Calculus Equation Sheet Generated by Austin M Shearin

Derivatives

$(x^n)' = nx^{n-1}$
$(\sin x)' = \cos x$
$(\cos x) = -\sin x$
$(\tan x)' = sec^2 x$
$(\sec x)' = \sec x \tan x$
$(\csc x)' = -\csc x \cot x$
$(\cot x)' = -csc^2 x$
$(\ln x)' = \frac{1}{x}$
$(e^x)' = e^x$
$(log_a x)' = \frac{1}{x}\frac{1}{\ln a}$
$(a^x)' = a^x \ln a$
$(sin^{-1}x)' = \frac{1}{\sqrt{1-x^2}}$
$(cos^{-1}x)' = -\frac{1}{\sqrt{1-x^2}}$
$(tan^{-1}x)' = \frac{1}{1+x^2}$

Product rule:
$\left(f(x)g(x)\right)' = f'g + fg'$

Quotient rule:
$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{gf' - fg'}{g(x)^2}$

Integrals

$\int x^n = \frac{x^{n+1}}{n+1}$
$\int \frac{1}{x} = ln|x|$
$\int e^x = e^x$
$\int a^x = \frac{a^x}{\ln a}$
$\int \cos x = \sin x$
$\int \sin x = -\cos x$
$\int \tan x = -\ln|\cos x|$
$\int \sec x = \ln|\sec x + \tan x|$
$\int \csc x = -ln|cscx + cotx|$
$\int \cot x = ln|\sin x|$
$\int \sinh x = \cosh x$
$\int \cosh x = \sinh x$
$\int tanhx = ln(coshx)$
$\int sechx = tan^{-1}(sinhx)$
$\int xe^x = (x-1)e^x$
$\int xe^{ax} = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$
$\int x^2 e^{ax} = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)$
$\int \frac{x}{\sqrt{x \pm a}} = \frac{2}{3}(x \pm 2a)\sqrt{x \pm a}$
$\int \frac{x}{(x+a)^2} = \frac{a}{a+x} + ln(a+x)$

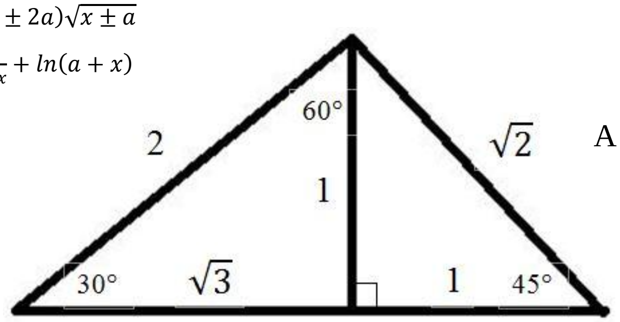
$\int \sec x \tan x = \sec x$
$\int \csc x \cot x = -\csc x$
$\int csc^2 x = -\cot x$
$\int sec^2 x = \tan x$
$\int sechx\ tanhx = -sechx$
$\int cschx\ cothx = -cschx$
$\int csch^2 x = -cothx$
$\int sech^2 x = tanhx$
$\int \frac{1}{\sqrt{a^2-x^2}} = sin^{-1}\left(\frac{x}{a}\right)$
$\int \frac{1}{a^2+x^2} = \frac{1}{a}tan^{-1}\left(\frac{x}{a}\right)$
$\int \frac{1}{|x|\sqrt{x^2-a^2}} = \frac{1}{a}sec^{-1}x$
$\int \frac{1}{a^2-x^2} = \frac{1}{2a}ln\left|\frac{x+a}{x-a}\right|$
$\int \frac{1}{x^2-a^2} = \frac{1}{2a}ln\left|\frac{x-a}{x+a}\right|$
$\int \frac{x^2}{a^2+x^2} = x - atan^{-1}\left(\frac{x}{a}\right)$
$\int \frac{x^3}{a^2+x^2} = \frac{1}{2}x^2 - \frac{1}{2}a^2 ln(a^2+x^2)$

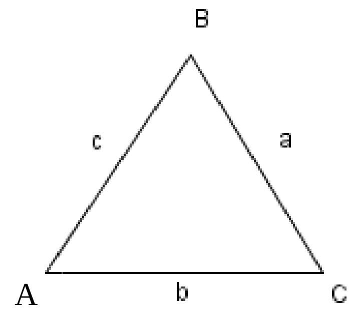
Substitution
If $\int f'(g(x))g'(x)dx$ let $u = g(x)$
and $du = g'(x)dx$

