude is

$$F = |q|vB_\perp \qquad (27.3)$$

This form may be more convenient, especially in problems involving *currents* rather than individual particles. We'll discuss forces on currents later in this chapter.

From Eq. (27.1) the *units* of B must be the same as the units of F/qv. Therefore the SI unit of B is equivalent to $1\ \text{N} \cdot \text{s/C} \cdot \text{m}$, or, since one ampere is one coulomb per second $(1\ \text{A} = 1\ \text{C/s})$, $1\ \text{N/A} \cdot \text{m}$. This unit is called the **tesla** (abbreviated T), in honor of Nikola Tesla (1856–1943), the prominent Serbian-American scientist and inventor:

$$1\ \text{tesla} = 1\ \text{T} = 1\ \text{N/A} \cdot \text{m}$$

Another unit of B, the **gauss** $(1\ \text{G} = 10^{-4}\ \text{T})$, is also in common use.

The magnetic field of the earth is of the order of 10^{-4} T or 1 G. Magnetic fields of the order of 10 T occur in the interior of atoms and are important in the analysis of atomic spectra. The largest steady magnetic field that can be produced at present in the laboratory is about 45 T. Some pulsed-current electromagnets can produce fields of the order of 120 T for millisecond time intervals.

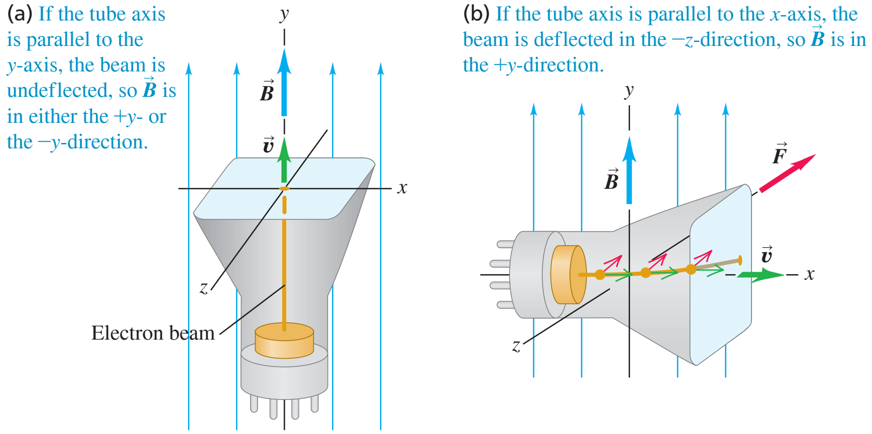
BIO APPLICATION Magnetic Fields of the Body All living cells are electrically active, and the feeble electric currents within your body produce weak but measurable magnetic fields. The fields produced by skeletal muscles have magnitudes less than 10^{-10} T, about one-millionth as strong as the earth's magnetic field. Your brain produces magnetic fields that are far weaker, only about 10^{-12} T.

Measuring Magnetic Fields with Test Charges

To explore an unknown magnetic field, we can measure the magnitude and direction of the force on a *moving* test charge and then use Eq. (27.2) to determine $\vec{B}$. The electron beam in a cathode-ray tube, such as that in an older television set (not a flat-screen TV), is a convenient device for this. The electron gun shoots out a narrow beam of electrons at a known speed. If there is no force to deflect the beam, it strikes the center of the screen.

If a magnetic field is present, in general the electron beam is deflected. But if the beam is parallel or antiparallel to the field, then $\phi = 0$ or π in Eq. (27.1) and $F = 0$; there is no force and hence no deflection. If we find that the electron beam is not deflected when its direction is parallel to a certain axis as in **Fig. 27.9a**, the $\vec{B}$ vector must point either up or down along that axis.

Figure **27.9** Determining the direction of a magnetic field by using a cathode-ray tube. Because electrons have a negative charge, the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ in part (b) points opposite to the direction given by the right-hand rule (see Fig. 27.7b).



(a) If the tube axis is parallel to the y-axis, the beam is undeflected, so $\vec{B}$ is in either the $+y$- or the $-y$-direction.

(b) If the tube axis is parallel to the x-axis, the beam is deflected in the $-z$-direction, so $\vec{B}$ is in the $+y$-direction.

If we then turn the tube 90° (Fig. 27.9b), $\phi = \pi/2$ in Eq. (27.1) and the magnetic force is maximum; the beam is deflected in a direction perpendicular to the plane of $\vec{B}$ and $\vec{v}$. The direction and magnitude of the deflection determine the direction and magnitude of $\vec{B}$. We can perform additional experiments in which the angle between $\vec{B}$ and $\vec{v}$ is between 0° and 90° to confirm Eq. (27.1). We note that the electron has a negative charge; the force in Fig. 27.9b is opposite in direction to the force on a positive charge.

When a charged particle moves through a region of space where *both* electric and magnetic fields are present, both fields exert forces on the particle. The total force $\vec{F}$ is the vector sum of the electric and magnetic forces:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad (27.4)$$

CAUTION **No perpendicular field component, no magnetic force** When a charged particle moves in a magnetic field $\vec{B}$, the particle experiences a magnetic force *only* if there is a component of $\vec{B}$ that is perpendicular to the particle's velocity. If $\vec{B}$ and the velocity are in the same direction (as in Fig. 27.9a) or in opposite directions, there is no perpendicular component of $\vec{B}$ and hence no magnetic force.

PROBLEM-SOLVING STRATEGY 27.1 Magnetic Forces

IDENTIFY *the relevant concepts:* The equation $\vec{F} = q\vec{v} \times \vec{B}$ allows you to determine the magnetic force on a moving charged particle.

SET UP *the problem* using the following steps:

1. Draw the velocity $\vec{v}$ and magnetic field $\vec{B}$ with their tails together so that you can visualize the plane that contains them.
2. Determine the angle ϕ between $\vec{v}$ and $\vec{B}$.
3. Identify the target variables.

EXECUTE *the solution* as follows:

1. Use Eq. (27.2), $\vec{F} = q\vec{v} \times \vec{B}$, to express the magnetic force. Equation (27.1) gives the magnitude of the force, $F = qvB\sin\phi$.
2. Remember that $\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. The right-hand rule (see Fig. 27.7) gives the direction of $\vec{v} \times \vec{B}$. If q is negative, $\vec{F}$ is *opposite* to $\vec{v} \times \vec{B}$.

EVALUATE *your answer:* Whenever possible, solve the problem in two ways to confirm that the results agree. Do it directly from the geometric definition of the vector product. Then find the components of the vectors in some convenient coordinate system and calculate the vector product from the components. Verify that the results agree.

EXAMPLE 27.1 Magnetic force on a proton

WITH VARIATION PROBLEMS

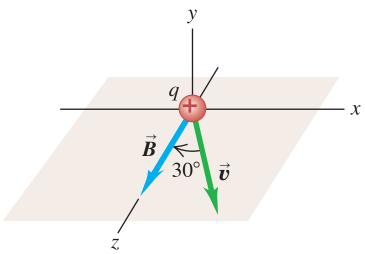
A beam of protons ($q = 1.6 \times 10^{-19}$ C) moves at 3.0×10^5 m/s through a uniform 2.0 T magnetic field directed along the positive z-axis (**Fig. 27.10**). The velocity of each proton lies in the xz-plane and is directed at 30° to the $+z$-axis. Find the force on a proton.

IDENTIFY and SET UP This problem uses the expression $\vec{F} = q\vec{v} \times \vec{B}$ for the magnetic force $\vec{F}$ on a moving charged particle. The target variable is $\vec{F}$.

EXECUTE The charge is positive, so the force is in the same direction as the vector product $\vec{v} \times \vec{B}$. From the right-hand rule, this direction is along the negative y-axis. The magnitude of the force, from Eq. (27.1), is

$$\begin{aligned} F &= qvB\sin\phi \\ &= (1.6 \times 10^{-19}\ \text{C})(3.0 \times 10^5\ \text{m/s})(2.0\ \text{T})(\sin 30°) \\ &= 4.8 \times 10^{-14}\ \text{N} \end{aligned}$$

Figure **27.10** Directions of $\vec{v}$ and $\vec{B}$ for a proton in a magnetic field.



Continued

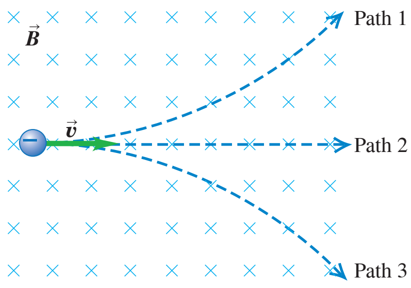
EVALUATE To check our result, we evaluate the force by using vector language and Eq. (27.2). We have

$$\begin{aligned}\vec{v} &= (3.0 \times 10^5\ \text{m/s})(\sin 30°)\hat{\imath} + (3.0 \times 10^5\ \text{m/s})(\cos 30°)\hat{k}\\ \vec{B} &= (2.0\ \text{T})\hat{k}\\ \vec{F} &= q\vec{v} \times \vec{B}\\ &= (1.6 \times 10^{-19}\ \text{C})(3.0 \times 10^5\ \text{m/s})(2.0\ \text{T})\\ &\quad \times (\sin 30°\hat{\imath} + \cos 30°\hat{k}) \times \hat{k}\\ &= (-4.8 \times 10^{-14}\ \text{N})\hat{\jmath}\end{aligned}$$

(Recall that $\hat{\imath} \times \hat{k} = -\hat{\jmath}$ and $\hat{k} \times \hat{k} = \mathbf{0}$.) We again find that the force is in the negative y-direction with magnitude 4.8×10^{-14} N.

If the beam consists of *electrons* rather than protons, the charge is negative ($q = -1.6 \times 10^{-19}$ C) and the direction of the force is reversed. The force is now directed along the *positive* y-axis, but the magnitude is the same as before, $F = 4.8 \times 10^{-14}$ N.

KEYCONCEPT A charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ experiences a magnetic force that depends on its charge, its speed, the magnitude of $\vec{B}$, and the angle between $\vec{v}$ and $\vec{B}$. The force is zero if the directions of $\vec{v}$ and $\vec{B}$ are the same or opposite; otherwise, the force is nonzero and in a direction perpendicular to both $\vec{v}$ and $\vec{B}$.



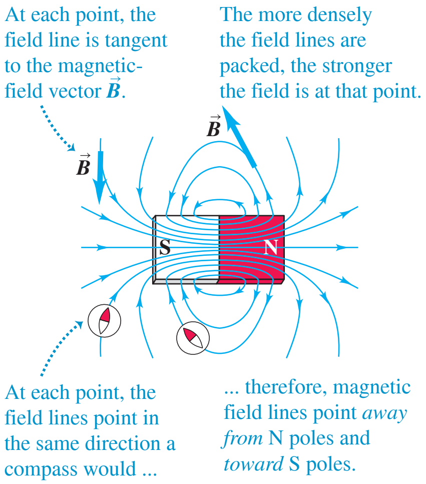
TEST YOUR UNDERSTANDING OF SECTION 27.2 The accompanying figure shows a uniform magnetic field $\vec{B}$ directed into the plane of the paper (shown by the blue ×'s). A particle with a negative charge moves in the plane. Which path—1, 2, or 3—does the particle follow?

ANSWER

path 3 Applying the right-hand rule to the vectors $\vec{v}$ (which points to the right) and $\vec{B}$ (which points into the plane of the figure) says that the force $\vec{F} = q\vec{v} \times \vec{B}$ on a *positive* charge would point *upward*. Since the charge is *negative*, the force points *downward* and the particle follows a trajectory that curves downward.

27.3 MAGNETIC FIELD LINES AND MAGNETIC FLUX

Figure **27.11** Magnetic field lines of a permanent magnet. Note that the field lines pass through the interior of the magnet.



We can represent any magnetic field by **magnetic field lines,** just as we did for the earth's magnetic field in Fig. 27.3. The idea is the same as for the electric field lines we introduced in Section 21.6. We draw the lines so that the line through any point is tangent to the magnetic-field vector $\vec{B}$ at that point (**Fig. 27.11**). Just as with electric field lines, we draw only a few representative lines; otherwise, the lines would fill up all of space. Where adjacent field lines are close together, the field magnitude is large; where these field lines are far apart, the field magnitude is small. Also, because the direction of $\vec{B}$ at each point is unique, field lines never intersect.

CAUTION **Magnetic field lines are not "lines of force"** Unlike electric field lines, magnetic field lines *do not* point in the direction of the force on a charge (**Fig. 27.12**). Equation (27.2) shows that the force on a moving charged particle is always perpendicular to the magnetic field and hence to the magnetic field line that passes through the particle's position. The direction of the force depends on the particle's velocity and the sign of its charge, so just looking at magnetic field lines cannot tell you the direction of the force on an arbitrary moving charged particle. Magnetic field lines *do* have the direction that a compass needle would point at each location; this may help you visualize them.

Figure **27.12** Magnetic field lines are *not* "lines of force."

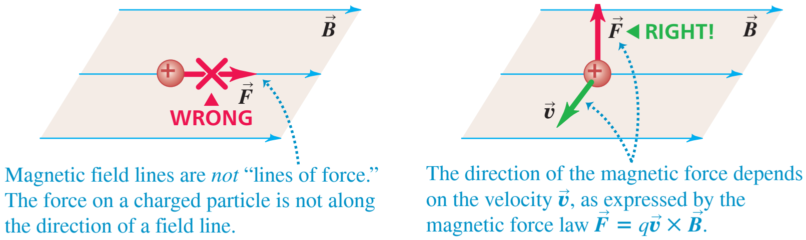
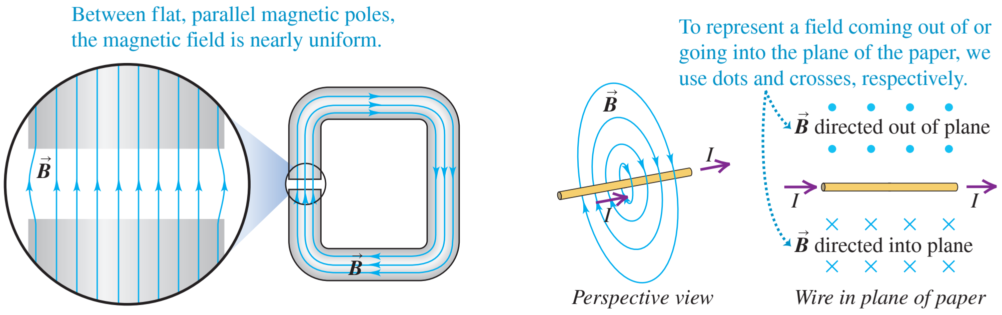
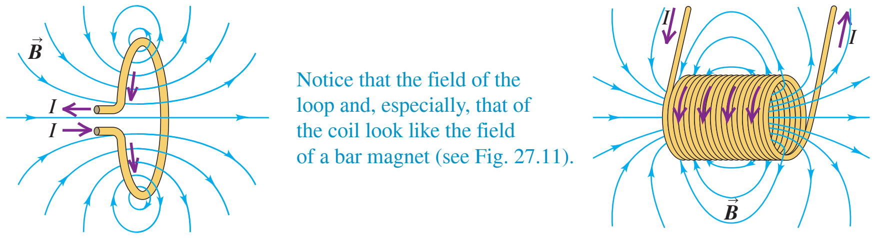


Figure **27.13** Magnetic field lines produced by some common sources of magnetic field.

(a) Magnetic field of a C-shaped magnet (b) Magnetic field of a straight current-carrying wire



(c) Magnetic fields of a current-carrying loop and a current-carrying coil (solenoid)

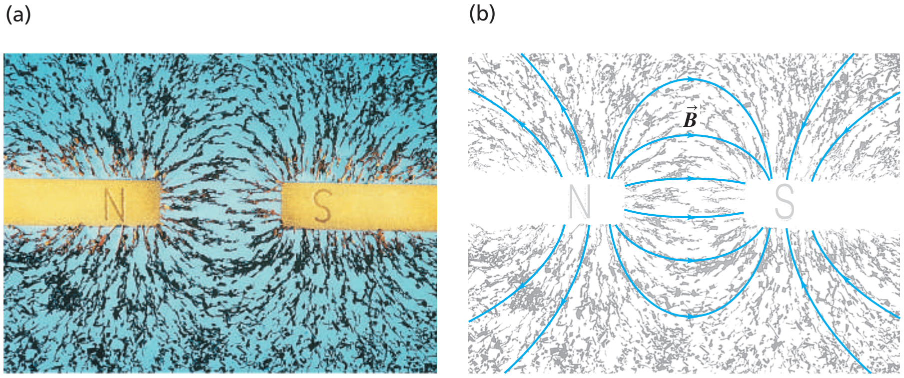


Figures 27.11 and 27.13 show magnetic field lines produced by several common sources of magnetic field. In the gap between the poles of the magnet shown in **Fig. 27.13a**, the field lines are approximately straight, parallel, and equally spaced, showing that the magnetic field in this region is approximately *uniform* (that is, constant in magnitude and direction).

Because magnetic-field patterns are three-dimensional, it's often necessary to draw magnetic field lines that point into or out of the plane of a drawing. To do this we use a dot (•) to represent a vector directed out of the plane and a cross (×) to represent a vector directed into the plane (Fig. 27.13b). To remember these, think of a dot as the head of an arrow coming directly toward you, and think of a cross as the feathers of an arrow flying directly away from you.

Iron filings, like compass needles, tend to align with magnetic field lines. Hence they provide an easy way to visualize field lines (**Fig. 27.14**).

Figure **27.14** (a) Like little compass needles, iron filings line up tangent to magnetic field lines. (b) Drawing of field lines for the situation shown in (a).



Magnetic Flux and Gauss's Law for Magnetism

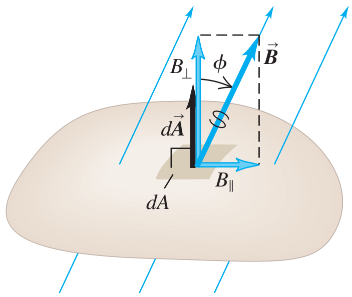


Figure **27.15** The magnetic flux through an area element dA is defined to be $d\Phi_B = B_\perp dA$.

We define the **magnetic flux** Φ_B through a surface just as we defined electric flux in connection with Gauss's law in Section 22.2. We can divide any surface into elements of area dA (**Fig. 27.15**). For each element we determine $B_\perp$, the component of $\vec{B}$ normal to the surface at the position of that element, as shown. From the figure, $B_\perp = B\cos\phi$, where ϕ is the angle between the direction of $\vec{B}$ and a line perpendicular to the surface. (Be careful not to confuse ϕ with Φ_B.) In general, this component varies from point to point on the surface. We define the magnetic flux $d\Phi_B$ through this area as

$$d\Phi_B = B_\perp dA = B\cos\phi\, dA = \vec{B}\cdot d\vec{A} \qquad (27.5)$$

The *total* magnetic flux through the surface is the sum of the contributions from the individual area elements:

$$\underset{\textbf{Magnetic flux through a surface}}{\Phi_B} = \int \overset{\text{Magnitude of magnetic field }\vec{B}}{B}\cos\underset{\text{Angle between }\vec{B}\text{ and normal to surface}}{\phi}\, dA = \int \overset{\text{Component of }\vec{B}\text{ perpendicular to surface}}{B_\perp}\, \underset{\text{Element of surface area}}{dA} = \int \vec{B}\cdot \underset{\text{Vector element of surface area}}{d\vec{A}} \qquad (27.6)$$

(Review the concepts of vector area and surface integral in Section 22.2.)

Magnetic flux is a *scalar* quantity. If $\vec{B}$ is uniform over a plane surface with total area A, then $B_\perp$ and ϕ are the same at all points on the surface, and

$$\Phi_B = B_\perp A = BA\cos\phi \qquad (27.7)$$

If $\vec{B}$ is also perpendicular to the surface (parallel to the area vector), then $\cos\phi = 1$ and Eq. (27.7) reduces to $\Phi_B = BA$. We'll use the concept of magnetic flux extensively during our study of electromagnetic induction in Chapter 29.

The SI unit of magnetic flux is equal to the unit of magnetic field (1 T) times the unit of area $(1\ \text{m}^2)$. This unit is called the **weber** (1 Wb), in honor of the German physicist Wilhelm Weber (1804–1891):

$$1\ \text{Wb} = 1\ \text{T}\cdot\text{m}^2$$

Also, $1\ \text{T} = 1\ \text{N/A}\cdot\text{m}$, so

$$1\ \text{Wb} = 1\ \text{T}\cdot\text{m}^2 = 1\ \text{N}\cdot\text{m/A}$$

In Gauss's law the total *electric* flux through a closed surface is proportional to the total electric charge enclosed by the surface. For example, if the closed surface encloses an electric dipole, the total electric flux is zero because the total charge is zero. (You may want to review Section 22.3 on Gauss's law.) By analogy, if there were such a thing as a single magnetic charge (magnetic monopole), the total *magnetic* flux through a closed surface would be proportional to the total magnetic charge enclosed. But we have mentioned that no magnetic monopole has ever been observed, despite intensive searches. This leads us to **Gauss's law for magnetism:**

Gauss's law for magnetism:

$$\overbrace{\oint}^{\text{The total magnetic flux through } any \textbf{ closed surface} \ldots}\vec{B}\cdot d\vec{A} = \underset{\ldots\text{ equals zero.}}{0} \qquad (27.8)$$

You can verify Gauss's law for magnetism by examining Figs. 27.11 and 27.13; if you draw a closed surface anywhere in any of the field maps shown in those figures, you'll see that every field line that enters the surface also exits from it; the net flux through the surface is zero. It also follows from Eq. (27.8) that magnetic field lines always form closed loops.

CAUTION Magnetic field lines have no ends Unlike electric field lines, which begin and end on electric charges, magnetic field lines *never* have endpoints; such a point would indicate the presence of a monopole. You might be tempted to draw magnetic field lines that begin at the north pole of a magnet and end at a south pole. But as Fig. 27.11 shows, a magnet's field lines continue through the interior of the magnet. Like all other magnetic field lines, they form closed loops.

For Gauss's law, which always deals with *closed* surfaces, the vector area element $d\vec{A}$ in Eq. (27.6) always points *out of* the surface. However, some applications of *magnetic* flux involve an *open* surface with a boundary line; there is then an ambiguity of sign in Eq. (27.6) because of the two possible choices of direction for $d\vec{A}$. In these cases we choose one of the two sides of the surface to be the "positive" side and use that choice consistently.

If the element of area dA in Eq. (27.5) is at right angles to the field lines, then $B_\perp = B$; calling the area $dA_\perp$, we have

$$B = \frac{d\Phi_B}{dA_\perp} \tag{27.9}$$

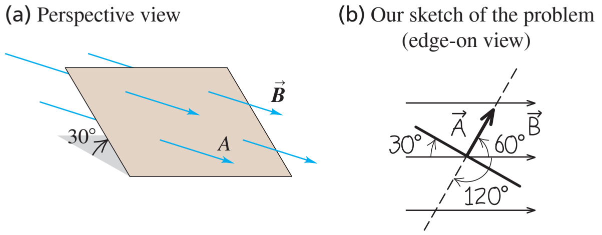
That is, the magnitude of the magnetic field is equal to *flux per unit area* across an area at right angles to the magnetic field. For this reason, magnetic field $\vec{B}$ is sometimes called **magnetic flux density.**

EXAMPLE 27.2 Magnetic flux calculations

Figure 27.16a is a perspective view of a flat surface with area 3.0 cm^2 in a uniform magnetic field $\vec{B}$. The magnetic flux through this surface is +0.90 mWb. Find the magnitude of the magnetic field and the direction of the area vector $\vec{A}$.

IDENTIFY and SET UP Our target variables are the field magnitude B and the direction of the area vector. Because $\vec{B}$ is uniform, B and ϕ are the same at all points on the surface. Hence we can use Eq. (27.7), $\Phi_B = BA\cos\phi$.

Figure 27.16 (a) A flat area A in a uniform magnetic field $\vec{B}$. (b) The area vector $\vec{A}$ makes a 60° angle with $\vec{B}$. (If we had chosen $\vec{A}$ to point in the opposite direction, ϕ would have been 120° and the magnetic flux Φ_B would have been negative.)


(a) Perspective view

(b) Our sketch of the problem (edge-on view)

EXECUTE The area A is 3.0×10^{-4} m^2; the direction of $\vec{A}$ is perpendicular to the surface, so ϕ could be either 60° or 120°. But Φ_B, B, and A are all positive, so $\cos\phi$ must also be positive. This rules out 120°, so $\phi = 60°$ (Fig. 27.16b). Hence we find

$$B = \frac{\Phi_B}{A\cos\phi} = \frac{0.90 \times 10^{-3}\ \text{Wb}}{(3.0 \times 10^{-4}\ \text{m}^2)(\cos 60°)} = 6.0\ \text{T}$$

EVALUATE In many problems we are asked to calculate the flux of a given magnetic field through a given area. This example is somewhat different: It tests your understanding of the definition of magnetic flux.

