

Financial Transaction Taxes and the Informational Efficiency of Financial Markets: A Structural Estimation

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Abstract

We develop a new methodology to estimate the impact of a financial transaction tax (FTT) on informational efficiency, liquidity and volatility. In our sequential trading model there are price elastic noise traders and traders with private information of heterogeneous quality. We estimate the model without a tax and then quantify the effect of an FTT. In our sample, noise traders are price elastic but less so than informed traders. The introduction of an FTT changes the composition of the market, lowering informational efficiency. Even a small, 5 bps, FTT impedes correct price convergence on a sizeable percentage of days.

JEL Classification: G14, D82, C13

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1 Introduction

In August 2012, France introduced a 0.2% financial transaction tax (FTT) on the purchase of shares of companies with a market capitalization above €1 billion. A similar tax was introduced in Italy in March 2013. A proposal by the European Commission for the introduction of a 0.1% *ad valorem* FTT for the entire European Union has triggered lengthy debates among member states; as of 2018, a decision by the European parliament has yet to be made. Proposals to introduce the tax have also been advanced in many other countries, including the United States, especially after the 2007 – 8 financial crisis. Transaction taxes are not a recent policy innovation. The United Kingdom’s Stamp Duty, implemented in 1694 to finance the war against France, is recognized as the first financial transaction tax. Although reformed several times, it is still in force. The United States levied a transaction tax from 1914 to 1966.¹ Sweden did so from 1984 until 1990.

The debate on the merits of FTTs dates back to Keynes’s *General Theory*, which proposed the use of these taxes to reduce stock market volatility and speculation. Tobin (1978) followed up on Keynes’s tax proposal after the collapse of the Bretton Woods system: he suggested a 1% tax on all foreign exchange transactions (the so-called “Tobin tax”) to reduce capital flow and exchange rate volatility. Later, Stiglitz (1989) and Summers (1989) advocated the use of FTTs to avoid the build-up of dangerous asset bubbles and reduce market volatility. The economic argument underlying all these proposals is that financial markets suffer from too much “noise trading” activity that is not based on fundamentals with adverse consequences for both price volatility and informational efficiency. An FTT would mainly affect such traders with short investment horizons, leaving long-term investors who trade for fundamental reasons unaffected. As a result, the effect of the tax would be socially beneficial. The opposite view is that an FTT, by introducing a friction into the trading process, slows down price discovery, thereby lowering liquidity and potentially increasing price volatility. This view has been advocated, for instance, by Edwards (1993) and Schwert and Seguin (1993), who observe that informed traders stabilize markets by offsetting the effects of noise traders. Similarly, Kupiec (1996) and Sørensen (2017) show that an FTT increases price volatility and lowers market liquidity and welfare.

A recent theoretical literature has tried to rationalize both views by studying the mech-

¹ Currently, in the US, a very small fee of \$21.80 per million dollars of securities transactions supports the operation costs of the Securities and Exchange Commission (“Section 31 fee”).

anisms through which an FTT impacts different classes of market participants. Dávila and Parlatore (2019) show that, in a CARA-Normal set up, the impact of the tax on price informativeness depends on the relative elasticity of informed traders and noise traders (hedgers).

² Song and Zhang (2005) propose a model of noise trading in which the FTT has a compositional effect on trading activity: a tax discourages noise trading and fundamental trading to a different extent depending on market conditions; in an economy with low (high) volatility and low (high) noise traders' participation, an increase in the FTT reduces (increases) volatility. ³

Overall, while clarifying the mechanisms through which an FTT affects market outcomes, this recent theoretical literature acknowledges that the overall effect of the tax is theoretically indeterminate. It depends on the tax's differential impact on classes of traders, that is, on its compositional effect. Whether the tax mainly impacts noise traders' activity and, therefore, has a positive effect on market outcomes, or informed traders' activity and, therefore, worsens price discovery is an empirical question. Unfortunately, the existing empirical studies, mainly event studies or difference in differences analyses, are only partially able to answer this question (see, e.g., the discussion in Habermeier and Kirilenko, 2003). There is strong evidence on the negative impact of the FTT on trading volumes. The best known case is the Swedish transaction tax of 1986, which led to a migration of 60% of trading volume in the eleven most traded Swedish shares from Stockholm to London (see Umlauf, 1993). The results on price volatility and market informational efficiency, however, are more ambiguous. While some empirical studies find that an FTT reduces price volatility (Umlauf, 1993; Jones and Seguin, 1997), others find either no significant or a negative effect on volatility and market quality (e.g. Deng *et al.* 2018).⁴

