Sorting

CPE 212 -- Lecture 18

Sorting

- Arranging a collection of data values into an order
  - Examples:
    - Alphabetizing a list of names
    - Sorting exams by grade received

Sorting Algorithm Efficiency

- Speed
  - Number of comparisons made
  - Number of swaps required
- Space
  - Memory space required
- Typical Tradeoff
  - More memory => less time required

$O(N^2)$ Sorting Algorithms

- Selection Sort
- Bubble Sort
- Insertion Sort

Selection Sort Example

Number Of Elements N = 4

Unsorted

Sorted

values (i=0)

[0]	18
[1]	9
[2]	5
[3]	10

i=0

values (i=1)

[0]	5
[1]	9
[2]	18
[3]	10

i=1

values (i=2)

[0]	5
[1]	9
[2]	18
[3]	10

i=2

values (i=3)

[0]	5
[1]	9
[2]	10
[3]	18

i=3

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Selection Sort Algorithm

Concept: Repeatedly select smallest unsorted element and move it to the front of the unsorted elements

Suppose the unsorted elements are in variables A_1 through A_N

For each pass i for 1 <= i <= N-1,

- Find the smallest unsorted element in the range A_i through A_N
- Swap that element with element A_i
- Now A_1 through A_i contain the i smallest elements
- Repeat for each i listed above

Selection Sort Implementation

```
template<class ItemType>
void Swap(ItemType& item1, ItemType& item2)
// Post: Contents of item1 and item2 have been swapped
{
  ItemType tempItem;

  tempItem = item1;
  item1 = item2;
  item2 = tempItem;
}

template<class ItemType>
int MinIndex(ItemType values[], int startIndex, int endIndex)
// Post: Returns the index of the smallest value in
//       values[startIndex]..values[endIndex].
{
  int indexOfMin = startIndex;
  for (int index = startIndex + 1; index <= endIndex; index++)
    if (values[index] < values[indexOfMin])
      indexOfMin = index;
  return indexOfMin;
}

template<class ItemType>
void SelectionSort(ItemType values[], int numValues)
// Post: The elements in the array values are sorted by key.
{
  int endIndex = numValues-1;
  for (int current = 0; current < endIndex; current++)
    Swap(values[current], values[MinIndex(values, current, endIndex)]);
}
```

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Selection Sort Analysis

- Comparison Analysis
  - First position requires N-1 comparisons
  - Second position requires N-2 comparisons
  - Third position requires N-3 comparisons

  …
  - Position N-1 requires 1 comparison

$$(N-1) + (N-2) + (N-3) + \ldots + 1 = N(N-1)/2 = O(N^2)$$

- Swap Analysis
  - Number of Swaps Performed = N - 1 = O(N)

Bubble Sort Example

Number Of Unsorted Elements
N = 4

Unsorted
Sorted

values

[0]	18
[1]	10
[2]	9
[3]	5

values

[0]	18
[1]	10
[2]	5
[3]	9

i=0

3 comparisons

values

[0]	18
[1]	5
[2]	10
[3]	9

values

[0]	5
[1]	18
[2]	10
[3]	9

Bubble Sort Example

Number Of Unsorted Elements N = 3

Unsorted

Sorted

values

[0]	5
[1]	18
[2]	10
[3]	9

values

[0]	5
[1]	18
[2]	9
[3]	10

i=1

2 comparisons

values

[0]	5
[1]	9
[2]	18
[3]	10

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Bubble Sort Example

Number Of Unsorted Elements N = 2

☐ **Unsorted**
■ **Sorted**

values

[0]	5
[1]	9
[2]	18
[3]	10

i=2

1 comparison

values

[0]	5
[1]	9
[2]	10
[3]	18

Bubble Sort Algorithm

Concept: “Lighter” elements bubble to the top

Suppose the unsorted elements are in variables A_1 through A_N

For each pass i for 1 <= i <= N-1,

- Bubble the smallest unsorted element in the range A_i through A_N up to position A_i by swapping as needed
- Now A_1 through A_i contain the i smallest elements
- Shrink the unsorted porition of the array by incrementing i across the range listed above

