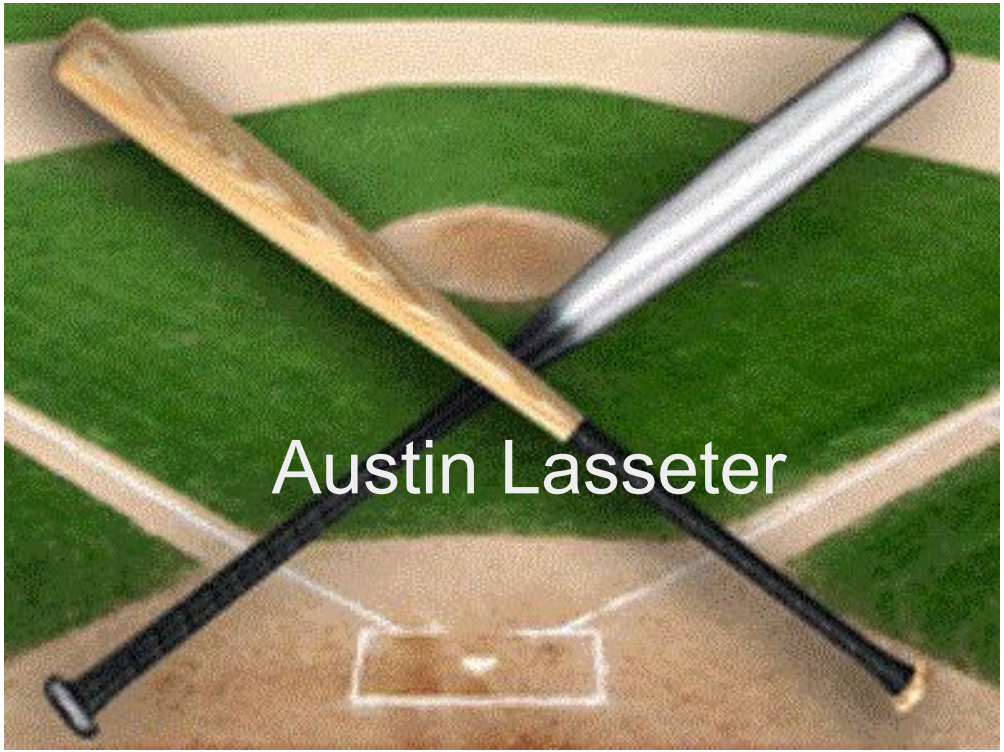
Hypothesis Testing



Five steps to hypothesis testing

1. Construct your hypothesis.
2. Specify a level of significance.
3. Calculate your point estimate.
4. Calculate your test statistic.
5. Find your p-value and make a conclusion.

Aluminum vs. Wooden Bats

Brandon Patch, a young pitcher in Montana, died in 2003 after a ball hit off an aluminum bat hit him in the head.

His parents sued the manufacturer (Louisville Slugger) for $850K.

How would their lawyer make the case against metal bats?

Crisco-Greenwald Batting Cage Study

bat	ω_{knob} rad/s	v_f m/s [mph]	v_f^* m/s [mph]	e_A	e	f_{hoop} Hz
W	43.1(2)	40.9(3) [91.4(7)]	42.7(2) [95.6(5)]	0.193(4)	0.452(5)	—
M1	44.8(4)	42.3(4) [94.6(8)]	42.6(2) [95.4(5)]	0.208(6)	0.494(6)	2334
M2	45.7(5)	45.4(4) [101.5(9)]	43.2(4) [96.7(8)]	0.233(4)	0.545(6)	1720
M3	46.1(4)	43.3(7) [96.8(15)]	42.6(2) [95.4(5)]	0.204(9)	0.515(11)	1908
M4	46.4(7)	43.9(5) [98.3(11)]	42.4(2) [94.9(5)]	0.221(11)	0.531(9)	1848
M5	44.4(4)	43.0(3) [96.1(7)]	42.6(2) [95.4(5)]	0.197(4)	0.505(5)	2233