KEYCONCEPT If a uniform magnetic field $\vec{B}$ is present over a surface, the magnetic flux through that surface equals the area of the surface multiplied by the component of $\vec{B}$ normal (perpendicular) to the surface. The net magnetic flux Φ_B through a closed surface is always zero.

TEST YOUR UNDERSTANDING OF SECTION 27.3 Imagine moving along the axis of the current-carrying loop in Fig. 27.13c, starting at a point well to the left of the loop and ending at a point well to the right of the loop. (a) How would the magnetic field strength vary as you moved along this path? (i) It would be the same at all points along the path; (ii) it would increase and then decrease; (iii) it would decrease and then increase. (b) Would the magnetic field direction vary as you moved along the path?

ANSWER

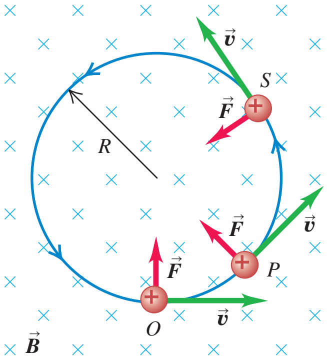
(a) **(ii)**, (b) **no** The magnitude of $\vec{B}$ would increase as you moved to the right, reaching a maximum as you passed through the plane of the loop. As you moved beyond the plane of the loop, the field magnitude would decrease. You can tell this from the spacing of the field lines: The closer the field lines, the stronger the field. The direction of the field would be to the right at all points along the path, since the path is along a field line and the direction of $\vec{B}$ at any point is tangent to the field line through that point.

27.4 MOTION OF CHARGED PARTICLES IN A MAGNETIC FIELD

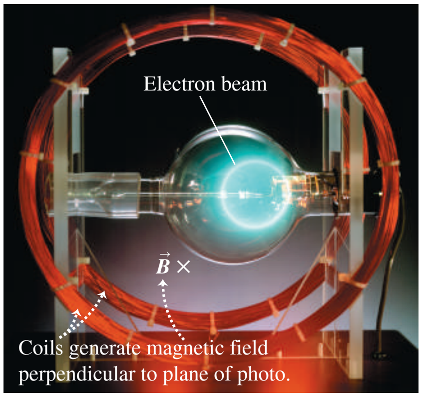
When a charged particle moves in a magnetic field, it is acted on by the magnetic force given by Eq. (27.2), and the motion is determined by Newton's laws. **Figure 27.17a** shows a simple example. A particle with positive charge q is at point O, moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$ directed into the plane of the figure. The vectors $\vec{v}$ and $\vec{B}$ are perpendicular, so the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ has magnitude $F = qvB$ and a direction as shown in the figure. The force is *always* perpendicular to $\vec{v}$, so it cannot change the *magnitude* of the velocity, only its direction. To put it differently, the magnetic force never has a component parallel to the particle's motion, so the magnetic force can never do *work* on the particle. This is true even if the magnetic field is not uniform.

Figure 27.17 A charged particle moves in a plane perpendicular to a uniform magnetic field $\vec{B}$.

(a) The orbit of a charged particle in a uniform magnetic field


A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.

(b) An electron beam (seen as a white arc) curving in a magnetic field


Electron beam

$\vec{B}$ ×

Coils generate magnetic field perpendicular to plane of photo.

Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.

Using this principle, we see that in Fig. 27.17a the magnitudes of both $\vec{F}$ and $\vec{v}$ are constant. As the particle of mass m moves from O to P to S, the directions of force and velocity change but their magnitudes stay the same. So the particle is acted on by a constant-magnitude force that is always at right angles to the velocity of the particle. It follows from our discussion of circular motion in Sections 3.4 and 5.4 that the particle moves in a *circle* of radius R with constant speed v. The centripetal acceleration is v^2/R and only the magnetic force acts, so from Newton's second law,

$$F = |q|vB = m\frac{v^2}{R} \tag{27.10}$$

We solve Eq. (27.10) for R:

Radius of a circular orbit in a magnetic field

$$R = \frac{mv}{|q|B} \tag{27.11}$$

Particle's mass

Particle's speed

Magnetic-field magnitude

Particle's charge

If the charge q is negative, the particle moves *clockwise* around the orbit in Fig. 27.17a. Figure 27.17b shows a beam of negatively charged electrons following just such an orbit.

The angular speed ω of the particle can be found from Eq. (9.13), $v = R\omega$. Combining this with Eq. (27.11), we get

$$\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m} \tag{27.12}$$

The number of revolutions per unit time is $f = \omega/2\pi$. This frequency f is independent of the radius R of the path. It is called the **cyclotron frequency;** in a particle accelerator called a *cyclotron,* particles moving in nearly circular paths are given a boost twice each revolution, increasing their energy and their orbital radii but not their angular speed or frequency. Similarly, one type of *magnetron,* a common source of microwave radiation for microwave ovens and radar systems, emits radiation with a frequency equal to the frequency of circular motion of electrons in a vacuum chamber between the poles of a magnet.

If the direction of the initial velocity is *not* perpendicular to the field, the velocity *component* parallel to the field is constant because there is no force parallel to the field. Then the particle moves in a helix (**Fig. 27.18**). The radius of the helix is given by Eq. (27.11), where v is now the component of velocity perpendicular to the $\vec{B}$ field.

Figure 27.18 The general case of a charged particle moving in a uniform magnetic field $\vec{B}$. The magnetic field does no work on the particle, so its speed and kinetic energy remain constant.

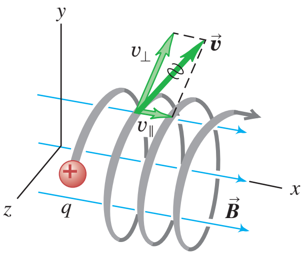

This particle's motion has components both parallel ($v_\parallel$) and perpendicular ($v_\perp$) to the magnetic field, so it moves in a helical path.

Figure 27.19 A magnetic bottle. Particles near either end of the region experience a magnetic force toward the center of the region. This is one way of containing an ionized gas that has a temperature of the order of 10^6 K, which would vaporize any material container.

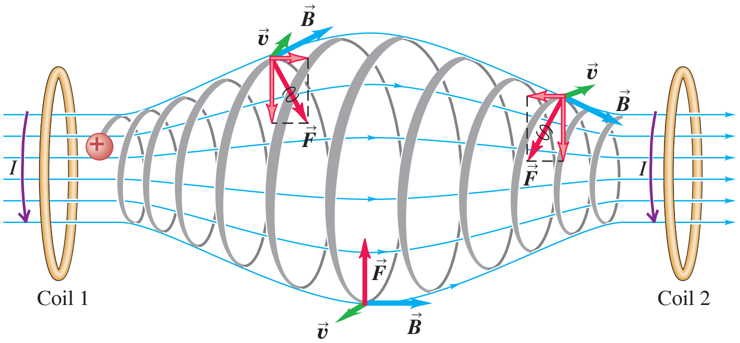
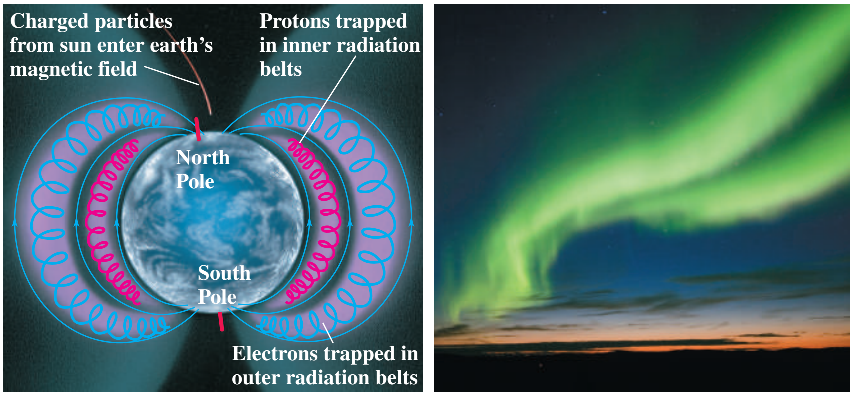


Figure 27.20 (a) The Van Allen radiation belts around the earth. Near the poles, charged particles from these belts can enter the atmosphere, producing the aurora borealis ("northern lights") and aurora australis ("southern lights"). (b) A photograph of the aurora borealis.

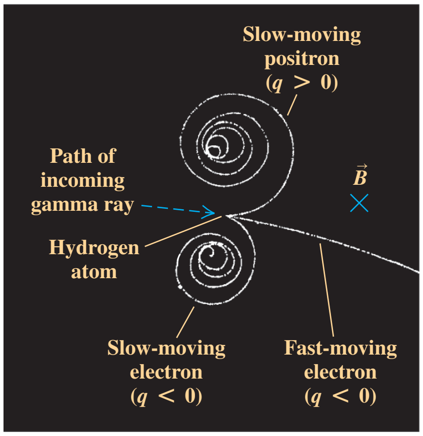
(a) (b)



Motion of a charged particle in a nonuniform magnetic field is more complex. **Figure 27.19** shows a field produced by two circular coils separated by some distance. Particles near either coil experience a magnetic force toward the center of the region; particles with appropriate speeds spiral repeatedly from one end of the region to the other and back. Because charged particles can be trapped in such a magnetic field, it is called a *magnetic bottle*. This technique is used to confine very hot plasmas with temperatures of the order of 10^6 K. In a similar way the earth's nonuniform magnetic field traps charged particles coming from the sun in doughnut-shaped regions around the earth, as shown in **Fig. 27.20**. These regions, called the *Van Allen radiation belts,* were discovered in 1958 from data obtained by instruments aboard the Explorer I satellite.

Magnetic forces on charged particles play an important role in studies of elementary particles. **Figure 27.21** shows a chamber filled with liquid hydrogen and with a magnetic field directed into the plane of the photograph. A high-energy gamma ray dislodges an electron from a hydrogen atom, sending it off at high speed and creating a visible track in the liquid hydrogen. The track shows the electron curving downward due to the magnetic force. The energy of the collision also produces another electron and a *positron* (a positively charged electron). Because of their opposite charges, the trajectories of the electron and the positron curve in opposite directions. As these particles plow through the liquid hydrogen, they collide with other charged particles, losing energy and speed. As a result, the radius of curvature decreases as suggested by Eq. (27.11). (The electron's speed is comparable to the speed of light, so Eq. (27.11) isn't directly applicable here.) Similar experiments allow physicists to determine the mass and charge of newly discovered particles.

Figure 27.21 This bubble chamber image shows the result of a high-energy gamma ray (which does not leave a track) that collides with an electron in a hydrogen atom. This electron flies off to the right at high speed. Some of the energy in the collision is transformed into a second electron and a positron (a positively charged electron). A magnetic field is directed into the plane of the image, which makes the positive and negative particles curve off in different directions.



PROBLEM-SOLVING STRATEGY 27.2 Motion in Magnetic Fields

IDENTIFY *the relevant concepts:* In analyzing the motion of a charged particle in electric and magnetic fields, you'll apply Newton's second law of motion, $\sum\vec{F} = m\vec{a}$, with the net force given by $\sum\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. Often other forces such as gravity can be ignored. Many of the problems are similar to the trajectory and circular-motion problems in Sections 3.3, 3.4, and 5.4; it would be a good idea to review those sections.

SET UP *the problem* using the following steps:

1. Determine the target variable(s).
2. Often the use of components is the most efficient approach. Choose a coordinate system and then express all vector quantities in terms of their components in this system.

EXECUTE *the solution* as follows:

1. If the particle moves perpendicular to a uniform magnetic field, the trajectory is a circle with a radius and angular speed given by Eqs. (27.11) and (27.12), respectively.
2. If your calculation involves a more complex trajectory, use $\sum\vec{F} = m\vec{a}$ in component form: $\sum F_x = ma_x$, and so forth. This approach is particularly useful when both electric and magnetic fields are present.

EVALUATE *your answer:* Check whether your results are reasonable.

EXAMPLE 27.3 Electron motion in a magnetron

WITH VARIATION PROBLEMS

A magnetron in a microwave oven emits electromagnetic waves with frequency $f = 2450$ MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency?

IDENTIFY and SET UP The problem refers to circular motion as shown in Fig. 27.17a. We use Eq. (27.12) to solve for the field magnitude B.

EXECUTE The angular speed that corresponds to the frequency f is $\omega = 2\pi f = (2\pi)(2450 \times 10^6\ \text{s}^{-1}) = 1.54 \times 10^{10}\ \text{s}^{-1}$. Then from Eq. (27.12),

$$B = \frac{m\omega}{|q|} = \frac{(9.11 \times 10^{-31}\ \text{kg})(1.54 \times 10^{10}\ \text{s}^{-1})}{1.60 \times 10^{-19}\ \text{C}} = 0.0877\ \text{T}$$

EVALUATE This is a moderate field strength, easily produced with a permanent magnet. Incidentally, 2450 MHz electromagnetic waves are useful for heating and cooking food because they are strongly absorbed by water molecules.

KEYCONCEPT A charged particle that travels perpendicular to a uniform magnetic field $\vec{B}$ undergoes uniform circular motion. The frequency of the motion is independent of the speed of the particle: It depends on only the magnetic field strength and the particle's charge and mass.

EXAMPLE 27.4 Helical particle motion in a magnetic field

WITH VARIATION PROBLEMS

In a situation like that shown in Fig. 27.18, the charged particle is a proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) and the uniform, 0.500 T magnetic field is directed along the x-axis. At $t = 0$ the proton has velocity components $v_x = 1.50 \times 10^5$ m/s, $v_y = 0$, and $v_z = 2.00 \times 10^5$ m/s. Only the magnetic force acts on the proton. (a) At $t = 0$, find the force on the proton and its acceleration. (b) Find the radius of the resulting helical path, the angular speed of the proton, and the *pitch* of the helix (the distance traveled along the helix axis per revolution).

IDENTIFY and SET UP The magnetic force is $\vec{F} = q\vec{v} \times \vec{B}$; Newton's second law gives the resulting acceleration. Because $\vec{F}$ is perpendicular to $\vec{v}$, the proton's speed does not change. Hence Eq. (27.11) gives the radius of the helical path if we replace v with the velocity component perpendicular to $\vec{B}$. Equation (27.12) gives the angular speed ω, which yields the time T for one revolution (the *period*). Given the velocity component parallel to the magnetic field, we can then determine the pitch.

EXECUTE (a) With $\vec{B} = B\hat{\imath}$ and $\vec{v} = v_x\hat{\imath} + v_z\hat{k}$, Eq. (27.2) yields

$$\begin{aligned}\vec{F} &= q\vec{v} \times \vec{B} = q(v_x\hat{\imath} + v_z\hat{k}) \times B\hat{\imath} = qv_zB\hat{\jmath}\\ &= (1.60 \times 10^{-19}\ \text{C})(2.00 \times 10^5\ \text{m/s})(0.500\ \text{T})\hat{\jmath}\\ &= (1.60 \times 10^{-14}\ \text{N})\hat{\jmath}\end{aligned}$$

(Recall: $\hat{\imath} \times \hat{\imath} = \mathbf{0}$ and $\hat{k} \times \hat{\imath} = \hat{\jmath}$.) The resulting acceleration is

$$\vec{a} = \frac{\vec{F}}{m} = \frac{1.60 \times 10^{-14}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}}\hat{\jmath} = (9.58 \times 10^{12}\ \text{m/s}^2)\hat{\jmath}$$

(b) Since $v_y = 0$, the component of velocity perpendicular to $\vec{B}$ is v_z; then from Eq. (27.11),

$$\begin{aligned}R &= \frac{mv_z}{|q|B} = \frac{(1.67 \times 10^{-27}\ \text{kg})(2.00 \times 10^5\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.500\ \text{T})}\\ &= 4.18 \times 10^{-3}\ \text{m} = 4.18\ \text{mm}\end{aligned}$$

From Eq. (27.12) the angular speed is

$$\omega = \frac{|q|B}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(0.500\ \text{T})}{1.67 \times 10^{-27}\ \text{kg}} = 4.79 \times 10^7\ \text{rad/s}$$

The period is $T = 2\pi/\omega = 2\pi/(4.79 \times 10^7\ \text{s}^{-1}) = 1.31 \times 10^{-7}$ s. The pitch is the distance traveled along the x-axis in this time, or

$$\begin{aligned}v_xT &= (1.50 \times 10^5\ \text{m/s})(1.31 \times 10^{-7}\ \text{s})\\ &= 0.0197\ \text{m} = 19.7\ \text{mm}\end{aligned}$$

EVALUATE Although the magnetic force has a tiny magnitude, it produces an immense acceleration because the proton mass is so small. Note that the pitch of the helix is almost five times greater than the radius R, so this helix is much more "stretched out" than that shown in Fig. 27.18.

KEYCONCEPT The most general path that a charged particle follows in a uniform magnetic field $\vec{B}$ is a helix: straight-line, constant-speed motion parallel to $\vec{B}$ combined with uniform circular motion perpendicular to $\vec{B}$.

TEST YOUR UNDERSTANDING OF SECTION 27.4 (a) If you double the speed of the charged particle in Fig. 27.17a while keeping the magnetic field the same (as well as the charge and the mass), how does this affect the radius of the trajectory? (i) The radius is unchanged; (ii) the radius is twice as large; (iii) the radius is four times as large; (iv) the radius is $\frac{1}{2}$ as large; (v) the radius is $\frac{1}{4}$ as large. (b) How does this affect the time required for one complete circular orbit? (i) The time is unchanged; (ii) the time is twice as long; (iii) the time is four times as long; (iv) the time is $\frac{1}{2}$ as long; (v) the time is $\frac{1}{4}$ as long.

ANSWER

(a) **(ii)**, (b) **(i)** The radius of the orbit as given by Eq. (27.11) is directly proportional to the speed, so doubling the particle speed causes the radius to double as well. The particle has twice as far to travel to complete one orbit but is traveling at double the speed, so the time for one orbit is unchanged. This result also follows from Eq. (27.12), which states that the angular speed ω is independent of the linear speed v. Hence the time per orbit, $T = 2\pi/\omega$, likewise does not depend on v.

27.5 APPLICATIONS OF MOTION OF CHARGED PARTICLES

This section describes several applications of the principles introduced in this chapter. Study them carefully, watching for applications of Problem-Solving Strategy 27.2 (Section 27.4).

Velocity Selector

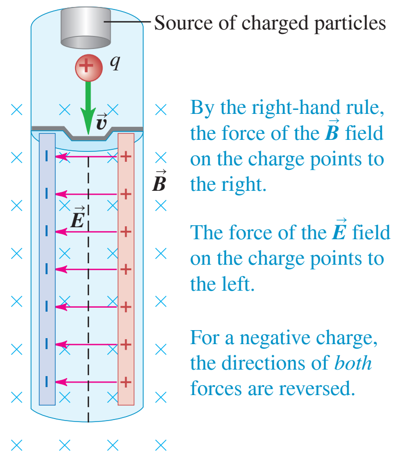
In a beam of charged particles produced by a heated cathode or a radioactive material, not all particles move with the same speed. Many applications, however, require a beam in which all the particle speeds are the same. Particles of a specific speed can be selected from the beam by using an arrangement of electric and magnetic fields called a *velocity selector.* In **Fig. 27.22a** a charged particle with mass m, charge q, and speed v enters a region of space where the electric and magnetic fields are perpendicular to the particle's velocity and to each other. The electric field $\vec{E}$ is to the left, and the magnetic field $\vec{B}$ is into the plane of the figure. If q is positive, the electric force is to the left, with magnitude qE, and the magnetic force is to the right, with magnitude qvB. For given field magnitudes E and B, for a particular value of v the electric and magnetic forces will be equal in magnitude; the total force is then zero, and the particle travels in a straight line with constant velocity. This will be the case if $qE = qvB$ (Fig. 27.22b), so the speed v for which there is no deflection is

$$v = \frac{E}{B} \qquad (27.13)$$

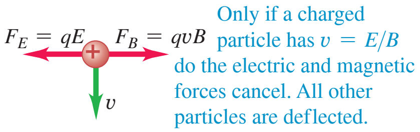
Only particles with speeds equal to E/B can pass through without being deflected by the fields. By adjusting E and B appropriately, we can select particles having a particular speed for use in other experiments. Because q divides out in Eq. (27.13), a velocity selector for positively charged particles also works for electrons or other negatively charged particles.

Figure **27.22** (a) A velocity selector for charged particles uses perpendicular $\vec{E}$ and $\vec{B}$ fields. Only charged particles with $v = E/B$ move through undeflected. (b) The electric and magnetic forces on a positive charge. Both forces are reversed if the charge is negative.

(a) Schematic diagram of velocity selector



(b) Free-body diagram for a positive particle



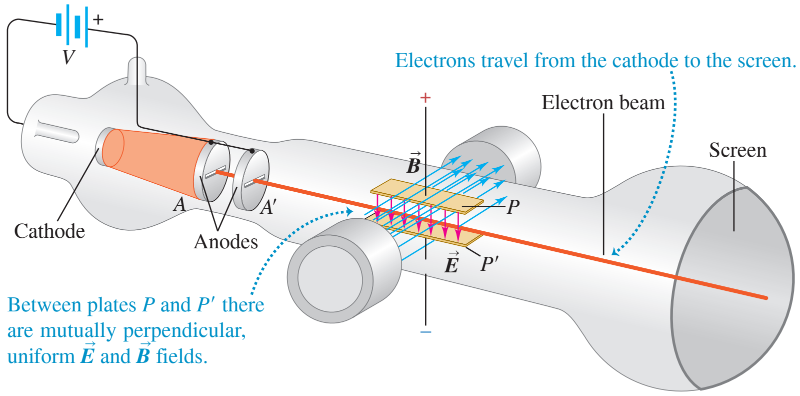
Thomson's *e*/*m* Experiment

In one of the landmark experiments in physics at the end of the 19th century, J. J. Thomson (1856–1940) used the idea just described to measure the ratio of charge to mass for the electron. For this experiment, carried out in 1897 at the Cavendish Laboratory in Cambridge, England, Thomson used the apparatus shown in **Fig. 27.23** (next page). In a highly evacuated glass container, electrons from the hot cathode are accelerated and formed into a beam by a potential difference V between the two anodes A and A'. The speed v of the electrons is determined by the accelerating potential V. The gained kinetic energy $\frac{1}{2}mv^2$ equals the lost electric potential energy eV, where e is the magnitude of the electron charge:

$$\frac{1}{2}mv^2 = eV \qquad \text{or} \qquad v = \sqrt{\frac{2eV}{m}} \qquad (27.14)$$

The electrons pass between the plates P and P' and strike the screen at the end of the tube, which is coated with a material that fluoresces (glows) at the point of impact. The

Figure **27.23** Thomson's apparatus for measuring the ratio e/m for the electron.



Electrons travel from the cathode to the screen.

Between plates P and P' there are mutually perpendicular, uniform $\vec{E}$ and $\vec{B}$ fields.

electrons pass straight through the plates when Eq. (27.13) is satisfied; combining this with Eq. (27.14), we get

$$\frac{E}{B} = \sqrt{\frac{2eV}{m}} \qquad \text{so} \qquad \frac{e}{m} = \frac{E^2}{2VB^2} \tag{27.15}$$

All the quantities on the right side can be measured, so the ratio e/m of charge to mass can be determined. It is *not* possible to measure e or m separately by this method, only their ratio.

The most significant aspect of Thomson's e/m measurements was that he found a *single value* for this quantity. It did not depend on the cathode material, the residual gas in the tube, or anything else about the experiment. This independence showed that the particles in the beam, which we now call electrons, are a common constituent of all matter. Thus Thomson is credited with the first discovery of a subatomic particle, the electron.

The most precise value of e/m available as of this writing is

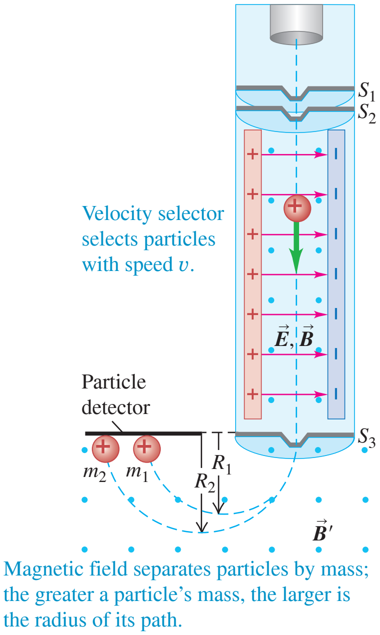
$$e/m = 1.758820024(11) \times 10^{11}\ \text{C/kg}$$

In this expression, (11) indicates the likely uncertainty in the last two digits, 24.

