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```
% Austin Welch
% EC503 HW7.2
% Ordinary Least Squares (OLS) versus Robust Linear Regression
```

load data

```
clear; clc;
rng default; % For reproducibility
load('linear_data.mat');
```

(a)

Implement ordinary least squares (OLS) linear regression for input (xdata) and output (ydata) provided in the linear_data.mat file. Do not use MATLAB's robustfit or regress. (i) Will the input data matrix (xdata) yield a unique solution? Why or why not? (ii) If h_OLS(x) = x^T*W_OLS + b_OLS, report the values of w_OLS and b_OLS and the resulting mean-squared error (MSE) and mean absolute error (MAE)

```
fprintf('Part (a):\n\n');
```

```
Part (a):
```

Normal equation:

$\theta = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$ (n << 10000 so normal equations should outperform gradient descent)

```
xData = xData';
yData = yData';
xDataExt = [xData; ones(1,length(xData))];
theta_ols = inv(xDataExt*xDataExt')*xDataExt*yData';
w_ols = theta_ols(1);
b_ols = theta_ols(2);

% uniqueness of solution
fprintf(['There should be a unique solution to the input data matrix
\n',...
    'xdata because (X*X^T) is non-singular/invertible/full-rank.\n
\n']);
```

```
% parameters
fprintf('w_ols: %0.4f\n', w_ols); % slope
fprintf('b_ols: %0.4f\n\n', b_ols); % intercept

% decision rule
h_ols = xData*w_ols+b_ols;

% quantify error
MSE_ols = sum((yData-h_ols).^2)/length(yData);
MAE_ols = sum(abs(yData-h_ols))/length(yData);
fprintf('MSE_ols: %0.4f\n', MSE_ols);
fprintf('MAE_ols: %0.4f\n\n', MAE_ols);
```

```
There should be a unique solution to the input data matrix
xdata because (X*X^T) is non-singular/invertible/full-rank.

w_ols: 0.0234
b_ols: 2.9738

MSE_ols: 0.2588
MAE_ols: 0.3175
```

visualize OLS regression line

```
%{
figure(1);
scatter(xData,yData);
hold on;
plot(xData,h_ols)
%}
% to check: above line and 'lsline' function are collinear
%lsline
```

(b)

Use MATLAB's robustfit function to implement robust linear regression for the input and output in linear_data.mat. There are two outliers in the dataset which skew the OLS regression model because it is not robust to outliers.

```
fprintf('Part (b):\n\n');

% loss functions (const, coefficient) with default tuning constants
cauchy = robustfit(xData,yData,'cauchy'); % w = 1 ./ (1 + r.^2)
fair = robustfit(xData,yData,'fair'); % w = 1 ./ (1 + abs(r))
huber = robustfit(xData,yData,'huber'); % w = 1 ./ max(1, abs(r))
talwar = robustfit(xData,yData,'talwar'); % w = 1 * (abs(r)<1)
ols = robustfit(xData,yData,'ols'); % same as (a), repeated for
 consistency

% estimates
hCauchy = xData*cauchy(2) + cauchy(1);
hFair = xData*fair(2) + fair(1);
```

```
hHuber = xData*huber(2) + huber(1);
hTalwar = xData*talwar(2) + talwar(1);
hOls = xData*ols(2) + ols(1);

% MSEs
MSEcauchy = mse(yData,hCauchy);
MSEfair = mse(yData,hFair);
MSEhuber = mse(yData,hHuber);
MSEtalwar = mse(yData,hTalwar);
MSEols = mse(yData,hOls);

% MAEs
MAEcauchy = mae(yData,hCauchy);
MAEfair = mae(yData,hFair);
MAEhuber = mae(yData,hHuber);
MAEtalwar = mae(yData,hTalwar);
MAEols = mae(yData,hOls);

% Compare robust errors to OLS error
Cauchy = [MSEcauchy; MAEcauchy];
Fair = [MSEfair; MAEfair];
Huber = [MSEhuber; MAEhuber];
Talwar = [MSEtalwar; MAEtalwar];
OLS = [MSEols; MAEols];
T = table(Cauchy, Fair, Huber, Talwar, OLS, 'RowNames',
 {'MSE', 'MAE'});
disp(T);
fprintf('OLS has lowest MSE, but highest MAE\n\n');

