Fn	$(f\circ g)(x) = f(g(x))$, composite		Domain: $\{x \mid x \in A$ and $f(x) \in B\}$	
	domain: $\mathbb{R}$, range: $\{x \mid x \geq 0\} = \mathbb{R}^+ \cup \{0\} = [0, \infty)$		diff: $A \setminus B = \{x \mid x \in A$ and $x \notin B\}$	
	fn is increasing if $a < b \Rightarrow f(a) < f(b)$ for any a, b		fn is decreasing if $a < b \Rightarrow f(a) > f(b)$ for any a, b	
	even fn: $f(-x) = f(x)$ odd fn: $f(-x) = -f(x)$ (if not odd/even, proof by counter e.g.) power fn: x^n, $n \in \mathbb{Z}^+$		symmetric about y-axis symmetric about origin if n is odd, fn is odd | if n is even, fn is even	
Trigo	$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\sin(-x) = -\sin x$ $\cos(-x) = \cos(x)$ $\tan(-x) = -\tan x$	$\csc(-x) = -\csc(x)$ $\sec(-x) = \sec(x)$ $\cot(-x) = -\cot(x)$
	$1 - x^2 \leq \cos x$ for $-\pi/2 < x < \pi/2$ $x < \tan x$ for $0 < x < \pi/2$	If a,b,c are sides of triangle and θ is angle opp c, then $c^2 = a^2 + b^2 - 2ab\cos\theta$	$-\lvert\theta\rvert \leq \sin\theta \leq \lvert\theta\rvert$ $-\lvert\theta\rvert \leq 1 - \cos\theta \leq \lvert\theta\rvert$	
	$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$	$\csc 2\theta = \frac{\sec\theta\csc\theta}{2}$ $\sec 2\theta = \frac{\sec^2\theta}{2-\sec^2\theta}$ $\cot 2\theta = \frac{\cot^2\theta-1}{2\cot\theta}$	$\cos^2\theta = \frac{1+\cos 2\theta}{2}$ $\sin^2\theta = \frac{1-\cos 2\theta}{2}$	$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ $\lim_{\theta\to 0}\frac{\cos\theta-1}{\theta} = 0$ $\lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1$
	$\sin(\pi - x) = \sin x$ $\cos(\pi - x) = -\cos x$ $\tan(\pi - x) = -\tan x$	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ $y-x = (y^{1/n}-x^{1/n})(y^{(n-1)/n} + y^{(n-2)/n}x^{1/n} + \ldots + x^{(n-1)/n})$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$	
Limits	slope = gradient = m = $\frac{y_1-y_0}{x_1-x_0}$		tangent line at y_0: $(y - y_0) = m(x - x_0)$	
Intuitive defn	only depends on values of f(x) for x near a, (not at a)		$\lim_{x\to a} f(x) = L$, if value of f(x) is arbitrarily close to L by taking x sufficiently close to a (intuitive definition)	
Properties	If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ 1. $\lim_{x\to a}[f(x) + g(x)] = L + M$ 2. $\lim_{x\to a}[f(x) - g(x)] = L - M$ 3. $\lim_{x\to a} cf(x) = cL$		4. $\lim_{x\to a}[f(x) \cdot g(x)] = L \cdot M$ 5. $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{L}{M}$, $M \neq 0$ 6. $\lim_{x\to a}[f(x)]^n = L^n$, $n \in \mathbb{Z}^+$ 7. $\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{L}$, $n \in \mathbb{Z}^+$ and if n is even, $f(x) \geq 0$ for all x near a	
Finding limit	If a is in domain of f, then $\lim_{x\to a} f(x) = f(a)$		If a not in domain, try simplifying/rationalise fraction	
1-sided lim	$\lim_{x\to a} f(x) = L \Leftrightarrow \lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$		if $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x) \Rightarrow \lim_{x\to a} f(x)$ DNE	
Infinite lim	If taking x sufficiently close to a, value of f(x) is arbitrarily large/small, Infinite limits **make sense**, but **do not exist**		$\lim_{x\to a} H(x) = \pm\infty$	