Fifteen years after Thomson's experiments, the American physicist Robert Millikan succeeded in measuring the charge of the electron precisely (see Challenge Problem 23.81). This value, together with the value of e/m, enables us to determine the *mass* of the electron. The most precise value available at present is

$$m = 9.10938356(11) \times 10^{-31}\ \text{kg}$$

Figure **27.24** Bainbridge's mass spectrometer utilizes a velocity selector to produce particles with uniform speed v. In the region of magnetic field B', particles with greater mass $(m_2 > m_1)$ travel in paths with larger radius $(R_2 > R_1)$.



Velocity selector selects particles with speed v.

Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

Mass Spectrometers

Techniques similar to Thomson's e/m experiment can be used to measure masses of ions and thus measure atomic and molecular masses. In 1919, Francis Aston (1877–1945), a student of Thomson's, built the first of a family of instruments called **mass spectrometers.** A variation built by Bainbridge is shown in **Fig. 27.24**. Positive ions from a source pass through the slits S_1 and S_2, forming a narrow beam. Then the ions pass through a velocity selector with crossed $\vec{E}$ and $\vec{B}$ fields, as we have described, to block all ions except those with speeds v equal to E/B. Finally, the ions pass into a region with a magnetic field $\vec{B}'$ perpendicular to the figure, where they move in circular arcs with radius R determined by Eq. (27.11): $R = mv/qB'$. Ions with different masses strike the detector at different points, and the values of R can be measured. We assume that each ion has lost one electron, so the net charge of each ion is just $+e$. With everything known in this equation except m, we can compute the mass m of the ion.

One of the earliest results from this work was the discovery that neon has two species of atoms, with atomic masses 20 and 22 g/mol. We now call these species **isotopes** of the element. Later experiments have shown that many elements have several isotopes—atoms with identical chemical behaviors but different masses due to differing numbers of neutrons in their nuclei. This is just one of the many applications of mass spectrometers in chemistry and physics.

EXAMPLE 27.5 An e/m demonstration experiment

WITH **VARIATION** PROBLEMS

You set out to reproduce Thomson's e/m experiment with an accelerating potential of 150 V and a deflecting electric field of magnitude 6.0×10^6 N/C. (a) How fast do the electrons move? (b) What magnetic-field magnitude will yield zero beam deflection? (c) With this magnetic field, how will the electron beam behave if you increase the accelerating potential above 150 V?

IDENTIFY and SET UP This is the situation shown in Fig. 27.23. We use Eq. (27.14) to determine the electron speed and Eq. (27.13) to determine the required magnetic field B.

EXECUTE (a) From Eq. (27.14), the electron speed v is

$$\begin{aligned} v &= \sqrt{2(e/m)V} = \sqrt{2(1.76 \times 10^{11}\ \text{C/kg})(150\ \text{V})} \\ &= 7.27 \times 10^6\ \text{m/s} = 0.024c \end{aligned}$$

(b) From Eq. (27.13), the required field strength is

$$B = \frac{E}{v} = \frac{6.0 \times 10^6\ \text{N/C}}{7.27 \times 10^6\ \text{m/s}} = 0.83\ \text{T}$$

(c) Increasing the accelerating potential V increases the electron speed v. In Fig. 27.23 this doesn't change the upward electric force eE, but it increases the downward magnetic force evB. Therefore the electron beam will turn *downward* and will hit the end of the tube below the undeflected position.

EVALUATE The required magnetic field is relatively large because the electrons are moving fairly rapidly (2.4% of the speed of light). If the maximum available magnetic field is less than 0.83 T, the electric field strength E would have to be reduced to maintain the desired ratio E/B in Eq. (27.15).

KEYCONCEPT A charged particle can experience both electric and magnetic forces if both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ are present. These forces can cancel if $\vec{E}$, $\vec{B}$, and the particle's velocity are all mutually perpendicular and the particle travels at one particular speed [Eq. (27.14)].

EXAMPLE 27.6 Finding leaks in a vacuum system

WITH **VARIATION** PROBLEMS

There is almost no helium in ordinary air, so helium sprayed near a leak in a vacuum system will quickly show up in the output of a vacuum pump connected to such a system. You are designing a leak detector that uses a mass spectrometer to detect He^+ ions (charge $+e = +1.60 \times 10^{-19}$ C, mass 6.65×10^{-27} kg). Ions emerge from the velocity selector with a speed of 1.00×10^5 m/s. They are curved in a semicircular path by a magnetic field B' and are detected at a distance of 10.16 cm from the slit S_3 in Fig. 27.24. Calculate the magnitude of the magnetic field B'.

IDENTIFY and SET UP After it passes through the slit, the ion follows a circular path as described in Section 27.4 (see Fig. 27.17). We solve Eq. (27.11) for B'.

EXECUTE The distance given is the *diameter* of the semicircular path shown in Fig. 27.24, so the radius is $R = \frac{1}{2}(10.16 \times 10^{-2}\ \text{m})$. From Eq. (27.11), $R = mv/qB'$, we get

$$B' = \frac{mv}{qR} = \frac{(6.65 \times 10^{-27}\ \text{kg})(1.00 \times 10^5\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(5.08 \times 10^{-2}\ \text{m})} = 0.0818\ \text{T}$$

EVALUATE Helium leak detectors are widely used with high-vacuum systems. Our result shows that only a small magnetic field is required, so leak detectors can be relatively compact.

KEYCONCEPT The radius of the circular path that a charged particle follows in a uniform magnetic field depends on the particle's charge, mass, and speed. This dependence can be used to separate ions according to their mass.

TEST YOUR UNDERSTANDING OF SECTION 27.5 In Example 27.6 He^+ ions with charge $+e$ move at 1.00×10^5 m/s in a straight line through a velocity selector. Suppose the He^+ ions were replaced with He^{2+} ions, in which both electrons have been removed from the helium atom and the ion charge is $+2e$. At what speed must the He^{2+} ions travel through the same velocity selector in order to move in a straight line? (i) 4.00×10^5 m/s; (ii) 2.00×10^5 m/s; (iii) 1.00×10^5 m/s; (iv) 0.50×10^5 m/s; (v) 0.25×10^5 m/s.

ANSWER

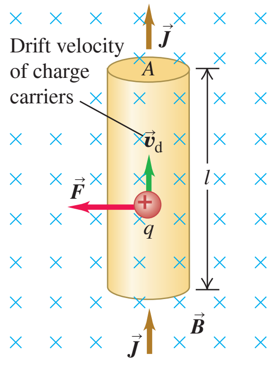
(iii) From Eq. (27.13), the speed $v = E/B$ at which particles travel straight through the velocity selector does not depend on the magnitude or sign of the charge or the mass of the particle. All that is required is that the particles (in this case, ions) have a nonzero charge.

27.6 MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR

What makes an electric motor work? Within the motor are conductors that carry currents (that is, whose charges are in motion), as well as magnets that exert forces on the moving charges. Hence there is a magnetic force on each current-carrying conductor, and these forces make the motor turn. The d'Arsonval galvanometer (Section 26.3) also uses magnetic forces on conductors.

We can compute the force on a current-carrying conductor starting with the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ on a single moving charge. **Figure 27.25** shows a straight segment of a conducting wire, with length l and cross-sectional area A; the current is from bottom to

Figure **27.25** Forces on a moving positive charge in a current-carrying conductor.



top. The wire is in a uniform magnetic field $\vec{B}$, perpendicular to the plane of the diagram and directed *into* the plane. Let's assume first that the moving charges are positive. Later we'll see what happens when they are negative.

The drift velocity $\vec{v}_d$ is upward, perpendicular to $\vec{B}$. The average force on each charge is $\vec{F} = q\vec{v}_d \times \vec{B}$, directed to the left as shown in the figure; since $\vec{v}_d$ and $\vec{B}$ are perpendicular, the magnitude of the force is $F = qv_dB$.

We can derive an expression for the *total* force on all the moving charges in a length l of conductor with cross-sectional area A by using the same language we used in Eqs. (25.2) and (25.3) of Section 25.1. The number of charges per unit volume, or charge concentration, is n; a segment of conductor with length l has volume Al and contains a number of charges equal to nAl. The total force $\vec{F}$ on *all* the moving charges in this segment has magnitude

$$F = (nAl)(qv_dB) = (nqv_dA)(lB) \qquad (27.16)$$

From Eq. (25.3) the current density is $J = nqv_d$. The product JA is the total current I, so we can rewrite Eq. (27.16) as

$$F = IlB \qquad (27.17)$$

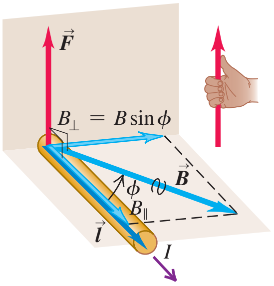
If the $\vec{B}$ field is not perpendicular to the wire but makes an angle ϕ with it, as in **Fig. 27.26**, we handle the situation the same way we did in Section 27.2 for a single charge. Only the component of $\vec{B}$ perpendicular to the wire (and to the drift velocities of the charges) exerts a force; this component is $B_\perp = B\sin\phi$. The magnetic force on the wire segment is then

$$F = IlB_\perp = IlB\sin\phi \qquad (27.18)$$

Figure 27.26 A straight wire segment of length $\vec{l}$ carries a current I in the direction of $\vec{l}$. The magnetic force on this segment is perpendicular to both $\vec{l}$ and the magnetic field $\vec{B}$.

Force $\vec{F}$ on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field $\vec{B}$:

- Magnitude is $F = IlB_\perp = IlB\sin\phi$.
- Direction of $\vec{F}$ is given by the right-hand rule.



The force is always perpendicular to both the conductor and the field, with the direction determined by the same right-hand rule we used for a moving positive charge (Fig. 27.26). Hence this force can be expressed as a vector product, like the force on a single moving charge. We represent the segment of wire with a vector $\vec{l}$ along the wire in the direction of the current; then the force $\vec{F}$ on this segment is

Magnetic force on a straight wire segment Current

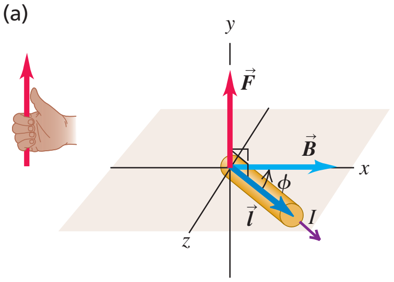
$$\vec{F} = I\vec{l} \times \vec{B} \qquad (27.19)$$

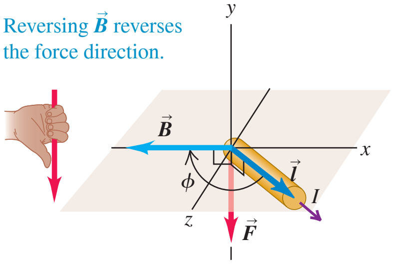
Magnetic field

Vector length of segment (points in current direction)

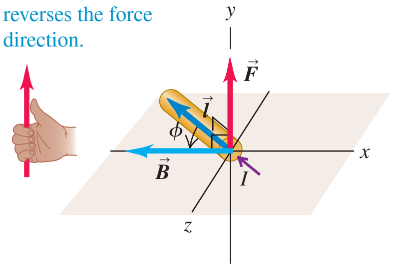
Figure 27.27 illustrates the directions of $\vec{B}$, $\vec{l}$, and $\vec{F}$ for several cases.

Figure 27.27 Magnetic field $\vec{B}$, length $\vec{l}$, and force $\vec{F}$ vectors for a straight wire carrying a current I.




(b)

Reversing $\vec{B}$ reverses the force direction.


(c)

Reversing the current [relative to (b)] reverses the force direction.

If the conductor is not straight, we can divide it into infinitesimal segments $d\vec{l}$. The force $d\vec{F}$ on each segment is

Magnetic force on an infinitesimal wire segment Current

$$d\vec{F} = I\,d\vec{l} \times \vec{B} \qquad (27.20)$$

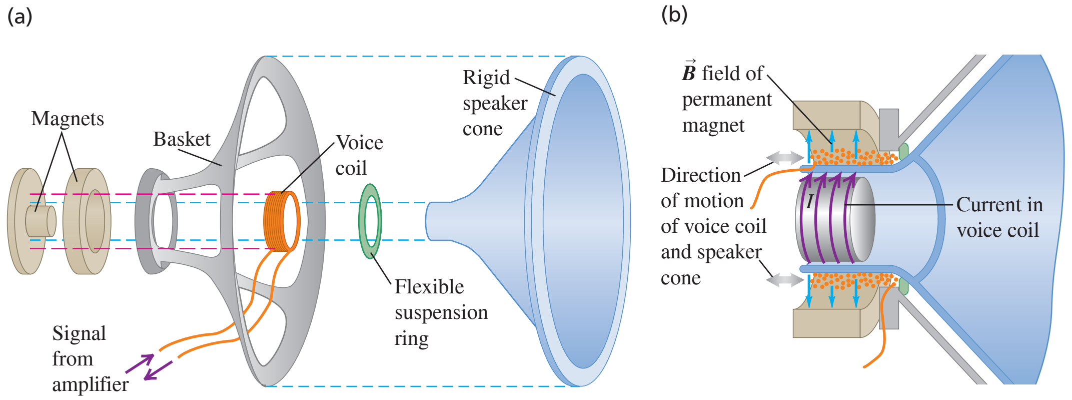
Magnetic field

Vector length of segment (points in current direction)

Then we can integrate this expression along the wire to find the total force on a conductor of any shape. The integral is a *line integral,* the same mathematical operation we have used to define work (Section 6.3) and electric potential (Section 23.2).

CAUTION **Current is not a vector** Recall from Section 25.1 that the current I is not a vector. The direction of current flow is described by $d\vec{l}$, not I. If the conductor is curved, I is the same at all points along its length, but $d\vec{l}$ changes direction—it is always tangent to the conductor.

Figure **27.28** (a) Components of a loudspeaker. (b) The permanent magnet creates a magnetic field that exerts forces on the current in the voice coil; for a current I in the direction shown, the force is to the right. If the electric current in the voice coil oscillates, the speaker cone attached to the voice coil oscillates at the same frequency.



Finally, what happens when the moving charges are negative, such as electrons in a metal? Then in Fig. 27.25 an upward current corresponds to a downward drift velocity. But because q is now negative, the direction of the force $\vec{F}$ is the same as before. Thus Eqs. (27.17) through (27.20) are valid for *both* positive and negative charges and even when *both* signs of charge are present at once. This happens in some semiconductor materials and in ionic solutions.

A common application of the magnetic forces on a current-carrying wire is found in loudspeakers (**Fig. 27.28**). The radial magnetic field created by the permanent magnet exerts a force on the voice coil that is proportional to the current in the coil; the direction of the force is either to the left or to the right, depending on the direction of the current. The signal from the amplifier causes the current to oscillate in direction and magnitude. The coil and the speaker cone to which it is attached respond by oscillating with an amplitude proportional to the amplitude of the current in the coil. Turning up the volume knob on the amplifier increases the current amplitude and hence the amplitudes of the cone's oscillation and of the sound wave produced by the moving cone.

EXAMPLE 27.7 Magnetic force on a straight conductor

WITH VARIATION PROBLEMS

A straight horizontal copper rod carries a current of 50.0 A from west to east in a region between the poles of a large electromagnet. In this region there is a horizontal magnetic field toward the northeast (that is, 45° north of east) with magnitude 1.20 T. (a) Find the magnitude and direction of the force on a 1.00 m section of rod. (b) While keeping the rod horizontal, how should it be oriented to maximize the magnitude of the force? What is the force magnitude in this case?

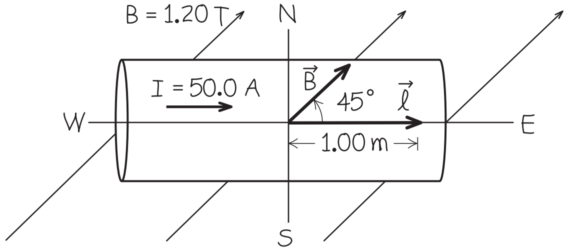
IDENTIFY and SET UP **Figure 27.29** shows the situation. This is a straight wire segment in a uniform magnetic field, as in Fig. 27.26. Our target variables are the force $\vec{F}$ on the segment and the angle ϕ for which the force magnitude F is greatest. We find the magnitude of the magnetic force from Eq. (27.18) and the direction from the right-hand rule.

EXECUTE (a) The angle ϕ between the directions of current and field is 45°. From Eq. (27.18) we obtain

$$F = IlB\sin\phi = (50.0\ \text{A})(1.00\ \text{m})(1.20\ \text{T})(\sin 45°) = 42.4\ \text{N}$$

The direction of the force is perpendicular to the plane of the current and the field, both of which lie in the horizontal plane. Thus the force must be vertical; the right-hand rule shows that it is vertically *upward* (out of the plane of the figure).

Figure **27.29** Our sketch of the copper rod as seen from overhead.



(b) From $F = IlB\sin\phi$, F is maximum for $\phi = 90°$, so that $\vec{l}$ and $\vec{B}$ are perpendicular. To keep $\vec{F} = I\vec{l} \times \vec{B}$ upward, we rotate the rod *clockwise* by 45° from its orientation in Fig. 27.29, so that the current runs toward the southeast. Then $F = IlB = (50.0\ \text{A})(1.00\ \text{m})(1.20\ \text{T}) = 60.0\ \text{N}$.

EVALUATE We check the result in part (a) by using Eq. (27.19) to calculate the force vector. If we use a coordinate system with the

Continued

x-axis pointing east, the y-axis north, and the z-axis upward, we have $\vec{l} = (1.00\text{ m})\hat{\imath}$, $\vec{B} = (1.20\text{ T})[(\cos 45°)\hat{\imath} + (\sin 45°)\hat{\jmath}]$, and

$$\begin{aligned}\vec{F} &= I\vec{l} \times \vec{B}\\ &= (50.0\text{ A})(1.00\text{ m})\hat{\imath} \times (1.20\text{ T})[(\cos 45°)\hat{\imath} + (\sin 45°)\hat{\jmath}]\\ &= (42.4\text{ N})\hat{k}\end{aligned}$$

Note that the maximum upward force of 60.0 N can hold the conductor in midair against the force of gravity—that is, *magnetically levitate* the conductor—if its weight is 60.0 N and its mass is $m = w/g = (60.0\text{ N})/(9.8\text{ m/s}^2) = 6.12$ kg. Magnetic levitation is used in some high-speed trains to suspend the train over the tracks. Eliminating rolling friction in this way allows the train to achieve speeds of over 400 km/h.

KEYCONCEPT A straight, current-carrying conductor in a magnetic field $\vec{B}$ experiences a magnetic force that depends on its current, its length, the magnitude of $\vec{B}$, and the angle between the current direction and $\vec{B}$. The force is zero if the directions of the current and $\vec{B}$ are the same or opposite; otherwise, the force is nonzero and in a direction perpendicular to both the current and $\vec{B}$.

EXAMPLE 27.8 Magnetic force on a curved conductor

In **Fig. 27.30** the magnetic field $\vec{B}$ is uniform and perpendicular to the plane of the figure, pointing out of the page. The conductor, carrying current I to the left, has three segments: (1) a straight segment with length L perpendicular to the plane of the figure, (2) a semicircle with radius R, and (3) another straight segment with length L parallel to the x-axis. Find the total magnetic force on this conductor.

IDENTIFY and SET UP The magnetic field $\vec{B} = B\hat{k}$ is uniform, so we find the forces $\vec{F}_1$ and $\vec{F}_3$ on the straight segments (1) and (3) from Eq. (27.19). We divide the curved segment (2) into infinitesimal straight segments and find the corresponding force $d\vec{F}_2$ on each straight segment from Eq. (27.20). We then integrate to find $\vec{F}_2$. The total magnetic force on the conductor is then $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$.

EXECUTE For segment (1), $\vec{L} = -L\hat{k}$. Hence from Eq. (27.19), $\vec{F}_1 = I\vec{L} \times \vec{B} = \mathbf{0}$. For segment (3), $\vec{L} = -L\hat{\imath}$, so $\vec{F}_3 = I\vec{L} \times \vec{B} = I(-L\hat{\imath}) \times (B\hat{k}) = ILB\hat{\jmath}$.

For the curved segment (2), Fig. 27.30 shows a segment $d\vec{l}$ with length $dl = R\,d\theta$, at angle θ. The right-hand rule shows that the direction of $d\vec{l} \times \vec{B}$ is radially outward from the center; make sure you can verify this. Because $d\vec{l}$ and $\vec{B}$ are perpendicular, the magnitude dF_2 of the force on the segment $d\vec{l}$ is $dF_2 = I\,dl\,B = I(R\,d\theta)B$. The components of the force on this segment are

$$dF_{2x} = IR\,d\theta\,B\cos\theta \qquad dF_{2y} = IR\,d\theta\,B\sin\theta$$

To find the components of the total force, we integrate these expressions with respect to θ from $\theta = 0$ to $\theta = \pi$ to take in the whole semicircle. The results are

$$F_{2x} = IRB\int_0^{\pi} \cos\theta\,d\theta = 0$$

$$F_{2y} = IRB\int_0^{\pi} \sin\theta\,d\theta = 2IRB$$

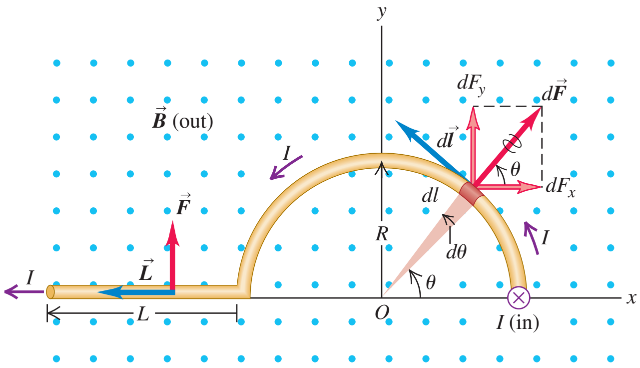
Hence $\vec{F}_2 = 2IRB\hat{\jmath}$. Finally, adding the forces on all three segments, we find that the total force is in the positive y-direction:

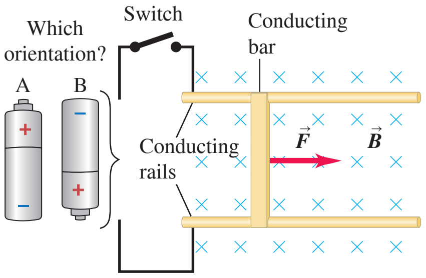
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 + 2IRB\hat{\jmath} + ILB\hat{\jmath} = IB(2R + L)\hat{\jmath}$$

EVALUATE We could have predicted from symmetry that the x-component of $\vec{F}_2$ would be zero: On the right half of the semicircle the x-component of the force is positive (to the right) and on the left half it is negative (to the left); the positive and negative contributions to the integral cancel. The result is that $\vec{F}_2$ is the force that would be exerted if we replaced the semicircle with a *straight* segment of length $2R$ along the x-axis. Do you see why?

KEYCONCEPT To find the magnetic force on a curved, current-carrying conductor in a magnetic field, first divide the conductor into infinitesimally small straight segments. Then find the force on one such segment. Finally, integrate over all segments in the conductor to find the net force.

Figure **27.30** What is the total magnetic force on the conductor?





TEST YOUR UNDERSTANDING OF SECTION 27.6 The accompanying figure shows a top view of two conducting rails on which a conducting bar can slide. A uniform magnetic field is directed perpendicular to the plane of the figure as shown. A battery is to be connected to the two rails so that when the switch is closed, current will flow through the bar and cause a magnetic force to push the bar to the right. In which orientation, A or B, should the battery be placed in the circuit?

ANSWER

I A This orientation will cause current to flow clockwise around the circuit and hence through the conducting bar in the direction from the top to the bottom of the figure. From the right-hand rule, the magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ on the bar will then point to the right.

27.7 FORCE AND TORQUE ON A CURRENT LOOP

Current-carrying conductors usually form closed loops, so it is worthwhile to use the results of Section 27.6 to find the *total* magnetic force and torque on a conductor in the form of a loop. Many practical devices make use of the magnetic force or torque on a conducting loop, including loudspeakers (see Fig. 27.28) and galvanometers (see Section 26.3). Hence the results of this section are of substantial practical importance. These results will also help us understand the behavior of bar magnets described in Section 27.1.

As an example, let's look at a rectangular current loop in a uniform magnetic field. We can represent the loop as a series of straight line segments. We'll find that the total *force* on the loop is zero but that there can be a net *torque* acting on the loop, with some interesting properties.

