MTH 411, Fall 2018, Quiz 1 (Thursday Long Quiz/12 pts)

1. **(6 points)** An integer c is *trodd* if there is an integer b such that $3b+1=c$. An integer c is *trodder* if there is an integer b such that $3b+2=c$. Prove that the sum of two trodd integers is trodder.

    - **Solution.** Suppose that m and n are trodd. Then, there are integers x and y such that $m=3x+1$ and $n=3y+1$. Let $z:=x+y$, then z is an integer as it is the sum of two integers. Then

    $$m+n=3x+1+3y+1=3(x+y)+2=3z+2,$$

    and so $m+n$ is trodder.

2. **(9 points)** Let $n\in\mathbb{Z}^+$ and let us define a relation $\sim$ on $\mathbb{Z}$ as follows: Given $a,b\in\mathbb{Z}$, $a\sim b$ if $a-b$ is divisible by n. In other words, $a\sim b$ if there exists $q\in\mathbb{Z}$ such that $a-b=qn$. Prove $\sim$ is an equivalence relation on $\mathbb{Z}$.

    - **Solution.** We need to prove that $\sim$ is an equivalence relation, i.e. that it is reflexive, symmetric and transitive. To show that $\sim$ is reflexive, let $x\in\mathbb{Z}$. Then $0\in\mathbb{Z}$ satisfies $x-x=0=0n$, and so $x\sim x$ and we are done. Next, to show that $\sim$ is symmetric, let $x,y\in\mathbb{Z}$. If $x\sim y$, then $x-y=qn$ for some $q\in\mathbb{Z}$. Then $-q\in\mathbb{Z}$ satisfies $y-x=(-q)n$ and so $y\sim x$ and we are done. Finally, to show that $\sim$ is transitive, let $x,y,z\in\mathbb{Z}$. If $x\sim y$ and $y\sim z$, then $x-y=sn$ and $y-z=tn$ for some $s,t\in\mathbb{Z}$. Then $s+t\in\mathbb{Z}$ satisfies $x-z=(x-y)+(y-z)=sn+tn=(s+t)n$ and so $x\sim z$ and we are done. Thus $\sim$ is an equivalence relation.