In this paper, we offer a different empirical strategy to understand the impact of an FTT. We develop a market-microstructure model of trading in financial markets featuring both informed and noise traders that is amenable to structural estimation. In the model, both types of traders have price-elastic demands; as a result, an FTT affects their behavior. Its

² Dupont and Lee (2007) show that, in a static model of a competitive specialist market, an FTT decreases (increases) market depth and liquidity when informational asymmetry is high (low). Subrahmanyam (1998) shows that, in a strategic model, an FTT increases market liquidity with a monopolist-informed trader but decreases with multiple informed traders

³ Other papers study the effect of the FTT on different types of traders using agent-based model, see, e.g., Mannaro *et al.* (2008) and Pellizzari and Westerhoff (2009).

⁴ The impact of a financial transaction tax on volatility and market efficiency has also been studied in the laboratory (e.g., Noussair *et al.*, 1998, and Cipriani and Guarino, 2008a).

impact on informational efficiency, liquidity, and volatility depends on the model’s parameter values. We resolve this indeterminacy by estimating the model using transaction data. Given parameter estimates, we obtain unique predictions for the impact of an FTT. The structural-estimation approach has several advantages. It enables us to measure the effects of an FTT by directly estimating its compositional effects; that is, we can immediately address the policy debate on whether the tax affects noise traders to a greater than informed traders. Moreover, we gauge the impact of a tax beyond observable market data, such as the price levels or intraday price volatility. In particular, we can recover market participants’ beliefs and preferences from the structural parameter estimates. We use these objects to construct direct measures of informational efficiency with and without an FTT, something that could not be achieved without a structural estimation. Finally, our methodology allows us to carry out counterfactual policy experiments to evaluate the impact of an FTT on financial markets or asset classes for which a tax has not been implemented yet.

To understand the compositional effect of an FTT, we develop a market microstructure model of financial market trading *à la* Glosten and Milgrom (1985) in which *both* informed and noise traders are price elastic. In previous structural estimations of market microstructure models — e.g., Easley *et al.* (1996, 1997) and the following voluminous literature on the PIN (probability of informed trading) — noise traders trade for exogenous reasons (e.g., liquidity shocks) independently of the price. Moreover, informed traders receive a perfectly informative signal, so that they buy or sell independently of the price level. In these models, an FTT has no effect on market activity as traders’ behavior does not depend on the price they are facing. Recently, Cipriani and Guarino (2014) have studied herd behavior through a structural estimation of a market microstructure model in which informed traders (but not noise traders) are price elastic as they receive a signal of finite precision.⁵ Here, we build on Cipriani and Guarino (2014) and introduce price-elastic noise traders, who receive a shock to their asset valuation. Such a shock may be interpreted as the result of liquidity or hedging motives; it can also stem from speculative non-information based reasons to trade.⁶ The presence of both price-elastic informed traders and price-elastic noise traders allows us to measure the compositional effect of the tax.

⁵ Herding in financial markets was first studied at a theoretical level by Gervais (1997) and Avery and Zemsky (1998).

⁶ The latter interpretation is similar to Stiglitz (1989)’s, according to whom noise trades “are based on (the belief of) differential information”. See also Glosten and Putnins (2016).

We estimate the model by maximum likelihood on intra-day transaction data for a set of stocks in the STAR segment of the Italian Stock Exchange (“Borsa Italiana”) for the sample period June 2012 to December 2013. In particular, we estimate the likelihood of an informational event, the proportion of informed traders in the market, the precision of the private signals received by informed traders, and the price elasticity of noise traders. In our theoretical model, an FTT generates situations in which informed traders stop trading altogether, a situation similar to what in the social learning literature is called an informational cascade (Bikhchandani *et al.* 1992; Welch, 1992; Gale, 1996; Hirshleifer and Teoh, 2003). As a result, with some probability, the price can diverge from the fundamental asset value. We derive this probability analytically and then use the parameter estimates to compute it for each stock in our sample. Across all stocks, even with a relatively small FTT of 5bps, the average probability of the price diverging from the fundamental value is significant. This happens on around 10% of days on which there is news about changes in the asset values.