Squeeze Theorem	If $f(x) \leq g(x) \leq h(x)$ for all x near a (except at a) and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x)$ exists and = L			
	$\lim_{x\to a} g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} h(x)$, where $g(x)/h(x) = f(x)$		Use if a not in domain of f(x)	
Precise defn	$\lim_{x\to a} f(x) = L$, if for every $\epsilon > 0$, $\exists$ a $\delta > 0$ such that $\lvert f(x) - L\rvert < \epsilon$ whenever $0 < \lvert x-a\rvert < \delta$			
Triangle Inequality	For any $a,b \in \mathbb{R}$, $\lvert a\rvert - \lvert b\rvert \leq \lvert a+b\rvert \leq \lvert a\rvert + \lvert b\rvert$			
Precise defn	Right hand limit: for every $\epsilon > 0$, there exists a num $\delta > 0$ s.t. $0 < x-a < \delta \Rightarrow \lvert f(x) - L\rvert < \epsilon$ Left hand limit: for every $\epsilon > 0$, there exists a num $\delta > 0$ s.t. $0 < a-x < \delta \Rightarrow \lvert f(x) - L\rvert < \epsilon$ $+\infty$ limit: for every $M > 0$, there exists a num $\delta > 0$ s.t. $0 < \lvert x-a\rvert < \delta \Rightarrow f(x) > M$ $-\infty$ limit: for every $M < 0$, there exists a num $\delta > 0$ s.t. $0 < \lvert x-a\rvert < \delta \Rightarrow f(x) < M$			
Precise defn as $x \to \pm\infty$	$x \to \infty$: for every $\epsilon > 0$, there exists a num M s.t. $x > M \Rightarrow \lvert f(x) - L\rvert < \epsilon$ $x \to -\infty$: for every $\epsilon > 0$, there exists a num M s.t. $x < M \Rightarrow \lvert f(x) - L\rvert < \epsilon$			
Prove limits	1. Let $\epsilon > 0$ 2. Choose a δ (found in working) to prove that $0 < \lvert x-a\rvert < \delta \Rightarrow \lvert f(x)-L\rvert < \epsilon$			
Continuous at a pt	Let f be a polynomial or rational fn If f has the direct substitution property at a, i.e. $\lim_{x\to a} f(x) = f(a)$, then f is continuous at a Opp of continuous is discontinuous		Defn of continuity: 1. f(a) is well-defined, i.e. a is in domain of f; and 2. $\lim_{x\to a} f(x)$ exists, i.e. it is a real number; and 3. $\lim_{x\to a} f(x) = f(a)$, precise defn of limits apply here as well	
Removable discontinuity	2. true but 1. false only at pt a, then f have removable discontinuity Let $f_1(x) = \begin{cases} f(x) & if\ x \neq a \\ \lim_{x\to a} f(x) & if\ x = a \end{cases}$. Then f_1 is the *continuous extension* of f at a			
Infinite discontinuity	Suppose f has at least 1 1-sided infinite limit at a: $\lim_{x\to a^+} f(x) = \pm\infty$; or $\lim_{x\to a^-} f(x) = \pm\infty$ Then vertical line x = a is an asymptote of y = f(x) and f is said to have an *infinite discontinuity* at a			
Jump discontinuity	Suppose $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exists, but $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ Then f is said to have a *jump discontinuity* at a			
1-sided continuity	$\lim_{x\to a^-} f(x) = f(a)$. Then f is continuous from the left at a $\lim_{x\to a^+} f(x) = f(a)$. Then f is continuous from the right at a f is continuous at a iff f is continuous from the left at a and from the right at a			
Continuity on intervals	f is continuous on [a,b] if f is continuous at every $x \in (a,b)$, and f is continuous from the right at a, and f is continuous from the left at b			

