16-720 Homework 6: Photometric Stereo

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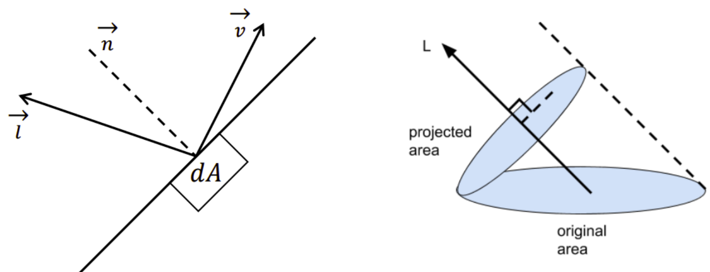
1 Calibrated Photometric Stereo

a)

When there is n-dot-l lighting, the surface radiance equation is below, where $\cos\theta = \vec{n} \cdot \vec{s}$, with the dot product of the incident direction and the normal with θ as the angle between the two vectors.

$$L = \frac{\rho_d}{\pi} I \cos\theta \tag{1}$$



(a) Geometry of photometric stereo (b) Projected area

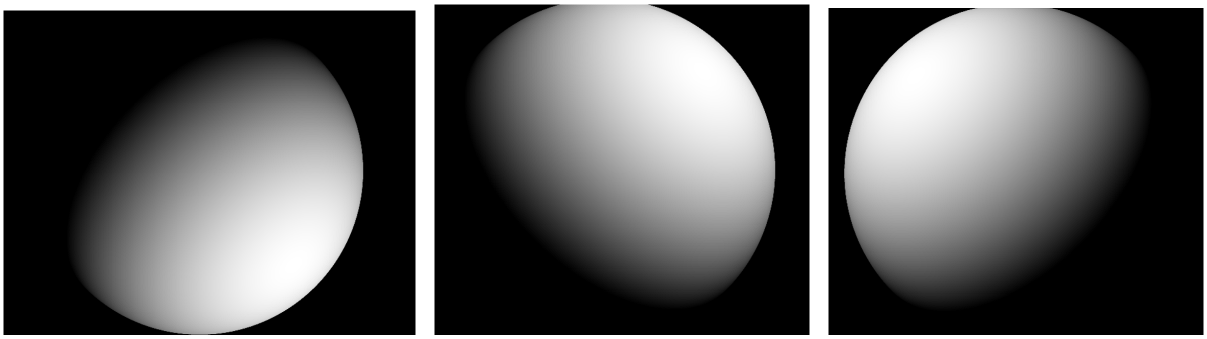
Figure 1: Geometry of Photometric Stereo

This θ is then also the angle between the projected area and original area in the figure below. this means that the projected area multiplied by the cosine of θ is the original area.

Also, the viewing direction does not matter because the surface radiance is not dependent on the angle to the observer, so the surface is bright for all directions.

b)

The results for the three different renderings are below.



(a) $(1,1,1)/\sqrt{3}$ (b) $(1,-1,1)/\sqrt{3}$ (c) $(-1,-1,1)/\sqrt{3}$

Figure 2: Rendering results

c) Code is included in the submission, and a screenshot of the function is below.

```python
def loadData(path = "../data/"):



    I = None
    L = None
    s = None

    for n in range(1,8):
        # get imiage
        image = cv2.imread(path+"input_"+str(n)+".tif", -1)
        # convert to xyz
        image = cv2.cvtColor(image, cv2.COLOR_BGR2XYZ)
        Irow = image[:,:,1]
        # set empty I and get shape
        if n ==1:
            h, w = Irow.shape
            s = (h,w)
            I = np.zeros((7, h*w))

        I[n-1,:] = np.reshape(Irow, (1,h*w))

    # get L
    L = np.load(path + "sources.npy")
    L = L.T


    return I, L, s
```

Figure 3: Load Function Code

d)

