# Measures of Central Tendency, Spread, and Shape

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## Nominal Scaling

- The lowest level of scaling
- In Nominal Scaling, each of the values serves as a representation.
- Each observation belongs to one mutually exclusive category and has no logical order
- Examples:
  - Gender
  - Ethnicity
  - School



## Ordinal Scaling

- In Ordinal Scaling, each of the values is in rank order.
- Each observation belongs to one mutually exclusive category, but we now have logical order.
- Examples:
  - Letter Grades
  - Place Finished in a Race
  - Likert-type Scaling



## Interval Scaling

- In Interval Scaling, each of the values has a specific order that reflects equal differences between observations.
- Each observation belongs to one mutually exclusive category, with logical order, and equal differences between each of the points and no absolute zero.
- Examples:
  - Temperature (Fahrenheit and Celcius)
  - IQ Scores
  - SAT/GRE Scores

## Ratio Scaling

- In Ratio Scaling, each of the values has a specific order that reflects equal differences and a "true" zero.
- Each observation belongs to one mutually exclusive category, with logical order, equal differences between each of the points, and has a "true" zero.
- Examples:
  - Kelvin Scale
  - Height and Weight
  - Speed



## Measures of Central Tendency

- The measures of central tendency try and give us a picture of what is going on at the middle of the distribution
- There are 3 types of measures of central tendency
- The mode - this is the most frequently appearing score
- The median - this is also called the "middle" score
- The mean - this is also called the "average" score



## Outline



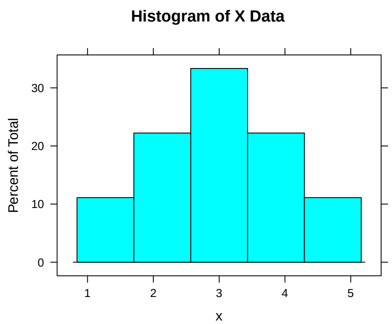


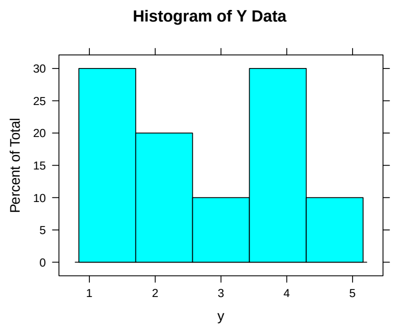
## The Mode

- The mode is the most frequently appearing score
- There can be as many modes as there are pieces of data in a dataset
- Datasets with two modes are usually referred to as bimodal

# Representing the Mode

- Suppose we have 2 separate datasets
- X = 1,2,2,3,3,3,4,4,5
- Y = 1,1,1,2,2,3,4,4,4,5






# Outline





# The Median

- The median is often called the "middle" score
- To compute the median, you first arrange the scores in ascending order
- For datasets with an odd number of scores, the median is the middle score.
- For datasets with an even number of scores, the median is the score halfway between the two middle scores
- If X = 1,2,3,4,10, the median would be 3
- If X = 1,2,3,10, the median would be 2.5
- If X = 3,2,10,1, the median would still be 2.5



# Outline

## The Mean

- The mean is also called the "average" score

$$\overline{X} = \sum_{i=1}^{n}(X_i)/n = \frac{\sum_{i=1}^{n}(X_i)}{n}$$

- Given the dataset X = 1,2,3,4

$$\overline{X} = \frac{1+2+3+4}{4} = \frac{10}{4} = 2.5$$



## Outline





## Range

- The range is used to describe the number of units on the scale of measurement
- It is computed as:
  - (Highest score - Lowest score)
- Suppose we have the dataset X = 1,2,3,4, the range would be:
  - (4 - 1) = 3



