Modeling an Optical Cloak Using Metamaterials

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ABSTRACT

This project aims to model a system for an optical cloaking device using metamaterials. The system should be able to reflect a wave that is equal in magnitude but offset in phases from the transmitted waves so that the two waves destructively interfere and thus the object becomes indetectable. This project first explains the derivation of the equations used in the experiment and then shows the results of a model done in Matlab, which prove the concepts derived in the analysis.

Introduction

Stealth technology is a technology which could have many different applications as the idea of being undetectable is a sought after one for many different fields. Current modes of stealth technology aim to disrupt waves from being received by a receiver which deciphers where the object is. This is normally done by causing waves to be scattered in a way such that the receiver cannot have enough waves reflected towards it to decipher the location of the object, however with this technology, the waves have to be scattered somewhere and there will always be some location from which the object can still be seen.

A theoretical solution to this current mode of stealth technology is the use of cloaking technology. The idea for cloaking is that instead of scattering the waves to ensure a receiver is unable to detect reflected waves, a material is used to send out a reflected wave which destructively interferes with the transmitted wave. This destructive interference then cancels out the transmitted wave and it is as if the wave never hits an object which causes reflectance in the first place.

There have been different methods conjectured and tested for the use of optical cloaking, but this particular project evaluates the use of a metamaterial cloak. This metamaterial would be a successful cloak if it first could have a reflective coefficient that would give a reflected wave with equal magnitude of the one transmitted and also if it could shift the phase of the reflected wave so that it could be completely out of phase with the one transmitted. If these two criteria are met, a reflected wave could successfully destructively interfere with the transmitted wave and effectively cloak the object. This project aims to model a system that can successfully follow those two criteria.

Analysis

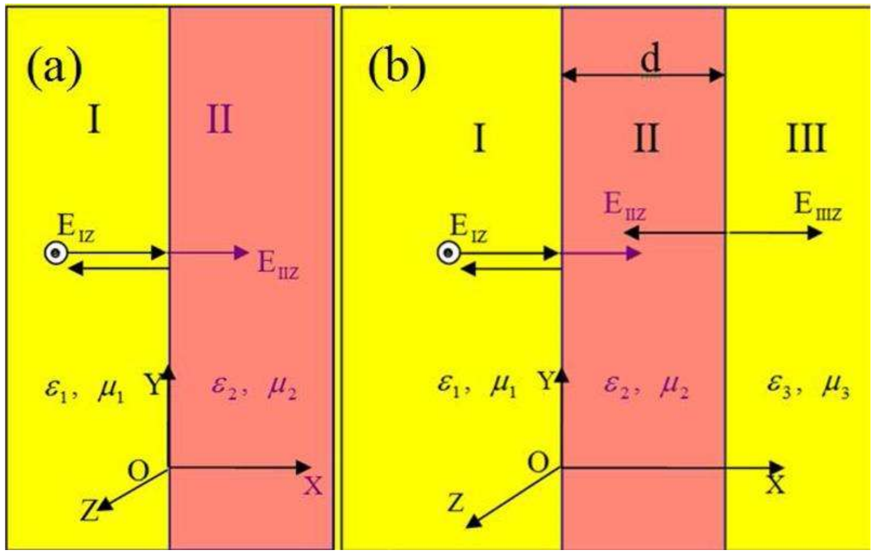

Figure 1. Electromagnetic propagation of a wave through (a) two semi-infinite media I & II and (b) sandwich media I, II, & III

This project focused on the example shown in Figure 1.b, the sandwich media. This media has permittivity ε_1, ε_2, and ε_3, and permeability μ_1, μ_2, μ_3 in the three respective regions. With this being a transverse electromagnetic wave, the equations for the electric fields in each region can be seen as follows.

$$E_{IZ} = e^{ikx} + R_1 e^{-ikx} \quad (Eq. 1)$$

$$E_{IIZ} = T_1 e^{ikn_1 x} + R_2 e^{-ikn_1 x} \quad (Eq. 2)$$

$$E_{IIZ} = T_2 e^{ikn_2 x} \quad (Eq. 3)$$

These equations also give that the derivatives of these functions for the electric field are as follows.

