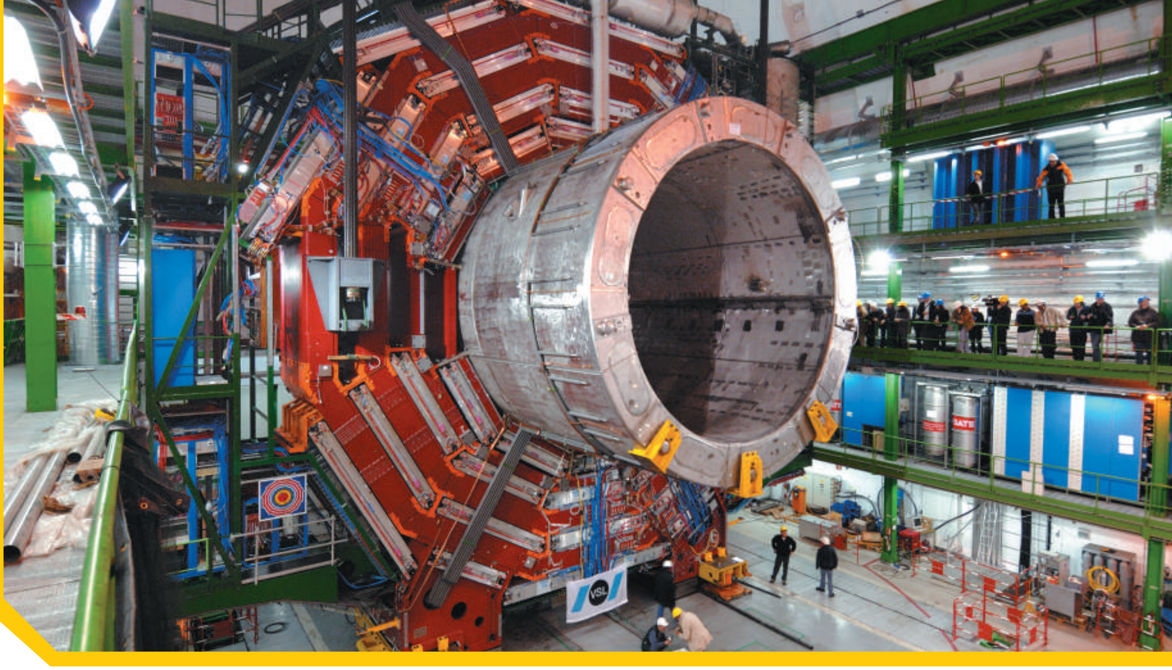
? The immense cylinder in this photograph is a current-carrying coil, or solenoid, that generates a uniform magnetic field in its interior as part of an experiment at CERN, the European Organization for Nuclear Research. If two such solenoids were joined end to end, the magnetic field along their common axis would (i) become four times stronger; (ii) double in strength; (iii) become $\sqrt{2}$ times stronger; (iv) not change; (v) weaken.



28 Sources of Magnetic Field

LEARNING OUTCOMES

In this chapter, you'll learn...

28.1 The nature of the magnetic field produced by a single moving charged particle.

28.2 How to describe the magnetic field produced by an element of a current-carrying conductor.

28.3 How to calculate the magnetic field produced by a long, straight, current-carrying wire.

28.4 Why wires carrying current in the same direction attract, while wires carrying opposing currents repel.

28.5 How to calculate the magnetic field produced by a current-carrying wire bent into a circle.

28.6 What Ampere's law is, and what it tells us about magnetic fields.

28.7 How to use Ampere's law to calculate the magnetic field of symmetric current distributions.

28.8 How microscopic currents within materials give them their magnetic properties.

You'll need to review...

10.5 Angular momentum of a particle.

21.3–21.5 Coulomb's law and electric-field calculations.

22.4 Solving problems with Gauss's law.

27.2–27.9 Magnetic field and magnetic force.

In Chapter 27 we studied the forces exerted on moving charges and on current-carrying conductors in a magnetic field. We didn't worry about how the magnetic field got there; we simply took its existence as a given fact. But how are magnetic fields *created*? We know that both permanent magnets and electric currents in electromagnets create magnetic fields. In this chapter we'll study these sources of magnetic field in detail.

We've learned that a charge creates an electric field and that an electric field exerts a force on a charge. But a *magnetic* field exerts a force on only a *moving* charge. Similarly, we'll see that only *moving* charges *create* magnetic fields. We'll begin our analysis with the magnetic field created by a single moving point charge. We can use this analysis to determine the field created by a small segment of a current-carrying conductor. Once we can do that, we can in principle find the magnetic field produced by *any* shape of conductor.

Then we'll introduce Ampere's law, which plays a role in magnetism analogous to the role of Gauss's law in electrostatics. Ampere's law lets us exploit symmetry properties in relating magnetic fields to their sources.

Moving charged particles within atoms respond to magnetic fields and can also act as sources of magnetic field. We'll use these ideas to understand how certain magnetic materials can be used to intensify magnetic fields as well as why some materials such as iron act as permanent magnets.

28.1 MAGNETIC FIELD OF A MOVING CHARGE

Let's start with the basics, the magnetic field of a single point charge q moving with a constant velocity $\vec{v}$. In practical applications, such as the solenoid shown in the photo that opens this chapter, magnetic fields are produced by tremendous numbers of charged particles moving together in a current. But once we understand how to calculate the magnetic field due to a single point charge, it's a small leap to calculate the field due to a current-carrying wire or collection of wires.

As we did for electric fields, we call the location of the moving charge at a given instant the **source point** and the point P where we want to find the field the **field point.** In Section 21.4 we found that at a field point a distance r from a point charge q, the magnitude of the *electric* field $\vec{E}$ caused by the charge is proportional to the charge magnitude $|q|$ and to $1/r^2$, and the direction of $\vec{E}$ (for positive q) is along the line from source point to field point. The corresponding relationship for the *magnetic* field $\vec{B}$ of a point charge q moving with constant velocity has some similarities and some interesting differences.

Experiments show that the magnitude of $\vec{B}$ is also proportional to $|q|$ and to $1/r^2$. But the *direction* of $\vec{B}$ is *not* along the line from source point to field point. Instead, $\vec{B}$ is perpendicular to the plane containing this line and the particle's velocity vector $\vec{v}$, as shown in **Fig. 28.1**. Furthermore, the field *magnitude* B is also proportional to the particle's speed v and to the sine of the angle ϕ. Thus the magnetic-field magnitude at point P is

$$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2} \qquad (28.1)$$

The quantity μ_0 (read as "mu-nought" or "mu-sub-zero") is called the **magnetic constant.** The reason for including the factor of 4π will emerge shortly. We did something similar with Coulomb's law in Section 21.3.

Moving Charge: Vector Magnetic Field

We can incorporate both the magnitude and direction of $\vec{B}$ into a single vector equation by using the vector product. To avoid having to say "the direction from the source q to the field point P" over and over, we introduce a *unit* vector $\hat{r}$ ("r-hat") that points from the source point to the field point. (We used $\hat{r}$ for the same purpose in Section 21.4.) This unit vector is equal to the vector $\vec{r}$ from the source to the field point divided by its magnitude: $\hat{r} = \vec{r}/r$. Then

Magnetic constant

Charge

Velocity

Magnetic field due to a point charge with constant velocity

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad (28.2)$$

Unit vector from point charge toward where field is measured

Distance from point charge to where field is measured

Figure 28.1 shows the relationship of $\hat{r}$ to P and shows the magnetic field $\vec{B}$ at several points in the vicinity of the charge. At all points along a line through the charge parallel to the velocity $\vec{v}$, the field is zero because $\sin\phi = 0$ at all such points. At any distance r from q, $\vec{B}$ has its greatest magnitude at points lying in the plane perpendicular to $\vec{v}$, because there $\phi = 90°$ and $\sin\phi = 1$. If q is negative, the directions of $\vec{B}$ are opposite to those shown in Fig. 28.1.

Moving Charge: Magnetic Field Lines

A point charge in motion also produces an *electric* field, with field lines that radiate outward from a positive charge. The *magnetic* field lines are completely different. For a point charge moving with velocity $\vec{v}$, the magnetic field lines are *circles* centered on the line of $\vec{v}$ and lying in planes perpendicular to this line. The field-line directions for a positive charge are given by the following *right-hand rule,* one of several that we'll encounter in this chapter: Grasp the velocity vector $\vec{v}$ with your right hand so that your right thumb points in the direction of $\vec{v}$; your fingers then curl around the line of $\vec{v}$ in the same sense as the magnetic field lines, assuming q is positive. Figure 28.1a shows parts of a few field lines; Fig. 28.1b shows some field lines in a plane through q, perpendicular to $\vec{v}$. If the point charge is negative, the directions of the field and field lines are the opposite of those shown in Fig. 28.1.

Equations (28.1) and (28.2) describe the $\vec{B}$ field of a point charge moving with *constant* velocity. If the charge *accelerates,* the field can be much more complicated. We won't need these more complicated results for our purposes. (The moving charged particles that make up a current in a wire accelerate at points where the wire bends and the direction of $\vec{v}$ changes. But because the magnitude v_d of the drift velocity in a conductor is typically very small, the centripetal acceleration v_d^2/r is so small that we can ignore its effects.)

As we discussed in Section 27.2, the unit of B is one tesla (1 T):

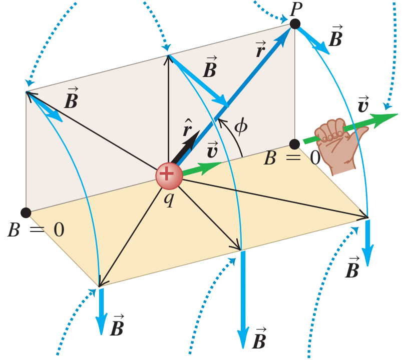
$$1\text{ T} = 1\text{ N}\cdot\text{s/C}\cdot\text{m} = 1\text{ N/A}\cdot\text{m}$$

Figure **28.1** (a) Magnetic-field vectors due to a moving positive point charge q. At each point, $\vec{B}$ is perpendicular to the plane of $\vec{r}$ and $\vec{v}$, and its magnitude is proportional to the sine of the angle between them. (b) Magnetic field lines in a plane containing a moving positive charge.

(a) Perspective view

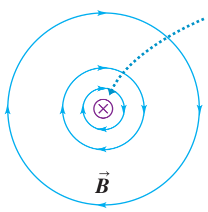
Right-hand rule for the magnetic field due to a positive charge moving at constant velocity: Point the thumb of your right hand in the direction of the velocity. Your fingers now curl around the charge in the direction of the magnetic field lines. (If the charge is negative, the field lines are in the opposite direction.)

For these field points, $\vec{r}$ and $\vec{v}$ both lie in the beige plane, and $\vec{B}$ is perpendicular to this plane.



For these field points, $\vec{r}$ and $\vec{v}$ both lie in the gold plane, and $\vec{B}$ is perpendicular to this plane.

(b) View from behind the charge



The × symbol indicates that the charge is moving into the plane of the page (away from you).

Using this with Eq. (28.1) or (28.2), we find that the units of the constant μ_0 are

$$1\ \mathrm{N \cdot s^2/C^2} = 1\ \mathrm{N/A^2} = 1\ \mathrm{Wb/A \cdot m} = 1\ \mathrm{T \cdot m/A}$$

In SI units the numerical value of μ_0 is, to nine significant figures,

$$\begin{aligned}\mu_0 &= 1.25663706 \times 10^{-6}\ \mathrm{N \cdot s^2/C^2} = 1.25663706 \times 10^{-6}\ \mathrm{Wb/A \cdot m} \\ &\cong 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \end{aligned} \tag{28.3}$$

It may seem incredible that μ_0 is equal to $4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ to nine significant figures! In fact this is a consequence of how the ampere and coulomb were previously defined. We'll explore this further in Section 28.4.

Here's another remarkable aspect of the value of the magnetic constant μ_0. Recall from Section 21.3 that the *electric* constant ϵ_0 that appears in Coulomb's law and Gauss's law has the value $\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$. If we multiply ϵ_0 by $\mu_0 = 1.257 \times 10^{-6}\ \mathrm{N \cdot s^2/C^2}$, take the square root of this product, then take the reciprocal of the result, we get

$$\frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.998 \times 10^8\ \mathrm{m/s}$$

But 2.998×10^8 m/s is just the value of the speed of light in vacuum! We conclude that

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{28.4}$$

This suggests that electric and magnetic fields are intimately related to the nature of light. We'll explore this relationship when we study electromagnetic waves in Chapter 32.

EXAMPLE 28.1 Forces between two moving protons

WITH VARIATION PROBLEMS

Two protons move parallel to the x-axis in opposite directions (**Fig. 28.2**) at the same speed v (small compared to the speed of light c). At the instant shown, find the electric and magnetic forces on the upper proton and compare their magnitudes.

IDENTIFY and SET UP Coulomb's law [Eq. (21.2)] gives the electric force F_E on the upper proton. The magnetic force law [Eq. (27.2)] gives the magnetic force on the upper proton; to use it, we must first use Eq. (28.2) to find the magnetic field that the lower proton produces at the position of the upper proton. The unit vector from the lower proton (the source) to the position of the upper proton is $\hat{\boldsymbol{r}} = \hat{\boldsymbol{\jmath}}$.

EXECUTE From Coulomb's law, the magnitude of the electric force on the upper proton is

$$F_E = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

The forces are repulsive, and the force on the upper proton is vertically upward (in the $+y$-direction).

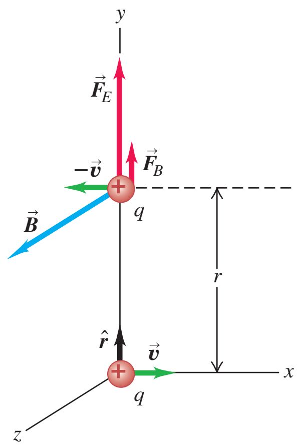
The velocity of the lower proton is $\vec{\boldsymbol{v}} = v\hat{\boldsymbol{\imath}}$. From the right-hand rule for the cross product $\vec{\boldsymbol{v}} \times \hat{\boldsymbol{r}}$ in Eq. (28.2), the $\vec{\boldsymbol{B}}$ field due to the lower proton at the position of the upper proton is in the $+z$-direction (see Fig. 28.2). From Eq. (28.2), the field is

$$\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q(v\hat{\boldsymbol{\imath}}) \times \hat{\boldsymbol{\jmath}}}{r^2} = \frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{\boldsymbol{k}}$$

The velocity of the upper proton is $-\vec{\boldsymbol{v}} = -v\hat{\boldsymbol{\imath}}$, so the magnetic force on it is

$$\vec{\boldsymbol{F}}_B = q(-\vec{\boldsymbol{v}}) \times \vec{\boldsymbol{B}} = q(-v\hat{\boldsymbol{\imath}}) \times \frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{\boldsymbol{k}} = \frac{\mu_0}{4\pi}\frac{q^2v^2}{r^2}\hat{\boldsymbol{\jmath}}$$

Figure 28.2 Electric and magnetic forces between two moving protons.



The magnetic interaction in this situation is also repulsive.

The ratio of the force magnitudes is

$$\frac{F_B}{F_E} = \frac{\mu_0 q^2 v^2/4\pi r^2}{q^2/4\pi\epsilon_0 r^2} = \frac{\mu_0 v^2}{1/\epsilon_0} = \epsilon_0\mu_0 v^2$$

With the relationship $\epsilon_0\mu_0 = 1/c^2$, Eq. (28.4), this becomes

$$\frac{F_B}{F_E} = \frac{v^2}{c^2}$$

When v is small in comparison to the speed of light, the magnetic force is much smaller than the electric force.

EVALUATE We have described the velocities, fields, and forces as they are measured by an observer who is stationary in the coordinate system of Fig. 28.2. In a coordinate system that moves with one of the charges, one of the velocities would be zero, so there would be *no* magnetic force. The explanation of this apparent paradox provided one of the paths that led to the special theory of relativity.

KEYCONCEPT A moving charged particle produces a magnetic field $\vec{B}$. The magnitude of $\vec{B}$ at a given point is proportional to the charge magnitude and to the particle's speed, and inversely proportional to the square of the distance from the charge; it also depends on the direction from the particle to that point compared to the direction of the particle's velocity. The direction of $\vec{B}$ is given by a right-hand rule.

TEST YOUR UNDERSTANDING OF SECTION 28.1 (a) If two protons are traveling parallel to each other in the *same* direction and at the same speed, is the magnetic force between them (i) attractive or (ii) repulsive? (b) Is the net force between them (i) attractive, (ii) repulsive, or (iii) zero? (Assume that the protons' speed is much slower than the speed of light.)

ANSWER

(a) **(i)**, (b) **(ii)** The situation is the same as shown in Fig. 28.2 except that the upper proton has velocity $\vec{v}$ rather than $-\vec{v}$. The magnetic field due to the lower proton is the same as shown in Fig. 28.2, but the direction of the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ on the upper proton is reversed. Hence the magnetic force is attractive. Since the speed v is small compared to c, the magnetic force is much smaller in magnitude than the repulsive electric force and the net force is still repulsive.

28.2 MAGNETIC FIELD OF A CURRENT ELEMENT

As for electric fields, there is a *principle of superposition of magnetic fields:*

PRINCIPLE OF SUPERPOSITION OF MAGNETIC FIELDS The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.

We can use this principle with the results of Section 28.1 to find the magnetic field produced by a current in a conductor.

We begin by calculating the magnetic field caused by a short segment $d\vec{l}$ of a current-carrying conductor, as shown in **Fig. 28.3a**. The volume of the segment is $A\,dl$, where A is the cross-sectional area of the conductor. If there are n moving charged particles per unit volume, each of charge q, the total moving charge dQ in the segment is

$$dQ = nqA\,dl$$

The moving charges in this segment are equivalent to a single charge dQ, traveling with a velocity equal to the *drift* velocity $\vec{v}_d$. (Magnetic fields due to the *random* motions of the charges will, on average, cancel out at every point.) From Eq. (28.1) the magnitude of the resulting field $d\vec{B}$ at any field point P is

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_d \sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_d A\,dl \sin\phi}{r^2}$$

But from Eq. (25.2), $n|q|v_d A$ equals the current I in the element. So

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\phi}{r^2} \tag{28.5}$$

Current Element: Vector Magnetic Field

In vector form, using the unit vector $\hat{r}$ as in Section 28.1, we have

Magnetic field due to an infinitesimal current element

Magnetic constant

Current

Vector length of element (points in current direction)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2} \tag{28.6}$$

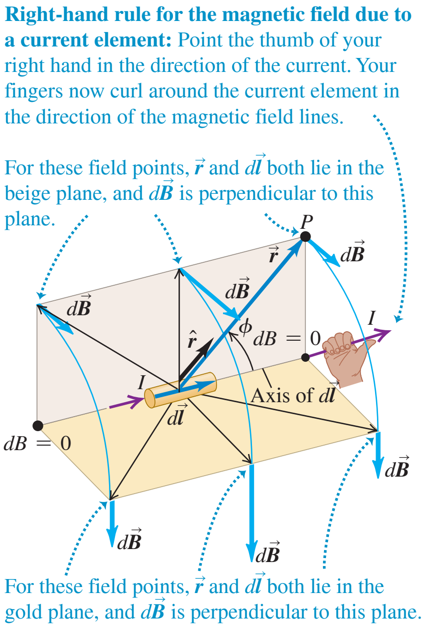
Unit vector from element toward where field is measured

Distance from element to where field is measured

where $d\vec{l}$ is a vector with length dl, in the same direction as the current.

Figure **28.3** (a) Magnetic-field vectors due to a current element $d\vec{l}$. (b) Magnetic field lines in a plane containing the current element $d\vec{l}$. Compare this figure to Fig. 28.1 for the field of a moving point charge.

(a) Perspective view

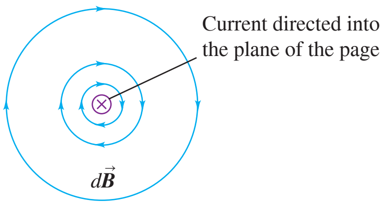


Right-hand rule for the magnetic field due to a current element: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the current element in the direction of the magnetic field lines.

For these field points, $\vec{r}$ and $d\vec{l}$ both lie in the beige plane, and $d\vec{B}$ is perpendicular to this plane.

For these field points, $\vec{r}$ and $d\vec{l}$ both lie in the gold plane, and $d\vec{B}$ is perpendicular to this plane.

(b) View along the axis of the current element



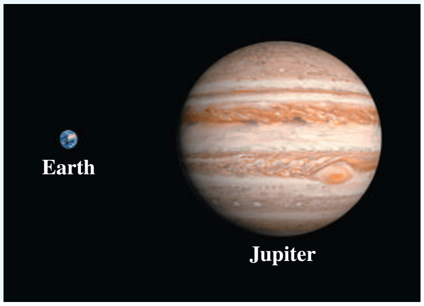
Current directed into the plane of the page

Equations (28.5) and (28.6) are called the **law of Biot and Savart** (pronounced “Bee-oh” and “Suh-var”). We can use this law to find the total magnetic field $\vec{B}$ at any point in space due to the current in a complete circuit. To do this, we integrate Eq. (28.6) over all segments $d\vec{l}$ that carry current; symbolically,

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l} \times \hat{r}}{r^2} \qquad (28.7)$$

In the following sections we’ll carry out this vector integration for several examples.

APPLICATION Currents and Planetary Magnetism The earth’s magnetic field is caused by currents circulating within its molten, conducting interior. These currents are stirred by our planet’s relatively rapid spin (one rotation per 24 hours). The planet Jupiter’s internal currents are much stronger: Jupiter is much larger than the earth, has an interior that is mostly liquid hydrogen (which is a very good conductor under high pressures), and spins very rapidly (one rotation per 10 hours). Hence at the same distance from the center of each planet, Jupiter’s magnetic field is about 2×10^4 times greater than the earth’s field.



Current Element: Magnetic Field Lines

As Fig. 28.3 shows, the field vectors $d\vec{B}$ and the magnetic field lines of a current element are exactly like those set up by a positive charge dQ moving in the direction of the drift velocity $\vec{v}_d$. The field lines are circles in planes perpendicular to $d\vec{l}$ and centered on the line of $d\vec{l}$. Their directions are given by the same right-hand rule that we introduced for point charges in Section 28.1.

We can’t verify Eq. (28.5) or (28.6) directly because we can never experiment with an isolated segment of a current-carrying circuit. What we measure experimentally is the *total* $\vec{B}$ for a complete circuit. But we can still verify these equations indirectly by calculating $\vec{B}$ for various current configurations with Eq. (28.7) and comparing the results with experimental measurements.

If matter is present in the space around a current-carrying conductor, the field at a field point P in its vicinity will have an additional contribution resulting from the *magnetization* of the material. We’ll return to this point in Section 28.8. However, unless the material is iron or some other ferromagnetic material, the additional field is small and is usually negligible. Additional complications arise if time-varying electric or magnetic fields are present or if the material is a superconductor; we’ll return to these topics later.

PROBLEM-SOLVING STRATEGY 28.1 Magnetic-Field Calculations

IDENTIFY *the relevant concepts:* The law of Biot and Savart [Eqs. (28.5) and (28.6)] allows you to calculate the magnetic field at a field point P due to a current-carrying wire of any shape. The idea is to calculate the field element $d\vec{B}$ at P due to a representative current element in the wire and integrate all such field elements to find the field $\vec{B}$ at P.