Bubble Sort Implementation

```
template<class ItemType>
void BubbleUp(ItemType values[], int startIndex, int endIndex)
// Post: Adjacent pairs that are out of order have been
//       switched between values[startIndex]..values[endIndex]
//       beginning at values[endIndex].
{
  for (int index = endIndex; index > startIndex; index--)
    if (values[index] < values[index-1])
      Swap(values[index], values[index-1]);
}


template<class ItemType>
void BubbleSort(ItemType values[], int numValues)
// Post: The elements in the array values are sorted by key.
{
  int current = 0;

  while (current < numValues - 1)
  {
    BubbleUp(values, current, numValues-1);
    current++;
  }
}
```

Bubble Sort Analysis

- Comparison Analysis
  - First position requires N-1 comparisons
  - Second position requires N-2 comparisons
  - Third position requires N-3 comparisons

  …
  - Position N-1 requires 1 comparison

$$(N-1) + (N-2) + (N-3) + \ldots + 1 = N(N-1)/2 = O(N^2)$$

- Swap Analysis
  - Number of Swaps in Worst Case = $N(N-1)/2 \approx N^2/2$
  - Average Number of Swaps $\approx 0.5*(N^2/2) = N^2/4$

ShortBubble Modification

```
template<class ItemType>
void ShortBubble(ItemType values[], int numValues)
// Post: The elements in the array values are sorted by key.
//       The process stops as soon as values is sorted.
{
  int current = 0;
  bool sorted = false;
  while (current < numValues - 1 && !sorted)
  {
    BubbleUp2(values, current, numValues-1, sorted);
    current++;
  }
}


template<class ItemType>
void BubbleUp2(ItemType values[], int startIndex, int endIndex, bool& sorted)
// Post: Adjacent pairs that are out of order have been switched
//       between values[startIndex]..values[endIndex] beginning at
//       values[endIndex].
//       sorted is false if a swap was made; otherwise, true.
{
  sorted = true;
  for (int index = endIndex; index > startIndex; index--)
    if (values[index] < values[index-1])
    {
      Swap(values[index], values[index-1]);
      sorted = false;
    }
}
```

ShortBubble Efficiency

- Also an $O(N^2)$ algorithm
- Only algorithm that recognizes an already sorted list and terminates

- See text for more details

Insertion Sort Example

Number Of Elements N = 4

Unsorted

Sorted

	values
[0]	18
[1]	9
[2]	5
[3]	10

i=0

	values
[0]	9
[1]	18
[2]	5
[3]	10

i=1

	values
[0]	5
[1]	9
[2]	18
[3]	10

i=2

	values
[0]	5
[1]	9
[2]	10
[3]	18

i=3

Insertion Sort Algorithm

Concept:

Create in place a sorted list by taking the values and inserting them in order into the list

Suppose the unsorted elements are in variables A_1 through A_N

For each pass i for 1 <= i <= N-1,

- Take the current element A_i and insert it into it proper position among the elements A_1 through A_{i-1}
- Now A_1 through A_i contain the i smallest elements in sorted order
- Repeat for each i listed above

Insertion Sort Implementation

```
template<class ItemType>
void InsertItem(ItemType values[], int startIndex, int endIndex)
// Post: values[0]..values[endIndex] are now sorted.
{
  bool finished = false;
  int current = endIndex;
  bool moreToSearch = (current != startIndex);

  while (moreToSearch && !finished)
  {
    if (values[current] < values[current-1])
    {
      Swap(values[current], values[current-1]);
      current--;
      moreToSearch = (current != startIndex);
    }
    else
      finished = true;
  }
}


template<class ItemType>
void InsertionSort(ItemType values[], int numValues)
// Post: The elements in the array values are sorted by key.
{
  for (int count = 0; count < numValues; count++)
    InsertItem(values, 0, count);
}
```

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Insertion Sort Analysis

- Comparison Analysis
  - Similar to previous examples
    $O(N^2)$

- Swap Analysis
  - Average Number of Swaps ≈ $N^2/4$

Coping with High Swap Cost

- If elements are large records, an algorithm which requires $O(N^2)$ swaps will have a significant impact on overall execution time
- Instead of swapping the records, maintain an array of pointers to the records, and reorder the array of pointers using the sorting algorithm
- Once the pointers have been rearranged, the records themselves can be reordered at $O(N)$ cost