## Outline

## Standard Deviation

- The standard deviation is also sometimes referred to as the mean deviation
- The SD represents the "average distance" of each score from the mean

$$SD_X = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}}$$



## Computing the Standard Deviation

- Suppose that we have a dataset where X = 1,2,3,4

$$\begin{aligned} SD_X &= \sqrt{\frac{(1-2.5)^2+(2-2.5)^2+(3-2.5)^2+(4-2.5)^2}{4-1}} \\ &= \sqrt{\frac{(-1.5)^2+(-0.5)^2+(0.5)^2+(1.5)^2}{3}} \\ &= \sqrt{\frac{2.25+0.25+0.25+2.25}{3}} \\ &= \sqrt{\frac{5}{3}} \\ &= \sqrt{1.67} \\ &= 1.29 \end{aligned}$$



## Outline





## Z-Scores

- As opposed to the other measure of variation, the z-score is a measure of deviation for a single individual, as opposed to a group of scores.
- The Z-score represents the number of Standard Deviation units a given piece of data is from the mean.

$$Z_i = \frac{X_i - \overline{X}}{SD_X}$$

| Raw Score | Z-Score |
|---|---|
| 1 | -1.16 |
| 2 | -0.39 |
| 3 | 0.39 |
| 4 | 1.16 |

## Outline





## Standard Error of the Mean

- The standard error of the mean is really just computed for the purposes of computing Confidence Intervals

$$SEM_X = \frac{SD_X}{\sqrt{n}}$$

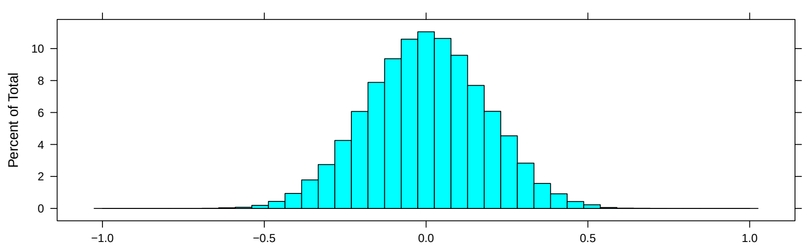
- For our dataset, X = 1,2,3,4 ,

$$SEM_X = \frac{1.29}{\sqrt{4}}$$
$$= 0.645$$



## Confidence Intervals

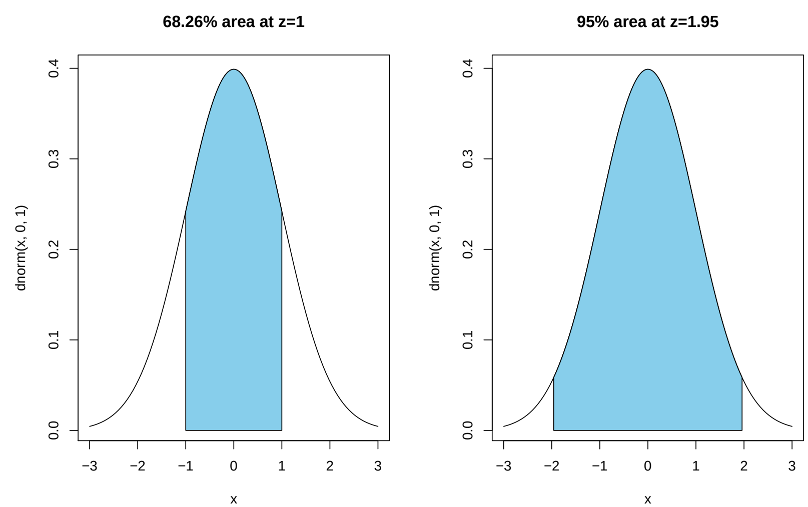
- Suppose that we have a uniform normal distribution of population scores between -1 and +1 $[U(-1,1)]$ and we take many samples of n=10.




```
> mean(res)
[1] 6.570396e-05
> sd(res)
[1] 0.1825853
```