Proving	Use limits to show that no matter value of a, pt 3 holds
Composite fn	Let f and g be 2 fn. Suppose $\lim_{x\to a} f(x)$ = b and $\lim_{y\to b} g(y)$ = c. Let y = f(x) and z = g(y). We know $x \to a$ (x ≠ a) ⇒ y→ b and y→ b (y ≠ b) ⇒ z→ c. But $\lim_{x\to a} g(f(x))$ = c is true only if 1. 1st part change to $x \to a$ (x ≠ a) ⇒ y→ b (y ≠ b) OR 2. 2nd part change to $y \to b$ ⇒ z→ c Condition 1 means $\lim_{x\to a} f(x)$ = b and f(x) ≠ b ∀ x in an open interval containing a except at a $\lim_{x\to a} g(f(x))$ = c = $\lim_{y\to b} g(y)$ Condition 2 means g is continuous at b $\lim_{x\to a} g(f(x))$ = c = g(b) = g$\left(\lim_{x\to a} f(x)\right)$
Substitution in limits	- $\lim_{x\to a} f(x) = \lim_{h\to 0} f(a+h)$, where h = x - a. (Derived from condition 1 of composite fn, x → a (x ≠ a) ⇒ h→ 0 (h ≠ 0)) - In particular, if f is continuous at a ⇔ $\lim_{x\to a} f(x)$ = f(a) ⇔ $\lim_{h\to 0} f(a+h)$ = f(a)
Continuous composite fn	Suppose f is continuous at a and g is continuous at f(a). Then g∘f is continuous at a $\lim_{x\to a} g(f(x)) = g(\lim_{x\to a} f(x))$ = g(f(a))
Root fn	Root fn, $\sqrt[n]{x}$ is continuous on (-∞,∞) if n is odd [0,∞) if n is even
Trigo fn	sin x and cos x are continuous on ℝ tan x = (sin x / cos x) and sec x = (1 / cos x) are continuous whenever cos x ≠ 0 ℝ \ {±π/2, ±3π/2, ±5π/2,...} cot x = cos x / sin x and csc x = 1 / sin x are continuous whenever sin x ≠ 0 ℝ \ {0, ±π, ±2π, ±3π,...}
IVT	Let f be a continuous fn on [a,b] Suppose f(a) < 0 and f(b) > 0 or f(a)> 0 and f(b)<0, then ∃ c ∈ (a,b) s.t. f(c) = 0 Suppose f(a) ≠ f(b) and N is btw f(a) and f(b), then ∃ c ∈ (a,b) s.t. f(c) = N Only prove there can be more than 1 root
Derivative	Slope/gradient = m = $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ = f'(a) = $\frac{dy}{dx} = \left.\frac{dy}{dx}\right\|_{x=a} = \frac{d}{dx}$f(a) = $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ Since $\lim_{h\to 0} f(a+h) = \lim_{x\to a} f(x)$, $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ Suppose f'(a) exists, then tangent line at x = a: y = f'(a)(x-a) + f(a) If f is differentiable at a, then f'(a) = $\frac{f(x)-f(a)}{x-a}$, then f is continuous at a Converse may not be true. f is continuous at a does not imply f is differentiable at a f differentiable at a means $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ exists
Formulas	(cf)' = cf'; $\frac{d}{dx}$(x^n) = nx^{n-1}, if n < 0, x cannot be 0; $\frac{d}{dx}$(sin x) = cos x; $\frac{d}{dx}$(cot x) = -$\csc^2$x (f ± g)' = f' ± g'; (g^{-1})' = (1/g)' = -g'/g^2; $\frac{d}{dx}$(cos x) = -sin x; $\frac{d}{dx}$(sec x) = sec x tan x (f^2)' = 2f f'; $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$, assuming g(x) ≠ 0; $\frac{d}{dx}$(tan x) = $\sec^2$x; $\frac{d}{dx}$(csc x) = -csc x cot x (fg)' = f'g + fg'
Chain rule	$\frac{dz}{dx} = \frac{dz}{dy}\cdot\frac{dy}{dx}$ If f differentiable at x, g differentiable at f(x), then g∘f is differentiable at x and (g∘f)'(x) = g'(f(x))f'(x)
Implicit differentiation	Implicit dy/dx assume dy/dx exists (cannot prove differentiability) E.g. $x^3 + y^3 = 3xy \Rightarrow 3x^2 + 3y^2\cdot\frac{dy}{dx} = 3y + 3x\cdot\frac{dy}{dx}$
EVT	Suppose f is continuous on [a,b], ∃ c, d ∈ [a,b] s.t. f(c) ≤ f(x) ≤ f(d) ∀ x ∈ [a,b] Extreme values: abs max, min
Fermat's Theorem	If f has local extreme value at c, then either f'(c) don't exist or f'(c) exist and = 0 This c is called a critical point. (Stationary pt if f'(c) = 0)
Closed Interval Mtd	Let f be continuous on [a,b] 1. Evalute values of f at endpoints: f(a) and f(b) 2. Evaluate values of f at critical points on (a,b) s.t. f'(c) don't exist or s.t. f'(c) = 0 3. Compare values obtained in 1 and 2 (largest value = absolute max; smallest = absolute min)
Rolle's Theorem	Suppose fn f is continuous on [a,b] and differentiable on (a,b) and f(a) = f(b) There must be c ∈ (a,b) s.t. f'(c) = 0 If f'(x) = 0 ∀ x ∈ (a,b), then f(x) = C ∀ x ∈ (a,b), where C is constant If f'(x) = g'(x) ∀ x ∈ (a,b), then f(x) = g(x) + C ∀ x ∈ (a,b), where C is constant
MVT	Suppose fn f is continuous on [a,b] and differentiable on (a,b) ∃ c ∈ (a,b) s.t. f'(c) = $\frac{f(b)-f(a)}{b-a}$
Increasing Test	Let fn f be continuous on close interval I, differentiable on interior of I and f'(x) > 0 (< 0) for every x in interior of I Then, f is increasing (decreasing) on I If f is non-decreasing on I, then f'(x) ≥ 0 ∀ x ∈ I
1st derivative test	Let fn f be continuous at critical pt c, differentiable on open interval containing c except at c If f' changes from -ve to +ve at c, local min at c. If f' change from +ve to -ve at c, local max If f' don't change sign at c, no local extreme val at c
2nd derivative test	Lemma. If $\lim_{x\to a} f(x)$ exists and +ve (-ve), then f(x) > 0 (< 0)∀ x in an open interval containing a, except at a Suppose f'(c) = 0. If f''(c) > 0, f has local min at c. If f''(c) < 0, f has local max at c 2nd derivative test is inconclusive if f'(c) = f''(c) = 0
Concavity	Suppose f is differentiable on open interval I, where f(x) - f(c) = f'(c)(x-c) If graph of f lies above all its tangent lines on I, f concave up on I, f(b) - f(a) > f'(a)(b-a) and f' increasing on I If graph of f lies below all its tangent lines on I, f concave down on I, f(b) - f(a) < f'(a)(b-a) and f' decreasing on I
Concavity Test	Suppose f is twice differentiable on open interval I If f''(x) > 0 ∀ x ∈ I ⇒ f concave up on I. If f''(x) < 0 ∀ x ∈ I ⇒ f concave down on I

