3.01 Let $X = \{(x, 0) \in \mathbb{R}^2 | x \in \mathbb{R}\}$ , the $x$ -axis in the plane. Describe the topology that $X$ inherits as a subspace of $\mathbb{R}^2$ with the standard topology.

3.02 Let $Y = [-1, 1]$ have the standard topology. Which of the following sets are open in $Y$ and which are open in $\mathbb{R}$?

$$A = (-1, -1/2) \cup (1/2, 1)$$
$$B = (-1, -1/2] \cup [1/2, 1)$$
$$C = [-1, -1/2) \cup (1/2, 1]$$
$$D = [-1, -1/2] \cup [1/2, 1]$$
$$E = \bigcup_{n=1}^{\infty} \left( \frac{1}{1+n}, \frac{1}{n} \right)$$

3.03 **Prove Theorem** 3.4 : Let $X$ be a topological space, and let $Y \subset X$ have the subspace topology. Then $C \subset Y$ is closed in $Y$ if and only if $C = D \cap Y$ for some closed set $D$ in $X$

3.07 Let $X$ be a Hausdorff topological space, and $Y$ be a subset of $X$. Prove that the subspace topology on $Y$ is Hausdorff.

3.08 Let $X$ be a topological space, and let $Y \subset X$ have the subspace topology.

(a) If $A$ is open in $Y$, and $Y$ is open in $X$, show that $A$ is open in $X$.

(b) If $A$ is closed in $Y$, and $Y$ is closed in $X$, show that $A$ is closed in $X$.

3.15 **Prove Theorem** 3.9 : Let $X$ and $Y$ be topological spaces, and assume that $A \subset X$ and $B \subset Y$. Then the topology on $A \times B$ as a subspace of the product $X \times Y$ is the same as the product topology on $A \times B$, where $A$ has the subspace topology inherited from $X$, and $B$ has the subspace topology inherited from $Y$.

3.16 Let $S^2$ be the sphere, $D$ be the disk, $T$ be the torus, $S^1$ be the circle, and $I = [0, 1]$ with the standard topology. Draw pictures of the product spaces $S^2 \times I, T \times I, S^1 \times I \times I$, and $S^1 \times D$

3.18 Show that if $X$ and $Y$ are Hausdorff spaces, then so is the product space $X \times Y$

3.19 Show that if $A$ is closed in $X$ and $B$ is closed in $Y$, then $A \times B$ is closed in $X \times Y$.

3.20 Show that if $A \subset X$ and $B \subset Y$, then $\mathrm{Cl}(A \times B) = Cl(A) \times Cl(B)$

3.23 If $\mathbb{R}$ has the standard topology, define

$$p : \mathbb{R} \to \{a, b, c, d, e\} \text{ by } p(x) = \begin{cases} a \text{ if } x > 2 \\ b \text{ if } x > 2 \\ b \text{ if } x = 2 \\ d \text{ if } 0 \leq x < 2 \\ d \text{ if } -1 < x < 0 \\ e \text{ if } x \leq -1 \end{cases}$$

(a) List the open sets in the quotient topology on $\{a, b, c, d, e\}$

(b) Now assume that $\mathbb{R}$ has the lower limit topology. What are the open sets in the resulting quotient topology on $\{a, b, c, d, e\}$?

3.24 Let $X = \mathbb{R}$ in the standard topology. Take the partition

$$X^* = \{\dots, (-1, 0], (0, 1], (1, 2], \dots\}$$

Describe the open sets in the resulting quotient topology on $X^*$ .

3.25 Define a partition of $X = \mathbb{R}^2 - \{O\}$ by taking each ray emanating from the origin as an element in the partition. (See Figure 3.25. Which topological space that we have previously encountered appears to be topologically equivalent to the quotient space that results from this partition?

3.27 Provide an example showing that a quotient space of a Hausdorff space need not be a Hausdorff space.

3.29 Consider the equivalence relation on $\mathbb{R}^2$ defined by $(x_1, x_2) \sim (w_1, w_2)$ if $x_1 + x_2 = w_1 + w_2$. Describe the quotient space that results from the partition of $\mathbb{R}^2$ into the equivalence classes in this equivalence relation.

3.30 Consider the equivalence relation on $\mathbb{R}^2$ defined by $(x_1, x_2) \sim (w_1, w_2)$ if $x_1^2 + x_2^2 = w_1^2 + w_2^2$. Describe the quotient space that results from the partition of $\mathbb{R}^2$ into the equivalence classes in this equivalence relation.

3.35 On a sketch of the surface $T\#T$ , illustrate where the glued edges of the octagon in Figure 3.33 appear.

3.36 (a) Show that a hexagon with opposite edges glued together straight across yields a torus.

(b) Show that a hexagon with opposite edges glued together with a flip yields a projective plane.

3.38 Show that the quotient space in Example 3.27 is topologically equivalent to $S^1 \times P$, the product of a circle and a projective plane.