http://baseball.physics.illinois.edu/ComparativeBatStudy.pdf

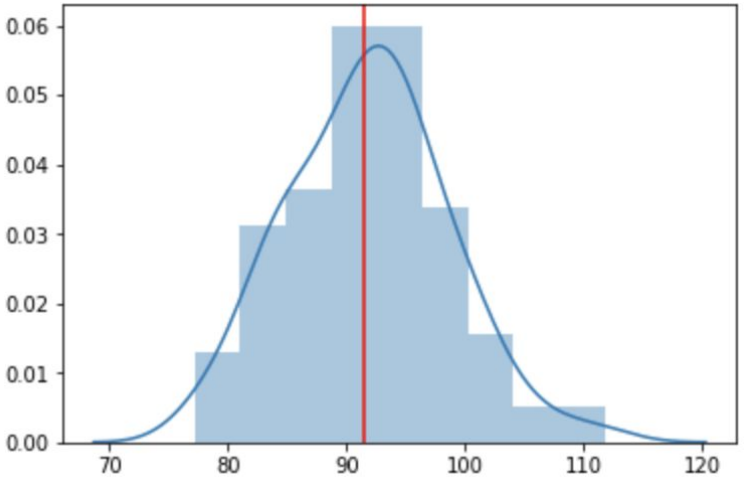
https://www.ncbi.nlm.nih.gov/pubmed/12370571

Let’s take a sample of 100 swings with each bat

Wooden Bat

Mean Velocity: 91.4 mph

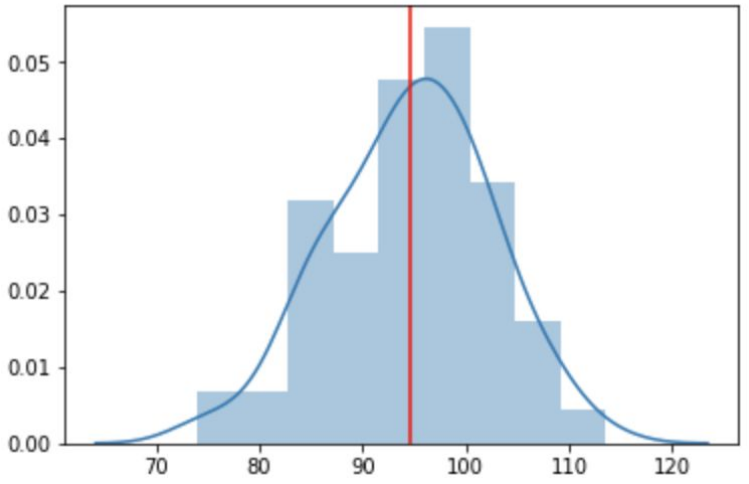
Standard Deviation: 7 mph



Aluminum Bat

Mean Velocity: 94.6 mph

Standard Deviation: 8 mph

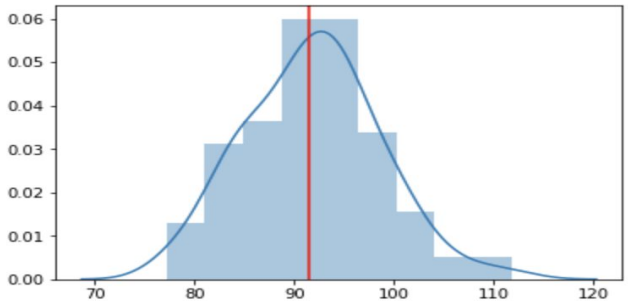


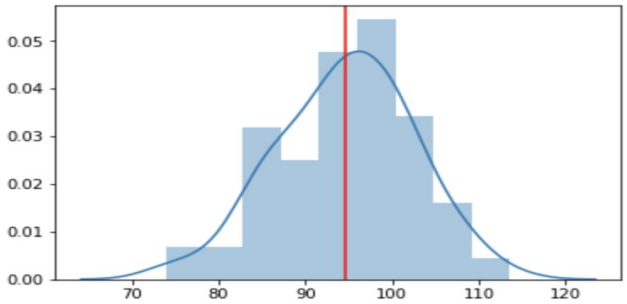
Study by Crisco & Greenwald (2003)

What can the sample tell us about *all* bat swings?

All swings with **wooden** bats

All swings with **metal** bats





Step 1. Hypothesize

“Null” Hypothesis:

The average velocity of all balls hit by a wooden bat is the SAME as the average velocity of all balls hit by metal bats.

“Alternative” Hypothesis:

The average velocity of all balls hit by a wooden bat is NOT THE SAME as the average velocity of all balls hit by metal bats.

Step 2. How sure do you want to be?

How much evidence do we need before we're going to ask for $850,000?

This is the called the “significance level” (alpha).

95% sure = .05 alpha

A higher alpha level means:

- We’re likelier to detect a difference (that’s good for **our** lawyers)
- We’re likelier to detect a difference *by mistake* (that’s good for **their** lawyers)

Step 3. What's our estimated difference?

