2.14 For each $n \in \mathbb{Z}_+$, let $B_n = \{n, n+1, n+2, \ldots\}$, and consider the collection $\mathcal{B} = \{B_n | n \in \mathbb{Z}_+\}$

(a) Show that $\mathcal{B}$ is a basis for a topology on $\mathbb{Z}_+$

(b) Show that the topology on X generated by $\mathcal{B}$ is not Hausdorff.

(c) Show that the sequence $(2, 4, 6, 8, \ldots)$ converges to every point in $\mathbb{Z}_+$ with the topology generated by $\mathcal{B}$

(d) Prove that every injective sequence converges to every point in $\mathbb{Z}_+$ with the topology generated by $\mathcal{B}$

2.15 Determine the set of limit points of $[0, 1]$ in the finite complement topology on $\mathbb{R}$

2.17 (a) Let $\mathcal{B} = \{[a, b) \subset \mathbb{R} | a, b \in \mathbb{Q} \text{ and } a < b\}$. Show that $\mathcal{B}$ is a basis for a topology on $\mathbb{R}$. The resulting topology is called the rational lower limit topology and is denoted $\mathbb{R}_{rl}$

(b) Determine the closures of $A = (0, \sqrt{2})$ and $B = (\sqrt{2}, 3)$ in $\mathbb{R}_l$ and in $\mathbb{R}_{rl}$

2.21 Determine the set of limit points of the set

$$S = \left\{ \left(x, \sin\left(\frac{1}{x}\right) \right) \in \mathbb{R}^2 | 0 < x \leq 1 \right\}$$

as a subset of $\mathbb{R}^2$ in the standard topology. (The closure of S in the plane is known as the topologist's sine curve.

2.27 Determine $\partial([0, 1])$ in $\mathbb{R}$ with the finite complement topology. Justify your result.

2.28 Prove Theorem 2.15 : Let A be a subset of a topological space X.

(a) ∂A is closed.

(b) $\partial A = \text{Cl}(A) \cap \text{Cl}(X - A)$

(c) $\partial A \cap \text{In}\, t(A) = \varnothing$

(d) $\partial A \cup \text{Int}(A) = \text{Cl}(A)$

(e) $\partial A \subset A$ if and only if A is closed.

(f) $\partial A \cap A = \varnothing$ if and only if A is open.

(g) $\partial A = \varnothing$ if and only if A is both open and closed.