$O(N^2)$ Sorting Algorithms

- Selection Sort
- Bubble Sort
  - ShortBubble
- Insertion Sort

Sorting Algorithms

Utilizing Divide and Conquer

More Sorting Algorithms

- General Tradeoff
  - Algorithm complexity versus Performance

- Quicksort
- Mergesort

Quicksort

- ***Quicksort*** Algorithm

if there is more than one item in ***values[first]…values[last]***

Select ***splitVal***

Split the array so that

values[first]…values[splitPoint-1] <= ***splitVal***

values[splitPoint] = ***splitVal***

values[splitPoint+1]…values[last] > ***splitVal***

QuickSort the left half

QuickSort the right half

Quicksort Implementation - 1

```
typedef int ItemType;



template<class ItemType>
void QuickSort(ItemType values[], int first, int last)
{
  if (first < last)
  {
    int splitPoint;

    Split(values, first, last, splitPoint);
    // values[first]..values[splitPoint-1] <= splitVal
    // values[splitPoint] = splitVal
    // values[splitPoint+1]..values[last] > splitVal

    QuickSort(values, first, splitPoint-1);
    QuickSort(values, splitPoint+1, last);
  }
}
```

Quicksort Example

Initial Order

9	20	6	10	14	8	60	11

Split Element
(first element)

9	20	6	10	14	8	60	11

9	8	6	10	14	20	60	11

<= Split

> Split

Swap Split
With Element
At Split Point

6	8	9	10	14	20	60	11

6	8	9

10	14	20	60	11

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Quicksort Implementation - 2

```
void Split(ItemType values[], int first, int last, int& splitPoint)
{
  ItemType splitVal = values[first];
  int saveFirst = first;
  bool onCorrectSide;

  first++;
  do
  {
    onCorrectSide = true;
    while (onCorrectSide)              // Move first toward last.
      if (values[first] > splitVal)
        onCorrectSide = false;
      else
      {
        first++;
        onCorrectSide = (first <= last);
      }

    onCorrectSide = (first <= last);
    while (onCorrectSide)             // Move last toward first.
      if (values[last] <= splitVal)
        onCorrectSide = false;
      else
      {
        last--;
        onCorrectSide = (first <= last);
      }

    if (first < last)
    {
      Swap(values[first], values[last]);
      first++;
      last--;
    }
  } while (first <= last);

  splitPoint = last;
  Swap(values[saveFirst], values[splitPoint]);
}
```

Split Function - 1

Initialization

9	20	6	10	14	8	60	11
Save First	first						last

Increment first
Until
value[first]>splitVal

9	20	6	10	14	8	60	11
Save First	first						last

Decrement last
Until
value[last]<=splitVal

9	20	6	10	14	8	60	11
Save First	first				last		

Split Function - 2

Swap value[first] & value[last]
Then move first and last
toward each other

9	8	6	10	14	20	60	11
Save First		first		last			

Increment first Until value[first]>splitVal
Decrement last Until value[last]<=splitVal
Or first > last

9	8	6	10	14	20	60	11
Save First		last	first				

first>last so no swap within loop
Swap value[savefirst] & value[last]

6	8	9	10	14	20	60	11
Save First		last Split point					

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Comments on Quicksort

- In-place sorting algorithm
- Non-recursive case (base case)
  - Segment contains at most one element
- Each recursive case splits problem into smaller, not necessarily even, pieces
- Algorithm works even if splitting value is the smallest or largest element in the subarray
  - Efficiency suffers
- Worst case: array sorted or mostly sorted
  - $O(N^2)$
- Best case: each split produces equal size subarrays
  - $O(N\log_2 N)$

