7.04 List the elements of the subgroup generated by the given subset: $\{12, 30\}$ of $\mathbb{Z}_{12}$
As the gcd(12,30) = 6, notice all elements are multiples of the gcd. 0,6,12,18,24,30

7.05 List the elements of the subgroup generated by the given subset: $\{12, 42\}$ of $\mathbb{Z}$
$\cdots, -12-6, 0, 6, 12, \cdots$

7.06 List the elements of the subgroup generated by the given subset: $\{18, 24, 39\}$ of $\mathbb{Z}$
$\cdots, -6, -3, 0, 3, 6, \cdots$

7.07 Compute these products using Fig. 7.11(b).

(a) $(a^2b)a^3$. Just follow three arcs of a ending up at a^3b

(b) $(ab)(a^3b)$. Just follow three arcs of a and one arc of b ending up at a^2

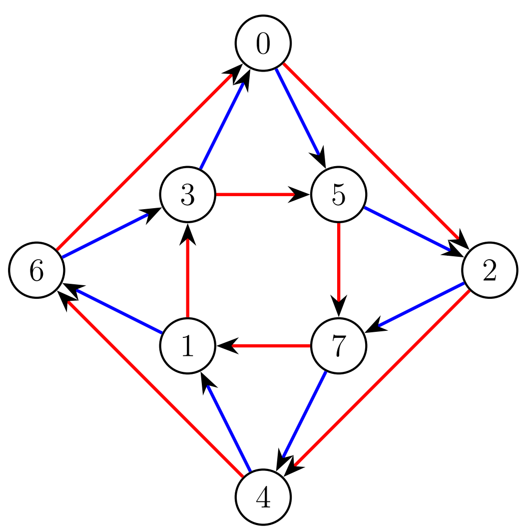
(c) $b(a^2b)$. Just follow two arcs of a and one arc of b ending up at a^2

7.10 Table for diagraph in Fig. 7.13(c)

	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	c	f	e	b	d
b	b	d	e	f	a	c
c	c	e	d	a	f	b
d	d	f	c	b	e	a
f	f	b	a	d	c	e

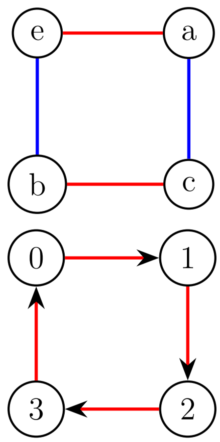
7.12 Determine whether or not the group corresponding to the Cayley diagraph in Fig. 7.11(b) is commutative.
Not commutative as a followed by b gives ab, while b followed by a gives a^3b

7.16 Draw a Cayley digraph for $\mathbb{Z}_8$ taking as generating set $S = \{2, 5\}$



Red = 2, Blue = 5

7.18 Draw digraphs for the two possible structurally different groups of order 4.



$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix} \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix} \quad \mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$$

8.02 Compute the indicated product: $\tau^2\sigma$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 6 & 3 \end{pmatrix}$$

8.04 Compute the indicated product: $\sigma^{-2}\tau$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 6 & 2 & 4 & 3 \end{pmatrix}$$

8.08 Compute the expression shown for σ^{100}

$$\sigma^{100} = (\sigma^6)^{16}\sigma^4 = \sigma^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 6 & 3 \end{pmatrix}$$

8.12 Find the orbit of 1 under the permutation defined earlier: τ

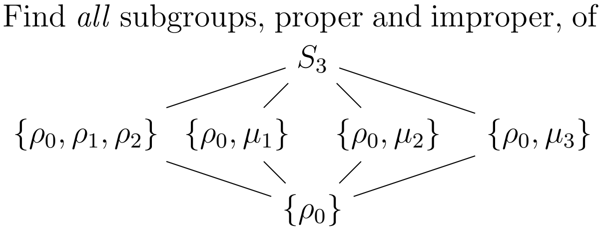
$$\{1, 2, 3, 4\}$$

8.18a Find the cyclic subgroups $\langle\rho_1\rangle, \langle\rho_2\rangle$, and $\langle\mu_1\rangle$ of S_3

$$\langle\rho_1\rangle = \langle\rho_2\rangle = \{\rho_0, \rho_1, \rho_2\}$$

$$\langle\mu_1\rangle = \{\rho_0, \mu_1\}$$

8.18b Find *all* subgroups, proper and improper, of S_3 and give the subgroup diagram for them.



