

MTH 411, Fall 2018, Quiz 2 (Thursday Long Quiz/12 pts)

1. (12 points) Suppose that \star is an associative binary operation on a set X . Let us define

$$A := \{a \in X \mid a \star x = x \star a \text{ for every } x \in X\}.$$

Prove that A is closed under \star .

- **Solution.** To prove that A is closed under \star , we need to show that for every $a, b \in A$, we have $a \star b \in A$. So, let $a, b \in A$. We need to show that $a \star b \in A$, i.e. $(a \star b) \star x = x \star (a \star b)$ for every $x \in X$. So, let $x \in X$. Then

$$\begin{aligned} (a \star b) \star x &= a \star (b \star x) && \text{since } \star \text{ is associative} \\ &= a \star (x \star b) && \text{since } b \in A \\ &= (a \star x) \star b && \text{since } \star \text{ is associative} \\ &= (x \star a) \star b && \text{since } a \in A \\ &= x \star (a \star b) && \text{since } \star \text{ is associative} \end{aligned}$$

Thus, it follows that A is closed under \star .