ME8135 - State Estimation - Assignment 3

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Repeat Assignment 2 with Particle Filtering.

1 Particle Filter

Use pyGame, or any other similar libraries, to simulate a simplified 2D robot and perform state estimation using a Particle Filter. Motion Model:

$$\dot{x} = \frac{r}{2}(u_r + u_l) + w_x \quad , \quad \dot{y} = \frac{r}{2}(u_r + u_l) + w_y \tag{1}$$

$r = 0.1$ m, is the radius of the wheel, u_r and u_l are control signals applied to the right and left wheels. $w_x = N(0, 0.1)$ and $w_y = N(0, 0.15)$

Simulate the system such that the robot is driven 1 m to the right. Assume the speed of each wheel is fixed and is 0.1 m/s Use these initial values

$$x_0 = 0 \ , \ y_0 = 0 \ , \ P_0 = \mathbf{\Sigma} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ (initial covariance matrix)} \tag{2}$$

and assume the motion model is computed 8 times a second. Assume every second a measurement is given:

$$z = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} r_x \\ r_y \end{bmatrix} \tag{3}$$

where $r_x = N(0, 0.05)$ and $r_y = N(0, 0.075)$

Deliverable: Plot and animate the trajectory along with covariance ellipse for all motion models and measurement updates. Report what happens when a measurement arrives. Note: you need to discretize the system: $x = (x_k - x_{k-1})/T$, where T is 1/8 sec. In your animation, show both ground-truth and estimates.

1.1 Report

When a measurement arrives it is used to weight the particles based on how well they match the measurement. This is found by first computing the distance between the measurement and each particle. The Euclidean distance to the mean measurement is used here. These distances are then used to determine normalized weights to inform resampling. This is done by formulating the normalized weights as a CDF and sampling uniformly for the desired effect of higher weights being more likely to be sampled. The resampling step brings the particle distribution towards that of the measurement. Visually this appears as the particles "jumping" towards the measurement.

The measurement experiences a y-axis drift as the true state moves away from the origin due to the given measurement matrix. The result is that particles follow this measurement drift.

In this demo I have initialized the particles at the same position as the true state. Compared to initializing uniformly, the difference here is minor as the particles converge onto the posterior distribution in one measurement.

2 Non-linear Particle Filter

Repeat the previous assignment, this time with a classic motion model and range observations made from a landmark located at $M = [10, 10]$. L is the distance between the wheel, known as wheelbase, and is 0.3m.

$$\dot{x} = \frac{r}{2}(u_r + u_l)\cos(\theta) + w_x \quad , \quad \dot{y} = \frac{r}{2}(u_r + u_l)\sin(\theta) + w_y \quad , \quad \dot{\theta} = \frac{r}{L}(u_r - u_l) \tag{4}$$

Assume

$$u_\omega = \frac{1}{2}(u_r + u_l) \quad , \quad u_\psi = (u_r - u_l) \tag{5}$$

Then the equations become:

$$\dot{x} = ru_\omega\cos(\theta) + w_\omega \quad , \quad \dot{y} = ru_\omega\sin(\theta) + w_\omega \quad , \quad dot\theta = \frac{r}{L}u_\psi + w_\psi \tag{6}$$

$w_\psi = N(0, 0.01)$ and $w_\omega = N(0, 0.1)$. Program the robot such that it loops around point M.

(a) Compute the EKF with the linear measurement model in the previous assignment.

(b) Compute the EKF with range/bearing measurements of point M. Assume range noise is N(0,0.1) and bearing noise is N(0,0.01). Range is in meters, and bearing is in radians. Visualize the measurements as well.

2.1 Report (a)

The animation is zoomed out farther so the location of the measurements can been seen. At 10 units along the y-axis, the measurement reads 20 units. The effect of the particles being drawn closer to the measurement can be clearly seen here. The operation is the same as in Part 1.

2.2 Report (b)

Note: I discovered an unnoticed issue that carried over from the previous assignment. The visualization is evidently incorrect.

The change to a range/bearing measurements eliminates the y-axis drift of the given linear model. Instead partricles are expected to follow the new non-linear measurement distribution.