Finally, we simulate the model without an FTT and with FTTs of different rates. This allows us to compare trading volume, bid and ask spreads, price volatility and informational efficiency under different tax regimes. For all stocks, a tax reduces trading activity, in particular on days with information event and especially for informed traders. Price volatility also decreases with the tax. An important focus of our analysis is on informational efficiency, measured as the distance of the price from the fundamental value during the day. For almost all stocks, the tax worsens informational efficiency, in some cases by a significant amount. Overall, our results lend weight to the scepticism about the FTT, which generally seems to worsen market outcomes. Such scepticism is, however, qualified by the fact that the impact of the tax varies substantially across stocks.

The rest of the paper is organized as follows. Sections 2 and 3 describe the theoretical model and its predictions. Section 4 explains the effect of an FTT. Section 5 explains the likelihood function. Section 6 describes the data. Section 7 presents the parameter estimates. Section 8 reports the results on the impact of an FTT. Section 9 concludes. The Appendix contains further estimation results and other supplementary material.

2 The Model

Building on Cipriani and Guarino (2014), we present a model in which informed traders and noise traders interact with a market maker (specialist market). Trading happens over many days, indexed by $d = 1, 2, 3, \dots$. Time within each day is discrete and indexed by

$t = 1, 2, 3, \dots$

2.1 The asset

We denote the fundamental value of the asset on day d by V^d . The asset value does not change during the day, but can change from one day to the next. At the beginning of the day, with probability $1 - \alpha$ the asset value remains the same as on the previous day ($V^d = v^{d-1}; v^{d-1} > 0$), and with probability $0 < \alpha < 1$ it changes.⁷ Each day d , the value of the asset on the previous day, v^{d-1} , is known to all market participants. On days with changes in the asset value, the trading population includes informed traders as will be explained later on; we say that an information event has occurred. On event days, with probability $1 - \delta$ the asset value decreases to $v_L^d = v^{d-1} + \lambda_L v^{d-1}$ where $-1 < \lambda_L < 0$; we refer to such days as bad information event days. With probability $0 < \delta < 1$, it increases to $v_H^d = v^{d-1} + \lambda_H v^{d-1}$ where $\lambda_H > 0$; these are good information event days. Information events are independently distributed across trading days. Finally, we assume that $(1 - \delta)\lambda_L = -\delta\lambda_H$, which guarantees that the closing price is a martingale.

We assume that the asset value increases or decreases in proportion to the previous day's value (multiplicative change). In the existing literature, for instance in the seminal paper by Easley *et al.*, 1997, the typical assumption is that the change in fundamental value between days is a constant (additive change). Neither choice is relevant for the estimation of the model's parameters when there is no tax. However, assuming that the change in fundamental value from one day to the next is multiplicative is preferable when studying transaction taxes: since transaction taxes are usually levied as *ad-valorem* taxes, when the change in fundamental value is multiplicative, the impact of a tax is independent of the asset price (i.e., it is the same on a share whose price fluctuates around \$1 and on a share whose price fluctuates around \$100). Additionally, a multiplicative change is consistent with the observation that the variance of asset returns does not decrease as the price of the asset increases (as would be the case with additive changes); moreover, it guarantees that the asset fundamental value never becomes negative.

⁷ Note that v^{d-1} is the realization of the random variable V^{d-1} . Throughout the text, we will denote random variables by capital letters and their realizations with lower-case letters.

2.2 The market

At any time t during the day, traders can buy, sell, or decide not to trade with the market maker. Each trade consists of the exchange of one unit of the asset for cash. A trader's action space is, therefore, $\mathcal{A} = \{\text{buy}, \text{sell}, \text{no trade}\}$. We denote the action of the trader at time t on day d by X_t^d and the history of trades and prices until time $t - 1$ of day d by h_t^d .

2.3 The market maker

At any time t of day d , the market maker sets the prices at which a trader can buy or sell the asset. When posting these prices, the market maker takes into account the possibility of trading with traders who, as we shall see, have some private information about the asset value. The market maker sets different prices for buying and for selling, that is, there is a bid-ask spread (Glosten and Milgrom, 1985). We denote the ask price at time t , that is the price at which a trader can buy, by a_t^d and the bid price, the price at which a trader can sell, by b_t^d .