	If f''(x) = 0, inconclusive
Inflection point	If f continuous at c, and change concavity at c ⇒ inflection pt at c If f is twice differentiable at c, f''(c) = 0
l'Hôpital's Rule	Assume fn f,g are differentiable at a and $\lim_{x\to a} f(x)$ = f(a) = 0 and $\lim_{x\to a} g(x)$ = g(a) = 0. Then f,g are continuous at a $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)} = \lim_{x\to a}\frac{[f(x)-f(a)]/(x-a)}{[g(x)-g(a)]/(x-a)} = \frac{\lim_{x\to a}[f(x)-f(a)]/(x-a)}{\lim_{x\to a}[g(x)-g(a)]/(x-a)} = \frac{f'(a)}{g'(a)}$, provided g'(a) ≠ 0 (simple version)
	Let fn f,g s.t. $\lim_{x\to a} f(x)$ = 0 and $\lim_{x\to a} g(x)$ = 0 and $\lim_{x\to a}\frac{f'(x)}{g'(x)}$ exists or = ±∞. (Does not matter if a is finite or infinite) Then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ (0/0 version)
	Let fn f,g s.t. $\lim_{x\to a} \lvert f(x)\rvert$ = ∞ and $\lim_{x\to a} \lvert g(x)\rvert$ = ∞ and $\lim_{x\to\infty}\frac{f'(x)}{g'(x)}$ exists or ±∞ Then $\lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{f'(x)}{g'(x)}$ (∞/∞ form)
Cauchy's MVT	Suppose f,g are continuous on [a,b] and differentiable on (a,b) and g'(x) ≠ 0 for any x ∈ (a,b) Then ∃ c ∈ (a,b) s.t. $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$ (Generalized MVT) Let g(x) = x. Then g'(x) = 1 ∀ x ∈ ℝ. Then ∃ c ∈ (a,b) s.t. f'(c) = $\frac{f(b)-f(a)}{b-a}$
	Suppose f' exist and is cts on [a,b] and f'' exists on (a,b) where a < b. Then ∃ c ∈ (a,b) s.t. f(b) = f(a) + (b-a)f'(a) + $\frac{(b-a)^2}{2}$f''(c)
Area under graph	Let f be non-negative continuous fn on [a,b] 1. Divide [a,b] into n equal subintervals 2. Construct rectangle on each subinterval whose height is value of f at left/right endpoint of subinterval 3. Find total area of n rectangles, L_n / R_n 4. L_n / R_n approaches actual area as n → ∞
Definite Integral	$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx$
Integrability of Continuous Fn	If fn f is continuous over interval [a,b], or if f has a finite num of jump discontinuities, then $\int_a^b f(x)\,dx$ exists and f is integrable over [a,b]
Geometric properties	Let f be nonnegative continuous fn on [a,b] Then $\int_a^b f(x)\,dx$ = area btw y = f(x) and x-axis from a to b Let f be a nonpositive continuous fn on [a,b] Then -f is nonnegative and continuous on [a,b] $\int_a^b -f(x)\,dx$ = area btw y = f(x) and x-axis from a to b Let f be continuous fn on [a,b]. Let A_1 represent area above x-axis and A_2 area below x-axis $\int_a^b f(x)\,dx$ = A_1 - A_2 = net area of region btw y = f(x) and x-axis from a to b $\int_a^b \lvert f(x)\rvert\,dx$ = A_1 + A_2 = area of region
	1. $\int_a^b c\,dx$ = c(b-a) 2. If f(x) ≥ g(x) ∀ x ∈ [a,b], then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$ 3. Let m,M be min,max val of f on [a,b], then m(b-a) ≤ $\int_a^b$ f(x) dx ≤ M(b-a) 4. $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$ 5. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ 6. If f is defined at a, $\int_a^a f(x)\,dx$ = 0 7. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$ 8. $\int_a^b$ f(x) + g(x) dx = $\int_a^b$ f(x) dx + $\int_a^b g$(x) dx
Anti-derivative	F is antiderivative of f on interval I if F'(x) = f(x) ∀ x ∈ I

