Laboratory study of the cosmic-ray muon lifetime

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V. CONCLUSION

It is known from Ehrenfest's theorem[25] that the center ($\langle x \rangle$) of any wave packet follows the *exact* classical path only when the potential is harmonic or uniform. If we stick to the two properties, namely, (i) $\Delta x \Delta p = \hbar/2$ for all time and (ii) $\langle x \rangle$ follows the classical path, to define coherent states, then the Schrödinger coherent states, treated above, are the only such states, that is, we can talk about coherent states only for a harmonic oscillator.[26]

There are three equivalent ways of describing a minimum uncertainty state, by Eqs. (12), (26), and (28). For the time evolution of such states, in many cases, it is simpler to start from an equation of the type (12) and obtain (37), in which $x_H(-t)$ and $p_H(-t)$ can be calculated from the classical equations of motion. When the resulting equation is linear in x and p, the time-developed states can easily be obtained. Such states are not in general coherent. Even a phase change, such as in (52) destroys the coherence of the state.

ACKNOWLEDGMENTS

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Laboratory study of the cosmic-ray muon lifetime

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The cosmic-ray muon lifetime was measured with a variety of counters designed to study both the free and μ^- capture lifetimes. The data were obtained using scintillation detectors and a lead glass detector. These data show the dependence of μ^- capture on the atomic number of the chemical element in the detector. The Z dependence of the weak interaction capture process is discussed in terms of the familiar Fermi ($\Delta J = 0$) and Gamow–Teller ($\Delta J = 1$) decays. This experiment was designed for use in advanced undergraduate physics laboratories.

I. INTRODUCTION

The study of the cosmic-ray muon lifetime has been a standard undergraduate exercise in modern physics laboratories for almost two decades.[1–4] This classic experiment has been both instructive and enlightening to students since it involves modern experimental techniques, simple electronic and detector configurations, and relies on a secondary cosmic-ray background flux. Hall, Lind, and Ristinen[2] simplified the lifetime experiment by using a single scintillator detector instead of a three-counter system. More recently, Owens and Macgregor[3] and Lewis[4] have developed new and simple fast logic automated systems for the measurement of the muon lifetime. Lewis[4] constructed a new microcomputer based experimental apparatus that interfaces a Radio-Shack TRS-80 to the electronics and detectors, eliminating elaborate and expensive data acquisition systems. The parameter measured in these experiments is the muon lifetime. The free space value[5] is $\tau = 2.198\ \mu s$ whereas the capture lifetime depends on the atomic number of the capturing element varying from 357 μs ($Z = 3$) to 77 ns ($Z = 92$). Previous studies[2–4] used plastic scintillator detectors which measure a composite lifetime of stopped free muons and μ^- capture on the carbon in the scintillator. The present study was designed to measure the Z dependence of the muon capture lifetime by comparing results of various detector types, i.e., plastic scintillator, liquid scintillator, lead glass, and a water detector with various chemicals dissolved in it. The water detector was found to be as efficient as either plastic or liquid scintillators. The reduced cost and increased flexibility of the water make it an ideal detector for use in modern physics laboratories.

II. EXPERIMENTAL PROCEDURE

Cosmic-ray muons interact with matter through electromagnetic or weak interactions making them easy to detect using scintillator detectors with simple electronic systems. The great majority of muons have very high energies allowing them to pass through detectors with thicknesses of 10–20 cm. In the present experiment about 1 in 250 muons that enter the detector were stopped and decayed inside it producing two electronic signals. The first pulse results from the passage of the muon in the detector signaling the particle's arrival. A second pulse results from the weak decay of the muon after it has come to rest in the detector. The time interval (Δt) between the start and stop (decay) signals is a measure of the muon lifetime.

The detectors were coupled to a 5-in. 4522-RCA photo multiplier tube. Signals from the muon passage (start) and decay (stop) were processed through a timing filter amplifier (TFA), a constant fraction discriminator (CFD), a time-to-pluse height converter (TAC), and stored in a multichannel analyzer (MCA) in the form of start–stop events versus Δt. A comprehensive discussion of methods are given in Refs. 1 and 2. The low-energy room background radiation was discriminated against by using a ^{226}Ra γ-ray source and raising the lower level discriminator of the CFD to eliminate such low-energy signals.

