9.01 Show that the identity map from the disk D in the plane to itself is homotopic to the map that takes D to the origin.

9.02 Show that if $f_1, f_2 : X \to Y$ are homotopic and $g_1, g_2 : Y \to Z$ are homotopic, then $g_2 \circ f_2$ is homotopic to $g_1 \circ f_1$.

9.03 Let f and g be paths in $\mathbb{R}$. Show that f is homotopic to g. .

9.04 Let f and g be paths in $\mathbb{R}^2 - \{O\}$. Show that f is homotopic to g . (Hint: Show that every path is homotopic to the constant path that sends the entire interval to the path's starting point. Then show that two constant paths are homotopic using the fact that $\mathbb{R}^2 - \{O\}$ is path connected.)

9.06 Show that if X is a topological space, and D is the disk in the plane, then there is only one homotopy class of continuous functions from X to D .

9.07 Consider the following definition:

DEFINITION 9.4. A topological space X is said to be **contractible** if the identity function on X is homotopic to a constant function.

(a) Prove that contractibility is a topological invariant. That is, prove that if X and Y are homeomorphic, then X is contractible if and only if Y is contractible.

(b) Prove that $\mathbb{R}^n$ is contractible.

(c) Let X be a contractible space. Prove that X is path connected.

9.09 Determine the degree of each of the following circle functions:

(a) The antipodal map, $A : S^1 \to S^1, A(\theta) = \theta + \pi$, that maps each point on the circle to the point opposite it through the center.

(b) A function that wraps the circle around itself once in one direction and then back around once in the other direction,

$$f : S^1 \to S^1, f(\theta) = \begin{cases} 2\theta & \text{for } \theta \in [0, \pi] \\ -2\theta & \text{for } \theta \in [\pi, 2\pi] \end{cases}$$

9.11 Prove Theorem 9.10: If a circle function $f : S^1 \to S^1$ has a nonzero degree, then f is surjective. (Hint: Prove the contrapositive.)

9.12 Let X be a connected Hausdorff space and B be a two-point subset of X. Prove that B is not a retract of X.

9.13 Let X be a Hausdorff space and A be a retract of X. Prove that A is a closed subset of X.

Summary