For this scenario, the normals vector is in 3D coordinates, so there should be three light sources from three different directions to determine the normals vector. This means that the intensity matrix can be determined with three different light sources and should have a rank of 3. But, when I do SVD for the **I**, I get the 7 singular values of

$$\begin{bmatrix} 16523752.0198 & 1965510.301 & 1376601.14512517 & \\ 420211.611 & 317502.425 & 250349.083 & 203873.380 \end{bmatrix} \tag{2}$$

This means the matrix has a rank of 7 and does not agree with the rank-3 requirement. This is probably the result of the image capture not being ideal, which could be because of blurriness or reflected lights causing issues. This is causing more independent measurements which leads to a rank greater than 3.

e)

To find an equation for the psuedonormals, we can rearrange the first equation below to get the second equation.

$$\mathbf{I} = \mathbf{L^T B} \tag{3}$$

$$\mathbf{B} = \left(\mathbf{L^T}\right)^{-\mathbf{1}} \mathbf{I} \tag{4}$$

But, the issue is that L is not a square matrix and thus not invertible, so we need to find $\left(\mathbf{L^T}\right)^{-\mathbf{1}}$, and then we can arrange the equation correctly. So, I used a common pseudoinverse method to find $\left(\mathbf{L^T}\right)^{-\mathbf{1}}$ below.

$$\left(\mathbf{LL^T}\right)^{-\mathbf{1}} = \left(\mathbf{L^T}\right)^{-\mathbf{1}} \mathbf{L}^{-1} \tag{5}$$

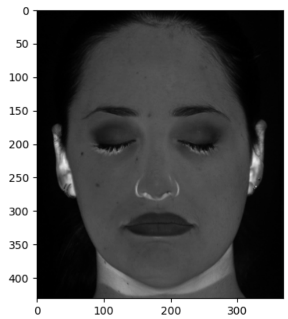
$$\left(\mathbf{L^T}\right)^{-\mathbf{1}} = \left(\mathbf{LL^T}\right)^{-\mathbf{1}} \mathbf{L} \tag{6}$$

Now, with $\left(\mathbf{L^T}\right)^{-\mathbf{1}}$, we get the equation for the pseudonormals below. This equation is now in the $\mathbf{x} = \mathbf{A^{-1}y}$ format, where we can see that A is $\mathbf{LL^T}$ and y is $\mathbf{LI}$

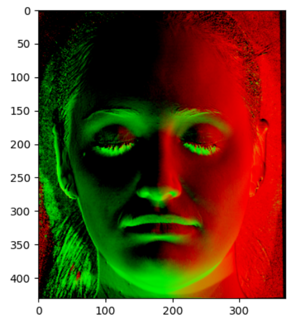
$$\mathbf{B} = \left(\mathbf{LL^T}\right)^{-\mathbf{1}} \mathbf{LI} \tag{7}$$

f)

The albedo and normals images are below. The albedo image had unusual features at the ears, neck, and nostrils. I think that is because these are areas where there are shadows and somehow these shadows are causing these unusual features. The normals matched my expectation of the curvature of the face. I was expecting flatter in the middle and then to turn to two different colors as they moved away from the center of the face in the horizontal direction since that is how a face isshaped. .



(a) Albedo Image



(b) Normals Image

Figure 4: Albedos and Normals for 1f

g)

Given $n = (n_1, n_2, n_3)$, and $z = f(x, y)$, we have the partial derivatives of f below.

$$f_x = \frac{\partial z}{\partial x} \tag{8}$$

$$f_y = \frac{\partial z}{\partial y} \tag{9}$$

If we only move a small amount dx in the x-direction, then we can find the line tangent to the surface as $(dx, 0, f_x dx)$. We can also do the same, but in the y direction and move by dy, this gives us a line tangent to the surface with the equation $(0, dy, f_y dy)$. We then know that the cross product of these two lines will be our normal line since it is perpendicular to the surface. Its equation is below.

$$n = (-f_x dxdy, -f_y dxdy, dxdy) = (n_1, n_2, n_3) \tag{10}$$

From this, we can then see that the following relationship holds.

