Optimal Absorption and Scattering in Embedded Design Problems

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***Abstract*—A multi-objective optimization problem is constructed to determine bounds on absorption and scattering characteristics of structures confined to regions of arbitrary shape and material characteristics. The problem takes the form of a quadratically constrained quadratic program and is solvable by deterministic means. The method is extended to incorporate the effects of arbitrary proximity-coupled objects through numerical Green's function methods, allowing for the calculation of bounds on embedded design performance.**

***Index Terms*—antenna theory, physical bounds, optimization.**

I. Introduction

Many properties of electromagnetic systems are subject to fundamental bounds based on electrical size and material properties, in some cases greatly limiting the potential performance of realized systems. Recently, a variety of methods have been developed enabling the rigorous calculation of physical bounds on Q-factor [1], efficiency [2]–[4], gain [5], and various cross sections [6], [7] for a wide class of problems involving arbitrary geometries and inhomogeneous media. Additionally, several of these bounds can be combined into multi-objective optimization problems [8], [9], enabling the study of trade-offs between optimal performance characteristics [10], [11].

In this paper, we extend the multi-objective calculation of bounds on absorption and scattering cross sections to cases involving embedded design regions, i.e., scenarios where a designer has the ability to alter part, but not all, of a system. This situation frequently arises in chassis- or vehicle-mounted antennas and the underlying mathematical methods are closely related to the numerical evaluation of problem-specific Green's functions [12], [13].

II. Partially controllable systems

Consider the situation where an electric field is incident upon two regions Ω_{c} and Ω_{u}. We consider the region Ω_{u} to be *uncontrollable* in the sense that its geometry and material properties may not be manipulated by a designer. Within the *controllable* region Ω_{c}, however, we allow for the arbitrary configurations of vacuum and a material with prescribed properties. To study bounds on the scattering and absorption responses of all possible design configurations within the controllable region, we begin by forming the electric field integral equation for the entire controllable and uncontrollable regions as a partitioned matrix system

(1)

where $\mathbf{V}$ and $\mathbf{I}$ are representations of the incident field and induced current while $\mathbf{Z}$ is the method of moments impedance matrix [14]. The uncontrollable region current $\mathbf{I}_{\mathrm{u}}$ may be written in terms of the controllable region currents $\mathbf{I}_{\mathrm{c}}$ and excitation $\mathbf{V}$. Doing so enables the rewriting of any quadratic form in the total current $\mathbf{I}$ solely in terms of controllable region current, i.e.,

$$\mathbf{I}^{\mathrm{H}}\mathbf{A}\mathbf{I} = \mathbf{I}_{\mathrm{c}}^{\mathrm{H}}\tilde{\mathbf{A}}\mathbf{I}_{\mathrm{c}} + \mathrm{Re}\left\{\mathbf{I}_{\mathrm{c}}^{\mathrm{H}}\tilde{\mathbf{a}}\right\} + \tilde{a}_0, \tag{2}$$

where the new terms $\tilde{\mathbf{A}}$, $\tilde{\mathbf{a}}$, and $\tilde{a}_0$ all implicitly depend on the excitation field. Many physical quantities of interest can be constructed in this way, including radiated power, absorbed power, and stored energy [15].

III. Optimization problems

With functionals of the form of (2), we may construct optimization problems seeking to maximize or minimize certain physical quantities by searching over all possible controllable region currents $\mathbf{I}_{\mathrm{c}}$. Assuming a basis that allows for arbitrary local portions of the current $\mathbf{I}_{\mathrm{c}}$ to be set to zero, such problems inherently cover all possible substructures within this region. Naturally, these optimization problems serve as tight, realistic bounds only when appropriate constraints are applied. In the scenario studied here, the relationship between the controllable and uncontrollable regions may be enforced by one such constraint. Additionally, conservation of complex extincted power is enforceable by including a relaxed form of (**??**) tested with the total current. Both single- and multi-objective problems may be formed in this way using weighted objective functionals.