The detectors used in this study were a NE 102 plastic scintillator ($15\times15\times100$ cm^3), a NE 213 liquid scintillator (10 cm $\times 324$ cm^3), a Pb glass Cerenkov detector ($15\times15\times30$ cm^3), and a plastic box (3800 cm^3) filled with distilled water or a chemical solution. The chemical forms dissolved in the water were nitrate and acetate salts of various elements such as Ca, Zn, or Ba whose concentrations were approximately 1%–5% by weight. The percentage capture on the elements in the water is simply proportional to the concentration of the element in the solution.

III. RESULTS AND DISCUSSION

In Fig. 1 are shown results of a typical run using the 1524-cm^2 plastic scintillator. A total of 41 211 events were recorded with an average count rate of about 7 events/min. A least-squares fit to the muon lifetime was $\tau = 2.29 \pm 0.17$ μs which compares well with the free space value of 2.20 μs. However, as noted by Hall, Lind, and Ristinen,[2] approximately 50% of the negative muons are captured by the carbon with a mean lifetime of 2.04 μs. Therefore, the measured muon lifetime should be somewhat shorter than the free space value of about 2.15 μs. Our uncertainty only reflects the statistical uncertainty of the fit to the data whereas drift in the TAC is probably the main source of error in these simple measurements. The liquid scintillator tests included using alcohol and mineral oil base, yielding results comparable to the plastic scintillator as given in Table I.

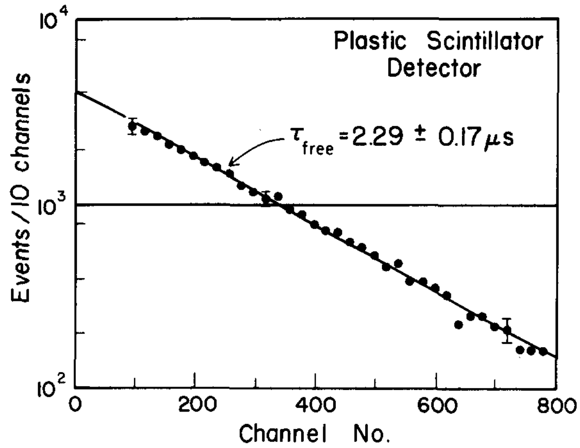

Fig. 1. Cosmic ray muon lifetime measured with a plastic scintillator detector. The TAC spectrum calibration was 9.15 ns/channel.

When a liquid scintillator with 5.5% cadmium ($Z = 48$) by weight is used then a TAC spectrum like that shown in Fig. 2 is obtained. The muon lifetime has two components of 2.35 ± 0.22 μs and 111 ± 15 ns corresponding to the free decay and the Cd capture, respectively. The accepted[6] capture lifetime on Cd is 95 ± 5 ns. The largest uncertainty in the short-lived capture decay results from the subtraction of the long-lived free-decay component coupled with the TAC drift.

The lead glass Cerenkov detector was composed of 30% Pb by weight. The free decay and the capture decay both result in the production of Cerenkov radiation primarily through the interaction of relativistic electrons produced by the primary decay as in $\mu^+ \to e^+ \nu e \nu \mu$ or through secondary pair production by high-energy γ rays produced in the

Table I. Comparison of muon detection rates in various scintillation detectors.

Detector type	Lifetime (μs)	Rate (events/cm^2·s)
Plastic	2.15 ± 0.16	8.0×10^{-5}
	2.51 ± 0.51	2.0×10^{-5}
	2.18 ± 0.25	8.6×10^{-5}
	2.29 ± 0.17	5.7×10^{-5}
Liquid w/alcohol	2.27 ± 0.30	3.8×10^{-5}
Liquid w/mineral oil	2.33 ± 0.29	5.5×10^{-5}
Liquid w/5.5% Cd	2.35 ± 0.22	1.0×10^{-4}
	0.111 ± 0.015	
Water	2.37 ± 0.18	9.6×10^{-5}
Water w/Ca	2.27 ± 0.21	9.7×10^{-5}
	0.367 ± 0.049	
Water w/Co	2.35 ± 0.30	8.0×10^{-5}
	0.187 ± 0.030	
Water w/Zn	2.23 ± 0.51	6.2×10^{-5}
	0.167 ± 0.038	
Water w/Ba	2.25 ± 0.65	9.7×10^{-5}
	0.090 ± 0.017	
Water w/Pb	2.10 ± 0.28	7.3×10^{-5}
	0.077 ± 0.015	