1. $\int x^n = \frac{1}{n+1}x^{n+1}$ + C, n ≠ 1	5. $\int \csc^2 kx = -\frac{1}{k}\cot kx$ + C	9. $\int \cot kx = \frac{1}{k}\ln\lvert\sin kx\rvert$ + C
2. $\int \sin kx = -\frac{1}{k}\cos kx$ + C	6. $\int \sec kx\tan kx = \frac{1}{k}\sec kx$ + C	10. $\int \sec kx = \frac{1}{k}\ln\lvert\sec kx + \tan kx\rvert$ + C
3. $\int \cos kx = \frac{1}{k}\sin kx$ + C	7. $\int \csc kx\cot kx = -\frac{1}{k}\csc kx$ + C	11. $\int \csc kx = -\frac{1}{k}\ln\lvert\csc kx + \cot kx\rvert$ + C
4. $\int \sec^2 kx = \frac{1}{k}\tan kx$ + C	8. $\int \tan kx = -\frac{1}{k}\ln\lvert\cos kx\rvert$ + C	

Fundamental Theorem of Calculus	Suppose f is continuous on [a,b]. Let g(x) = $\int_a^x f(t)$ dt Then g is continuous on [a,b], differentiable on (a,b) and g'(x) = f(x) ∀ x ∈ (a,b) $\int_a^b f(x)\,dx$ = F(b) - F(a) = F(x)$\Big\|_{x=a}^{x=b}$ $\frac{d}{dx}\int_a^x f(t)dt$ = f(x) $\frac{du}{dx}\frac{d}{du}\int_a^u f(t)dt = \frac{du}{dx}$ · f(u)
MVT for definite integrals	If f is continuous on [a,b], then ∃ c in [a,b] s.t. f(c) = $\frac{1}{b-a}\int_a^b f(x)\,dx$
Indefinite Integrals	Collection of all antiderivatives of f is called the indefinite integral of f w.r.t x and is denoted by $\int f(x)$ dx
Substi-tuition Rule	Suppose u = g(x) is differentiable, whose range is interval I. Suppose g' is continuous and f is continuous on I. Then $\int f$(g(x))g'(x) dx = $\int f(u)$ du And $\int_a^b f$(g(x))g'(x) dx = $\int_{g(a)}^{g(b)} f(u)$ du E.g. $\int 2x\sqrt{1-x^2}$ dx. Let u = 1-x^2. $\frac{du}{dx}$ = -2x $\int 2x\sqrt{1-x^2}$ dx = $\int -\frac{du}{dx}\sqrt{u}$ dx = -$\int\sqrt{u}$ du = -$\frac{u^{3/2}}{3/2}$ + C = -$\frac{2}{3}$(1-x^2)$^{3/2}$ + C
Odd/ Even Fn	Let f be continuous fn on [-a,a] If f is odd, then $\int_{-a}^a f(x)$dx = 0 If f is even, then $\int_{-a}^a f(x)$dx = 2$\int_0^a f(x)$dx
Disconti-nuous Fn	Let f be continuous on [a,b), discontinuous at b from left, $\int_a^b f(x)$dx = $\lim_{t\to b^-}\int_a^t f(x)$dx Let f be continuous on (a,b], discontinuous at a from right, $\int_a^b f(x)$dx = $\lim_{t\to a^+}\int_t^b f(x)$dx $\int_a^b f(x)$dx is convergent is limit exist / divergent is limit DNE Suppose f is discontinuous at c ∈ (a,b), and $\int_a^b f(x)$dx = $\int_a^c f(x)$dx + $\int_c^b f(x)$dx $\int_a^b f(x)$dx is convergent if both integrals exists, divergent if at least one integral is divergent Let f be fn continuous on (a,b) and $\lim_{x\to a^+} f(x)$dx and $\lim_{x\to b^-} f(x)$dx exist Let f_1 be the continuous extension of f, then $\int_a^b f(x)$dx = $\int_a^b f_1(x)$dx