Mergesort

- $O(N \cdot \log_2 N)$
- ***Mergesort*** Algorithm

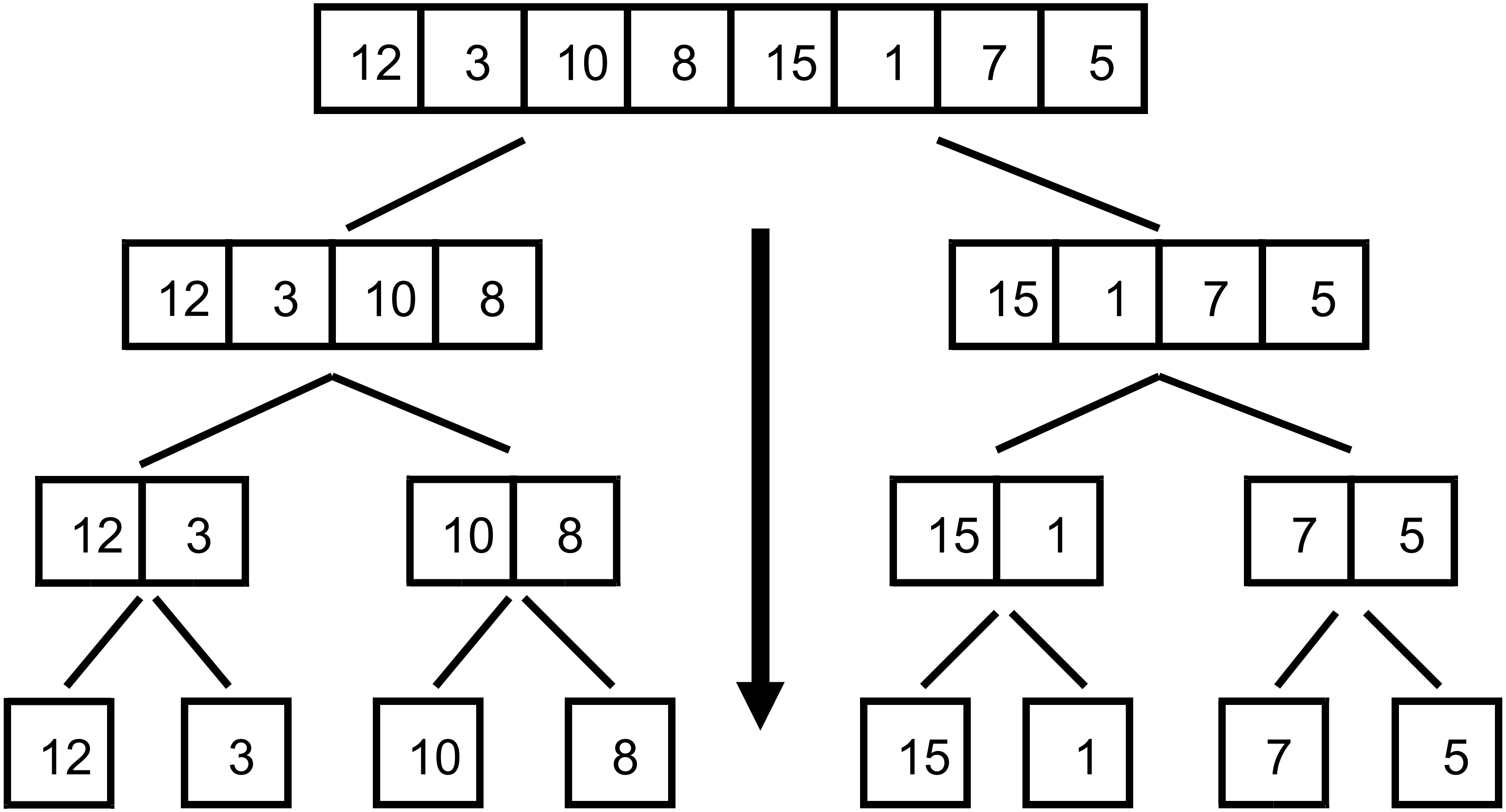
  Cut the array in half

  MergeSort the left half

  MergeSort the right half