Due to (unmodelled) potential competition, the market maker makes zero profits in expectation by setting the ask and bid prices equal to the asset's expected value conditional on the information available to him at time t . Moreover, again due to potential competition, the ask and the bid are, respectively, the smallest and largest price satisfying this zero-profit condition, that is,

$$a_t^d = \min_a \{a : a - \mathbb{E}(V|X_t^d = \text{buy}, a, h_t^d) = 0\}, \quad (1)$$

$$b_t^d = \max_b \{b : \mathbb{E}(V|X_t^d = \text{sell}, b, h_t^d) - b = 0\}. \quad (2)$$

We refer to the market maker's expectation conditional on the history of trades as the "price" of the asset, and we will denote it by $p_t^d = \mathbb{E}(V^d|h_t^d)$.⁸

2.4 The traders

There are a countable number of traders. Traders act in an exogenous sequential order. On day d , each trader is assigned a randomly chosen time t at which he takes a single action: buy, sell, or no trade. Traders are of two types, informed or noise.

⁸ Standard arguments show that $b_t^d \leq p_t^d \leq a_t^d$ (see Glosten and Milgrom, 1985).

2.4.1 Informed traders

The trader's own type is private information. On days without information event (no-event days), all traders are noise traders. On days with an information event (event days), at any time t an informed trader is chosen to trade with probability $0 < \mu < 1$ and a noise trader with probability $1 - \mu$. Informed traders have private information about the asset value. In addition to observing the history of prices and trades, they receive a private signal about the current asset value V^d .⁹ The private signal S_t^d is a random variable with the following value-contingent densities:

$$\begin{aligned} f^H(s_t^d | V^d = v_H^d) &= 1 + \tau(2s_t^d - 1), \\ f^L(s_t^d | V^d = v_L^d) &= 1 - \tau(2s_t^d - 1), \end{aligned} \tag{3}$$

with $\tau \in (0, 2]$ (see Figure 1).

For $\tau \in (0, 1]$, the support of the densities is $[0, 1]$. For $\tau > 1$, the support shrinks to $[1 - \frac{1}{\tau}, 1]$ for $f^H(\cdot | v_H^d)$ and to $[0, \frac{1}{\tau}]$ for $f^L(\cdot | v_L^d)$. In this case, the support is bounded away from 0 for $f^H(\cdot | v_H^d)$ and bounded away from 1 for $f^L(\cdot | v_L^d)$. At each time t , the likelihood ratio after receiving the signal s_t^d is

$$\frac{\Pr(V^d = v_H^d | h_t^d, s_t^d)}{\Pr(V^d = v_L^d | h_t^d, s_t^d)} = \begin{cases} +\infty & \text{if } s_t^d \geq \frac{1}{\tau}, \\ \frac{1+\tau(2s_t^d-1)}{1-\tau(2s_t^d-1)} \frac{\mathbb{P}(V^d=v_H^d|h_t^d)}{\mathbb{P}(V^d=v_L^d|h_t^d)} & \text{if } s_t^d \in (\frac{\tau-1}{\tau}, \frac{1}{\tau}), \\ 0 & \text{if } s_t^d \leq \frac{\tau-1}{\tau}, \end{cases}$$

The signal satisfies the monotone likelihood ratio property, as it is weakly increasing in s_t^d : the higher the signal realization, the higher a trader's asset valuation. Moreover, the likelihood ratio increases after receiving a signal if $s_t^d > 0.5$ and decreases if $s_t^d < 0.5$; for this reason we refer to a signal larger than 0.5 as a “good signal” and to a signal smaller than 0.5 as a “bad signal”.

The parameter τ measures the informativeness of the signals. When $\tau \rightarrow 0$, the densities become uniform, and signals are completely uninformative. As τ increases, the signals become more and more informative. For any $\tau \in (0, 1)$, the support of the distribution

⁹ As we will explain later, in the model, there is a one-to-one mapping from trades to prices. For this reason, in bringing the model to the data, we only need to assume that traders observe the history of past trades.