Figure 27.31a shows a rectangular loop of wire with side lengths a and b. A line perpendicular to the plane of the loop (i.e., a *normal* to the plane) makes an angle ϕ with the direction of the magnetic field $\vec{B}$, and the loop carries a current I. The wires leading the current into and out of the loop and the source of emf are omitted to keep the diagram simple.

The force $\vec{F}$ on the right side of the loop (length a) is to the right, in the $+x$-direction as shown. On this side, $\vec{B}$ is perpendicular to the current direction, and the force on this side has magnitude

$$F = IaB \tag{27.21}$$

A force $-\vec{F}$ with the same magnitude but opposite direction acts on the opposite side of the loop, as shown in the figure.

The sides with length b make an angle $(90° - \phi)$ with the direction of $\vec{B}$. The forces on these sides are the vectors $\vec{F}'$ and $-\vec{F}'$; their magnitude F' is given by

$$F' = IbB\sin(90° - \phi) = IbB\cos\phi$$

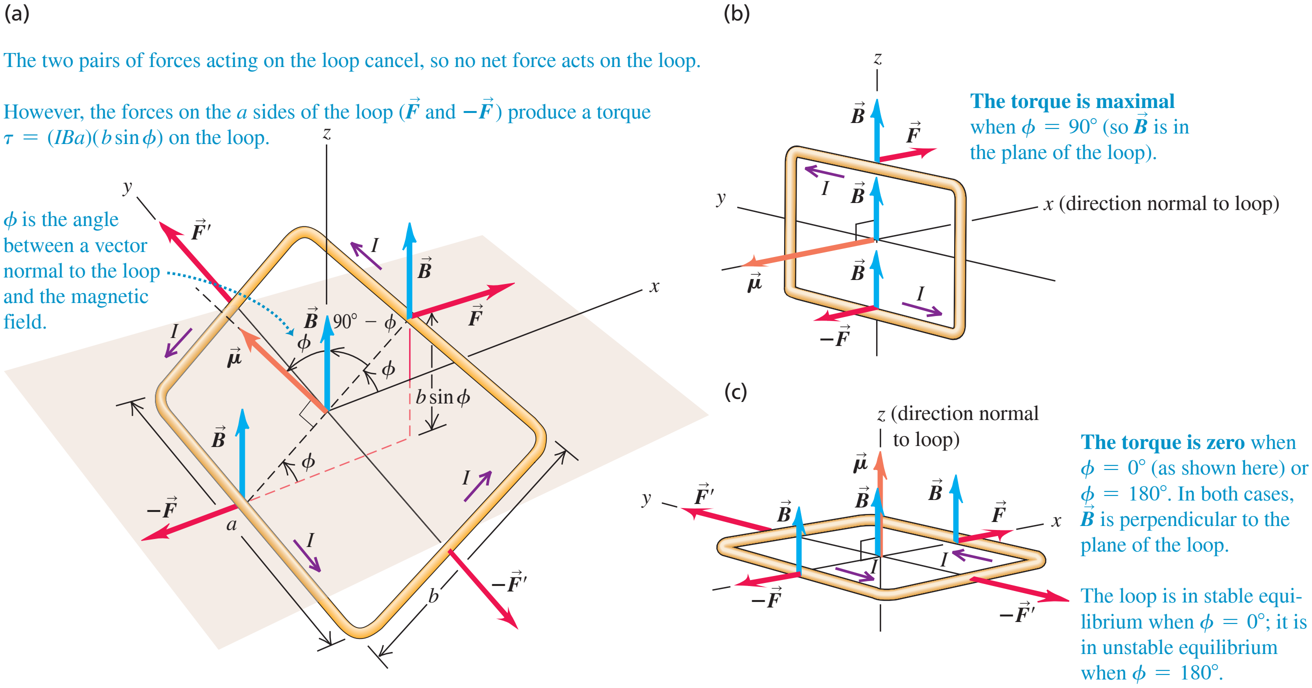
The lines of action of both forces lie along the y-axis.

The *total* force on the loop is zero because the forces on opposite sides cancel out in pairs.

The net force on a current loop in a uniform magnetic field is zero. However, the net torque is not in general equal to zero.

(You may find it helpful to review the discussion of torque in Section 10.1.) The two forces $\vec{F}'$ and $-\vec{F}'$ in Fig. 27.31a lie along the same line and so give rise to zero net torque with

Figure **27.31** Finding the torque on a current-carrying loop in a uniform magnetic field.



The two pairs of forces acting on the loop cancel, so no net force acts on the loop.

However, the forces on the a sides of the loop ($\vec{F}$ and $-\vec{F}$) produce a torque $\tau = (IBa)(b\sin\phi)$ on the loop.

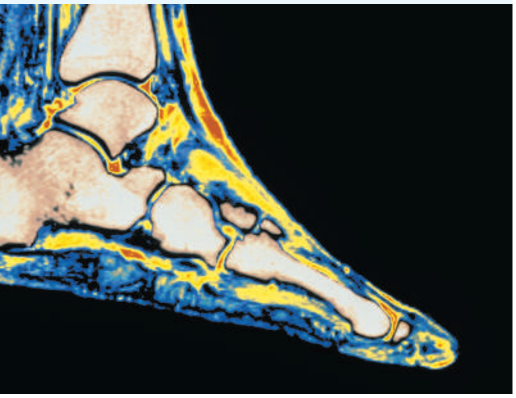
ϕ is the angle between a vector normal to the loop and the magnetic field.

The torque is maximal when $\phi = 90°$ (so $\vec{B}$ is in the plane of the loop).

The torque is zero when $\phi = 0°$ (as shown here) or $\phi = 180°$. In both cases, $\vec{B}$ is perpendicular to the plane of the loop.

The loop is in stable equilibrium when $\phi = 0°$; it is in unstable equilibrium when $\phi = 180°$.

BIO APPLICATION Magnetic Resonance Imaging In magnetic resonance imaging (MRI), a patient is placed in a strong magnetic field. Each hydrogen nucleus in the patient acts like a miniature current loop with a magnetic dipole moment that tends to align with the applied field. Radio waves of just the right frequency then flip these magnetic moments out of alignment. The extent to which the radio waves are absorbed is proportional to the amount of hydrogen present. This makes it possible to image details in hydrogen-rich soft tissue that cannot be seen in x-ray images. (X rays are superior to MRI for imaging bone, which is hydrogen deficient.)



respect to any point. The two forces $\vec{F}$ and $-\vec{F}$ lie along different lines, and each gives rise to a torque about the y-axis. According to the right-hand rule for determining the direction of torques, the vector torques due to $\vec{F}$ and $-\vec{F}$ are both in the $+y$-direction; hence the net vector torque $\vec{\tau}$ is in the $+y$-direction as well. The moment arm for each of these forces (equal to the perpendicular distance from the rotation axis to the line of action of the force) is $(b/2)\sin\phi$, so the torque due to each force has magnitude $F(b/2)\sin\phi$. If we use Eq. (27.21) for F, the magnitude of the net torque is

$$\tau = 2F(b/2)\sin\phi = (IBa)(b\sin\phi) \qquad (27.22)$$

The torque is greatest when $\phi = 90°$, $\vec{B}$ is in the plane of the loop, and the normal to this plane is perpendicular to $\vec{B}$ (Fig. 27.31b). The torque is zero when ϕ is 0° or 180° and the normal to the loop is parallel or antiparallel to the field (Fig. 27.31c). The value $\phi = 0°$ is a stable equilibrium position because the torque is zero there, and when the loop is rotated slightly from this position, the resulting torque tends to rotate it back toward $\phi = 0°$. The position $\phi = 180°$ is an *unstable* equilibrium position; if displaced slightly from this position, the loop tends to move farther away from $\phi = 180°$. Figure 27.31 shows rotation about the y-axis, but because the net force on the loop is zero, Eq. (27.22) for the torque is valid for *any* choice of axis. The torque always tends to rotate the loop in the direction of *decreasing* ϕ—that is, toward the stable equilibrium position $\phi = 0°$.

The area A of the loop is equal to ab, so we can rewrite Eq. (27.22) as

$$\underset{\substack{\textbf{Magnitude of}\\\textbf{magnetic torque}\\\textbf{on a current loop}}}{\tau} = \overset{\text{Current}}{I}\,\overset{\text{Magnetic-field magnitude}}{B}\,\underset{\text{Area of loop}}{A}\sin\underset{\substack{\text{Angle between}\\\text{normal to loop plane}\\\text{and field direction}}}{\phi} \qquad (27.23)$$

The product IA is called the **magnetic dipole moment** or **magnetic moment** of the loop, for which we use the symbol μ (the Greek letter mu):

$$\mu = IA \qquad (27.24)$$

It is analogous to the electric dipole moment introduced in Section 21.7. In terms of μ, the magnitude of the torque on a current loop is

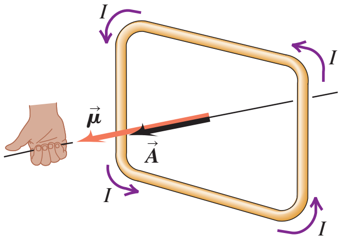
$$\tau = \mu B\sin\phi \qquad (27.25)$$

where ϕ is the angle between the normal to the loop (the direction of the vector area $\vec{A}$) and $\vec{B}$. A current loop, or any other object that experiences a magnetic torque given by Eq. (27.25), is also called a **magnetic dipole.**

Magnetic Torque: Vector Form

We can also define a vector magnetic moment $\vec{\mu}$ with magnitude IA: This is shown in Fig. 27.31. The direction of $\vec{\mu}$ is defined to be perpendicular to the plane of the loop, with a sense determined by a right-hand rule, as shown in **Fig. 27.32**. Wrap the fingers of your right hand around the perimeter of the loop in the direction of the current. Then extend your thumb so that it is perpendicular to the plane of the loop; its direction is the direction of $\vec{\mu}$ (and of the vector area $\vec{A}$ of the loop). The torque is greatest when $\vec{\mu}$ and $\vec{B}$ are perpendicular and is zero when they are parallel or antiparallel. In the stable equilibrium position, $\vec{\mu}$ and $\vec{B}$ are parallel.

Figure 27.32 The right-hand rule determines the direction of the magnetic moment of a current-carrying loop. This is also the direction of the loop's area vector $\vec{A}$; $\vec{\mu} = I\vec{A}$ is a vector equation.



Finally, we can express this interaction in terms of the torque vector $\vec{\tau}$, which we used for *electric*-dipole interactions in Section 21.7. From Eq. (27.25) the magnitude of $\vec{\tau}$ is equal to the magnitude of $\vec{\mu} \times \vec{B}$, and reference to Fig. 27.31 shows that the directions are also the same. So we have

$$\underset{\substack{\textbf{Vector magnetic torque}\\\textbf{on a current loop}}}{\vec{\tau}} = \overset{\text{Magnetic dipole moment}}{\vec{\mu}} \times \underset{\text{Magnetic field}}{\vec{B}} \qquad (27.26)$$

This result is directly analogous to the result we found in Section 21.7 for the torque exerted by an *electric* field $\vec{E}$ on an *electric* dipole with dipole moment $\vec{p}$.

Potential Energy for a Magnetic Dipole

When a magnetic dipole changes orientation in a magnetic field, the field does work on it. In an infinitesimal angular displacement $d\phi$, the work dW is given by $\tau\, d\phi$, and there is a corresponding change in potential energy. As the above discussion suggests, the potential energy U is least when $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$ are parallel and greatest when they are antiparallel. To find an expression for U as a function of orientation, note that the torque on an *electric* dipole in an *electric* field is $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}}$; we found in Section 21.7 that the corresponding potential energy is $U = -\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{E}}$. The torque on a *magnetic* dipole in a *magnetic* field is $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$, so we can conclude immediately that

Potential energy for a magnetic dipole in a magnetic field

Magnetic dipole moment

$$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}} = -\mu B \cos\phi \qquad (27.27)$$

Angle between $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$

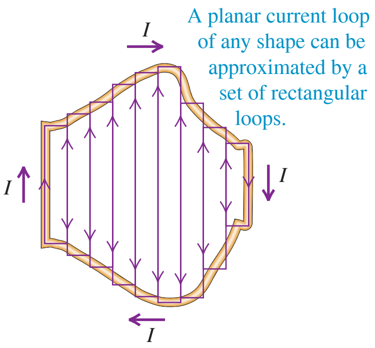
Magnetic field

With this definition, U is zero when the magnetic dipole moment is perpendicular to the magnetic field ($\phi = 90°$); then $U = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}} = -\mu B \cos 90° = 0$.

Magnetic Torque: Loops and Coils

Although we have derived Eqs. (27.21) through (27.27) for a rectangular current loop, all these relationships are valid for a plane loop of any shape at all. Any planar loop may be approximated as closely as we wish by a very large number of rectangular loops, as shown in **Fig. 27.33**. If these loops all carry equal currents in the same clockwise sense, then the forces and torques on the sides of two loops adjacent to each other cancel, and the only forces and torques that do not cancel are due to currents around the boundary. Thus all the above relationships are valid for a plane current loop of any shape, with the magnetic moment $\vec{\boldsymbol{\mu}} = I\vec{\boldsymbol{A}}$.

Figure **27.33** The collection of rectangles exactly matches the irregular plane loop in the limit as the number of rectangles approaches infinity and the width of each rectangle approaches zero.


A planar current loop of any shape can be approximated by a set of rectangular loops.

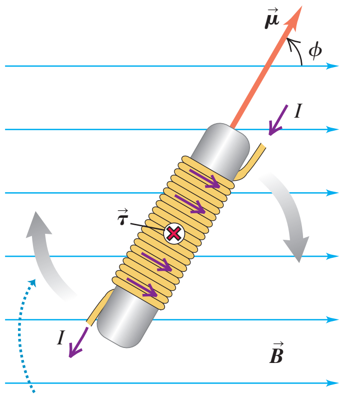
We can also generalize this whole formulation to a coil consisting of N planar loops close together; the effect is simply to multiply each force, the magnetic moment, the torque, and the potential energy by a factor of N.

An arrangement of particular interest is the **solenoid,** a helical winding of wire, such as a coil wound on a circular cylinder (**Fig. 27.34**). If the windings are closely spaced, the solenoid can be approximated by a number of circular loops lying in planes at right angles to its long axis. The total torque on a solenoid in a magnetic field is simply the sum of the torques on the individual turns. For a solenoid with N turns in a uniform field B, the magnetic moment is $\mu = NIA$ and

$$\tau = NIAB \sin\phi \qquad (27.28)$$

where ϕ is the angle between the axis of the solenoid and the direction of the field. The magnetic-moment vector $\vec{\boldsymbol{\mu}}$ is along the solenoid axis. The torque is greatest when the solenoid axis is perpendicular to the magnetic field and zero when they are parallel. The effect of this torque is to tend to rotate the solenoid into a position where its axis is parallel to the field. Solenoids are also useful as *sources* of magnetic field, as we'll discuss in Chapter 28.

Figure **27.34** The torque $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$ on this solenoid in a uniform magnetic field is directed straight into the page. An actual solenoid has many more turns, wrapped closely together.



The torque tends to make the solenoid rotate clockwise in the plane of the page, aligning magnetic moment $\vec{\boldsymbol{\mu}}$ with field $\vec{\boldsymbol{B}}$.

The d'Arsonval galvanometer, described in Section 26.3, makes use of a magnetic torque on a coil carrying a current. As Fig. 26.14 shows, the magnetic field is not uniform but is *radial,* so the side thrusts on the coil are always perpendicular to its plane. Thus the angle ϕ in Eq. (27.28) is always 90°, and the magnetic torque is directly proportional to the current, no matter what the orientation of the coil. A restoring torque proportional to the angular displacement of the coil is provided by two hairsprings, which also serve as current leads to the coil. When current is supplied to the coil, it rotates along with its attached pointer until the restoring spring torque just balances the magnetic torque. Thus the pointer deflection is proportional to the current.

EXAMPLE 27.9 Magnetic torque on a circular coil

A circular coil 0.0500 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a counterclockwise (as viewed from above) current of 5.00 A. The coil is in a uniform 1.20 T magnetic field directed toward the right. Find the magnitudes of the magnetic moment and the torque on the coil.

IDENTIFY and SET UP This problem uses the definition of magnetic moment and the expression for the torque on a magnetic dipole in a magnetic field. **Figure 27.35** shows the situation. Equation (27.24) gives the magnitude μ of the magnetic moment of a single turn of wire; for N turns, the magnetic moment is N times greater. Equation (27.25) gives the magnitude τ of the torque.

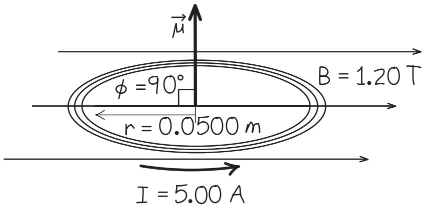


Figure 27.35 Our sketch for this problem.

EXECUTE The area of the coil is $A = \pi r^2$. From Eq. (27.24), the total magnetic moment of all 30 turns is

$$\mu_{\text{total}} = NIA = 30(5.00\ \text{A})\pi(0.0500\ \text{m})^2 = 1.18\ \text{A}\cdot\text{m}^2$$

The angle ϕ between the direction of $\vec{B}$ and the direction of $\vec{\mu}$ (which is along the normal to the plane of the coil) is 90°. From Eq. (27.25), the torque on the coil is

$$\begin{aligned}\tau &= \mu_{\text{total}}B\sin\phi = (1.18\ \text{A}\cdot\text{m}^2)(1.20\ \text{T})(\sin 90°)\\ &= 1.41\ \text{N}\cdot\text{m}\end{aligned}$$

EVALUATE The torque tends to rotate the right side of the coil down and the left side up, into a position where the normal to its plane is parallel to $\vec{B}$.

KEYCONCEPT A current-carrying coil in a magnetic field $\vec{B}$ experiences a magnetic torque. The torque tends to orient the coil so its magnetic dipole moment $\vec{\mu}$ (a vector perpendicular to the plane of the coil, with magnitude equal to the product of the number of turns, the current, and the coil area) is in the same direction as $\vec{B}$.

EXAMPLE 27.10 Potential energy for a coil in a magnetic field

If the coil in Example 27.9 rotates from its initial orientation to one in which its magnetic moment $\vec{\mu}$ is parallel to $\vec{B}$, what is the change in potential energy?

IDENTIFY and SET UP Equation (27.27) gives the potential energy for each orientation. The initial position is as shown in Fig. 27.35, with $\phi_1 = 90°$. In the final orientation, the coil has been rotated 90° clockwise so that $\vec{\mu}$ and $\vec{B}$ are parallel, so the angle between these vectors is $\phi_2 = 0$.

EXECUTE From Eq. (27.27), the potential energy change is

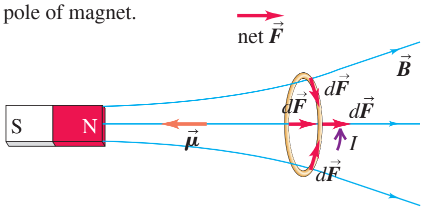
$$\begin{aligned}\Delta U = U_2 - U_1 &= -\mu B\cos\phi_2 - (-\mu B\cos\phi_1)\\ &= -\mu B(\cos\phi_2 - \cos\phi_1)\\ &= -(1.18\ \text{A}\cdot\text{m}^2)(1.20\ \text{T})(\cos 0° - \cos 90°) = -1.41\ \text{J}\end{aligned}$$

EVALUATE The potential energy decreases because the rotation is in the direction of the magnetic torque that we found in Example 27.9.

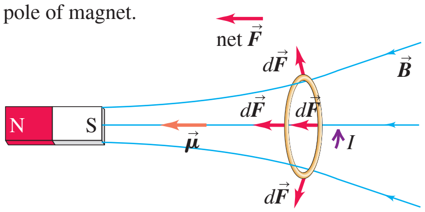
KEYCONCEPT The potential energy of a current-carrying coil in a magnetic field $\vec{B}$ depends on the magnitudes and directions of $\vec{B}$ and of the coil's magnetic dipole moment $\vec{\mu}$. The potential energy is minimum when $\vec{\mu}$ and $\vec{B}$ are in the same direction.

Figure 27.36 Forces on current loops in a nonuniform $\vec{B}$ field. In each case the axis of the bar magnet is perpendicular to the plane of the loop and passes through the center of the loop.

(a) Net force on this coil is away from north pole of magnet.



(b) Net force on same coil is toward south pole of magnet.



Magnetic Dipole in a Nonuniform Magnetic Field

We have seen that a current loop (that is, a magnetic dipole) experiences zero net force in a uniform magnetic field. **Figure 27.36** shows two current loops in the *nonuniform* $\vec{B}$ field of a bar magnet; in both cases the net force on the loop is *not* zero. In Fig. 27.36a the magnetic moment $\vec{\mu}$ is in the direction opposite to the field, and the force $d\vec{F} = I\,d\vec{l} \times \vec{B}$ on a segment of the loop has both a radial component and a component to the right. When these forces are summed to find the net force $\vec{F}$ on the loop, the radial components cancel so that the net force is to the right, away from the magnet. Note that in this case the force is toward the region where the field lines are farther apart and the field magnitude B is less. The polarity of the bar magnet is reversed in Fig. 27.36b, so $\vec{\mu}$ and $\vec{B}$ are parallel; now the net force on the loop is to the left, toward the region of greater field magnitude near the magnet. Later in this section we'll use these observations to explain why bar magnets can pick up unmagnetized iron objects.

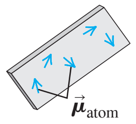
Magnetic Dipoles and How Magnets Work

The behavior of a solenoid in a magnetic field (see Fig. 27.34) resembles that of a bar magnet or compass needle; if free to turn, both the solenoid and the magnet orient themselves with their axes parallel to the $\vec{B}$ field. In both cases this is due to the interaction of moving electric charges with a magnetic field; the difference is that in a bar magnet the motion of charge occurs on the microscopic scale of the atom.

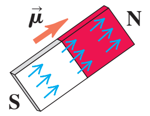
Think of an electron as being like a spinning ball of charge. In this analogy the circulation of charge around the spin axis is like a current loop, and so the electron has a net magnetic moment. (This analogy, while helpful, is inexact; an electron isn't really a spinning sphere. A full explanation of the origin of an electron's magnetic moment involves quantum mechanics, which is beyond our scope here.) In an iron atom a substantial fraction of the electron magnetic moments align with each other, and the atom has a nonzero magnetic moment. (By contrast, the atoms of most elements have little or no net magnetic moment.) In an unmagnetized piece of iron there is no overall alignment of the magnetic moments of the atoms; their vector sum is zero, and the net magnetic moment is zero (**Fig. 27.37a**). But in an iron bar magnet the magnetic moments of many of the atoms are parallel, and there is a substantial net magnetic moment $\vec{\mu}$ (Fig. 27.37b). If the magnet is placed in a magnetic field $\vec{B}$, the field exerts a torque given by Eq. (27.26) that tends to align $\vec{\mu}$ with $\vec{B}$ (Fig. 27.37c). A bar magnet tends to align with a $\vec{B}$ field so that a line from the south pole to the north pole of the magnet is in the direction of $\vec{B}$; hence the real significance of a magnet's north and south poles is that they represent the head and tail, respectively, of the magnet's dipole moment $\vec{\mu}$.

Figure **27.37** (a) An unmagnetized piece of iron. (Only a few representative atomic moments are shown.) (b) A magnetized piece of iron (bar magnet). The net magnetic moment of the bar magnet points from its south pole to its north pole. (c) A bar magnet in a magnetic field.

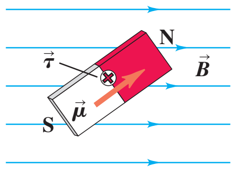
(a) Unmagnetized iron: magnetic moments are oriented randomly.



(b) In a bar magnet, the magnetic moments are aligned.



(c) A magnetic field creates a torque on the bar magnet that tends to align its dipole moment with the $\vec{B}$ field.



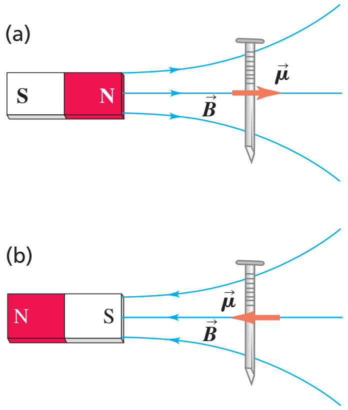
The torque experienced by a current loop in a magnetic field also explains how an unmagnetized iron object like that in Fig. 27.37a becomes magnetized. If an unmagnetized iron paper clip is placed next to a powerful magnet, the magnetic moments of the paper clip's atoms tend to align with the $\vec{B}$ field of the magnet. When the paper clip is removed, its atomic dipoles tend to remain aligned, and the paper clip has a net magnetic moment. The paper clip can be demagnetized by being dropped on the floor or heated; the added internal energy jostles and re-randomizes the atomic dipoles.