SET UP *the problem* using the following steps:

1. Make a diagram showing a representative current element and the field point P.
2. Draw the current element $d\vec{l}$, being careful that it points in the direction of the current.
3. Draw the unit vector $\hat{r}$ directed *from* the current element (the source point) to P.
4. Identify the target variable (usually $\vec{B}$).

EXECUTE *the solution* as follows:

1. Use Eq. (28.5) or (28.6) to express the magnetic field $d\vec{B}$ at P from the representative current element.
2. Add up all the $d\vec{B}$’s using the rules of *vector* addition to find the total field at point P. In some situations the $d\vec{B}$’s at point P have the same direction for all the current elements; then the magnitude of the total $\vec{B}$ field is the sum of the magnitudes of the $d\vec{B}$’s. But often the $d\vec{B}$’s have different directions for different current elements. Then you have to set up a coordinate system and represent each $d\vec{B}$ in terms of its components. The integral for the total $\vec{B}$ is then expressed in terms of an integral for each component.
3. Sometimes you can use the symmetry of the situation to prove that one component of $\vec{B}$ must vanish. Always be alert for ways to use symmetry to simplify the problem.
4. Look for ways to use the principle of superposition of magnetic fields. Later in this chapter we’ll determine the fields produced by certain simple conductor shapes; if you encounter a conductor of a complex shape that can be represented as a combination of these simple shapes, you can use superposition to find the field of the complex shape. Examples include a rectangular loop and a semicircle with straight line segments on both sides.

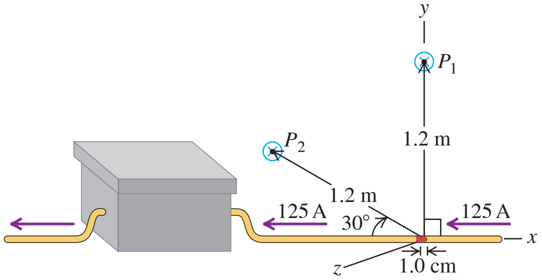
EVALUATE *your answer:* Often your answer will be a mathematical expression for $\vec{B}$ as a function of the position of the field point. Check the answer by examining its behavior in as many limits as you can.

EXAMPLE 28.2 Magnetic field of a current segment

WITH VARIATION PROBLEMS

A copper wire carries a steady 125 A current to an electroplating tank (**Fig. 28.4**). Find the magnetic field due to a 1.0 cm segment of this wire at a point 1.2 m away from it, if the point is (a) point P_1, straight out to the side of the segment, and (b) point P_2, in the xy-plane and on a line at 30° to the segment.

Figure **28.4** Finding the magnetic field at two points due to a 1.0 cm segment of current-carrying wire (not shown to scale).



IDENTIFY and SET UP Although Eqs. (28.5) and (28.6) apply only to infinitesimal current elements, we may use either of them here because the segment length is much less than the distance to the field point. The current element is shown in red in Fig. 28.4 and points in the $-x$-direction (the direction of the current), so $d\vec{l} = dl(-\hat{\imath})$. The unit vector $\hat{r}$ for each field point is directed from the current element toward that point: $\hat{r}$ is in the $+y$-direction for point P_1 and at an angle of 30° above the $-x$-direction for point P_2.

EXECUTE (a) At point P_1, $\hat{r} = \hat{\jmath}$, so

$$\begin{aligned}\vec{B} &= \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl(-\hat{\imath})\times\hat{\jmath}}{r^2} = -\frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\hat{k}\\ &= -(10^{-7}\,\mathrm{T\cdot m/A})\frac{(125\,\mathrm{A})(1.0\times10^{-2}\,\mathrm{m})}{(1.2\,\mathrm{m})^2}\hat{k}\\ &= -(8.7\times10^{-8}\,\mathrm{T})\hat{k}\end{aligned}$$

The direction of $\vec{B}$ at P_1 is into the xy-plane of Fig. 28.4.

(b) At P_2, the unit vector is $\hat{r} = (-\cos 30°)\hat{\imath} + (\sin 30°)\hat{\jmath}$. From Eq. (28.6),

$$\begin{aligned}\vec{B} &= \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl(-\hat{\imath})\times(-\cos 30°\hat{\imath} + \sin 30°\hat{\jmath})}{r^2}\\ &= -\frac{\mu_0 I}{4\pi}\frac{dl\sin 30°}{r^2}\hat{k}\\ &= -(10^{-7}\,\mathrm{T\cdot m/A})\frac{(125\,\mathrm{A})(1.0\times10^{-2}\,\mathrm{m})(\sin 30°)}{(1.2\,\mathrm{m})^2}\hat{k}\\ &= -(4.3\times10^{-8}\,\mathrm{T})\hat{k}\end{aligned}$$

The direction of $\vec{B}$ at P_2 is also into the xy-plane of Fig. 28.4.

EVALUATE We can check our results for the direction of $\vec{B}$ by comparing them with Fig. 28.3. The xy-plane in Fig. 28.4 corresponds to the beige plane in Fig. 28.3, but here the direction of the current and hence of $d\vec{l}$ is the reverse of that shown in Fig. 28.3. Consequently the direction of the magnetic field is reversed as well. It follows that the field at points in the xy-plane in Fig. 28.4 must point *into,* not out of, that plane, just as we concluded above.

KEYCONCEPT The magnetic field $\vec{B}$ due to a short current segment of length dl is like that of a moving charged particle. The difference is that the product of the moving particle's charge and velocity is replaced by the product of the current and the vector $d\vec{l}$ in the direction of the current.

TEST YOUR UNDERSTANDING OF SECTION 28.2 An infinitesimal current element located at the origin ($x = y = z = 0$) carries current I in the positive y-direction. Rank the following locations in order of the strength of the magnetic field that the current element produces at that location, from largest to smallest value. (i) $x = L$, $y = 0$, $z = 0$; (ii) $x = 0$, $y = L$, $z = 0$; (iii) $x = 0$, $y = 0$, $z = L$; (iv) $x = L/\sqrt{2}$, $y = L/\sqrt{2}$, $z = 0$.

ANSWER

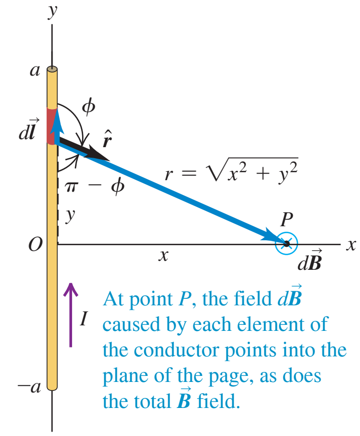
(i) and (iii) (tie), (iv), (ii) From Eq. (28.5), the magnitude of the field dB due to a current element of length dl carrying current I is $dB = (\mu_0/4\pi)(I\,dl\sin\phi/r^2)$. In this expression r is the distance from the element to the field point, and ϕ is the angle between the direction of the current and a vector from the current element to the field point. All four points are the same distance $r = L$ from the current element, so the value of dB is proportional to the value of $\sin\phi$. For the four points the angle is (i) $\phi = 90°$, (ii) $\phi = 0$, (iii) $\phi = 90°$, and (iv) $\phi = 45°$, so the values of $\sin\phi$ are (i) 1, (ii) 0, (iii) 1, and (iv) $1/\sqrt{2}$.

28.3 MAGNETIC FIELD OF A STRAIGHT CURRENT-CARRYING CONDUCTOR

Let's use the law of Biot and Savart to find the magnetic field produced by a straight current-carrying conductor. This result is useful because straight conducting wires are found in essentially all electric and electronic devices. **Figure 28.5** (next page) shows such a conductor with length $2a$ carrying a current I. We'll find $\vec{B}$ at a point a distance x from the conductor on its perpendicular bisector.

We first use the law of Biot and Savart, Eq. (28.5), to find the field $d\vec{B}$ caused by the element of conductor of length $dl = dy$ shown in Fig. 28.5. From the figure, $r = \sqrt{x^2 + y^2}$ and $\sin\phi = \sin(\pi - \phi) = x/\sqrt{x^2 + y^2}$. The right-hand rule for the vector product $d\vec{l} \times \hat{r}$

Figure **28.5** Magnetic field produced by a straight current-carrying conductor of length $2a$.



At point P, the field $d\vec{B}$ caused by each element of the conductor points into the plane of the page, as does the total $\vec{B}$ field.

shows that the *direction* of $d\vec{B}$ is into the plane of the figure, perpendicular to the plane; furthermore, the directions of the $d\vec{B}$'s from *all* elements of the conductor are the same. Thus in integrating Eq. (28.7), we can just add the *magnitudes* of the $d\vec{B}$'s, a significant simplification.

Putting the pieces together, we find that the magnitude of the total $\vec{B}$ field is

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x\,dy}{(x^2 + y^2)^{3/2}}$$

We can integrate this by trigonometric substitution or by using an integral table:

$$B = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}} \qquad (28.8)$$

When the length $2a$ of the conductor is much greater than its distance x from point P, we can consider it to be infinitely long. When a is much larger than x, $\sqrt{x^2 + a^2}$ is approximately equal to a; hence in the limit $a \rightarrow \infty$, Eq. (28.8) becomes

$$B = \frac{\mu_0 I}{2\pi x}$$

The physical situation has axial symmetry about the y-axis. Hence $\vec{B}$ must have the same *magnitude* at all points on a circle centered on the conductor and lying in a plane perpendicular to it, and the *direction* of $\vec{B}$ must be everywhere tangent to such a circle (**Fig. 28.6**). Thus, at all points on a circle of radius r around the conductor, the magnitude B is

Magnetic constant

Magnetic field near a long, straight, current-carrying conductor

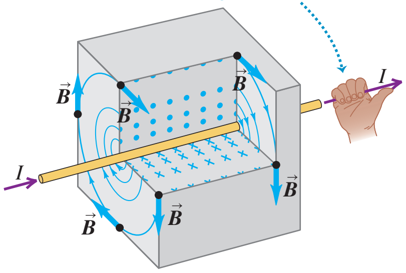
$$B = \frac{\mu_0 I}{2\pi r} \qquad (28.9)$$

Current

Distance from conductor

Figure **28.6** Magnetic field around a long, straight, current-carrying conductor. The field lines are circles, with directions determined by the right-hand rule.

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.



The geometry in this case is similar to that of Example 21.10 (Section 21.5), in which we solved the problem of the *electric* field caused by an infinite line of charge. The same integral appears in both problems, and the field magnitudes in both problems are proportional to $1/r$. But the lines of $\vec{B}$ in the magnetic problem have completely different shapes than the lines of $\vec{E}$ in the analogous electrical problem. Electric field lines radiate outward from a positive line charge distribution (inward for negative charges). By contrast, magnetic field lines *encircle* the current that acts as their source. Electric field lines due to charges begin and end at those charges, but magnetic field lines always form closed loops and *never* have endpoints, irrespective of the shape of the current-carrying conductor that sets up the field. As we discussed in Section 27.3, this is a consequence of Gauss's law for magnetism, which states that the total magnetic flux through *any* closed surface is always zero:

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(magnetic flux through any closed surface)} \qquad (28.10)$$

Any magnetic field line that enters a closed surface must emerge from that surface.

EXAMPLE 28.3 Magnetic field of a single wire

WITH VARIATION PROBLEMS

A long, straight conductor carries a 1.0 A current. At what distance from the axis of the conductor does the resulting magnetic field have magnitude $B = 0.5 \times 10^{-4}$ T (about that of the earth's magnetic field in Pittsburgh)?

IDENTIFY and SET UP The length of a "long" conductor is much greater than the distance from the conductor to the field point. Hence we can use the ideas of this section. The geometry is the same as that of Fig. 28.6, so we use Eq. (28.9). All of the quantities in this equation are known except the target variable, the distance r.

EXECUTE We solve Eq. (28.9) for r:

$$r = \frac{\mu_0 I}{2\pi B} = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(1.0\ \mathrm{A})}{(2\pi)(0.5 \times 10^{-4}\ \mathrm{T})} = 4 \times 10^{-3}\ \mathrm{m} = 4\ \mathrm{mm}$$

EVALUATE As we saw in Example 26.14, currents of an ampere or more are typical of those found in the wiring of home appliances. This example shows that the magnetic fields produced by these appliances are very weak even very close to the wire; the fields are proportional to $1/r$, so they become even weaker at greater distances.

KEYCONCEPT The magnetic field $\vec{B}$ of a long, straight, current-carrying wire is proportional to the current and inversely proportional to the distance from the wire. The field lines are concentric circles around the wire, with the field direction given by a right-hand rule.

EXAMPLE 28.4 Magnetic field of two wires

WITH VARIATION PROBLEMS

Figure 28.7a is an end-on view of two long, straight, parallel wires perpendicular to the xy-plane, each carrying a current I but in opposite directions. (a) Find $\vec{B}$ at points P_1, P_2, and P_3. (b) Find an expression for $\vec{B}$ at any point on the x-axis to the right of wire 2.

IDENTIFY and SET UP We can find the magnetic fields $\vec{B}_1$ and $\vec{B}_2$ due to wires 1 and 2 at each point by using the ideas of this section. By the superposition principle, the magnetic field at each point is then $\vec{B} = \vec{B}_1 + \vec{B}_2$. We use Eq. (28.9) to find the magnitudes B_1 and B_2 of these fields and the right-hand rule to find the corresponding directions. Figure 28.7a shows $\vec{B}_1$, $\vec{B}_2$, and $\vec{B} = \vec{B}_{\text{total}}$ at each point; you should confirm that the directions and relative magnitudes shown are correct. Figure 28.7b shows some of the magnetic field lines due to this two-wire system.

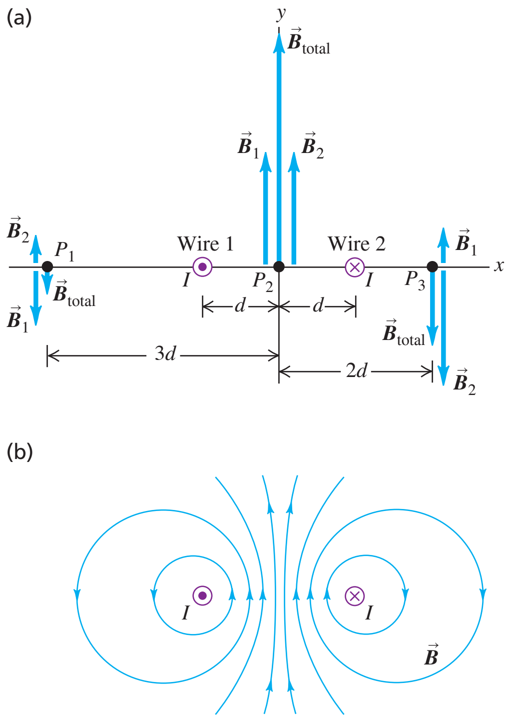
EXECUTE (a) Since point P_1 is a distance $2d$ from wire 1 and a distance $4d$ from wire 2, $B_1 = \mu_0 I/2\pi(2d) = \mu_0 I/4\pi d$ and $B_2 = \mu_0 I/2\pi(4d) = \mu_0 I/8\pi d$. The right-hand rule shows that $\vec{B}_1$ is in the negative y-direction and $\vec{B}_2$ is in the positive y-direction, so

$$\vec{B}_{\text{total}} = \vec{B}_1 + \vec{B}_2 = -\frac{\mu_0 I}{4\pi d}\hat{\jmath} + \frac{\mu_0 I}{8\pi d}\hat{\jmath} = -\frac{\mu_0 I}{8\pi d}\hat{\jmath} \qquad \text{(point } P_1\text{)}$$

At point P_2, a distance d from both wires, $\vec{B}_1$ and $\vec{B}_2$ are both in the positive y-direction, and both have the same magnitude $B_1 = B_2 = \mu_0 I/2\pi d$. Hence

$$\vec{B}_{\text{total}} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{2\pi d}\hat{\jmath} + \frac{\mu_0 I}{2\pi d}\hat{\jmath} = \frac{\mu_0 I}{\pi d}\hat{\jmath} \qquad \text{(point } P_2\text{)}$$

Figure **28.7** (a) Two long, straight conductors carrying equal currents in opposite directions. The conductors are seen end-on. (b) Map of the magnetic field produced by the two conductors. The field lines are closest together between the conductors, where the field is strongest.



Finally, at point P_3 the right-hand rule shows that $\vec{B}_1$ is in the positive y-direction and $\vec{B}_2$ is in the negative y-direction. This point is a distance $3d$ from wire 1 and a distance d from wire 2, so $B_1 = \mu_0 I/2\pi(3d) = \mu_0 I/6\pi d$ and $B_2 = \mu_0 I/2\pi d$. The total field at P_3 is

$$\vec{B}_{\text{total}} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{6\pi d}\hat{\jmath} - \frac{\mu_0 I}{2\pi d}\hat{\jmath} = -\frac{\mu_0 I}{3\pi d}\hat{\jmath} \qquad \text{(point } P_3\text{)}$$

The same technique can be used to find $\vec{B}_{\text{total}}$ at any point; for points off the x-axis, caution must be taken in vector addition, since $\vec{B}_1$ and $\vec{B}_2$ need no longer be simply parallel or antiparallel.

(b) At any point on the x-axis to the right of wire 2 (that is, for $x > d$), $\vec{B}_1$ and $\vec{B}_2$ are in the same directions as at P_3. Such a point is a distance $x + d$ from wire 1 and a distance $x - d$ from wire 2, so the total field is

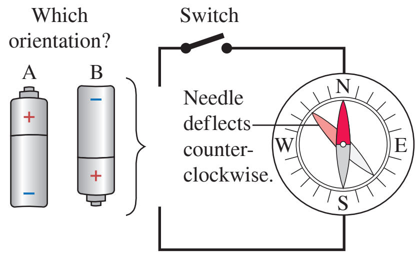
$$\vec{B}_{\text{total}} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{2\pi(x+d)}\hat{\jmath} - \frac{\mu_0 I}{2\pi(x-d)}\hat{\jmath}$$
$$= -\frac{\mu_0 I d}{\pi(x^2 - d^2)}\hat{\jmath}$$

where we used a common denominator to combine the two terms.

EVALUATE Consider our result from part (b) at a point very far from the wires, so that x is much larger than d. Then the d^2 term in the denominator can be ignored, and the magnitude of the total field is approximately $B_{\text{total}} = \mu_0 I d/\pi x^2$. For one wire, Eq. (28.9) shows that the magnetic field decreases with distance in proportion to $1/x$; for two wires carrying opposite currents, $\vec{B}_1$ and $\vec{B}_2$ partially cancel each other, and so B_{total} decreases more rapidly, in proportion to $1/x^2$. This effect is used in communication systems such as telephone or computer networks. The wiring is arranged so that a conductor carrying a signal in one direction and the conductor carrying the return signal are side by side, as in Fig. 28.7a, or twisted around each other (**Fig. 28.8**). As a result, the magnetic field due to these signals *outside* the conductors is very small, making it less likely to exert unwanted forces on other information-carrying currents.

Figure **28.8** Computer cables, or cables for audio-video equipment, create little or no magnetic field. This is because within each cable, closely spaced wires carry current in both directions along the length of the cable. The magnetic fields from these opposing currents cancel each other.

KEYCONCEPT Magnetic fields obey the superposition principle: The net magnetic field due to two or more sources is the vector sum of the fields due to the individual sources.



TEST YOUR UNDERSTANDING OF SECTION 28.3 The accompanying figure shows a circuit that lies on a horizontal table. A compass is placed on top of the circuit as shown. A battery is to be connected to the circuit so that when the switch is closed, the compass needle deflects counterclockwise. In which orientation, A or B, should the battery be placed in the circuit?

ANSWER

A | This orientation will cause current to flow clockwise around the circuit. Hence current will flow south through the wire that lies under the compass. From the right-hand rule for the magnetic field produced by a long, straight, current-carrying conductor, this will produce a magnetic field that points to the left at the position of the compass (which lies atop the wire). The combination of the northward magnetic field of the earth and the westward field produced by the current gives a net magnetic field to the northwest, so the compass needle will swing counterclockwise to align with this field.

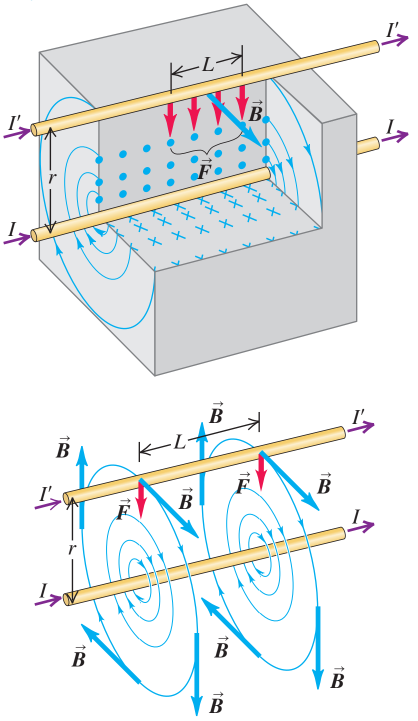
28.4 FORCE BETWEEN PARALLEL CONDUCTORS

Now that we know how to calculate the magnetic field produced by a long, current-carrying conductor, we can find the *magnetic force* that one such conductor exerts on another. This force plays a role in many practical situations in which current-carrying wires are close to each other. **Figure 28.9** shows segments of two long, straight, parallel conductors separated by a distance r and carrying currents I and I' in the same direction. Each conductor lies in the magnetic field set up by the other, so each experiences a force. The figure shows some of the field lines set up by the current in the lower conductor.

Figure 28.9 Parallel conductors carrying currents in the same direction attract each other. The diagrams show how the magnetic field $\vec{B}$ caused by the current in the lower conductor exerts a force $\vec{F}$ on the upper conductor.

The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

If the wires had currents in *opposite* directions, they would *repel* each other.



From Eq. (28.9) the lower conductor produces a $\vec{B}$ field that, at the position of the upper conductor, has magnitude

$$B = \frac{\mu_0 I}{2\pi r}$$

From Eq. (27.19) the force that this field exerts on a length L of the upper conductor is $\vec{F} = I'\vec{L} \times \vec{B}$, where the vector $\vec{L}$ is in the direction of the current I' and has magnitude L. Since $\vec{B}$ is perpendicular to the length of the conductor and hence to $\vec{L}$, the magnitude of this force is

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi r}$$

and the force *per unit length* F/L is

Magnetic force per unit length between two long, straight, parallel, current-carrying conductors

Magnetic constant
Current in first conductor
Current in second conductor

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} \qquad (28.11)$$

Distance between conductors

Applying the right-hand rule to $\vec{F} = I'\vec{L} \times \vec{B}$ shows that the force on the upper conductor is directed *downward*.

The current in the *upper* conductor also sets up a $\vec{B}$ field at the position of the *lower* conductor. Two successive applications of the right-hand rule for vector products (one to find the direction of the $\vec{B}$ field due to the upper conductor, as in Section 28.2, and one to find the direction of the force that this field exerts on the lower conductor, as in Section 27.6) show that the force on the lower conductor is *upward.* Thus *two parallel conductors carrying current in the same direction attract each other.* If the direction of either current is reversed, the forces also reverse. *Parallel conductors carrying currents in opposite directions repel each other.*

Magnetic Forces and the Value of μ_0

We saw in Section 28.1 that the magnetic constant μ_0 is very nearly equal to $4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$. If μ_0 had *exactly* this value, Eq. (28.11) shows that the force per unit length on each of two infinitely long parallel conductors one meter apart, each carrying a current of one ampere, would be

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1\ \text{A})(1\ \text{A})}{2\pi(1\ \text{m})} = 2 \times 10^{-7}\ \text{T}\cdot\text{A} = 2 \times 10^{-7}\ \text{N/m}$$

Prior to 2018 the ampere was defined so that F/L would have exactly this value in this situation, and μ_0 was indeed exactly equal to $4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$. The coulomb was defined to be the amount of charge transferred in one second by a current of one ampere.

