ME8135 - State Estimation - Assignment 5

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Due: Jun 23, 4 PM. Write a summary of the estimation methods and algorithms, with their key differences, advantages and disadvantages. Max 2 pages, in Overleaf.

1 Report

This summary of estimation methods and algorithms will focus primarily on those implemented in previous assignments, namely: Bayes filters in general, the Kalman and extended Kalman Filters, and particle filters. The structure will follow a narrative starting with the Kalman Filter and progress by looking at model assumptions and comparative computational costs. The intent is to highlight some key aspects rather than being a thorough or exhaustive treatment of the topic.

At a high level, the objective of state estimation is to efficiently determine the state of a system based on inputs. These inputs carry degrees of noise and uncertainty so the goal is to effectively "filter" these errors out to produce strong estimates. The methods detailed in the texts up to this point are applications of "Bayes filters" which are based on Bayesian inference. In general, state predictions (priors) are made based on motion models which are updated using measurements to give corrected states (posteriors). The classic algorithm to do this is the Kalman Filter.

The core process of the Kalman Filter occurs when a measurement is received. Again, the goal is to effectively combine predictions and measurements to make a strong estimate. Here this is done by computing the Kalman gain, K:

$$K = P_{pred} C^{\top} (C P_{pred} C^{\top} + Q)^{-1}$$

Where P_{pred} is the predicted state, C is the measurement matrix, and Q is the measurement covariance. The Kalman gain is a weighting where the more certain of the prediction and measurement values is preferred. Under the assumptions of linear models and Gaussian noise the result is an optimal estimate in that it is unbiased and the mean squared error is minimized. With relatively low dimensions the Kalman Filter has a likewise low computational cost. If we have nonlinear models however, a modification is needed.

The Extended Kalman Filter (EKF) is a conceptually straightforward modification to the Kalman Filter that addresses the linear assumption by approximating, or linearizing, the nonlinear model. For instance, if the measurement model is nonlinear we can linearize the model using a Jacobian matrix and simply substitute the resulting measurement matrix into the Kalman gain formula. While the EKF is more flexible by allowing for nonlinear models there is the added issue of approximation errors and increased computational cost. One way to address both is further modifications to take samples of the model rather than compute the Jacobian at each step.

This is the approach of the Unscented Kalman Filter which operates by computing sigma points that represent the distributions. These points are passed through the nonlinear function as opposed to linearizing the function as in the EKF. The overall result is less computational cost, especially with higher dimensionality, and added control over the approximation error through the choice of sigma points. The Kalman filter and its variants assume that noise is Gaussian. If the noise is multimodal or nonparametric, a different approach may be suitable.

Nonparametric filters are those that do not need a parametric assumption of the models, such as that the noise is Gaussian. One example of this is the particle filter. Rather than attempting to parametrically model the distributions the predicted states are modeled by particles where each represents a possible state and together approximate the distribution. Like in the Kalman filters, weighting is also required to effectively incorporate measurements. These so-called "importance weights" are based on how likely a given particle is to being the true state based on the measurement. These weights are then used to inform resampling where more likely particles are retained with replacement. The result is that the particle distribution becomes closer to that of the measurement distribution. While this approach is typically more computationally expensive than Kalman Filter-based approaches, they are more robust to nonlinear and non-Gaussian models. The expense is also partially alleviated by the "anytime" property where the number of particles can be adapted based on available resources. Care must be taken in the choice of particles in order to avoid a lack of diversity that results in a poor approximation of the distribution.

In summary, Kalman Filter-based approaches are computationally cheaper at lower dimensions and suitable if the uncertainties are relatively Gaussian. While the basic Kalman Filter is optimal assuming linear models its variants can be applied to nonlinear models for a degree of approximation error. Nonparametric filters, like the particle filter, may be suitable if the models are nonlinear and non-Gaussian. Although they are more computationally expensive at lower dimensions, they scale well to higher dimensional states.