The magnetic-dipole picture of a bar magnet explains the attractive and repulsive forces between bar magnets shown in Fig. 27.1. The magnetic moment $\vec{\mu}$ of a bar magnet points from its south pole to its north pole, so the current loops in Figs. 27.36a and 27.36b are both equivalent to a magnet with its north pole on the left. Hence the situation in Fig. 27.36a is equivalent to two bar magnets with their north poles next to each other; the resultant force is repulsive, as in Fig. 27.1b. In Fig. 27.36b we again have the equivalent of two bar magnets end to end, but with the south pole of the left-hand magnet next to the north pole of the right-hand magnet. The resultant force is attractive, as in Fig. 27.1a.

Finally, we can explain how a magnet can attract an unmagnetized iron object (see Fig. 27.2). It's a two-step process. First, the atomic magnetic moments of the iron tend to align with the $\vec{B}$ field of the magnet, so the iron acquires a net magnetic dipole moment $\vec{\mu}$ parallel to the field. Second, the nonuniform field of the magnet attracts the magnetic dipole. **Figure 27.38a** shows an example. The north pole of the magnet is closer to the nail (which contains iron), and the magnetic dipole produced in the nail is equivalent to a loop with a current that circulates in a direction opposite to that shown in Fig. 27.36a. Hence the net magnetic force on the nail is opposite to the force on the loop in Fig. 27.36a, and the nail is attracted toward the magnet. Changing the polarity of the magnet, as in Fig. 27.38b, reverses the directions of both $\vec{B}$ and $\vec{\mu}$. The situation is now equivalent to that shown in Fig. 27.36b; like the loop in that figure, the nail is attracted toward the magnet. Hence a previously unmagnetized object containing iron is attracted to *either* pole of a magnet. By contrast, objects made of brass, aluminum, or wood hardly respond at all to a magnet; the atomic magnetic dipoles of these materials, if present at all, have less tendency to align with an external field.

Our discussion of how magnets and pieces of iron interact has just scratched the surface of a diverse subject known as *magnetic properties of materials*. We'll discuss these properties in more depth in Section 28.8.

Figure **27.38** A bar magnet attracts an unmagnetized iron nail in two steps. First, the $\vec{B}$ field of the bar magnet gives rise to a net magnetic moment in the nail. Second, because the field of the bar magnet is not uniform, this magnetic dipole is attracted toward the magnet. The attraction is the same whether the nail is closer to (a) the magnet's north pole or (b) the magnet's south pole.



TEST YOUR UNDERSTANDING OF SECTION 27.7 Figure 27.13c depicts the magnetic field lines due to a circular current-carrying loop. (a) What is the direction of the magnetic moment of this loop? (b) Which side of the loop is equivalent to the north pole of a magnet, and which side is equivalent to the south pole?

ANSWER

❙ (a) **to the right;** (b) **north pole on the right, south pole on the left** If you wrap the fingers of your right hand around the coil in the direction of the current, your right thumb points to the right (perpendicular to the plane of the coil). This is the direction of the magnetic moment $\vec{\mu}$. The magnetic moment points from the south pole to the north pole, so the right side of the loop is equivalent to a north pole and the left side is equivalent to a south pole.

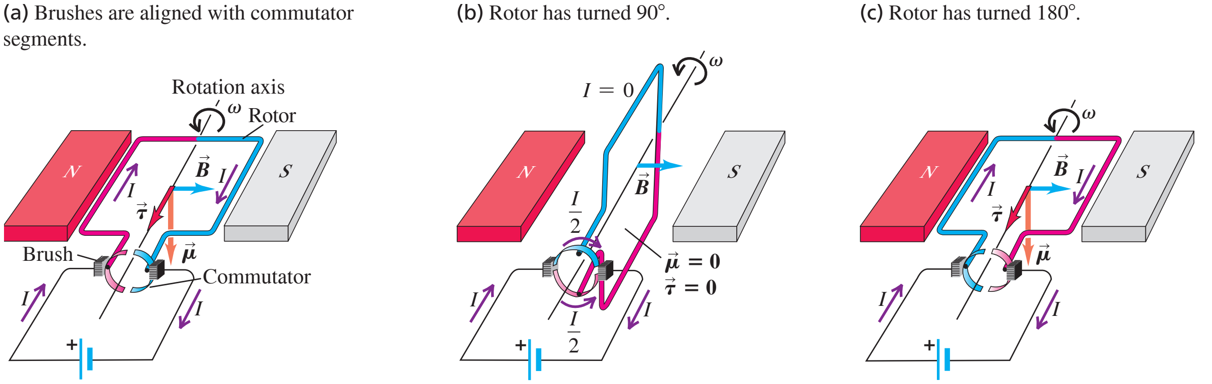
27.8 THE DIRECT-CURRENT MOTOR

Electric motors play an important role in contemporary society. In a motor a magnetic torque acts on a current-carrying conductor, and electric energy is converted to mechanical energy. As an example, let's look at a simple type of direct-current (dc) motor, shown in **Fig. 27.39**.

The moving part of the motor is the *rotor,* a length of wire formed into an open-ended loop and free to rotate about an axis. The ends of the rotor wires are attached to circular conducting segments that form a *commutator.* In Fig. 27.39a, each of the two commutator segments makes contact with one of the terminals, or *brushes,* of an external circuit that includes a source of emf. This causes a current to flow into the rotor on one side, shown in red, and out of the rotor on the other side, shown in blue. Hence the rotor is a current loop with a magnetic moment $\vec{\mu}$. The rotor lies between opposing poles of a permanent magnet, so there is a magnetic field $\vec{B}$ that exerts a torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ on the rotor. For the rotor orientation shown in Fig. 27.39a the torque causes the rotor to turn counterclockwise, in the direction that will align $\vec{\mu}$ with $\vec{B}$.

In Fig. 27.39b the rotor has rotated by 90° from its orientation in Fig. 27.39a. If the current through the rotor were constant, the rotor would now be in its equilibrium orientation; it would simply oscillate around this orientation. But here's where the commutator comes into play; each brush is now in contact with *both* segments of the commutator. There is no potential difference between the commutators, so at this instant no current flows through the rotor, and the magnetic moment is zero. The rotor continues to rotate counterclockwise because of its inertia, and current again flows through the rotor as in Fig. 27.39c. But now current enters on the *blue* side of the rotor and exits on the *red* side, just the opposite of the situation in Fig. 27.39a. While the direction of the current has reversed with respect

Figure **27.39** Schematic diagram of a simple dc motor. The rotor is a wire loop that is free to rotate about an axis; the rotor ends are attached to the two curved conductors that form the commutator. (The rotor halves are colored red and blue for clarity.) The commutator segments are insulated from one another.



(a) Brushes are aligned with commutator segments.

- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

(b) Rotor has turned 90°.

- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

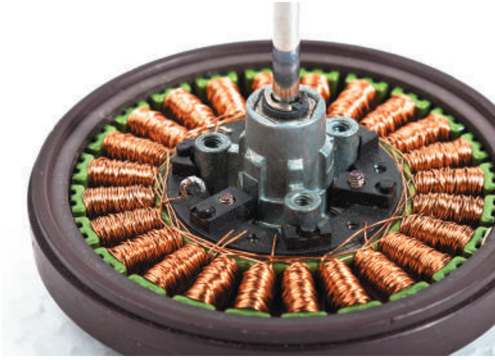
(c) Rotor has turned 180°.

- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.

to the rotor, the rotor itself has rotated 180° and the magnetic moment $\vec{\mu}$ is in the same direction with respect to the magnetic field. Hence the magnetic torque $\vec{\tau}$ is in the same direction in Fig. 27.39c as in Fig. 27.39a. Thanks to the commutator, the current reverses after every 180° of rotation, so the torque is always in the direction to rotate the rotor counterclockwise. When the motor has come "up to speed," the average magnetic torque is just balanced by an opposing torque due to air resistance, friction in the rotor bearings, and friction between the commutator and brushes.

The simple motor shown in Fig. 27.39 has only a single turn of wire in its rotor. In practical motors, the rotor has many turns; this increases the magnetic moment and the torque so that the motor can spin larger loads. The torque can also be increased by using a stronger magnetic field, which is why many motor designs use electromagnets instead of a permanent magnet. Another drawback of the simple design in Fig. 27.39 is that the magnitude of the torque rises and falls as the rotor spins. This can be remedied by having the rotor include several independent coils of wire oriented at different angles (**Fig. 27.40**).

Figure **27.40** This electric motor has multiple current-carrying coils. They interact with permanent magnets on the rotor (not shown) to make the rotor spin around the motor's shaft. (This design is the reverse of the design in Fig. 27.39, in which the permanent magnets are stationary and the coil rotates.) Because there are multiple coils, the magnetic torque is very nearly constant and the rotor spins at a constant rate. Motors of this kind are used in drones, electric cars, and many other applications.



Power for Electric Motors

Because a motor converts electric energy to mechanical energy or work, it requires electric energy input. If the potential difference between its terminals is V_{ab} and the current is I, then the power input is $P = V_{ab}I$. Even if the motor coils have negligible resistance, there must be a potential difference between the terminals if P is to be different from zero. This potential difference results principally from magnetic forces exerted on the currents in the conductors of the rotor as they rotate through the magnetic field. The associated electromotive force $\mathcal{E}$ is called an *induced* emf; it is also called a *back* emf because its sense is opposite to that of the current. In Chapter 29 we'll study induced emfs resulting from motion of conductors in magnetic fields.

In a *series* motor the rotor is connected in series with the electromagnet that produces the magnetic field; in a *shunt* motor they are connected in parallel. In a series motor with internal resistance r, V_{ab} is greater than $\mathcal{E}$, and the difference is the potential drop Ir across the internal resistance. That is,

$$V_{ab} = \mathcal{E} + Ir \tag{27.29}$$

Because the magnetic force is proportional to velocity, $\mathcal{E}$ is *not* constant but is proportional to the speed of rotation of the rotor.

EXAMPLE 27.11 A series dc motor

A dc motor with its rotor and field coils connected in series has an internal resistance of 2.00 Ω. When running at full load on a 120 V line, it draws a 4.00 A current. (a) What is the emf in the rotor? (b) What is the power delivered to the motor? (c) What is the rate of dissipation of energy in the internal resistance? (d) What is the mechanical power developed? (e) What is the motor's efficiency? (f) What happens if the machine being driven by the motor jams, so that the rotor suddenly stops turning?

IDENTIFY and SET UP This problem uses the ideas of power and potential drop in a series dc motor. We are given the internal resistance $r = 2.00\ \Omega$, the voltage $V_{ab} = 120$ V across the motor, and the current $I = 4.00$ A through the motor. We use Eq. (27.29) to determine the emf $\mathcal{E}$ from these quantities. The power delivered to the motor is $V_{ab}I$, the rate of energy dissipation is I^2r, and the power output by the motor is the difference between the power input and the power dissipated. The efficiency e is the ratio of mechanical power output to electric power input.

EXECUTE (a) From Eq. (27.29), $V_{ab} = \mathcal{E} + Ir$, we have

$$120\text{ V} = \mathcal{E} + (4.00\text{ A})(2.00\ \Omega) \quad \text{and so} \quad \mathcal{E} = 112\text{ V}$$

(b) The power delivered to the motor from the source is

$$P_{\text{input}} = V_{ab}I = (120\text{ V})(4.00\text{ A}) = 480\text{ W}$$

(c) The power dissipated in the resistance r is

$$P_{\text{dissipated}} = I^2r = (4.00\text{ A})^2(2.00\ \Omega) = 32\text{ W}$$

(d) The mechanical power output is the electric power input minus the rate of dissipation of energy in the motor's resistance (assuming that there are no other power losses):

$$P_{\text{output}} = P_{\text{input}} - P_{\text{dissipated}} = 480\text{ W} - 32\text{ W} = 448\text{ W}$$

(e) The efficiency e is the ratio of mechanical power output to electric power input:

$$e = \frac{P_{\text{output}}}{P_{\text{input}}} = \frac{448\text{ W}}{480\text{ W}} = 0.93 = 93\%$$

(f) With the rotor stalled, the back emf $\mathcal{E}$ (which is proportional to rotor speed) goes to zero. From Eq. (27.29) the current becomes

$$I = \frac{V_{ab}}{r} = \frac{120\text{ V}}{2.00\ \Omega} = 60\text{ A}$$

and the power dissipated in the resistance r becomes

$$P_{\text{dissipated}} = I^2r = (60\text{ A})^2(2.00\ \Omega) = 7200\text{ W}$$

Continued

EVALUATE If this massive overload doesn't blow a fuse or trip a circuit breaker, the coils will quickly melt. When the motor is first turned on, there's a momentary surge of current until the motor picks up speed. This surge causes greater-than-usual voltage drops $(V = IR)$ in the power lines supplying the current. Similar effects are responsible for the momentary dimming of lights that can occur in a house when an air conditioner or dishwasher motor starts.

KEYCONCEPT When an electric motor is in operation, a large emf is induced in the rotor coils. To find the mechanical power developed by the motor, multiply this emf by the current in the coils.

TEST YOUR UNDERSTANDING OF SECTION 27.8 In the circuit shown in Fig. 27.39, you add a switch in series with the source of emf so that the current can be turned on and off. When you close the switch and allow current to flow, will the rotor begin to turn no matter what its original orientation?

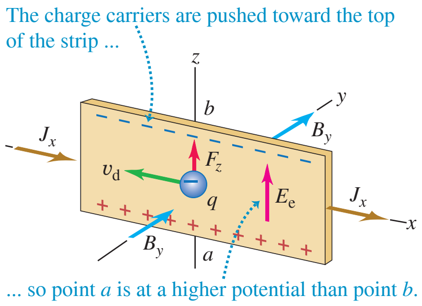
ANSWER

no The rotor will not begin to turn when the switch is closed if the rotor is initially oriented as shown in Fig. 27.39b. In this case there is no current through the rotor and hence no magnetic torque. This situation can be remedied by using multiple rotor coils oriented at different angles around the rotation axis. With this arrangement, there is always a magnetic torque no matter what the orientation.

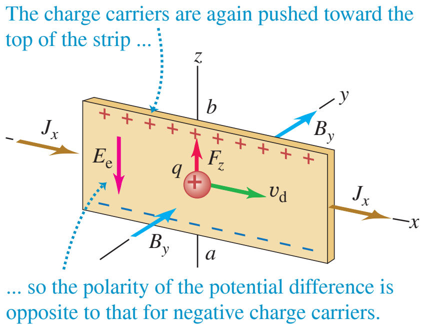
27.9 THE HALL EFFECT

Figure **27.41** Forces on charge carriers in a conductor in a magnetic field.

(a) Negative charge carriers (electrons)



(b) Positive charge carriers



The reality of the forces acting on the moving charges in a conductor in a magnetic field is strikingly demonstrated by the *Hall effect,* an effect analogous to the transverse deflection of an electron beam in a magnetic field in vacuum. (The effect was discovered by the American physicist Edwin Hall in 1879 while he was still a graduate student.) To describe this effect, let's consider a conductor in the form of a flat strip, as shown in **Fig. 27.41**. The current is in the direction of the $+x$-axis and there is a uniform magnetic field $\vec{B}$ perpendicular to the plane of the strip, in the $+y$-direction. The drift velocity of the moving charges (charge magnitude $|q|$) has magnitude v_d. Figure 27.41a shows the case of negative charges, such as electrons in a metal, and Fig. 27.41b shows positive charges. In both cases the magnetic force is upward, just as the magnetic force on a conductor is the same whether the moving charges are positive or negative. In either case a moving charge is driven toward the *upper* edge of the strip by the magnetic force $F_z = |q|v_dB$.

If the charge carriers are electrons, as in Fig. 27.41a, an excess negative charge accumulates at the upper edge of the strip, leaving an excess positive charge at its lower edge. This accumulation continues until the resulting transverse electrostatic field $\vec{E}_e$ becomes large enough to cause a force (magnitude $|q|E_e$) that is equal and opposite to the magnetic force (magnitude $|q|v_dB$). After that, there is no longer any net transverse force to deflect the moving charges. This electric field causes a transverse potential difference between opposite edges of the strip, called the *Hall voltage* or the *Hall emf.* The polarity depends on whether the moving charges are positive or negative. Experiments show that for metals the upper edge of the strip in Fig. 27.41a *does* become negatively charged, showing that the charge carriers in a metal are indeed negative electrons.

However, if the charge carriers are *positive,* as in Fig. 27.41b, then *positive* charge accumulates at the upper edge, and the potential difference is *opposite* to the situation with negative charges. Soon after the discovery of the Hall effect in 1879, it was observed that some materials, particularly some *semiconductors,* show a Hall emf opposite to that of the metals, as if their charge carriers were positively charged. We now know that these materials conduct by a process known as *hole conduction.* Within such a material there are locations, called *holes,* that would normally be occupied by an electron but are actually empty. A missing negative charge is equivalent to a positive charge. When an electron moves in one direction to fill a hole, it leaves another hole behind it. The hole migrates in the direction opposite to that of the electron.

In terms of the coordinate axes in Fig. 27.41b, the electrostatic field $\vec{E}_e$ for the positive-q case is in the $-z$-direction; its z-component E_z is negative. The magnetic field is in the $+y$-direction, and we write it as B_y. The magnetic force (in the $+z$-direction) is qv_dB_y. The current density J_x is in the $+x$-direction. In the steady state, when the forces qE_z and qv_dB_y sum to zero,

$$qE_z + qv_dB_y = 0 \qquad \text{or} \qquad E_z = -v_dB_y$$

This confirms that when q is positive, E_z is negative. From Eq. (25.4),

$$J_x = nqv_\mathrm{d}$$

Eliminating v_d between these equations, we find

Hall effect:

$$\underset{\text{Charge per carrier}}{\overset{\text{Concentration of mobile charge carriers}}{nq}} = \frac{-\overset{\text{Current density}}{J_x}\overset{\text{Magnetic field}}{B_y}}{\underset{\text{Electrostatic field in conductor}}{E_z}} \qquad (27.30)$$

Note that this result (as well as the entire derivation) is valid for both positive and negative q. When q is negative, E_z is positive, and conversely.

We can measure J_x, B_y, and E_z, so we can compute the product nq. In both metals and semiconductors, q is equal in magnitude to the electron charge, so the Hall effect permits a direct measurement of n, the concentration of current-carrying charges in the material. The *sign* of the charges is determined by the polarity of the Hall emf, as we have described.

The Hall effect can also be used for a direct measurement of electron drift speed v_d in metals. As we saw in Chapter 25, these speeds are very small, often of the order of 1 mm/s or less. If we move the entire conductor in the opposite direction to the current with a speed equal to the drift speed, then the electrons are at rest with respect to the magnetic field, and the Hall emf disappears. Thus the conductor speed needed to make the Hall emf vanish is equal to the drift speed.

BIO APPLICATION Exercise Machines and the Hall Effect This athlete is making a flywheel spin inside a rowing machine. A gear attached to the flywheel has magnetic teeth, and every time a tooth on the spinning gear passes a current-carrying sensor, the tooth's magnetic field deflects the current and produces a Hall voltage in the sensor. The harder the athlete works, the faster the flywheel spins and the more voltage pulses there are per minute. The voltage information is passed to the rowing machine's display and tells the athlete her power output.

EXAMPLE 27.12 A Hall-effect measurement

You place a strip of copper, 2.0 mm thick and 1.50 cm wide, in a uniform 0.40 T magnetic field as shown in Fig. 27.41a. When you run a 75 A current in the $+x$-direction, you find that the potential at the bottom of the slab is 0.81 μV higher than at the top. From this measurement, determine the concentration of mobile electrons in copper.

IDENTIFY and SET UP This problem describes a Hall-effect experiment. We use Eq. (27.30) to determine the mobile electron concentration n.

EXECUTE First we find the current density J_x and the electric field E_z:

$$J_x = \frac{I}{A} = \frac{75\ \mathrm{A}}{(2.0 \times 10^{-3}\ \mathrm{m})(1.50 \times 10^{-2}\ \mathrm{m})} = 2.5 \times 10^6\ \mathrm{A/m^2}$$

$$E_z = \frac{V}{d} = \frac{0.81 \times 10^{-6}\ \mathrm{V}}{1.5 \times 10^{-2}\ \mathrm{m}} = 5.4 \times 10^{-5}\ \mathrm{V/m}$$

Then, from Eq. (27.30),

$$n = \frac{-J_xB_y}{qE_z} = \frac{-(2.5 \times 10^6\ \mathrm{A/m^2})(0.40\ \mathrm{T})}{(-1.60 \times 10^{-19}\ \mathrm{C})(5.4 \times 10^{-5}\ \mathrm{V/m})}$$
$$= 11.6 \times 10^{28}\ \mathrm{m^{-3}}$$

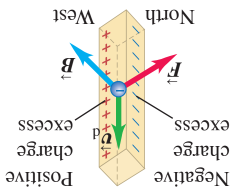
EVALUATE The actual value of n for copper is $8.5 \times 10^{28}\ \mathrm{m^{-3}}$. The difference shows that our simple model of the Hall effect, which ignores quantum effects and electron interactions with the ions, must be used with caution. This example also shows that with good conductors, the Hall emf is very small even with large current densities. In practice, Hall-effect devices for magnetic-field measurements use semiconductor materials, for which moderate current densities give much larger Hall emfs.

KEYCONCEPT In the Hall effect, a current in a conductor flows in a direction perpendicular to a magnetic field $\vec{B}$. The magnetic force on the current produces a charge separation in the conductor. This separation is perpendicular to both the current and $\vec{B}$, and produces a transverse electric field and potential difference in the conductor.

TEST YOUR UNDERSTANDING OF SECTION 27.9 A copper wire of square cross section is oriented vertically. The four sides of the wire face north, south, east, and west. There is a uniform magnetic field directed from east to west, and the wire carries current downward. Which side of the wire is at the highest electric potential? (i) North side; (ii) south side; (iii) east side; (iv) west side.

ANSWER

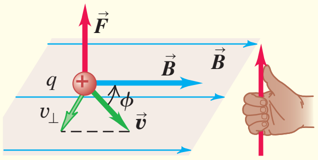
(ii) The mobile charge carriers in copper are negatively charged electrons, which move upward through the wire to give a downward current. From the right-hand rule, the force on a positively charged particle moving upward in a westward-pointing magnetic field would be to the south; hence the force on a negatively charged particle is to the north. The result is an excess of negative charge on the north side of the wire, leaving an excess of positive charge—and hence a higher electric potential—on the south side.



CHAPTER 27 SUMMARY

Magnetic forces: Magnetic interactions are fundamentally interactions between moving charged particles. These interactions are described by the vector magnetic field, denoted by $\vec{B}$. A particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{v}$ and $\vec{B}$. The SI unit of magnetic field is the tesla $(1\ \text{T} = 1\ \text{N/A}\cdot\text{m})$. (See Example 27.1.)

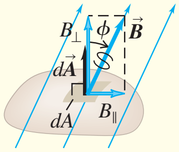
$$\vec{F} = q\vec{v} \times \vec{B} \qquad (27.2)$$



Magnetic field lines and flux: A magnetic field can be represented graphically by magnetic field lines. At each point a magnetic field line is tangent to the direction of $\vec{B}$ at that point. Where field lines are close together, the field magnitude is large, and vice versa. Magnetic flux Φ_B through an area is defined in an analogous way to electric flux; its SI unit is the weber $(1\ \text{Wb} = 1\ \text{T}\cdot\text{m}^2)$. The net magnetic flux through any closed surface is zero (Gauss's law for magnetism), so magnetic field lines always close on themselves. (See Example 27.2.)

$$\Phi_B = \int B\cos\phi\, dA = \int B_\perp\, dA = \int \vec{B}\cdot d\vec{A} \qquad (27.6)$$

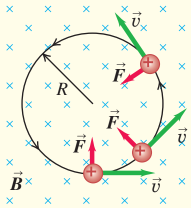
$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \text{(closed surface)} \qquad (27.8)$$



Motion in a magnetic field: The magnetic force is always perpendicular to $\vec{v}$; a particle moving under the action of a magnetic field alone moves with constant speed. In a uniform field, a particle with initial velocity perpendicular to the field moves in a circle with radius R that depends on the magnetic field strength B and the particle mass m, speed v, and charge q. (See Examples 27.3 and 27.4.)

Crossed electric and magnetic fields can be used as a velocity selector. The electric and magnetic forces exactly cancel when $v = E/B$. (See Examples 27.5 and 27.6.)