As we described in Section 21.3, the coulomb is now defined in terms of e, the magnitude of charge on an electron or proton. Hence the value of μ_0 has changed very slightly from $4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ and has a fractional uncertainty of about 2×10^{-10}. The difference between the old and new values of μ_0 is so miniscule, however, that we can safely use $4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ in calculations.

Mutual forces of attraction exist not only between *wires* carrying currents in the same direction, but also between the elements of a single current-carrying conductor. If the conductor is a liquid or an ionized gas (a plasma), these forces result in a constriction of the conductor called the *pinch effect*. The pinch effect in a plasma has been used in one technique to bring about nuclear fusion.

EXAMPLE 28.5 Forces between parallel wires

WITH VARIATION PROBLEMS

Two straight, parallel, superconducting wires 4.5 mm apart carry equal currents of 15,000 A in opposite directions. What force, per unit length, does each wire exert on the other?

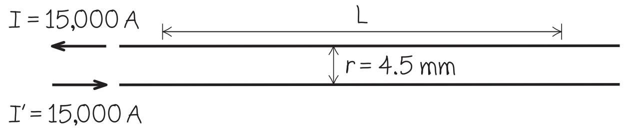
IDENTIFY and SET UP **Figure 28.10** shows the situation. We find F/L, the magnetic force per unit length of wire, from Eq. (28.11).

EXECUTE The conductors *repel* each other because the currents are in opposite directions. From Eq. (28.11) the force per unit length is

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(15{,}000\ \mathrm{A})^2}{(2\pi)(4.5 \times 10^{-3}\ \mathrm{m})}$$

$$= 1.0 \times 10^4\ \mathrm{N/m}$$

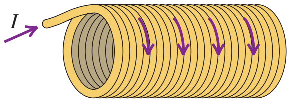
Figure **28.10** Our sketch for this problem.



EVALUATE This is a large force, more than one ton per meter. Currents and separations of this magnitude are used in superconducting electromagnets in particle accelerators, and mechanical stress analysis is a crucial part of the design process.

KEYCONCEPT Two long, straight, parallel, current-carrying conductors attract each other if they carry current in the same direction. They repel each other if they carry current in opposite directions.

TEST YOUR UNDERSTANDING OF SECTION 28.4 A solenoid is a wire wound into a helical coil. The accompanying figure shows a solenoid that carries a current I. (a) Is the *magnetic* force that one turn of the coil exerts on an adjacent turn (i) attractive, (ii) repulsive, or (iii) zero? (b) Is the *electric* force that one turn of the coil exerts on an adjacent turn (i) attractive, (ii) repulsive, or (iii) zero? (c) Is the *magnetic* force between opposite sides of the same turn of the coil (i) attractive, (ii) repulsive, or (iii) zero? (d) Is the *electric* force between opposite sides of the same turn of the coil (i) attractive, (ii) repulsive, or (iii) zero?



ANSWER

(a) **(i)**, (b) **(iii)**, (c) **(ii)**, (d) **(iii)** Current flows in the same direction in adjacent turns of the coil, so the magnetic forces between these turns are attractive. Current flows in opposite directions on opposite sides of the same turn, so the magnetic forces between these sides are repulsive. Thus the magnetic forces on the solenoid turns squeeze them together in the direction along its axis but push them apart radially. The *electric* forces are zero because the wire is electrically neutral, with as much positive charge as there is negative charge.

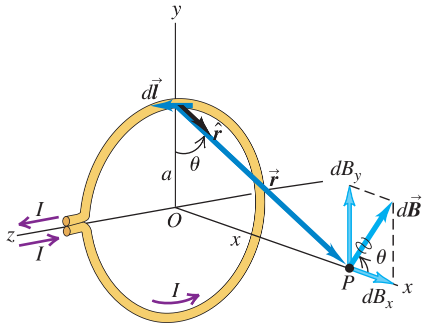
28.5 MAGNETIC FIELD OF A CIRCULAR CURRENT LOOP

If you look inside a doorbell, a transformer, an electric motor, or an electromagnet (**Fig. 28.11**), you'll find coils of wire with a large number of turns, spaced so closely that each turn is very nearly a planar circular loop. A current in such a coil is used to establish a magnetic field. In Section 27.7 we considered the force and torque on such a current loop placed in an external magnetic field produced by other currents; we are now about to find the magnetic field produced by such a loop or by a collection of closely spaced loops forming a coil.

Figure 28.12 (next page) shows a circular conductor with radius a. A current I is led into and out of the loop through two long, straight wires side by side; the currents in these straight wires are in opposite directions, and their magnetic fields very nearly cancel each other (see Example 28.4 in Section 28.3).

Figure **28.11** This electromagnet contains a current-carrying coil with numerous turns of wire. The resulting magnetic field can pick up large quantities of steel bars and other iron-bearing items.

Figure **28.12** Magnetic field on the axis of a circular loop. The current in the segment $d\vec{l}$ causes the field $d\vec{B}$, which lies in the xy-plane. The currents in other $d\vec{l}$'s cause $d\vec{B}$'s with different components perpendicular to the x-axis; these components add to zero. The x-components of the $d\vec{B}$'s combine to give the total $\vec{B}$ field at point P.



We can use the law of Biot and Savart, Eq. (28.5) or (28.6), to find the magnetic field at a point P on the axis of the loop, at a distance x from the center. As the figure shows, $d\vec{l}$ and $\hat{r}$ are perpendicular, and the direction of the field $d\vec{B}$ caused by this particular element $d\vec{l}$ lies in the xy-plane. Since $r^2 = x^2 + a^2$, the magnitude dB of the field due to element $d\vec{l}$ is

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \tag{28.12}$$

The components of the vector $d\vec{B}$ are

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{a}{(x^2 + a^2)^{1/2}} \tag{28.13}$$

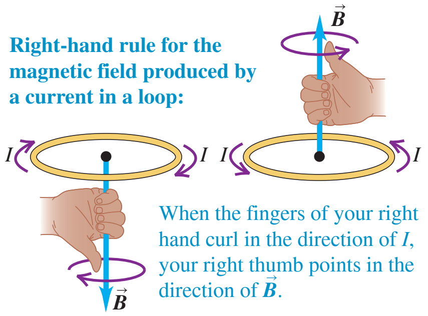
$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{x}{(x^2 + a^2)^{1/2}} \tag{28.14}$$

The *total* field $\vec{B}$ at P has only an x-component (it is perpendicular to the plane of the loop). Here's why: For every element $d\vec{l}$ there is a corresponding element on the opposite side of the loop, with opposite direction. These two elements give equal contributions to the x-component of $d\vec{B}$, given by Eq. (28.13), but *opposite* components perpendicular to the x-axis. Thus all the perpendicular components cancel and only the x-components survive.

To obtain the x-component of the total field $\vec{B}$, we integrate Eq. (28.13), including all the $d\vec{l}$'s around the loop. Everything in this expression except dl is constant and can be taken outside the integral, and we have

$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{a\,dl}{(x^2 + a^2)^{3/2}} = \frac{\mu_0 I a}{4\pi(x^2 + a^2)^{3/2}} \int dl$$

Figure **28.13** The right-hand rule for the direction of the magnetic field produced on the axis of a current-carrying coil.



The integral of dl is just the circumference of the circle, $\int dl = 2\pi a$, and so

Magnetic field on axis of a circular current-carrying loop

Magnetic constant; Current; Radius of loop; Distance along axis from center of loop to field point

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \tag{28.15}$$

The *direction* of this magnetic field is given by a right-hand rule. If you curl the fingers of your right hand around the loop in the direction of the current, your right thumb points in the direction of the field (**Fig. 28.13**).

Magnetic Field on the Axis of a Coil

Now suppose that instead of the single loop in Fig. 28.12 we have a coil consisting of N loops, all with the same radius. The loops are closely spaced so that the plane of each loop is essentially the same distance x from the field point P. Then the total field is N times the field of a single loop:

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}} \quad \text{(on the axis of } N \text{ circular loops)} \tag{28.16}$$

The factor N in Eq. (28.16) is the reason coils of wire, not single loops, are used to produce strong magnetic fields; for a desired field strength, using a single loop might require a current I so great as to exceed the rating of the loop's wire.

Figure **28.14** Graph of the magnetic field along the axis of a circular coil with N turns. When x is much larger than a, the field magnitude decreases approximately as $1/x^3$.

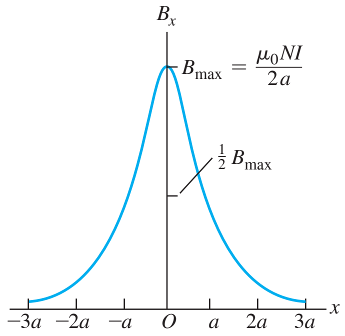


Figure 28.14 shows a graph of B_x as a function of x. The maximum value of the field is at $x = 0$, the center of the loop or coil:

Magnetic field at center of N circular current-carrying loops

Magnetic constant; Number of loops; Current; Radius of loop

$$B_x = \frac{\mu_0 N I}{2a} \tag{28.17}$$

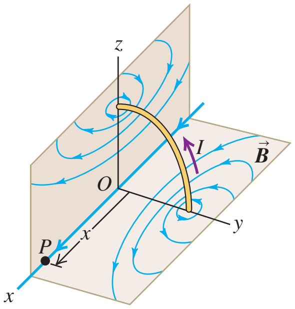
In Section 27.7 we defined the *magnetic dipole moment* μ (or *magnetic moment*) of a current-carrying loop to be equal to IA, where A is the cross-sectional area of the loop. If there are N loops, the total magnetic moment is NIA. The circular loop in Fig. 28.12 has area $A = \pi a^2$, so the magnetic moment of a single loop is $\mu = I\pi a^2$; for N loops, $\mu = NI\pi a^2$. Substituting these results into Eqs. (28.15) and (28.16), we find

$$B_x = \frac{\mu_0 \mu}{2\pi(x^2 + a^2)^{3/2}} \quad \text{(on the axis of any number of circular loops)} \qquad (28.18)$$

We described a magnetic dipole in Section 27.7 in terms of its response to a magnetic field produced by currents outside the dipole. But a magnetic dipole is also a *source* of magnetic field; Eq. (28.18) describes the magnetic field *produced* by a magnetic dipole for points along the dipole axis. This field is directly proportional to the magnetic dipole moment μ. Note that the field at all points on the x-axis is in the same direction as the vector magnetic moment $\vec{\boldsymbol{\mu}}$.

Figure 28.15 shows some of the magnetic field lines surrounding a circular current loop (magnetic dipole) in planes through the axis. The directions of the field lines are given by the same right-hand rule as for a long, straight conductor. Grab the conductor with your right hand, with your thumb in the direction of the current; your fingers curl around in the same direction as the field lines. The field lines for the circular current loop are closed curves that encircle the conductor; they are *not* circles, however.

Figure **28.15** Magnetic field lines produced by the current in a circular loop. At points on the axis the $\vec{\boldsymbol{B}}$ field has the same direction as the magnetic moment of the loop.



CAUTION **Magnetic field of a coil** Equations (28.15), (28.16), and (28.18) are valid only on the *axis* of a loop or coil. Don't attempt to apply these equations at other points!

BIO APPLICATION **Magnetic Fields for MRI** Magnetic resonance imaging (see Section 27.7) requires a magnetic field of about 1.5 T. In a typical MRI scan, the patient lies inside a coil that produces the intense field. The currents required are very high, so the coils are bathed in liquid helium at a temperature of 4.2 K to keep them from overheating.

EXAMPLE 28.6 Magnetic field of a coil

A coil consisting of 100 circular loops with radius 0.60 m carries a 5.0 A current. (a) Find the magnetic field at a point along the axis of the coil, 0.80 m from the center. (b) Along the axis, at what distance from the center of the coil is the field magnitude $\frac{1}{8}$ as great as it is at the center?

IDENTIFY and SET UP This problem concerns the magnetic-field magnitude B along the axis of a current-carrying coil, so we can use Eq. (28.16). We are given $N = 100$, $I = 5.0$ A, and $a = 0.60$ m. In part (a) our target variable is B_x at a given value of x. In part (b) the target variable is the value of x at which the field has $\frac{1}{8}$ of the magnitude that it has at the origin.

EXECUTE (a) Using $x = 0.80$ m, from Eq. (28.16) we have

$$B_x = \frac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(100)(5.0\ \text{A})(0.60\ \text{m})^2}{2[(0.80\ \text{m})^2 + (0.60\ \text{m})^2]^{3/2}}$$
$$= 1.1 \times 10^{-4}\ \text{T}$$

(b) Considering Eq. (28.16), we seek a value of x such that

$$\frac{1}{(x^2 + a^2)^{3/2}} = \frac{1}{8}\frac{1}{(0^2 + a^2)^{3/2}}$$

To solve this for x, we take the reciprocal of the whole thing and then take the $\frac{2}{3}$ power of both sides; the result is

$$x = \pm\sqrt{3}a = \pm 1.04\ \text{m}$$

EVALUATE We check our answer in part (a) by finding the coil's magnetic moment and substituting the result into Eq. (28.18):

$$\mu = NI\pi a^2 = (100)(5.0\ \text{A})\pi(0.60\ \text{m})^2 = 5.7 \times 10^2\ \text{A} \cdot \text{m}^2$$

$$B_x = \frac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(5.7 \times 10^2\ \text{A} \cdot \text{m}^2)}{2\pi[(0.80\ \text{m})^2 + (0.60\ \text{m})^2]^{3/2}} = 1.1 \times 10^{-4}\ \text{T}$$

The magnetic moment μ is relatively large, yet it produces a rather small field, comparable to that of the earth. This illustrates how difficult it is to produce strong fields of 1 T or more.

KEYCONCEPT The magnetic field $\vec{\boldsymbol{B}}$ on the axis of a current-carrying coil points along the axis, with a direction given by a right-hand rule. The magnitude of $\vec{\boldsymbol{B}}$ is proportional to the magnetic dipole moment of the coil and is greatest at the center of the coil.

TEST YOUR UNDERSTANDING OF SECTION 28.5 Figure 28.12 shows the magnetic field $d\vec{B}$ produced at point P by a segment $d\vec{l}$ that lies on the positive y-axis (at the top of the loop). This field has components $dB_x > 0$, $dB_y > 0$, $dB_z = 0$. (a) What are the signs of the components of the field $d\vec{B}$ produced at P by a segment $d\vec{l}$ on the negative y-axis (at the bottom of the loop)? (i) $dB_x > 0$, $dB_y > 0$, $dB_z = 0$; (ii) $dB_x > 0$, $dB_y < 0$, $dB_z = 0$; (iii) $dB_x < 0$, $dB_y > 0$, $dB_z = 0$; (iv) $dB_x < 0$, $dB_y < 0$, $dB_z = 0$; (v) none of these. (b) What are the signs of the components of the field $d\vec{B}$ produced at P by a segment $d\vec{l}$ on the negative z-axis (at the right-hand side of the loop)? (i) $dB_x > 0$, $dB_y > 0$, $dB_z = 0$; (ii) $dB_x > 0$, $dB_y < 0$, $dB_z = 0$; (iii) $dB_x < 0$, $dB_y > 0$, $dB_z = 0$; (iv) $dB_x < 0$, $dB_y < 0$, $dB_z = 0$; (v) none of these.

ANSWER

(a) **(ii)**, (b) **(v)** The vector $d\vec{B}$ is in the direction of $d\vec{l} \times \vec{r}$. For a segment on the negative y-axis, $d\vec{l} = -\hat{k}\,dl$ points in the negative z-direction and $\vec{r} = x\hat{\imath} + a\hat{\jmath}$. Hence $d\vec{l} \times \vec{r} = (a\,dl)\hat{\imath} - (x\,dl)\hat{\jmath}$, which has a positive x-component, a negative y-component, and zero z-component. For a segment on the negative z-axis, $d\vec{l} = \hat{\jmath}\,dl$ points in the positive y-direction and $\vec{r} = x\hat{\imath} + a\hat{k}$. Hence $d\vec{l} \times \vec{r} = 1(a\,dl)\hat{\imath} - 1(x\,dl)\hat{k}$, which has a positive x-component, zero y-component, and a negative z-component.

28.6 AMPERE'S LAW

So far our calculations of the magnetic field due to a current have involved finding the infinitesimal field $d\vec{B}$ due to a current element and then summing all the $d\vec{B}$'s to find the total field. This approach is directly analogous to our *electric*-field calculations in Chapter 21.

For the electric-field problem we found that in situations with a highly symmetric charge distribution, it was often easier to use Gauss's law to find $\vec{E}$. There is likewise a law that allows us to more easily find the *magnetic* fields caused by highly symmetric *current* distributions. But the law that allows us to do this, called *Ampere's law,* is rather different in character from Gauss's law.

Gauss's law for electric fields (Chapter 22) involves the flux of $\vec{E}$ through a closed surface; it states that this flux is equal to the total charge enclosed within the surface, divided by the constant ϵ_0. Thus this law relates electric fields and charge distributions. By contrast, Gauss's law for *magnetic* fields, Eq. (28.10), is *not* a relationship between magnetic fields and current distributions; it states that the flux of $\vec{B}$ through *any* closed surface is always zero, whether or not there are currents within the surface. So Gauss's law for $\vec{B}$ can't be used to determine the magnetic field produced by a particular current distribution.

Ampere's law is formulated not in terms of magnetic flux, but rather in terms of the *line integral* of $\vec{B}$ around a closed path, denoted by

$$\oint \vec{B} \cdot d\vec{l}$$

We used line integrals to define work in Chapter 6 and to calculate electric potential in Chapter 23. To evaluate this integral, we divide the path into infinitesimal segments $d\vec{l}$, calculate the scalar product of $\vec{B} \cdot d\vec{l}$ for each segment, and sum these products. In general, $\vec{B}$ varies from point to point, and we must use the value of $\vec{B}$ at the location of each $d\vec{l}$. An alternative notation is $\oint B_\parallel\,dl$, where $B_\parallel$ is the component of $\vec{B}$ parallel to $d\vec{l}$ at each point. The circle on the integral sign indicates that this integral is always computed for a *closed* path, one whose beginning and end points are the same.

Ampere's Law for a Long, Straight Conductor

To introduce the basic idea of Ampere's law, let's consider again the magnetic field caused by a long, straight conductor carrying a current I. We found in Section 28.3 that the field at a distance r from the conductor has magnitude

$$B = \frac{\mu_0 I}{2\pi r}$$

The magnetic field lines are circles centered on the conductor. Let's take the line integral of $\vec{B}$ around a circle with radius r, as in **Fig. 28.16a**. At every point on the circle, $\vec{B}$ and $d\vec{l}$ are parallel, and so $\vec{B}\cdot d\vec{l} = B\,dl$; since r is constant around the circle, B is constant as well. Alternatively, we can say that $B_{\parallel}$ is constant and equal to B at every point on the circle. Hence we can take B outside of the integral. The remaining integral is just the circumference of the circle, so

$$\oint \vec{B}\cdot d\vec{l} = \oint B_{\parallel}\,dl = B\oint dl = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

The line integral is thus independent of the radius of the circle and is equal to μ_0 multiplied by the current passing through the area bounded by the circle.

In Fig. 28.16b the situation is the same, but the integration path now goes around the circle in the opposite direction. Now $\vec{B}$ and $d\vec{l}$ are antiparallel, so $\vec{B}\cdot d\vec{l} = -B\,dl$ and the line integral equals $-\mu_0 I$. We get the same result if the integration path is the same as in Fig. 28.16a, but the direction of the current is reversed. Thus $\oint \vec{B}\cdot d\vec{l}$ equals μ_0 multiplied by the current passing through the area bounded by the integration path, with a positive or negative sign depending on the direction of the current relative to the direction of integration.

There's a simple rule for the sign of the current; you won't be surprised to learn that it uses your right hand. Curl the fingers of your right hand around the integration path so that they curl in the direction of integration (that is, the direction that you use to evaluate $\oint \vec{B}\cdot d\vec{l}$). Then your right thumb indicates the positive current direction. Currents that pass through the integration path in this direction are positive; those in the opposite direction are negative. Using this rule, convince yourself that the current is positive in Fig. 28.16a and negative in Fig. 28.16b. Here's another way to say the same thing: Looking at the surface bounded by the integration path, integrate counterclockwise around the path as in Fig. 28.16a. Currents moving toward you through the surface are positive, and those going away from you are negative.

An integration path that does *not* enclose the conductor is used in Fig. 28.16c. Along the circular arc ab of radius r_1, $\vec{B}$ and $d\vec{l}$ are parallel and $B_{\parallel} = B_1 = \mu_0 I/2\pi r_1$; along the circular arc cd of radius r_2, $\vec{B}$ and $d\vec{l}$ are antiparallel and $B_{\parallel} = -B_2 = -\mu_0 I/2\pi r_2$. The $\vec{B}$ field is perpendicular to $d\vec{l}$ at each point on the straight sections bc and da, so $B_{\parallel} = 0$ and these sections contribute zero to the line integral. The total line integral is then

$$\oint \vec{B}\cdot d\vec{l} = \oint B_{\parallel}\,dl = B_1\int_a^b dl + (0)\int_b^c dl + (-B_2)\int_c^d dl + (0)\int_d^a dl$$

$$= \frac{\mu_0 I}{2\pi r_1}(r_1\theta) + 0 - \frac{\mu_0 I}{2\pi r_2}(r_2\theta) + 0 = 0$$

The magnitude of $\vec{B}$ is greater on arc cd than on arc ab, but the arc length is less, so the contributions from the two arcs exactly cancel. Even though there is a magnetic field everywhere along the integration path, the line integral $\oint \vec{B}\cdot d\vec{l}$ is zero if there is no current passing through the area bounded by the path.

We can also derive these results for more general integration paths, such as the one in **Fig. 28.17a**. At the position of the line element $d\vec{l}$, the angle between $d\vec{l}$ and $\vec{B}$ is ϕ, and

$$\vec{B}\cdot d\vec{l} = B\,dl\cos\phi$$

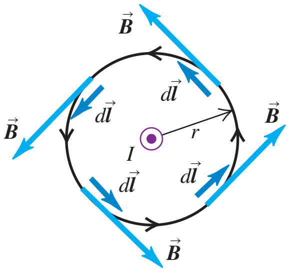
From the figure, $dl\cos\phi = r\,d\theta$, where $d\theta$ is the angle subtended by $d\vec{l}$ at the position of the conductor and r is the distance of $d\vec{l}$ from the conductor. Thus

$$\oint \vec{B}\cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi r}(r\,d\theta) = \frac{\mu_0 I}{2\pi}\oint d\theta$$

Figure **28.16** Three integration paths for the line integral of $\vec{B}$ in the vicinity of a long, straight conductor carrying current I *out* of the plane of the page (as indicated by the circle with a dot). The conductor is seen end-on.

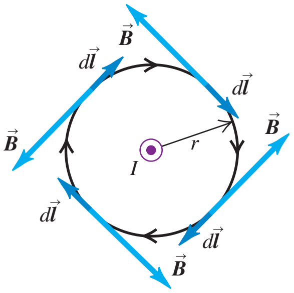
(a) Integration path is a circle centered on the conductor; integration goes around the circle counterclockwise.

