Formulae and Facts

1. Given $X = x$, the predictor of Y with the least MSE is $\mathrm{E}[Y|x]$, and its MSE is $\mathrm{var}[Y|x]$.
2. Let $X_1, \ldots, X_n$ be IID $\mathrm{N}(\mu, \sigma^2)$ RV's.

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

   Then $\bar{X}$ and S^2 are independent,

$$\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right), \qquad \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}, \qquad \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$$

3. Let $Z_1, \ldots, Z_k$ be IID N(0,1) RV's. Then $Y = \sum_{i=1}^{k} Z_i^2 \sim \chi^2_k$. $\mathrm{E}(Y) = k$ and $\mathrm{var}(Y) = 2k$.
4. Consider a population of size N with mean μ and variance σ^2. For $n \le N$, let $X_1, \ldots, X_n$ be a simple random sample from the population, and let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

$$\mathrm{E}(\bar{X}) = \mu, \qquad \mathrm{var}(\bar{X}) = \frac{N-n}{N-1}\frac{\sigma^2}{n}$$

5. The Multinomial$(n, (p_1, \ldots, p_r))$ probability mass function is:

$$\frac{n!}{x_1! \cdots x_r!} p_1^{x_1} \cdots p_r^{x_r}$$

   where $x_1, \ldots, x_r$ are non-negative integers summing to n.
6. For an estimator $\hat{\theta}$ of θ, its mean squared error (MSE) is

$$\mathrm{E}\{(\hat{\theta}-\theta)^2\} = \{\mathrm{E}(\hat{\theta}) - \theta\}^2 + \mathrm{var}(\hat{\theta})$$

7. Let $\hat{\theta}_n$ be the ML estimator of $\theta \in \mathbb{R}^p$ based on IID random variables $X_1, \ldots, X_n$ with density $f(x|\theta)$. For large n, approximately

$$\hat{\theta} \sim \mathrm{Normal}\left(\theta, \frac{\mathscr{I}(\theta)^{-1}}{n}\right)$$

   where the Fisher information is

$$\mathscr{I}(\theta) = -\mathrm{E}\left(\frac{\mathrm{d}^2 \log f(X_i|\theta)}{\mathrm{d}\theta^2}\right)$$

8. n IID RV's density is defined by $\theta \in \Omega$ with k_1 independent parameters. Let L_1 be the maximum likelihood value over Ω. Let Ω_0 be a subset defined by $k_0 < k_1$ parameters. Let L_0 be the maximum likelihood value over Ω_0. If $\theta \in \Omega_0$, for large n, approximately

$$G = 2\log\left(\frac{L_1}{L_0}\right) = 2(\ell_1 - \ell_0) \sim \chi^2_{k_1-k_0}$$

   $\ell_1 = \log L_1$, $\ell_0 = \log L_0$.

END OF PAPER