Lab 13

[HW7-1]

```
"/Users/Austin/Desktop/PHY 40/Homework/7/cmake-build-debug/7"
dimension of the matrix: 3
enter a 3 x 3 matrix (separated by space):
2 0 0
0 4 1
0 1 4

eigen problem for matix A:
  2.000  0.000  0.000
  0.000  4.000  1.000
  0.000  1.000  4.000

number of Jacobi applied: 1
eigenvalues:
  2.000  3.000  5.000

eigenvectors:
  1.000  0.000  0.000
  0.000  0.707  0.707
  0.000 -0.707  0.707
```

[HW 7 - 1]

$$K = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix} \qquad K - \lambda I = \begin{bmatrix} 2-\lambda & 0 & 0 \\ 0 & 4-\lambda & 1 \\ 0 & 1 & 4-\lambda \end{bmatrix} = 0$$

$$2-\lambda\left((4-\lambda)^2 - 1\right) - 0 - 0 = 0$$

$$2-\lambda(\lambda^2 - 8\lambda + 16 - 1) = 0$$

$$2-\lambda(\lambda^2 - 8\lambda + 15) = 0$$

$$(2-\lambda)(\lambda-5)(\lambda-3) = 0$$

$$\lambda = 2, 3, 5$$

Values check out with the Jacobi ✓.

HW7-2

S_y : $S_y = \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $|S_y - \lambda I| = \begin{vmatrix} -\lambda & -i\hbar/2 \\ i\hbar/2 & -\lambda \end{vmatrix} = 0$

$\lambda^2 - \frac{\hbar^2}{4} = 0$ $\boxed{\lambda = \frac{\hbar}{2}, \frac{-\hbar}{2}}$ – E-value for S_y

$+\hbar/2$ $\begin{bmatrix} -\hbar/2 & -i\hbar/2 \\ \frac{i\hbar}{2} & -\hbar/2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0$

$\cancel{+}\frac{\cancel{\hbar}}{\cancel{2}} a \cancel{+} \frac{i\hbar}{2} b = 0$

$a + ib = 0$

$|a|^2 + |b|^2 = 1$

$|a|^2 + |ib|^2 = 1$

$|a|^2 + |a|^2 = 1$

$\boxed{|a|^2 = \frac{1}{2}}$

$-\hbar/2$: $\begin{bmatrix} \hbar/2 & -i\hbar/2 \\ i\hbar/2 & \hbar/2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0$

$\frac{i\hbar}{\cancel{2}} a + \frac{\hbar}{\cancel{2}} b = 0$

$ia + b = 0$

$a = ib$

$|a|^2 + |b|^2 = 1$

$|ib|^2 + |b|^2 = 1$

$b^2 + b^2 = 1$

$\boxed{|b|^2 = \frac{1}{2}}$

[HW7-2]

If a measurement is observable, it must be Hermitian

S_x : $S_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $|S_x - \lambda I| = \begin{vmatrix} -\lambda & \hbar/2 \\ \hbar/2 & -\lambda \end{vmatrix}$

$\lambda^2 - \frac{\hbar^2}{4} = 0$ $\Rightarrow \boxed{\lambda = \frac{\hbar}{2}, -\frac{\hbar}{2}}$ – E-values for S_x

$\lambda = +\frac{\hbar}{2}$ $\begin{bmatrix} -\hbar/2 & \hbar/2 \\ \hbar/2 & -\hbar/2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0$ $-\frac{\cancel{\hbar}}{\cancel{2}} a + \frac{\cancel{\hbar}}{\cancel{2}} b = 0$

$b = a$

$|a|^2 + |b|^2 = 1$

$|a|^2 + |a|^2 = 1$

$\boxed{|a|^2 = \frac{1}{2}}$

$\lambda = -\frac{\hbar}{2}$ $\begin{bmatrix} \hbar/2 & \hbar/2 \\ \hbar/2 & \hbar/2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0$ $a\frac{\hbar}{2} + b\frac{\hbar}{2} = 0$

$a = -b$

$|-b|^2 + |b|^2 = 1$

$2|b|^2 = 1$

$\boxed{|b|^2 = \frac{1}{2}}$

$\underline{S_z}$:

$$\frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\hbar}{2} - \lambda & 0 \\ 0 & -\frac{\hbar}{2} - \lambda \end{bmatrix} \quad \left(\frac{\hbar}{2} - \lambda\right)\left(-\frac{\hbar}{2} - \lambda\right) \quad -\frac{\hbar^2}{2} + \lambda^2 = 0$$

$$\lambda = \pm \frac{\hbar}{2}$$

$$+\frac{\hbar}{2}: \begin{bmatrix} \frac{\hbar}{2} - \frac{\hbar}{2} & 0 \\ 0 & -\hbar/2 - \hbar/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\hbar \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$a(0) + b(0) = 0$

