### MATH 115 (PROBABILITY & STATISTICS)

N. PSOMAS

Final Exam Review

- 1. You measure the heights of a random sample of 400 high school sophomore males in a Midwestern state. The sample mean is 66.2 inches. Suppose that the heights of the population of all high school sophomore males follow a normal distribution with unknown mean  $\mu$  and standard deviation  $\sigma$ = 4.1 inches. You compute a 95% confidence interval for  $\mu$ . Suppose you had measured the heights of a random sample of 100 sophomore males rather than 400. Which of the following statements is true?
  - A) The margin of error for our 95% confidence interval would decrease.
  - **B**)  $\sigma$  would decrease.
  - C) The margin of error for our 95% confidence interval would increase.
  - **D)** The margin of error for our 95% confidence interval would stay the same, because the level of confidence has not changed.
- 2. You measure the heights of a random sample of 20 high school sophomore males in a Midwestern state. The sample mean is 68.2 inches and the sample standard deviation is 4.3 inches. Suppose that the heights of the population of all high school sophomore males follow a normal distribution. Compute 90%, 95% and 98% confidence intervals for  $\mu$ .
- 3. The nicotine content in cigarettes of a certain brand is normally distributed. The brand advertises that the mean nicotine content of their cigarettes is 1.5 milligrams, but measurements on a random sample of 100 cigarettes of this brand gave a sample mean  $\bar{x} = 1.53$  milligrams with a sample standard deviation s = 0.1 milligrams. Is this evidence that the mean nicotine content is actually higher than advertised?

### Use the information below to answer question 4-5

The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures the motivation, attitude, and study habits of college students Scores range from 0 to 200 and follow (approximately) a normal distribution with mean  $\mu$ =115 and standard deviation  $\sigma$ = 25. You suspect that incoming freshman have a mean  $\mu$  that is different from115 because they are often excited yet anxious about entering college.

- 4. You decide to perform a hypothesis test to see if your suspicion is correct. What are the null and alternative hypotheses that you need to test?
- 5. You give the SSHA to 36 students who are incoming freshmen and find their mean score  $\bar{x}$  is 116.2.
  - a. What is a 95 % confidence interval for the true mean SSHA score µ for all incoming freshmen?
  - b. At the 5% significance level, is the true mean SSHA score  $\mu$  for all incoming students (freshmen) different than 115, the true mean SSHA score for the rest of the students?
- 6. The amount spent on books by freshmen in their first year at a major university has a normal distribution with a known standard deviation  $\sigma$ = \$30. How large a sample is required in order to compute a 92% confidence interval for the average amount spent on books by freshmen in their first year at a major university, if the margin of error of the interval should not exceed \$2?

7. The heights of young American women, in inches, are normally distributed. I select a simple random sample of four (4) young American women and measure their heights. The four heights, in inches, are:

63 69 62 66

- (a) Compute a 99% confidence interval for  $\mu$ .
- (b) At the 1% level of significance, is the true population mean height of young American women different than 70 inches?
- 8. The yield per acre for a new variety of corn follows a normal distribution with an unknown mean  $\mu$  and a known standard deviation  $\sigma$ . An agricultural researcher plants 25 plots with this new variety of corn and finds that the average yield for these plots is 150 bushels per acre. The researcher reports that a 90% confidence interval for  $\mu$  was found to be 150 ± 3.29. What can you tell about the standard deviation  $\sigma$  of the yield per acre for the new variety of corn?
- 9. Suppose that the population of the scores of all high school seniors who took the SAT Math (SAT-M) test this year follows a Normal distribution with standard deviation  $\sigma$ = 100. You read a report that says, "On the basis of a simple random sample of 100 high school seniors that took the SAT-M test this year, a confidence interval for  $\mu$  is found to be 512.00 ± 25.76." What was the confidence level used to calculate this confidence interval?
- 10. The water diet requires one to drink two cups of water every half hour from the time that one gets up until one goes to bed, but otherwise allows one to eat whatever one likes. Four adult volunteers agree to test the diet. They are weighed prior to beginning the diet and after six weeks on the diet. The weights (in pounds) are

Person	1	2	3	4
Weight before the diet	180	125	240	150
Weight after six weeks	170	130	215	152

For the population of all adults, assume that the weight loss after six weeks on the diet (weight before beginning the diet – weight after six weeks on the diet) is normally distributed with mean  $\mu$ . To determine if the diet leads to weight loss, you test the hypotheses:  $H_0$ :  $\mu = 0$ ,  $H_a$ :  $\mu > 0$ .