$$E'_{IZ} = ike^{ikx} - ikR_1 e^{-ikx} \quad (Eq. 4)$$

$$E'_{IIZ} = ikn_1 T_1 e^{ikn_1 x} - ikn_1 R_2 e^{ikn_1 x} \quad (Eq. 5)$$

$$E'_{IIZ} = ikn_2 T_2 e^{ikn_2 x} \quad (Eq. 6)$$

The constants T_1, R_1, and R_2 are coefficients that can be solved for by applying four boundary conditions. These four boundary conditions are that at $x = 0$, $E_{IZ} = E_{IIZ}$ and $E'_{IZ} = E'_{IIZ}$, and at $x = d$, $E_{IIZ} = E_{IIIZ}$ and $E'_{IIZ} = E'_{IIZ}$. Applying the first of these boundary conditions at $x = 0$, gives two equations:

$$1 + R_1 = T_1 + R_2 \quad (Eq. 7)$$

$$T_1 - R_2 = \frac{1 - R_1}{n_1} \quad (Eq. 8)$$

Adding these two equations together will give an equation for T_1 in terms of R_1, which is known to be -1, and n_1, which is also a known quantity as follows.

$$T_1 = \frac{1 + R_1 + \frac{1 - R_1}{n_1}}{2} \quad (Eq. 9)$$

The second boundary condition at x = d, gives two more equations as follows.

$$T_2 e^{ikLn_2} = T_1 e^{ikn_1L} + R_2 e^{-ikn_1L} \quad (Eq. 10)$$

$$T_2 e^{ikLn_2} = n_1\left(T_1 e^{ikn_1L} - R_2 e^{-ikn_1L}\right) \quad (Eq. 11)$$

Subtracting Eq. 11 from Eq. 10 gives us an equation for R_2 in terms of T_1, and n_2.

$$R_2 = \frac{T_1[e^{ikdn_2}(1 - n_1)]}{[(1 + n_1)e^{-ikn_2L}]} \quad (Eq. 12)$$

Finally, rearranging Eq. 10 can give us an equation to find T_2 as follows.

$$T_2 = \frac{T_1 e^{ikL} + R_2 e^{-ikL}}{e^{ikL}} \quad (Eq. 13)$$

This project has a second part to it beyond the coefficients for the electric fields and that has to do with a phase shifter. In order for the cloaking to occur, the aim is for the transmitted wave and the reflected wave to be equal but opposite. In other words, the magnitudes need to be equal, but the waves need to be completely out of phase so as to cause destructive interference among the waves and cancel one another out. The coefficients ensure the magnitudes of the waves are equal, and the phases are controlled by a phase shifter.

In order to determine the phase shifter, it is necessary to understand the relationship between the reflection coefficient and the reflected wave. The equation for the reflection coefficient is given as follows based on the boundary conditions.

$$r_{TE} = \frac{r_1 + r_2 e^{i2k_{2x}d}}{1 + r_1 r_2 e^{i2k_{2x}d}} = |r|e^{i\emptyset_{TE}} \quad (Eq. 14)$$

Because the reflected wave must be equal but opposite, $e^{i\emptyset_{TE}}$ must be equal to -1. Because of Euler's Identity, $e^{i\pi} + 1 = 0$, we can say that $\emptyset_{TE}$ must therefore be equal to pi. In order to attain a value for the phase shifter, the use of the permitivities and permeabilities must also be used in Eq. 14. These relations that will be used are all listed as follows.

