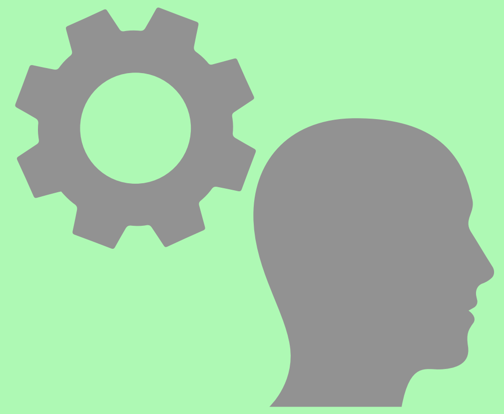
A more expressive preference model

- A *Mahalanobis metric* $\mathbf{M} \succ \mathbf{0} \in \mathbb{R}^{D \times D}$ overcomes these drawbacks:

$$\|\mathbf{x}_i - \mathbf{u}\|_{\mathbf{M}}^2 = (\mathbf{x}_i - \mathbf{u})^\top \mathbf{M} (\mathbf{x}_i - \mathbf{u})$$

- ***New goal:*** Estimate ideal item $\mathbf{u}$ and metric $\mathbf{M}$

Most preferred item



How users make preference judgements

Constraints for metric estimation

$$\mathbf{d} \in \mathbb{R}^N, \quad \mathbf{d}[i] = \|\mathbf{x}_i - \mathbf{u}\|^2_{\mathbf{M}}$$

- Assuming $\mathbf{M}$ and $\mathbf{d}$ are known, we have a *closed form expression for* $\mathbf{u}$:

$$\mathbf{u} = \frac{1}{2}\mathbf{M}^\dagger\mathbf{R}^\dagger(\mathbf{a}_\mathbf{M} - \mathbf{Q}_\Gamma\mathbf{d})$$

$\mathbf{R}$: "Feature matrix" that depends on item feature vectors **(known)**

$\mathbf{a}_\mathbf{M}$: "Item-metric vector" constructed from ground truth metric and item features

$\mathbf{Q}_\Gamma$: "Selection matrix" selects rows based on items used in comparisons **(known)**

- This leads to a condition that our metric and distances must satisfy:

$$\boxed{\mathbf{0} = (\mathbf{I} - \mathbf{R}\mathbf{R}^\dagger)(\mathbf{a}_\mathbf{M} - \mathbf{Q}_\Gamma\mathbf{d})}$$