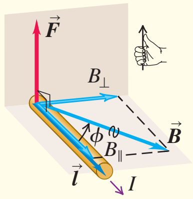
$$R = \frac{mv}{|q|B} \qquad (27.11)$$



Magnetic force on a conductor: A straight segment of a conductor carrying current I in a uniform magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{B}$ and the vector $\vec{l}$, which points in the direction of the current and has magnitude equal to the length of the segment. A similar relationship gives the force $d\vec{F}$ on an infinitesimal current-carrying segment $d\vec{l}$. (See Examples 27.7 and 27.8.)

$$\vec{F} = I\vec{l} \times \vec{B} \qquad (27.19)$$

$$d\vec{F} = I\, d\vec{l} \times \vec{B} \qquad (27.20)$$

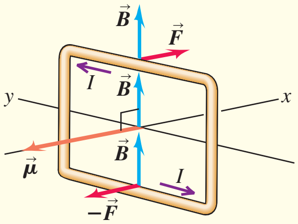


Magnetic torque: A current loop with area A and current I in a uniform magnetic field $\vec{B}$ experiences no net magnetic force, but does experience a magnetic torque of magnitude τ. The vector torque $\vec{\tau}$ can be expressed in terms of the magnetic moment $\vec{\mu} = I\vec{A}$ of the loop, as can the potential energy U of a magnetic moment in a magnetic field $\vec{B}$. The magnetic moment of a loop depends only on the current and the area; it is independent of the shape of the loop. (See Examples 27.9 and 27.10.)

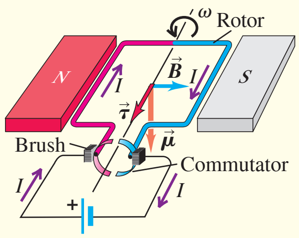
$$\tau = IBA\sin\phi \qquad (27.23)$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad (27.26)$$

$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi \qquad (27.27)$$

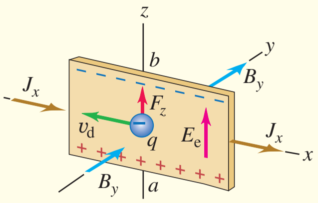


Electric motors: In a dc motor a magnetic field exerts a torque on a current in the rotor. Motion of the rotor through the magnetic field causes an induced emf called a back emf. For a series motor, in which the rotor coil is in series with coils that produce the magnetic field, the terminal voltage is the sum of the back emf and the potential drop Ir across the internal resistance. (See Example 27.11.)



The Hall effect: The Hall effect is a potential difference perpendicular to the direction of current in a conductor, when the conductor is placed in a magnetic field. The Hall potential is determined by the requirement that the associated electric field must just balance the magnetic force on a moving charge. Hall-effect measurements can be used to determine the sign of charge carriers and their concentration n. (See Example 27.12.)

$$nq = \frac{-J_x B_y}{E_z} \qquad (27.30)$$



Chapter 27 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE ✓ARIATION PROBLEMS

Be sure to review EXAMPLE 27.1 (Section 27.2) before attempting these problems.

VP27.1.1 An electron (charge -1.60×10^{-19} C) moves at 2.20×10^5 m/s through a uniform 1.55 T magnetic field that points in the $+y$-direction. The velocity of the electron lies in the xy-plane and is directed at 40.0° to the $+x$-axis and 50.0° to the $+y$-axis. Find (a) the magnitude and (b) the direction of the magnetic force on the electron.

VP27.1.2 A charged particle moving in the presence of a 0.750 T magnetic field experiences a magnetic force of magnitude 2.50×10^{-10} N. The particle is moving at 1.95×10^5 m/s at an angle of 36.0° to the direction of the magnetic field. Find the magnitude of the charge on the particle.

VP27.1.3 A particle (charge $+3.20 \times 10^{-19}$ C) with velocity $v_x = 3.75 \times 10^5$ m/s is in the presence of a $\vec{B}$ field with only a z-component. The magnetic force on the particle is 2.10×10^{-15} N in the $-y$-direction. Find (a) the magnitude and (b) the direction of $\vec{B}$.

VP27.1.4 Highly ionized iron atoms are found in the sun's outer atmosphere, or corona. One such ion has lost 13 electrons and so has charge $+13e = +2.08 \times 10^{-18}$ C. This ion experiences a magnetic force of 2.30×10^{-15} N when it moves at 7.30×10^3 m/s in the xz-plane, 36.9° from the $+x$-axis and 53.1° from the $+z$-axis. The magnetic field is in the $+z$-direction. Find (a) the magnitude of the magnetic field and (b) the direction of the force on the ion.

Be sure to review EXAMPLES 27.3 and 27.4 (Section 27.4) and EXAMPLES 27.5 and 27.6 (Section 27.5) before attempting these problems.

VP27.6.1 A typical magnetic field in sunspots (highly magnetized regions on the surface of the sun) is 0.300 T. For a proton (charge $+1.60 \times 10^{-19}$ C, mass 1.67×10^{-27} kg) moving at 1.25×10^4 m/s in a direction perpendicular to such a field, find (a) the radius of its circular orbit, (b) its angular speed in its orbit, and (c) the frequency of its orbital motion.

VP27.6.2 In the situation of the previous problem, suppose the 0.300 T magnetic field is in the $+y$-direction and the proton's motion is *not* perpendicular to the field: Initially its velocity has components $v_x = 1.00 \times 10^4$ m/s, $v_y = 7.50 \times 10^3$ m/s, $v_z = 0$. Find (a) the radius of the proton's helical path, (b) how far the proton moves along the helix axis per revolution, and (c) the magnitude of the magnetic force on the proton.

VP27.6.3 A velocity selector uses an electric field $\vec{E} = (2.80 \times 10^4\ \mathrm{N/C})\hat{\imath}$ and a magnetic field $\vec{B} = (0.0350\ \mathrm{T})\hat{k}$. (a) What particle speed will yield zero deflection? (b) In what direction should a charged particle travel through these fields to have zero deflection? (c) A positively charged particle travels through these fields with the speed you found in part (a) and the direction you found in part (b), but the magnetic-field magnitude is now greater than 0.0350 T. In what direction will the particle initially be deflected?

VP27.6.4 You send a beam of oxygen ions through a mass spectrometer like the one shown in Fig. 27.24. The nuclei emerge from the velocity selector at 2.00×10^4 m/s and encounter a magnetic field of magnitude $B' = 0.0500$ T. All the ions have charge $+1.60 \times 10^{-19}$ C, but some are the isotope ^{16}O (mass 2.66×10^{-26} kg) and some are the isotope ^{18}O (mass 2.99×10^{-26} kg). Find the radius of the semicircular path followed by (a) the ^{16}O ions and (b) the ^{18}O ions.

Be sure to review EXAMPLE 27.7 (Section 27.6) before attempting these problems.

VP27.7.1 A straight wire 0.150 m in length carries a current of 3.50 A in the $+x$-direction. The wire is in a uniform 0.0136 T magnetic field in the xy-plane that points in a direction 20.0° from the $-x$-axis and 70.0° from the $+y$-axis. Find (a) the magnitude and (b) the direction of the magnetic force on the wire.

VP27.7.2 A vertical straight wire 25.0 cm in length carries a current. You do not know either the magnitude of the current or whether the current is moving upward or downward. If there is a uniform horizontal magnetic field of 0.0350 T that points due north, the wire experiences a horizontal magnetic force to the west of 0.0140 N. Find (a) the magnitude and (b) the direction of the current.

VP27.7.3 A straight wire carries a current of 1.20 A. The length vector of the wire is $\vec{l} = (0.200\ \mathrm{m})\hat{\imath} + (-0.120\ \mathrm{m})\hat{\jmath}$, and the wire is in a uniform magnetic field $\vec{B} = (0.0175\ \mathrm{T})\hat{k}$. Find (a) the x-component, (b) the y-component, (c) the z-component, and (d) the magnitude of the magnetic force on the wire.

VP27.7.4 A straight wire 0.280 m in length carries a current of 3.40 A. What are the *two* angles between the direction of the current and the direction of a uniform 0.0400 T magnetic field for which the magnetic force on the wire has magnitude 0.0250 N?

BRIDGING PROBLEM Magnetic Torque on a Current-Carrying Ring

A circular ring with area 4.45 cm^2 is carrying a current of 12.5 A. The ring, initially at rest, is immersed in a region of uniform magnetic field given by $\vec{\boldsymbol{B}} = (1.15 \times 10^{-2}\ \text{T})(12\hat{\imath} + 3\hat{\jmath} - 4\hat{\boldsymbol{k}})$. The ring is positioned initially such that its magnetic moment is given by $\vec{\boldsymbol{\mu}}_{\text{i}} = \mu(-0.800\hat{\imath} + 0.600\hat{\jmath})$, where μ is the (positive) magnitude of the magnetic moment. (a) Find the initial magnetic torque on the ring. (b) The ring (which is free to rotate around one diameter) is released and turns through an angle of 90.0°, at which point its magnetic moment is given by $\vec{\boldsymbol{\mu}}_{\text{f}} = -\mu\hat{\boldsymbol{k}}$. Determine the decrease in potential energy. (c) If the moment of inertia of the ring about a diameter is $8.50 \times 10^{-7}\ \text{kg} \cdot \text{m}^2$, determine the angular speed of the ring as it passes through the second position.

SOLUTION GUIDE

IDENTIFY and SET UP

1. The current-carrying ring acts as a magnetic dipole, so you can use the equations for a magnetic dipole in a uniform magnetic field.
2. There are no nonconservative forces acting on the ring as it rotates, so the sum of its rotational kinetic energy (discussed in Section 9.4) and the potential energy is conserved.

EXECUTE

3. Use the vector expression for the torque on a magnetic dipole to find the answer to part (a). (*Hint:* Review Section 1.10.)
4. Find the change in potential energy from the first orientation of the ring to the second orientation.
5. Use your result from step 4 to find the rotational kinetic energy of the ring when it is in the second orientation.
6. Use your result from step 5 to find the ring's angular speed when it is in the second orientation.

EVALUATE

7. If the ring were free to rotate around *any* diameter, in what direction would the magnetic moment point when the ring is in a state of stable equilibrium?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q27.1 Can a charged particle move through a magnetic field without experiencing any force? If so, how? If not, why not?

Q27.2 At any point in space, the electric field $\vec{\boldsymbol{E}}$ is defined to be in the direction of the electric force on a positively charged particle at that point. Why don't we similarly define the magnetic field $\vec{\boldsymbol{B}}$ to be in the direction of the magnetic force on a moving, positively charged particle?

Q27.3 Section 27.2 describes a procedure for finding the direction of the magnetic force using your right hand. If you use the same procedure, but with your left hand, will you get the correct direction for the force? Explain.

Q27.4 The magnetic force on a moving charged particle is always perpendicular to the magnetic field $\vec{\boldsymbol{B}}$. Is the trajectory of a moving charged particle always perpendicular to the magnetic field lines? Explain your reasoning.

Q27.5 A charged particle is fired into a cubical region of space where there is a uniform magnetic field. Outside this region, there is no magnetic field. Is it possible that the particle will remain inside the cubical region? Why or why not?

Q27.6 If the magnetic force does no work on a charged particle, how can it have any effect on the particle's motion? Are there other examples of forces that do no work but have a significant effect on a particle's motion?

Q27.7 A charged particle moves through a region of space with constant velocity (magnitude and direction). If the external magnetic field is zero in this region, can you conclude that the external electric field in the region is also zero? Explain. (By "external" we mean fields other than those produced by the charged particle.) If the external electric field is zero in the region, can you conclude that the external magnetic field in the region is also zero?

Q27.8 How might a loop of wire carrying a current be used as a compass? Could such a compass distinguish between north and south? Why or why not?

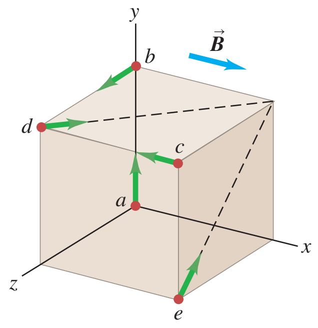
Q27.9 How could the direction of a magnetic field be determined by making only *qualitative* observations of the magnetic force on a straight wire carrying a current?

Q27.10 A loose, floppy loop of wire is carrying current I. The loop of wire is placed on a horizontal table in a uniform magnetic field $\vec{\boldsymbol{B}}$ perpendicular to the plane of the table. This causes the loop of wire to expand into a circular shape while still lying on the table. In a diagram, show all possible orientations of the current I and magnetic field $\vec{\boldsymbol{B}}$ that could cause this to occur. Explain your reasoning.

Q27.11 Several charges enter a uniform magnetic field directed into the page. (a) What path would a positive charge q moving with a velocity of magnitude v follow through the field? (b) What path would a positive charge q moving with a velocity of magnitude $2v$ follow through the field? (c) What path would a negative charge $-q$ moving with a velocity of magnitude v follow through the field? (d) What path would a neutral particle follow through the field?

Q27.12 Each of the lettered points at the corners of the cube in **Fig. Q27.12** represents a positive charge q moving with a velocity of magnitude v in the direction indicated. The region in the figure is in a uniform magnetic field $\vec{\boldsymbol{B}}$, parallel to the x-axis and directed toward the right. Which charges experience a force due to $\vec{\boldsymbol{B}}$? What is the direction of the force on each charge?

Figure **Q27.12**



Q27.13 A student claims that if lightning strikes a metal flagpole, the force exerted by the earth's magnetic field on the current in the pole can be large enough to bend it. Typical lightning currents are of the order of 10^4 to 10^5 A. Is the student's opinion justified? Explain your reasoning.

Q27.14 Could an accelerator be built in which *all* the forces on the particles, for steering and for increasing speed, are magnetic forces? Why or why not?

Q27.15 The magnetic force acting on a charged particle can never do work because at every instant the force is perpendicular to the velocity. The torque exerted by a magnetic field can do work on a current loop when the loop rotates. Explain how these seemingly contradictory statements can be reconciled.

Q27.16 When the polarity of the voltage applied to a dc motor is reversed, the direction of motion does *not* reverse. Why not? How *could* the direction of motion be reversed?

Q27.17 In a Hall-effect experiment, is it possible that *no* transverse potential difference will be observed? Under what circumstances might this happen?

Q27.18 Hall-effect voltages are much greater for relatively poor conductors (such as germanium) than for good conductors (such as copper), for comparable currents, fields, and dimensions. Why?

EXERCISES

Section 27.2 Magnetic Field

27.1 • A particle with a charge of -1.24×10^{-8} C is moving with instantaneous velocity $\vec{\boldsymbol{v}} = (4.19 \times 10^4\ \text{m/s})\hat{\boldsymbol{\imath}} + (-3.85 \times 10^4\ \text{m/s})\hat{\boldsymbol{\jmath}}$. What is the force exerted on this particle by a magnetic field (a) $\vec{\boldsymbol{B}} = (1.40\ \text{T})\hat{\boldsymbol{\imath}}$ and (b) $\vec{\boldsymbol{B}} = (1.40\ \text{T})\hat{\boldsymbol{k}}$?

27.2 • A particle of mass 0.195 g carries a charge of -2.50×10^{-8} C. The particle is given an initial horizontal velocity that is due north and has magnitude 4.00×10^4 m/s. What are the magnitude and direction of the minimum magnetic field that will keep the particle moving in the earth's gravitational field in the same horizontal, northward direction?

27.3 • In a 1.25 T magnetic field directed vertically upward, a particle having a charge of magnitude 8.50 μC and initially moving northward at 4.75 km/s is deflected toward the east. (a) What is the sign of the charge of this particle? Make a sketch to illustrate how you found your answer. (b) Find the magnetic force on the particle.

27.4 • A particle with mass 1.81×10^{-3} kg and a charge of 1.22×10^{-8} C has, at a given instant, a velocity $\vec{\boldsymbol{v}} = (3.00 \times 10^4\ \text{m/s})\hat{\boldsymbol{\jmath}}$. What are the magnitude and direction of the particle's acceleration produced by a uniform magnetic field $\vec{\boldsymbol{B}} = (1.63\ \text{T})\hat{\boldsymbol{\imath}} + (0.980\ T)\hat{\boldsymbol{\jmath}}$?

27.5 • An electron is moving in the *xy*-plane. If at time t a magnetic field $B = 0.200$ T in the $+z$-direction exerts a force on the electron equal to $F = 5.50 \times 10^{-18}$ N in the $-y$-direction, what is the velocity (magnitude and direction) of the electron at this instant?

27.6 • An electron moves at 1.40×10^6 m/s through a region in which there is a magnetic field of unspecified direction and magnitude 7.40×10^{-2} T. (a) What are the largest and smallest possible magnitudes of the acceleration of the electron due to the magnetic field? (b) If the actual acceleration of the electron is one-fourth of the largest magnitude in part (a), what is the angle between the electron velocity and the magnetic field?

27.7 •• A group of particles is traveling in a magnetic field of unknown magnitude and direction. You observe that a proton moving at 1.50 km/s in the $+x$-direction experiences a force of 2.25×10^{-16} N in the $+y$-direction, and an electron moving at 4.75 km/s in the $-z$-direction experiences a force of 8.50×10^{-16} N in the $+y$-direction. (a) What are the magnitude and direction of the magnetic field? (b) What are the magnitude and direction of the magnetic force on an electron moving in the $-y$-direction at 3.20 km/s?

27.8 •• **CP** A particle with charge -5.60 nC is moving in a uniform magnetic field $\vec{\boldsymbol{B}} = -(1.25\ \text{T})\hat{\boldsymbol{k}}$. The magnetic force on the particle is measured to be $\vec{\boldsymbol{F}} = -(3.40 \times 10^{-7}\ \text{N})\hat{\boldsymbol{\imath}} + (7.40 \times 10^{-7}\ \text{N})\hat{\boldsymbol{\jmath}}$. (a) Calculate all the components of the velocity of the particle that you can from this information. (b) Are there components of the velocity that are not determined by the measurement of the force? Explain. (c) Calculate the scalar product $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{F}}$. What is the angle between $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{F}}$?

Section 27.3 Magnetic Field Lines and Magnetic Flux

27.9 • A circular area with a radius of 6.50 cm lies in the *xy*-plane. What is the magnitude of the magnetic flux through this circle due to a uniform magnetic field $B = 0.230$ T (a) in the $+z$-direction; (b) at an angle of 53.1° from the $+z$-direction; (c) in the $+y$-direction?

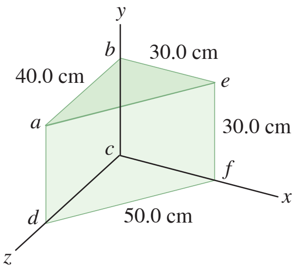
27.10 • A flat, square surface with side length 3.40 cm is in the *xy*-plane at $z = 0$. Calculate the magnitude of the flux through this surface produced by a magnetic field $\vec{\boldsymbol{B}} = (0.200\ \text{T})\hat{\boldsymbol{\imath}} + (0.300\ \text{T})\hat{\boldsymbol{\jmath}} - (0.500\ \text{T})\hat{\boldsymbol{k}}$.

27.11 •• An open plastic soda bottle with an opening diameter of 2.5 cm is placed on a table. A uniform 1.75 T magnetic field directed upward and oriented 25° from vertical encompasses the bottle. What is the total magnetic flux through the plastic of the soda bottle?

27.12 • A horizontal rectangular surface has dimensions 2.80 cm by 3.20 cm and is in a uniform magnetic field that is directed at an angle of 30.0° above the horizontal. What must the magnitude of the magnetic field be to produce a flux of 3.10×10^{-4} Wb through the surface?

27.13 •• The magnetic field $\vec{\boldsymbol{B}}$ in a certain region is 0.128 T, and its direction is that of the $+z$-axis in **Fig. E27.13**. (a) What is the magnetic flux across the surface *abcd* in the figure? (b) What is the magnetic flux across the surface *befc*? (c) What is the magnetic flux across the surface *aefd*? (d) What is the net flux through all five surfaces that enclose the shaded volume?

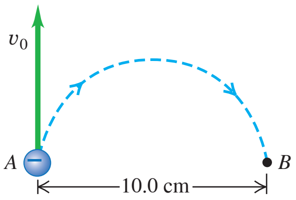
Figure **E27.13**



Section 27.4 Motion of Charged Particles in a Magnetic Field

27.14 •• An electron at point A in **Fig. E27.14** has a speed v_0 of 1.41×10^6 m/s. Find (a) the magnitude and direction of the magnetic field that will cause the electron to follow the semicircular path from A to B, and (b) the time required for the electron to move from A to B.

Figure **E27.14**



27.15 •• Repeat Exercise 27.14 for the case in which the particle is a proton rather than an electron.

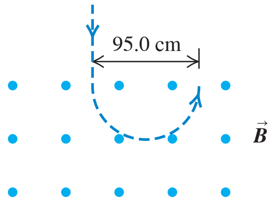
27.16 • An alpha particle (a He nucleus, containing two protons and two neutrons and having a mass of 6.64×10^{-27} kg) traveling horizontally at 35.6 km/s enters a uniform, vertical, 1.80 T magnetic field. (a) What is the diameter of the path followed by this alpha particle? (b) What effect does the magnetic field have on the speed of the particle? (c) What are the magnitude and direction of the acceleration of the alpha particle while it is in the magnetic field? (d) Explain why the speed of the particle does not change even though an unbalanced external force acts on it.

27.17 • **CP** A 150 g ball containing 4.00×10^8 excess electrons is dropped into a 125 m vertical shaft. At the bottom of the shaft, the ball suddenly enters a uniform horizontal magnetic field that has magnitude 0.250 T and direction from east to west. If air resistance is negligibly small, find the magnitude and direction of the force that this magnetic field exerts on the ball just as it enters the field.

27.18 • **BIO** Cyclotrons are widely used in nuclear medicine for producing short-lived radioactive isotopes. These cyclotrons typically accelerate H^- (the *hydride* ion, which has one proton and two electrons) to an energy of 5 MeV to 20 MeV. This ion has a mass very close to that of a proton because the electron mass is negligible—about $\frac{1}{2000}$ of the proton's mass. A typical magnetic field in such cyclotrons is 1.9 T. (a) What is the speed of a 5.0 MeV H^-? (b) If the H^- has energy 5.0 MeV and $B = 1.9$ T, what is the radius of this ion's circular orbit?

27.19 • In an experiment with cosmic rays, a vertical beam of particles that have charge of magnitude $3e$ and mass 12 times the proton mass enters a uniform horizontal magnetic field of 0.250 T and is bent in a semicircle of diameter 95.0 cm, as shown in **Fig. E27.19**. (a) Find the speed of the particles and the sign of their charge. (b) Is it reasonable to ignore the gravity force on the particles? (c) How does the speed of the particles as they enter the field compare to their speed as they exit the field?

Figure **E27.19**

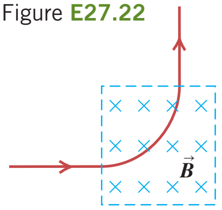


27.20 • A deuteron (the nucleus of an isotope of hydrogen) has a mass of 3.34×10^{-27} kg and a charge of $+e$. The deuteron travels in a circular path with a radius of 6.96 mm in a magnetic field with magnitude 2.50 T. (a) Find the speed of the deuteron. (b) Find the time required for it to make half a revolution. (c) Through what potential difference would the deuteron have to be accelerated to acquire this speed?

27.21 • An electron in the beam of a cathode-ray tube is accelerated by a potential difference of 2.00 kV. Then it passes through a region of transverse magnetic field, where it moves in a circular arc with radius 0.180 m. What is the magnitude of the field?

27.22 •• A beam of protons traveling at 1.20 km/s enters a uniform magnetic field, traveling perpendicular to the field. The beam exits the magnetic field, leaving the field in a direction perpendicular to its original direction (**Fig. E27.22**). The beam travels a distance of 1.18 cm *while in the field*. What is the magnitude of the magnetic field?

Figure **E27.22**



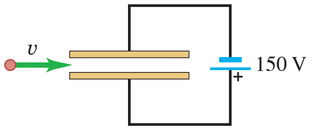
Section 27.5 Applications of Motion of Charged Particles

27.23 • **Crossed $\vec{E}$ and $\vec{B}$ Fields.** A particle with initial velocity $\vec{v}_0 = (5.85 \times 10^3 \text{m/s})\hat{\jmath}$ enters a region of uniform electric and magnetic fields. The magnetic field in the region is $\vec{B} = -(1.35\text{ T})\hat{k}$. Calculate the magnitude and direction of the electric field in the region if the particle is to pass through undeflected, for a particle of charge (a) +0.640 nC and (b) −0.320 nC. You can ignore the weight of the particle.