Infinity	If $\int_a^t f(x)dx$ exists ∀ t ≥ a, then $\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$ If $\int_t^b f(x)dx$ exists ∀ t ≤ b, then $\int_{-\infty}^b f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$ Integral is convergent if limits exist else divergent	$\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^a f(x)dx \int_a^\infty f(x)dx$ Convergent if both improper integrals on right are convergent, else divergent

1-1 fn	Fn is 1-1 (one-to-one) if horizontal line intersects graph ≤ 1 time OR a ≠ b ⇒ f(a) ≠ f(b) for any a,b in domain, D of f OR f(a) = f(b) ⇒ a = b for any a, b ∈ D Let f be 1-1 fn. Then $f^{-1}(y) = x \Leftrightarrow y = f(x)$.	f : D → R, f^{-1} : R → D. R is domain of f^{-1} and D is range of f^{-1} $(f^{-1})^{-1} = f$ $f^{-1}(f(x)) = x$ for any x ∈ D. $f(f^{-1}(y)) = y$ for any y ∈ R In general, $f^{-1} \circ f \neq f \circ f^{-1}$
Finding inverse	f and f^{-1} are symmetric (reflection) w.r.t line y = x	1. Express x in terms of y, $x = f^{-1}(y)$ 2. Interchange x and y to express f^{-1} as fn in x, $y = f^{-1}(x)$
Properties	f is cts. f is 1-1 ⇔ f is monotonic (incr^{ing} or decr^{ing}) f is 1-1. f is continuous ⇒ f^{-1} is continuous	f is 1-1 cts fn. f incr^{ing} (decr^{ing}) ⇒ f^{-1} incr^{ing} (decr^{ing}) $(f^{-1})'(b) = \frac{1}{f'(a)}$, f'(a) ≠ 0

Inverse Trigo

fn	domain	range	continuous	differentiable	derivative
$\sin^{-1}$	[-1, 1]	[-π/2, π/2]	[-1,1]	(-1,1)	$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$, x ∈ (-1,1)
$\cos^{-1}$	[-1, 1]	[0, π]	[-1,1]	(-1,1)	$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$, x ∈ (-1,1)
$\tan^{-1}$	ℝ	(-π/2, π/2)	ℝ		$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$
$\cot^{-1}$	ℝ	(0, π)	ℝ		$\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$
$\sec^{-1}$	(-∞, -1] ∪ [1, ∞)	$[0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$	(-∞, -1] ∪ [1, ∞)	(-∞, -1) ∪ (1, ∞)	$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$, |x| > 1
$\csc^{-1}$	(-∞, -1] ∪ [1, ∞)	$(0, \frac{\pi}{2}] \cup (\pi, \frac{3\pi}{2}]$	(-∞, -1] ∪ [1, ∞)	(-∞, -1) ∪ (1, ∞)	$\frac{d}{dx}\csc^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$, |x| > 1

$\sin^{-1}x + \cos^{-1}x = \pi/2$, x ∈ [-1,1]	$\tan^{-1}x + \cot^{-1}x = \pi/2$	$\sec^{-1}x + \csc^{-1}x = \begin{cases} \pi/2 & if\ x \geq 1 \\ 5\pi/2 & if\ x \leq -1 \end{cases}$

Logarithmic fn

$\ln x = \int_1^x \frac{1}{t}dt$ (x > 0)	ln x: (0, ∞) → ℝ	ln x is cts, differentiable, incr^{ing} and concave down on $\mathbb{R}^+$
ln 1 = 0	$\lim_{x\to0^+} \ln x = -\infty$, $\lim_{x\to\infty} \ln x = \infty$,	Let x > 0 and a > 0. Then ln(ax) = ln a + ln x
Let x > 0 and r ∈ ℚ. Then $\ln(x^r) = r \ln x$		For any x ≠ 0, $\frac{d}{dx}\ln\|x\| = \frac{1}{x}$ and $\int \frac{1}{x}dx = \ln\|x\| + C$

Logarithmic differentiation	1. Take abs value: $\|y\| = \|f_1(x)\|^{r1} \cdots \|f_n(x)\|^{rn}$ 2. Take natural log: $\ln\|y\| = r1 \ln\|f_1(x)\| + \cdots + rn \ln\|f_n(x)\|$	3. Differentiate w.r.t x Cannot use logarithmic differentiation if y = 0
Exponential fn	Euler's num = e = 2.71828 and Let n ∈ ℕ. Then $e^n = e*e...*e$ and $e^{-n} = 1/e^n$	$e^x = \exp(x) = f^{-1}$: ℝ → $\mathbb{R}^+$ where f(x) = ln x Let x ∈ ℚ. Then $f(e^x) = \ln(e^x) = x \ln e = x = e^{\ln x}$