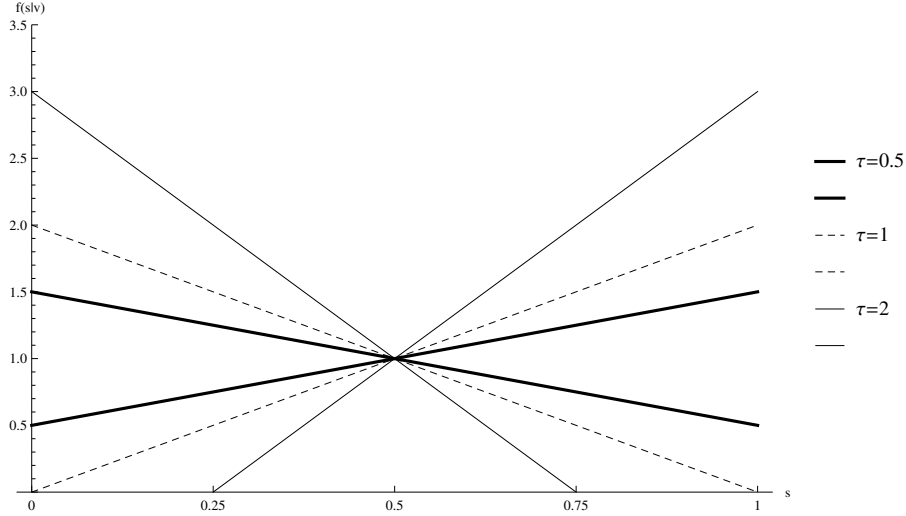


Figure 1: *The figure shows the signal's value-contingent density functions for different values of τ .*

of the likelihood ratio is bounded away from 0 and infinity. In contrast, for $\tau \geq 1$, there is a region of the supports where the likelihood ratio is 0 and a region where it is infinity; this stems from the fact that the supports of the high and low density functions are bounded away from 0 and 1 respectively. Following Smith and Sørensen (2000), when $\tau \in (0, 1)$, we say that beliefs are bounded and, when $\tau \in [1, 2]$, that they are unbounded. With bounded beliefs, no signal realization, even the most extreme one, reveals the asset value with probability one. In contrast, with unbounded beliefs some high (low) signal realizations are possible only when the asset value is high (low), and, therefore, the signal can be perfectly informative.¹⁰ As τ tends to 2, the measure of perfectly informative signals tends to one.

An informed trader knows that an information event has occurred and that, as a result, the asset value has changed with respect to the previous day. Moreover, his signal is informative on whether the event is good or bad. Nevertheless, depending on the signal realization and the precision τ , he may still be uncertain about the direction of the change. For instance, he may know that there has been a change in the investment strategy of a company, but may not be sure whether this change will affect the asset value in a positive or negative way. The parameter τ can be interpreted as measuring the precision of the information that the trader receives, or the ability of the trader to process such private information. Finally, note

¹⁰ In particular, any signal greater than $\frac{1}{\tau}$ reveals that the asset value is v_d^H , whereas a signal lower than $\frac{\tau-1}{\tau}$ reveals that the asset value is v_d^L .

that, given our signal structure, informed traders are heterogeneous since they receive signal realizations with different degrees of informativeness about the asset's fundamental value.

Informed traders are risk-neutral. An informed trader's payoff function, $U : \{v_L^d, v_H^d\} \times \mathcal{A} \times [v_L^d, v_H^d]^2 \rightarrow \mathbf{R}_+$, is

$$U(v^d, X_t^d, a_t^d, b_t^d) = \begin{cases} v^d - a_t^d & \text{if } X_t^d = \text{buy}, \\ 0 & \text{if } X_t^d = \text{no trade}, \\ b_t^d - v^d & \text{if } X_t^d = \text{sell}. \end{cases} \quad (4)$$

An informed trader chooses X_t^d to maximize $\mathbb{E}[U(V^d, X_t^d, a_t^d, b_t^d) | h_t^d, s_t^d]$. Therefore, he finds it optimal to buy whenever $\mathbb{E}(V^d | h_t^d, s_t^d) > a_t^d$ and to sell whenever $\mathbb{E}(V^d | h_t^d, s_t^d) < b_t^d$. He chooses not to trade when $b_t^d < \mathbb{E}(V^d | h_t^d, s_t^d) < a_t^d$. Otherwise, he is indifferent between buying and not trading, or selling and not trading.

2.4.2 Noise traders

As mentioned above, on event days, at any time t a noise trader is chosen to trade with probability $(1 - \mu)$. Noise traders are traders who, in addition to the common value V^d , have a private value from holding the asset. In contrast to conventional empirical market microstructure models of sequential trading, we allow noise traders to be price-elastic. Whereas the conventional price-inelastic noise traders always have a private value (e.g., liquidity or hedging need) so large that they want to buy or sell independently of the price, price-elastic noise traders' private values can be small enough to make their decision to trade depend on the price.