  Merge the two sorted halves into one sorted array

Mergesort Example



Recursive Splitting

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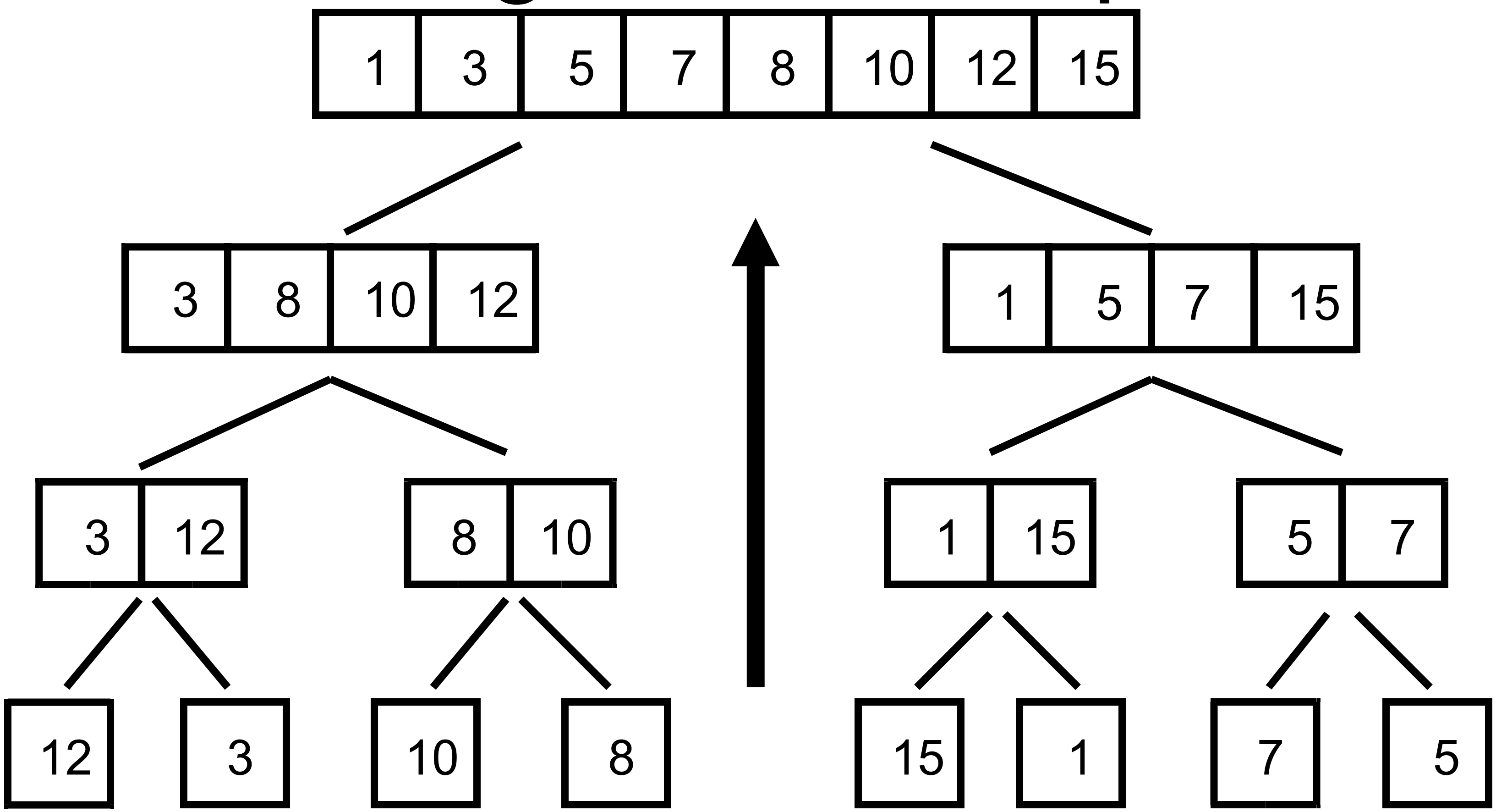
Mergesort Example



