# Section 3.1

## Def 3.1: Subspace Topology

DEFINITION 3.1. Let X be a topological space and let Y be a subset of $X$.
Define $\mathcal{T}_Y = \{U \cap Y | U \text{ is open in } X\}$. This is called the subspace topology on $Y$ and, with this topology, $Y$ is called a subspace of $X$. We say that $V \subset Y$ is open in $Y$ if $V$ is an open set in the subspace topology on $Y$.

## Def 3.2: Standard Topology on a subspace $Y$

DEFINITION 3.2. Let Y be a subset of $\mathbb{R}^n$. The standard topology on $Y$ is the topology that $Y$ inherits as a subspace of $\mathbb{R}^n$ with the standard topology.

## Def 3.3: Closed in a Subspace

DEFINITION 3.3. Let X be a topological space, and let $Y \subset X$ have the subspace topology. We say that a set $C \subset Y$ is closed in $Y$ if $C$ is closed in the subspace topology on $Y$

## Thm 3.4: Showing a set is closed in a subspace

THEOREM 3.4. Let X be a topological space, and let $Y \subset X$ have the subspace topology. Then $C \subset Y$ is closed in $Y$ if and only if $C = D \cap Y$ for some closed set $D$ in $X$

## Thm 3.5: Basis for subspace

THEOREM 3.5. Let X be a topological space and B be a basis for the topology on $X$. If $Y \subset X$, then the collection

$$B_Y = \{B \cap Y | B \in \mathcal{B}\}$$

is a basis for the subspace topology on $Y$

# Section 3.2

## Def 3.6: Product Topology

DEFINITION 3.6. Let $X$ and $Y$ be topological spaces and $X \times Y$ be their product. The product topology on $X \times Y$ is the topology generated by the basis

$$\mathcal{B} = \{U \times V | U \text{ is open in } X \text{ and } V \text{ is open in } Y\}$$

## Thm 3.7: Basis for product of two topologies

THEOREM 3.7. Let $X$ and $Y$ be topological spaces and $X \times Y$ be their product. Define

$$\mathcal{B} := \{U \times V | U \text{ is open in } X \text{ and } V \text{ is open in } Y\}$$

The collection $\mathcal{B}$ is a basis for a topology on $X \times Y$

### Thm 3.8: Products of bases form a basis

THEOREM 3.8. If $\mathcal{C}$ is a basis for $X$ and $\mathcal{D}$ is a basis for $Y$, then

$$\mathcal{E} = \{C \times D | C \in \mathcal{C} \text{ and } D \in \mathcal{D}\}$$

is a basis that generates the product topology on $X \times Y$.

### Thm 3.9: Subspace of a Product Topology

THEOREM 3.9. Let $X$ and $Y$ be topological spaces, and assume that $A \subset X$ and $B \subset Y$. Then the topology on $A \times B$ as a subspace of the product $X \times Y$ is the same as the product topology on $A \times B$, where A has the subspace topology inherited from $X$, and $B$ has the subspace topology inherited from $Y$.

### Thm 3.10: Interior of Product Topologies

THEOREM 3.10. Let $A$ and $B$ be subsets of topological spaces $X$ and $Y$, respectively. Then $\text{Int}(A \times B) = \text{Int}(A) \times \text{Int}(B)$

## Section 3.3

### Def 3.11: Quotient Topology, Quotient Map, and Quotient Space

DEFINITION 3.11. Let $X$ be a topological space and $A$ be a set (that is not necessarily a subset of $X$ ). Let $p : X \to A$ be a surjective map. Define a subset $U$ of $A$ to be open in $A$ if and only if $p^{-1}(U)$ is open in $X$. The resultant collection of open sets in $A$ is called the **quotient topology induced by** $p$, and the function $p$ is called a **quotient map**. The topological space $A$ is called a **quotient space.**

### Thm 3.12: Quotient Maps induce a Quotient Topology

THEOREM 3.12. Let $p : X \to A$ be a quotient map. The quotient topology on $A$ induced by $p$ is a topology.