MTH 411, Fall 2018, Quiz 5 (Tuesday Short Quiz/6 pts)

1. **(2 points)** Given a group G and an element $x \in G$, define the **cyclic subgroup generated by** x.

   - **Solution.** The cyclic subgroup generated by x is defined as $\langle x \rangle := \{x^n \mid n \in \mathbb{Z}\}$.

2. **(2 points)** Define the **order of an element** x in a group G.

   - **Solution.** Given $x \in G$, either $\langle x \rangle$ has finite or infinite order. In the first case, the order of x is defined to be the order of $\langle x \rangle$; and in the second case, x is said to be of infinite order.

     My handout on subgroups of finite cyclic groups has an equivalent characterization:

     > Given $x \in G$, the **order of** x, denoted by $\mathrm{ord}(x)$, is the smallest positive integer such that $a^{\mathrm{ord}(x)} = e$. If no such positive integer exists, we say x is of **infinite order**.

3. **(2 points)** State the **left cancellation law** in a group G.

   - **Solution.** Let G be a group. For all $a, b, c \in G$ we have that $ab = ac \Rightarrow b = c$.

4. **(Bonus throwback to SQ2)** Fix $n \in \mathbb{Z}^+$. Define U_n (the set whose elements are the $\mathbf{n}^{th}$ **roots of unity**). Next, explicitly describe its elements.

   - **Solution.** We define $U_n := \{z \in \mathbb{C} \ : \ z^n = 1\}$. It can be proved[1] that

$$U_n = \left\{ \cos\left(\frac{2\pi k}{n}\right) + i \sin\left(\frac{2\pi k}{n}\right) \ : \ k \in \{0, 1, 2, \ldots, n-1\} \right\}$$

[1] Fraleigh says on p.18: "Using the technique of Examples 1.6 and 1.7, we see that the elements of this set are . . . ". This is an invitation to the reader to prove this fact for themselves.