Merging Sorted SubArrays

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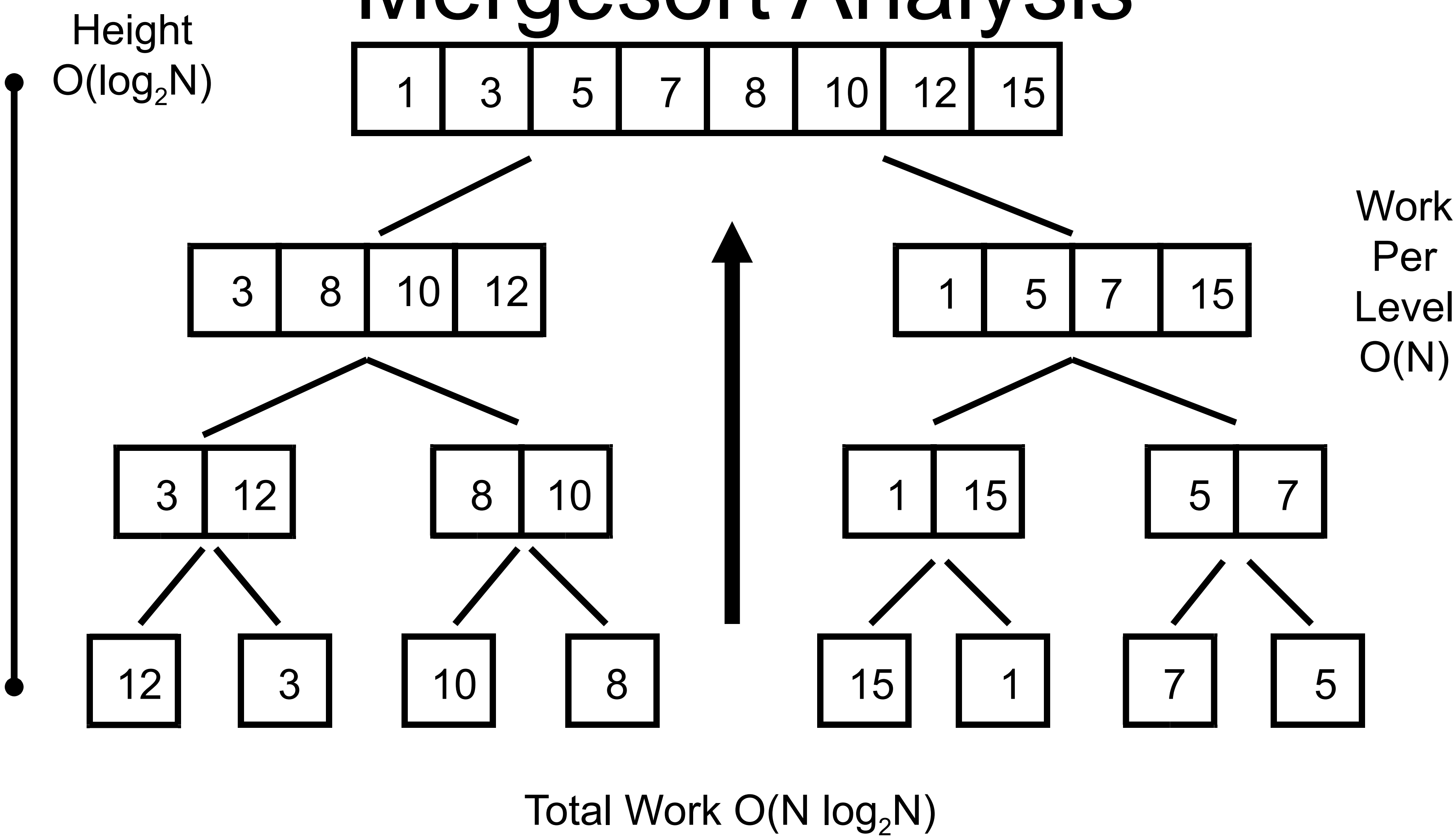
Mergesort Analysis



Mergesort Implementation

```
template<class ItemType>
void MergeSort(ItemType values[], int first, int last)
// first = starting index
// last = ending index
// Post: The elements in values are sorted by key.
{
  if (first < last)
  {
    int middle = (first + last) / 2;             // Calculate split index
    MergeSort(values, first, middle);            // Sort left half of array
    MergeSort(values, middle + 1, last);         // Sort right half of array
    Merge(values, first, middle, middle + 1, last); // Combine the sorted halves
  }
}
```

```cpp
template<class ItemType>
void Merge (ItemType values[], int leftFirst, int leftLast, int rightFirst, int rightLast)
// Post: values[leftFirst]..values[leftLast] and values[rightFirst]..values[rightLast] have been merged.
//       values[leftFirst]..values[rightLast] is now sorted.
{
  ItemType tempArray[SIZE];
  int index = leftFirst;
  int saveFirst = leftFirst;

  while ((leftFirst <= leftLast) && (rightFirst <= rightLast))
  {
    if (values[leftFirst] < values[rightFirst])
    {
      tempArray[index] = values[leftFirst];
      leftFirst++;
    }
    else
    {
      tempArray[index] = values[rightFirst];
      rightFirst++;
    }
    index++;
  }

  while (leftFirst <= leftLast)
  // Copy remaining items from left half.

  {
    tempArray[index] = values[leftFirst];
    leftFirst++;
    index++;
  }

  while (rightFirst <= rightLast)
  // Copy remaining items from right half.
  {
    tempArray[index] = values[rightFirst];
    rightFirst++;
    index++;
  }

  for (index = saveFirst; index <= rightLast; index++)
    values[index] = tempArray[index];
}
```

Big-O Comparisons

N	$\log_2 N$	N^2	$N\log_2 N$
32	5	1024	160
64	6	4096	384
128	7	16,384	896
256	8	65,536	2,048
512	9	262,144	4,608
1024	10	1,048,576	10,240
2048	11	4,194,304	22,528
4096	12	16,777,216	49,152