8.20 Give the multiplication table for the cyclic subgroup of S_5 generated by

$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$$

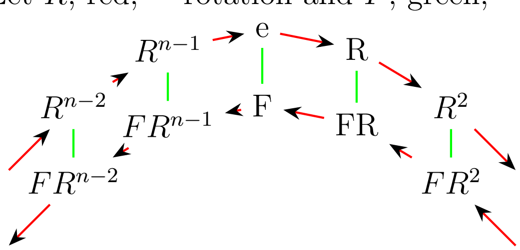
There will be six elements. Let them be $\rho, \rho^2, \rho^3, \rho^4, \rho^5$, and $\rho^0 = \rho^6$. Is this group isomorphic to S_3?
Not isomorphic, as the group is abelian, while S_3 is not abelian.

8.28 A *permutation* of a set S is a one-to-one map from S to S A *permutation* of a set S is a one-to-one map of S onto S.

8.29 The *left regular representation* of a group G is the map of G into S_G whose value at $g \in G$ is the permutation of G that carries each $x \in G$ into gx
Correct as stated.

8.36 Show by an example that every proper subgroup of a nonabelian group may be abelian.
S_3 is a good example. S_3 is nonabelian, yet all of its proper subgroups are abelian.

8.38 Indicate schematically a Cayley digraph for D_n using a generating set consisting of a rotation through $2\pi/n$ radians and a reflection.
Let R, red, = rotation and F, green, = flip.



8.46 Show that S_n is a nonabelian group for $n \geq 3$.
Let $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$. Then, by permutation multiplication we get $\tau\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and $\sigma\tau = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$
Thus, they do not commute
Thus, the group S_3 is nonabelian.
This hold true for all $n > 3$

8.48 Let $a, b \in A$ and $\sigma \in S_a$. Show that if $\mathcal{O}_{a,\sigma}$ and $\mathcal{O}_{b,\sigma}$ have an element in common, then $\mathcal{O}_{a,\sigma} = \mathcal{O}_{b,\sigma}$.
Let $c \in \mathcal{O}_{a,\sigma} \cap \mathcal{O}_{b,\sigma}$. Let $r, s \in \mathbb{Z}$ and $\sigma^r(a) = c$ and $\sigma^s(b) = c$. Then,

$$\sigma^{r-s}(a) = \sigma^{-s}(\sigma^r(a)) = \sigma^{-s}(c) = b$$

Then $\forall n \in \mathbb{Z}$, we have $\sigma^n(b) = \sigma^{n+r-s}(a)$. Thus, $\mathcal{O}_{a,\sigma} = \mathcal{O}_{b,\sigma}$

8.50 Show that for $\sigma \in S_a$, $\langle\sigma\rangle$ is transitive on A if and only if $\mathcal{O}_{a,\sigma} = A$ for some $a \in A$
Let $\langle\sigma\rangle$ be transitive on A and let $a \in A$. Then $\{\sigma^n(a) | n \in \mathbb{Z}\} = \mathcal{O}_{a,\sigma} = A$.

Going the other way, let $a \in A$ and assume $\mathcal{O}_{a,\sigma} = A$. Then $\{\sigma^n(a) | n \in \mathbb{Z}\} = A$. Let $b, c \in A$ with $b = \sigma^r(a)$and $c = \sigma^s(a)$. Then

$$\sigma^{s-r}(b) = \sigma^s(\sigma^{-r}(b)) = \sigma^s(a) = c$$

Thus, $\langle\sigma\rangle$ is transitive on A