

7.04 List the elements of the subgroup generated by the given subset: $\{12, 30\}$ of \mathbb{Z}_{12}
 As the $\gcd(12, 30) = 6$, notice all elements are multiples of the gcd. $0, 6, 12, 18, 24, 30$

7.05 List the elements of the subgroup generated by the given subset: $\{12, 42\}$ of \mathbb{Z}
 $\dots, -12 - 6, 0, 6, 12, \dots$

7.06 List the elements of the subgroup generated by the given subset: $\{18, 24, 39\}$ of \mathbb{Z}
 $\dots, -6, -3, 0, 3, 6, \dots$

7.07 Compute these products using Fig. 7.11(b).

(a) $(a^2b)a^3$. Just follow three arcs of a ending up at a^3b

(b) $(ab)(a^3b)$. Just follow three arcs of a and one arc of b ending up at a^2

(c) $b(a^2b)$. Just follow two arcs of a and one arc of b ending up at a^2

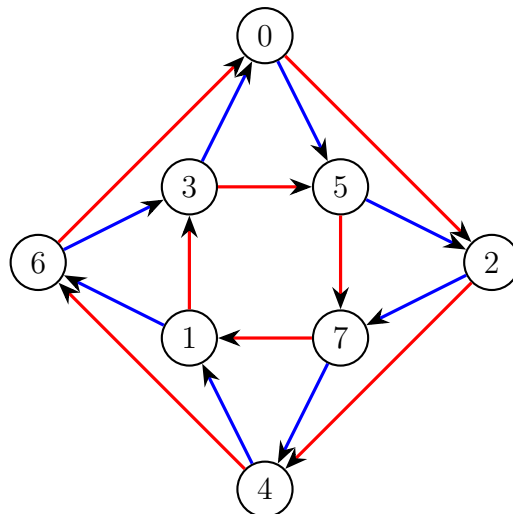
7.10 Table for diagraph in Fig. 7.13(c)

	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	c	f	e	b	d
b	b	d	e	f	a	c
c	c	e	d	a	f	b
d	d	f	c	b	e	a
f	f	b	a	d	c	e

7.12 Determine whether or not the group corresponding to the Cayley diagram in Fig. 7.11(b) is commutative.

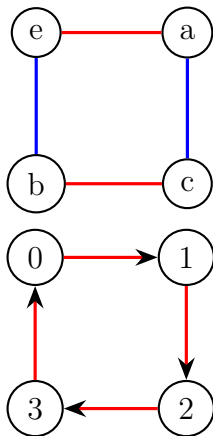
Not commutative as a followed by b gives ab , while b followed by a gives a^3b

7.16 Draw a Cayley digraph for \mathbb{Z}_8 taking as generating set $S = \{2, 5\}$



Red = 2, Blue = 5

7.18 Draw digraphs for the two possible structurally different groups of order 4.



$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix} \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix} \quad \mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$$

8.02 Compute the indicated product: $\tau^2\sigma$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 6 & 3 \end{pmatrix}$$

8.04 Compute the indicated product: $\sigma^{-2}\tau$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 6 & 2 & 4 & 3 \end{pmatrix}$$

8.08 Compute the expression shown for σ^{100}

$$\sigma^{100} = (\sigma^6)^{16}\sigma^4 = \sigma^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 6 & 3 \end{pmatrix}$$

8.12 Find the orbit of 1 under the permutation defined earlier: τ

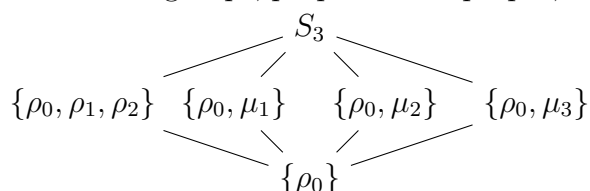
$$\{1, 2, 3, 4\}$$

8.18a Find the cyclic subgroups $\langle \rho_1 \rangle$, $\langle \rho_2 \rangle$, and $\langle \mu_1 \rangle$ of S_3

$$\langle \rho_1 \rangle = \langle \rho_2 \rangle = \{\rho_0, \rho_1, \rho_2\}$$

$$\langle \mu_1 \rangle = \{\rho_0, \mu_1\}$$

8.18b Find *all* subgroups, proper and improper, of S_3 and give the subgroup diagram for them.



8.20 Give the multiplication table for the cyclic subgroup of S_5 generated by

$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$$

There will be six elements. Let them be $\rho, \rho^2, \rho^3, \rho^4, \rho^5$, and $\rho^0 = \rho^6$. Is this group isomorphic to S_3 ?

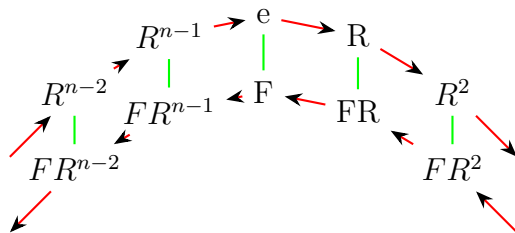
Not isomorphic, as the group is abelian, while S_3 is not abelian.

8.28 A *permutation* of a set S is a one-to-one map from S to S . A *permutation* of a set S is a one-to-one map of S onto S .

8.29 The *left regular representation* of a group G is the map of G into S_G whose value at $g \in G$ is the permutation of G that carries each $x \in G$ into gx .
Correct as stated.

8.36 Show by an example that every proper subgroup of a nonabelian group may be abelian. S_3 is a good example. S_3 is nonabelian, yet all of its proper subgroups are abelian.

8.38 Indicate schematically a Cayley digraph for D_n using a generating set consisting of a rotation through $2\pi/n$ radians and a reflection.
Let R , red, = rotation and F , green, = flip.



8.46 Show that S_n is a nonabelian group for $n \geq 3$.

8.48 Let $a, b \in A$ and $\sigma \in S_A$. Show that if $\mathcal{O}_{a,\sigma}$ and $\mathcal{O}_{b,\sigma}$ have an element in common, then $\mathcal{O}_{a,\sigma} = \mathcal{O}_{b,\sigma}$.

8.50 Show that for $\sigma \in S_A$, $\langle \sigma \rangle$ is transitive on A if and only if $\mathcal{O}_{a,\sigma} = A$ for some $a \in A$.