## Area Under the Curve



## Computing 95% Confidence Intervals

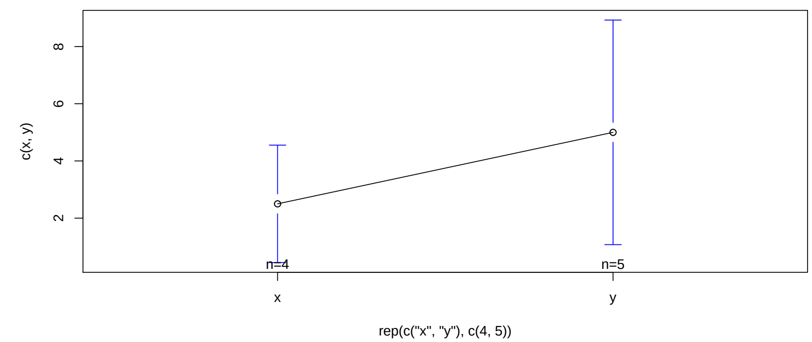
Let's look back at our previous dataset where X=1,2,3,4. In this case, we noted that the mean of X = 2.5 and the SD of X = 1.29. This would make the SEM = 1.29/2 = 0.645. 95% confidence intervals for any dataset are computed as:

$$
\begin{aligned}
CI_{95\%} &= \overline{X} \pm 1.96 * SEM \\
&= 2.5 \pm 1.96 * 0.645 \\
&= 2.5 \pm 1.264
\end{aligned}
$$



## Graphing 95% Confidence Intervals

```
> x <- c(1, 2, 3, 4)
> y <- c(1, 3, 5, 7, 9)
> plotmeans(c(x, y) ~ rep(c("x", "y"), c(4, 5)))
```






## Thinking Through Confidence Intervals

Suppose that we have two different datasets with the following properties:

| | Study 1 | Study 2 |
|---|---|---|
| mean | 10 | 10 |
| SD | 5 | 10 |
| n | 30 | 30 |

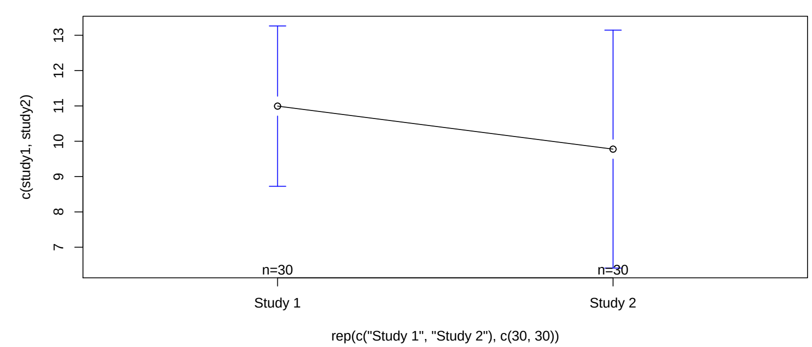
### Thought Question

Given the properties of the above two studies, which study would have the LARGER confidence intervals?



## Thinking Through Confidence Intervals

```
> study1 <- rnorm(30, 10, 5)
> study2 <- rnorm(30, 10, 10)
> plotmeans(c(study1, study2) ~ rep(c("Study 1",
+     "Study 2"), c(30, 30)))
```



## Thinking Through Confidence Intervals

Suppose that we have two different datasets with the following properties:

| | Study 1 | Study 2 |
|---|---|---|
| mean | 10 | 10 |
| SD | 10 | 10 |
| n | 300 | 30 |

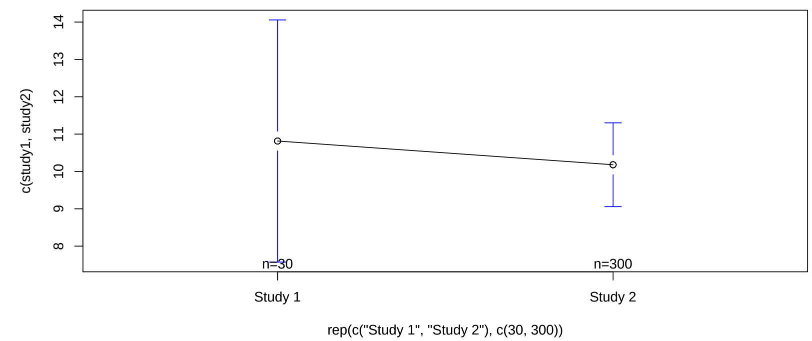
### Thought Question

Given the properties of the above two studies, which study would have the LARGER confidence intervals?