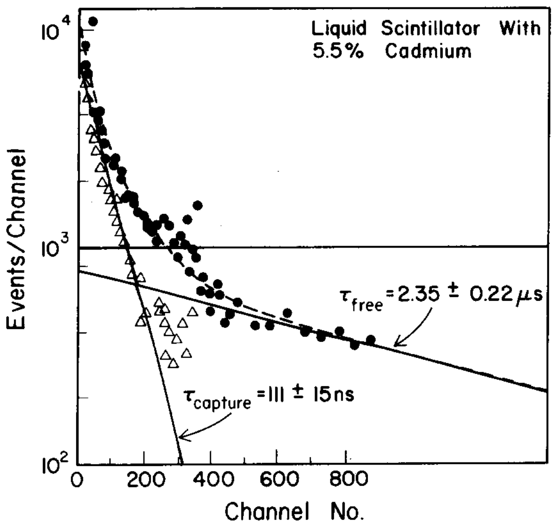

Fig. 2. Muon TAC spectrum taken with a liquid scintillator detector with 5.5% cadmium. The short-lived component is due to the muon capture on Cd.

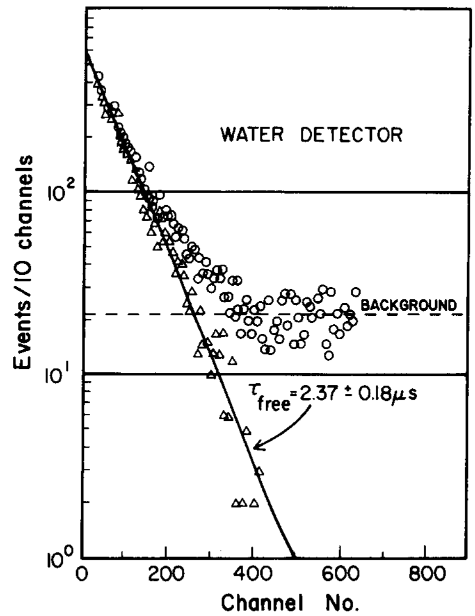

Fig. 3. TAC spectrum of muons decaying in a water detector. The TAC calibration was 38 ns/channel.

capture process. A typical spectrum taken with the lead glass showed two components. The two components were analyzed to be 79 ± 15 ns and 2.15 ± 0.23 μs. The short 79-ns component corresponds to the capture lifetime in Pb whose accepted[6] value is 76 ± 6 ns.

Encouraged by the results of the Cd-scintillator and Pb glass experiments we undertook to develop a detector which could be used routinely to survey lifetimes in several heavy elements. Since a liquid scintillator doped with Cd or other heavy elements was too expensive and glass mixtures were too prohibitive we decided to use water as a detector with various heavy element chemicals (typically nitrates or acetates) dissolved in it to a concentration of 1%–5% by weight. The detector was constructed of plastic with a total volume of 3.8 liters or about 1 gallon. Inside the plastic box were NE 102 plastic scintillator shelfs (approx. $\frac{1}{8}$-in. thickness) to act as a light guide to the photomultiplier tube. In Fig. 3 is shown a spectrum of the water detector without any heavy element dissolved in it. The measured lifetime in water was 2.37 ± 0.18 μs with a total number of events of 6954. The constant background observed in the spectrum is due to accidental coincidence recorded when two muons pass through the detector almost simultaneously, one recording a start pulse and the second registering as a stop pulse. A more complicated electronic anticoincident shield above and below the detector could eliminate this background. However, it never constituted more than 5%–10% of the events registered and could easily be subtracted since it was constant in the TAC spectrum. Spectra were also obtained for Ca, Co, Zn, Sr, Ba, and Pb dissolved in water. The results of those measurements were in good agreement ($\pm$ 20%) with the accepted[6,7] capture lifetimes of those elements.