$$r_1 = (\mu_2 k_{1x} - \mu_1 k_{2x})/(\mu_2 k_{1x} + \mu_1 k_{2x} \quad (Eq. 15)$$

$$r_2 = (\mu_3 k_{2x} - \mu_2 k_{3x})/(\mu_3 k_{2x} + \mu_2 k_{3x} \quad (Eq. 16)$$

$$\mu_1 = \mu_2 = \mu_3 = 1 \quad (Eq. 17)$$

Lastly, before deriving the equation for the phase shifter, it must be known that, $k_{1x} = k_0\sqrt{\varepsilon_1}$, $k_{2x} = k_0\sqrt{\varepsilon_2}$, $k_{3x} = k_0\sqrt{\varepsilon_3}$, and $\varepsilon_3 \to \infty$. Using these relations and the relationship between the phase shifter and the reflection coefficient, we attain the following equation for the phase shifter.

$$\tan\varphi_{TE} = \frac{(\sqrt{\varepsilon_2}\sin(2k_0\sqrt{\varepsilon_2}d)}{[\varepsilon_2\cos^2(k_0\sqrt{\varepsilon_2}d) - \sin^2(k_0\sqrt{\varepsilon_2}d)]} \quad (Eq.18)$$

Results & Conclusions

In order to test the derivations done in the analysis section, the derived equations were plotted in Matlab. First the electric fields in each of the regions are plotted with given initial conditions as follows.

$$n_1 = 3.7514, n_2 = 0.225$$

$$k = \frac{0.1\ x\ 10^9}{7.30\ x\ 10^{-9}}, k_0 = \frac{\omega}{2.99\ x\ 10^8}, k_2 = k_0\sqrt{11.68}, k_3 = k_0\sqrt{10^{11}}$$

$$d = 7.0\ x\ 10^{-7}, i = \sqrt{-1}, \omega = 2\pi\ x\ 10^8$$

This gave us three models for electric fields in the three differing regions.

