Chapter Thirteen

Tsunamis

13.1 MATHEMATICAL MODEL OF TSUNAMI PROPAGATION (TRANSIENT WAVES)

> A tsunami is a series of ocean waves generated by sudden displacements in the sea floor, landslides, or volcanic activity. In the deep ocean, the tsunami wave may only be a few inches high. The tsunami wave will increase in height to become a fast moving wall of turbulent water several meters high on reaching the coastline. They can cause great destruction and are most frequent in the Pacific Ocean and Indonesia primarily because of the large number of active submarine earthquake zones around the Pacific Rim.
>
> [143]

> A tsunami is a series of ocean waves with very long wavelengths (typically hundreds of kilometres) caused by large-scale disturbances of the ocean, such as earthquakes; landslide; volcanic eruptions; explosions; meteorites. These disturbances can either be from below (e.g., underwater earthquakes with large vertical displacements, submarine landslides) or from above (e.g., meteorite impacts).
>
> [143]

> Tsunami is a Japanese word with the English translation: “harbour wave”. In the past, tsunamis have been referred to as “tidal waves” or “seismic sea waves”. The term “tidal wave” is misleading; even though a tsunami’s impact upon a coastline is dependent upon the tidal level at the time a tsunami strikes, tsunamis are unrelated to the tides. (Tides result from the gravitational influences of the moon, sun, and planets.) The term “seismic sea wave” is also misleading. “Seismic” implies an earthquake-related generation mechanism, but a tsunami can also be caused by a non-seismic event, such as a landslide or meteorite impact.
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> Tsunamis are also often confused with storm surges, even though they are quite different phenomena. A storm surge is a rapid rise in coastal sea-level caused by a significant meteorological event—these are often associated with tropical cyclones.
>
> [210]

We assume a two-dimensional ocean of (undisturbed) constant depth h and no rigid boundaries other than the seafloor. In this chapter we develop the basic features discussed earlier and incorporate a fluctuation on the ocean floor that will generate and drive the tsunami wave. We shall examine the space and time development of the resulting disturbance but deviate a little from the notation used in chapter 11, simply to relieve the reader from frequent page-turning back to that chapter. In addition, it affords some notational continuity with the sources cited below. Thus the variable Φ (instead of ϕ) will be used to signify the velocity potential and, instead of η, the free surface displacement will be denoted here by $\xi(x, z, t)$. Consequently the governing equations are [43], [112]

$$\nabla^2\Phi = 0, \, \Phi = \Phi(x, z, t), \tag{13.1}$$

and on the free upper surface,

$$\frac{\partial \xi}{\partial t} = \frac{\partial \Phi}{\partial z}, \; z = 0, \tag{13.2}$$

and

$$\frac{\partial \Phi}{\partial t} + g\xi = \frac{-P_a(x, t)}{\rho}, \; z = 0, \tag{13.3}$$

where $P_a(x, t)$ is the prescribed atmospheric pressure. On the seafloor we introduce a displacement $H(x, t)$ due to an earthquake or other disturbance such that now

$$z = -h + H(x, t). \tag{13.4}$$

If the ground motion is known, we may write the lower boundary implicitly as $F(x, z, t) = 0$, where

$$F(x, z, t) = z + h - H(x, t). \tag{13.5}$$

Thus we may write the total derivative of F as

$$\frac{dF}{dt} + \mathbf{v} \cdot \nabla F = 0, \tag{13.6}$$

so that on $F = 0$ continuity of the normal component of velocity requires that

$$\frac{\partial \Phi}{\partial z} = \frac{\partial H}{\partial t} + \frac{\partial \Phi}{\partial x}\frac{\partial H}{\partial x}; \quad z = -h + H, \tag{13.7}$$

or neglecting quadratically small terms,

$$\frac{\partial \Phi}{\partial z} = \frac{\partial H}{\partial t} \equiv W(x, t) \text{ on } z \approx -h. \tag{13.8}$$

Initial conditions will be prescribed as needed in the following Laplace transform approach. To that end we define the Laplace transform of the function $f(t)$ by

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt, \tag{13.9}$$

and its inverse transform by

$$f(t) = \frac{1}{2\pi i}\int_\Gamma e^{st}\bar{f}(s)ds, \tag{13.10}$$

where Γ is a vertical line to the right of all singularities in the complex s-plane. Then we can transform equations (13.1) and (13.8) to get

$$\nabla^2\bar{\Phi}(x, z, s) = 0, \; -h < z < 0 \tag{13.11}$$