In Table I is shown a comparison of the detection rates of various systems used in this experiment, except for the lead glass whose density is substantially greater. Clearly, the plastic scintillator (PS), liquid scintillator (LS), LS with alcohol, LS with mineral oil, LS with cadium, and water detectors all have comparable detection efficiencies of about $6\text{–}8 \times 10^{-5}$ events/cm^2·s. These values are comparable to earlier count rates observed in previous experiments.[2–4] We conclude that a water detector is more versatile and considerably less expensive than plastic scintillators or liquid scintillators.

There are a number of good scientific reviews of muon capture[6–10] and muon physics.[11] The classic paper by Primakoff,[8] *Theory of Muon Capture* and the report by Mukhopadkyay[10] are required reading for the more serious physics student. The present study was intended to be more than an electronic exercise for the advanced modern physics student since the interpretation of these simple lifetime measurements are a valuable introduction into the theories of weak interactions and elementary particle models. There are only four basic interactions or forces by which two particles can interact with one another and there are only two families or genre of matter, namely quarks and leptons. The muon as a member of the lepton family interacts with all other matter through weak and electromagnetic forces. The photon and leptons have no strong interactions. Fermi's theory of weak interactions[12–14] has been well established for the past half-century and is recognized as a universal phenomenological description of weak interactions. More recently, the *Unified Gauge Theory* of Weinberg–Salam[15,16] unites the theories of weak and electromagnetic interactions into a single fundamental interaction. One result of this fundamental convolution of weak and electromagnetic interactions was the predicted existence of new elementary particles known as the intermediate vector bosons (W^+, W^-, and Z°) which, like the photon, mediates the interaction. The interaction of quarks and leptons through weak processes is known as flavordynamics in quantum chromodynamics (QCD). The reader is referred to reports in Refs. 17, 18, and 19 for a more detailed discussion of this topic than can be allowed for in this article.

The lifetimes[6,7] for muon capture and decay are commonly plotted as the log of the decay constant ($\lambda = 1/\tau$) versus the log of the atomic number (Z) of the capturing element as shown in Fig. 4. The free muon lifetime ($\tau = 2.198$ μs) dominates the total lifetime at low Z since the total lifetime is given by

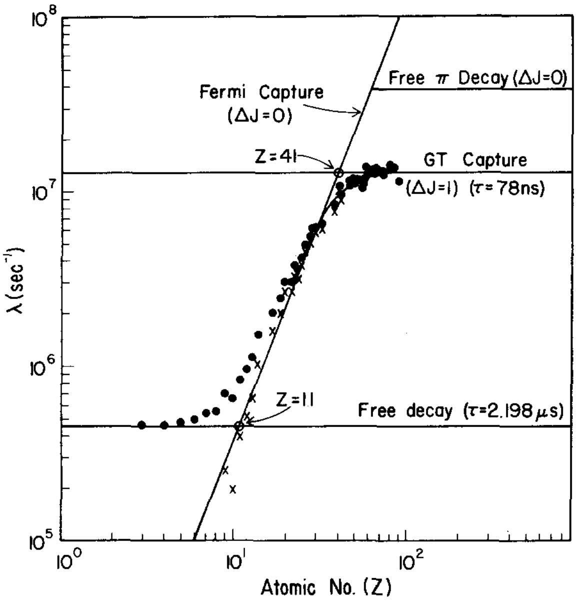

Fig. 4. Plot of the muon capture decay constants versus Z of the capturing element. The lifetimes were taken from Refs. 6 and 7.

$$\lambda_{\text{total}} = \lambda_{\text{free}} + \lambda_{\text{capture}}. \tag{1}$$