## Thinking Through Confidence Intervals

```
> study1 <- rnorm(30, 10, 10)
> study2 <- rnorm(300, 10, 10)
> plotmeans(c(study1, study2) ~ rep(c("Study 1",
+     "Study 2"), c(30, 300)))
```



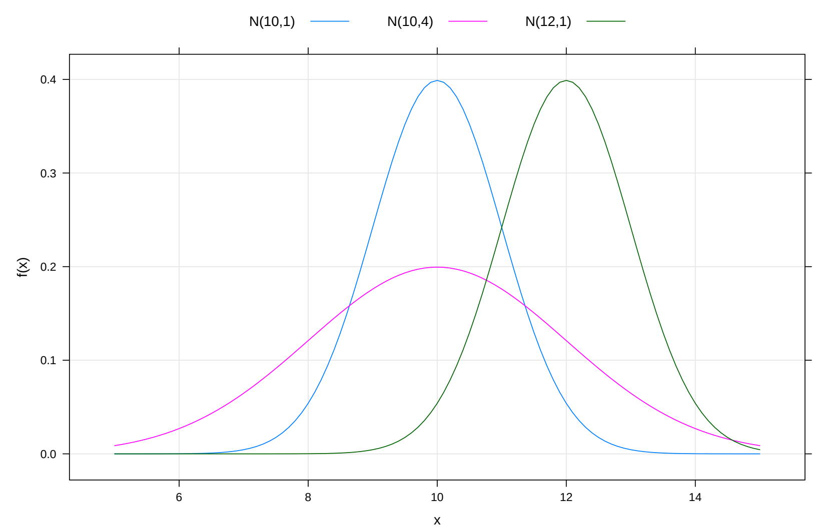



## Outline





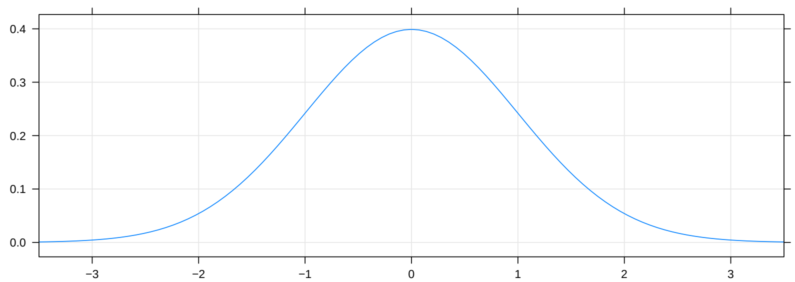
## Specific Normal Distributions



## The standard normal distribution

- The *standard* normal distribution is the one with $\mu = 0$ and $\sigma = 1$. We often use $Z$ to denote it, i.e. $Z \sim \mathcal{N}(0, 1^2)$.
- Its density function is often written $\phi(z)$ and

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \qquad -\infty < z < \infty$$






## Evaluating the Normal Distribution

- As given from the previous slide, we assume that the normal distribution has a structure such that mean meadian mode.
- We can also evaluate the shape of our distribution relative to it's deviation from the standard normal distribution through multiple means.
  - Skewness - a measure of shape determining whether or not the "left half" looks like the "right half"
  - Kurtosis - a measure of shape determining relative hieght to width
  - Gaussian probability density plot