At the 8% level, what is your conclusion?

11. Researchers compared two groups of competitive rowers: a group of skilled rowers and a group of novices. The researchers measured the angular velocity of each subject's right knee, which describes the rate at which the knee joint opens as the legs push the body back on the sliding seat. The sample size *n*, the sample means, and the sample standard deviations for the two groups are given below.

Group	n	Mean	Standard Deviation
Skilled	16	4.2	0.6
Novice	16	3.2	0.8

The researchers want to test the hypotheses:

 $H_0$ : The mean knee velocities for skilled and novice rowers are the same.

 $H_a$ : The mean knee velocity for skilled rowers is larger than for novice rowers.

Write the null and alternative hypotheses stated above in terms of the two populations means and carry out the test. At the 10% level, what is the test conclusion? What is the p-value?

- 12. The candy company that makes M&M's claims that 10% of the M&M's it produces are green. Suppose that the candies are packaged at random in large bags of 300 M&M's. In a randomly selected bag of M&M's you find that there are only 18 green M&M's. On the basis of that:
  - (a) Compute a 98% confidence interval for p, the population proportion of green M&M's.
  - (b) At the 5% level, is the true proportion of green M&M's different than 10%?

#### Use the information below to answer questions 13 - 16

A sociologist is studying the effect of having children within the first two years of marriage on the divorce rate. Using hospital birth records, she selects a simple random sample of 200 couples who had children within the first two years of marriage. Following up on these couples, she finds that 80 couples are divorced within five years. Let p = the population proportion of couples who had children within the first two years of marriage and are divorced within five years.

- 13. What is a 90% confidence interval for *p*, the true population proportion of couples who have children within the first two years of marriage and are divorced within five years?
- 14. What is a 90% plus-four confidence interval for *p*, the true proportion of couples who have children within the first two years of marriage and are divorced within five years?
- 15. How large a sample is required if you want to compute a 90% confidence interval for *p* with a margin of error that does not exceed 0.02 (or 2%)? Use the estimate 80/200 for p\* in the formula for calculating the required sample size.
- 16. Suppose the sociologist wishes to test  $H_0$ :  $p = \frac{1}{3}$  versus  $H_a$ :  $p \neq \frac{1}{3}$ . At the 10% level of significance, does the sociologist have enough evidence to reject the null hypothesis? What is the p-Value?

## Use the information below to answer question 17 – 18

When the term "Spring Break" is mentioned, many of us think "Florida!". A survey among college students found that 812 out of 1500 college students asked responded that they would go to Florida during Spring Break.

- 17. Calculate a 95% confidence interval for the population proportion of students who go to Florida during Spring Break.
- 18. On the basis of this survey, can we conclude that the majority of students go to Florida during Spring Break? To answer this, test the hypotheses  $H_0$ : p = 0.50 versus  $H_a$ : p > 0.50. What is the *P*-value of the test? At the 5% significance level, are the data statistically significant?

### Use the information below to answer question 19

An inspector inspects large truckloads of potatoes to determine the proportion p in the shipment with major defects prior to using the potatoes to make potato chips. Unless there is clear evidence that this proportion is less than 0.10, she will reject the shipment. She selects a simple random sample (SRS) of 150 potatoes from the more than 3000 potatoes on the truck. Six (6) of the potatoes sampled are found to have major defects.

19. Test the hypotheses:  $H_0$ : p = 0.10, vs.  $H_a$ : p < 0.10. At the 5% level, can you reject the null hypothesis? What is the P-value of the test? Does this truckload of potatoes meet the inspector's standards for making potato chips?

# Use the information below to answer question 20

A simple random sample of 100 flights of a large airline (call this Airline 1) showed that 64 were on time. A simple random sample of 100 flights of another large airline (call this Airline 2) showed that 80 were on time.

20. Let  $p_1$  and  $p_2$  be the proportion of all flights that are on time for these two airlines. On the basis of the data above, can we conclude that the on-time rates  $p_1$  and  $p_2$  for the two airlines are different? To determine this, test the hypotheses  $H_0$ :  $p_1 = p_2$  versus  $H_a$ :  $p_1 \neq p_2$ . What is the p-value? At the 5% level, what is your conclusion?