Relative Algorithm Efficiency

Sort	Best Case	Average Case	Worst Case
SelectionSort	$O(N^2)$	$O(N^2)$	$O(N^2)$
BubbleSort	$O(N^2)$	$O(N^2)$	$O(N^2)$
ShortBubble	$O(N)$*	$O(N^2)$	$O(N^2)$
InsertionSort	$O(N)$*	$O(N^2)$	$O(N^2)$
MergeSort	$O(N\log_2 N)$	$O(N\log_2 N)$	$O(N\log_2 N)$
QuickSort	$O(N\log_2 N)$	$O(N\log_2 N)$	$O(N^2)$
HeapSort	$O(N\log_2 N)$	$O(N\log_2 N)$	$O(N\log_2 N)$

* Data Almost Sorted

Radix Sort

- Not a ***comparison*** sort
- Makes use of the values in the keys to order the items
- Radix
  - The number of possibilities for each position

    or
  - The number of digits in a number system

Radix Sort Example - 1

Unsorted Array

762
124
432
761
800
402
976
100
001
999

Radix Sort Procedure
- Divide by radix subgroup starting with least significant position first
- Recombine the values in order
- Repeat through most significant position

Radix Sort Example - 2

[0]	[1]	[2]	[3]	[4]
800	761	762		124
100	001	432		
		402		

[5]	[6]	[7]	[8]	[9]
	976			999

Array of Queues for Pass 1

Array After Pass 1

800
100
761
001
762
432
402
124
976
999

Radix Sort Example - 3

[0]	[1]	[2]	[3]	[4]
800		124	432	
100				
001				
402				

[5]	[6]	[7]	[8]	[9]
	761	976		999
	762			

Array of Queues for Pass 2

Array After Pass 2

800
100
001
402
124
432
761
762
976
999

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Radix Sort Example - 4

[0]	[1]	[2]	[3]	[4]
001	100			402
	124			432

[5]	[6]	[7]	[8]	[9]
		761	800	976
		762		999

Array of Queues for Pass 3

Array After Pass 3

001
100
124
402
432
761
762
800
976
999

```
// This file contains the functions for Radix Sort.
// In the RadixSort function, the parameters have these meanings:
//
// values        is the array to be sorted
// numValues     is the size of the array to be sorted
// numPositions  is the size of the key measured in digits, characters etc..
//               If keys have 3 digits, this has the value 3,
//               If 10 digit keys, this has the value 10.
//               If word keys, then this is the number of characters in
//                           the longest word.
// radix         is the radix of the key, 10 in the case of decimal digit keys
//               26 for case-insensitive letters, 52 if case-sensitive letters.

#include "QueType.h"
typedef int ItemType;

template<class ItemType>
void RadixSort(ItemType values[], int numValues, int numPositions, int radix)
// Post: Elements in values are in order by key.
{
  QueType<ItemType> queues[radix];
  // With default constructor, each queue size is 500
  int whichQueue;

  for (int position = 1; position <= numPositions; position++)
  {
    for (int counter = 0; counter < length; counter++)
    {
      whichQueue = values[counter].SubKey(position);   // SubKey extracts digit at specified position
      queues[whichQueue].Enque(values[counter]);
    }
    CollectQueues(values, queues, radix);
  }
}
```

```
template<class ItemType>
void CollectQueues(ItemType values[], QueType<ItemType> queues[], int radix)

// Post: queues are concatenated with queue[0]'s on top and
//       queue[9]'s on the bottom and copied into values.
{
  int index = 0;
  ItemType item;

  for (int counter = 0; counter < radix; counter++)
  {
    while (!queues[counter].IsEmpty())
    {
      queues[counter].Deque(item);
      values[index] = item;
      index++;
    }
  }
}
```

SubKey

- Suppose itemKey = 8749

  Position 1: itemKey % 10 = 9

  Position 2: (itemKey / 10) % 10 = 4

  Position 3: (itemKey / 100) % 10 = 7

  Position 4: (itemKey / 1000) % 10 = 8

- values[counter].SubKey(position)

  Digit = (itemKey / $10^{position-1}$) % 10

Radix Sort Efficiency

- Does not compare items
- Efficiency depends upon implementation of supporting data structures
  – List of values
  – Radix queues

Heap Sort

- General approach
  - Take the root (maximum) element off the heap and put it in its place
  - Reheap remaining elements
  - Repeat until no more elements left in heap
- Sounds similar to selection sort
  - Heap order property makes locating next element faster $O(\log_2 N)$

