ECONOMICS 261
STATISTICAL METHODS
PROF. KARSTENSSON

EXAM 2
SPRING SEMESTER 2002
SOLUTIONS

Part I: Concept questions.

1. Relative frequency approach.

2. Experiment.

3. Continuous variable.

4. Normal distribution.

5. Standard normal variable.

Part II: Problem questions.

6. Probability problems:

Contingency table:

Major	Affected by the 9/11 Tragedy (A)	Not Affected by the 9/11 Tragedy (N)	Total
Business (B)	44	29	73
Hotel (H)	31	5	36
Other (O)	5	2	7
Total	80	36	116

(a) Probability matrix:

Major	Affected by the 9/11 Tragedy (A)	Not Affected by the 9/11 Tragedy (N)	Total
Business (B)	.379	.250	.629
Hotel (H)	.267	.043	.310
Other (O)	.043	.017	.060
Total	.690	.310	1.000

(b) Marginal probability:

$$P(A) = \frac{n_A}{n} = \frac{80}{116} = .6897 \approx 68.9\%$$

(c) Marginal probability:

$$P(N) = \frac{n_N}{n} = \frac{36}{116} = .310 = 31.0\%$$

(d) Conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.379}{.629} = .603 = 60.3\%$$

(e) Conditional probability:

$$P(A|H) = \frac{P(A \cap H)}{P(H)} = \frac{.267}{.310} = .861 \approx 86.1\%$$

7. Discrete probability distribution problem:

X = Number of high-end deluxe suites at the Bellagio;
P(X) = The probability that a given number of suites will be occupied.

X	P(X)	XP(X)	[X-E(X)]	$[X-E(X)]^2$	$[X-E(X)]^2P(X)$
0	.10	.00	-2.5	6.25	0.625
1	.15	.15	-1.5	2.25	0.3375
2	.20	.40	-0.5	0.25	0.05
3	.25	.75	0.5	0.25	0.0625
4	.30	1.20	1.5	2.25	0.675

$E(X) = 2.50$ $\qquad$ $\sigma^2(X) = 1.75$

$\sigma(X) = \sqrt{1.75} \approx 1.323$

(a) $E(X) = \Sigma XP(X) = 2.5$ suites occupied

(b) $\sigma(X) = \Sigma\{[X-E(X)]^2P(X)\} = 1.75$ suites occupied squared

(c) $\sigma(X) = \sqrt{\Sigma\{[X-E(X)]^2P(X)\}} \approx 1.323$ suites occupied

8. Binomial distribution problems:

(a) $$P(X=4|n=5,\pi=.4) = \frac{5!}{4!(5-4)!}(.4)^4(1-.4)^{5-4}$$

$$= \frac{120}{24}(.0256)(.6)$$

$$= (5)(.0256)(.6) = .0768$$

(b) $P(X=4|n=10,\pi=.4) = .2508$

(From Binomial Table)

(c) $P(X<4|n=12,\pi=.4) = .1419 + .0639 + .0174 + .0022 = .2254$

(From Table B, Binomial Distribution)

9. Normal distribution problems:

Given: μ = 6 million dollars

σ = 1.2 million dollars

(a) $P(6 \le X \le 7)$ =

$$Z(6) = \frac{6-6}{1.2} = 0.00 \qquad Z(7) = \frac{7-6}{1.2} + 0.83$$

From Standard Normal Table:

$P(6 \le X \le 7) = P(0.00 \le Z \le +0.83) = .2967 = 29.67\%$

The probability that the Bellagio gross income will be between $6 million and $7 million on a randomly selected day is .2967; thus, there is a 29.67% chance that the gross income will be between $6 million and $7 million on a given day.

(b) $P(4 \le X \le 5)$ =

$$Z(4) = \frac{4-6}{1.2} = -1.67 \qquad Z(5) = \frac{5-6}{1.2} = -0.83$$

From Standard Normal Table:

$P(4 \le X \le 5) = P(-1.67 \le Z \le -0.83) = .4525 - .2967 = .1558 = 15.58\%$

The Bellagio's gross income will be between $4 million and $5 million on 15.58 percent of the days.

(c) $P(X > 8)$ =

$$Z(8) = \frac{8-6}{1.2} = +1.67$$

From Standard Normal Table:

$P(X > 8) = P(Z > +1.67) = .5 - .4525 = .0475 = 4.75\%$

$\rightarrow (.0475)(365) = 17.34$ days

The Bellagio's gross income will exceed $8 million on 17.34 days per year.

Part III: Interpretation questions.

10. The standard deviation measures the variation, or spread, in a distribution. So the variation in the distribution of Bellagio suits occupied per day is 1.323 suites. In other words, the typical observation in this distribution deviates from the expected number of suites occupied (2.5) by 1.323 suites.

11. Types of variables:

    (a) Question 7: Number of high-end suites at the Bellagio, a counted variable, is a discrete variable.

    (b) Question 9: The gross income of the Bellagio, a variable measured in monetary units, is a continuous variable.

Part IV: Optional extra credit question on the movie, *A Beautiful Mind*.

12. Alicia Larde was a student and wife of John Forbes Nash. She was played by Jennifer Connelly.