Result: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$



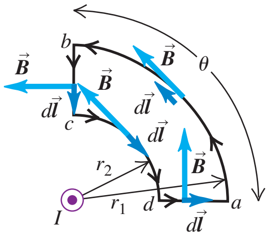
(b) Same integration path as in (a), but integration goes around the circle clockwise.

Result: $\oint \vec{B}\cdot d\vec{l} = -\mu_0 I$



(c) An integration path that does not enclose the conductor

Result: $\oint \vec{B}\cdot d\vec{l} = 0$



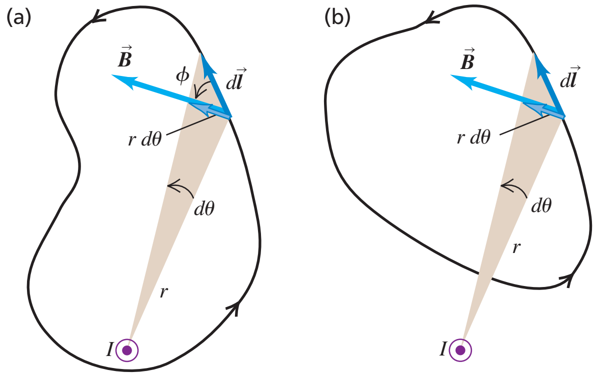


Figure 28.17 (a) A more general integration path for the line integral of $\vec{B}$ around a long, straight conductor carrying current I *out* of the plane of the page. The conductor is seen end-on. (b) A more general integration path that does not enclose the conductor.

But $\oint d\theta$ is just equal to 2π, the total angle swept out by the radial line from the conductor to $d\vec{l}$ during a complete trip around the path. So we get

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \tag{28.19}$$

This result doesn't depend on the shape of the path or on the position of the wire inside it. If the current in the wire is opposite to that shown, the integral has the opposite sign. But if the path doesn't enclose the wire (Fig. 28.17b), then the net change in θ during the trip around the integration path is zero; $\oint d\theta$ is zero instead of 2π and the line integral is zero.

Ampere's Law: General Statement

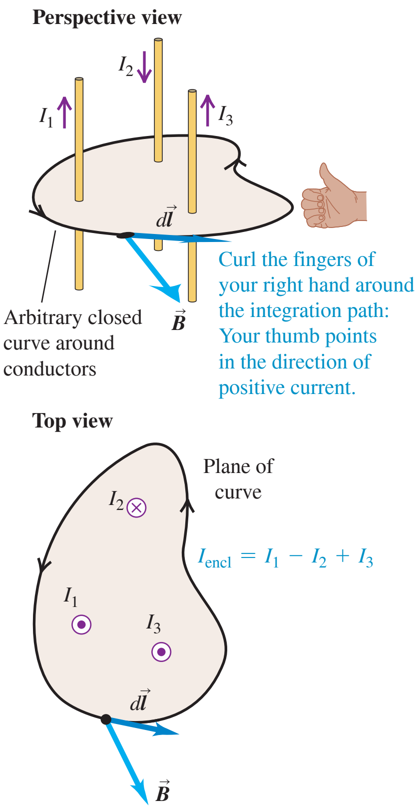
We can generalize Ampere's law even further. Suppose *several* long, straight conductors pass through the surface bounded by the integration path. The total magnetic field $\vec{B}$ at any point on the path is the vector sum of the fields produced by the individual conductors. Thus the line integral of the total $\vec{B}$ equals μ_0 times the *algebraic sum* of the currents. In calculating this sum, we use the sign rule for currents described above. If the integration path does not enclose a particular wire, the line integral of the $\vec{B}$ field of that wire is zero, because the angle θ for that wire sweeps through a net change of zero rather than 2π during the integration. Any conductors present that are not enclosed by a particular path may still contribute to the value of $\vec{B}$ at every point, but the *line integrals* of their fields around the path are zero.

Thus we can replace I in Eq. (28.19) with I_{encl}, the algebraic sum of the currents *enclosed* or *linked* by the integration path, with the sum evaluated by using the sign rule just described (**Fig. 28.18**). Then **Ampere's law** says

Ampere's law:

$$\overbrace{\oint}^{\text{Line integral around a closed path}} \underbrace{\vec{B} \cdot d\vec{l}}_{\text{Scalar product of magnetic field and vector segment of path}} = \overset{\text{Magnetic constant}}{\mu_0}\,\overset{\text{Net current enclosed by path}}{I_{\text{encl}}} \tag{28.20}$$

Figure 28.18 Ampere's law.

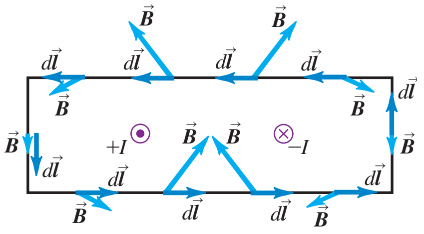


Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

While we have derived Ampere's law only for the special case of the field of several long, straight, parallel conductors, Eq. (28.20) is in fact valid for conductors and paths of *any* shape. The general derivation is no different in principle from what we have presented, but the geometry is more complicated.

If $\oint \vec{B} \cdot d\vec{l} = 0$, it *does not* necessarily mean that $\vec{B} = \mathbf{0}$ everywhere along the path, only that the total current through an area bounded by the path is zero. In Figs. 28.16c and 28.17b, the integration paths enclose no current at all; in **Fig. 28.19** there are positive and negative currents of equal magnitude through the area enclosed by the path. In both cases, $I_{\text{encl}} = 0$ and the line integral is zero.

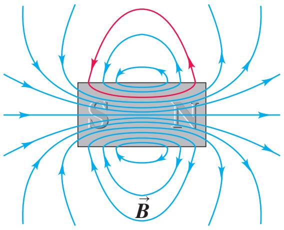
Figure 28.19 An end-on view of two long, straight conductors carrying equal currents in opposite directions. The line integral $\oint \vec{B} \cdot d\vec{l}$ gets zero contribution from the upper and lower segments, a positive contribution from the left segment, and a negative contribution from the right segment; the net integral is zero.



Equation (28.20) turns out to be valid *only* if the currents are steady and if no time-varying electric fields are present. In Chapter 29 we'll see how to generalize Ampere's law for time-varying fields.

CAUTION **Line integrals of electric and magnetic fields** In Chapter 23 we saw that the line integral of the electrostatic field $\vec{E}$ around any closed path is equal to zero; this is a statement that the electrostatic force $\vec{F} = q\vec{E}$ on a point charge q is conservative, so this force does zero work on a charge that moves around a closed path that returns to the starting point. The value of the line integral $\oint \vec{B} \cdot d\vec{l}$ is not similarly related to the question of whether the *magnetic* force is conservative. Remember that the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ on a moving charged particle is always *perpendicular* to $\vec{B}$, so $\oint \vec{B} \cdot d\vec{l}$ is *not* related to the work done by the magnetic force; as stated in Ampere's law, this integral is related only to the total current through a surface bounded by the integration path. In fact, the magnetic force on a moving charged particle is *not* conservative. A conservative force depends on only the position of the object on which the force is exerted, but the magnetic force on a moving charged particle also depends on the *velocity* of the particle. ❙

TEST YOUR UNDERSTANDING OF SECTION 28.6 The accompanying figure shows magnetic field lines through the center of a permanent magnet. The magnet is not connected to a source of emf. One of the field lines is colored red. What can you conclude about the currents inside the permanent magnet within the region enclosed by this field line? (i) There are no currents inside the magnet; (ii) there are currents directed out of the plane of the page; (iii) there are currents directed into the plane of the page; (iv) not enough information is given to decide.



ANSWER

❙ **(ii)** Imagine carrying out the integral $\oint \vec{B} \cdot d\vec{l}$ along an integration path that goes counterclockwise around the red magnetic field line. At each point along the path the magnetic field $\vec{B}$ and the infinitesimal segment $d\vec{l}$ are both tangent to the path, so $\vec{B} \cdot d\vec{l}$ is positive at each point and the integral $\oint \vec{B} \cdot d\vec{l}$ is likewise positive. It follows from Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ and the right-hand rule that the integration path encloses a current directed out of the plane of the page. There are no currents in the empty space outside the magnet, so there must be currents inside the magnet (see Section 28.8).

28.7 APPLICATIONS OF AMPERE'S LAW

Below are several examples of using Ampere's law to calculate the magnetic field due to a current distribution. Problem-Solving Strategy 28.2 is directly analogous to Problem-Solving Strategy 22.1 (Section 22.4) for using Gauss's law to calculate the electric field due to a charge distribution; we suggest you review that strategy now.

CAUTION **Ampere's law is powerful but limited** Like Gauss's law for $\vec{E}$, Ampere's law for $\vec{B}$ can be used to calculate the field *only* in highly symmetric situations. As the next examples show, in such situations you can exploit the symmetry to calculate the line integral of $\vec{B}$ around a simple path. ❙

PROBLEM-SOLVING STRATEGY 28.2 Ampere's Law

IDENTIFY *the relevant concepts:* Ampere's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$, can yield the magnitude of $\vec{B}$ as a function of position if the geometry of the field-generating electric current is highly symmetric.

SET UP *the problem* using the following steps:

1. Determine the target variable(s). Usually one will be the magnitude of the $\vec{B}$ field as a function of position.
2. Select the integration path to use with Ampere's law. To determine the magnetic field at a certain point, the path must pass through that point. The integration path doesn't have to be any actual physical boundary; it may be in empty space, embedded in a solid object, or some of each. The integration path has to have enough *symmetry* to make evaluation of the integral possible. Ideally the path will be tangent to $\vec{B}$ in regions of interest; elsewhere the path should be perpendicular to $\vec{B}$ or should run through regions where $\vec{B} = \mathbf{0}$.

EXECUTE *the solution* as follows:

1. Carry out the integral $\oint \vec{B} \cdot d\vec{l}$ along the chosen path. If $\vec{B}$ is tangent to all or some portion of the path and has the same magnitude B at every point, then its line integral is the product of B and the length of that portion of the path. If $\vec{B}$ is perpendicular to some portion of the path, or if $\vec{B} = \mathbf{0}$, that portion makes no contribution to the integral.
2. In the integral $\oint \vec{B} \cdot d\vec{l}$, $\vec{B}$ is the *total* magnetic field at each point on the path; it can be caused by currents enclosed *or not enclosed* by the path. If *no* net current is enclosed by the path, the field at points on the path need not be zero, but the integral $\oint \vec{B} \cdot d\vec{l}$ is always zero.
3. Determine the current I_{encl} enclosed by the integration path. A right-hand rule gives the sign of this current: If you curl the fingers of your right hand so that they follow the path in the direction of integration, then your right thumb points in the direction of positive current. If $\vec{B}$ is tangent to the path everywhere and I_{encl} is positive, the direction of $\vec{B}$ is the same as the direction of integration. If instead I_{encl} is negative, $\vec{B}$ is in the direction opposite to that of the integration.
4. Use Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ to solve for the target variable.

EVALUATE *your answer:* If your result is an expression for the field magnitude as a function of position, check it by examining how the expression behaves in different limits.

EXAMPLE 28.7 Field of a long, straight, current-carrying conductor

In Section 28.6 we derived Ampere's law from Eq. (28.9) for the field $\vec{B}$ of a long, straight, current-carrying conductor. Reverse this process, and use Ampere's law to find $\vec{B}$ for this situation.

IDENTIFY and SET UP The situation has cylindrical symmetry, so in Ampere's law we take our integration path to be a circle with radius r centered on the conductor and lying in a plane perpendicular to it, as in Fig. 28.16a. The field $\vec{B}$ is everywhere tangent to this circle and has the same magnitude B everywhere on the circle.

EXECUTE With our choice of the integration path, Ampere's law [Eq. (28.20)] becomes

$$\oint \vec{B} \cdot d\vec{l} = \oint B_{\parallel}\, dl = B(2\pi r) = \mu_0 I$$

Equation (28.9), $B = \mu_0 I/2\pi r$, follows immediately.

Ampere's law determines the direction of $\vec{B}$ as well as its magnitude. Since we chose to go counterclockwise around the integration path, the positive direction for current is out of the plane of Fig. 28.16a; this is the same as the actual current direction in the figure, so I is positive and the integral $\oint \vec{B} \cdot d\vec{l}$ is also positive. Since the $d\vec{l}$'s run counterclockwise, the direction of $\vec{B}$ must be counterclockwise as well, as shown in Fig. 28.16a.

EVALUATE Our results are consistent with those in Section 28.6.

KEYCONCEPT Ampere's law is easiest to use if the magnetic field $\vec{B}$ is everywhere tangent to an integration path and has the same magnitude at all points along that path.

EXAMPLE 28.8 Field of a long cylindrical conductor

WITH VARIATION PROBLEMS

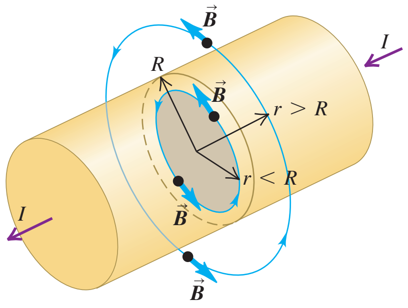
A cylindrical conductor with radius R carries a current I (**Fig. 28.20**). The current is uniformly distributed over the cross-sectional area of the conductor. Find the magnetic field as a function of the distance r from the conductor axis for points both inside $(r < R)$ and outside $(r > R)$ the conductor.

IDENTIFY and SET UP As in Example 28.7, the current distribution has cylindrical symmetry, and the magnetic field lines must be circles concentric with the conductor axis. To find the magnetic field inside and outside the conductor, we choose circular integration paths with radii $r < R$ and $r > R$, respectively (see Fig. 28.20).

EXECUTE In either case the field $\vec{B}$ has the same magnitude at every point on the circular integration path and is tangent to the path. Thus the magnitude of the line integral is simply $B(2\pi r)$. To find the current I_{encl} enclosed by a circular integration path inside the conductor $(r < R)$, note that the current density (current per unit area) is $J = I/\pi R^2$ so $I_{\text{encl}} = J(\pi r^2) = Ir^2/R^2$. Hence Ampere's law gives $B(2\pi r) = \mu_0 I r^2/R^2$, or

$$B = \frac{\mu_0 I}{2\pi}\frac{r}{R^2} \qquad \text{(inside the conductor, } r < R) \qquad (28.21)$$

Figure 28.20 To find the magnetic field at radius $r < R$, we apply Ampere's law to the circle enclosing the gray area. The current through the gray area is $(r^2/R^2)I$. To find the magnetic field at radius $r > R$, we apply Ampere's law to the circle enclosing the entire conductor.



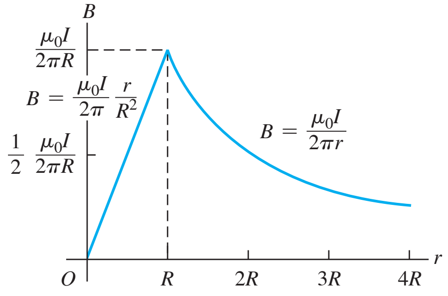
A circular integration path outside the conductor encloses the total current in the conductor, so $I_{\text{encl}} = I$. Applying Ampere's law gives the same equation as in Example 28.7, with the same result for B:

$$B = \frac{\mu_0 I}{2\pi r} \qquad \text{(outside the conductor, } r > R) \qquad (28.22)$$

Outside the conductor, the magnetic field is the same as that of a long, straight conductor carrying current I, independent of the radius R over which the current is distributed. Indeed, the magnetic field outside *any* cylindrically symmetric current distribution is the same as if the entire current were concentrated along the axis of the distribution. This is analogous to the results of Examples 22.5 and 22.9 (Section 22.4), in which we found that the *electric* field outside a spherically symmetric *charged* object is the same as though the entire charge were concentrated at the center.

EVALUATE At the surface of the conductor $(r = R)$, Eqs. (28.21) and (28.22) agree, as they must. **Figure 28.21** shows a graph of B as a function of r.

Figure 28.21 Magnitude of the magnetic field inside and outside a long, straight cylindrical conductor with radius R carrying a current I.



KEYCONCEPT The magnetic field lines produced by a solid, cylindrical, current-carrying conductor are concentric circles around the conductor, with the field direction given by a right-hand rule. Inside the conductor the field magnitude increases with increasing distance from the center; outside the conductor the magnitude decreases with distance.

EXAMPLE 28.9 Field of a solenoid

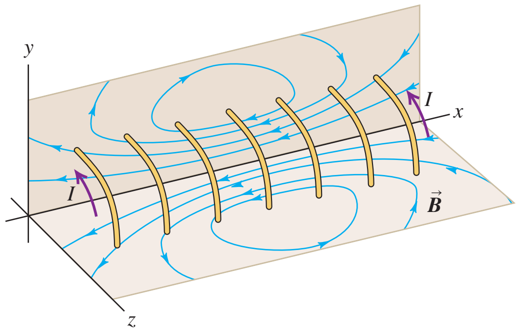
WITH VARIATION PROBLEMS

A **solenoid** consists of a helical winding of wire on a cylinder, usually circular in cross section. There can be thousands of closely spaced turns (often in several layers), each of which can be regarded as a circular loop. For simplicity, **Fig. 28.22** shows a solenoid with only a few turns. All turns carry the same current I, and the total $\vec{B}$ field at every point is the vector sum of the fields caused by the individual turns. The figure shows field lines in the xy- and xz-planes. We draw field lines that are uniformly spaced at the center of the solenoid. Exact calculations show that for a long, closely wound solenoid, half of these field lines emerge from the ends and half "leak out" through the windings between the center and the end, as the figure suggests.

If the solenoid is long in comparison with its cross-sectional diameter and the coils are tightly wound, the field inside the solenoid near its midpoint is very nearly uniform over the cross section and parallel to the axis; the *external* field near the midpoint is very small.

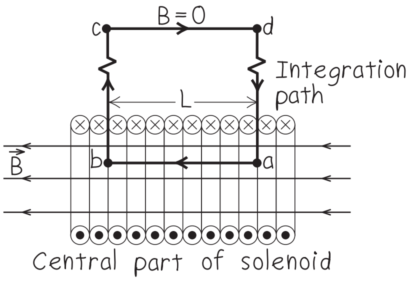
Use Ampere's law to find the field at or near the center of such a solenoid if it has n turns per unit length and carries current I.

Figure 28.22 Magnetic field lines produced by the current in a solenoid. For clarity, only a few turns are shown.



IDENTIFY and SET UP We assume that $\vec{B}$ is uniform inside the solenoid and zero outside. **Figure 28.23** shows the situation and our chosen integration path, rectangle $abcd$. Side ab, with length L, is parallel to the axis of the solenoid. Sides bc and da are taken to be very long so that side cd is far from the solenoid; then the field at side cd is negligibly small.

Figure 28.23 Our sketch for this problem.



EXECUTE Along side ab, $\vec{B}$ is parallel to the path and is constant. Our Ampere's-law integration takes us along side ab in the same direction as $\vec{B}$, so here $B_{\parallel} = +B$ and

$$\int_a^b \vec{B} \cdot d\vec{l} = BL$$

Along sides bc and da, $\vec{B}$ is perpendicular to the path, and so $B_{\parallel} = 0$; along side cd, $\vec{B} = \mathbf{0}$ and so $B_{\parallel} = 0$. Around the entire closed path, then, we have $\oint \vec{B} \cdot d\vec{l} = BL$.

In a length L there are nL turns, each of which passes once through $abcd$ carrying current I. Hence the total current enclosed by the rectangle is $I_{\text{encl}} = nLI$. The integral $\oint \vec{B} \cdot d\vec{l}$ is positive, so from Ampere's law I_{encl} must be positive as well. This means that the current passing through the surface bounded by the integration path must be in the direction shown in Fig. 28.23. Ampere's law then gives $BL = \mu_0 nLI$, or

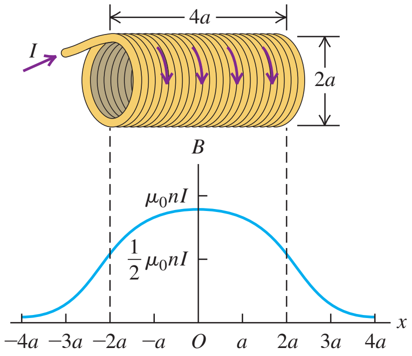
$$B = \mu_0 nI \qquad \text{(solenoid)} \qquad (28.23)$$

Side ab need not lie on the axis of the solenoid, so this result demonstrates that the field is uniform over the entire cross section at the center of the solenoid's length.

EVALUATE Note that the *direction* of $\vec{B}$ inside the solenoid is in the same direction as the solenoid's vector magnetic moment $\vec{\mu}$, as we found in Section 28.5 for a single current-carrying loop.

For points along the axis, the field is strongest at the center of the solenoid and drops off near the ends. For a solenoid very long in comparison to its diameter, the field magnitude at each end is exactly half that at the center. This is approximately the case even for a relatively short solenoid, as **Fig. 28.24** shows.

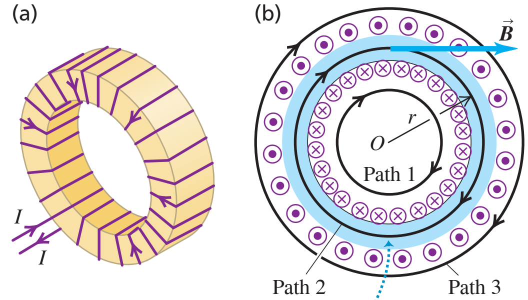
Figure 28.24 Magnitude of the magnetic field at points along the axis of a solenoid with length $4a$, equal to four times its radius a. The field magnitude at each end is about half its value at the center. (Compare with Fig. 28.14 for the field of N circular loops.)



KEYCONCEPT The magnetic field $\vec{B}$ of an ideal solenoid is uniform inside the solenoid and zero outside. The direction of $\vec{B}$ inside the solenoid is given by the same right-hand rule as for the field along the axis of a current-carrying coil (see Example 28.6).

EXAMPLE 28.10 Field of a toroidal solenoid

WITH VARIATION PROBLEMS



The magnetic field is confined almost entirely to the space enclosed by the windings (in blue).

Figure 28.25 (a) A toroidal solenoid. For clarity, only a few turns of the winding are shown. (b) Integration paths (black circles) used to compute the magnetic field $\vec{B}$ set up by the current (shown as dots and crosses).

Figure 28.25a shows a doughnut-shaped **toroidal solenoid,** tightly wound with N turns of wire carrying a current I. (In a practical solenoid the turns would be much more closely spaced than they are in the figure.) Find the magnetic field at all points.

IDENTIFY and SET UP Ignoring the slight pitch of the helical windings, we can consider each turn of a tightly wound toroidal solenoid as a loop lying in a plane perpendicular to the large, circular axis of the toroid. The symmetry of the situation then tells us that the magnetic field lines must be circles concentric with the toroid axis. Therefore we choose circular integration paths for use with Ampere's law, so that the field $\vec{B}$ (if any) is tangent to each path at all points along the path. Figure 28.25b shows three such paths.

EXECUTE Along each path, $\oint \vec{B} \cdot d\vec{l}$ equals the product of B and the path circumference $l = 2\pi r$. The total current enclosed by path 1 is zero, so from Ampere's law the field $\vec{B} = \mathbf{0}$ everywhere on this path.

Each turn of the winding passes *twice* through the area bounded by path 3, carrying equal currents in opposite directions. The *net* current enclosed is therefore zero, and hence $\vec{B} = \mathbf{0}$ at all points on this path as well. We conclude that *the field of an ideal toroidal solenoid is confined to the space enclosed by the windings.* We can think of such a solenoid as a tightly wound, straight solenoid that has been bent into a circle.