Integration by parts
$\int f(x)g'(x) = f(x)g(x) - \int g(x)f'(x)$

Partial fractions example:
$\int \frac{x^2+x+2}{x^2(x+1)} = \int \frac{A}{x} + \frac{Bx+C}{x^2} + \frac{D}{(x+1)}$

Trig substitution
If $\int \frac{1}{\sqrt{a^2-x^2}}dx$ then let $x = asin\Theta$
If $\int \frac{1}{\sqrt{x^2+a^2}}dx$ then let $x = atan\Theta$
If $\int \frac{1}{\sqrt{x^2-a^2}}dx$ then let $x = asec\Theta$





Law of cosines
$a^2 = b^2 + c^2 - 2bc\ cosA$
Law of sines
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{Sin\ C}$

Trigonometric identities

Fundamental
$\sin x = \frac{1}{\csc x}$
$\cos x = \frac{1}{\sec x}$
$\tan x = \frac{1}{\cot x}$
$\tan x = \frac{\sin x}{\cos x}$
$\cot x = \frac{\cos x}{\sin x}$
$sin^2 x + cos^2 x = 1$
$1 + tan^2 x = sec^2 x$
$1 + cot^2 x = csc^2 x$

Negatives
Odd functions
$\sin(-x) = -\sin x$
$\tan(-x) = -\tan x$
$\cot(-x) = -\cot x$
$\csc(-x) = -\csc x$
Even functions
$\cos(-x) = \cos x$
$\sec(-x) = \sec x$

Cofunction
$sin\left(\frac{\pi}{2} - x\right) = \cos x$
$cos\left(\frac{\pi}{2} - x\right) = \sin x$
$tan\left(\frac{\pi}{2} - x\right) = \cot x$
$cot\left(\frac{\pi}{2} - x\right) = \tan x$
$sec\left(\frac{\pi}{2} - x\right) = \csc x$
$csc\left(\frac{\pi}{2} - x\right) = \sec x$

Double angle
$\sin 2x = 2\sin x \cos x$
$\cos 2x = cos^2 x - sin^2 x$
$= 1 - 2sin^2 x$
$= 2cos^2 x - 1$
$\tan 2x = \frac{2\tan x}{1 - tan^2 x}$

Half angle
$sin^2 x = \frac{1 - \cos 2x}{2}$
$cos^2 x = \frac{1 + \cos 2x}{2}$
$tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$

Addition
$\sin(u+v) = sinu\ cosv + cosu\ sinv$
$\cos(u+v) = cosu\ cosv - sinu\ sinv$
$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$

Subtraction
$\sin(u-v) = sinu\ cosv - cosu\ sinv$
$\cos(u-v) = cosu\ cosv + sinu\ sinv$
$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$

Half angle
$sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{2}}$
$cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$
$tan\left(\frac{x}{2}\right) = \pm\frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$

Product-To-Sum
$sinu\ cosv = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$
$cosu\ sinv = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$
$cosu\ cosv = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$
$sinu\ sinv = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$

Sum-To-Product
$sinu + sinv = 2sin\left(\frac{u+v}{2}\right)cos\left(\frac{u-v}{2}\right)$
$sinu - sinv = 2cos\left(\frac{u+v}{2}\right)sin\left(\frac{u-v}{2}\right)$
$cosu + cosv = 2cos\left(\frac{u+v}{2}\right)cos\left(\frac{u-v}{2}\right)$
$cosu - cosv = -2sin\left(\frac{u+v}{2}\right)sin\left(\frac{u-v}{2}\right)$

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Exponentials

$x^m x^n = x^{m+n}$

$\dfrac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$

$x^{-m} = \dfrac{1}{x^m}$

$(xy)^m = x^m y^m$

$\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$

$x^{1/n} = \sqrt[n]{x}$

$x^{m/n} = \sqrt[n]{x^m}$

$\sqrt[m]{xy} = \sqrt[m]{x}\sqrt[m]{y}$

$\sqrt[m]{x/y} = \sqrt[m]{x}/\sqrt[m]{y}$

Logarithms

$log_a b = c \rightarrow a^c = b$

$log_a 1 = 0$

$log_a a = 1$

$log_a a^r = r$

$log_a M^r = r log_a M$

$a^x = e^{xlna}$

$log_a(MN) = log_a M + log_a N$

$log_a\left(\dfrac{M}{N}\right) = log_a M - log_a N$

If $M = N$ then $log_a M = log_a N$

Change of base formula:

$log_a M = \dfrac{log_b M}{log_b a}$

Summations

$\sum_{i=m}^{n} ca_i = c\sum_{i=m}^{n} a_i$ $\quad \sum_{i=m}^{n}(a_i + b_i) = \sum_{i=m}^{n} a_i + \sum_{i=m}^{n} b_i$ $\quad \sum_{i=m}^{n}(a_i - b_i) = \sum_{i=m}^{n} a_i - \sum_{i=m}^{n} b_i$ $\quad \sum_{i=m}^{n} c = cn$

$\sum_{i=m}^{n} i = \dfrac{n(n+1)}{2}$ $\quad \sum_{i=m}^{n} i^2 = \dfrac{n(n+1)(n+2)}{6}$ $\quad \sum_{i=m}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

2-D lines

$y - y_0 = m(x - x_o)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

3-D lines

Point (x_o, y_o, z_0) vector (a, b, c)

Parametric $(x_o + at, y_o + bt, z_o + ct)$

Symmetric $\frac{x - x_0}{a} = \frac{y - y_o}{b} = \frac{z - z_o}{c}$

Planes

Normal vector $\vec{n} = (a, b, c)$

Point on plane $P_o = (x_o, y_o, z_o)$

$(x - x_o)a + (y - y_o)b + (z - z_o)c = 0$

Critical points $\quad D = \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \frac{\partial^2 f}{\partial xy}$

$$\begin{bmatrix} D_p > 0 \; and \; if \; f_{xx} > 0 \; \rightarrow local \; minimum \\ D_p > 0 \; and \; if \; f_{xx} < 0 \; \rightarrow local \; maximum \\ D_p < 0 \rightarrow saddle \; point \\ D_p = 0 \rightarrow nonconclusive \end{bmatrix}$$

Fundamental theorem of calculus

$\int_a^b F'(x)dx = F(b) - F(a)$

Fundamental theorem of line integrals

$\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

Green's theorem

$\oint_C Pdx + Qdy = \iint_D \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}\right) dA$

Stokes theorem

$\int_C \vec{F} \cdot d\vec{l} = \iint_S \nabla x \vec{F} \cdot d\vec{S}$

Divergence theorem

$\iint_S \vec{F} \cdot d\,\vec{S} = \iiint_E \nabla \cdot \vec{F} dV$

Complex numbers

$z = a + bi$

$\Theta = tan^{-1}\left(\dfrac{b}{a}\right)$

$R = \sqrt{a^2 + b^2}$

$z = Re^{i\Theta} = Rcos\Theta + Ri\;sin\Theta$

Trig circular/hyperbolic relations

$cos\Theta = \dfrac{e^{i\Theta} + e^{-i\Theta}}{2}$

$sin\Theta = \dfrac{e^{i\Theta} - e^{-i\Theta}}{2i}$

$cosh\Theta = \dfrac{e^{\Theta} + e^{-\Theta}}{2}$

$sinh\Theta = \dfrac{e^{\Theta} - e^{-\Theta}}{2}$

$cosh\Theta = \cos(i\Theta)$

$sinh\Theta = \dfrac{1}{i}\sin(i\Theta)$

$cos\Theta = \cosh(i\Theta)$

$sin\Theta = \dfrac{1}{i}\sinh(i\Theta)$

L'hospitals rule

$\lim_{x\to a}\dfrac{f(x)}{g(x)} = \lim_{x\to a}\dfrac{f'(x)}{g'(x)}$

Dot product

$\vec{a} \cdot \vec{b} = |a||b|cos\Theta$

Cross product

$\vec{a}x\vec{b} = \begin{bmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$