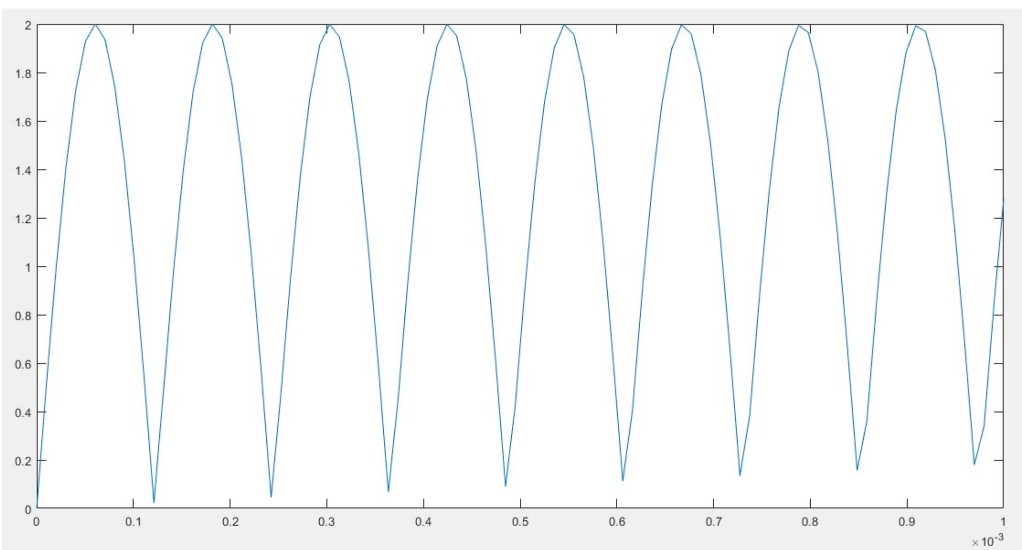

Figure 2. The electric field in Region I as a function of x

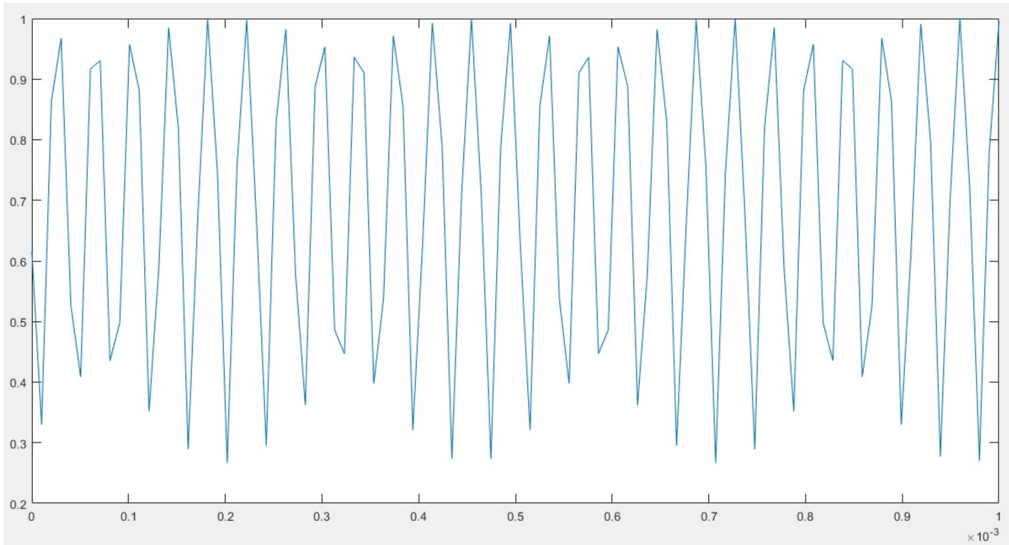

Figure 3. The electric field in Region II as a function of x

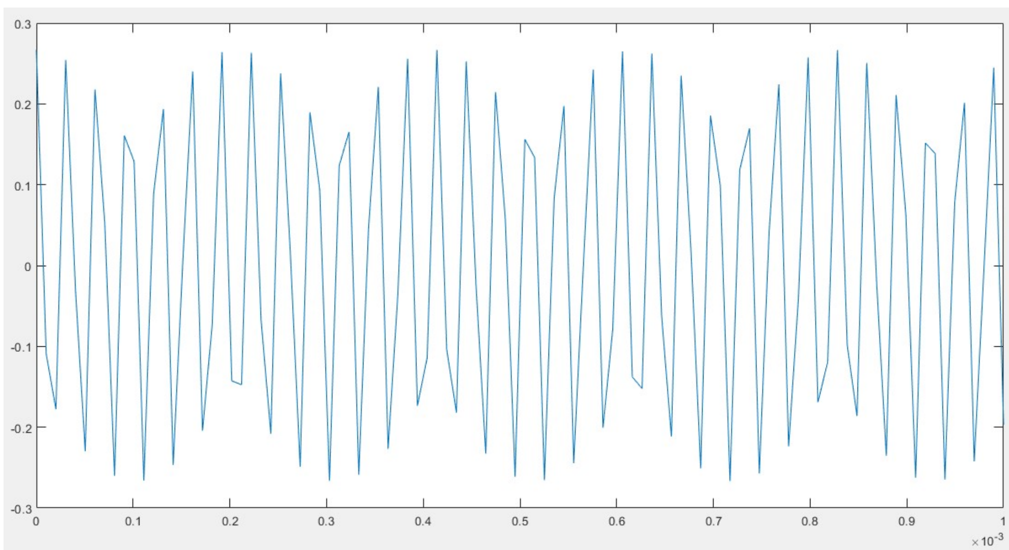

Figure 4. The electric field in Region III as a function of x

Graphing the electric fields in the three different regions is helpful in envisioning the fact that there would be three different waves propagating through the three different media, but it does not actually show the success of the optical cloak that has been derived in the analysis of this paper. In order to this, a graph of the transmitted wave and the reflected wave must be looked at. This can be seen in Figure 5.