	ln e = 1	$e^0 = 1$	$e^r = e^{m/n} = \sqrt[n]{e^m}$	$\lim_{x\to-\infty} e^x = 0$	$\lim_{x\to\infty} e^x = \infty$
	$e^xe^y = e^{x+y}$		$(e^x)^y = e^{xy}$	$e^{-x} = 1/e^x$	$\frac{d}{dx}e^x = e^x$
Let a > 0	$a^r = e^{r \ln a}$	$a^xa^y = a^{x+y}$	$(a^x)^y = a^{xy}$	$a^{-x} = 1/a^x$	$\frac{d}{dx}a^x = a^x\ln a$
$\frac{d}{dx}x^a = ax^{a-1}$ (x > 0, and for any a)			$\int x^a\,dx = \begin{cases} \ln\|x\| + C, if\ a = -1 \\ \frac{x^{a+1}}{a+1} + C,\ \ if\ a \neq -1 \end{cases}$ If a ∉ ℚ, then domain of $f(x) = x^a$ is [0, ∞]		$e = \lim_{x\to0}(1+x)^{\frac{1}{x}}$

To find $\lim_{x\to0} f(x)^{g(x)}$, where f(x) > 0	1. Express $f(x)^{g(x)} = \exp[g(x)\ln f(x)]$ 2. Interchange lim and exp function

Hyperbolic Trigo fn	Hyperbolic sine fn: $\sinh x = \frac{e^x - e^{-x}}{2}$ Hyperbolic cosine fn: $\cosh x = \frac{e^x + e^{-x}}{2}$ $\cosh^2 x - \sinh^2 x = 1$	sinh(x + y) = sinh x cosh y + cosh x sinh y cosh(x + y) = cosh x cosh y + sinh x sinh y $\frac{d}{dx}\sinh x = \cosh x$ and $\frac{d}{dx}\cosh x = \sinh x$

fn	shape	domain	range	continuous	differentiable	derivative
$\sinh^{-1}$	increasing	ℝ		ℝ		$\frac{d}{dx}\sinh^{-1}x = \frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1}$	increasing, concave up	[1,∞)	[0,∞)	[1,∞)	(1,∞)	$\frac{d}{dx}\cosh^{-1}x = \frac{1}{\sqrt{x^2-1}}$ for x > 1

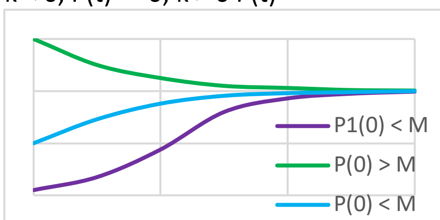
Inverse substitution rule	Let f be continuous fn. Suppose x = g(t) is 1-1 and g' is continuous, then ∫ f(x) dx = ∫ f(g(t))g'(t) dt	Just substitution but expanding the integral
Integration by parts	$\int u\frac{dv}{dx}dx = uv - \int \frac{du}{dx}v\,dx$	∫ u dv = uv - ∫ v du
	$\int \frac{dx}{(1+x^2)^n} = \int (\cos t)^{2n-2}dt$	$n\int (\cos x)^n dx = (\cos x)^{n-1}\sin x + (n-1)\int (\cos x)^{n-2} dx$
Trigo sub	1. $\sqrt{a^2 - x^2}$ (a > 0). Let x = a sin t, t ∈ [-π/2, π/2] 2. $\sqrt{a^2 + x^2}$ (a > 0). Let x = a tan t, t ∈ (-π/2, π/2) 3. $\sqrt{x^2 - a^2}$ (a > 0). Let x = a sec t, t ∈ [0, π/2) ∪ [π, 3π/2)	$\sqrt{a^2 - x^2}$ = a cos t $\sqrt{a^2 + x^2}$ = a sec t $\sqrt{x^2 - a^2}$ = a tan t
Integration by partial fractions	Just factorise denominator into product of real linear factors and real irreducible quadratic factors	$\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+d}{x^2+1}$, $x^6 - 1 = (x-1)(x+1)(x^2 + x + 1)(x^2 - x + 1)$
Universal trigo sub	Let f be a rational expression in 2 var. ∫ f(sin x, cos x) dx, -π < x < π, can be evaluated by	$\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$

	t = tan(x/2), i.e. x = 2$\tan^{-1}$ t. $\frac{dx}{dt} = \frac{2}{1+t^2}$	∫ f(sin x, cos x) dx = ∫ f($\frac{2t}{1+t^2}$, $\frac{1-t^2}{1+t^2}$) $\frac{2}{1+t^2}$ dt

