4.4.1b Solve the linear congruence, $5x \equiv 2 \pmod{26}$

$$\begin{aligned} 5x - 26y &= 2 \\ 5(-10) - 26(-2) &= 2 \end{aligned}$$

Thus, $x_0 = -10$
$x = -10 - 26(0)$, as gcd $= 1$
Thus,

$$\begin{aligned} x &\equiv -10 \pmod{26} \\ &\equiv 16 \pmod{26} \end{aligned}$$

4.4.4c Solve the following sets of simultaneous congruences,
$x \equiv 5 \pmod{6}$,
$x \equiv 4 \pmod{11}$,
$x \equiv 3 \pmod{17}$
$n_k = 6 \cdot 11 \cdot 17 = 1122$

$$N_1 = 187, N_2 = 102, N_3 = 66$$

$$\begin{aligned} 187x &\equiv 1 \pmod{6} & 102x &\equiv 1 \pmod{11} & 66x &\equiv 1 \pmod{17} \\ 186x + 1x &\equiv 1 \pmod{6} & 99x + 3x &\equiv 1 \pmod{11} & 51x + 15x &\equiv 1 \pmod{17} \\ 1x &\equiv 1 \pmod{6} & 3x &\equiv 1 \pmod{11} & 15x &\equiv 1 \pmod{17} \\ x_1 &= 1 & x_2 &= 4 & x_3 &= 8 \end{aligned}$$

$$\begin{aligned} \bar{x} &= 187 \cdot 5 \cdot 1 + 102 \cdot 4 \cdot 4 + 66 \cdot 3 \cdot 8 \\ &= 935 + 1632 + 1584 \\ &= 4141 \end{aligned}$$

$$\begin{aligned} \bar{x} &\equiv 4151 \bmod(1122) \\ &\equiv 785 \bmod(1122) \end{aligned}$$

4.4.10 A band of 17 pirates stole a sack of gold coins. When they tried to divide the fortune into equal portions, 3 coins remained. In the ensuing brawl over who should get the extra coins, one pirate was killed. The wealth was redistributed, nut this time an equal division left 10 coins. Again an argument developed in which another pirate was killed. But now the total fortune was evenly distributed among the survivors. What was the least number of coins that could have been stolen? $n_k = 17 \cdot 16 \cdot 15 = 4080$

$$N_1 = 240, N_2 = 255, N_3 = 272$$

$$\begin{aligned} 240x &\equiv 1 \pmod{17} \\ 2x &\equiv 1 \pmod{17} \\ x &\equiv 9 \pmod{17} \\ x_1 &= 9 \end{aligned} \qquad \begin{aligned} 255x &\equiv 1 \pmod{16} \\ 15x &\equiv 1 \pmod{16} \\ x &\equiv 15 \pmod{16} \\ x_2 &= 15 \end{aligned} \qquad \begin{aligned} 272x &\equiv 1 \pmod{15} \\ 2x &\equiv 1 \pmod{15} \\ x &\equiv 8 \pmod{15} \\ x_3 &= 8 \end{aligned}$$

$$\begin{aligned} \bar{x} &= 240 \cdot 3 \cdot 9 + 255 \cdot 10 \cdot 15 + 272 \cdot 0 \cdot 8 \\ &= 6480 + 38250 + 0 \\ &= 44730 \end{aligned}$$

$$\begin{aligned} \bar{x} &\equiv 44730 \text{ mod}(4080) \\ &\equiv 3930 \text{ mod}(4080) \end{aligned}$$

3930 stolen coins!

4.4.13 If $x \equiv a \pmod{n}$, prove that either $x \equiv a \pmod{2n}$ or $x \equiv a + n$ (mode $2n$)

Proof. Let $x \equiv a \pmod{n}$ Then, $x = a + kn$ for $k \in \mathbb{Z}$

k is even Then, $k = 2l$ for $l \in \mathbb{Z}$

$$x = a + 2ln \Rightarrow x \equiv a \text{ (mod 2n)}$$

k is odd Then, $k = 2l + 1$ for $l \in \mathbb{Z}$

$$x = a + (2l + 1)n = a + n + l2n \Rightarrow x \equiv a + n \text{ (mod 2n)}$$

Therefore, if $x \equiv a \pmod{n}$, either $x \equiv a \pmod{2n}$ or $x \equiv a + n$ (mode $2n$) □

4.4.20a Find the solutions of the following system of congruences,
$5x + 3y \equiv 1 \pmod{7}$
$3x + 2y \equiv 4 \pmod{7}$
Solving for y.

$$15x + 9y = 3$$
$$15x + 10y = 20$$

$y \equiv 17 \pmod{7}$
$y \equiv 3 \pmod{7}$

Solving for x.

$$10x + 6y = 2$$
$$9x + 6y = 12$$

$x \equiv -10 \pmod{7}$
$x \equiv 4 \pmod{7}$
Thus $x = 4, y = 3$