CS 453 Project 3

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Problem 1.

0.1 A

$$\dot{x} = \frac{dx(t)}{dt}$$
$$\dot{y} = \frac{dy(t)}{dt}$$

$$\begin{aligned}
\frac{dx(t)}{dt} &= -1 \\
dx(t) &= -dt \\
\int dx(t) &= -\int dt \\
x &= -t + C
\end{aligned}$$

$$\begin{aligned}
\frac{dy(t)}{dt} &= 1 \\
dy(t) &= dt \\
\int dy(t) &= \int dt \\
y &= t + C
\end{aligned}$$

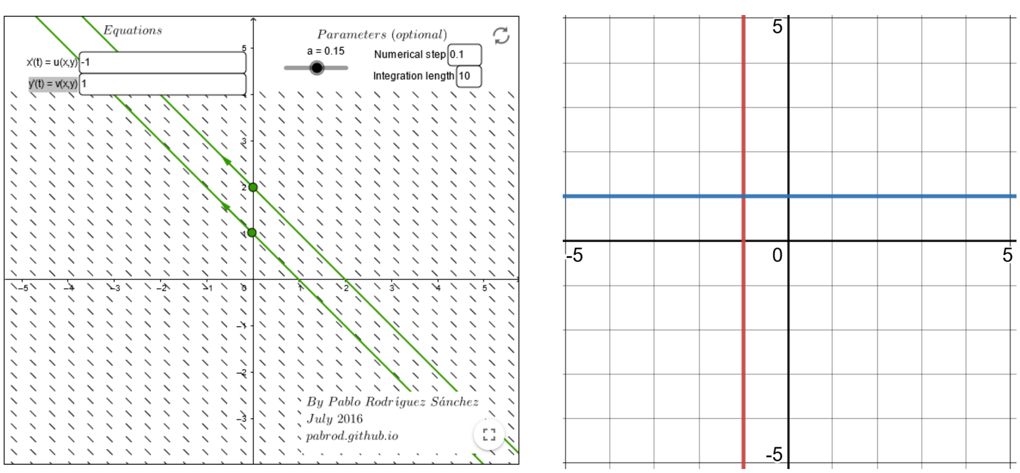


Figure 1: Phase plane and Curves

0.2 B

When looking at $\dot{x} = y$

$$\frac{dx}{dt} - y(t) = 0 \tag{1}$$

$$\dot{y} = x \tag{2}$$

Using equation 1 we can go through the process of solving the first order linear differential equation. On step 3 we can subsitute in equation 2

$$\begin{aligned} \frac{dx}{dt} - y(t) &= 0 \\ \frac{d^2x}{dt^2} - \frac{dy}{dt} &= 0 \\ \frac{d^2x}{dt^2} - x(t) &= 0 \\ r^2 - 1 &= 0 \end{aligned}$$

This gives us a polynomial equation whose roots are solutions to $x = e^{rx}$. We can say $r = \pm 1$.

$$x = c_1e^t + c_2e^{-t} \tag{3}$$

Then we can rewrite this as

$$\begin{aligned} \frac{d}{dt}(x) &= \frac{d}{dx}(c_1e^t + c_2e^{-t}) \\ &= c_1e^t - c_2e^{-t} \\ c_3 &= 0 \end{aligned}$$

Such that

$$\dot{y} = c_1e^t + c_2e^{-t} \tag{4}$$

$$\begin{aligned} y &= \int c_1e^t + c_2e^{-t}dt \\ &= c_1e^t - c_2e^{-t} + c3 \\ y &= c_1e^t - c_2e^{-t} \end{aligned}$$

This gives us the forms

$$y = c_1e^t - c_2e^{-t} \tag{5}$$

and

$$x = c_1e^t + c_2e^{-t} \tag{6}$$

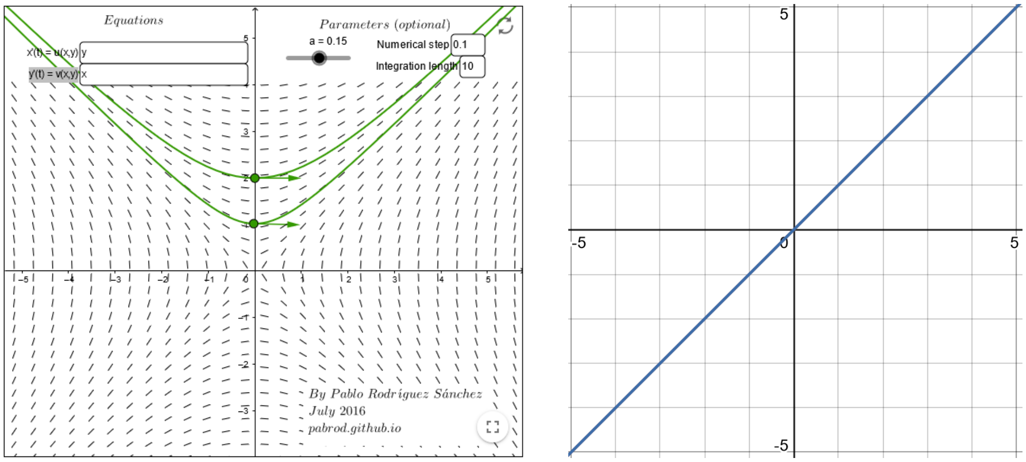
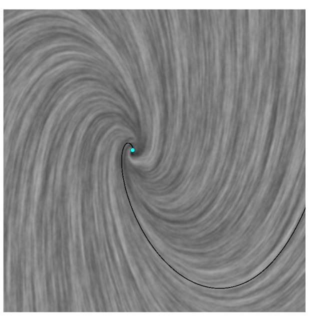
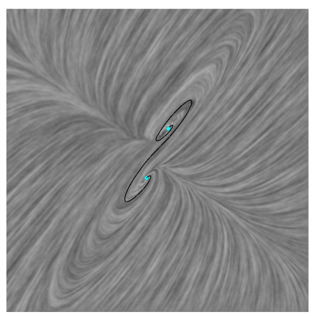


