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1.

(a) Geometric series:

$$S = \sum_{i=0}^{\infty} x^i = \frac{1}{1-x} = \frac{1}{1-\frac{1}{3}} = \frac{2}{3}$$

(b)

$$\sum_{i=4}^N 5^i = \frac{5^{N+1}}{4} - \frac{625}{4}$$

(c)

$$\sum_{i=0}^{10^N} i^3 + 9i - 2 = \frac{1}{4}(10^N + 1)(9 \times 2^{N+1} 5^N + 100^N + 1000^N - 9)$$

(d)

$$\sum_{i=1}^{10^N} \log_5 i = \log_5(10^N!) = 10^N \log_5(10^N) - 10^N + O(\log_5(10^N))$$

2.

(a)

$$x \cdot x^2 \cdot \dots \cdot x^N = x^{\frac{n(n+1)}{2}}$$

(b)

$$\log_n(n^6) = \frac{6 \log(n)}{\log(n)} = 6$$

(c)

$$17^{\log_{17} 280} = 280$$

(d)

$$\ln(\ln(e^{e^N})) = N$$

3.

(a)

$$\frac{n!}{(n-k)!} = \frac{10!}{(10-3)!} = 720$$

(b)

$$\frac{39!}{4!(39-4)!} = 82,250$$