A multi-objective problem bounding all possible absorption and scattering cross sections (see [11]) of an embedded design may be written as

$$\begin{aligned} \max \quad & w_{\mathrm{a}}P_{\mathrm{a}}(\mathbf{I}) + w_{\mathrm{s}}P_{\mathrm{s}}(\mathbf{I}) \\ \text{s.t.} \quad & P_{\mathrm{a}}(\mathbf{I}) + P_{\mathrm{s}}(\mathbf{I}) = P_{\mathrm{t}}(\mathbf{I},\mathbf{V}) \\ & W(\mathbf{I}) = W_{\mathrm{t}}(\mathbf{I},\mathbf{V}) \\ & \mathbf{I}_{\mathrm{u}} = \mathbf{C}\mathbf{V}_{\mathrm{u}} + \mathbf{D}\mathbf{I}_{\mathrm{c}} \end{aligned} \tag{3}$$

where w_{a} and w_{s} are real weights, all quantities of the form $A(\mathbf{I})$ are expressible in the form of (2), and quantities of

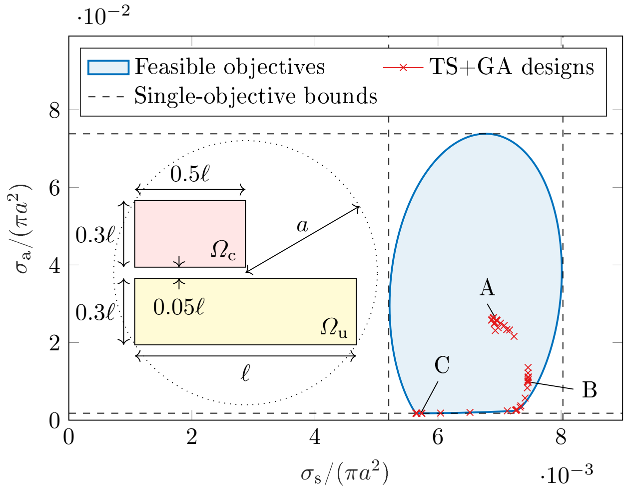

Fig. 1. Feasible and realized objectives in scattering and absorption cross sections σ_s and σ_a. In this example, the entire system has electrical size $ka = 0.5$ and the controllable and uncontrollable regions have surface resistivity $R_{s,c} = 10\ \Omega$ and $R_{s,u} = 1\ \Omega$, respectively.

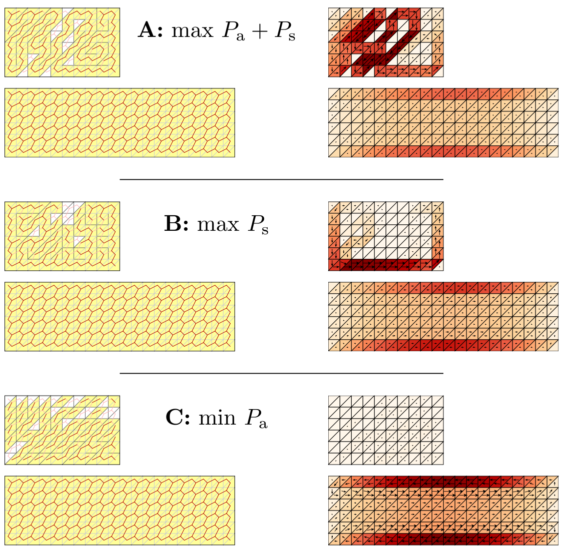

Fig. 2. Example designs produced by TS+GA (left) and their driven current distributions (right), labelled to correspond to points on Fig. 1. Dark red lines within each design represent current paths (basis functions) not removed by the inverse design algorithm.