## Outline



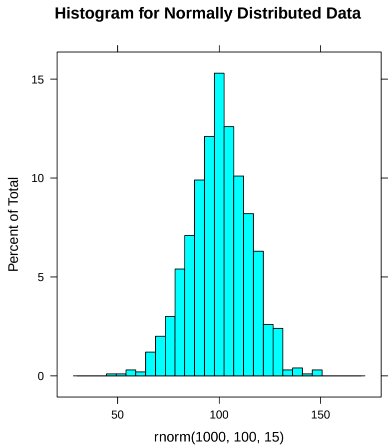


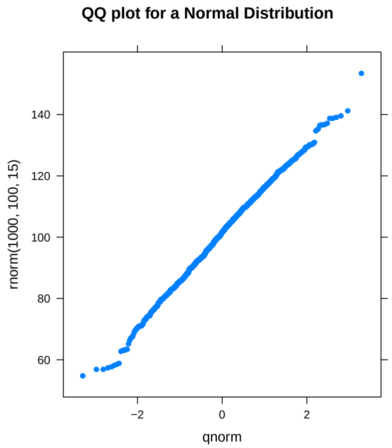
## Skewness

- Skewness measures typically range from -3 to +3
- A skewness of 0 means that the left half looks exactly like the right half
- Positively skewed means that the mean is influenced by outliers making it "seem" larger than the relative mean of the population from which the sample was drawn

$$skew = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left( \frac{x_i - \bar{x}}{SD_x} \right)^3 = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} z_i^3$$

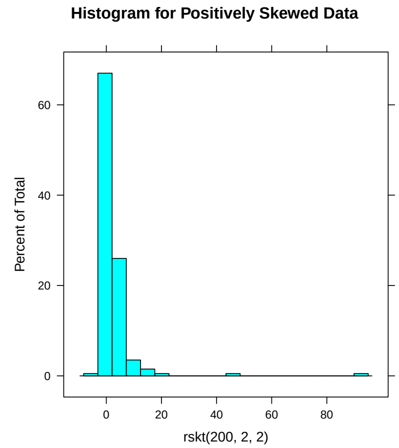
## Skewness Examples - Normal Distribution

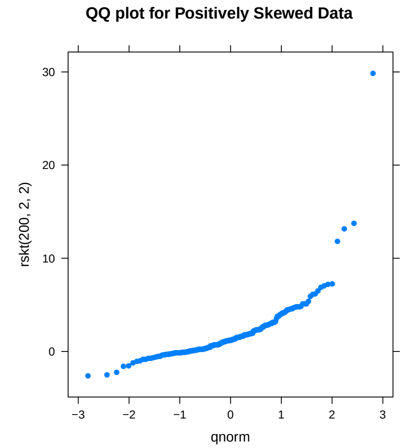




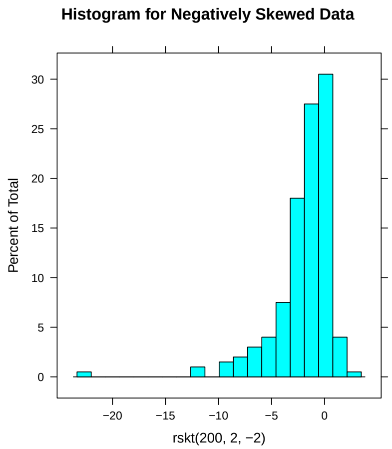

## Skewness Examples - Positively Skewed Distribution

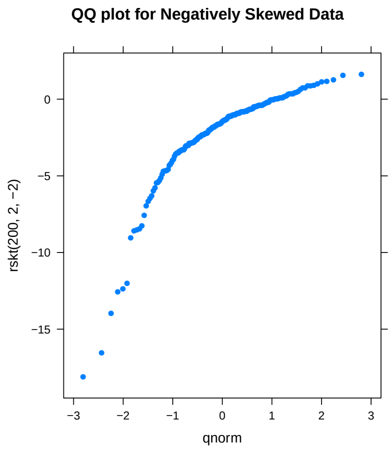





## Skewness Examples - Negatively Skewed Distribution






## Outline

## Kurtosis

- Kurtosis measures typically range from -3 to +3
- A kurtosis of 0 means that your distribution has the same relative height to width properties as a normal distribution
- Positive kurtosis means that your distribution is leptokurtic (taller and skinnier)
- Negative kurtosis means that your distribution is platykurtic (shorter and fatter)

$$kurt = \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=1}^{n} \left( \frac{x_i - \bar{x}}{SD_x} \right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)}$$



## Graphical Examples of Kurtosis

- Although all of these theoretically could be normal, they illustrate leptokurtic and platykurtic distributions.