Formula Sheet – Final Exam

Regression Model

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \qquad \text{where} \quad \varepsilon_i \sim N(0,\sigma) \text{ and independent}$$

Sum of Squares

$$SS_{XX} = \sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 \qquad SS_{XY} = \sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y}) \qquad SS_{YY} = \sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2$$

Ordinary Least Squares Estimators

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} \qquad \hat{\beta}_0 = \overline{y} - \hat{\beta}_1\overline{x} \qquad \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \qquad y_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + e_i$$

Sum of Square Errors, Variance and Standard Error

$$SSE = \sum (e_i)^2 = \sum (y_i - \hat{y})^2 = SS_{YY} - \hat{\beta}_1 SS_{XY}$$

$$s^2 = \frac{SSE}{n-2} \qquad s = \sqrt{\frac{SSE}{n-2}}$$

Sampling Distribution of Estimated Slope

$$\sigma_{\hat{\beta}_1} = \frac{\sigma}{\sqrt{SS_{XX}}} \qquad \text{(population)}$$

$$s_{\hat{\beta}_1} = \frac{s}{\sqrt{SS_{XX}}} \qquad \text{(estimated value)}$$

$$t = \frac{\hat{\beta}_1}{s_{\hat{\beta}_1}}$$ (has a t-distribution with n–k-1 degrees of freedom)

$$\hat{\beta}_1 \pm t_{\alpha/2} s_{\hat{\beta}_1}$$ $(1-\alpha)$ confidence interval for β_1

Correlation and Fit

$$R = \frac{SS_{XY}}{\sqrt{SS_{XX}SS_{YY}}} \qquad \text{(Correlation Coefficient)}$$

$$R^2 = 1 - \frac{SSE}{SS_{YY}} \qquad \text{(Coefficient of Determination)}$$

Estimation

A $1-\alpha$ confidence interval for the mean value of y at $x = x_t$ is

$$\hat{\beta}_0 + \hat{\beta}_1 x_t \pm t_{\alpha/2} s \sqrt{\frac{1}{n} + \frac{(x_t - \overline{x})^2}{SS_{XX}}}$$

A $1-\alpha$ confidence interval for the value of y at $x = x_t$ is

$$\hat{\beta}_0 + \hat{\beta}_1 x_t \pm t_{\alpha/2} s \sqrt{1 + \frac{1}{n} + \frac{(x_t - \overline{x})^2}{SS_{XX}}}$$

Multiple Regression (Ch. 13)

Standard Error

$$s = \sqrt{\frac{SSE}{n-k-1}}$$

Global Test Statistic (F)

$$F = \frac{\sum (\hat{y}_t - \overline{y})^2 / k}{\sum (y_t - \hat{y}_t)^2 / (n-k-1)} = \frac{R^2/k}{(1-R^2)/(n-k-1)}$$ which has $v_1 = k$ & $v_2 = n-k-1$ degrees of freedom.