For path 2, we have $\oint \vec{B} \cdot d\vec{l} = 2\pi r B$. Each turn of the winding passes *once* through the area bounded by this path, so $I_{\text{encl}} = NI$. We note that I_{encl} is positive for the clockwise direction of integration in Fig. 28.25b, so $\vec{B}$ is in the direction shown. Ampere's law then says that $2\pi r B = \mu_0 N I$, so

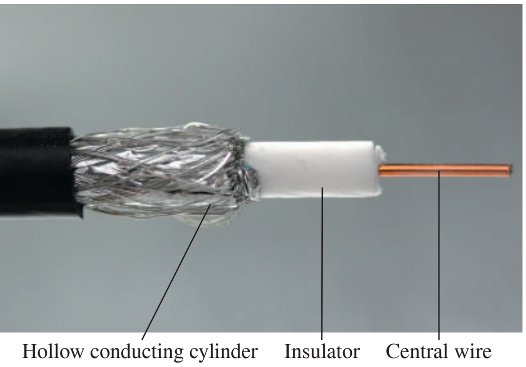
$$B = \frac{\mu_0 N I}{2\pi r} \quad \text{(toroidal solenoid)} \qquad (28.24)$$

EVALUATE Equation (28.24) indicates that B is *not* uniform over the interior of the core, because different points in the interior are different distances r from the toroid axis. However, if the radial extent of the core is small in comparison to r, the variation is slight. In that case, considering that $2\pi r$ is the circumferential length of the toroid and that $N/2\pi r$ is the number of turns per unit length n, the field may be written as $B = \mu_0 n I$, just as it is at the center of a long, *straight* solenoid.

In a real toroidal solenoid the turns are not precisely circular loops but rather segments of a bent helix. As a result, the external field is not exactly zero. To estimate its magnitude, we imagine Fig. 28.25a as being *very* roughly equivalent, for points outside the torus, to a *single-turn* circular loop with radius r. At the center of such a loop, Eq. (28.17) gives $B = \mu_0 I/2r$; this is smaller than the field inside the solenoid by the factor N/π.

The equations we have derived for the field in a closely wound straight or toroidal solenoid are strictly correct only for windings in *vacuum*. For most practical purposes, however, they can be used for windings in air or on a core of any nonmagnetic, nonsuperconducting material. In the next section we'll show how these equations are modified if the core is a magnetic material.

KEYCONCEPT Inside the space enclosed by the windings of an ideal toroidal solenoid, the magnetic field lines are circles that follow the curvature of the toroid. Outside the windings, the magnetic field $\vec{B}$ is zero.



TEST YOUR UNDERSTANDING OF SECTION 28.7 In a *coaxial cable* (see photo), a conducting wire runs along the central axis of a hollow conducting cylinder. (The cable that connects a television set to a local cable provider is an example of a coaxial cable.) In such a cable a current I runs in one direction along the hollow conducting cylinder and is spread uniformly over the cylinder's cross-sectional area. An equal current runs in the opposite direction along the central wire. How does the magnitude B of the magnetic field outside such a cable depend on the distance r from the central axis of the cable? (i) B is proportional to $1/r$; (ii) B is proportional to $1/r^2$; (iii) B is zero at all points outside the cable.

ANSWER

(iii) By symmetry, any $\vec{B}$ field outside the cable must circulate around the cable, with circular field lines like those surrounding the solid cylindrical conductor in Fig. 28.20. Choose an integration path like the one shown in Fig. 28.20 with radius $r > R$, so that the path completely encloses the cable. As in Example 28.8, the integral $\oint \vec{B} \cdot d\vec{l}$ for this path has magnitude $B(2\pi r)$. From Ampere's law this is equal to $\mu_0 I_{\text{encl}}$. The net enclosed current I_{encl} is zero because it includes two currents of equal magnitude but opposite direction: one in the central wire and one in the hollow cylinder. Hence $B(2\pi r) = 0$, and so $B = 0$ for any value of r outside the cable. (The field is nonzero *inside* the cable; see Exercise 28.39.)

28.8 MAGNETIC MATERIALS

In discussing how currents cause magnetic fields, we have assumed that the conductors are surrounded by vacuum. But the coils in transformers, motors, generators, and electromagnets nearly always have iron cores to increase the magnetic field and confine it to desired regions. Permanent magnets, magnetic recording tapes, and computer disks depend directly on the magnetic properties of materials; when you store information on a computer disk, you are actually setting up an array of microscopic permanent magnets on the disk. So it is worthwhile to examine some aspects of the magnetic properties of materials. After describing the atomic origins of magnetic properties, we'll discuss three broad classes of magnetic behavior that occur in materials; these are called *paramagnetism, diamagnetism,* and *ferromagnetism.*

The Bohr Magneton

As we discussed briefly in Section 27.7, the atoms that make up all matter contain moving electrons, and these electrons form microscopic current loops that produce magnetic fields of their own. In many materials these currents are randomly oriented and cause no net magnetic field. But in some materials an external field (a field produced by currents outside the material) can cause these loops to become oriented preferentially with the field, so their magnetic fields *add* to the external field. We then say that the material is *magnetized.*

Let's look at how these microscopic currents come about. **Figure 28.26** shows a primitive model of an electron in an atom. We picture the electron (mass m, charge $-e$) as moving in a circular orbit with radius r and speed v. This moving charge is equivalent to a current loop. In Section 27.7 we found that a current loop with area A and current I has a magnetic dipole moment μ given by $\mu = IA$; for the orbiting electron the area of the loop is $A = \pi r^2$. To find the current associated with the electron, we note that the orbital period T (the time for the electron to make one complete orbit) is the orbit circumference divided by the electron speed: $T = 2\pi r/v$. The equivalent current I is the total charge passing any point on the orbit per unit time, which is just the magnitude e of the electron charge divided by the orbital period T:

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

The magnetic moment $\mu = IA$ is then

$$\mu = \frac{ev}{2\pi r}(\pi r^2) = \frac{evr}{2} \tag{28.25}$$

It is useful to express μ in terms of the *angular momentum* L of the electron. For a particle moving in a circular path, the magnitude of angular momentum equals the magnitude of momentum mv multiplied by the radius r—that is, $L = mvr$ (see Section 10.5). Comparing this with Eq. (28.25), we can write

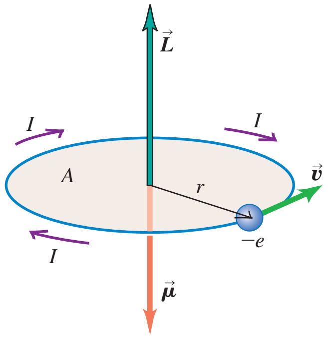
$$\mu = \frac{e}{2m}L \tag{28.26}$$

Equation (28.26) is useful in this discussion because atomic angular momentum is *quantized;* its component in a particular direction is always an integer multiple of $h/2\pi$, where h is a fundamental physical constant called *Planck's constant.* The numerical value of h is

$$h = 6.62607015 \times 10^{-34}\ \text{J}\cdot\text{s}$$

The quantity $h/2\pi$ thus represents a fundamental unit of angular momentum in atomic systems, just as e is a fundamental unit of charge. Associated with the quantization of $\vec{L}$ is a fundamental uncertainty in the *direction* of $\vec{L}$ and therefore of $\vec{\mu}$. In the following discussion, when we speak of the magnitude of a magnetic moment, a more precise statement would be "maximum component in a given direction." Thus, to say that a magnetic moment $\vec{\mu}$ is aligned with a magnetic field $\vec{B}$ really means that $\vec{\mu}$ has its maximum possible component in the direction of $\vec{B}$; such components are always quantized.

Figure **28.26** An electron moving with speed v in a circular orbit of radius r has an angular momentum $\vec{L}$ and an oppositely directed orbital magnetic dipole moment $\vec{\mu}$. It also has a spin angular momentum and an oppositely directed spin magnetic dipole moment.



CAUTION **Planck's constant and SI units** We first introduced Planck's constant h in Section 1.3. Since h has units of $\text{J}\cdot\text{s} = \text{kg}\cdot\text{m}^2/\text{s}$, its *defined* value, combined with the definitions of the meter and the second, serves as the basis for the definition of the kilogram. Physicists define the kilogram this way because the value of h is relatively easy to measure in any physics laboratory, so it provides a useful standard for determining masses in kilograms.

Equation (28.26) shows that associated with the fundamental unit of angular momentum is a corresponding fundamental unit of magnetic moment. If $L = h/2\pi$, then

$$\mu = \frac{e}{2m}\left(\frac{h}{2\pi}\right) = \frac{eh}{4\pi m} \qquad (28.27)$$

This quantity is called the **Bohr magneton,** denoted by μ_B. Its numerical value is

$$\mu_B = 9.274 \times 10^{-24}\ \text{A}\cdot\text{m}^2 = 9.274 \times 10^{-24}\ \text{J/T}$$

You should verify that these two sets of units are consistent. The second set is useful when we compute the potential energy $U = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}}$ for a magnetic moment in a magnetic field.

Electrons also have an intrinsic angular momentum, called *spin,* that is not related to orbital motion but that can be pictured in a classical model as spinning on an axis. This angular momentum also has an associated magnetic moment, and its magnitude turns out to be almost exactly one Bohr magneton. (Effects having to do with quantization of the electromagnetic field cause the spin magnetic moment to be about 1.001 μ_B.)

Paramagnetism

In an atom, most of the various orbital and spin magnetic moments of the electrons add up to zero. However, in some cases the atom has a net magnetic moment that is of the order of μ_B. When such a material is placed in a magnetic field, the field exerts a torque on each magnetic moment, as given by Eq. (27.26): $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$. These torques tend to align the magnetic moments with the field, as we discussed in Section 27.7. In this position, the directions of the current loops are such as to *add* to the externally applied magnetic field.

We saw in Section 28.5 that the $\vec{\boldsymbol{B}}$ field produced by a current loop is proportional to the loop's magnetic dipole moment. In the same way, the additional $\vec{\boldsymbol{B}}$ field produced by microscopic electron current loops is proportional to the total magnetic moment $\vec{\boldsymbol{\mu}}_{\text{total}}$ per unit volume V in the material. We call this vector quantity the **magnetization** of the material, denoted by $\vec{\boldsymbol{M}}$:

$$\vec{\boldsymbol{M}} = \frac{\vec{\boldsymbol{\mu}}_{\text{total}}}{V} \qquad (28.28)$$

The additional magnetic field due to magnetization of the material turns out to be equal simply to $\mu_0\vec{\boldsymbol{M}}$, where μ_0 is the same constant that appears in the law of Biot and Savart and Ampere's law. When such a material completely surrounds a current-carrying conductor, the total magnetic field $\vec{\boldsymbol{B}}$ in the material is

$$\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_0 + \mu_0\vec{\boldsymbol{M}} \qquad (28.29)$$

where $\vec{\boldsymbol{B}}_0$ is the field caused by the current in the conductor.

To check that the units in Eq. (28.29) are consistent, note that magnetization $\vec{\boldsymbol{M}}$ is the magnetic moment per unit volume. The units of magnetic moment are current times area $(\text{A}\cdot\text{m}^2)$, so the units of magnetization are $(\text{A}\cdot\text{m}^2)/\text{m}^3 = \text{A/m}$. From Section 28.1, the units of the constant μ_0 are $\text{T}\cdot\text{m/A}$. So the units of $\mu_0\vec{\boldsymbol{M}}$ are the same as the units of $\vec{\boldsymbol{B}}$: $(\text{T}\cdot\text{m/A})(\text{A/m}) = \text{T}$.

A material showing the behavior just described is said to be **paramagnetic.** The result is that the magnetic field at any point in such a material is greater by a dimensionless factor K_m, called the **relative permeability** of the material, than it would be if the material were replaced by vacuum. The value of K_m is different for different materials; for common paramagnetic solids and liquids at room temperature, K_m typically ranges from 1.00001 to 1.003.

All of the equations in this chapter that relate magnetic fields to their sources can be adapted to the situation in which the current-carrying conductor is embedded in a paramagnetic material. All that need be done is to replace μ_0 by $K_m\mu_0$. This product is usually denoted as μ and is called the **permeability** of the material:

$$\mu = K_m\mu_0 \qquad (28.30)$$

CAUTION **Two meanings of the symbol μ** Equation (28.30) involves dangerous notation because we use μ for magnetic dipole moment as well as for permeability, as is customary. But beware: From now on, every time you see a μ, make sure you know whether it is permeability or magnetic moment. You can usually tell from the context.

The amount by which the relative permeability differs from unity is called the **magnetic susceptibility,** denoted by χ_m:

$$\chi_m = K_m - 1 \tag{28.31}$$

Both K_m and χ_m are dimensionless quantities. **Table 28.1** lists values of magnetic susceptibility for several materials. For example, for aluminum, $\chi_m = 2.2 \times 10^{-5}$ and $K_m = 1.000022$. The first group in the table consists of paramagnetic materials; we'll soon discuss the second group, which contains *diamagnetic* materials.

The tendency of atomic magnetic moments to align themselves parallel to the magnetic field (where the potential energy is minimum) is opposed by random thermal motion, which tends to randomize their orientations. For this reason, paramagnetic susceptibility always decreases with increasing temperature. In many cases it is inversely proportional to the absolute temperature T, and the magnetization M can be expressed as

$$M = C\frac{B}{T} \tag{28.32}$$

This relationship is called *Curie's law,* after its discoverer, Pierre Curie (1859–1906). The quantity C is a constant, different for different materials, called the *Curie constant.*

As we described in Section 27.7, an object with atomic magnetic dipoles is attracted to the poles of a magnet. In most paramagnetic substances this attraction is very weak due to thermal randomization of the atomic magnetic moments. But at very low temperatures the thermal effects are reduced, the magnetization increases in accordance with Curie's law, and the attractive forces are greater.

TABLE 28.1 Magnetic Susceptibilities of Paramagnetic and Diamagnetic Materials at $T = 20°\text{C}$

Material	$\chi_m = K_m - 1\ (\times 10^{-5})$
Paramagnetic	
Iron ammonium alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	−16.6
Mercury	−2.9
Silver	−2.6
Carbon (diamond)	−2.1
Lead	−1.8
Sodium chloride	−1.4
Copper	−1.0

EXAMPLE 28.11 Magnetic dipoles in a paramagnetic material

Nitric oxide (NO) is a paramagnetic compound. The magnetic moment of each NO molecule has a maximum component in any direction of about one Bohr magneton. Compare the interaction energy of such magnetic moments in a 1.5 T magnetic field with the average translational kinetic energy of molecules at 300 K.

IDENTIFY and SET UP This problem involves the energy of a magnetic moment in a magnetic field and the average thermal kinetic energy. We have Eq. (27.27), $U = -\vec{\mu}\cdot\vec{B}$, for the interaction energy of a magnetic moment $\vec{\mu}$ with a $\vec{B}$ field, and Eq. (18.16), $K = \frac{3}{2}kT$, for the average translational kinetic energy of a molecule at temperature T.

EXECUTE We can write $U = -\mu_{\parallel}B$, where $\mu_{\parallel}$ is the component of the magnetic moment $\vec{\mu}$ in the direction of the $\vec{B}$ field. Here the maximum value of $\mu_{\parallel}$ is about μ_B, so

$$|U|_{\max} \approx \mu_B B = (9.27 \times 10^{-24}\ \text{J/T})(1.5\ \text{T})$$
$$= 1.4 \times 10^{-23}\ \text{J} = 8.7 \times 10^{-5}\ \text{eV}$$

The average translational kinetic energy K is

$$K = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(300\ \text{K})$$
$$= 6.2 \times 10^{-21}\ \text{J} = 0.039\ \text{eV}$$

EVALUATE At 300 K the magnetic interaction energy is only about 0.2% of the thermal kinetic energy, so we expect only a slight degree of alignment. This is why paramagnetic susceptibilities at ordinary temperature are usually very small.

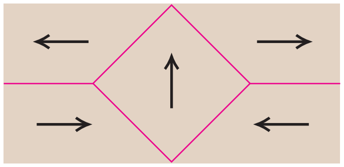
KEYCONCEPT In a paramagnetic material the magnetic dipole moments of its atoms tend to align with an externally applied magnetic field. Random thermal motions tend to cancel out this alignment, however. Paramagnetic materials are weakly attracted to the poles of a magnet.

Diamagnetism

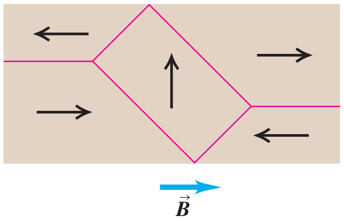
In some materials the total magnetic moment of all the atomic current loops is zero when no magnetic field is present. But even these materials have magnetic effects because an external field alters electron motions within the atoms, causing additional current loops and induced magnetic dipoles comparable to the induced *electric* dipoles we studied in Section 28.5. In this case the additional field caused by these current loops is always *opposite* in direction to that of the external field. (This behavior is explained by Faraday's law of induction, which we'll study in Chapter 29. An induced current always tends to cancel the field change that caused it.)

Figure 28.27 In this drawing adapted from a magnified photo, the arrows show the directions of magnetization in the domains of a single crystal of nickel. Domains that are magnetized in the direction of an applied magnetic field grow larger.

(a) No field



(b) Weak field



(c) Stronger field

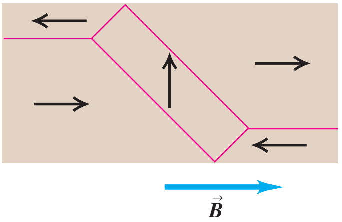
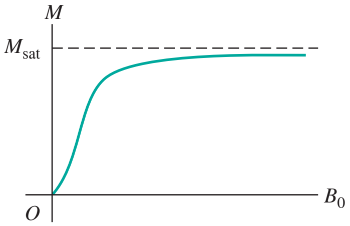


Figure 28.28 A magnetization curve for a ferromagnetic material. The magnetization M approaches its saturation value M_{sat} as the magnetic field B_0 (caused by external currents) becomes large.



Such materials are said to be **diamagnetic.** They always have negative susceptibility, as shown in Table 28.1, and relative permeability K_m slightly *less* than unity, typically of the order of 0.99990 to 0.99999 for solids and liquids. Diamagnetic susceptibilities are very nearly temperature independent.

Ferromagnetism

There is a third class of materials, called **ferromagnetic** materials, that includes iron, nickel, cobalt, and many alloys containing these elements. In these materials, strong interactions between atomic magnetic moments cause them to line up parallel to each other in regions called **magnetic domains,** even when no external field is present. **Figure 28.27** shows an example of magnetic domain structure. Within each domain, nearly all of the atomic magnetic moments are parallel.

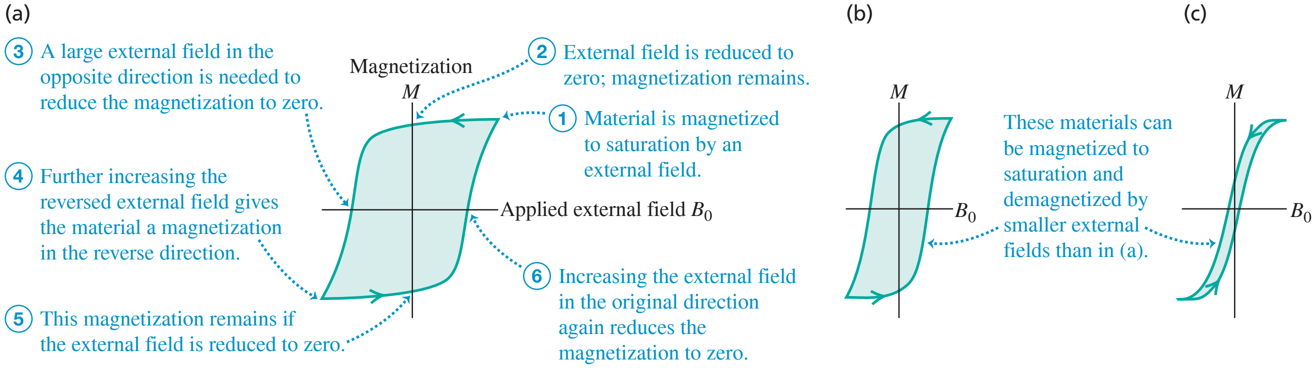
When there is no externally applied field, the domain magnetizations are randomly oriented. But when a field $\vec{B}_0$ (caused by external currents) is present, the domains tend to orient themselves parallel to the field. The domain boundaries also shift; the domains that are magnetized in the field direction grow, and those that are magnetized in other directions shrink. Because the total magnetic moment of a domain may be many thousands of Bohr magnetons, the torques that tend to align the domains with an external field are much stronger than occur with paramagnetic materials. The relative permeability K_m is *much* larger than unity, typically of the order of 1000 to 100,000. As a result, an object made of a ferromagnetic material such as iron is strongly magnetized by the field from a permanent magnet and is attracted to the magnet (see Fig. 27.38). A paramagnetic material such as aluminum is also attracted to a permanent magnet, but K_m for paramagnetic materials is so much smaller for such a material than for ferromagnetic materials that the attraction is very weak. Thus a magnet can pick up iron nails, but not aluminum cans.

As the external field is increased, a point is eventually reached at which nearly *all* the magnetic moments in the ferromagnetic material are aligned parallel to the external field. This condition is called *saturation magnetization;* after it is reached, further increase in the external field causes no increase in magnetization or in the additional field caused by the magnetization.

Figure 28.28 shows a "magnetization curve," a graph of magnetization M as a function of external magnetic field B_0, for soft iron. An alternative description of this behavior is that K_m is not constant but decreases as B_0 increases. (Paramagnetic materials also show saturation at sufficiently strong fields. But the magnetic fields required are so large that departures from a linear relationship between M and B_0 in these materials can be observed only at very low temperatures, 1 K or so.)

For many ferromagnetic materials the relationship of magnetization to external magnetic field is different when the external field is increasing from when it is decreasing. **Figure 28.29a** shows this relationship for such a material. When the material is magnetized to saturation and then the external field is reduced to zero, some magnetization

Figure 28.29 Hysteresis loops. The materials of both (a) and (b) remain strongly magnetized when B_0 is reduced to zero. Since (a) is also hard to demagnetize, it would be good for permanent magnets. Since (b) magnetizes and demagnetizes more easily, it could be used as a computer memory material. The material of (c) would be useful for transformers and other alternating-current devices where zero hysteresis would be optimal.



③ A large external field in the opposite direction is needed to reduce the magnetization to zero.

② External field is reduced to zero; magnetization remains.

① Material is magnetized to saturation by an external field.

④ Further increasing the reversed external field gives the material a magnetization in the reverse direction.

⑤ This magnetization remains if the external field is reduced to zero.

⑥ Increasing the external field in the original direction again reduces the magnetization to zero.

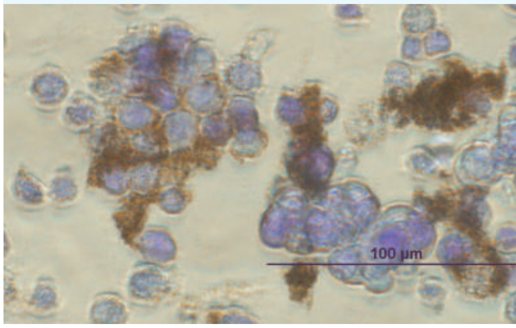
These materials can be magnetized to saturation and demagnetized by smaller external fields than in (a).

remains. This behavior is characteristic of permanent magnets, which retain most of their saturation magnetization when the magnetizing field is removed. To reduce the magnetization to zero requires a magnetic field in the reverse direction.

