MTH 411, Fall 2018, Quiz 2 (Tuesday Short Quiz/6 pts)

1. **(1 points)** Fix $n \in \mathbb{Z}^+$. Define the equivalence relation on $\mathbb{Z}$ known as **congruence modulo** n.

   - **Solution.** Denote the equivalence relation by $\sim$. For $a, b \in \mathbb{Z}$, $a \sim b$ if $a - b$ is divisible by n.

2. **(1 points)** Fix $n \in \mathbb{Z}^+$. Define U_n. Next, write out all the elements that belong to U_3.

   - **Solution.** $U_n := \{z \in \mathbb{C} \ : \ z^n = 1\}$ and $U_3 = \{1, e^{i2\pi/3}, e^{i4\pi/3}\}$.
     Comments: Note that it can be proved[1] that

     $$U_n = \left\{ \cos\left(\frac{2\pi k}{n}\right) + i \sin\left(\frac{2\pi k}{n}\right) \ : \ k \in \{0, 1, 2, \ldots, n-1\} \right\}.$$

     By Euler's formula $\big(e^{ix} = \cos(x) + i\sin(x)\big)$, we can also write this as

     $$U_n = \left\{ e^{i\left(\frac{2\pi k}{n}\right)} \ : \ k \in \{0, 1, 2, \ldots, n-1\} \right\}.$$

3. **(1 points)** Fix $n \in \mathbb{Z}^+$. Define $\mathbb{Z}_n$. Next, write out all the elements that belong to $\mathbb{Z}_3$.

   - **Solution.** $\mathbb{Z}_n := \{0, 1, 2, \ldots, n-1\}$ and $\mathbb{Z}_3 := \{0, 1, 2\}$.

4. **(1 points)** Define a **binary operation** $\star$ on a set S.

   - **Solution.** A binary operation $\star$ on a set S is a map from $S \times S$ to S, i.e.

     $$\star : S \times S \to S$$

     $$(x, y) \mapsto x \star y$$

5. **(1 points)** Define what it means for a binary operation $\star$ on a set S to be **commutative**.

   - **Solution.** A binary operation $\star$ on a set S is commutative if for all $x, y \in S$ we have that

     $$x \star y = y \star x \ .$$

6. **(1 points)** Define what it means for a binary operation $\star$ on a set S to be **associatative**.

   - **Solution.** A binary operation $\star$ on a set S is associative if for all $x, y, z \in S$ we have that

     $$x \star (y \star z) = (x \star y) \star z \ .$$

[1] Fraleigh says on p.18: "Using the technique of Examples 1.6 and 1.7, we see that the elements of this set are ...". This is an invitation to the reader to prove this fact for themselves.