Figure 2: Phase plane and Curves

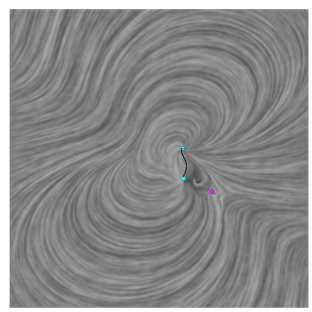
Problem 2.



(a) v1 (-2, 1)



(b) v5 (1, 3)

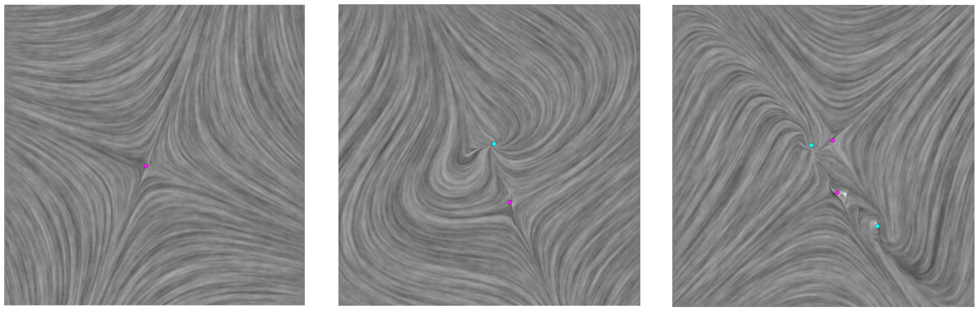


(c) v10 (1.7, 0)

Figure 3: Solutions to User Input of Vector Fields

I colored classified my singularities in this equation and then was able to draw the solution to the dynamic system given some point. This involved using bi-linear interpolation, solving dynamic systems of equations given some point, and projecting both forward and backwards with a streamline.

Problem 3.

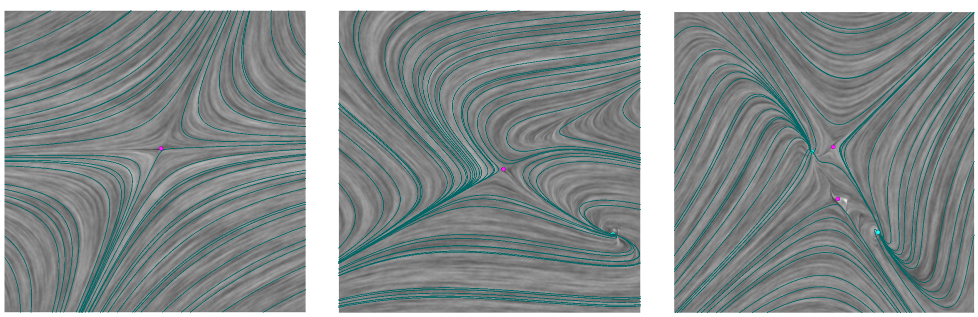


(a) v3 (b) v8 (c) v9

Figure 4: Classification of singularities in vector fields

I was able to classify the singularities in the vector field by taking the determinant of the Jacobian matrix and depicting what the classification is. Color classification, saddles are purple, sources are cyan, higher order singularities are yellow (not pictured).

Problem 4.



(a) v3 (b) v8 (c) v9

Figure 5: Visualization of Streamlines

I generated random streamlines to display in the above figures which allow the user to follow the vector field flow and see how they interact around singularities and behave in general. Visualizing the streamlines give an okay impression on being able to follow a vector through the data set and seeing how some vector flows intersect together in certain parts. However, IBFV gives a better view on how the overall data is interacting since the streamlines can only display a finite amount of data comparatively. IBFV's however cannot allow the user to trace the flow through time leading to users following a certain streamline getting lost.