This behavior is called **hysteresis,** and the curves in Fig. 28.29 are called *hysteresis loops.* Magnetizing and demagnetizing a material that has hysteresis involve the dissipation of energy, and the temperature of the material increases during such a process.

Ferromagnetic materials are widely used in electromagnets, transformer cores, and motors and generators, in which it is desirable to have as large a magnetic field as possible for a given current. Because hysteresis dissipates energy, materials that are used in these applications should usually have as narrow a hysteresis loop as possible. Soft iron is often used; it has high permeability without appreciable hysteresis. For permanent magnets a broad hysteresis loop is usually desirable, with large zero-field magnetization and large reverse field needed to demagnetize. Many kinds of steel and many alloys, such as Alnico, are used for permanent magnets. The remaining magnetic field in such a material, after it has been magnetized to near saturation, is typically of the order of 1 T, corresponding to a remaining magnetization $M = B/\mu_0$ of about 800,000 A/m.

BIO APPLICATION Ferromagnetic Nanoparticles for Cancer Therapy The violet blobs in this microscope image are cancer cells that have broken away from a tumor and threaten to spread throughout a patient's body. An experimental technique for fighting these cells uses particles of a ferromagnetic material (shown in brown) injected into the body. These particles are coated with a chemical that preferentially attaches to cancer cells. A magnet outside the patient then "steers" the particles out of the body, taking the cancer cells with them. (Photo courtesy of cancer researcher Dr. Kenneth Scarberry.)



EXAMPLE 28.12 A ferromagnetic material

A cube-shaped permanent magnet is made of a ferromagnetic material with a magnetization M of about 8×10^5 A/m. The side length is 2 cm. (a) Find the magnetic dipole moment of the magnet. (b) Estimate the magnetic field due to the magnet at a point 10 cm from the magnet along its axis.

IDENTIFY and SET UP This problem uses the relationship between magnetization M and magnetic dipole moment μ_{total} and the idea that a magnetic dipole produces a magnetic field. We find μ_{total} from Eq. (28.28). To estimate the field, we approximate the magnet as a current loop with this same magnetic moment and use Eq. (28.18).

EXECUTE (a) From Eq. (28.28),

$$\mu_{\text{total}} = MV = (8 \times 10^5\ \text{A/m})(2 \times 10^{-2}\ \text{m})^3 = 6\ \text{A} \cdot \text{m}^2$$

(b) From Eq. (28.18), the magnetic field on the axis of a current loop with magnetic moment μ_{total} is

$$B = \frac{\mu_0 \mu_{\text{total}}}{2\pi(x^2 + a^2)^{3/2}}$$

where x is the distance from the loop and a is its radius. We can use this expression here if we take a to refer to the size of the permanent magnet. Strictly speaking, there are complications because our magnet does not have the same geometry as a circular current loop. But because $x = 10$ cm is fairly large in comparison to the 2 cm size of the magnet, the term a^2 is negligible in comparison to x^2 and can be ignored. So

$$B \approx \frac{\mu_0 \mu_{\text{total}}}{2\pi x^3} = \frac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(6\ \text{A} \cdot \text{m}^2)}{2\pi(0.1\ \text{m})^3} = 1 \times 10^{-3}\ \text{T} = 10\ \text{G}$$

which is about ten times stronger than the earth's magnetic field.

EVALUATE We calculated B at a point *outside* the magnetic material and therefore used μ_0, not the permeability μ of the magnetic material, in our calculation. You would substitute permeability μ for μ_0 if you were calculating B *inside* a material with relative permeability K_m, for which $\mu = K_m \mu_0$.

KEYCONCEPT In a ferromagnetic material there are strong interactions between the magnetic dipole moments of neighboring atoms, and the material can maintain a large overall magnetic dipole even if there is no externally applied magnetic field.

TEST YOUR UNDERSTANDING OF SECTION 28.8 Which of the following materials are attracted to a magnet? (i) Sodium; (ii) bismuth; (iii) lead; (iv) uranium.

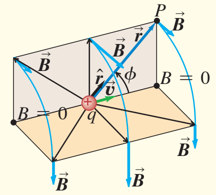
ANSWER

| **(i), (iv)** Sodium and uranium are paramagnetic materials and hence are attracted to a magnet, while bismuth and lead are diamagnetic materials that are repelled by a magnet. (See Table 28.1.)

CHAPTER 28 SUMMARY

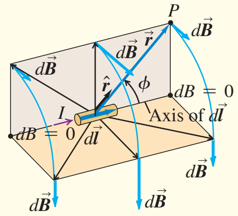
Magnetic field of a moving charge: The magnetic field $\vec{B}$ created by a charge q moving with velocity $\vec{v}$ depends on the distance r from the source point (the location of q) to the field point (where $\vec{B}$ is measured). The $\vec{B}$ field is perpendicular to $\vec{v}$ and to $\hat{r}$, the unit vector directed from the source point to the field point. The principle of superposition of magnetic fields states that the total $\vec{B}$ field produced by several moving charges is the vector sum of the fields produced by the individual charges. (See Example 28.1.)

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \tag{28.2}$$



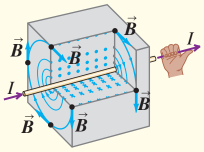
Magnetic field of a current-carrying conductor: The law of Biot and Savart gives the magnetic field $d\vec{B}$ created by an element $d\vec{l}$ of a conductor carrying current I. The field $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$, the unit vector from the element to the field point. The $\vec{B}$ field created by a finite current-carrying conductor is the integral of $d\vec{B}$ over the length of the conductor. (See Example 28.2.)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \tag{28.6}$$



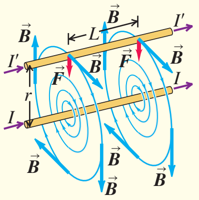
Magnetic field of a long, straight, current-carrying conductor: The magnetic field $\vec{B}$ at a distance r from a long, straight conductor carrying a current I has a magnitude that is inversely proportional to r. The magnetic field lines are circles coaxial with the wire, with directions given by the right-hand rule. (See Examples 28.3 and 28.4.)

$$B = \frac{\mu_0 I}{2\pi r} \tag{28.9}$$



Magnetic force between current-carrying conductors: Two long, parallel, current-carrying conductors attract if the currents are in the same direction and repel if the currents are in opposite directions. The magnetic force per unit length between the conductors depends on their currents I and I' and their separation r. The definition of the ampere is based on this relationship. (See Example 28.5.)

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} \tag{28.11}$$



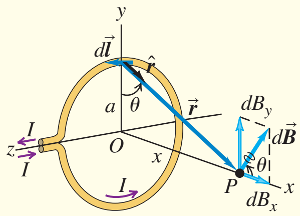
Magnetic field of a current loop: The law of Biot and Savart allows us to calculate the magnetic field produced along the axis of a circular conducting loop of radius a carrying current I. The field depends on the distance x along the axis from the center of the loop to the field point. If there are N loops, the field is multiplied by N. At the center of the loop, $x = 0$. (See Example 28.6.)

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \tag{28.15}$$

(circular loop)

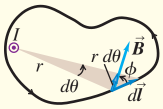
$$B_x = \frac{\mu_0 N I}{2a} \tag{28.17}$$

(center of N circular loops)



Ampere's law: Ampere's law states that the line integral of $\vec{B}$ around any closed path equals μ_0 times the net current through the area enclosed by the path. The positive sense of current is determined by a right-hand rule. (See Examples 28.7–28.10.)

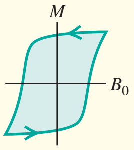
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \tag{28.20}$$



Magnetic fields due to current distributions: The table lists magnetic fields caused by several current distributions. In each case the conductor is carrying current I.

Current Distribution	Point in Magnetic Field	Magnetic-Field Magnitude
Long, straight conductor	Distance r from conductor	$B = \frac{\mu_0 I}{2\pi r}$
Circular loop of radius a (for N loops, multiply these expressions for magnetic-field amplitude by N)	On axis of loop	$B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$
	At center of loop	$B = \frac{\mu_0 I}{2a}$
Long cylindrical conductor of radius R	Inside conductor, $r < R$	$B = \frac{\mu_0 I}{2\pi}\frac{r}{R^2}$
	Outside conductor, $r > R$	$B = \frac{\mu_0 I}{2\pi r}$
Long, closely wound solenoid with n turns per unit length, near its midpoint	Inside solenoid, near center	$B = \mu_0 n I$
	Outside solenoid	$B \approx 0$
Tightly wound toroidal solenoid (toroid) with N turns	Within the space enclosed by the windings, distance r from symmetry axis	$B = \frac{\mu_0 N I}{2\pi r}$
	Outside the space enclosed by the windings	$B \approx 0$

Magnetic materials: When magnetic materials are present, the magnetization of the material causes an additional contribution to $\vec{B}$. For paramagnetic and diamagnetic materials, μ_0 is replaced in magnetic-field expressions by $\mu = K_m \mu_0$, where μ is the permeability of the material and K_m is its relative permeability. The magnetic susceptibility χ_m is defined as $\chi_m = K_m - 1$. Magnetic susceptibilities for paramagnetic materials are small positive quantities; those for diamagnetic materials are small negative quantities. For ferromagnetic materials, K_m is much larger than unity and is not constant. Some ferromagnetic materials are permanent magnets, retaining their magnetization even after the external magnetic field is removed. (See Examples 28.11 and 28.12.)



Chapter 28 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE √ARIATION PROBLEMS

Be sure to review EXAMPLE 28.1 (Section 28.1) and EXAMPLE 28.2 (Section 28.2) before attempting these problems.

VP28.2.1 A proton with charge $+1.60 \times 10^{-19}$ C is at the origin ($x = y = z = 0$) and is moving at 2.00×10^5 m/s in the $+x$-direction. Find the magnetic field $\vec{B}$ that this proton produces at (a) $x = 0$, $y = 1.00$ mm, $z = 0$; (b) $x = 0$, $y = 0$, $z = 2.00$ mm; (c) $x = 1.00$ mm, $y = 0$, $z = 0$; (d) $x = 1.00$ mm, $y = 1.00$ mm, $z = 0$.

VP28.2.2 A proton (charge $+1.60 \times 10^{-19}$ C) and an electron (charge -1.60×10^{-19} C) are both moving in the xy-plane with the same speed, 4.20×10^5 m/s. The proton is moving in the $+y$-direction along the line $x = 0$, and the electron is moving in the $-y$-direction along the line $x = +2.00$ mm. At the instant when the proton and electron are at their closest approach, what are the magnitude and direction of the magnetic force that (a) the proton exerts on the electron and (b) the electron exerts on the proton?

VP28.2.3 A segment of wire centered at the origin ($x = y = z = 0$) is 2.00 mm in length and carries a current of 6.00 A in the $+y$-direction. You measure the magnetic field due to this segment at the point $x = 3.00$ m, $y = 4.00$ m, $z = 0$. Find (a) the unit vector from the wire segment to this point and (b) the magnetic field at this point.

VP28.2.4 A segment of wire centered at the origin ($x = y = z = 0$) is 8.00 mm in length and carries a current of 4.00 A in the $+z$-direction. Find the magnetic force that this segment exerts on an electron (charge -1.60×10^{-19} C) at the point $x = 1.25$ m, $y = 0$, $z = 0$ that is moving at 3.00×10^5 m/s in the $-x$-direction.

Be sure to review EXAMPLES 28.3 and 28.4 (Section 28.3) and EXAMPLE 28.5 (Section 28.4) before attempting these problems.

VP28.5.1 How much current must a long, straight wire carry in order for the magnetic field that it produces 1.40 cm from the central axis of the wire to have magnitude 3.20×10^{-5} T?

VP28.5.2 Two long, straight, conducting wires are both perpendicular to the xy-plane. Wire 1 passes through the point $x = 0$, $y = 1.00$ cm, $z = 0$ and carries current 1.00 A in the $+z$-direction. Wire 2 passes through the point $x = 0$, $y = -1.00$ cm, $z = 0$ and carries current 4.00 A also in the $+z$-direction. Find the net magnetic field due to both wires at (a) $x = 0$, $y = 0$, $z = 0$; (b) $x = 0$, $y = 2.00$ cm, $z = 0$; (c) $x = 0$, $y = -2.00$ cm, $z = 0$.

VP28.5.3 A long, straight, conducting wire carries a current of 2.00 A. A second, identical wire is parallel to the first and separated from it by 3.00 cm. The magnetic field at a point halfway between the two wires has magnitude 4.00×10^{-5} T. What is the current in the second wire if its direction is (a) the same as in the first wire? (b) Opposite to that in the first wire?

VP28.5.4 Two straight, parallel, superconducting wires are 0.900 m apart. Both wires are oriented straight up and down. One wire carries a current of 1.30×10^4 A upward and experiences a magnetic force per unit length of 95.0 N/m that attracts it to the other wire. What are the magnitude and direction of the current in the other wire?

Be sure to review EXAMPLES 28.8, 28.9, and 28.10 (Section 28.7) before attempting these problems.

VP28.10.1 A solid cylindrical conductor with radius 4.50 cm carries a current of 2.00 A. The current is uniformly distributed over the cross-sectional area of the conductor. Find the magnitude of the magnetic field at the following distances from the conductor axis: (a) 2.50 cm; (b) 4.50 cm; (c) 6.00 cm.

VP28.10.2 A long, hollow cylinder with inner radius R_1 and outer radius R_2 carries current along its length. The current is uniformly distributed over the cross-sectional area of the cylinder and has current density J. (a) Find the magnetic-field magnitude B as a function of the distance r from the conductor axis for points inside the hollow interior ($r < R_1$), inside the solid conductor ($R_2 > r > R_1$), and outside the conductor ($r > R_2$). (b) For which value of r is B greatest?

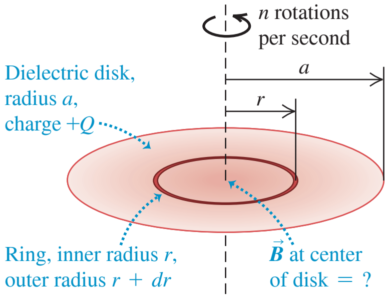
VP28.10.3 A very long solenoid with tightly wound coils has 8.00×10^3 turns of wire per meter of length. What must the current be in the solenoid in order for the field inside the solenoid to have magnitude 0.0320 T?

VP28.10.4 A copper wire carries a current of 0.840 A. You wrap this wire around a hollow toroidal core, forming a toroidal solenoid. The inner and outer radii of the core are 6.00 cm and 8.00 cm, respectively. (a) How many turns of wire around the core are required to produce a magnetic field of magnitude 2.00×10^{-3} T at a distance of 7.00 cm from the toroid axis? (b) With this number of turns, what are the maximum and minimum field magnitudes in the interior of the core?

BRIDGING PROBLEM Magnetic Field of a Charged, Rotating Dielectric Disk

A thin dielectric disk with radius a has a total charge $+Q$ distributed uniformly over its surface (**Fig. 28.30**). It rotates n times per second about an axis perpendicular to the surface of the disk and passing through its center. Find the magnetic field at the center of the disk.

Figure 28.30 Finding the $\vec{B}$ field at the center of a uniformly charged, rotating disk.



SOLUTION GUIDE

IDENTIFY and SET UP

1. Think of the rotating disk as a series of concentric rotating rings. Each ring acts as a circular current loop that produces a magnetic field at the center of the disk.
2. Use the results of Section 28.5 to find the magnetic field due to a single ring. Then integrate over all rings to find the total field.

EXECUTE

3. Find the charge on a ring with inner radius r and outer radius $r + dr$ (Fig. 28.30).
4. How long does it take the charge found in step 3 to make a complete trip around the rotating ring? Use this to find the current of the rotating ring.
5. Use a result from Section 28.5 to determine the magnetic field that this ring produces at the center of the disk.
6. Integrate your result from step 5 to find the total magnetic field from all rings with radii from $r = 0$ to $r = a$.

EVALUATE

7. Does your answer have the correct units?
8. Suppose all of the charge were concentrated at the rim of the disk (at $r = a$). Would this increase or decrease the field at the center of the disk?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q28.1 A topic of current interest in physics research is the search (thus far unsuccessful) for an isolated magnetic pole, or magnetic *monopole*. If such an entity were found, how could it be recognized? What would its properties be?

Q28.2 Streams of charged particles emitted from the sun during periods of solar activity create a disturbance in the earth's magnetic field. How does this happen?

Q28.3 The text discussed the magnetic field of an infinitely long, straight conductor carrying a current. Of course, there is no such thing as an infinitely long *anything*. How do you decide whether a particular wire is long enough to be considered infinite?

Q28.4 Two parallel conductors carrying current in the same direction attract each other. If they are permitted to move toward each other, the forces of attraction do work. From where does the energy come? Does this contradict the assertion in Chapter 27 that magnetic forces on moving charges do no work? Explain.

Q28.5 Pairs of conductors carrying current into or out of the power-supply components of electronic equipment are sometimes twisted together to reduce magnetic-field effects. Why does this help?

Q28.6 Suppose you have three long, parallel wires arranged so that in cross section they are at the corners of an equilateral triangle. Is there any way to arrange the currents so that all three wires attract each other? So that all three wires repel each other? Explain.

Q28.7 In deriving the force on one of the long, current-carrying conductors in Section 28.4, why did we use the magnetic field due to only one of the conductors? That is, why didn't we use the *total* magnetic field due to *both* conductors?

Q28.8 Two concentric, coplanar, circular loops of wire of different diameter carry currents in the same direction. Describe the nature of the force exerted on the inner loop by the outer loop and on the outer loop by the inner loop.

Q28.9 A current was sent through a helical coil spring. The spring contracted, as though it had been compressed. Why?

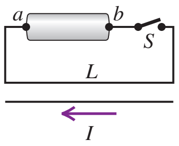
Q28.10 What are the relative advantages and disadvantages of Ampere's law and the law of Biot and Savart for practical calculations of magnetic fields?

Q28.11 Magnetic field lines never have a beginning or an end. Use this to explain why it is reasonable for the field of an ideal toroidal solenoid to be confined entirely to its interior, while a straight solenoid *must* have some field outside.

Q28.12 Two very long, parallel wires carry equal currents in opposite directions. (a) Is there any place that their magnetic fields completely cancel? If so, where? If not, why not? (b) How would the answer to part (a) change if the currents were in the same direction?

Q28.13 In the circuit shown in **Fig. Q28.13**, when switch S is suddenly closed, the wire L is pulled toward the lower wire carrying current I. Which (a or b) is the positive terminal of the battery? How do you know?

Figure **Q28.13**



Q28.14 A metal ring carries a current that causes a magnetic field B_0 at the center of the ring and a field B at point P a distance x from the center along the axis of the ring. If the radius of the ring is doubled, find the magnetic field at the center. Will the field at point P change by the same factor? Why?

Q28.15 Show that the units $\text{A} \cdot \text{m}^2$ and J/T for the Bohr magneton are equivalent.

Q28.16 Why should the permeability of a paramagnetic material be expected to decrease with increasing temperature?

Q28.17 If a magnet is suspended over a container of liquid air, it attracts droplets to its poles. The droplets contain only liquid oxygen; even though nitrogen is the primary constituent of air, it is not attracted to the magnet. Explain what this tells you about the magnetic susceptibilities of oxygen and nitrogen, and explain why a magnet in ordinary, room-temperature air doesn't attract molecules of oxygen *gas* to its poles.

Q28.18 What features of atomic structure determine whether an element is diamagnetic or paramagnetic? Explain.

Q28.19 The magnetic susceptibility of paramagnetic materials is quite strongly temperature dependent, but that of diamagnetic materials is nearly independent of temperature. Why the difference?

Q28.20 A cylinder of iron is placed so that it is free to rotate around its axis. Initially the cylinder is at rest, and a magnetic field is applied to the cylinder so that it is magnetized in a direction parallel to its axis. If the direction of the *external* field is suddenly reversed, the direction of magnetization will also reverse and the cylinder will begin rotating around its axis. (This is called the *Einstein–de Haas effect*.) Explain why the cylinder begins to rotate.

EXERCISES

Section 28.1 Magnetic Field of a Moving Charge

28.1 •• A $+6.00\ \mu\text{C}$ point charge is moving at a constant 8.00×10^6 m/s in the $+y$-direction, relative to a reference frame. At the instant when the point charge is at the origin of this reference frame, what is the magnetic-field vector $\vec{B}$ it produces at the following points: (a) $x = 0.500$ m, $y = 0$, $z = 0$; (b) $x = 0$, $y = -0.500$ m, $z = 0$; (c) $x = 0$, $y = 0$, $z = +0.500$ m; (d) $x = 0$, $y = -0.500$ m, $z = +0.500$ m?

28.2 • **Fields Within the Atom.** In the Bohr model of the hydrogen atom, the electron moves in a circular orbit of radius 5.3×10^{-11} m with a speed of 2.2×10^6 m/s. If we are viewing the atom in such a way that the electron's orbit is in the plane of the paper with the electron moving clockwise, find the magnitude and direction of the electric and magnetic fields that the electron produces at the location of the nucleus (treated as a point).

28.3 • An electron moves at $0.100c$ as shown in **Fig. E28.3**. Find the magnitude and direction of the magnetic field this electron produces at the following points, each 2.00 μm from the electron: (a) points A and B; (b) point C; (c) point D.

28.4 •• An alpha particle (charge $+2e$) and an electron move in opposite directions from the same point, each with the speed of 2.50×10^5 m/s (**Fig. E28.4**). Find the magnitude and direction of the total magnetic field these charges produce at point P, which is 8.65 nm from each charge.

Figure **E28.3**

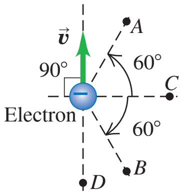
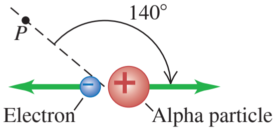


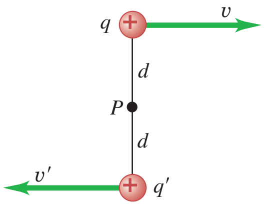
Figure **E28.4**



28.5 • A $-4.80\ \mu\text{C}$ charge is moving at a constant speed of 6.80×10^5 m/s in the $+x$-direction relative to a reference frame. At the instant when the point charge is at the origin, what is the magnetic-field vector it produces at the following points: (a) $x = 0.500$ m, $y = 0$, $z = 0$; (b) $x = 0$, $y = 0.500$ m, $z = 0$; (c) $x = 0.500$ m, $y = 0.500$ m, $z = 0$; (d) $x = 0$, $y = 0$, $z = 0.500$ m?

28.6 • Positive point charges $q = +8.00\ \mu\text{C}$ and $q' = +3.00\ \mu\text{C}$ are moving relative to an observer at point P, as shown in **Fig. E28.6**. The distance d is 0.120 m, $v = 4.50 \times 10^6$ m/s, and $v' = 9.00 \times 10^6$ m/s. (a) When the two charges are at the locations shown in the figure, what are the magnitude and direction of the net magnetic field they produce at point P? (b) What are the magnitude and direction of the electric and magnetic forces that each charge exerts on the other, and what is the ratio of the magnitude of the electric force to the magnitude of the magnetic force? (c) If the direction of $\vec{v}\,'$ is reversed, so both charges are moving in the same direction, what are the magnitude and direction of the magnetic forces that the two charges exert on each other?