We model noise traders by following an approach similar to that of Glosten and Putnins (2016). In particular, with probability $0 < \psi < 1$, noise traders receive a signal (or shock) n_t^d , which is distributed uniformly on the interval $[0, 1]$, independently of the asset fundamental value. With probability $(1 - \psi)$ they receive no signal. Although the signal n_t^d is uniformly distributed, upon receiving it a noise trader computes the expected value of the asset *as if* the signal were distributed according to the following value-contingent densities:

$$\begin{aligned} g^H(n_t^d | V^d = v_H^d) &= 1 + \nu(2n_t^d - 1), \\ g(n_t^d | V^d = v^{d-1}) &= \frac{\nu}{2 - \nu} \text{ for } \nu < 2, \\ g^L(n_t^d | V^d = v_L^d) &= 1 - \nu(2n_t^d - 1), \end{aligned} \quad (5)$$

with $1 \leq \nu < 2$ and supports respectively $[\frac{\nu-1}{\nu}, 1]$, $[\frac{\nu-1}{\nu}, \frac{1}{\nu}]$ and $[0, \frac{1}{\nu}]$. For $\nu = 2$, $g(\cdot|v^{d-1})$ is not defined, and the noise trader computes the expected value only according to $g^H(\cdot|v_H^d)$ and $g^L(\cdot|v_L^d)$.¹¹ Noise traders maximize their expected payoff given their belief about the asset's fundamental value. When they do not receive a private-value shock, which happens with probability $1 - \psi$, their expected value is equal to that of the market maker. Therefore, they optimally decide not to trade whenever the bid-ask spread is positive. When they receive a private value shock, which happens with probability ψ , they update their valuation for the asset to $E(V^d|h_t^d, n_t^d)$ analogously to how informed traders update valuations upon observing signals. As shown by Glosten and Putnins (2016), one can interpret the private value n_t^d in different ways: it may stem from hedging or liquidity reasons; it may also stem from bounded rationality, i.e., noise traders wrongly believe that their signal is informative.

A few remarks are in order. First, noise traders act as if the signal were informative, whereas it is not, since it is uniformly distributed on $[0, 1]$ independently of the type of day. Second, the signal's density functions for good and bad-event days, $g^H(\cdot|v_H^d)$ and $g^L(\cdot|v_L^d)$, have the same functional form as those of informed traders (with a different parameter, ν , which is restricted to be greater than 1). This allows us to solve the model analytically. Third, noise traders believe that, on no-event days, the signal is uniformly distributed.¹² Moreover, the support of the density $g(\cdot|v^{d-1})$ on a no-event day is the intersection of the supports of $g^H(\cdot|v_H^d)$ and $g^L(\cdot|v_L^d)$ for good and bad-event days. This, along with the restriction $\nu \geq 1$, guarantees that, for any history, there is always a mass of noise traders whose asset valuation is either equal v_H^d or to v_L^d ; these traders want to buy and sell irrespective of the price, thus avoiding market breakdowns.¹³ Finally, at least for some signal realizations, if $\nu < 2$, noise traders' valuation of the asset is always between v_L^d and v_H^d , so that at any time t noise traders' demand is, indeed, price-elastic; for $\nu = 2$, noise traders act as if, with probability one, the day were a bad-event day (when $n_t^d < 0.5$) or a good-event day (when $n_t^d > 0.5$).¹⁴

¹¹ I.e., for $\nu = 2$ noise traders attach zero probability to being on a no-event day.

¹² We did not specify a distribution for informed traders' signal on no-event days, since informed traders only trade on event days.

¹³ Because of asymmetric information, the market would break down in the absence of noise traders. Note that from an empirical viewpoint, this is a natural assumption, since we do not observe such market breakdowns.

¹⁴ We could have modelled the noise traders as valuing the asset $\mathbb{E}(V^d|h_t^d) + \rho_t$ or $\rho_t \mathbb{E}(V^d|h_t^d)$, where ρ_t is an i.i.d. additive or multiplicative private value. In such cases, even if we chose the support of ρ_t such that at time 1 of each day the noise traders' are price elastic, they could nevertheless become price

Modelling noise traders' private values distributed as if they were informative signals is a simple and flexible way of introducing price-elastic noise traders into our economy. This encompasses the standard empirical market microstructure models, where noise traders are price inelastic: whereas in those model price inelasticity is assumed, in our analysis the elasticity of noise traders' demand is estimated.¹⁵ Allowing noise traders to be price-elastic creates an important trade off when analyzing the effect of a transaction tax: the tax discourages both informed and uninformed traders from trading, and therefore has ambiguous effects on informational efficiency.

