

$$(1) \quad \phi_{ij}^* = \left(w_j f_{ij} \left(\left(\frac{\theta}{\theta-1} w_i T_{ij} \right)^{1-\theta} \frac{1}{\theta} Y_j P_j^{\theta-1} \right)^{-1} \right)^{\frac{1}{\theta-1}}$$

$$(2) \quad Y_j = L_j w_j$$

$$(3) \quad P_j^{1-\theta} = \sum_{i \in S} N_i \left(\frac{\theta}{\theta-1} \right)^{1-\theta} w_i^{1-\theta} T_{ij}^{1-\theta} \int_{\phi_{ij}^*} \phi^{\theta-1} F(\phi) d\phi$$

$$(4) \quad w_i f_{ei} = \sum_j (\phi_{ij}^*)^{1-\theta} w_j f_{ij} \int_{\phi_{ij}^*} \phi^{\theta-1} F(\phi) d\phi - \sum_j \int_{\phi_{ij}^*} w_j f_{ij} F(\phi) d\phi$$

$$(5) \quad L_i = \sum_{j \in S} N_i (\theta-1) \frac{w_j}{w_i} (\phi_{ij}^*)^{1-\theta} F_{ij} \int_{\phi_{ij}^*} \phi^{\theta-1} F(\phi) d\phi \\ + \sum_{j \in S} N_j \int_{\phi_{ij}^*} F_{ji} F(\phi) d\phi \\ + N_i \frac{1}{1 - F(\min\{\phi_{ij}^*\})} F_{ei}$$

$$\int_{\phi_{ij}^*} \phi^{\theta-1} F(\phi) d\phi = \frac{\theta b_i^\theta}{\theta+1-\theta} (\phi_{ij}^*)^{(\theta-1)-\theta} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{given in class}$$

$$1 - F(\phi_{ij}^*) = b_i^\theta (\phi_{ij}^*)^{-\theta}$$

$$\int_{\phi_{ij}^*} F(\theta) d\theta = \int \theta b_i^\theta \phi^{-\theta-1} d\phi \\ = \left(\frac{b_i}{\phi_{ij}^*} \right)^\theta$$

$$\begin{aligned}
 (4) \quad w_i f_{ei} &= \sum_j (\phi_{ij}^*)^{1-\theta} w_j f_{ij} \frac{\theta b_i^\theta}{\theta+1-\theta} (\phi_{ij}^*)^{(\theta-1)-\theta} - \sum_j \int_{\phi_{ij}^*} w_j f_{ij} F(\theta) d\theta \\
 &= \sum_j w_j f_{ij} \left[(\phi_{ij}^*)^{-\theta} \frac{\theta b_i^\theta}{\theta+1-\theta} - \int_{\phi_{ij}^*} F(\theta) d\theta \right] \\
 &= \sum_j w_j f_{ij} \left[(\phi_{ij}^*)^{-\theta} \frac{\theta b_i^\theta}{\theta+1-\theta} - \left(\frac{b_i}{\phi_{ij}^*} \right)^\theta \right] \\
 &= \sum_j w_j f_{ij} \phi_{ij}^{\theta-\theta} b_i^\theta \left(\frac{\theta}{\theta+1-\theta} - 1 \right) \\
 &= \sum_j w_j f_{ij} \phi_{ij}^{\theta-\theta} b_i^\theta \left(\frac{\theta-1}{\theta+1-\theta} \right) \\
 f_{ei} &= \sum_j \frac{w_j}{w_i} f_{ij} \phi_{ij}^{\theta-\theta} b_i^\theta \left(\frac{\theta-1}{\theta+1-\theta} \right)
 \end{aligned}$$

$$\int_{\phi_{ij}^*} \phi^{\theta-1} F(\phi) d\phi = \frac{\theta b_i^\theta}{\theta+1-\theta} (\phi_{ij}^*)^{(\theta-1)-\theta}, \quad \int_{\phi_{ij}^*} F(\theta) d\theta = \left(\frac{b_i}{\phi_{ij}^*} \right)^\theta \Rightarrow$$

$$\begin{aligned}
 \theta_{ij}^{1-\theta} \int_{\phi_{ij}^*} F(\theta) d\theta &= \theta_{ij}^{1-\theta} \left(\frac{b_i}{\phi_{ij}^*} \right)^\theta = \frac{\theta+1-\theta}{\theta+1-\theta} \frac{\theta}{\theta} \theta_{ij}^{\theta-1} \left(\frac{b_i}{\phi_{ij}^*} \right)^\theta \\
 &= \frac{\theta+1-\theta}{\theta} \frac{\theta b_i^\theta}{\theta+1-\theta} (\phi_{ij}^*)^{(\theta-1)-\theta} \\
 &= \frac{\theta+1-\theta}{\theta} \int_{\phi_{ij}^*} \phi^{\theta-1} F(\phi) d\phi
 \end{aligned}$$