In our sample, the difference in the average velocity of balls hit by wooden and metal bats is **4.5 mph.**

Our null hypothesis is that the true difference is **zero**.

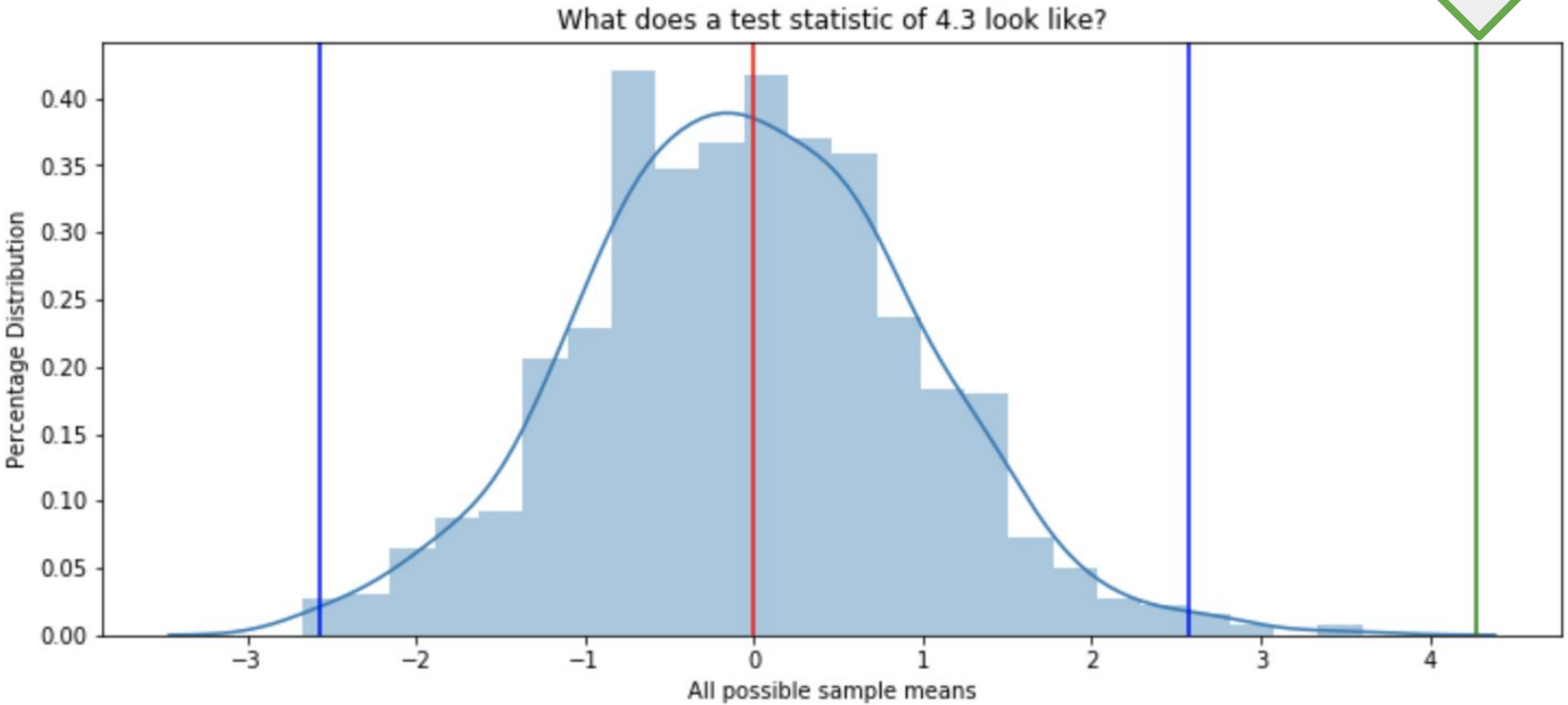
Step 4. How likely is it that we see a difference of ***4.5 mph*** by chance?

This number represents the distance (in standard deviations) between the observed mean and the expected mean

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Z = 4.3 standard deviations

Let’s visualize that test statistic!



99% of all possible sample means fall between the blue lines

Step 5. Draw your own conclusions

Our p-value: 0.00004% probability

The definition of p-value is "the probability that, given a re-run of our experiment, we get a test statistic that is as extreme as the test statistic we just received."

Let’s keep our little baseball guys safe!