$a(0) + \hbar(b) = 0$

<u>probability is Zero</u>

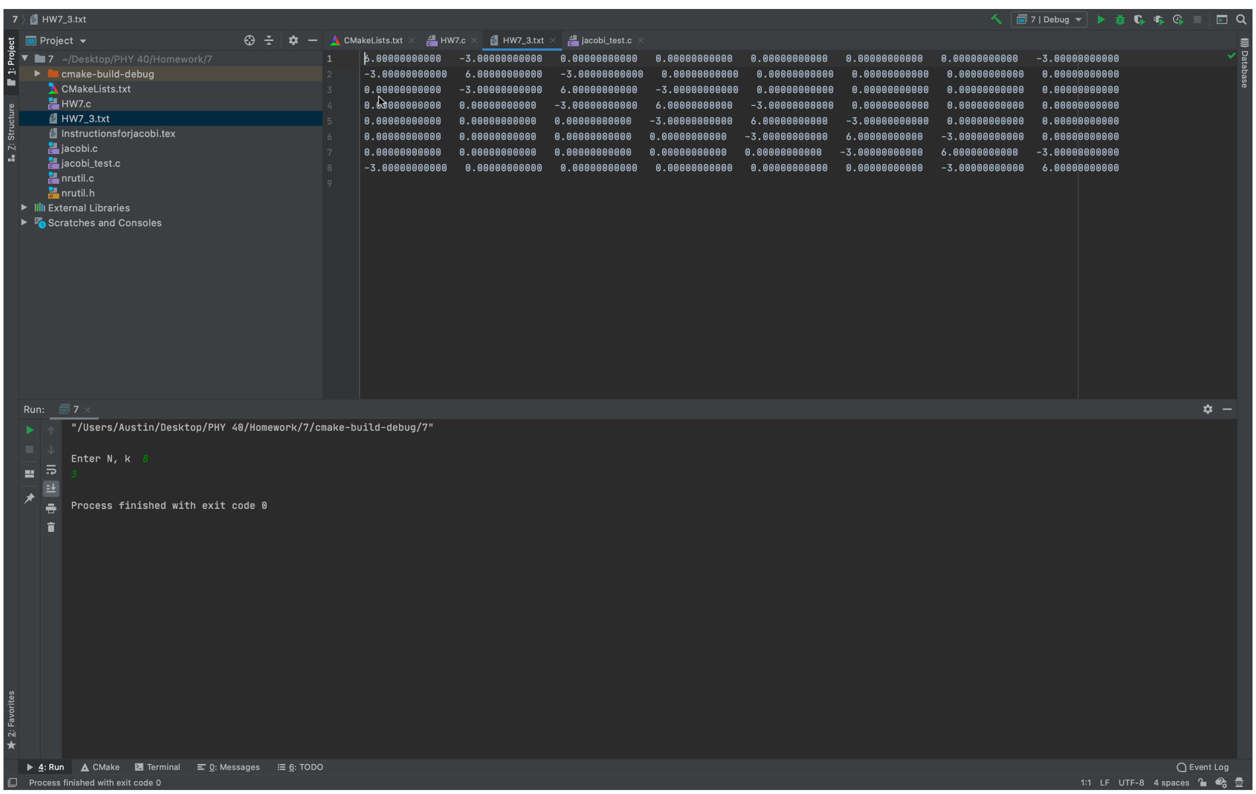
$$-\frac{\hbar}{2} \begin{bmatrix} \frac{\hbar}{2} - \left(-\frac{\hbar}{2}\right) & 0 \\ 0 & -\hbar/2 - \left(-\hbar/2\right) \end{bmatrix} \quad \begin{bmatrix} \hbar & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$a(\hbar) + b(0) = 0$

$a(0) + b(0) = 0$

<u>probability is Zero</u>

7_3/4



7_5

[HW 2-5]

$S_x \uparrow$: $|\psi\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$ $\quad S_x \downarrow$: $|\psi\rangle = \frac{1}{\sqrt{2}}|+x\rangle - \frac{1}{\sqrt{2}}|-x\rangle$

