1. A microcontroller system has a 12 bit AD converter, 10,000 bytes of available RAM, and samples stereo audio signal with sampling frequency of 20 KHz. How many seconds of audio signal can be stored in memory?

T = 10000 bytes / (2 channels * 2 bytes/sample * 20000 samples/s) = 0.125 s (125ms)

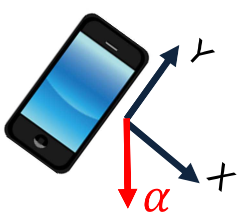
or optimized:

T = 10000 bytes / (2 channels * 1.5 bytes/sample * 20000 samples/s) = 0.166 s (166ms)

2. The output of a causal LTI system with the impulse response $h(t)$ to a causal input $x(t)$ is

$$y(t) = \int_0^t x(\tau)h(t-\tau)d\tau$$

3. Accelerometer (± 2g) with analog output and power supply of +3V is used in smartphone to determine orientation of the smartphone according to the figure below. What are the values of X and Y components of the accelerometer for $\alpha = 45°$.



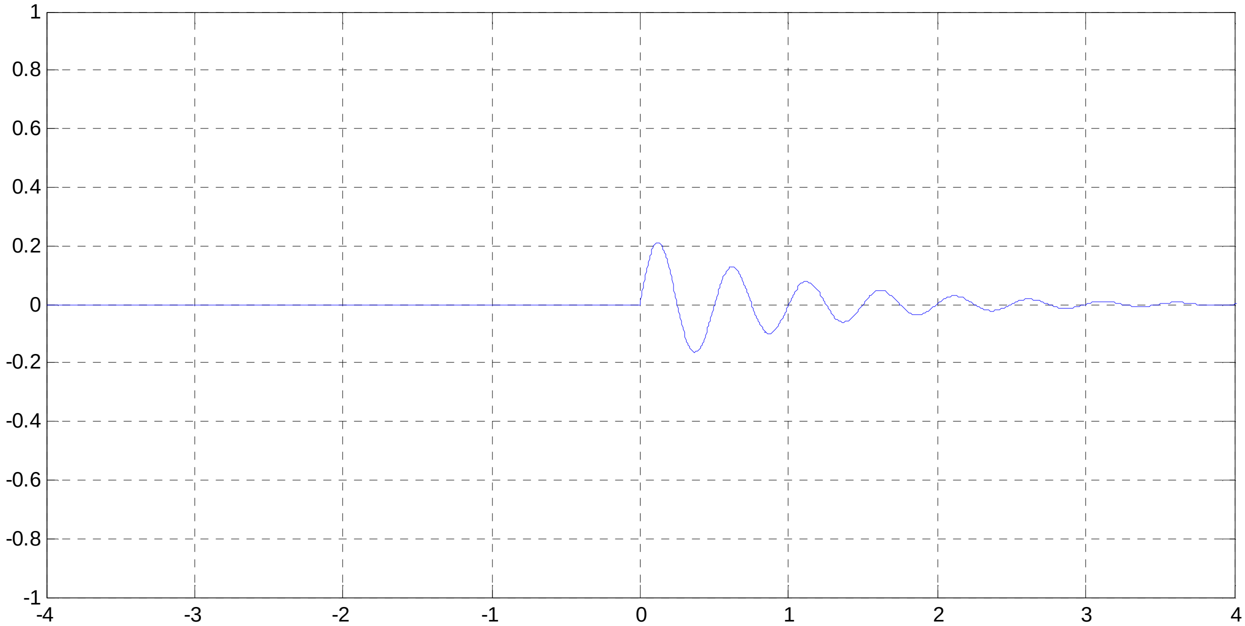
Sensitivity of the accelerometer 1*g* → *s = 3V / 4 g* = 0.75 [V / g]

