Austin Yu and Yijun Zhang
Professor Barbara
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Feasibility of Wondering Earth

RESEARCH QUESTION AND DESCRIPTION OF THE SYSTEM

In 1925, a German scientist called Walter Hohmann published his book whose name was The Attainability of Celestial Bodies. In the book, he stated a new way for orbit transferring. If the initial and final orbits are in the same plane, the moving object can change its orbit by accelerating itself at the initial orbit so that it could rotate in an elliptical orbit whose aphelion is just in the final orbit. Then when the object reaches aphelion, it could accelerate again to stay in the final orbit.

This technique has not only been commonly applied in the field of aerospace, but also been used by scientific fictions. In The Wandering Earth, a scientific novel written by Cixin Liu, the earth is suffering from the helium flash of the sun. The sun has become a red giant that the edge of the sun will reach the earth's orbit. Unless the earth moves to a farther orbit, it will be devoured by the sun. Therefore, people on earth come up with an idea that they insert a huge enough engine to push earth to a longer distance from the sun to survive.

Given such a situation, we are interested in the movement of the earth. If a certain condition of the sun is given, how far does the earth need to move in order to maintain a pleasant temperature? In order to discover it, we need to find out the relation between the intensity of the sun in red giant state and distance.

EXPLANATION

To do the calculation for the orbit, we need to assume that the original and final orbits are circle in order for much easier calculations. We also have to assume that the earth could accelerate in a very short time when it enters and exits the elliptical orbit so that the total energy of earth is conserved in the elliptical orbit. In such a way we are able to avoid influence of the time parameter.

We start form the sun. When the sun's core runs out of Hydrogen, it will leave the main sequence and move to the giants branch. The luminosity (L) of the sun will no longer be stable as result of the lack of the Hydrogen, but to us it is still stable in a relative long time. If the earth moves to a new circular orbit with a radius of r, the intensity of the radiation of the sun at distance r will be $I = \frac{L}{4\pi r^2}$. Neglect the influence of the atmosphere. That means there won't be reflection of the radiation from the atmosphere and the earth will absorb all the radiation from the sun. The power per unit area that the earth radiates back in to space is equal to $\frac{P}{A} = \frac{I}{4} = \frac{L}{16\pi r^2}$. With the Stefan-Boltzmann law, $p = \varepsilon\sigma AT^4$, we get $T = \left(\frac{P}{\varepsilon\sigma A}\right)^{\frac{1}{4}}$. For perfect black body, $\varepsilon = 1$. Therefore, $T = \left(\frac{P}{\sigma A}\right)^{\frac{1}{4}}$ where $\sigma = 5.67 \times 10^{-8}\frac{w}{m^2k^4}$. Plugging $\frac{P}{A} = \frac{I}{4} = \frac{L}{16\pi r^2}$ into $T = \left(\frac{P}{\sigma A}\right)^{\frac{1}{4}}$, we get $T = \left(\frac{L}{16\pi r^2\sigma}\right)^{\frac{1}{4}}$. By restricting the temperature T to the normal temperature on earth, we are able to find the approximate necessary distance of the final orbit.

Since we know the information of both orbits, we can start the analysis of the Hohmann transfer. At the beginning, we have assumed that two orbits are circle. Perihelion and aphelion intersect with original orbit and final orbit respectively. It is not hard to construct diagrams of Hohmann Transformation.

VALIDATION OF CODES

We show the validation of our codes by two approaches: comparing against the analytic solution and demonstrating that the angular momentum is conserved. The results are summarized in the table bellow.

We first compare our Numerical solutions to the analytical ones. For the three orbits, we have solved them both analytically and numerically. We first unzip the final results for the three orbits and then use phi array to analytically solve for r array. Then, we could compare the analytically and numerically solved r array to find the error. By applying The procedure np.mean(numericalSolution - analyticalSolution), we could find the accuracy of our numerical approximation. The table below summarize the results.

(The luminosity of the sun at different time comes from http://www.astronomy.ohio-state.edu/~pogge/Ast162/Unit6/futuresun.html)

		Error in r	Error in Angular Momentum
r_current -> r_hCore	**Initial Orbit**	-8.819580e-05	5.801514e-16
	Homann Transfer	-2.919997e-09	0.000000e+00
	Final Orbit	1.525879e-05	0.000000e+00
r_hCore -> r_redGiant	**Initial Orbit**	-3.326416e-05	-2.012028e-16
	Homann Transfer	-6.792331e-07	0.000000e+00
	Final Orbit	-7.841797e-03	0.000000e+00
r_redGiant -> r_HoriBranch	**Initial Orbit**	-7.841797e-03	-2.884903e-17
	Homann Transfer	-4.881573e-07	0.000000e+00
	Final Orbit	3.198242e-04	1.526364e-16
r_HoriBranch -> r_HeCore	**Initial Orbit**	3.198242e-04	-9.081863e-16
	Homann Transfer	-1.343372e-07	1.526364e-16
	Final Orbit	2.960938e-02	0.000000e+00

There is also a part of codes in the notebook showing the procedures of finding these errors.

We could conclude that the error between the analytical and numerical solutions are fairly low, considering our magnitude of masses of earth and sun, so our numerical approximation is pretty accurate.

Second, we could check the conservation of momentum in our numerical solutions. We calculate the angular momentum at each instant by $\mu r^2\dot{\phi}$. We could do this by modifying the derivative function to record $\dot{\phi}$ values, and then calculate angular momentum. Obtaining the array of angular momentum for the three orbits, we could now analyze the error.