3 Equilibrium Analysis

In this section, we describe the strategies of informed and noise traders.¹⁶ Since informed traders' signals respect the monotone likelihood ratio property, at each time t , the trading decision of an informed trader can be simply characterized by two thresholds, σ_t^d and β_t^d , satisfying the equalities

$$E[V^d|h_t^d, \sigma_t^d] = b_t^d, \quad (6)$$

and

$$E[V^d|h_t^d, \beta_t^d] = a_t^d. \quad (7)$$

An informed trader sells for any signal realization smaller than σ_t^d and buys for any signal realization greater than β_t^d . Obviously, the thresholds at each time t depend on the history of trades until that time and on the parameter values.¹⁷

inelastic (even for $\nu < 2$) as $\mathbb{E}(V^d|h_t^d)$ converges to the boundaries of its support, since their valuation would exceed even the highest value (or be below the lowest value).

¹⁵ In our model, for $\nu = 2$, noise traders behave as they do in previous empirical market microstructure models, that is, they buy or sell with fixed probabilities. It is important to notice though, that in the previous literature, noise traders do not change their behavior for *any* price. In our model, instead, even for $\nu = 2$, noise traders remain price-elastic, since, e.g., they do not buy when the price they incur exceeds the asset's highest value. This is the case when the ask price is close to such a value and, in addition, the trader has to pay an FTT. We will discuss this issue in more detail when we introduce an FTT in our model.

¹⁶ We solve for a Perfect Bayesian Equilibrium in which agents' strategies are optimal given everyone else's strategies and beliefs; and beliefs are consistent with strategies and updated using Bayes's rule.

¹⁷ Since noise traders buy and sell with probabilities bounded away from zero, standard arguments prove that the bid and ask prices exist and are unique (see, e.g., Cipriani and Guarino, 2008). Similar arguments can be used to prove existence and uniqueness of the thresholds (see Cipriani and Guarino, 2014).

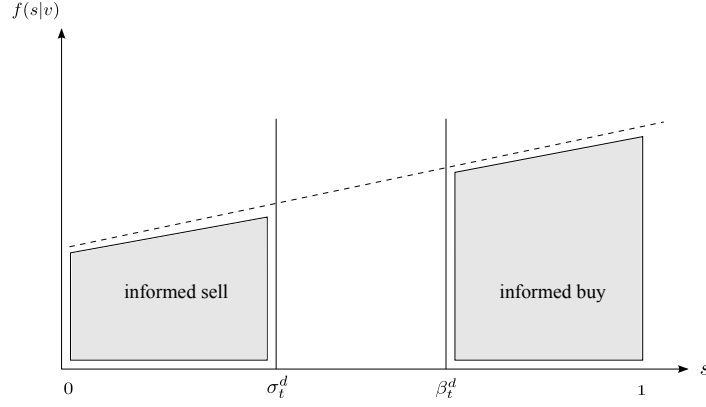


Figure 2: *The figure illustrates the signal realizations for which an informed trader decides to buy or sell when $V^d = v_h^d$.*

Figure 2 (drawn for the case of a good informational event) illustrates the decision of informed traders. An informed trader buys the asset with a signal higher than the threshold value β_t^d , sells it with a signal lower than σ_t^d , and does not trade otherwise. The measure of informed traders buying or selling is equal to the areas (labeled as “informed buy” and “informed sell”) below the line representing the signal density function.

A similar analysis holds for noise traders. At each trading time t on day d , their behavior can be characterized by a buy threshold κ_t^d and a sell threshold γ_t^d . The sell threshold γ_t^d for a noise trader satisfies the equality

$$E(V^d | h_t^d, \gamma_t^d) = b_t^d, \quad (8)$$

and the buy threshold κ_t^d the equality

$$E(V^d | h_t^d, \kappa_t^d) = a_t^d. \quad (9)$$

Figure 3 illustrates the resulting trading probabilities for noise traders. Notice that since the private value shocks N_t^d are drawn from a uniform distribution on $[0, 1]$ irrespective of the type of day, these probabilities only depend on the thresholds γ_t^d and κ_t^d but not on the realization of V^d unlike in the case of the informed traders.