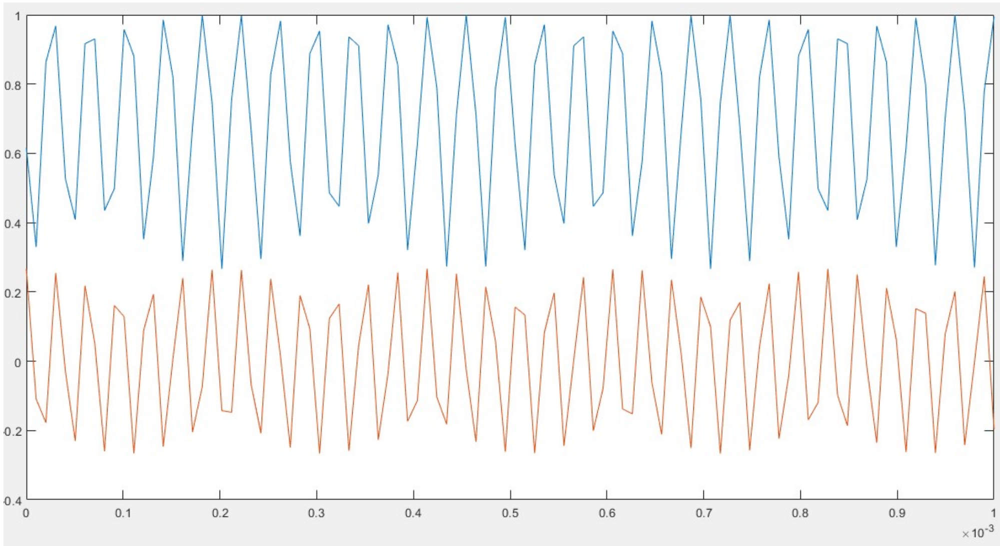

Figure 5 The transmitted wave (in blue) and the reflected wave (in red)

Figure 5 shows the deconstructive interference present between the transmitted and reflected wave. In order for this demonstration to be successful, the troughs of the blue graph must match up with the peaks of the red graph in order for destructive interference to occur, and this can be seen in the graph above.

The other graph to be looked at is that of the phase shifter. In order to prove the analysis of the phase shifter done, it would seem that the phase shift angle would have to be a completely linear function in order to satisfy the criteria done in the analysis.

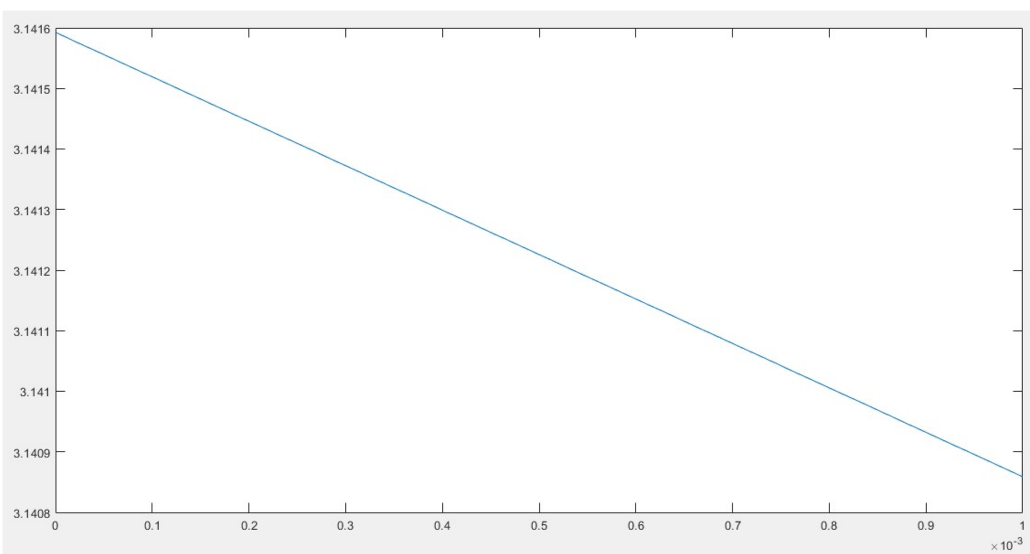

Figure 6 Value of the phase shifter as a function of x

Figure 6 shows that, as expected, changes in steps linearly. This proves the derivation for the phase shifter done in the analysis section of this paper.