MTH 411, Fall 2018, Quiz 1 (Tuesday Short Quiz/6 pts) *sample solutions*

1. **(2 points)** Given a map between sets $f : A \to B$, define what it means for f to be **injective**, and also what it means for f to be **surjective**.

f is injective if $\forall\, x, y \in A \quad x \neq y \Rightarrow f(x) \neq f(y)$.

f is surjective if $\forall\, z \in B \quad \exists\, x \in A$ such that $f(x) = z$.

2. **(2 points)** Define the set $\mathbb{R}_{2\pi}$. Next, define the operation $+_{2\pi}$ called **addition modulo** 2π on the set $\mathbb{R}_{2\pi}$.

$\mathbb{R}_{2\pi} := [0, 2\pi)$

Given $a, b \in \mathbb{R}_{2\pi}$ we define $a +_{2\pi} b := \begin{cases} a+b & \text{if } a+b < 2\pi \\ a+b-2\pi & \text{if } a+b \geq 2\pi \end{cases}$

3. **(2 points)** Define an **equivalence relation** $\mathcal{R}$ on a set S.

An equivalence relation $\mathcal{R}$ on a set S is a subset $\mathcal{R}$ of $S \times S$ that satisfies the following properties for all $x, y, z \in S$:-

1. $(x,x) \in \mathcal{R}$
2. $(x,y) \in \mathcal{R} \Rightarrow (y,x) \in \mathcal{R}$
3. $(x,y) \in \mathcal{R}$ and $(y,z) \in \mathcal{R} \Rightarrow (x,z) \in \mathcal{R}$

[Elts $(a,b) \in \mathcal{R}$ can also be written as $a \mathcal{R} b$.]

1. $x \mathcal{R} x$
2. $x \mathcal{R} y \Rightarrow y \mathcal{R} x$
3. $x \mathcal{R} y$ and $y \mathcal{R} z \Rightarrow x \mathcal{R} z$