Heap Sort

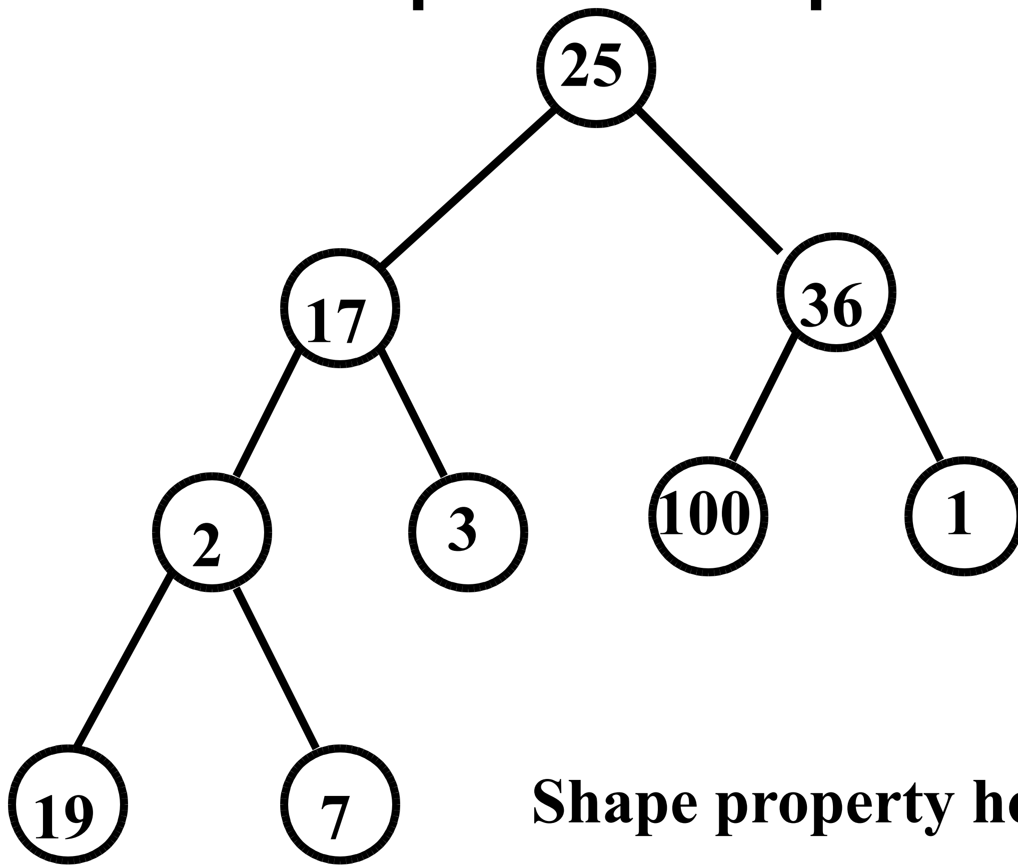
- Problem
  - Starting with an unsorted list
  - Must build a heap from this list of elements

Building a Heap

- Given an unsorted array of elements
- Construct a heap
  - Shape property
  - Order property

Build Heap Example

[0]	25
[1]	17
[2]	36
[3]	2
[4]	3
[5]	100
[6]	1
[7]	19
[8]	7



Shape property holds

Must reorder, but how?

Build Heap Example

- ReheapDown
  - Corrects order if each subtree is already ordered
- Apply ReheapDown to all subtrees on a level, then move up a level and ReheapDown again until root level reached

BuildHeap Algorithm

- FOR ***index*** going from first nonleaf node up to the root node

  ReheapDown(values, index, numValues-1)

Heap Sort Implementation

```
template<class ItemType>
void HeapSort(ItemType  values[], int numValues)
{
    int index;

    // Convert the array of values into a heap
    for(index = numValues/2 - 1; index >= 0; index--)
        ReheapDown(values, index, numValues-1);

    // Sort the array
    for(index = numValues-1; index >= 1; index--)
    {
        Swap(values[0], values[index]);
        ReheapDown(values, 0, index-1);
    }
}
```

Heap Sort Efficiency

- $O(N\log_2 N)$ comparisons
- Efficiency not affected by initial order of elements

Sorting Algorithm Efficiency

- $O(N\log_2 N)$ is theoretically optimal for key comparison sorting
- Other efficiency considerations
  - Actual size of N
  - Programmer time
  - Memory space
  - Eliminating function calls can improve proportionality constants
    - Swap(X,Y)
    - Temp = X;   X=Y;   Y=Temp;