Lab 9 Turntables 4/2/20

$G \equiv$ inertial frame ("Lab frame") $T \equiv$ turntable frame

$\vec{\Omega}$ ang. vel of T

$$m\{{}^G\vec{a} = {}^T\vec{a} + 2\vec{\Omega}\times{}^T\vec{v} + \vec{\Omega}\times\vec{\Omega}\times\vec{r}\}$$

$m\,{}^G\vec{a} = {}^G\vec{F}_{net}$ forces identified in inertial frame

Solve for ${}^T\vec{a}$ acceleration in noninertial frame T (turntable)

$$m\,{}^T\vec{a} = {}^G\vec{F}_{net} - 2m\vec{\Omega}\times{}^T\vec{v} - m(\vec{\Omega}\times\vec{\Omega}\times\vec{r})$$

* change order of cross product

$$\underbrace{m\,{}^T\vec{a}}_{\text{accel defined in Turntable frame}} = \underbrace{{}^G\vec{F}_{net}}_{\text{forces identified in inertial Ground frame}} + \underbrace{2m\,{}^T\vec{v}\times\vec{\Omega}}_{\text{coriolis force}} + \underbrace{m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}}_{\text{Centrifugal force}}$$

Coriolis force NOTE: depends on velocity of object in T-frame ($\perp$ to $\vec{v}$)

Centrifugal force depends on how far object is from axis rotation rotation (radially outward)

$|\vec{\Omega}|$ constant

$\dot{\Omega} = 0$

This lab focuses on coriolis + centrifugal pseudoforces

${}^G\vec{a} = 0$

$$\boxed{{}^T\vec{a} = 2\,{}^T\vec{v}\times\vec{\Omega} + \vec{\Omega}\times\vec{r}\times\vec{\Omega}}$$

$\vec{\Omega} = \Omega e_3$

$\vec{r} = x e_1 + y e_2$

$\vec{v} = \dot{x} e_1 + \dot{y} e_2$

$\vec{a} = \ddot{x} e_1 + \ddot{y} e_2$

do these cross products

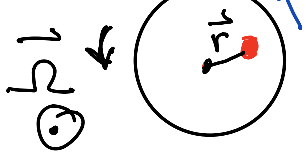
you need to code $\ddot{x}$. $\ddot{y}$

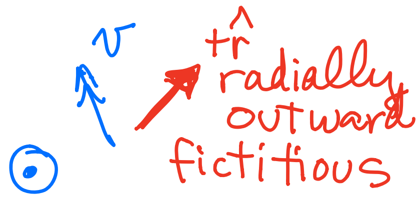
including F_{COR} F_{CF}

$$F_{CF} = m(\underline{\vec{\Omega} \times \vec{r}}) \times \vec{\Omega}$$

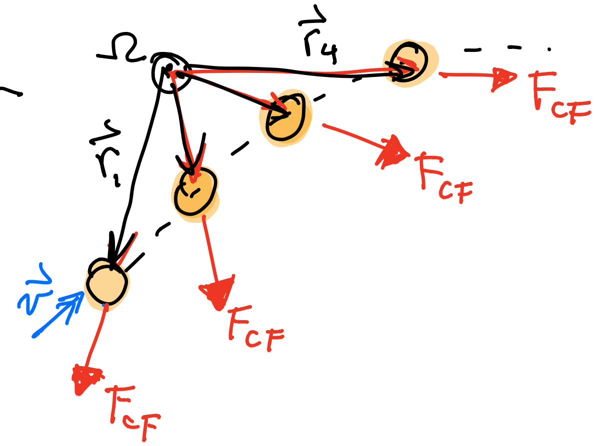
Note $\vec{v}_{Tang}$ is first cross product in F_{CF}

$$\vec{v} = \underline{\vec{\Omega} \times \vec{r}}$$

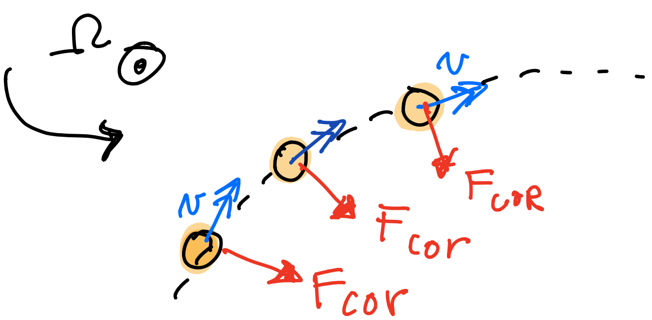




"pushes" object <u>AWAY</u> from the center of rotation



$$F_{COR} = 2m(\dot{\vec{r}} \times \vec{\Omega})$$



"pushes" object opposite to direction of rotation of turntable:

if Ω is ccw

F_{COR} is cw

(Ω: cw , F_{COR}: ccw)

For code:

$\ddot{x}_{COR}$ $\ddot{x}_{CF}$ $\ddot{y}_{COR}$ $\ddot{y}_{CF}$

add to deriv function

In your lab all init condns are given in T-frame : x_0 y_0 v_{x0} v_{y0}

To convert to G-frame you have to use the rule :

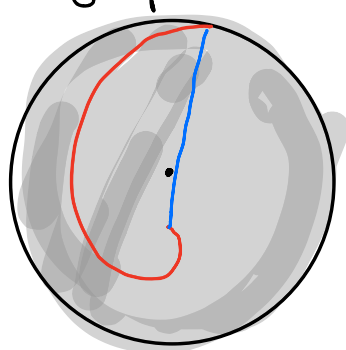
$$\boxed{\frac{d}{dt}_G \, \square = \frac{d}{dt}_T \, \square + \vec{\Omega} \times \square}$$

Here are some hints by which you can check your work early in the lab:

In Ex 9.5, you will [finally!] plot the trajectory of the puck in the T-frame.

The puck "appears out of nowhere" on the turntable at $r_0 = [1, 0]$ with $v_0 = [0, 1]$.

Your graph should look like red trace below:



If I superimpose the puck's trajectory as seen in the G-frame, it should move in a straight line that begins and ends in the same place, like the blue trace.

If you get these trajectories, your code is probably correct.