$$F_{e,i} = \sum_j \frac{w_j}{w_i} F_{ij} \phi_{ij}^{s-1} b_i^{\theta} \left(\frac{\theta-1}{\theta+1-s} \right) \quad P_j^{s-1} = \sum_{i \in S} N_i \left(\frac{\theta}{\theta-1} \right)^{s-1} w_i^{s-1} T_{ij}^{s-1} \int_{\phi_{ij}} \phi^{s-1} F(\phi) d\phi$$

$$F_{ij} = \phi_{ij}^{s-1} w_j^{-1} \left(\left(\frac{\theta}{\theta-1} w_i T_{ij} \right)^{s-1} \frac{1}{\theta} Y_j P_j^{s-1} \right)$$

$$\int_{\phi_{ij}} \phi^{s-1} F(\phi) d\phi = \frac{\theta+1-s}{\theta} \int_{\phi_{ij}} \phi^{s-1} F(\phi) d\phi$$

$$(5) L_i = \sum_{j \in S} N_j (\theta-1) \frac{w_j}{w_i} (\phi_{ij}^*)^{s-1} F_{ij} \int_{\phi_{ij}^*} \phi^{s-1} F(\phi) d\phi \\ + \sum_{j \in S} N_j \int_{\phi_{ij}^*} F_{ij} F(\phi) d\phi + N_i \frac{1}{1 - F(\min\{\phi_{ij}^*\})} F_{e,i}$$

$$= N_i \sum_{j \in S} (\theta-1) \frac{w_j}{w_i} (\phi_{ij}^*)^{s-1} F_{ij} \frac{\theta b_i^{\theta}}{\theta+1-s} (\phi_{ij}^*)^{(s-1)-s}$$

$$+ \sum_{j \in S} N_j \int_{\phi_{ij}^*} \phi_{ij}^{s-1} w_i^{-1} \left(\left(\frac{\theta}{\theta-1} w_i T_{ji} \right)^{s-1} \frac{1}{\theta} Y_i P_i^{s-1} \right) F(\phi) d\phi \\ + N_i \frac{1}{1 - F(\min\{\phi_{ij}^*\})} F_{e,i}$$

$$= N_i F_{e,i} \theta + N_i \frac{F_{e,i}}{1 - F(\min\{\phi_{ij}^*\})}$$

$$+ Y_i P_i^{s-1} w_i^{-1} \sum_{j \in S} N_j \left(\left(\frac{\theta}{\theta-1} w_i T_{ji} \right)^{s-1} \frac{1}{\theta} \right) \int_{\phi_{ij}^*} \phi_{ij}^{s-1} F(\phi) d\phi$$

$$= N_i F_{e,i} \left(\theta + \frac{1}{1 - F(\min\{\phi_{ij}^*\})} \right)$$

$$+ Y_i P_i^{s-1} w_i^{-1} \left(\frac{\theta+1-s}{\theta} \right) \sum_{j \in S} N_j \left(\frac{\theta}{\theta-1} w_i T_{ji} \right)^{s-1} \int_{\phi_{ij}^*} \phi^{s-1} F(\phi) d\phi$$

$$= N_i F_{e,i} \left(\theta + \frac{1}{1 - F(\min\{\phi_{ij}^*\})} \right) + Y_i w_i^{-1} \left(\frac{\theta+1-s}{\theta} \right) P_i^{s-1} P_i^{1-s}$$

$$= N_i \text{Fe}_i \left(\theta + \frac{1}{b_i^\theta (\min(\phi_{ij}))^\theta} \right) + L_i \left(\frac{\theta+1-\delta}{\delta\theta} \right)$$

\Rightarrow System is:

$$(5) \quad N_i = \frac{L_i \left(1 - \frac{\theta+1-\delta}{\delta\theta} \right)}{\text{Fe}_i \left(\theta + \frac{1}{b_i^\theta (\min(\phi_{ij}))^\theta} \right)}$$

$$(1) \quad \phi_{ij}^* = \left(\frac{\theta}{\delta-1} w_i T_{ij} \right) \left(w_j F_{ij} \left(\frac{1}{\delta} Y_j P_j^{\delta-1} \right)^{-1} \right)^{\frac{1}{\delta-1}}$$

$$(2) \quad Y_j = L_j w_j$$

$$(3) \quad P_j^{1-\delta} = \sum_{i \in S} N_i \left(\frac{\theta}{\delta-1} \right)^{1-\delta} w_i^{1-\delta} T_{ij}^{1-\delta} \frac{\theta b_i^\theta}{\theta+1-\delta} (\phi_{ij}^*)^{(\delta-1)-\theta}$$

$$(4) \quad w_i = b_i^\theta \left(\frac{\theta+1-\delta}{\theta+1-\delta} \right)^{\frac{1}{\delta}} \frac{1}{F_e} \sum_j w_j F_{ij} \phi_{ij}^{*\theta}$$

Code:

- i) Guess $P + w_j$
- ii) run (2) \rightarrow (1) \rightarrow (5)
- iii) update P_{ij}, w_i (3), (4)
- iv) repeat (ii), (iii) until convergence