27.24 • (a) What is the speed of a beam of electrons when the simultaneous influence of an electric field of 1.56×10^4 V/m and a magnetic field of 4.62×10^{-3} T, with both fields normal to the beam and to each other, produces no deflection of the electrons? (b) In a diagram, show the relative orientation of the vectors $\vec{v}$, $\vec{E}$, and $\vec{B}$. (c) When the electric field is removed, what is the radius of the electron orbit? What is the period of the orbit?

27.25 •• A 150 V battery is connected across two parallel metal plates of area 28.5 cm^2 and separation 8.20 mm. A beam of alpha particles (charge $+2e$, mass 6.64×10^{-27} kg) is accelerated from rest through a potential difference of 1.75 kV and enters the region between the plates perpendicular to the electric field, as shown in **Fig. E27.25**. What magnitude and direction of magnetic field are needed so that the alpha particles emerge undeflected from between the plates?

Figure **E27.25**



27.26 • In the Bainbridge mass spectrometer (see Fig. 27.22), the magnetic-field magnitude in the velocity selector is 0.510 T, and ions having a speed of 1.82×10^6 m/s pass through undeflected. (a) What is the electric-field magnitude in the velocity selector? (b) If the separation of the plates is 5.20 mm, what is the potential difference between the plates?

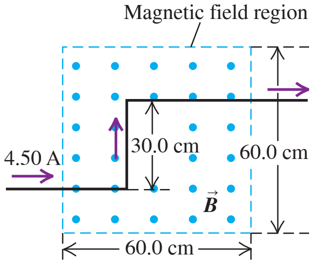
27.27 • Singly ionized (one electron removed) atoms are accelerated and then passed through a velocity selector consisting of perpendicular electric and magnetic fields. The electric field is 155 V/m and the magnetic field is 0.0315 T. The ions next enter a uniform magnetic field of magnitude 0.0175 T that is oriented perpendicular to their velocity. (a) How fast are the ions moving when they emerge from the velocity selector? (b) If the radius of the path of the ions in the second magnetic field is 17.5 cm, what is their mass?

27.28 •• **BIO** **Ancient Meat Eating.** The amount of meat in prehistoric diets can be determined by measuring the ratio of the isotopes ^{15}N to ^{14}N in bone from human remains. Carnivores concentrate ^{15}N, so this ratio tells archaeologists how much meat was consumed. For a mass spectrometer that has a path radius of 12.5 cm for ^{12}C ions (mass 1.99×10^{-26} kg), find the separation of the ^{14}N (mass 2.32×10^{-26} kg) and ^{15}N (mass 2.49×10^{-26} *kg*) isotopes at the detector.

Section 27.6 Magnetic Force on a Current-Carrying Conductor

27.29 •• A long wire carrying 4.50 A of current makes two 90° bends, as shown in **Fig. E27.29**. The bent part of the wire passes through a uniform 0.240 T magnetic field directed as shown in the figure and confined to a limited region of space. Find the magnitude and direction of the force that the magnetic field exerts on the wire.

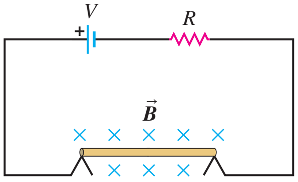
Figure **E27.29**



27.30 • A straight, 2.5 m wire carries a typical household current of 1.5 A (in one direction) at a location where the earth's magnetic field is 0.55 gauss from south to north. Find the magnitude and direction of the force that our planet's magnetic field exerts on this wire if it is oriented so that the current in it is running (a) from west to east, (b) vertically upward, (c) from north to south. (d) Is the magnetic force ever large enough to cause significant effects under normal household conditions?

27.31 • A thin, 50.0-cm-long metal bar with mass 750 g rests on, but is not attached to, two metallic supports in a uniform 0.450 T magnetic field, as shown in **Fig. E27.31**. A battery and a 25.0 Ω resistor in series are connected to the supports. (a) What is the highest voltage the battery can have without breaking the circuit at the supports? (b) The battery voltage has the maximum value calculated in part (a). If the resistor suddenly gets partially short-circuited, decreasing its resistance to 2.0 Ω, find the initial acceleration of the bar.

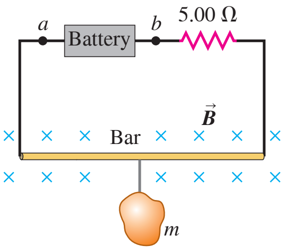
Figure **E27.31**



27.32 •• An electromagnet produces a magnetic field of 0.550 T in a cylindrical region of radius 2.50 cm between its poles. A straight wire carrying a current of 10.8 A passes through the center of this region and is perpendicular to both the axis of the cylindrical region and the magnetic field. What magnitude of force does this field exert on the wire?

27.33 • **Magnetic Balance.** The circuit shown in **Fig. E27.33** is used to make a magnetic balance to weigh objects. The mass *m* to be measured is hung from the center of the bar that is in a uniform magnetic field of 1.50 T, directed into the plane of the figure. The battery voltage can be adjusted to vary the current in the circuit. The horizontal bar is 60.0 cm long and is made of extremely lightweight material. It is connected to the battery by thin vertical wires that can support no appreciable tension; all the weight of the suspended mass *m* is supported by the magnetic force on the bar. A resistor with $R = 5.00\ \Omega$ is in series with the bar; the resistance of the rest of the circuit is much less than this. (a) Which point, *a* or *b*, should be the positive terminal of the battery? (b) If the maximum terminal voltage of the battery is 175 V, what is the greatest mass *m* that this instrument can measure?

Figure **E27.33**



27.34 • A straight, vertical wire carries a current of 2.60 A downward in a region between the poles of a large superconducting electromagnet, where the magnetic field has magnitude $B = 0.588$ T and is horizontal. What are the magnitude and direction of the magnetic force on a 1.00 cm section of the wire that is in this uniform magnetic field, if the magnetic field direction is (a) east; (b) south; (c) 30.0° south of west?

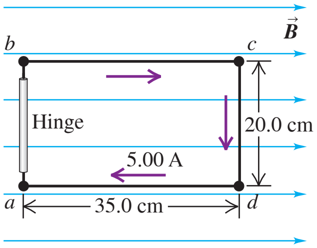
Section 27.7 Force and Torque on a Current Loop

27.35 • A flat circular coil carrying a current of 8.80 A has a magnetic dipole moment of 0.194 A · m^2 to the left. Its area vector $\vec{A}$ is 4.0 cm^2 to the left. (a) How many turns does the coil have? (b) An observer is on the coil's axis to the left of the coil and is looking toward the coil. Does the observer see a clockwise or counterclockwise current? (c) If a huge 45.0 T external magnetic field directed out of the paper is applied to the coil, what torque (magnitude and direction) results?

27.36 •• The plane of a 5.0 cm × 8.0 cm rectangular loop of wire is parallel to a 0.19 T magnetic field. The loop carries a current of 6.2 A. (a) What torque acts on the loop? (b) What is the magnetic moment of the loop? (c) What is the maximum torque that can be obtained with the same total length of wire carrying the same current in this magnetic field?

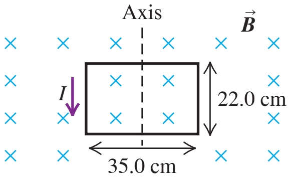
27.37 • The 20.0 cm × 35.0 cm rectangular circuit shown in **Fig. E27.37** is hinged along side *ab*. It carries a clockwise 5.00 A current and is located in a uniform 1.20 T magnetic field oriented perpendicular to two of its sides, as shown. (a) Draw a clear diagram showing the direction of the force that the magnetic field exerts on each segment of the circuit (*ab*, *bc*, etc.). (b) Of the four forces you drew in part (a), decide which ones exert a torque about the hinge *ab*. Then calculate only those forces that exert this torque. (c) Use your results from part (b) to calculate the torque that the magnetic field exerts on the circuit about the hinge axis *ab*.

Figure **E27.37**



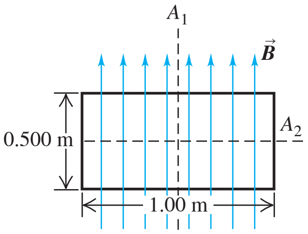
27.38 • A rectangular coil of wire, 22.0 cm by 35.0 cm and carrying a current of 1.95 A, is oriented with the plane of its loop perpendicular to a uniform 1.50 T magnetic field (**Fig. E27.38**). (a) Calculate the net force and torque that the magnetic field exerts on the coil. (b) The coil is rotated through a 30.0° angle about the axis shown, with the left side coming out of the plane of the figure and the right side going into the plane. Calculate the net force and torque that the magnetic field now exerts on the coil. (*Hint:* To visualize this three-dimensional problem, make a careful drawing of the coil as viewed along the rotation axis.)

Figure **E27.38**



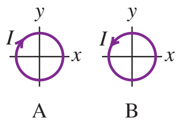
27.39 • **CP** A uniform rectangular coil of total mass 212 g and dimensions 0.500 m × 1.00 m is oriented with its plane parallel to a uniform 3.00 T magnetic field (**Fig. E27.39**). A current of 2.00 A is suddenly started in the coil. (a) About which axis (A_1 or A_2) will the coil begin to rotate? Why? (b) Find the initial angular acceleration of the coil just after the current is started.

Figure **E27.39**



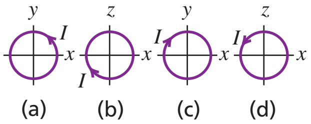
27.40 • Both circular coils A and B (**Fig. E27.40**) have area *A* and *N* turns. They are free to rotate about a diameter that coincides with the *x*-axis. Current *I* circulates in each coil in the direction shown. There is a uniform magnetic field $\vec{B}$ in the $+z$-direction. (a) What is the direction of the magnetic moment $\vec{\mu}$ for each coil? (b) Explain why the torque on both coils due to the magnetic field is zero, so the coil is in rotational equilibrium. (c) Use Eq. (27.27) to calculate the potential energy for each coil. (d) For each coil, is the equilibrium stable or unstable? Explain.

Figure **E27.40**



27.41 • A circular coil with area *A* and *N* turns is free to rotate about a diameter that coincides with the *x*-axis. Current *I* is circulating in the coil. There is a uniform magnetic field $\vec{B}$ in the positive *y*-direction. Calculate the magnitude and direction of the torque $\vec{\tau}$ and the value of the potential energy *U*, as given in Eq. (27.27), when the coil is oriented as shown in parts (a) through (d) of **Fig. E27.41**.

Figure **E27.41**

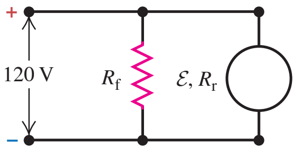


27.42 •• A coil with magnetic moment 1.45 A · m^2 is oriented initially with its magnetic moment antiparallel to a uniform 0.835 T magnetic field. What is the change in potential energy of the coil when it is rotated 180° so that its magnetic moment is parallel to the field?

Section 27.8 The Direct-Current Motor

27.43 •• In a shunt-wound dc motor with the field coils and rotor connected in parallel (**Fig. E27.43**), the resistance R_f of the field coils is 106 Ω, and the resistance R_r of the rotor is 5.9 Ω. When a potential difference of 120 V is applied to the brushes and the motor is running at full speed delivering mechanical power, the current supplied to it is 4.82 A. (a) What is the current in the field coils? (b) What is the current in the rotor? (c) What is the induced emf developed by the motor? (d) How much mechanical power is developed by this motor?

Figure **E27.43**

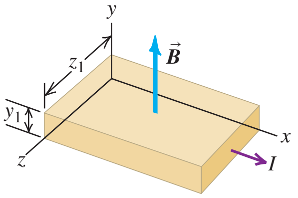


27.44 • A dc motor with its rotor and field coils connected in series has an internal resistance of 3.2 Ω. When the motor is running at full load on a 120 V line, the emf in the rotor is 105 V. (a) What is the current drawn by the motor from the line? (b) What is the power delivered to the motor? (c) What is the mechanical power developed by the motor?

Section 27.9 The Hall Effect

27.45 • Figure E27.45 shows a portion of a silver ribbon with $z_1 = 11.8$ mm and $y_1 = 0.23$ mm, carrying a current of 120 A in the $+x$-direction. The ribbon lies in a uniform magnetic field, in the y-direction, with magnitude 0.95 T. Apply the simplified model of the Hall effect presented in Section 27.9. If there are 5.85×10^{28} free electrons per cubic meter, find (a) the magnitude of the drift velocity of the electrons in the x-direction; (b) the magnitude and direction of the electric field in the z-direction due to the Hall effect; (c) the Hall emf.

Figure **E27.45**



27.46 • Let Fig. E27.45 represent a strip of an unknown metal of the same dimensions as those of the silver ribbon in Exercise 27.45. When the magnetic field is 2.29 T and the current is 78.0 A, the Hall emf is found to be 131 μV. What does the simplified model of the Hall effect presented in Section 27.9 give for the density of free electrons in the unknown metal?

PROBLEMS

27.47 •• CP A small particle with positive charge $q = +3.75 \times 10^{-4}$ C and mass $m = 5.00 \times 10^{-5}$ kg is moving in a region of uniform electric and magnetic fields. The magnetic field is $B = 4.00$ T in the $+z$-direction. The electric field is also in the $+z$-direction and has magnitude $E = 60.0$ N/C. At time $t = 0$ the particle is on the y-axis at $y = +1.00$ m and has velocity $v = 30.0$ m/s in the $+x$-direction. Neglect gravity. (a) What are the x-, y-, and z-coordinates of the particle at $t = 0.0200$ s? (b) What is the speed of the particle at $t = 0.0200$ s?

27.48 • A particle with charge 7.26×10^{-8} C is moving in a region where there is a uniform 0.650 T magnetic field in the $+x$-direction. At a particular instant, the velocity of the particle has components $v_x = -1.68 \times 10^4$ m/s, $v_y = -3.11 \times 10^4$ m/s, and $v_z = 5.85 \times 10^4$ m/s. What are the components of the force on the particle at this time?

27.49 ••• CP Fusion Reactor. If two deuterium nuclei (charge $+e$, mass 3.34×10^{-27} kg) get close enough together, the attraction of the strong nuclear force will fuse them to make an isotope of helium, releasing vast amounts of energy. The range of this force is about 10^{-15} m. This is the principle behind the fusion reactor. The deuterium nuclei are moving much too fast to be contained by physical walls, so they are confined magnetically. (a) How fast would two nuclei have to move so that in a head-on collision they would get close enough to fuse? (Assume their speeds are equal. Treat the nuclei as point charges, and assume that a separation of 1.0×10^{-15} m is required for fusion.) (b) What strength magnetic field is needed to make deuterium nuclei with this speed travel in a circle of diameter 2.50 m?

27.50 •• Magnetic Moment of the Hydrogen Atom. In the Bohr model of the hydrogen atom (see Section 39.3), in the lowest energy state the electron orbits the proton at a speed of 2.2×10^6 m/s in a circular orbit of radius 5.3×10^{-11} m. (a) What is the orbital period of the electron? (b) If the orbiting electron is considered to be a current loop, what is the current I? (c) What is the magnetic moment of the atom due to the motion of the electron?

27.51 •• You measure the charge-to-mass ratio q/m for a particle with positive charge in the following way: The particle starts from rest, is accelerated through a potential difference ΔV, and attains a velocity with magnitude v. It then enters a region of uniform magnetic field $B = 0.200$ T that is directed perpendicular to the velocity; the particle moves in a path that is an arc of a circle of radius R. You measure R as a function of ΔV. You plot your data as R^2 (in units of m^2) versus ΔV (in V) and find that the values lie close to a straight line that has slope 1.04×10^{-6} m^2/V. What is the value of q/m for this particle?

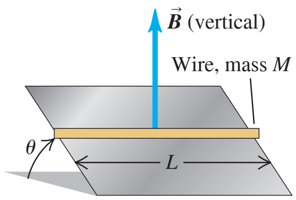
27.52 • The magnetic poles of a small cyclotron produce a magnetic field with magnitude 0.85 T. The poles have a radius of 0.40 m, which is the maximum radius of the orbits of the accelerated particles. (a) What is the maximum energy to which protons ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) can be accelerated by this cyclotron? Give your answer in electron volts and in joules. (b) What is the time for one revolution of a proton orbiting at this maximum radius? (c) What would the magnetic-field magnitude have to be for the maximum energy to which a proton can be accelerated to be twice that calculated in part (a)? (d) For $B = 0.85$ T, what is the maximum energy to which alpha particles ($q = 3.20 \times 10^{-19}$ C, $m = 6.64 \times 10^{-27}$ kg) can be accelerated by this cyclotron? How does this compare to the maximum energy for protons?

27.53 •• Suppose the electric field between the plates in Fig. 27.22 is 1.88×10^4 V/m and the magnetic field in both regions is 0.682 T. If the source contains the three isotopes of krypton, ^{82}Kr, ^{84}Kr, and ^{86}Kr, and the ions are singly charged, find the distance between the lines formed by the three isotopes on the particle detector. Assume the atomic masses of the isotopes (in atomic mass units) are equal to their mass numbers, 82, 84, and 86. (One atomic mass unit $= 1$ u $= 1.66 \times 10^{-27}$ kg.)

27.54 •• Mass Spectrograph. A mass spectrograph is used to measure the masses of ions, or to separate ions of different masses (see Section 27.5). In one design for such an instrument, ions with mass m and charge q are accelerated through a potential difference V. They then enter a uniform magnetic field that is perpendicular to their velocity, and they are deflected in a semicircular path of radius R. A detector measures where the ions complete the semicircle and from this it is easy to calculate R. (a) Derive the equation for calculating the mass of the ion from measurements of B, V, R, and q. (b) What potential difference V is needed so that singly ionized ^{12}C atoms will have $R = 50.0$ cm in a 0.150 T magnetic field? (c) Suppose the beam consists of a mixture of ^{12}C and ^{14}C ions. If v and B have the same values as in part (b), calculate the separation of these two isotopes at the detector. Do you think that this beam separation is sufficient for the two ions to be distinguished? (Make the assumption described in Problem 27.53 for the masses of the ions.)

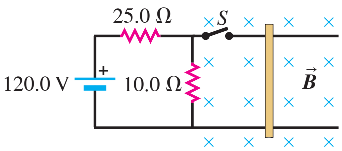
27.55 •• A straight piece of conducting wire with mass M and length L is placed on a frictionless incline tilted at an angle θ from the horizontal (**Fig. P27.55**). There is a uniform, vertical magnetic field $\vec{B}$ at all points (produced by an arrangement of magnets not shown in the figure). To keep the wire from sliding down the incline, a voltage source is attached to the ends of the wire. When just the right amount of current flows through the wire, the wire remains at rest. Determine the magnitude and direction of the current in the wire that will cause the wire to remain at rest. Copy the figure and draw the direction of the current on your copy. In addition, show in a free-body diagram all the forces that act on the wire.

Figure **P27.55**



27.56 •• **CP** A 2.60 N metal bar, 0.850 m long and having a resistance of 10.0 Ω, rests horizontally on conducting wires connecting it to the circuit shown in **Fig. P27.56**. The bar is in a uniform, horizontal, 1.60 T magnetic field and is not attached to the wires in the circuit. What is the acceleration of the bar just after the switch S is closed?

Figure **P27.56**

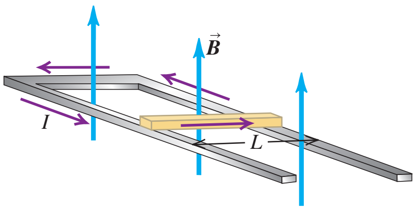


27.57 •• **BIO** **Determining Diet.** One method for determining the amount of corn in early Native American diets is the *stable isotope ratio analysis* (SIRA) technique. As corn photosynthesizes, it concentrates the isotope carbon-13, whereas most other plants concentrate carbon-12. Overreliance on corn consumption can then be correlated with certain diseases, because corn lacks the essential amino acid lysine. Archaeologists use a mass spectrometer to separate the ^{12}C and ^{13}C isotopes in samples of human remains. Suppose you use a velocity selector to obtain singly ionized (missing one electron) atoms of speed 8.50 km/s, and you want to bend them within a uniform magnetic field in a semicircle of diameter 25.0 cm for the ^{12}C. The measured masses of these isotopes are 1.99×10^{-26} kg (^{12}C) and 2.16×10^{-26} kg (^{13}C). (a) What strength of magnetic field is required? (b) What is the diameter of the ^{13}C semicircle? (c) What is the separation of the ^{12}C and ^{13}C ions at the detector at the end of the semicircle? Is this distance large enough to be easily observed?

27.58 •• **CP** A plastic circular loop has radius R, and a positive charge q is distributed uniformly around the circumference of the loop. The loop is then rotated around its central axis, perpendicular to the plane of the loop, with angular speed ω. If the loop is in a region where there is a uniform magnetic field $\vec{B}$ directed parallel to the plane of the loop, calculate the magnitude of the magnetic torque on the loop.

27.59 •• **CP** **An Electromagnetic Rail Gun.** A conducting bar with mass m and length L slides over horizontal rails that are connected to a voltage source. The voltage source maintains a constant current I in the rails and bar, and a constant, uniform, vertical magnetic field $\vec{B}$ fills the region between the rails (**Fig. P27.59**). (a) Find the magnitude and direction of the net force on the conducting bar. Ignore friction, air resistance, and electrical resistance. (b) If the bar has mass m, find the distance d that the bar must move along the rails from rest to attain speed v. (c) It has been suggested that rail guns based on this principle could accelerate payloads into earth orbit or beyond. Find the distance the bar must travel along the rails if it is to reach the escape speed for the earth (11.2 km/s). Let $B = 0.80$ T, $I = 2.0 \times 10^3$ A, $m = 25$ kg, and $L = 50$ cm. For simplicity assume the net force on the object is equal to the magnetic force, as in parts (a) and (b), even though gravity plays an important role in an actual launch in space.

Figure **P27.59**



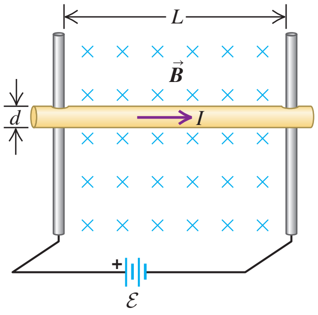
27.60 • A wire 25.0 cm long lies along the z-axis and carries a current of 7.40 A in the $+z$-direction. The magnetic field is uniform and has components $B_x = -0.242$ T, $B_y = -0.985$ T, and $B_z = -0.336$ T. (a) Find the components of the magnetic force on the wire. (b) What is the magnitude of the net magnetic force on the wire?

27.61 •• **CALC** Point a is on the $+y$-axis at $y = +0.200$ m and point b is on the $+x$-axis at $x = +0.200$ m. A wire in the shape of a circular arc of radius 0.200 m and centered on the origin goes from a to b and carries current $I = 5.00$ A in the direction from a to b. (a) If the wire is in a uniform magnetic field $B = 0.800$ T in the $+z$-direction, what are the magnitude and direction of the net force that the magnetic field exerts on the wire segment? (b) What are the magnitude and direction of the net force on the wire if the field is $B = 0.800$ T in the $+x$-direction?

27.62 •• Ceiling fans use electric motors that involve stationary permanent magnets (called stators) attached to the central hub and with a typical strength of 1.0 T, that supply torque to current-carrying coils (called rotors) fixed to the fan blades. Think of a ceiling fan at your home or workplace. (a) Estimate the diameter of the central hub at the inner edge of the blades. (b) If 12 rotors are fixed around the central hub, and if the diameters of the rotors make up one-third of the circumference of the hub, what is the diameter of a single rotor? (c) A typical ceiling fan supplies 1.4 N · m of torque. If this torque is supplied equally by the 12 rotors in the presence of 1.0 T stator fields parallel to the axis of each rotor, estimate the magnitude of the magnetic moment of each rotor. (d) The rotors are connected in parallel, so each receives $\frac{1}{12}$ of the fan current, which is typically 0.75 A. Estimate the number of windings in each rotor.

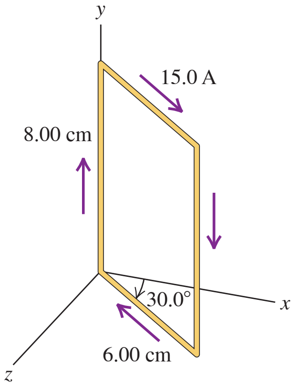
27.63 •• Earth's magnetic field near the ground is typically 0.50 G (0.50 gauss), where 1 G = 10^{-4} tesla. In temperate northern latitudes, this field is inclined downward at an angle of approximately 45°. Consider the feasibility of using the earth's magnetic field to enable the levitation device shown in **Fig. P27.63** (next page). (a) Estimate the current needed to lift a copper bar with diameter $d = 5.0$ mm using the horizontal component of the earth's magnetic field. The density of copper is 8900 kg/m^3. (b) A copper bar with diameter 5.0 mm will melt if it carries a current greater than the fusing current of 900 A. Is it feasible for our device? (c) Estimate the minimum strength of the magnetic field needed to levitate our copper bar. (d) Suppose we use easily obtainable 1.0 T permanent magnets to supply the horizontal magnetic field and suppose our bar has length 10 cm. Estimate how much extra weight we could levitate with our device using a current of 10 A.

