2.23 Let $\mathcal{T}$ be the collection of subsets of $\mathbb{R}$ consisting of the empty set and every set whose complement is countable.

(a) Show that $\mathcal{T}$ is a topology on $\mathbb{R}$. (It is called the countable complement topology.)
(b) Show that the point 0 is a limit point of the set $A = \mathbb{R} - \{0\}$ in the countable complement topology.
(c) Show that in $A = \mathbb{R} - \{0\}$ there is no sequence converging to 0 in the countable complement topology.

2.26 Determine the boundary of each of the following subsets of $\mathbb{R}^2$ in the standard topology:
(a) $A = \{(x, 0) \in \mathbb{R}^2 | x \in \mathbb{R}\}$

(b) $B = \{(x, y) \in \mathbb{R}^2 | x > 0, y \neq 0\}$

(c) $C = \left\{\left(\frac{1}{n}, 0\right) \in \mathbb{R}^2 | n \in \mathbb{Z}_+\right\}$

(d) $D = \{(x, y) \in \mathbb{R}^2 | 0 \leq x^2 - y^2 < 1\}$

PG: 2.28 Prove Theorem 2.15 : Let A be a subset of a topological space X.
(a) ∂A is closed.

(b) $\partial A = Cl(A) \cap Cl(X - A)$

(c) $\partial A \cap \mathrm{Int}(A) = \varnothing$

(d) $\partial A \cup \mathrm{Int}(A) = Cl(A)$

(e) $\partial A \subset A$ if and only if A is closed.

(i) $\partial A \cap A = \varnothing$ if and only if A is open.

(g) $\partial A = \varnothing$ if and only if A is both open and closed.

PN: 3.01 Let $X = \{(x, 0) \in \mathbb{R}^2 | x \in \mathbb{R}\}$, the x -axis in the plane. Describe the topology that X inherits as a subspace of $\mathbb{R}^2$ with the standard topology.

PN: 3.02 Let $Y = [-1, 1]$ have the standard topology. Which of the following sets are open in Y and which are open in $\mathbb{R}$?

$$\begin{aligned} A &= (-1, -1/2) \cup (1/2, 1) \\ B &= (-1, -1/2] \cup [1/2, 1) \\ C &= [-1, -1/2) \cup (1/2, 1] \\ D &= [-1, -1/2] \cup [1/2, 1] \\ E &= \bigcup_{n=1}^{\infty} \left(\frac{1}{1+n}, \frac{1}{n}\right) \end{aligned}$$

PB: 3.03 Prove Theorem 3.4 : Let X be a topological space, and let $Y \subset X$ have the subspace topology. Then $C \subset Y$ is closed in Y if and only if $C = D \cap Y$ for some closed set D in X.

Proof. $\square$

PN: 3.15 Prove Theorem 3.9: Let X and Y be topological spaces, and assume that $A \subset X$ and $B \subset Y$. Then the topology on $A \times B$ as a subspace of the product $X \times Y$ is the same as the product topology on $A \times B$, where A has the subspace topology inherited from X, and B has the subspace topology inherited from Y.

Proof. $\square$

PB: 3.16 Let S^2 be the sphere, D be the disk, T be the torus, S^1 be the circle, and $I = [0, 1]$ with the standard topology. Draw pictures of the product spaces $S^2 \times I, T \times I, S^1 \times I \times I$, and $S^1 \times D$

PN: 3.18 Show that if X and Y are Hausdorff spaces, then so is the product space $X \times Y$.

Proof. $\square$

PB: 3.19 Show that if A is closed in X and B is closed in Y, then $A \times B$ is closed in $X \times Y$.

Proof. $\square$