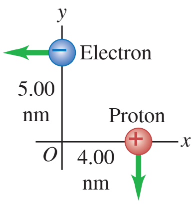
Figure **E28.6**



28.7 • At one instant, point P is 6.60 μm to the left of a proton that is moving at 3.30 km/s in vacuum. (a) What is the direction of the unit vector in Eq. (28.2)? (b) What is the magnetic field caused by this proton if it is moving to the right or to the left? (c) What is the magnetic field (magnitude and direction) caused by this proton if it is moving toward the top of the page? (d) What is the answer to part (c) for an electron instead of a proton?

28.8 •• An electron and a proton are each moving at 735 km/s in perpendicular paths as shown in **Fig. E28.8**. At the instant when they are at the positions shown, find the magnitude and direction of (a) the total magnetic field they produce at the origin; (b) the magnetic field the electron produces at the location of the proton; (c) the total electric force and the total magnetic force that the electron exerts on the proton.

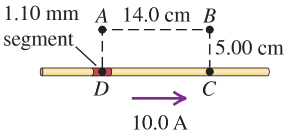
Figure **E28.8**



Section 28.2 Magnetic Field of a Current Element

28.9 • A straight wire carries a 10.0 A current (**Fig. E28.9**). *ABCD* is a rectangle with point *D* in the middle of a 1.10 mm segment of the wire and point *C* in the wire. Find the magnitude and direction of the magnetic field due to this segment at (a) point *A*; (b) point *B*; (c) point *C*.

Figure **E28.9**



28.10 • A short current element $d\vec{l} = (0.500\text{ mm})\hat{\jmath}$ carries a current of 5.40 A in the same direction as $d\vec{l}$. Point *P* is located at $\vec{r} = (-0.730\text{ m})\hat{\imath} + (0.390\text{ m})\hat{k}$. Use unit vectors to express the magnetic field at *P* produced by this current element.

28.11 •• A long, straight wire lies along the *z*-axis and carries a 4.00 A current in the $+z$-direction. Find the magnetic field (magnitude and direction) produced at the following points by a 0.500 mm segment of the wire centered at the origin: (a) $x = 2.00$ m, $y = 0$, $z = 0$; (b) $x = 0$, $y = 2.00$ m, $z = 0$; (c) $x = 2.00$ m, $y = 2.00$ m, $z = 0$; (d) $x = 0$, $y = 0$, $z = 2.00$ m,

28.12 •• Two parallel wires are 5.00 cm apart and carry currents in opposite directions, as shown in **Fig. E28.12**. Find the magnitude and direction of the magnetic field at point *P* due to two 1.50 mm segments of wire that are opposite each other and each 8.00 cm from *P*.

Figure **E28.12**

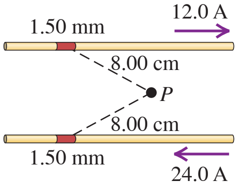
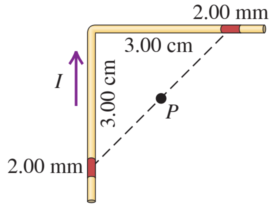


Figure **E28.13**



28.13 • A wire carrying a 28.0 A current bends through a right angle. Consider two 2.00 mm segments of wire, each 3.00 cm from the bend (**Fig. E28.13**). Find the magnitude and direction of the magnetic field these two segments produce at point *P*, which is midway between them.

Section 28.3 Magnetic Field of a Straight Current-Carrying Conductor

28.14 •• A long, straight wire lies along the *x*-axis and carries current $I = 60.0$ A in the $+x$-direction. A small particle with mass 3.00×10^{-6} kg and charge 8.00×10^{-3} C is traveling in the vicinity of the wire. At an instant when the particle is on the *y*-axis at $y = 8.00$ cm, its acceleration has components $a_x = -5.00 \times 10^3\text{ m/s}^2$ and $a_y = +9.00 \times 10^3\text{ m/s}^2$. At that instant what are the *x*- and *y*-components of the velocity of the particle?

28.15 • **The Magnetic Field from a Lightning Bolt.** Lightning bolts can carry currents up to approximately 20 kA. We can model such a current as the equivalent of a very long, straight wire. (a) If you were unfortunate enough to be 5.0 m away from such a lightning bolt, how large a magnetic field would you experience? (b) How does this field compare to one you would experience by being 5.0 cm from a long, straight household current of 10 A?

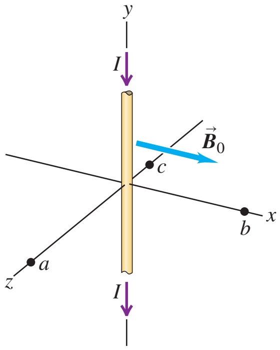
28.16 • A very long, straight horizontal wire carries a current such that 8.20×10^{18} electrons per second pass any given point going from west to east. What are the magnitude and direction of the magnetic field this wire produces at a point 4.00 cm directly above it?

28.17 • **BIO Currents in the Heart.** The body contains many small currents caused by the motion of ions in the organs and cells. Measurements of the magnetic field around the chest due to currents in the heart give values of about 10 μG. Although the actual currents are rather complicated, we can gain a rough understanding of their magnitude if we model them as a long, straight wire. If the surface of the chest is 5.0 cm from this current, how large is the current in the heart?

28.18 • Two long, straight wires, one above the other, are separated by a distance $2a$ and are parallel to the *x*-axis. Let the $+y$-axis be in the plane of the wires in the direction from the lower wire to the upper wire. Each wire carries current *I* in the $+x$-direction. What are the magnitude and direction of the net magnetic field of the two wires at a point in the plane of the wires (a) midway between them; (b) at a distance *a* above the upper wire; (c) at a distance *a* below the lower wire?

28.19 •• A long, straight wire lies along the *y*-axis and carries a current $I = 8.00$ A in the $-y$-direction (**Fig. E28.19**). In addition to the magnetic field due to the current in the wire, a uniform magnetic field $\vec{B}_0$ with magnitude 1.50×10^{-6} T is in the $+x$-direction. What is the total field (magnitude and direction) at the following points in the *xz*-plane: (a) $x = 0$, $z = 1.00$ m; (b) $x = 1.00$ m, $z = 0$; (c) $x = 0$, $z = -0.25$ m?

Figure **E28.19**



28.20 •• **BIO Transmission Lines and Health.** Currents in dc transmission lines can be 100 A or higher. Some people are concerned that the electromagnetic fields from such lines near their homes could pose health dangers. For a line that has current 150 A and a height of 8.0 m above the ground, what magnetic field does the line produce at ground level? Express your answer in teslas and as a percentage of the earth's magnetic field, which is 0.50 G. Is this value cause for worry?

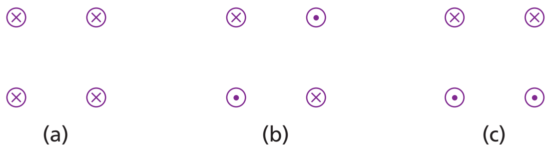
28.21 • Two long, straight, parallel wires, 10.0 cm apart, carry equal 4.00 A currents in the same direction, as shown in **Fig. E28.21**. Find the magnitude and direction of the magnetic field at (a) point P_1, midway between the wires; (b) point P_2, 25.0 cm to the right of P_1; (c) point P_3, 20.0 cm directly above P_1.

Figure **E28.21**

28.22 •• A rectangular loop with dimensions 4.20 cm by 9.50 cm carries current *I*. The current in the loop produces a magnetic field at the center of the loop that has magnitude 5.50×10^{-5} T and direction away from you as you view the plane of the loop. What are the magnitude and direction (clockwise or counterclockwise) of the current in the loop?

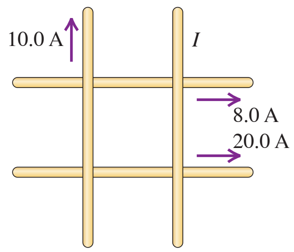
28.23 • Four long, parallel power lines each carry 100 A currents. A cross-sectional diagram of these lines is a square, 20.0 cm on each side. For each of the three cases shown in **Fig. E28.23**, calculate the magnetic field at the center of the square.

Figure **E28.23**



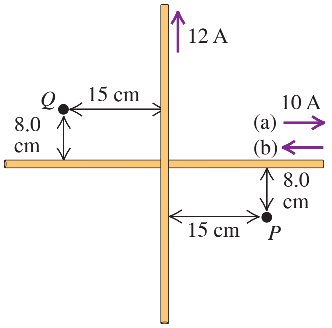
28.24 • Four very long, current-carrying wires in the same plane intersect to form a square 40.0 cm on each side, as shown in **Fig. E28.24**. Find the magnitude and direction of the current I so that the magnetic field at the center of the square is zero.

Figure **E28.24**



28.25 •• Two very long insulated wires perpendicular to each other in the same plane carry currents as shown in **Fig. E28.25**. Find the magnitude of the *net* magnetic field these wires produce at points P and Q if the 10.0 A current is (a) to the right or (b) to the left.

Figure **E28.25**

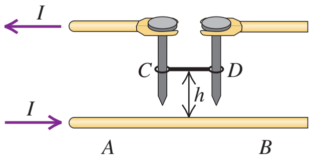


28.26 • A wire of length 20.0 cm lies along the x-axis with the center of the wire at the origin. The wire carries current $I = 8.00$ A in the $-x$-direction. (a) What is the magnitude B of the magnetic field of the wire at the point $y = 5.00$ cm on the y-axis? (b) What is the percent difference between the answer in (a) and the value you obtain if you assume the wire is infinitely long and use Eq. (28.9) to calculate B?

Section 28.4 Force Between Parallel Conductors

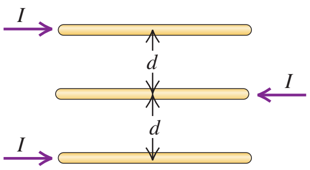
28.27 • A long, horizontal wire AB rests on the surface of a table and carries a current I. Horizontal wire CD is vertically above wire AB and is free to slide up and down on the two vertical metal guides C and D (**Fig. E28.27**). Wire CD is connected through the sliding contacts to another wire that also carries a current I, opposite in direction to the current in wire AB. The mass per unit length of the wire CD is λ. To what equilibrium height h will the wire CD rise, assuming that the magnetic force on it is due entirely to the current in the wire AB?

Figure **E28.27**



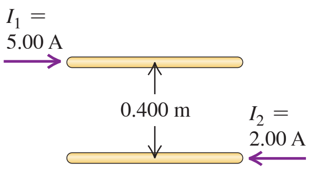
28.28 • Three very long parallel wires each carry current I in the directions shown in **Fig. E28.28**. If the separation between adjacent wires is d, calculate the magnitude and direction of the net magnetic force per unit length on each wire.

Figure **E28.28**



28.29 • Two long, parallel wires are separated by a distance of 0.400 m (**Fig. E28.29**). The currents I_1 and I_2 have the directions shown. (a) Calculate the magnitude of the force exerted by each wire on a 1.20 m length of the other. Is the force attractive or repulsive? (b) Each current is doubled, so that I_1 becomes 10.0 A and I_2 becomes 4.00 A. Now what is the magnitude of the force that each wire exerts on a 1.20 m length of the other?

Figure **E28.29**



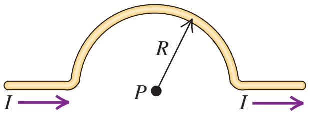
28.30 • Two long, parallel wires are separated by a distance of 2.50 cm. The force per unit length that each wire exerts on the other is 4.00×10^{-5} N/m, and the wires repel each other. The current in one wire is 0.600 A. (a) What is the current in the second wire? (b) Are the two currents in the same direction or in opposite directions?

Section 28.5 Magnetic Field of a Circular Current Loop

28.31 • **BIO** **Currents in the Brain.** The magnetic field around the head has been measured to be approximately 3.0×10^{-8} G. Although the currents that cause this field are quite complicated, we can get a rough estimate of their size by modeling them as a single circular current loop 16 cm (the width of a typical head) in diameter. What is the current needed to produce such a field at the center of the loop?

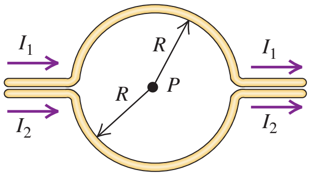
28.32 • Calculate the magnitude and direction of the magnetic field at point P due to the current in the semicircular section of wire shown in **Fig. E28.32**. (*Hint:* Does the current in the long, straight section of the wire produce any field at P?)

Figure **E28.32**



28.33 •• Calculate the magnitude of the magnetic field at point P of **Fig. E28.33** in terms of R, I_1, and I_2. What does your expression give when $I_1 = I_2$?

Figure **E28.33**



28.34 •• A closely wound, circular coil with radius 2.40 cm has 800 turns. (a) What must the current in the coil be if the magnetic field at the center of the coil is 0.0770 T? (b) At what distance x from the center of the coil, on the axis of the coil, is the magnetic field half its value at the center?

28.35 •• Two concentric circular loops of wire lie on a tabletop, one inside the other. The inner wire has a diameter of 20.0 cm and carries a clockwise current of 12.0 A, as viewed from above, and the outer wire has a diameter of 30.0 cm. What must be the magnitude and direction (as viewed from above) of the current in the outer wire so that the net magnetic field due to this combination of wires is zero at the common center of the wires?

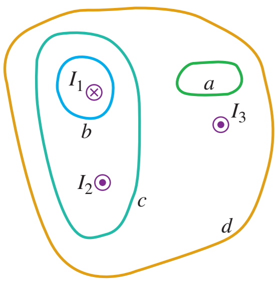
28.36 •• A closely wound coil has a radius of 6.00 cm and carries a current of 2.50 A. How many turns must it have if, at a point on the coil axis 6.00 cm from the center of the coil, the magnetic field is 6.39×10^{-4} T?

Section 28.6 Ampere's Law

28.37 • A closed curve encircles several conductors. The line integral $\oint \vec{B} \cdot d\vec{l}$ around this curve is $3.83 \times 10^{-4}\ \text{T} \cdot \text{m}$. (a) What is the net current in the conductors? (b) If you were to integrate around the curve in the opposite direction, what would be the value of the line integral? Explain.

28.38 • **Figure E28.38** shows, in cross section, several conductors that carry currents through the plane of the figure. The currents have the magnitudes $I_1 = 4.0$ A, $I_2 = 6.0$ A, and $I_3 = 2.0$ A, and the directions shown. Four paths, labeled a through d, are shown. What is the line integral $\oint \vec{B} \cdot d\vec{l}$ for each path? Each integral involves going around the path in the counterclockwise direction. Explain your answers.

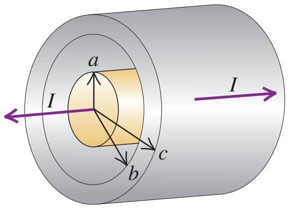
Figure **E28.38**



Section 28.7 Applications of Ampere's Law

28.39 • **Coaxial Cable.** A solid conductor with radius a is supported by insulating disks on the axis of a conducting tube with inner radius b and outer radius c (**Fig. E28.39**). The central conductor and tube carry equal currents I in opposite directions. The currents are distributed uniformly over the cross sections of each conductor. Derive an expression for the magnitude of the magnetic field (a) at points outside the central, solid conductor but inside the tube ($a < r < b$) and (b) at points outside the tube ($r > c$).

Figure **E28.39**



28.40 •• As a new electrical technician, you are designing a large solenoid to produce a uniform 0.150 T magnetic field near the center of the solenoid. You have enough wire for 4000 circular turns. This solenoid must be 55.0 cm long and 2.80 cm in diameter. What current will you need to produce the necessary field?

28.41 • Repeat Exercise 28.39 for the case in which the current in the central, solid conductor is I_1, the current in the tube is I_2, and these currents are in the same direction rather than in opposite directions.

28.42 •• A 15.0-cm-long solenoid with radius 0.750 cm is closely wound with 600 turns of wire. The current in the windings is 8.00 A. Compute the magnetic field at a point near the center of the solenoid.

28.43 •• A solenoid is designed to produce a magnetic field of 0.0270 T at its center. It has radius 1.40 cm and length 40.0 cm, and the wire can carry a maximum current of 12.0 A. (a) What minimum number of turns per unit length must the solenoid have? (b) What total length of wire is required?

28.44 • An ideal toroidal solenoid (see Example 28.10) has inner radius $r_1 = 15.0$ cm and outer radius $r_2 = 18.0$ cm. The solenoid has 250 turns and carries a current of 8.50 A. What is the magnitude of the magnetic field at the following distances from the center of the torus: (a) 12.0 cm; (b) 16.0 cm; (c) 20.0 cm?

28.45 • A magnetic field of 37.2 T has been achieved at the MIT Francis Bitter Magnet Laboratory. Find the current needed to achieve such a field (a) 2.00 cm from a long, straight wire; (b) at the center of a circular coil of radius 42.0 cm that has 100 turns; (c) near the center of a solenoid with radius 2.40 cm, length 32.0 cm, and 40,000 turns.

Section 28.8 Magnetic Materials

28.46 •• A toroidal solenoid with 400 turns of wire and a mean radius of 6.0 cm carries a current of 0.25 A. The relative permeability of the core is 80. (a) What is the magnetic field in the core? (b) What part of the magnetic field is due to the magnetic moments of the atoms in the core?

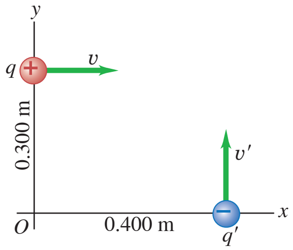
28.47 • A long solenoid with 60 turns of wire per centimeter carries a current of 0.15 A. The wire that makes up the solenoid is wrapped around a solid core of silicon steel ($K_\text{m} = 5200$). (The wire of the solenoid is jacketed with an insulator so that none of the current flows into the core.) (a) For a point inside the core, find the magnitudes of (i) the magnetic field $\vec{B}_0$ due to the solenoid current; (ii) the magnetization $\vec{M}$; (iii) the total magnetic field $\vec{B}$. (b) In a sketch of the solenoid and core, show the directions of the vectors $\vec{B}$, $\vec{B}_0$, and $\vec{M}$ inside the core.

28.48 • The current in the windings of a toroidal solenoid is 2.400 A. There are 500 turns, and the mean radius is 25.00 cm. The toroidal solenoid is filled with a magnetic material. The magnetic field inside the windings is found to be 1.940 T. Calculate (a) the relative permeability and (b) the magnetic susceptibility of the material that fills the toroid.

PROBLEMS

28.49 •• A pair of point charges, $q = +8.00\ \mu\text{C}$ and $q' = -5.00\ \mu\text{C}$, are moving as shown in **Fig. P28.49** with speeds $v = 9.00 \times 10^4$ m/s and $v' = 6.50 \times 10^4$ m/s. When the charges are at the locations shown in the figure, what are the magnitude and direction of (a) the magnetic field produced at the origin and (b) the magnetic force that q' exerts on q?

Figure **P28.49**



28.50 •• At a particular instant, charge $q_1 = +4.80 \times 10^{-6}$ C is at the point (0, 0.250 m, 0) and has velocity $\vec{v}_1 = (9.20 \times 10^5\ \text{m/s})\hat{\imath}$. Charge $q_2 = -2.90 \times 10^{-6}$ C is at the point (0.150 m, 0, 0) and has velocity $\vec{v}_2 = (-5.30 \times 10^5\ \text{m/s})\hat{\jmath}$. At this instant, what are the magnitude and direction of the magnetic force that q_1 exerts on q_2?

28.51 •• A long, straight wire lies along the x-axis and carries current $I_1 = 2.00$ A in the $+x$-direction. A second wire lies in the xy-plane and is parallel to the x-axis at $y = +0.800$ m. It carries current $I_2 = 6.00$ A, also in the $+x$-direction. In addition to $y \rightarrow \pm\infty$, at what point on the y-axis is the resultant magnetic field of the two wires equal to zero?

28.52 •• Repeat Problem 28.51 for I_2 in the $-x$-direction, with all the other quantities the same.

28.53 •• We can estimate the strength of the magnetic field of a refrigerator magnet in the following way: Imagine the magnet as a collection of current-loop magnetic dipoles. (a) Derive the force between two current loops with radius R and current I separated by distance $d \ll R$. Very close to the wire its magnetic field is about the same as for an infinitely long wire, and Eq.(28.11) can be used. (b) Using Eq. (28.17), express the current I in terms of the magnetic field at the middle of the loop, and express the radius R in terms of the area of the loop. In this way, derive an expression for the force F between two identical current loops separated by a small distance d in terms of their mutual area A and center magnetic field B. (c) Rearrange your result to obtain an expression for the magnetic field of a dipole with area A in terms of the force F from an identical dipole separated by a small distance d. (d) Now notice that the force it takes to separate one magnet from your refrigerator is nearly the same as the force it takes to separate two magnets stuck together. Estimate that force F. (e) Estimate the area of a refrigerator magnet. (f) Assume that when these magnets are stuck together or to the refrigerator, they are separated by an effective distance $d = 25\ \mu$m. Use the formula derived above to estimate the magnetic field strength of the magnet.

28.54 • A long, straight wire carries a current of 8.60 A. An electron is traveling in the vicinity of the wire. At the instant when the electron is 4.50 cm from the wire and traveling at a speed of 6.00×10^4 m/s directly toward the wire, what are the magnitude and direction (relative to the direction of the current) of the force that the magnetic field of the current exerts on the electron?

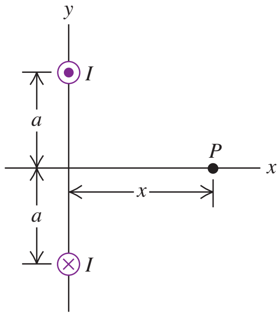
28.55 • **CP** A long, straight wire carries a 13.0 A current. An electron is fired parallel to this wire with a velocity of 250 km/s in the same direction as the current, 2.00 cm from the wire. (a) Find the magnitude and direction of the electron's initial acceleration. (b) What should be the magnitude and direction of a uniform electric field that will allow the electron to continue to travel parallel to the wire? (c) Is it necessary to include the effects of gravity? Justify your answer.

28.56 •• An electron is moving in the vicinity of a long, straight wire that lies along the x-axis. The wire has a constant current of 9.00 A in the $-x$-direction. At an instant when the electron is at point (0, 0.200 m, 0) and the electron's velocity is $\vec{v} = (5.00 \times 10^4 \text{ m/s})\hat{\imath} - (3.00 \times 10^4 \text{ m/s})\hat{\jmath}$, what is the force that the wire exerts on the electron? Express the force in terms of unit vectors, and calculate its magnitude.

28.57 •• **CP** (a) Determine the transmission power P of your cell phone. (This information is available online.) (b) A typical cell phone battery supplies a 1.5 V potential. If your phone battery supplies the power P, what is a good estimate of the current supplied by the battery? (c) Estimate the width of your head. (d) Estimate the diameter of the phone speaker that goes next to your ear. Model the current in the speaker as a current loop with the same diameter as the speaker. Use these values to estimate the magnetic field generated by your phone midway between the ears when it is held near one ear. (e) How does your answer compare to the earth's field, which is about 50 μT?

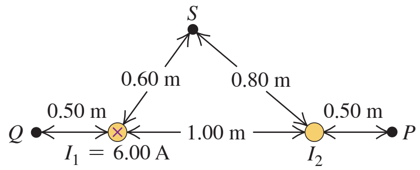
28.58 •• **Figure P28.58** shows an end view of two long, parallel wires perpendicular to the xy-plane, each carrying a current I but in opposite directions. (a) Copy the diagram, and draw vectors to show the $\vec{B}$ field of each wire and the net $\vec{B}$ field at point P. (b) Derive the expression for the magnitude of $\vec{B}$ at any point on the x-axis in terms of the x-coordinate of the point. What is the direction of $\vec{B}$? (c) Graph the magnitude of $\vec{B}$ at points on the x-axis. (d) At what value of x is the magnitude of $\vec{B}$ a maximum? (e) What is the magnitude of $\vec{B}$ when $x \gg a$?

