Determine weather the given map ϕ is a homomorphism.

13.07 Let $\phi_i : G_i \to G_1 \times G_2 \times \cdots \times G_i \times \cdots \times G_R$ be given by $\phi_i(g_i) = (e_1, e_2, \ldots, g_i, \ldots, e_R)$, where $g_i \in G_i$ and e_j is the identity element of G_j

13.08 Let G be any group and let $\phi : G \to G$ be given by $\phi(g) = g^{-1}$ for $g \in G$

13.12 Let M_n be the additive group of all $n \times n$ matrices with real entries, and let $\mathbb{R}$ be the additive group of real numbers. Let $\phi(A) = \det(A)$, the determinant of A, for $A \in M_n$

13.13 Let M_n and $\mathbb{R}$ be as in Exercise 12 . Let $\phi(A) = \mathrm{tr}(A)$ for $A \in M_n$, where the trace $\mathrm{tr}(A)$ is the sum of the elements on the main diagonal of A, from the upper-left to the lower-right corner.

13.14 Let $GL(n, \mathbb{R})$ be the multiplicative group of invertible $n \times n$ matrices, and let $\mathbb{R}$ be the additive group of real numbers. Let $\phi : GL(n, \mathbb{R}) \to \mathbb{R}$ be given by$\phi(A) = \mathrm{tr}(A)$, where tr (A) is defined in Exercise 13

13.30 A *homomorphism* is a map such that $\phi(xy) = \phi(x)\phi(y)$

13.31 Let $\phi : G \to G'$ be a homomorphism of groups. The kernel of ϕ is$\{x \in G | \phi(x) = e'\}$ where e' is the identity in G'

13.44 Let $\phi : G \to G'$ be a group homomorphism. Show that if $|G|$ is finite, then, $|\phi[G]|$ is finite and is a divisor of $|G|$

13.45 Let $\phi : G \to G'$ be a group homomorphism. Show that if $|G'|$ is finite, then, $|\phi[G]|$ is finite and is a divisor of $|G'|$

13.48 The sign of an even permutation is $+1$ and the sign of an odd permutation is -1. Observe that the map $\mathrm{sgn}_n : S_n \to \{1, -1\}$ defined by $\mathrm{sgn}_n(\sigma) =$ sign of σ is a homomorphism of S_n onto the multiplicative group$\{1, -1\}$
What is the Kernel?

13.50 Let $\phi : G \to H$ be a group homomorphism. Show that$\phi[G]$ is abelian if and only if for all$x, y \in G$, we have $xyx^{-1}y^{-1} \in \mathrm{Ker}(\phi)$

14.06

14.16

14.17

14.18

14.19

14.24

14.37

14.40