$$f_x = -\frac{n_1}{n_3} \tag{11}$$

$$f_y = -\frac{n_2}{n_3} \tag{12}$$

h)

$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \tag{13}$$

Given the matrix above we can find the gradients that are below.

$$g_x = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{14}$$

$$g_y = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix} \tag{15}$$

If we then use g_x to find the first row of g and then use g_y to find the rest, we get the results below.

$$g_{row1} = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \tag{16}$$

$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \tag{17}$$

If we then use g_y to find the first column of g and then use g_x to find the rest, we get the results below.

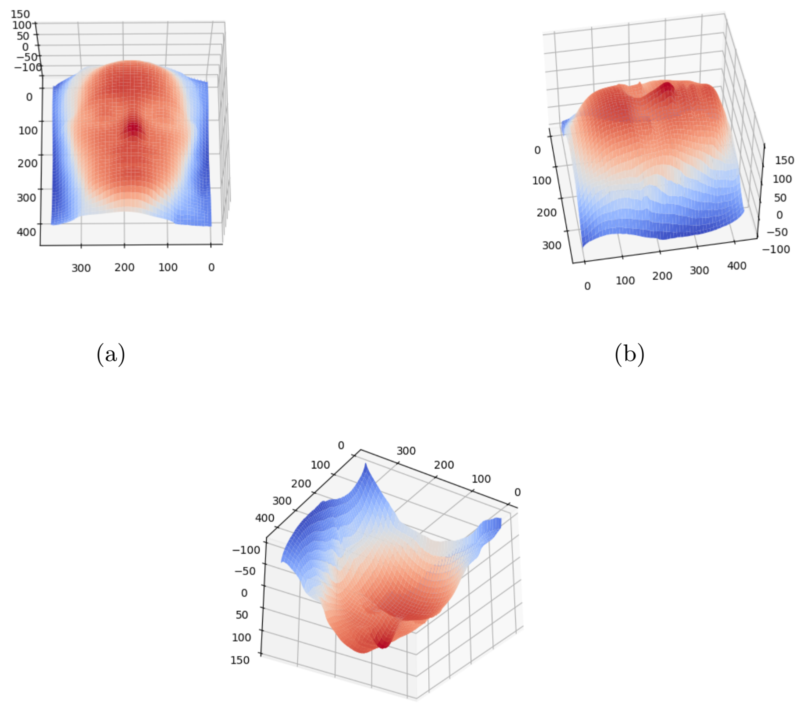
$$g_{column1} = \begin{bmatrix} 1 \\ 5 \\ 9 \\ 13 \end{bmatrix} \tag{18}$$

$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \tag{19}$$

We can see that these two procedures yield the same results. This means they are integrable. Two basic ways to change the gradients to be nonintergrable would to insert some negative values or just make it where the gradients had different values throughout. This could cause addition and subtraction to be done on different values leading to a different g. The gradients estimated in our original way could be nonintegrable because there could be disturbances that affect them and their application.

i)

Several views of the surface plot are shown below.



(a) (b) (c)

Figure 5: Surface Plot Viewpoints

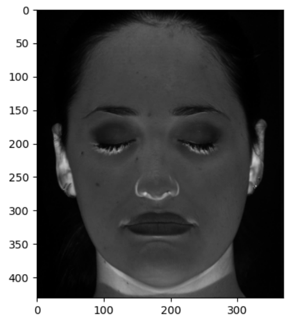
2 Uncalibrated Photometric Stereo

a)