Substracting the free component one is left with the capture lifetime as shown in the figure by the crosses. It is evident from the curve that the capture lifetime is equal to the free value for $Z \approx 11$ and approximately constant for $Z \geqslant 40$. For very low Z the lifetime is proportional to Z^4. The Z^4 law results from the fact that the capture probability is proportional to the number of protons (Z) in the nucleus and to the charge density of the muon–nucleus overlap (Z^3). Primakoff's formula[8] correctly predicts the Z dependence provided Z is not too low. The empirical formula is based on weak interaction theory and is given by

$$\lambda_{\text{cap}} = Z^4_{\text{eff}} \lambda ({}^1_1\text{H}) \gamma_\nu [1 - \zeta (N - Z)/A], \tag{2}$$

where Z_{eff} is the effective nuclear charge, γ_ν is the proportional to the emitted neutrino momentum, and the last factor takes into account the neutron excess of the heavy nuclei. The probability is given by

$$\lambda ({}^1_1\text{H}) = M^2_{\text{F}} + 3M^2_{\text{GT}}, \tag{3}$$

where M^2_{F} is the Fermi matrix element contribution ($\Delta J = 0$) and M^2_{GT} is the Gamow–Teller matrix element ($\Delta J = 1$). This form of the total transition matrix element is similar to one used in strong interactions of nuclei, i.e., (p,n) reactions at intermediate energies that are observed to have strong axial vector GT transition strength.[20–22] The GT matrix element is composed of axial vector and pseudoscalar components,[8]

$$M^2_{\text{GT}} = M^2_{\text{AV}} + \tfrac{1}{3} M^2_{\text{PS}} - \tfrac{2}{3} \text{M}_{\text{PS}} \text{M}_{\text{AV}}. \tag{4}$$

The Gamow–Teller axial vector component (M^2_{AV}) is commonly investigated in beta decay and in (p,n) reactions. The GT matrix elements are approximately constant at low to intermediate energies. Primakoff's formula [Eq. (2)] correctly predicts the low Z behavior up to $Z \sim 40$ but for elements greater than this one must use $Z_{\text{eff}} = 42$. Previous interpretations of the cause of this limited Z dependence focused on the muon–nucleus overlap since it was argued that for heavy nuclei the muon spends most of its time inside the nucleus which behaves as an infinite medium. An alternative explanation of the limiting Z behavior centers on the Gamow–Teller contributions to the transition matrix elements of Eqs. (3) and (4). Primakoff obtained $\lambda ({}^1_1\text{H})$ for muon capture by introducing the expression for the neutron β-decay rate which includes only the vector (Fermi) and axial vector (GT) components of the matrix element. A natural explanation for the constant lifetime is the pseudoscalar contribution. In Fig. 5 are shown two Feynman diagrams of the muon capture process. The first diagram involves Fermi transitions of the type $\mu^- p \rightarrow \nu_\mu n$ and includes higher order axial vector contributions. The second diagram involves the exchange current or pion cloud of the nucleus given by $p = p' + \pi^\circ$ for the neutral current and $p = n' + \pi^+$ for the charged current. The second diagram is given by the sequence $\mu^- p \rightarrow \mu^- p' \pi^\circ \rightarrow \nu_\mu p' \pi^- \rightarrow \nu_\mu n'$. This second interaction is known as the induced pseudoscalar transition since the meson or pion is a pseudoscalar particle whose intrinsic spin parity is $J^\pi = 0^-$. The matrix element for the second diagram is $\langle p' \pi^- \nu_\mu | H_{\text{PS}} | p' \pi^\circ \mu^- \rangle$ which can be related to the free pion decay ($\pi^+ \rightarrow \mu^+ \nu_\mu$ "vacuum") by the relation

$$(\text{M.E.})^2_{\text{PS}} = [1/(2\text{J} + 1)](\text{M.E.})^2_\pi, \tag{5}$$

where $(\text{M.E.})_\pi = \langle \text{“0”} \nu_\mu \mu^+ | H_\pi | \pi^+ \rangle$. Using crossing relations the free pion matrix element becomes $\langle \nu_\mu \pi^- | \text{H}_\pi | \text{“0”} \mu^- \rangle$ which is equivalent to the matrix element for the second diagram. The lifetime for the induced pseudoscaler decay simplifies to

$$\lambda_{\text{PS}} = [1/(2\text{J} + 1)] \lambda \pi, \tag{6}$$

where $\Delta J = 1$ for the GT transition yields $\lambda_{\text{PS}} = 1.28 \times 10^7\ \text{s}^{-1}$, a value in good agreement with the limiting lifetime observed for high Z atomic elements ($Z > 40$).