We analyze the error by the following algorithm. Since angular momentum should be conserved, if we subtract any number in the numerical array with the analytical value, we should get zero. We could then analyze the error by subtracting the numerical array by analytical value and dividing by the analytical value. The procedures are documented in the notebook.

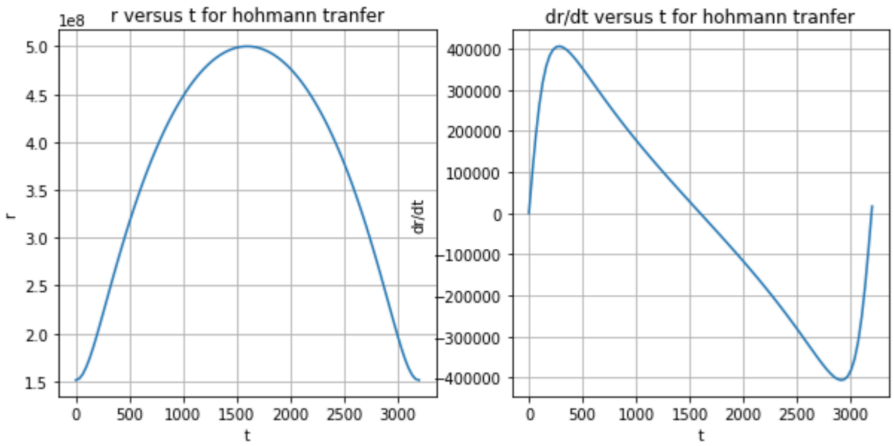
We can see that the result is pretty accurate in reflecting the conservation of momentum.

By comparing against the analytic solution and demonstrating that the angular momentum is conserved, we show that are codes are valid in analyzing the orbits of Hohmann Transfer.

		lambda
r_current -> r_hCore	Initial Orbit->Homann Transfer	1.062030
	Homann Transfer->Final Orbit	1.070825
r_hCore -> r_redGiant	Initial Orbit->Homann Transfer	1.393022
	Homann Transfer->Final Orbit	4.099912
r_redGiant -> r_HoriBranch	Initial Orbit->Homann Transfer	0.483104
	Homann Transfer->Final Orbit	0.752367
r_HoriBranch -> r_HeCore	Initial Orbit->Homann Transfer	1.338146
	Homann Transfer->Final Orbit	2.185478

FINDINGS

The most significant finding is the value for lambda, which is the ratio of final speed over the initial speed during each orbit transformation. The lambda values associated with each state of transition is summarized in the table above. The immediate conclusion is that though most transitions requires only a "moderate" modification of speed, some transitions do require drastic change in orbiting speed. The increase of speed by a factor of 4 is indeed practically impossible. Considering the gigantic mass of the earth, the required kinetic energy increase is even larger. Even worse, to let earth maintain a safe temperature, the total speed increase at the end of He core stage is almost by a factor of 7.



In addition, in the idealized situation, if we analyze the graph for $\frac{dr}{dt}$ Hohmann Transfer orbit for all transitions, there is a shorter duration when we should accelerate away from the sun and a longer period to accelerate towards the sun.

SUMMARY

We successfully calculated the radius of the new orbit and the Hohmann transfer at many time points. But these has a lot of errors. We made a lot of assumptions in order to reduce the difficulties in calculating and modeling, such as regarding the earth's orbit as a circle. Actually , it is not a perfect circle but an eclipse. We also neglect the effect of the atmosphere. Atmosphere could reflect part of the radiation from the sun and absorb more radiation after the sunlight goes through the atmosphere. If we take it into consideration, the model will be more complicated to be built up. It would be better if we can find a relationship between the luminosity of the sun and its mass or temperature, but due to the weird change when the sun becomes a red giant, we found it is not manageable. However, we were able to locate some important time points when the sun is suffering from huge change in its body.

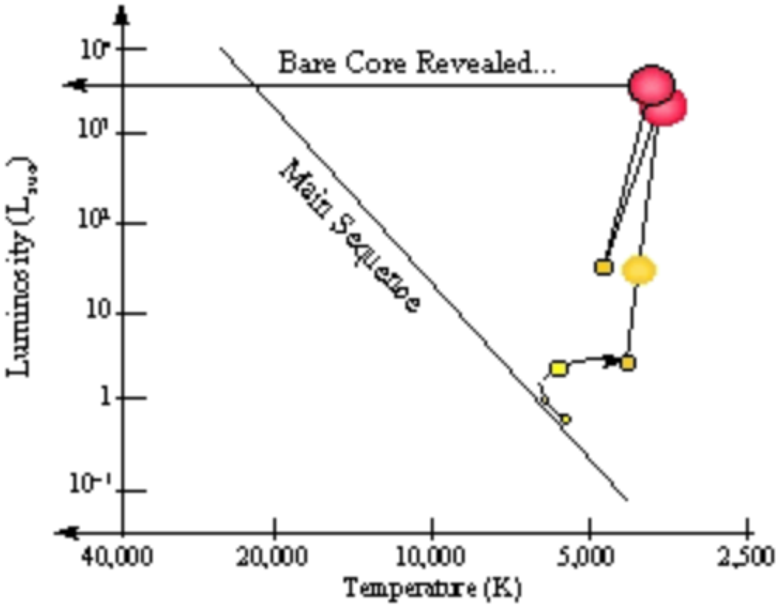


Figure 1(Luminosity vs Temperature for Sun) From http://www.astronomy.ohio-state.edu/~pogge/Ast162/Unit6/futuresun.html

Our analysis follows the common idea that the earth has to move farther from the sun when it becomes older. However, based on the current technology, human has no possibility to move the earth by Hohmann transfer. In the future, maybe human could achieve the goal.