Arc length

$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}$

Chain rule

If $f(x, y)$ and $x(t)$ and $y(t)$

Then $\dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial t}$

operation	Cartesian coordinates	Cylindrical coordinates	Spherical coordinates
Definition of coordinates		$\begin{bmatrix} x = \rho cos\varphi \\ y = \rho sin\varphi \\ z = z \end{bmatrix}$ $\begin{bmatrix} \rho = \sqrt{x^2 + y^2} \\ \varphi = tan^{-1}(y/x) \\ z = z \end{bmatrix}$	$\begin{bmatrix} x = rsin\Theta cos\varphi \\ y = rsin\Theta sin\varphi \\ z = rcos\Theta \end{bmatrix}$ $\begin{bmatrix} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = cos^{-1}(z/r) \\ \varphi = tan^{-1}(y/x) \end{bmatrix}$
Vector field $\vec{A}$	$A_X\hat{x} + A_Y\hat{y} + A_Z\hat{z}$	$A_\rho\hat{\rho} + A_\varphi\hat{\varphi} + A_Z\hat{z}$	$A_r\hat{r} + A_\Theta\hat{\Theta} + A_\varphi\hat{\varphi}$
Gradient ∇f	$\dfrac{\partial f}{\partial x}\hat{x} + \dfrac{\partial f}{\partial y}\hat{y} + \dfrac{\partial f}{\partial z}\hat{z}$	$\dfrac{\partial f}{\partial \rho}\hat{\rho} + \dfrac{1}{\rho}\dfrac{\partial f}{\partial \varphi}\hat{\varphi} + \dfrac{\partial f}{\partial z}\hat{z}$	$\dfrac{\partial f}{\partial r}\hat{r} + \dfrac{1}{r}\dfrac{\partial f}{\partial \Theta}\hat{\Theta} + \dfrac{1}{rsin\,\Theta}\dfrac{\partial f}{\partial \varphi}\hat{\varphi}$
Divergence $\nabla \cdot$ A	$\dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}$	$\dfrac{1}{\rho}\dfrac{\partial(\rho A_\rho)}{\partial \rho} + \dfrac{1}{\rho}\dfrac{\partial A_\varphi}{\partial \varphi} + \dfrac{\partial A_z}{\partial z}$	$\dfrac{1}{r^2}\dfrac{\partial(r^2 A_r)}{\partial r} + \dfrac{1}{rsin\,\Theta}\dfrac{\partial}{\partial \Theta}(A_\Theta \sin\Theta) + \dfrac{1}{rsin\,\Theta}\dfrac{\partial A_\varphi}{\partial \varphi}$
Differential displacement	$dl = dx\hat{x} + dy\hat{y} + dz\hat{z}$	$dl = d\rho\hat{\rho} + \rho d\varphi\hat{\varphi} + dz\hat{z}$	$dl = dr\hat{r} + rd\Theta\hat{\Theta} + rsin\Theta d\varphi\hat{\varphi}$
Differential normal area	$ds = dydz\hat{x} + dxdz\hat{y} + dxdy\hat{z}$	$ds = \rho d\varphi dz\hat{\rho} + d\rho dz\hat{\varphi} + \rho d\rho d\varphi\hat{z}$	$ds = r^2 sin\Theta d\Theta d\varphi\hat{r} + rsin\Theta drd\varphi\hat{\Theta} + rdrd\Theta\hat{\varphi}$
Dif. volume	$dv = dxdydz$	$dv = \rho d\rho d\varphi dz$	$dv = r^2 sin\Theta drd\Theta d\varphi$
Unit vectors		$\hat{x} = -sin\varphi\hat{\varphi} + cos\varphi\hat{\rho}$ $\hat{y} = cos\varphi\hat{\varphi} + sin\varphi\hat{\rho}$ $\hat{\rho} = cos\varphi\hat{x} + sin\varphi\hat{y}$ $\hat{\varphi} = -sin\varphi\hat{x} + cos\varphi\hat{y}$	$\hat{x} = sin\Theta cos\varphi\hat{r} + cos\Theta cos\varphi\hat{\Theta} - sin\varphi\hat{\varphi}$ $\hat{y} = sin\Theta sin\varphi\hat{r} + cos\Theta sin\varphi\hat{\Theta} + cos\varphi\hat{\varphi}$ $\hat{z} = cos\Theta\hat{r} - sin\Theta\hat{\Theta}$ $\hat{r} = sin\Theta cos\varphi\hat{x} + sin\Theta sin\varphi\hat{y} + cos\Theta\hat{z}$ $\hat{\Theta} = cos\Theta cos\varphi\hat{x} + cos\Theta sin\varphi\hat{y} - sin\Theta\hat{z}$ $\hat{\varphi} = -sin\Theta sin\varphi\hat{x} + sin\Theta cos\varphi\hat{y}$

Cylindrical $\nabla \times A$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\varphi} + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{z}$	Spherical $\nabla \times A$	$\frac{1}{rsin\theta}\left(\frac{\partial}{\partial \theta}(A_\varphi sin\theta) - \frac{\partial A_\theta}{\partial \varphi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right)\hat{\varphi}$
$\nabla^2 f$	$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$