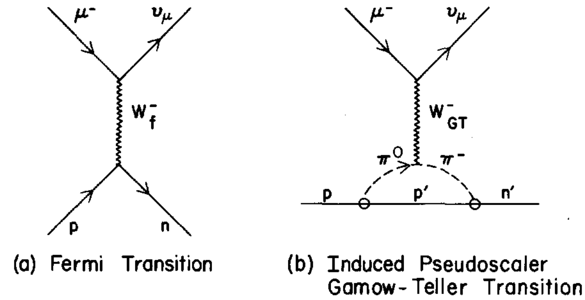

Fig. 5. Feynman diagrams of the Fermi and Gamow–Teller (induced pseudoscalar) muon capture interactions.

The role that quarks play in the muon capture process is self-evident when we substitute quarks for the composite mesons or baryons. The capture transitions become

$$\mu^- p = \mu^- (uud) \rightarrow \nu_\mu (dud) = \nu_\mu n$$

and

$$\mu^- \pi^\circ = \mu^- (1/\sqrt{2})(u\bar{u} + d\bar{d}) \rightarrow \nu_\mu (\bar{u} d) = \nu_\mu \pi^-.$$

The Fermi transition involves the transformation of an up quark into a down quark, $\mu^- u \rightarrow \nu_\mu d$ whereas the GT-induced pseudoscalar transition transforms either quark or antiquark components, $\mu^- u \rightarrow \nu_\mu d$ and $\mu^- \bar{d} \rightarrow \nu_\mu \bar{u}$. The weak interaction of leptons and quarks is known as flavordynamics[17–19] since one is only changing the flavor ($u \rightarrow d$ or $\bar{d} \rightarrow \bar{u}$) and not the color of the quark.

In summary the cosmic-ray muon lifetime was measured with a variety of counters designed to study both the free muon decay and the μ^- capture lifetime. A scintillator box filled with water and various chemical elements dissolved in it was used to investigate the Z dependence of the μ-capture decay. The water detector was found to be as efficient as plastic or liquid scintillators in detecting muons, less expensive, and more versatile, making it an ideal detector material for undergraduate modern physics laboratories.

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Temperature of incandescent lamps

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A method for determining filament temperatures of commercially available incandescent lamps has been presented. The light emitted by a filament at two well-defined wavelengths has been measured by means of two silicon solar cells with interference filters. Since the ratio r of the two signals so determined is a function of the filament temperature, an r against T calibration curve is needed, in order to obtain the filament temperature itself. This calibration curve is determined by means of the well-known dependence of tungsten resistivity on absolute temperature. Considerations about the color index of a star can be raised.

I. INTRODUCTION

In the experiment presented here we want to find the temperature of some commercially available incandescent lamps, analyzing the radiation emitted by their filament at two well-defined wavelengths.

The first step is to determine a calibration curve, using another commercially available incandescent lamp, arbitrarily chosen by us as the standard lamp. By means of a voltmeter and an ammeter, it is possible to find the electrical resistance of its filament, by increasing the voltage until the operating voltage is attained. Then, from the well-known variation law of tungsten resistivity with temperature, the filament temperature can be calculated. A simple relation[1] valid at high temperatures, that permits this calculation, is

$$T = T_0(R/R_0)^{1/1.2}, \tag{1}$$

where R_0 is the resistance of the tungsten filament at ambient temperature T_0.

For each filament temperature, by means of two silicon solar cells with interference filters, the signals $S(\lambda_1)$ and $S(\lambda_2)$ at the two wavelengths allowed by the two filters, can be determined. Then, the ratio $r = S(\lambda_1)/S(\lambda_2)$ of the two signals is plotted against the temperature, to obtain the calibration curve.

The last phase of this experiment consists in the determination of the temperatures of some other incandescent lamps, by measuring the ratio r of the two optical signals, a ratio that allows us to estimate their filament temperature by interpolation on the calibration curve. Then, the reliability of the values so found can be controlled by means of