

**Exercise 1.** Prove that the summation identity

$$1 + 4 + 9 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad (1)$$

holds for all  $n \in \mathbb{Z}^+$ .

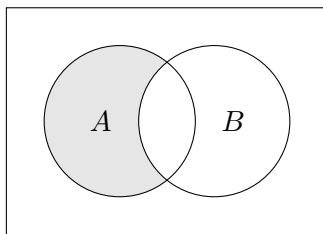
*Proof.* In the case of  $n = 1$ , we have the left hand side of equation (1) is 1, while the right hand side is  $\frac{1(2)(3)}{6} = 1$ . So equation (1) holds for  $n = 1$ .

Suppose equation (1) holds for some  $k \in \mathbb{Z}^+$ . We will now show that under this assumption, the identity holds for  $k + 1$ . We see that

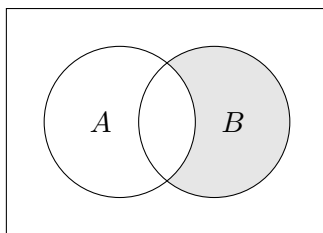
$$\begin{aligned} 1 + 4 + 9 + \cdots + (k+1)^2 &= 1 + 4 + 9 + \cdots + k^2 + (k+1)^2 \\ &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2, \text{ by assumption} \\ &= (k+1) \left[ \frac{k(2k+1)}{6} + (k+1) \right] \\ &= (k+1) \left[ \frac{k(2k+1) + 6(k+1)}{6} \right] \\ &= (k+1) \cdot \frac{2k^2 + 7k + 6}{6} \\ &= (k+1) \cdot \frac{(k+2)(2k+3)}{6} \\ &= \frac{(k+1)(k+2)(2(k+1)+1)}{6}. \end{aligned}$$

Therefore, by the Principle of Mathematical Induction, equation (1) holds for all  $n \in \mathbb{Z}^+$ . □

**Exercise 2.** Draw a Venn diagrams for  $A \setminus B$  and  $B \setminus A$ .



$A \setminus B$



$B \setminus A$

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TikZ Resources:

- Jacques Crémer, A very minimal introduction to TikZ:  
<https://cremeronline.com/LaTeX/minimaltikz.pdf>
- PGF and TikZ examples gallery:  
<http://www.texample.net/tikz/examples/>
- TeX - LaTeX Stack Exchange (covers all TeX related questions, not focused solely on TikZ):  
<https://tex.stackexchange.com>