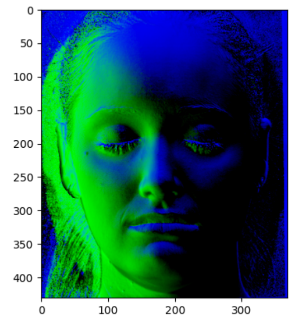
We want the same equation as before $\mathbf{I} = \mathbf{L^T B}$, but now we do not have L or B. In order to solve this issue we know that singular value decomposition gives us $\mathbf{M} = \mathbf{U \Sigma V^T}$, so if we did this for I, we would have $\mathbf{I} = \mathbf{U \Sigma V^T}$. We also want I to be of rank 3 when it is currently of rank 7. We can do this by changing the values from the SVD of I. Set all singular values except the top 3 of Σ to 0 to get $\hat{\Sigma}$. Then choose the top 3 vectors of U and L. With these three new values, we find that we can set $\hat{L}$ as our new three vector U and $\hat{B}$ as our new three vector V^T.

b)

The albedo and normals images are below.



(a) Albedo Image



(b) Normals Image

Figure 6: Albedos and Normals Image for Q2

c)

Screenshots of $\mathbf{L_0}$ and $\hat{\mathbf{L}}$ are below. As you can see the values are not the same. A simple change to correct this would be to multiply $\hat{\mathbf{L}}$ and $\hat{\mathbf{B}}$ by the $\hat{\mathbf{\Sigma}}$ we found in 2a.

```
[[-0.1418  0.1215 -0.069   0.067  -0.1627  0.      0.1478]
 [-0.1804 -0.2026 -0.0345 -0.0402  0.122   0.1194  0.1209]
 [-0.9267 -0.9717 -0.838  -0.9772 -0.979  -0.9648 -0.9713]]
```

Figure 7: $\mathbf{L_0}$

```
[[-0.3553  0.3534  0.6292 -0.5433 -0.1594 -0.1459 -0.1058]
 [-0.3986 -0.6298  0.3977  0.4242 -0.3118 -0.0095 -0.0951]
 [-0.3253  0.1788  0.1061  0.2529  0.2887 -0.1699  0.8215]]
```

Figure 8: $\hat{\mathbf{L}}$

d)

Surface plot of the face is below. The image does not look like a face yet.

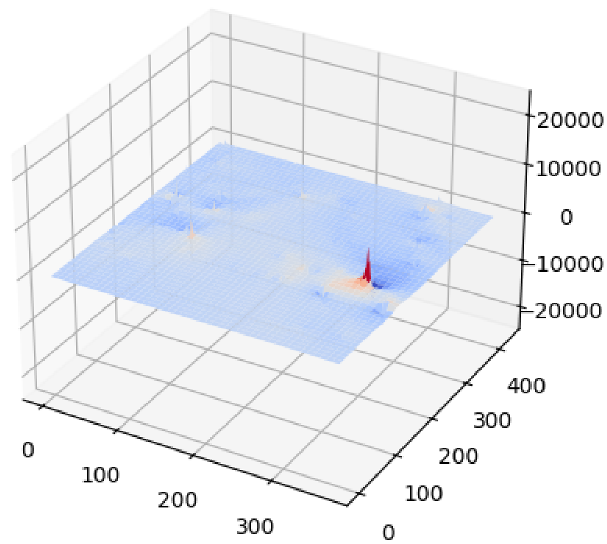
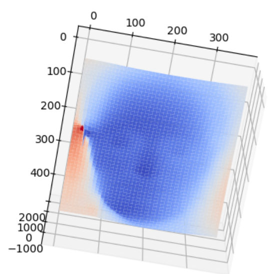


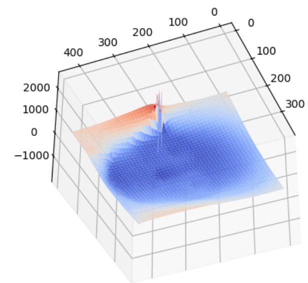
Figure 9: Reconstruction Attempt 1

e)

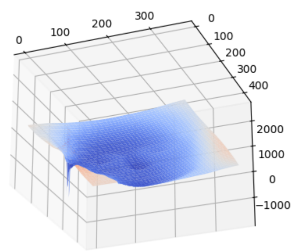
Surface plots are below after enforcing integrability. The images looks like a face, but it is just very flat. This will be fixed when we do bas-relief later.



