Proposal of investigating the factors that affect table salt dissolve time by factorial experiment

Objective

Table salt is an essential seasoning in daily meals. It dissolves at different times when it is added to different foods. In this experiment, I am interested in examining three factors that affect table salt dissolve time. The three factors are temperature, stirring rate, and amount of solvent. We have the following hypotheses:

1. null hypothesis: temperature has no effect on dissolve time;
   alternative hypothesis: temperature has effect on dissolve time;

2. null hypothesis: stirring rate has no effect on dissolve time;
   alternative hypothesis: stirring rate has effect on dissolve time;

3. null hypothesis: amount of solvent has no effect on dissolve time;
   alternative hypothesis: amount of solvent has effect on dissolve time.

Experimental Procedures

A factorial experimental design can be used. Each independent variable is a factor in the design. There are three factors, and each factor has two levels as follows:

Table 1. Three factors and their levels for table salt dissolve time

Factor	Level 1	Level 2
Temperature	Low(<50°C)	High(>50°C)
Stirring rate	Low(<120rpm[1])	High(>120rpm)
Amount of solvent	Small(<300ml)	Large(>300ml)

This is a 2×2×2, or 2^3 factorial design, which has 2^3=8 different experimental conditions. The experiment unit is each time I dissolve table salt in different experimental conditions. Table 2 below shows what the experimental conditions are.

[1] Revolutions per minute, 120rpm=2cycles per second

Table 2. Experimental conditions in the 2^3 factorial design for table salt dissolve time			
	Factor		
Experimental condition number	Temperature	Stirring rate	Amount of solvent
1	Low	Low	Small
2	Low	Low	Large
3	Low	High	Small
4	Low	High	Large
5	High	Low	Small
6	High	Low	Large
7	High	High	Small
8	High	High	Large

It will take 8 times to do a factorial experiment; additionally, a replicate factorial experiment will be conducted for another 8 trials for statistical analysis of variance for each unit; so there are total 16 experimental units. When carrying out the experiment, I will add table salt into solvent (water is used) to control the order of adding water or table salt, as well as I will use same cup, same type of water, and same amount of table salt to control other factors (make sure the cup is completely washed and dried so that each trial is independent). Besides, the time will be started whenever table salt is added to water, and it will be stopped when there is no more table salt can be observed in the water. The time recorded each trial is the response variable y_i for the ith experiment, where will have total 16 y_i when experiment is completed.

Statistical Analysis Plan

Firstly, I will build a linear model with the 3 factors (x1, x2, x3), and their interaction terms (x1x2, x1x3, x2x3, x1x2x3), total of 7 variables, to compute the main effect for three factors. The model can be written as:

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \hat{\beta}_3 x_{i3} + \hat{\beta}_4 x_{i1} x_{i2} + \hat{\beta}_5 x_{i1} x_{i3} + \hat{\beta}_6 x_{i2} x_{i3} + \hat{\beta}_7 x_{i1} x_{i2} x_{i3} + \hat{e}_i$$

Where the main effect for three factors x1, x2, x3 can be calculated as $2\times\hat{\beta}_1$, $2\times\hat{\beta}_2$, $2\times\hat{\beta}_3$, separately. In order to provide a data visualization, a cubeplot will be built to show the value of y_i for various combination of three factors at the corners of cube, which shows 12 comparisons along 12 edges of cube (four measurements for the effect of each factor change). When testing whether there exist obvious factor interactions, interaction plot should be given, which shows the main yield for each pair of factors. For example, If the interaction plot of two factors shows the two lines are parallel, then there is no interaction; otherwise, there might be a two factor interaction, where should have 3 interaction plots of two factors. Besides, variance of each response can be estimated by the replication experiment, $s_i^2 = \frac{(y_{i1}-y_{i2})^2}{2}$, where y_{i1} is the outcome from first experiment of the ith trial, y_{i2} is the outcome from the replication experiment of the ith trial. With the estimated variance of each effect, a confidence interval can be constructed for each factor, including the three factors and four interaction factors.