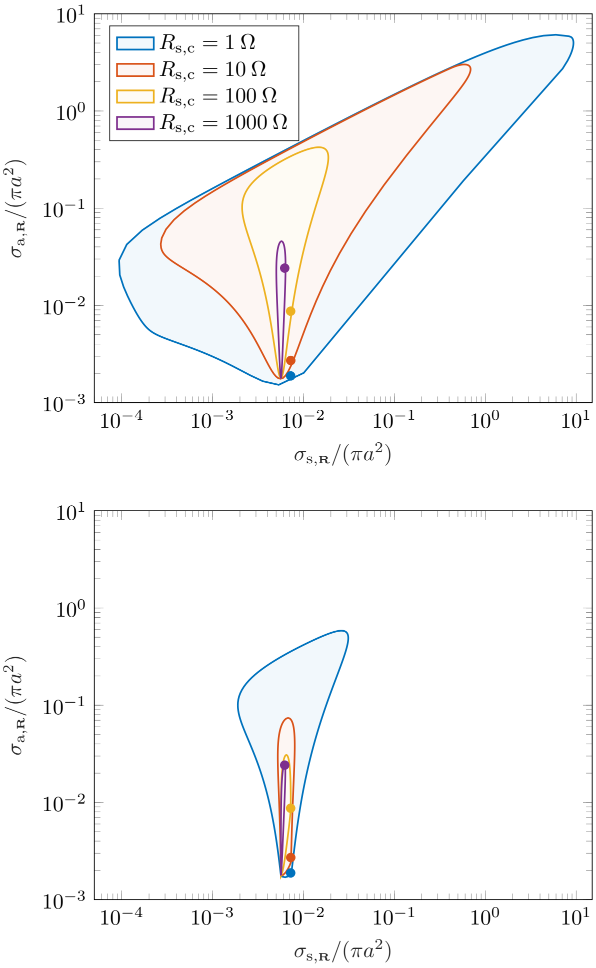

Fig. 3. Feasible objectives in scattering and absorption for the structure considered in Fig. 1 with electrical size $ka = 0.5$. In this example, the uncontrollable has a fixed value of $R_{s,u} = 1\,\Omega$ while the controllable region surface resistivity is swept with values $R_{s,c} \in \{1, 10, 100, 1000\}\,\Omega$. The top panel corresponds to optimization with only real power constraints, and the bottom considers complex power balance constraints. The markers correspond to the driven solution to the unaltered plates for each surface resistivity.

the form $B(\mathbf{I}, \mathbf{V})$ depend on both the current and excitation field. Here P_a, P_s, and P_t are absorbed, scattered, and extincted powers. The first two constraints in (3) enforce complex power balance, while the final constraint enforces the relationship between the controllable and uncontrollable regions. By sweeping over the weights w_a and w_s, a two-dimensional range of feasible scattering and absorption cross sections σ_s and σ_a may be found.

As an example, we consider the structure shown in Fig. 1 consisting of controllable and uncontrollable regions Ω_c and Ω_u. In this example, the controllable and uncontrollable regions are allowed to have independent surface resistivities $R_{s,c}$ and $R_{s,u}$, respectively. The weights in (3) are assigned as

$$w_a = \cos\phi, \quad w_s = \sin\phi \tag{4}$$

with $\phi \in [0, 2\pi]$ to sweep over all possible weighted maximizations and minimizations of scattered and absorbed power. For each set of weights, the problem in (3) is solved and the resulting scattering and absorption cross sections σ_s and σ_a are calculated. The locus of these points bounds a feasible region in σ_s-σ_a space, shown as a shaded region in Fig. 1. Only points within this feasible region may be reached by designs consisting of substructures of Ω_c, though the actual existence of such a substructure for each point within the feasible region is not guaranteed.

To examine how much of the feasible region is realizable, we synthesize candidate designs using a inverse design method (TS+GA) combining topology sensitivity [16] with genetic algorithms [17]. Several candidate designs, along with their induced current distributions, are shown in Fig. 2. Without the prior calculation of the bounds shown in Fig. 1, the relative optimality of any one of these synthesized designs would be unknown, i.e., the designer would have no indication if further design efforts may yield improved results. Hence, this example set of calculations and designs demonstrates the relevance of physical bounds to the development of inverse design methodologies.

As an extension of the first example, the effect of varying the controllable region resistivity and the resulting feasible objective spaces are shown in Fig. 3. The problem setup differs only in the controllable region surface resistivity is now analyzed over a set of values, while the uncontrollable region value remains fixed. The resulting feasible objective spaces exhibit the material dependence that scattering and absorption have for this particular structure. The top panel shows scattering and absorption objectives with only real power and $\Omega_{\mathrm{c}}/\Omega_{\mathrm{u}}$ current relation constraints. Inclusion of only the real power constraint leads to bounds on all possible materials with fixed losses, including those with tunable reactance. We observe in this case that the increase in controllable region loss predominately impacts the system's ability to control scattering, while absorption control is impacted to a much lesser degree. The bottom panel considers the same problem with an additional imaginary power balance constraint, i.e., fully prescribed material properties in both loss and reactance. In that case, we see similar trends though the overall feasible region is greatly reduced.

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