(a) Viewpoints 1



(b) Viewpoints 2



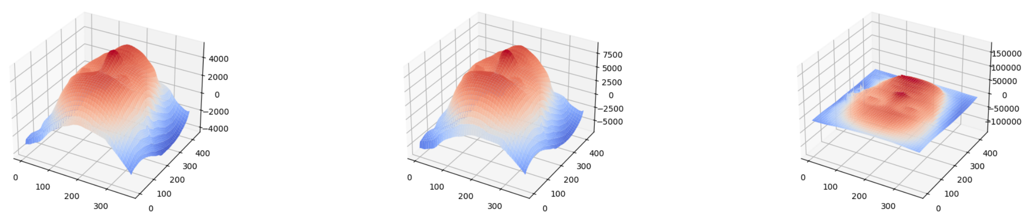
(c) Viewpoints 3

Figure 10: Reconstructed Attempt 2

f)

After playing around with the parameters μ, ν, λ, I found that 0.5, 1.5, and 0.1 worked very well. From those base parameters I varied each value to show the effects of the parameters.

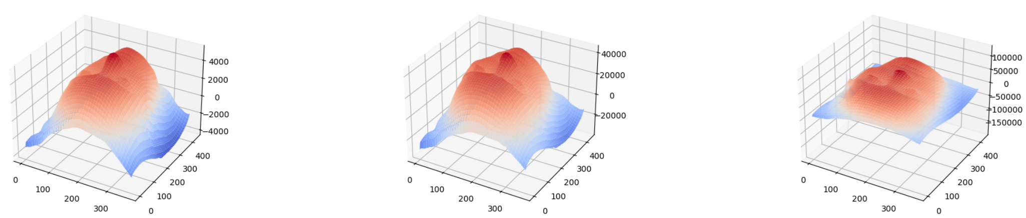
I first varied μ, and as you can see below as μ increased, the surface plot became flatter and the quality of the reconstruction decreased.



(a) $\mu = 0.5$ (b) $\mu = 4.0$ (c) $\mu = 10.0$

Figure 11: Changing μ

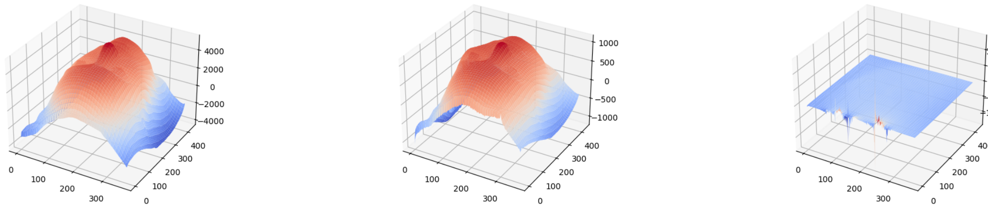
I then varied ν, and as you can see below, as μ increased, the surface plot became flatter and its quality of reconstruction decreased. It took much larger values of ν to see changes in the surface.



(a) $\nu = 1.5$ (b) $\nu = 20$ (c) $\nu = 30$

Figure 12: Changing ν

Lastly, I varied λ, and as it increased it too became flatter and its quality decreased. From this we can understand why it is called bas-relief, meaning low-relief. Because, when the parameters are low, the quality of the reconstruction is far better.



(a) $\lambda = 0.1$ (b) $\lambda = 0.8$ (c) $\lambda = 1.0$

Figure 13: Changing λ

g)

To make the estimated surface flattest, I would choose large values for λ, μ, and ν. From my results above, larger values of each caused the graph to lose its curvature, so increasing all of them should make the surface very flat.

h)

Acquiring more pictures from more lighting directions could help resolve some of the ambguity because it would provide more information to solve for the surface shape. There would still be ambiguity in how we calculated our $\hat{L}$ and $\hat{B}$, though. Also, these pictures may have noise that could cause even more ambiguity.