$V = (1/\sqrt{5}, 2/\sqrt{5})$

$$\begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix} \Rightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \langle \psi_+|V\rangle = \left(\frac{1\langle +z|}{\sqrt{2}} + \frac{1\langle -z|}{\sqrt{2}}\right)\left(\frac{1|+z\rangle}{\sqrt{5}} + \frac{2|-z\rangle}{\sqrt{5}}\right) = \frac{1}{\sqrt{10}} + \frac{2}{\sqrt{10}} = \frac{3}{\sqrt{10}}$$

$$\boxed{|\langle \psi_+|V\rangle|^2 = \frac{9}{10} \quad \text{for} \ +\frac{\hbar}{2}}$$

$$|\langle \psi_+|V\rangle|^2 + |\langle \psi_-|V\rangle|^2 = 1$$

$$|\langle \psi_-|V\rangle|^2 = 1 - \frac{9}{10}$$

$$\boxed{|\langle \psi_-|V\rangle|^2 = \frac{1}{2} \quad \text{for} \ -\frac{\hbar}{2}}$$

S_x

$\underline{S_z}$: $\langle +z|V\rangle = \frac{1}{\sqrt{5}}$ $\quad \boxed{|\langle +z|V\rangle|^2 = \frac{1}{5}}$

$$\boxed{|\langle -z|V\rangle|^2 = 1 - \frac{1}{5} = \frac{4}{5}}$$

7_6

[HW7-6]

$S_x : \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \frac{\cancel{\hbar}}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \lambda \cancel{\hbar}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$\det = \begin{vmatrix} -\lambda & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -\lambda & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -\lambda \end{vmatrix} = 0 = -\lambda(\lambda^2 - 1/2) + 1/2\,\lambda = 0$$

$-\lambda^3 + \frac{\lambda}{2} + \frac{\lambda}{2} = 0$

$\lambda^3 - \lambda = 0$

$\lambda(\lambda^2 - 1) = 0$

$\lambda = -1, 0, 1$

$\lambda = 1 : \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 1\begin{pmatrix} a \\ b \\ c \end{pmatrix} \Rightarrow \frac{b}{\sqrt{2}} = a \qquad \frac{a}{\sqrt{2}} + \frac{c}{\sqrt{2}} = b \qquad \frac{b}{\sqrt{2}} = c$

$c = a \qquad \frac{a+a}{\sqrt{2}} = b$

$\frac{2a}{\sqrt{2}} = b$

$|a|^2 + |b|^2 + |c|^2 = 1 \Rightarrow |a|^2 + \left|\frac{2a}{\sqrt{2}}\right|^2 + |a|^2 = 1 \qquad 2\frac{|2a|^2}{2} + \frac{|2a|^2}{2} = 1$

$\frac{8a^2}{2} + \frac{4a^2}{2} = 1$

$\frac{12|a^2|^2}{12} = \frac{2}{12} = \boxed{\frac{1}{6}}$

for $\lambda = -1$ $\boxed{|c|^2 = \frac{1}{6}}$

$|b|^2 = 1 - \frac{1}{6} - \frac{1}{6} \Rightarrow \boxed{|b|^2 = \frac{2}{3}}$

$\underline{S_y}$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i & 0 \\ +i & 0 & -i \\ 0 & +i & 0 \end{pmatrix} \Rightarrow \frac{\hbar}{2}\begin{pmatrix} -\lambda & -i & 0 \\ i & -\lambda & -i \\ 0 & i & -\lambda \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \lambda\hbar\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

we get $\lambda = -1, 0, +1$

$\lambda = +1$

$$\frac{\cancel{\hbar}}{2}\begin{pmatrix} 0 & -i & 0 \\ +i & 0 & -i \\ & +i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = +\cancel{\hbar}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$= -ib = \sqrt{2}\,a$$
$$i(a-c) = \sqrt{2}\,b$$
$$ib = \sqrt{2}\,c$$

$$c_1 = -a_1\ , \ b_1 = -\sqrt{2}\,i a$$

$$|a|^2 + |-\sqrt{2}\,i a|^2 + |-a|^2 = 1$$

$$|a|^2 + 2|a|^2 + |a|^2 = 1$$

$$4|a|^2 = 1$$

$$\therefore \boxed{|a|^2 = \frac{1}{4}} \text{ and } \boxed{|c|^2 = \frac{1}{4}}$$

$$|b|^2 = 1 - |a|^2 - |c|^2 \Rightarrow \boxed{|b|^2 = \frac{1}{2}}$$

$\underline{S_z}$

$$\frac{\hbar}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1-\lambda \end{pmatrix}$$

Probability is 0.

7_7

TO NORMALIZE A VECTOR MEANS TO TAKE A MAGNITUDE OF THE SUM OF IT'S COMPONENTS AND DIVIDE EACH COMPONENT BY THE MAGNITUDES VALUE.