Applications

Let f be a cts fn on [a, b], f(a) = c, f(b) = d, f ≥ g, $f^{-1} \geq g^{-1}$ (if fns on diff side of axis, just take lower one as 0/left as 0)		
y = f(x), x = f^{-1}(y)	Rotate abt x-axis	Rotate abt y-axis
Area	$\int_a^b f(x) - g(x)\, dx$	$\int_c^d f^{-1}(y) - g^{-1}(y)\, dy$
Volume	$\int_a^b A(x)\, dx$	$\int_c^d A(y)\, dy$
Solids of Revolution (Disk, perp to axis of revolution)	$\int_a^b \pi[f(x)^2 - g(x)^2]\, dx$	$\int_c^d \pi[f^{-1}(y)^2 - g^{-1}(y)^2]\, dy$
Solids of Revolution (Shell, parallel to axis of revolution)	$\int_c^d 2\pi y[f^{-1}(y) - g^{-1}(y)]\, dy$	$\int_a^b 2\pi x[f(x) - g(x)]\, dx$
Arc Length	$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	$\int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$
Surface Area	$\int_a^b 2\pi f(x)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	$\int_c^d 2\pi f^{-1}(y)\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

ODE

Name	Standard form	Eqn	
1st order ODE	$\frac{dy}{dx}$ = f(x)	y = ∫ f(x) dx	
	$\frac{dy}{dx}$ = g(y)	$\frac{dx}{dy} = \frac{1}{g(y)} \Rightarrow x = \int \frac{1}{g(y)}\, dy$	
1st order separable ODE	$\frac{dy}{dx}$ = f(x)g(y)	∫ f(x) dx = ∫ $\frac{1}{g(y)}$ dy, (g(y) ≠ 0)	
1st order homogeneous ODE	$\frac{dy}{dx}$ = F(x, y) where F(tx, ty) = F(x, y), i.e. have y/x in F(x, y)	1. Let z = $\frac{y}{x}$. Then y = xz and $\frac{dy}{dx} = z + x\frac{dz}{dx}$. F(x, y) = F(1, z) for x ≠ 0 2. ODE becomes z + x$\frac{dz}{dx}$ = F(1, z), which is separable	
1st order linear ODE	$\frac{dy}{dx}$ + p(x)y = q(x)	1. Evaluate ∫ p(x) dx = P(x) + C 2. Use integrating factor $e^{P(x)}$. Then $\frac{d}{dx}e^{P(x)} = p(x)e^{P(x)}$ 3. Multiply integrating factor to eqn ⇒ $\frac{d}{dx}[e^{P(x)}y] = e^{P(x)}q(x)$ 4. y = $\frac{1}{e^{P(x)}}\int e^{P(x)}q(x)\, dx$	
Bernoulli's Eqn	$\frac{dy}{dx}$ + p(x)y = q(x)y^n	For n ≠ 0, 1. Let z = y^{1-n}. $\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx}$ Multiply $(1-n)y^{-n}$ to DE, $(1-n)y^{-n}\frac{dy}{dx} + (1-n)y^{-n}p(x)y = (1-n)y^{-n}q(x)y^n$ $\frac{dz}{dx}$ + (1-n)p(x)z = (1-n)q(x), (linear ODE)	
Exponential growth & decay	$\frac{dy}{dt}$ = ky	y = Ce^{kt}	If k > 0: law of natural growth If k < 0: law of natural decay
Continuously compounded interest	Annually: A(t) = $A_0(1+r)^t$ n times per year: A(t) = $A_0(1+\frac{r}{n})^{nt}$	Continuously compounded: Let n → ∞, A(t) = $\lim_{n\to\infty} A_0\left(1+\frac{r}{n}\right)^{nt}$ = A_0e^{rt}, r = interest per annum, A_0 = initial amt, t = num of years	
Radiocarbon Dating	$\frac{dm}{dt}$ = km	m(t) = m(0)e^{kt}, k = $-\frac{\ln 2}{t_{1/2}}$ half-life: $t_{1/2}$ = time for half of qty to decay	
Logistic Population Growth M = carrying capacity	$\frac{dP}{dt}$ = kP (no M) $\frac{dP}{dt}$ = k(M-P)P, M > 0, k > 0 (Bernoulli's DE)	P(t) = P_0e^{kt} (if k is constant) P(t) = $\frac{M}{1+Ce^{-Mkt}}$ (logistic fn) P(0) = $\frac{M}{1+C}$ P = $\frac{M}{2}$ at t = $\frac{\ln(C)}{Mk}$	k < 0, P(t) → 0; k > 0 P(t) → ∞ 
Newton's Law of Cooling	$\frac{dT}{dt} = -r(T-T_S)$ (heat transfer model) where r > 0	T(t) = $T_S + (T_0 - T_S)e^{-rt}$. As t → ∞, T(t) → T_S	

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