Figure **P28.58**



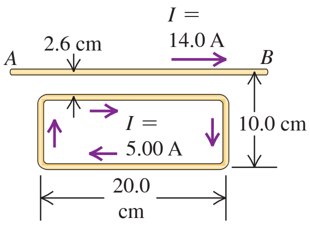
28.59 • Two long, straight, parallel wires are 1.00 m apart (**Fig. P28.59**). The wire on the left carries a current I_1 of 6.00 A into the plane of the paper. (a) What must the magnitude and direction of the current I_2 be for the net field at point P to be zero? (b) Then what are the magnitude and direction of the net field at Q? (c) Then what is the magnitude of the net field at S?

Figure **P28.59**



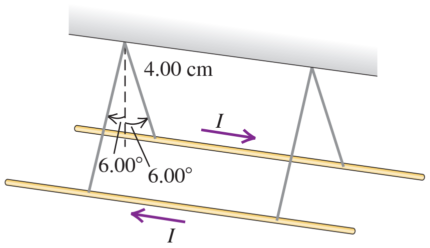
28.60 • The long, straight wire AB shown in **Fig. P28.60** carries a current of 14.0 A. The rectangular loop whose long edges are parallel to the wire carries a current of 5.00 A. Find the magnitude and direction of the net force exerted on the loop by the magnetic field of the wire.

Figure **P28.60**



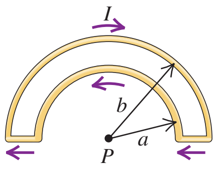
28.61 ••• **CP** Two long, parallel wires hang by 4.00-cm-long cords from a common axis (**Fig. P28.61**). The wires have a mass per unit length of 0.0125 kg/m and carry the same current in opposite directions. What is the current in each wire if the cords hang at an angle of 6.00° with the vertical?

Figure **P28.61**



28.62 • The wire semicircles shown in **Fig. P28.62** have radii a and b. Calculate the net magnetic field (magnitude and direction) that the current in the wires produces at point P.

Figure **P28.62**



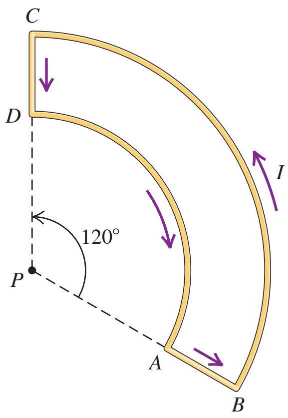
28.63 • A long, straight, solid cylinder, oriented with its axis in the z-direction, carries a current whose current density is $\vec{J}$. The current density, although symmetric about the cylinder axis, is not constant and varies according to the relationship

$$\begin{aligned} \vec{J} &= \left(\frac{b}{r}\right)e^{(r-a)/\delta}\,\hat{k} \quad && \text{for } r \le a \\ &= \mathbf{0} && \text{for } r \ge a \end{aligned}$$

where the radius of the cylinder is $a = 5.00$ cm, r is the radial distance from the cylinder axis, b is a constant equal to 600 A/m, and δ is a constant equal to 2.50 cm. (a) Let I_0 be the total current passing through the entire cross section of the wire. Obtain an expression for I_0 in terms of b, δ, and a. Evaluate your expression to obtain a numerical value for I_0. (b) Using Ampere's law, derive an expression for the magnetic field $\vec{B}$ in the region $r \ge a$. Express your answer in terms of I_0 rather than b. (c) Obtain an expression for the current I contained in a circular cross section of radius $r \le a$ and centered at the cylinder axis. Express your answer in terms of I_0 rather than b. (d) Using Ampere's law, derive an expression for the magnetic field $\vec{B}$ in the region $r \le a$. (e) Evaluate the magnitude of the magnetic field at $r = \delta$, $r = a$, and $r = 2a$.

28.64 •• Calculate the magnetic field (magnitude and direction) at a point P due to a current $I = 12.0$ A in the wire shown in **Fig. P28.64**. Segment BC is an arc of a circle with radius 30.0 cm, and point P is at the center of curvature of the arc. Segment DA is an arc of a circle with radius 20.0 cm, and point P is at its center of curvature. Segments CD and AB are straight lines of length 10.0 cm each.

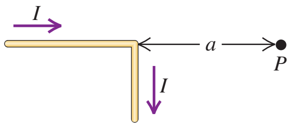
Figure **P28.64**



28.65 • **CALC** A long, straight wire with a circular cross section of radius R carries a current I. Assume that the current density is not constant across the cross section of the wire, but rather varies as $J = \alpha r$, where α is a constant. (a) By the requirement that J integrated over the cross section of the wire gives the total current I, calculate the constant α in terms of I and R. (b) Use Ampere's law to calculate the magnetic field $B(r)$ for (i) $r \leq R$ and (ii) $r \geq R$. Express your answers in terms of I.

28.66 • **CALC** The wire shown in **Fig. P28.66** is infinitely long and carries a current I. Calculate the magnitude and direction of the magnetic field that this current produces at point P.

Figure **P28.66**



28.67 • **CALC** A long, straight, solid cylinder, oriented with its axis in the z-direction, carries a current whose current density is $\vec{J}$. The current density, although symmetric about the cylinder axis, is not constant but varies according to the relationship

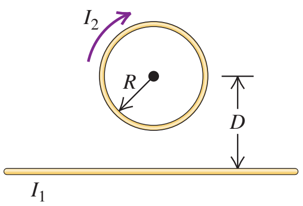
$$\vec{J} = \frac{2I_0}{\pi a^2}\left[1-\left(\frac{r}{a}\right)^2\right]\hat{k} \qquad \text{for } r \leq a$$

$$= 0 \qquad \text{for } r \geq a$$

where a is the radius of the cylinder, r is the radial distance from the cylinder axis, and I_0 is a constant having units of amperes. (a) Show that I_0 is the total current passing through the entire cross section of the wire. (b) Using Ampere's law, derive an expression for the magnitude of the magnetic field $\vec{B}$ in the region $r \geq a$. (c) Obtain an expression for the current I contained in a circular cross section of radius $r \leq a$ and centered at the cylinder axis. (d) Using Ampere's law, derive an expression for the magnitude of the magnetic field $\vec{B}$ in the region $r \leq a$. How do your results in parts (b) and (d) compare for $r = a$?

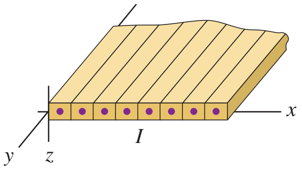
28.68 • A circular loop has radius R and carries current I_2 in a clockwise direction (**Fig. P28.68**). The center of the loop is a distance D above a long, straight wire. What are the magnitude and direction of the current I_1 in the wire if the magnetic field at the center of the loop is zero?

Figure **P28.68**



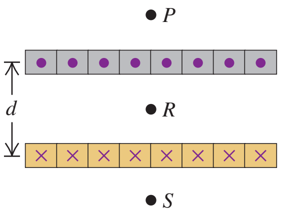
28.69 • **An Infinite Current Sheet.** Long, straight conductors with square cross sections and each carrying current I are laid side by side to form an infinite current sheet (**Fig. P28.69**). The conductors lie in the xy-plane, are parallel to the y-axis, and carry current in the $+y$-direction. There are n conductors per unit length measured along the x-axis. (a) What are the magnitude and direction of the magnetic field a distance a below the current sheet? (b) What are the magnitude and direction of the magnetic field a distance a above the current sheet?

Figure **P28.69**



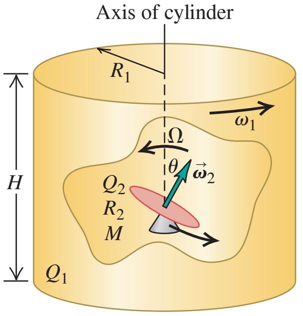
28.70 • Long, straight conductors with square cross section, each carrying current I, are laid side by side to form an infinite current sheet with current directed out of the plane of the page (**Fig. P28.70**). A second infinite current sheet is a distance d below the first and is parallel to it. The second sheet carries current into the plane of the page. Each sheet has n conductors per unit length. (Refer to Problem 28.69.) Calculate the magnitude and direction of the net magnetic field at (a) point P (above the upper sheet); (b) point R (midway between the two sheets); (c) point S (below the lower sheet).

Figure **P28.70**



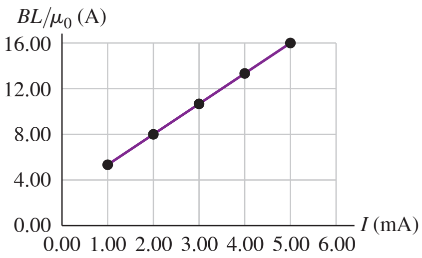
28.71 ••• **CP** A cylindrical shell with radius R_1 and height H has charge Q_1 and rotates around its axis with angular speed ω_1, as shown in **Fig. P28.71**. Inside the cylinder, far from its edges, sits a very small disk with radius R_2, mass M, and charge Q_2 mounted on a pivot, spinning with a large angular velocity $\vec{\omega}_2$ and oriented at angle θ with respect to the axis of the cylinder. The center of the disk is on the axis of the cylinder. The magnetic interaction between the cylinder and the disk causes a precession of the axis of the disk. (a) What is the magnitude of the enclosed current I_{encl} surrounded by a loop that has one vertical side that is along the axis of the cylinder and extends beyond the top and bottom of the cylinder? The other vertical side of the loop is very far outside the cylinder. (b) Assume the field is uniform within the cylinder and use Ampere's law to find the magnetic field at the center of the disk. (c) The magnetic moment of the disk has magnitude $\mu = \frac{1}{4}Q_2\omega_2 R_2^2$. What is the magnitude of the torque exerted on the disk? (d) What is the magnitude of the angular momentum of the disk?

Figure **P28.71**



28.72 •• **DATA** As a summer intern at a research lab, you are given a long solenoid that has two separate windings that are wound close together, in the same direction, on the same hollow cylindrical form. You must determine the number of turns in each winding. The solenoid has length $L = 40.0$ cm and diameter 2.80 cm. You let a 2.00 mA current flow in winding 1 and vary the current I in winding 2; both currents flow in the same direction. Then you measure the magnetic-field magnitude B at the center of the solenoid as a function of I. You plot your results as BL/μ_0 versus I. The graph in **Fig. P28.72** shows the best-fit straight line to your data. (a) Explain why the data plotted in this way should fall close to a straight line. (b) Use Fig. P28.72 to calculate N_1 and N_2, the number of turns in windings 1 and 2. (c) If the current in winding 1 remains 2.00 mA in its original direction and winding 2 has $I = 5.00$ mA in the opposite direction, what is B at the center of the solenoid?

Figure **P28.72**



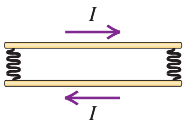
28.73 •• DATA You use a teslameter (a Hall-effect device) to measure the magnitude of the magnetic field at various distances from a long, straight, thick cylindrical copper cable that is carrying a large constant current. To exclude the earth's magnetic field from the measurement, you first set the meter to zero. You then measure the magnetic field B at distances x from the surface of the cable and obtain these data:

x (cm)	2.0	4.0	6.0	8.0	10.0
B (mT)	0.406	0.250	0.181	0.141	0.116

(a) You think you remember from your physics course that the magnetic field of a wire is inversely proportional to the distance from the wire. Therefore, you expect that the quantity Bx from your data will be constant. Calculate Bx for each data point in the table. Is Bx constant for this set of measurements? Explain. (b) Graph the data as x versus $1/B$. Explain why such a plot lies close to a straight line. (c) Use the graph in part (b) to calculate the current I in the cable and the radius R of the cable.

28.74 ••• DATA A pair of long, rigid metal rods, each of length 0.50 m, lie parallel to each other on a frictionless table. Their ends are connected by identical, very lightweight conducting springs with unstretched length l_0 and force constant k (**Fig. P28.74**). When a current I runs through the circuit consisting of the rods and springs, the springs stretch. You measure the distance x each spring stretches for certain values of I. When $I = 8.05$ A, you measure that $x = 0.40$ cm. When $I = 13.1$ A, you find $x = 0.80$ cm. In both cases the rods are much longer than the stretched springs, so it is accurate to use Eq. (28.11) for two infinitely long, parallel conductors. (a) From these two measurements, calculate l_0 and k. (b) If $I = 12.0$ A, what distance x will each spring stretch? (c) What current is required for each spring to stretch 1.00 cm?

Figure **P28.74**

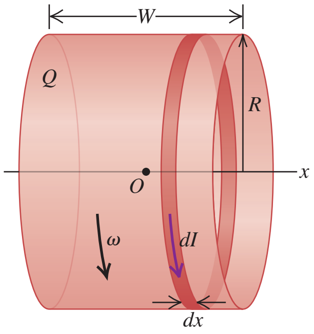


28.75 ••• CP CALC A plasma is a gas of ionized (charged) particles. When plasma is in motion, magnetic effects "squeeze" its volume, inducing inward pressure known as a pinch. Consider a cylindrical tube of plasma with radius R and length L moving with velocity $\vec{v}$ along its axis. If there are n ions per unit volume and each ion has charge q, we can determine the pressure felt by the walls of the cylinder. (a) What is the volume charge density ρ in terms of n and q? (b) The thickness of the cylinder "surface" is $n^{-1/3}$. What is the surface charge density σ in terms of n and q? (c) The current density inside the cylinder is $\vec{J} = \rho\vec{v}$. Use this result along with Ampere's law to determine the magnetic field on the surface of the cylinder. Denote the circumferential unit vector as $\hat{\phi}$. (d) The width of a differential strip of surface current is $R\,d\phi$. What is the differential current dI_{surface} that flows along this strip? (e) What differential force is felt by this strip due to the magnetic field generated by the volume current? (f) Integrate to determine the total force on the walls of the cylinder; then divide by the wall area to obtain the pressure in terms of n, q, R, and v. (g) If a plasma cylinder with radius 2.0 cm has a charge density of 8.0×10^{16} ions/cm^3, where each ion has a charge of $e = 1.6 \times 10^{-19}$ C and is moving axially with a speed of 20.0 m/s, what is its pinch pressure?

CHALLENGE PROBLEMS

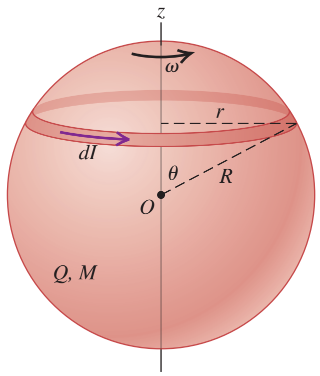
28.76 ••• CALC A cylindrical shell with radius R and length W carries a uniform charge Q and rotates about its axis with angular speed ω. The center of the cylinder lies at the origin O and its axis is coincident with the x-axis, as shown in **Fig. P28.76**. (a) What is the charge density σ? (b) What is the differential current dI on a circular strip of the cylinder centered at x and with width dx? (c) Use Eq. (28.15) to write an expression for the differential magnetic field $d\vec{B}$ at the origin due to this strip. (d) Integrate to determine the magnetic field at the origin.

Figure **P28.76**



28.77 ••• CALC When a rigid charge distribution with charge Q and mass M rotates about an axis, its magnetic moment $\vec{\mu}$ is linearly proportional to its angular momentum $\vec{L}$, with $\vec{\mu} = \gamma\vec{L}$. The constant of proportionality γ is called the gyromagnetic ratio of the object. We can write $\gamma = g(Q/2M)$, where g is a dimensionless number called the g-factor of the object. Consider a spherical shell with mass M and uniformly distributed charge Q centered on the origin O and rotating about the z-axis with angular speed ω. (a) A thin slice with latitude θ measured with respect to the positive z-axis describes a current loop with width $R\,d\theta$ and radius $r = R \sin\theta$, as shown in **Fig. P28.77**. What is the differential current dI carried by this loop, in terms of Q, ω, R, θ, and $d\theta$? (b) The differential magnetic moment contributed by that slice is $d\mu = A\,dI$, where $A = \pi r^2$ is the area enclosed by the loop. Express the differential magnetic moment in terms of Q, ω, R, θ, and $d\theta$. (c) Integrate over θ to determine the magnetic moment $\vec{\mu}$. (d) What is the magnitude of the angular momentum $\vec{L}$? (e) Determine the gyromagnetic ratio γ. (f) What is the g-factor for a spherical shell?

Figure **P28.77**



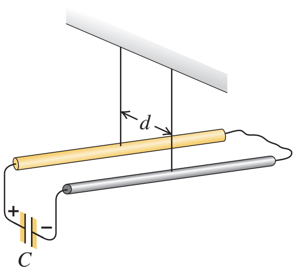
28.78 ••• CALC The law of Biot and Savart in Eq. (28.7) generalizes to the case of surface currents as

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\sigma\vec{v}\times\hat{r}}{r^2}da$$

where σ is the local charge density, $\vec{v}$ is the local velocity, and da is a differential area element. Re-visit Challenge Problem 28.76 and use the above equation as an alternative means to derive the magnetic field at the center of the cylinder. Use the following steps: (a) Write the charge density σ. (b) The origin is at the center of the cylinder. What is the vector $\vec{v}$ that points from the element with coordinates $(x, y, z) = (x, R\cos\phi, R\sin\phi)$ to the origin? (c) What is the velocity $\vec{v}$ of the element? (d) What is the vector product $\vec{v}\times\hat{r}$? (e) An area element on the cylinder may be written as $da = R\,dx\,d\phi$. Use this and the previously established information to write the generalized law of Biot and Savart as a double integral. Evaluate the integral to determine the magnetic field $\vec{B}$ at the center of the cylinder. (f) Is your result consistent with your result in Challenge Problem 28.76?

28.79 ••• **CP** Two long, straight conducting wires with linear mass density λ are suspended from cords so that they are each horizontal, parallel to each other, and a distance d apart. The back ends of the wires are connected to each other by a slack, low-resistance connecting wire. A charged capacitor (capacitance C) is now added to the system; the positive plate of the capacitor (initial charge $+Q_0$) is connected to the front end of one of the wires, and the negative plate of the capacitor (initial charge $-Q_0$) is connected to the front end of the other wire (**Fig. P28.79**). Both of these connections are also made by slack, low-resistance wires. When the connection is made, the wires are pushed aside by the repulsive force between the wires, and each wire has an initial horizontal velocity of magnitude v_0. Assume that the time constant for the capacitor to discharge is negligible compared to the time it takes for any appreciable displacement in the position of the wires to occur. (a) Show that the initial speed v_0 of either wire is given by

Figure **P28.79**



$$v_0 = \frac{\mu_0 Q_0^2}{4\pi\lambda RCd}$$

where R is the total resistance of the circuit. (b) To what height h will each wire rise as a result of the circuit connection?

28.80 ••• A wide, long, insulating belt has a uniform positive charge per unit area σ on its upper surface. Rollers at each end move the belt to the right at a constant speed v. Calculate the magnitude and direction of the magnetic field produced by the moving belt at a point just above its surface. (*Hint:* At points near the surface and far from its edges or ends, the moving belt can be considered to be an infinite current sheet like that in Problem 28.69.)

MCAT-STYLE PASSAGE PROBLEMS

BIO Studying Magnetic Bacteria. Some types of bacteria contain chains of ferromagnetic particles parallel to their long axis. The chains act like small bar magnets that align these *magnetotactic* bacteria with the earth's magnetic field. In one experiment to study the response of such bacteria to magnetic fields, a solenoid is constructed with copper wire 1.0 mm in diameter, evenly wound in a single layer to form a helical coil of length 40 cm and diameter 12 cm. The wire has a very thin layer of insulation, and the coil is wound so that adjacent turns are just touching. The solenoid, which generates a magnetic field, is in an enclosure that shields it from other magnetic fields. A sample of magnetotactic bacteria is placed inside the solenoid. The torque on an individual bacterium in the solenoid's magnetic field is proportional to the magnitude of the magnetic field and to the sine of the angle between the long axis of the bacterium and the magnetic-field direction.

28.81 What current is needed in the wire so that the magnetic field experienced by the bacteria has a magnitude of 150 μT? (a) 0.095 A; (b) 0.12 A; (c) 0.30 A; (d) 14 A.

28.82 To use a larger sample, the experimenters construct a solenoid that has the same length, type of wire, and loop spacing but twice the diameter of the original. How does the maximum possible magnetic torque on a bacterium in this new solenoid compare with the torque the bacterium would have experienced in the original solenoid? Assume that the currents in the solenoids are the same. The maximum torque in the new solenoid is (a) twice that in the original one; (b) half that in the original one; (c) the same as that in the original one; (d) one-quarter that in the original one.

28.83 The solenoid is removed from the enclosure and then used in a location where the earth's magnetic field is 50 μT and points horizontally. A sample of bacteria is placed in the center of the solenoid, and the same current is applied that produced a magnetic field of 150 μT in the lab. Describe the field experienced by the bacteria: The field (a) is still 150 μT; (b) is now 200 μT; (c) is between 100 and 200 μT, depending on how the solenoid is oriented; (d) is between 50 and 150 μT, depending on how the solenoid is oriented.

ANSWERS

Chapter Opening Question?

(iv) There would be *no* change in the magnetic field strength. From Example 28.9 (Section 28.7), the field inside a solenoid has magnitude $B = \mu_0 nI$, where n is the number of turns of wire per unit length. Joining two solenoids end to end doubles both the number of turns and the length, so the number of turns per unit length is unchanged.

Key Example VARIATION Problems

VP28.2.1 (a) $(3.20 \times 10^{-15}\ \text{T})\hat{k}$ (b) $(-8.00 \times 10^{-16}\ \text{T})\hat{j}$ (c) zero (d) $(1.13 \times 10^{-15}\ \text{T})\hat{k}$

VP28.2.2 (a) 1.13×10^{-28} N in the $-x$-direction (toward the proton) (b) 1.13×10^{-28} N in the $+x$-direction (toward the electron)

VP28.2.3 (a) $(0.600)\hat{\imath} + (0.800)\hat{\jmath}$ (b) $(-2.88 \times 10^{-11}\ \text{T})\hat{k}$

VP28.2.4 $(9.83 \times 10^{-23}\ \text{N})\hat{k}$

VP28.5.1 2.24 A

VP28.5.2 (a) $(-6.00 \times 10^{-5}\ \text{T})\hat{\imath}$ (b) $(-4.67 \times 10^{-5}\ \text{T})\hat{\imath}$ (c) $(8.67 \times 10^{-5}\ \text{T})\hat{\imath}$

VP28.5.3 (a) 5.00 A (b) 1.00 A

VP28.5.4 3.29×10^4 A, upward

VP28.10.1 (a) 4.94×10^{-6} T (b) 8.89×10^{-6} T (c) 6.67×10^{-6} T

VP28.10.2 (a) $B = 0$ for $r < R_1$, $B = (\mu_0 J/2r)(r^2 - R_1^2)$ for $R_2 > r > R_1$, $B = (\mu_0 J/2r)(R_2^2 - R_1^2)$ for $r > R_2$ (b) $r = R_2$

VP28.10.3 3.18 A

VP28.10.4 (a) 833 (b) 2.33×10^{-3} T maximum, 1.75×10^{-3} T minimum

Bridging Problem

$$B = \frac{\mu_0 nQ}{a}$$