% report values of wHuber and bHuber
fprintf('w_huber: %0.4f\n', hHuber(2));
fprintf('b_huber: %0.4f\n\n', hHuber(1));

% plot ols and robust methods to compare
figure(2);
scatter(xData,yData,'filled'); grid on; hold on
plot(xData, hOls, 'k', 'LineWidth', 1);
plot(xData, hCauchy, 'b', 'LineWidth', 1);
plot(xData, hFair, 'g', 'LineWidth', 1);
plot(xData, hHuber, 'r', 'LineWidth', 1);
plot(xData, hTalwar, 'y', 'LineWidth', 1);
legend({'Data','OLS', 'Cauchy', 'Fair', 'Huber', 'Talwar'},'Box','off')
set(legend,'position',[0.15  0.12  0.1286  0.3])
set(gca,'fontsize',7)
title('Comparison of OLS to robust linear regression loss
 functions', ...
    'fontsize',7);
xlabel('X'); ylabel('Y');

% observations
fprintf(['The Cauchy loss function has the lowest MAE. All of the
\n', ...
    'robust loss functions are much less sensitive to the outliers
\n',...
```

```
    'compared to ordinary least squares regression.\n\n']);
```

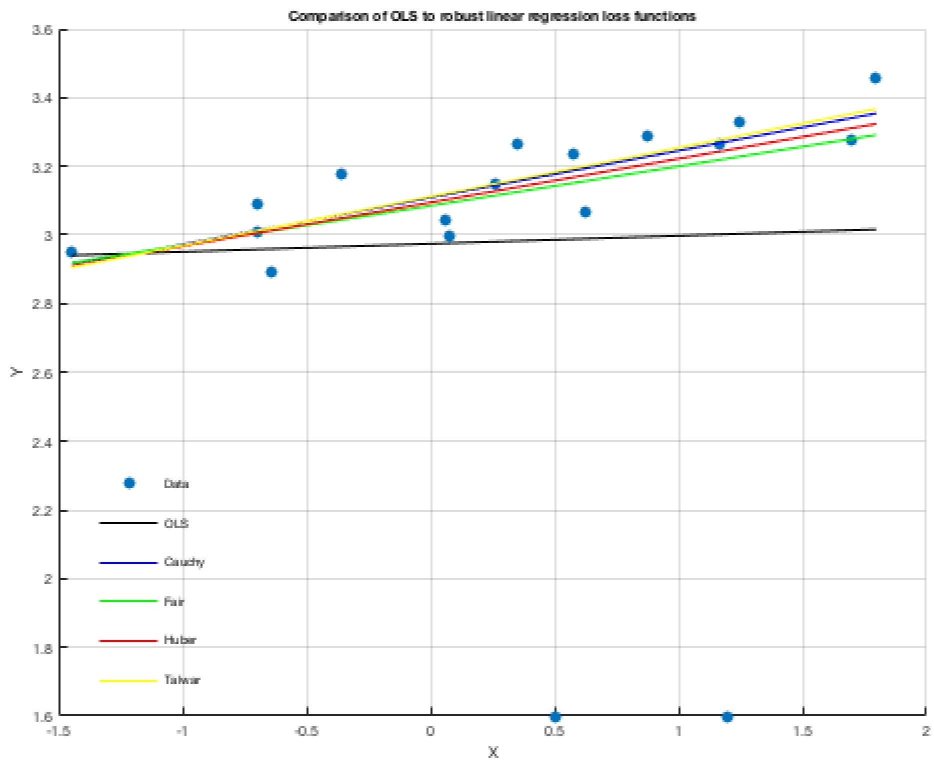
Part (b):

	Cauchy	Fair	Huber	Talwar	OLS
MSE	0.29955	0.28608	0.29218	0.30242	0.25884
MAE	0.24268	0.24683	0.24509	0.24349	0.31747

OLS has lowest MSE, but highest MAE

w_huber: 3.1743
b_huber: 3.2429

The Cauchy loss function has the lowest MAE. All of the robust loss functions are much less sensitive to the outliers compared to ordinary least squares regression.



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