A_0 (0 g) = 1.5V

$A_X = A_0 + 1g * \cos(\pi/4) = 2.03$ V

$A_Y = A_0 + -(1g * \sin(\pi/4)) = 0.97$ V

4. $x(t) = \frac{3}{4\pi} e^{-t} \cdot \sin(4\pi t) \cdot u(t)$



5. Consider the periodic signal x(t) = cos(0.4π·*t*) + 3·cos(2 π·*t/7*), −∞ < t < ∞..

Is x(t) periodic? If it is, what is the period T_0 of $x(t)$? T_0 = 35 s

$x(t) = x_1(t) + x_2(t)$

$T_1 = 2\pi / (0.2\pi) = 5$ s

$T_2 = 2\pi / (2\pi/7) = 7$ s

$T_0 = N*T_1 = M*T_2$ → The least common multiple of 5 and 7 is 35, therefore 7N = 5M → $T_0 = 7*5 = 35$ s

What is the average power of $x(t)$?

$$\int_0^x \cos^2(x)dx = \int_0^x \frac{1}{2}(1+\cos(2x)) = \frac{1}{2}\int_0^x dx + \frac{1}{4}\int_0^x \cos(y)dy = \left(\frac{x}{2}+\frac{1}{4}\sin(2x)\right)\Bigg|_0^x$$

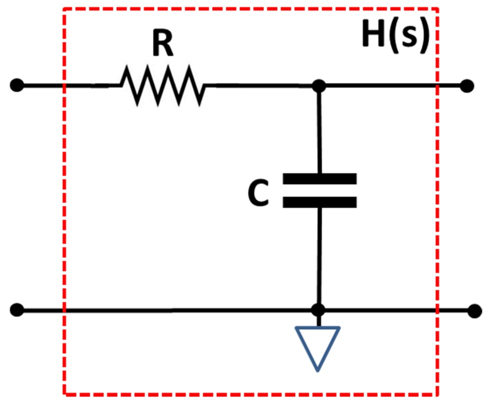
$$\int_0^t \cos^2(x)dx = \left(\frac{x}{2}+\frac{1}{4}\sin(2x)\right)\Bigg|_0^t = \frac{t}{2}+\frac{1}{4}\sin(2t) \Rightarrow \textit{ for } t = T \int_0^T \cos^2(x)dx = \frac{1}{2}$$

$$P_{x1} = \frac{1}{T_1}\int_0^{T_1} x_1^{\,2}(t)dt = \frac{1}{0.5}\cdot\left(\frac{x}{2}+\frac{1}{4}\sin(2x)\right)\Bigg|_0^{T_1} = \frac{1}{T_1}\left(\frac{T_1}{2}+\frac{1}{4}\sin\left(12\pi\cdot\frac{1}{6}\right)\right) = 1\cdot\frac{1}{2} = 0.5$$

$$P_{x2} = \frac{1}{T_2}\int_0^{T_2} x_2^{\,2}(t)dt = \frac{1}{T_2}\int_0^{T_2} (3\cos(16\pi t))^2\,dt = 9\cdot\frac{1}{T_2}\int_0^{T_2}\cos^2(16\pi\, t)dt = 9\cdot\frac{1}{2} = 4.5$$

$P = P_{x1} + P_{x2} = 0.5 + 4.5 = 5$

6. (4 points)



$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

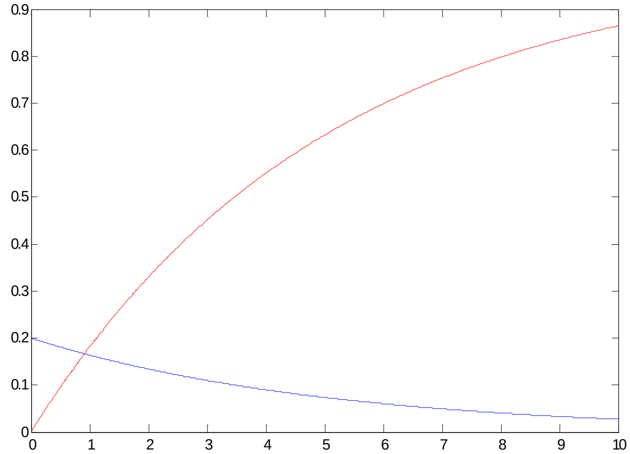
$$h(t) = \frac{1}{RC} \cdot e^{-\frac{t}{RC}}$$