Figure **P27.63**



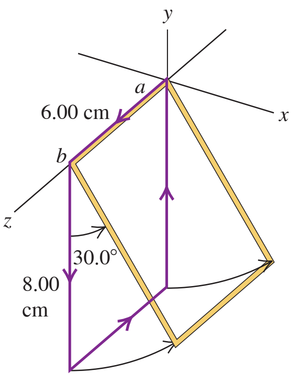
27.64 •• The rectangular loop shown in **Fig. P27.64** is pivoted about the *y*-axis and carries a current of 15.0 A in the direction indicated. (a) If the loop is in a uniform magnetic field with magnitude 0.48 T in the $+x$-direction, find the magnitude and direction of the torque required to hold the loop in the position shown. (b) Repeat part (a) for the case in which the field is in the $-z$-direction. (c) For each of the above magnetic fields, what torque would be required if the loop were pivoted about an axis through its center, parallel to the *y*-axis?

Figure **P27.64**



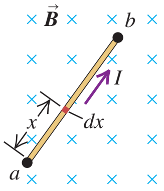
27.65 •• CP The rectangular loop of wire shown in **Fig. P27.65** has a mass of 0.15 g per centimeter of length and is pivoted about side *ab* on a frictionless axis. The current in the wire is 8.2 A in the direction shown. Find the magnitude and direction of the magnetic field parallel to the *y*-axis that will cause the loop to swing up until its plane makes an angle of 30.0° with the *yz*-plane.

Figure **P27.65**



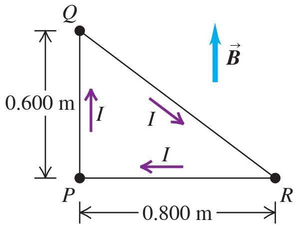
27.66 •• CALC A uniform bar of length *L* carries a current *I* in the direction from point *a* to point *b* (**Fig. P27.66**). The bar is in a uniform magnetic field that is directed into the page. Consider the torque about an axis perpendicular to the bar at point *a* that is due to the force that the magnetic field exerts on the bar. (a) Suppose that an infinitesimal section of the bar has length *dx* and is located a distance *x* from point *a*. Calculate the torque $d\tau$ about point *a* due to the magnetic force on this infinitesimal section. (b) Use $\tau = \int_a^b d\tau$ to calculate the total torque τ on the bar. (c) Show that τ is the same as though all of the magnetic force acted at the midpoint of the bar.

Figure **P27.66**



27.67 •• The loop of wire shown in **Fig. P27.67** forms a right triangle and carries a current $I = 5.00$ A in the direction shown. The loop is in a uniform magnetic field that has magnitude $B = 3.00$ T and the same direction as the current in side *PQ* of the loop. (a) Find the force exerted by the magnetic field on each side of the triangle. If the force is not zero, specify its direction. (b) What is the net force on the loop? (c) The loop is pivoted about an axis that lies along side *PR*. Use the forces calculated in part (a) to calculate the torque on each side of the loop (see Problem 27.66). (d) What is the magnitude of the net torque on the loop? Calculate the net torque from the torques calculated in part (c) and also from Eq. (27.28). Do these two results agree? (e) Is the net torque directed to rotate point *Q* into the plane of the figure or out of the plane of the figure?

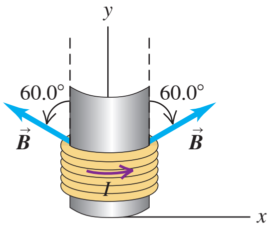
Figure **P27.67**



27.68 •• Repeat Problem 27.67 but with the magnetic field $B = 3.00$ T directed into the page in Fig. P27.67.

27.69 •• CALC A Voice Coil. It was shown in Section 27.7 that the net force on a current loop in a *uniform* magnetic field is zero. The magnetic force on the voice coil of a loudspeaker (see Fig. 27.28) is nonzero because the magnetic field at the coil is not uniform. A voice coil in a loudspeaker has 50 turns of wire and a diameter of 1.56 cm, and the current in the coil is 0.950 A. Assume that the magnetic field at each point of the coil has a constant magnitude of 0.220 T and is directed at an angle of 60.0° outward from the normal to the plane of the coil (**Fig. P27.69**). Let the axis of the coil be in the *y*-direction. The current in the coil is in the direction shown (counterclockwise as viewed from a point above the coil on the *y*-axis). Calculate the magnitude and direction of the net magnetic force on the coil.

Figure **P27.69**



27.70 •• CP A uniform bar has mass 0.0120 kg and is 30.0 cm long. It pivots without friction about an axis perpendicular to the bar at point *a* (**Fig. P27.70**). The gravitational force on the bar acts in the $-y$-direction. The bar is in a uniform magnetic field that is directed into the page and has magnitude $B = 0.150$ T. (a) What must be the current *I* in the bar for the bar to be in rotational equilibrium when it is at an angle $\theta = 30.0°$ above the horizontal? Use your result from Problem 27.66. (b) For the bar to be in rotational equilibrium, should *I* be in the direction from *a* to *b* or *b* to *a*?

Figure **P27.70**

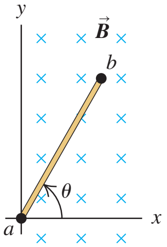
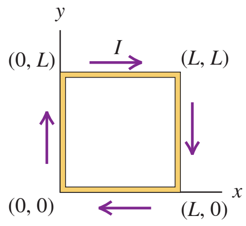
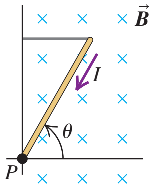


Figure **P27.71**



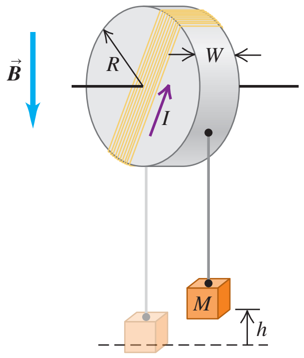
27.71 •• CALC Force on a Current Loop in a Nonuniform Magnetic Field. It was shown in Section 27.7 that the net force on a current loop in a *uniform* magnetic field is zero. But what if $\vec{B}$ is *not* uniform? **Figure P27.71** shows a square loop of wire that lies in the xy-plane. The loop has corners at $(0, 0)$, $(0, L)$, $(L, 0)$, and (L, L) and carries a constant current I in the clockwise direction. The magnetic field has no x-component but has both y- and z-components: $\vec{B} = (B_0 z/L)\hat{\jmath} + (B_0 y/L)\hat{k}$, where B_0 is a positive constant. (a) Sketch the magnetic field lines in the yz-plane. (b) Find the magnitude and direction of the magnetic force exerted on each of the sides of the loop by integrating Eq. (27.20). (c) Find the magnitude and direction of the net magnetic force on the loop.

Figure **P27.72**



27.72 •• CP The lower end of the thin uniform rod in **Fig. P27.72** is attached to the floor by a frictionless hinge at point P. The rod has mass 0.0840 kg and length 18.0 cm and is in a uniform magnetic field $B = 0.120$ T that is directed into the page. The rod is held at an angle $\theta = 53.0°$ above the horizontal by a horizontal string that connects the top of the rod to the wall. The rod carries a current $I = 12.0$ A in the direction toward P. Calculate the tension in the string. Use your result from Problem 27.66 to calculate the torque due to the magnetic-field force.

Figure **P27.73**



27.73 ••• CP A magnetic lift uses a cylinder with radius R and width W wrapped by conducting wire N times on a diameter, as shown in **Fig. P27.73**. A uniform external magnetic field $\vec{B}$ points downward while current I flows in the sense indicated. A mass M is hung from the cylinder by a cable attached to its rim *on the axis of the coil*. The height above the lowest possible position of the mass is h. (a) What is the minimum current I for which the mass can be suspended with $h > 0$? (b) What is the height of the mass $h = h_{\text{top}}$ when the cable attachment rotates one-half turn to the top of the cylinder, in terms of R? Define a dimensionless parameter $\sigma = 2NIWB/Mg$ and express subsequent results in terms of σ and other given quantities. (c) What is the net torque on the cylinder if $0 \le h \le R$? (d) What is the net torque on the cylinder if $R \le h \le h_{\text{top}}$? (e) Define potential energy $U(h)$ that is zero when $h = 0$. What is the potential energy for $0 \le h \le R$? (f) What is the potential energy for $R \le h \le h_{\text{top}}$? (g) If $\sigma > 1$, for what value of h will the mass remain suspended motionless? (h) For what values of σ will the cylinder rotate more than 180° if the mass is released from rest at $h = h_{\text{top}}$? (*Hint:* This corresponds to $U(h_{\text{top}}) > 0$.)

27.74 ••• CP CALC A particle with charge q and mass m is dropped at time $t = 0$ from rest at its origin in a region of constant magnetic field $\vec{B}$ that points horizontally. What happens? To answer, construct a Cartesian coordinate system with the y-axis pointing downward and the z-axis pointing in the direction of the magnetic field. At time $t \ge 0$ the particle has velocity $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$. The net force $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$ on the particle is the vector sum of its weight and the magnetic force. (a) Using Newton's second law, write equations for a_x and a_y, where $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$ is the acceleration of the particle. (b) Differentiate the second of these equations with respect to time. Then substitute your expression for $a_x = dv_x/dt$ to determine an equation for $d{v_y}^2/dt^2$ in terms of v_y. (c) This result shows that v_y is a simple harmonic oscillator. Use the initial conditions to determine $v_y(t)$. Write your answer in terms of the angular frequency $\omega = qB/m$. Note that $(dv_y/dt)_0 = g$. (d) Substitute your result for $v_y(t)$ into your equation for dv_x/dt. Integrate using the initial conditions to determine $v_x(t)$. (e) Integrate your expressions for $v_x(t)$ and $v_y(t)$ to determine $x(t)$ and $y(t)$. (f) If $m = 1.00$ mg, $q = 19.6$ μC, and $B = 10.0$ T, what maximum vertical distance does the particle drop before returning upward?

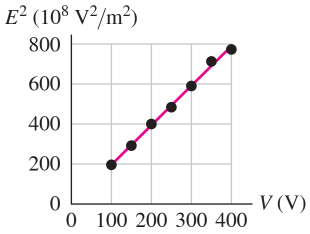
27.75 •• A circular loop of wire with area A lies in the xy-plane. As viewed along the z-axis looking in the $-z$-direction toward the origin, a current I is circulating clockwise around the loop. The torque produced by an external magnetic field $\vec{B}$ is given by $\vec{\tau} = D(4\hat{\imath} - 3\hat{\jmath})$, where D is a positive constant, and for this orientation of the loop the magnetic potential energy $U = -\vec{\mu} \cdot \vec{B}$ is negative. The magnitude of the magnetic field is $B_0 = 13D/IA$. (a) Determine the vector magnetic moment of the current loop. (b) Determine the components B_x, B_y, and B_z of $\vec{B}$.

27.76 •• DATA You are using a type of mass spectrometer to measure charge-to-mass ratios of atomic ions. In the device, atoms are ionized with a beam of electrons to produce positive ions, which are then accelerated through a potential difference V. (The final speed of the ions is great enough that you can ignore their initial speed.) The ions then enter a region in which a uniform magnetic field $\vec{B}$ is perpendicular to the velocity of the ions and has magnitude $B = 0.250$ T. In this $\vec{B}$ region, the ions move in a semicircular path of radius R. You measure R as a function of the accelerating voltage V for one particular atomic ion:

***V* (kV)**	10.0	12.0	14.0	16.0	18.0
***R* (cm)**	19.9	21.8	23.6	25.2	26.8

(a) How can you plot the data points so that they will fall close to a straight line? Explain. (b) Construct the graph described in part (a). Use the slope of the best-fit straight line to calculate the charge-to-mass ratio (q/m) for the ion. (c) For $V = 20.0$ kV, what is the speed of the ions as they enter the $\vec{B}$ region? (d) If ions that have $R = 21.2$ cm for $V = 12.0$ kV are singly ionized, what is R when $V = 12.0$ kV for ions that are doubly ionized?

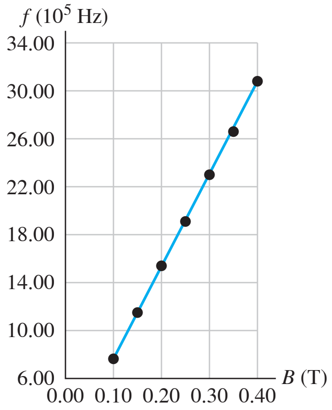
Figure **P27.77**



27.77 •• DATA You are a research scientist working on a high-energy particle accelerator. Using a modern version of the Thomson e/m apparatus, you want to measure the mass of a muon (a fundamental particle that has the same charge as an electron but greater mass). The magnetic field between the two charged plates is 0.340 T. You measure the electric field for zero particle deflection as a function of the accelerating potential V. This potential is large enough that you can assume the initial speed of the muons to be zero. **Figure P27.77** is an E^2-versus-V graph of your data. (a) Explain why the data points fall close to a straight line. (b) Use the graph in Fig. P27.77 to calculate the mass m of a muon. (c) If the two charged plates are separated by 6.00 mm, what must be the voltage between the plates in order for the electric field between the plates to be 2.00×10^5 V/m? Assume that the dimensions of the plates are much larger than the separation between them. (d) When $V = 400$ V, what is the speed of the muons as they enter the region between the plates?

27.78 •• DATA You are a technician testing the operation of a cyclotron. An alpha particle in the device moves in a circular path in a magnetic field $\vec{B}$ that is directed perpendicular to the path of the alpha particle. You measure the number of revolutions per second (the frequency f) of the alpha particle as a function of the magnetic field strength B. **Figure P27.78** shows your results and the best straight-line fit to your data. (a) Use the graph in Fig. P27.78 to calculate the charge-to-mass ratio of the alpha particle, which has charge $+2e$. On the basis of your data, what is the mass of an alpha particle? (b) With $B = 0.300$ T, what are the cyclotron frequencies f of a proton and of an electron? How do these f values compare to the frequency of an alpha particle? (c) With $B = 0.300$ T, what speed and kinetic energy does an alpha particle have if the radius of its path is 12.0 cm?

Figure **P27.78**



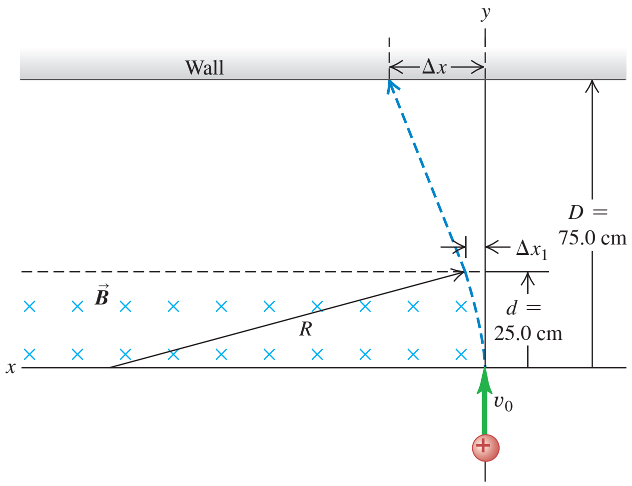
CHALLENGE PROBLEMS

27.79 ••• CALC Determine the magnetic moment $\vec{\mu}$ of a spherical shell with radius R and uniform charge Q rotating with angular speed $\vec{\Omega} = \omega\hat{k}$. Use the following steps: (a) Consider a coordinate system with the origin at the center of the sphere. Parameterize each latitude on the sphere with the angle θ measured from the positive z-axis. There is a circular current loop at each value of θ for $0 \leq \theta \leq \pi$. What is the radius of the loop at latitude θ? (b) The differential current carried by that loop is $dI = \sigma v\, dW$, where σ is the charge density of the sphere, v is the tangential speed of the loop, and $dW = R\, d\theta$ is its differential width. Express dI in terms of σ, R, ω, θ, and $d\theta$. (c) The differential magnetic moment of the loop is $d\mu = A\, dI$, where A is the area enclosed by the loop. Express $d\mu$ in terms of R, ω, θ, and $d\theta$. (d) Integrate over the sphere to determine the magnetic moment. Express your result as a vector; use the total charge Q rather than the charge density σ. (e) If the sphere is in a uniform magnetic field $\vec{B} = (\sin\alpha\hat{\imath} + \cos\alpha\hat{\jmath})B$, what is the torque on the sphere?

27.80 ••• An electron traveling from the sun as part of the solar wind strikes the earth's magnetosphere at latitude 80.0° N in a region where the magnetic field has a strength of 15.0 μT and is directed toward the earth's center. The electron has a speed of 400 km/s and is directed toward the earth's axis parallel to the equator. Magnetic forces send the electron on a helical trajectory. (a) What is the radius of this helix? (b) With what speed does the electron approach the surface of the earth? (c) If you are looking downward toward the earth from space, is the electron's motion clockwise or counterclockwise? (d) What is the frequency of the motion? (e) This electron strikes the ionosphere, where it is further accelerated by an electric field with strength 20.0 mV/m directed northward parallel to the earth's surface. What is the electron's new speed after it has been deflected 100 km southward by this field? (f) By what factor has its kinetic energy been increased by the electric field?

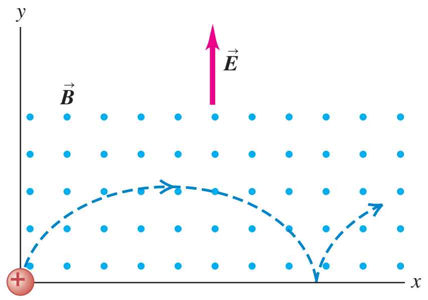
27.81 ••• A particle with charge 2.15 μC and mass 3.20×10^{-11} kg is initially traveling in the $+y$-direction with a speed $v_0 = 1.45 \times 10^5$ m/s. It then enters a region containing a uniform magnetic field that is directed into, and perpendicular to, the page in **Fig. P27.81**. The magnitude of the field is 0.420 T. The region extends a distance of 25.0 cm along the initial direction of travel; 75.0 cm from the point of entry into the magnetic field region is a wall. The length of the field-free region is thus 50.0 cm. When the charged particle enters the magnetic field, it follows a curved path whose radius of curvature is R. It then leaves the magnetic field after a time t_1, having been deflected a distance Δx_1. The particle then travels in the field-free region and strikes the wall after undergoing a total deflection Δx. (a) Determine the radius R of the curved part of the path. (b) Determine t_1, the time the particle spends in the magnetic field. (c) Determine Δx_1, the horizontal deflection at the point of exit from the field. (d) Determine Δx, the total horizontal deflection.

Figure **P27.81**



27.82 ••• CP A Cycloidal Path. A particle with mass m and positive charge q starts from rest at the origin shown in **Fig. P27.82**. There is a uniform electric field $\vec{E}$ in the $+y$-direction and a uniform magnetic field $\vec{B}$ directed out of the page. It is shown in more advanced books that the path is a *cycloid* whose radius of curvature at the top points is twice the y-coordinate at that level. (a) Explain why the path has this general shape and why it is repetitive. (b) Prove that the speed at any point is equal to $\sqrt{2qEy/m}$. (*Hint:* Use energy conservation.) (c) Applying Newton's second law at the top point and taking as given that the radius of curvature here equals $2y$, prove that the speed at this point is $2E/B$.

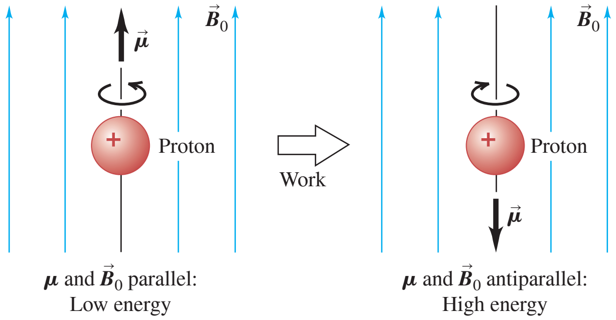
Figure **P27.82**



MCAT-STYLE PASSAGE PROBLEMS

BIO Magnetic Fields and MRI. *Magnetic resonance imaging* (MRI) is a powerful imaging method that, unlike x-ray imaging, allows sharp images of soft tissue to be made without exposing the patient to potentially damaging radiation. A rudimentary understanding of this method can be achieved by the relatively simple application of the classical (that is, non-quantum) physics of magnetism. The starting point for MRI is *nuclear magnetic resonance* (NMR), a technique that depends on the fact that protons in the atomic nucleus have a magnetic field $\vec{B}$. The origin of the proton's magnetic field is the spin of the proton. Being charged, the spinning proton constitutes an electric current analogous to a wire loop through which current flows. Like the wire loop, the proton has a magnetic moment $\vec{\mu}$; thus it will experience a torque when it is subjected to an external magnetic field $\vec{B}_0$. The magnitude of $\vec{\mu}$ is about 1.4×10^{-26} J/T. The proton can be thought of as being in one of two states, with $\vec{\mu}$ oriented parallel or antiparallel to the applied magnetic field, and work must be done to flip the proton from the low-energy state to the high-energy state, as the accompanying figure below shows.

An important consideration is that the net magnetic field of any nucleus, except for that of hydrogen (which has a proton only), consists of contributions from both protons and neutrons. If a nucleus has an even number of protons and neutrons, they will pair in such a way that half of the protons have spins in one orientation and half have spins in the other orientation. Thus the net magnetic moment of the nucleus is zero. Only nuclei with a net magnetic moment are candidates for MRI. Hydrogen is the atom that is most commonly imaged.



27.83 If a proton is exposed to an external magnetic field of 2 T that has a direction perpendicular to the axis of the proton's spin, what will be the torque on the proton? (a) 0; (b) 1.4×10^{-26} N · m; (c) 2.8×10^{-26} N · m; (d) 0.7×10^{-26} N · m.

27.84 Which of following elements is a candidate for MRI? (a) $^{12}C_6$; (b) $^{16}O_8$; (c) $^{40}Ca_{20}$; (d) $^{31}P_{15}$.

27.85 The large magnetic fields used in MRI can produce forces on electric currents within the human body. This effect has been proposed as a possible method for imaging "biocurrents" flowing in the body, such as the current that flows in individual nerves. For a magnetic field strength of 2 T, estimate the magnitude of the maximum force on a 1-mm-long segment of a single cylindrical nerve that has a diameter of 1.5 mm. Assume that the entire nerve carries a current due to an applied voltage of 100 mV (that of a typical action potential). The resistivity of the nerve is 0.6 Ω · m. (a) 6×10^{-7} N; (b) 1×10^{-6} N; (c) 3×10^{-4} N; (d) 0.3 N.

ANSWERS

Chapter Opening Question ?

(ii) A magnetized compass needle has a magnetic dipole moment along its length, and the earth's magnetic field (which points generally northward) exerts a torque that tends to align that dipole moment with the field. See Section 27.7 for details.

Key Example ✓ARIATION Problems

VP27.1.1 (a) 4.18×10^{-14} N (b) $-z$-direction
VP27.1.2 2.91×10^{-15} C
VP27.1.3 (a) 0.0175 T (b) $+z$-direction
VP27.1.4 (a) 0.189 T (b) $-y$-direction
VP27.6.1 (a) 0.435 mm (b) 2.87×10^{7} rad/s (c) 4.57×10^{6} Hz
VP27.6.2 (a) 0.348 mm (b) 1.64 mm (c) 4.80×10^{-16} N
VP27.6.3 (a) 8.00×10^{5} m/s (b) $-y$-direction (c) $-x$-direction
VP27.6.4 (a) 6.64 cm (b) 7.47 cm
VP27.7.1 (a) 2.44×10^{-3} N (b) $+z$-direction
VP27.7.2 (a) 1.60 A (b) upward
VP27.7.3 (a) -2.52×10^{-3} N (b) -4.20×10^{-3} N (c) 0 N (d) 4.90×10^{-3} N
VP27.7.4 41.0° and 139.0°

Bridging Problem

(a) $\tau_x = -1.54 \times 10^{-4}$ N · m,
$\tau_y = -2.05 \times 10^{-4}$ N · m,
$\tau_z = -6.14 \times 10^{-4}$ N · m
(b) -7.55×10^{-4} J
(c) 42.1 rad/s