Triangulating torus and sphere

1 Sphere and torus as surfaces of revolution

One way is to think of a torus or a sphere as surfaces of revolution. More precisely, to get a torus, we need to rotate the circle in the xz-plane centered at $(R, 0, 0)$ with radius $r < R$ (geometrically this means we don't want the origin inside the circle – a torus need to have a through-hole). In the case of a sphere, we rotate a half-circle like the one in Figure 1.

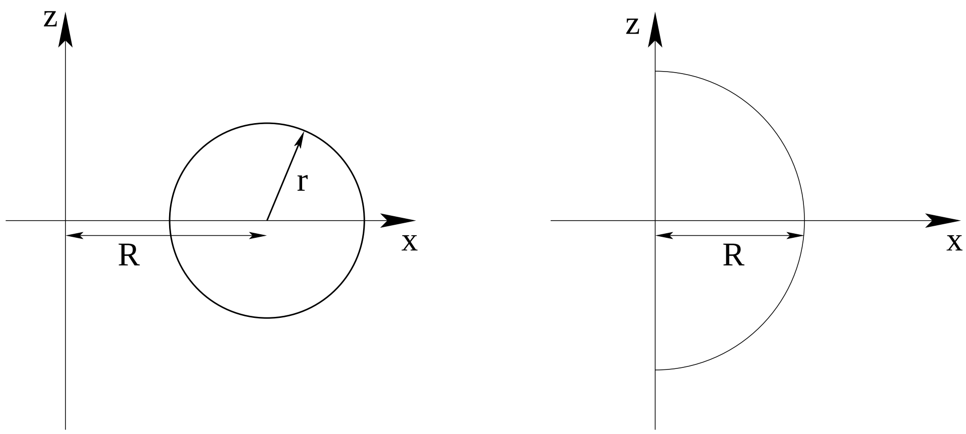

Figure 1: Circle and half-circle needed to be rotated around the z-axis in order to obtain a torus and a circle.

The points of sheet we will wrap can be described by two numbers (coordinates). These coordinates will be the rotation angle (ϕ) and an angular variable describing a point on the rotated curve (in this case, circle or half-circle). See Figure 2.

In the case of a sphere, the curve to be rotated around is described by

$$c(\psi) = (R\cos\psi, 0, R\sin\psi), \psi \in [-\pi/2, \pi/2].$$

For the torus, we get

$$c(\psi) = (R + r\cos\psi, 0, r\sin\psi), \psi \in [0, 2\pi].$$

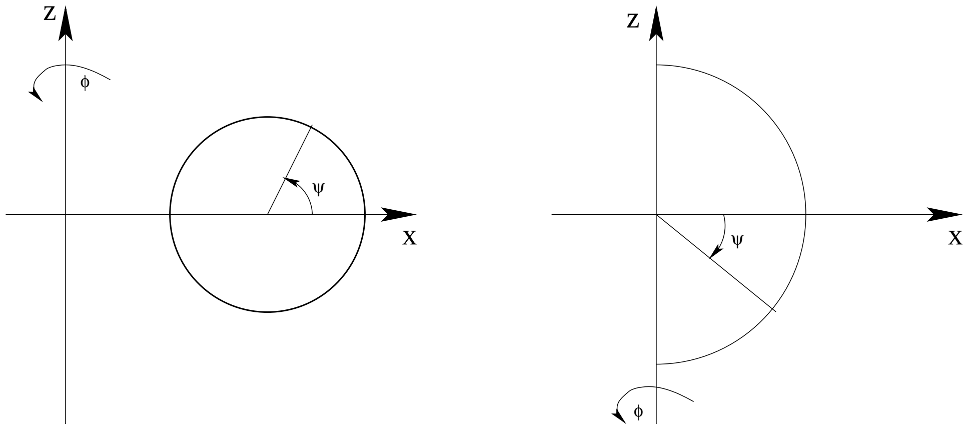

Figure 2: Parameters ϕ and ψ

Now, we apply the rotation matrix describing the rotation around the z-axis by the angle $\phi \in [0, 2\pi]$.

$$\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This leads to formulas:

$$P(\psi, \phi) = (R\cos\psi\cos\phi, R\cos\psi\sin\phi, Rsin\psi).$$

for the sphere and

$$P(\psi, \phi) = ((R + r\cos\psi)\cos\phi, (R + r\cos\psi)\sin\phi, rsin\psi).$$

for the torus. Mathematically speaking, these are formulas which assign a point on the surface to a point in the square. The domains are different: $[-\pi/2, \pi/2] \times [0, 2\pi]$ and $[0, 2\pi] \times [0, 2\pi]$.

2 Triangulating the sheet

This is easy. Just do something like in figure 3.

3 Carrying over the sheet triangulation to 3D

This is also simple. Just apply the formulas in Section 1 to the triangles with which you covered the sheet.

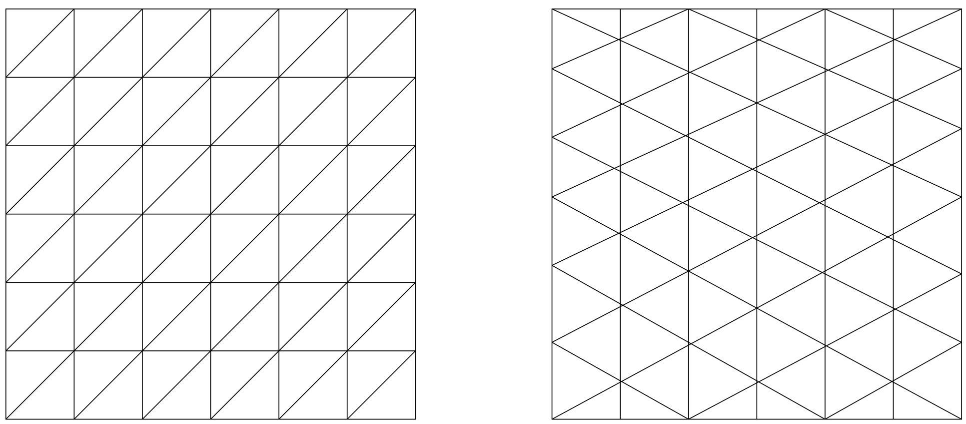

Figure 3: Sheet triangulation