b) (6 points) Step response:

$$\frac{1}{RC} = \frac{1}{5M\Omega \cdot 1\mu F} = 0.2$$

$$S(s) = \frac{1}{s} H(s) = \frac{1}{s} \frac{0.2}{s+0.2} = \frac{A}{s} + \frac{B}{s+0.2} = \frac{1}{s} - \frac{1}{s+0.2}$$

$$s(t) = \left(1 - e^{-0.2 \cdot t}\right) \cdot u(t)$$



7. A system with input x(t) and output y(t) is defined by the following differential equation:

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=x(t)$$

Find the impulse response *h(t)* and the unit-step response *s(t)*.

The Laplace transform of the differential equation gives

$$[s^2Y(s)-sy(0)-\frac{dy(t)}{dt}\,|_{t=0}]+3[sY(s)-y(0)]+2Y(s)=X(s)$$
$$Y(s)(s^2+3s+2)-(s+3)=X(s)$$

so we have that

$$\begin{aligned} Y(s) &= \frac{X(s)}{(s+1)(s+2)}+\frac{s+3}{(s+1)(s+2)} \\ &= \frac{1+3s+s^2}{s(s+1)(s+2)}=\frac{B_1}{s}+\frac{B_2}{s+1}+\frac{B_3}{s+2} \end{aligned}$$

We find B_1 = 0.5, B_2 = 1, and B_3 = - 0.5.

therefore:

$$y(t)=\left[0.5+e^{-t}-0.5\cdot e^{-2t}\right]\cdot u(t)$$

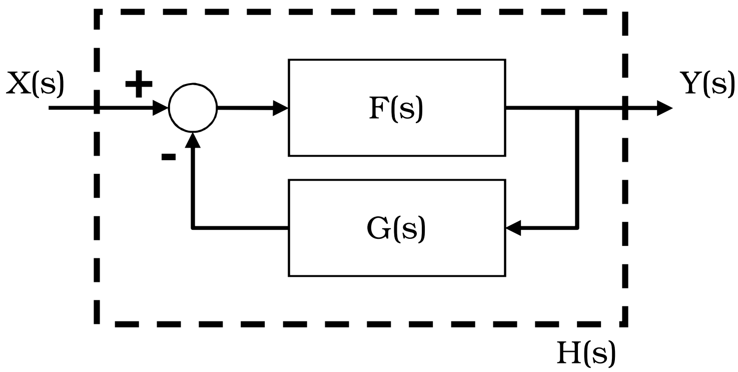
steady state response is

$$0.5\cdot u(t)$$

and transient response is

$$\left[e^{-t}-0.5\cdot e^{-2t}\right]\cdot u(t)$$

8. (5 points) What is the transfer function H(s) of the system represented below?



Y(s) = F(s)*(X(s) - G(s)*Y(s)) = F(s)*X(s) - F(s)*G(s)*Y(s)

$$H(s) = \frac{Y(s)}{X(s)} = \frac{F(s)}{1 + F(s) \cdot G(s)}$$

b) (10 points) Transfer function of the feedback block is

$$G(s) = \frac{Z_{R||C}}{Ls + Z_{R||C}}$$

$$Z_{R||C} = \frac{R\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{R}{RCs + 1}$$

$$G(s) = \frac{R}{R + Ls(RCs + 1)} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Transfer function of the system is:

$$H(s) = \frac{F(s)}{1 + F(s) \cdot G(s)}$$

$F(s) = A$ and $G(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$

$$H(s) = \frac{A}{1 + A\left(\frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\right)} for\ A \to \infty\ H(s) = LCs^2 + \frac{L}{R}s + 1$$

9.