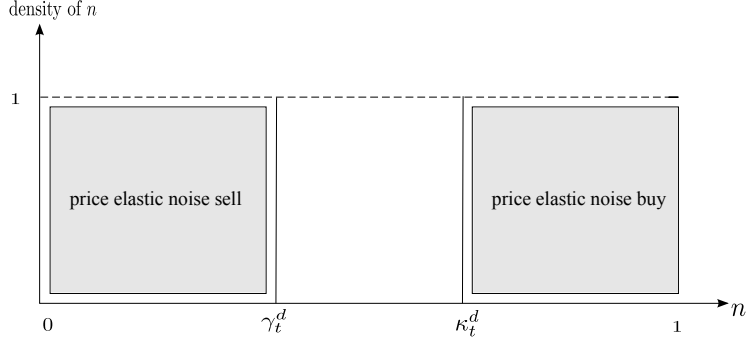


Figure 3: The figure illustrates the shock realizations for which a price-elastic noise trader decides to buy or sell.

Let us now derive the equilibrium buy thresholds. Obviously, (7) and (9) together imply that

$$E(V^d|h_t^d, \kappa_t^d) = E(V^d|h_t^d, \beta_t^d), \quad (10)$$

from which one can derive a linear relationship between the two thresholds,

$$\kappa_t^d = c_0(h_t^d) + c_1(h_t^d) \beta_t^d, \quad (11)$$

where $c_0(h_t^d)$ and $c_1(h_t^d)$ are functions of model parameters and the history of trades, h_t^d , only.¹⁸ As we know, the ask price has to yield zero expected profits to the market maker given his beliefs upon observing a buy order. It is thus given by the following expression:

$$a_t^d = v^{d-1}(1 + \Pr(v_H^d|\text{buy}_t^d, h_t^d, a_t^d, b_t^d) \lambda_H - \Pr(v_L^d|\text{buy}_t^d, h_t^d, a_t^d, b_t^d) \lambda_L), \quad (12)$$

where the probabilities of a good and bad event can be expressed as functions of the buy thresholds β_t^d and κ_t^d and h_t^d . Furthermore, using (11), we can express κ_t^d as a function of β_t^d .

Similarly, the expectation of an informed trader with signal β_t^d can be expressed as

$$E(V^d|h_t^d, \beta_t^d) = v^{d-1}(1 + \Pr(v_H^d|h_t^d, \beta_t^d) \lambda_H - \Pr(v_L^d|h_t^d, \beta_t^d) \lambda_L). \quad (13)$$

¹⁸ Remember that the densities of s_t^d and n_t^d are different, and therefore equation (10) does not imply $\kappa_t^d = \beta_t^d$.

After substituting (11) into (12) and then both (12) and (13) into (7), one obtains a quadratic equation in β_t^d . The threshold β_t^d is the smallest solution to this quadratic equation such that $\beta_t^d \in [0, 1]$.¹⁹ An analogous approach yields a quadratic equation for the equilibrium sell thresholds σ_t^d ; σ_t^d is the largest solution in the interval $[0, 1]$.²⁰ Once solved for β_t^d and σ_t^d , one can obtain the buy and sell thresholds for the noise traders, κ_t^d and γ_t^d , from (11).

As a final remark, note that in equilibrium, it may occur that $\beta_t^d < 0.5$ or $\sigma_t^d > 0.5$, that is, informed traders may herd (Cipriani and Guarino, 2014). When $\beta_t^d < 0.5$, informed traders “herd buy,” that is, they buy even for some bad signals; when $\sigma_t^d > 0.5$, informed traders “herd sell,” that is, they sell even for some good signals. Whereas informed traders may herd, this is never the case for noise traders (i.e., $\gamma_t^d \leq 0.5 \leq \kappa_t^d$); since, in contrast to informed traders, they receive a shock independently of whether an event has occurred or not, a noise trader with a shock equal to 0.5 has the same belief as the market maker; therefore, he buys for a shock higher than a threshold $\kappa_t^d \geq 0.5$ and sells for a shock lower than a threshold $\gamma_t^d \leq 0.5$.

4 The Financial Transaction Tax

We now consider the introduction of a financial transaction tax (FTT). We consider an FTT levied *ad valorem*, as are most of the proposed and implemented financial transaction taxes. Specifically, whenever a trader buys the asset, he pays a tax ρa_t^d ; and whenever he sells, he pays ρb_t^d . As a result, an informed trader with signal s_t^d finds it optimal to buy when $\mathbb{E}(V^d | h_t^d, s_t^d) > a_t^d(1 + \rho)$, and to sell when $\mathbb{E}(V^d | h_t^d, s_t^d) < b_t^d(1 - \rho)$; he chooses not to trade when $b_t^d(1 - \rho) < \mathbb{E}(V^d | h_t^d, s_t^d) < a_t^d(1 + \rho)$, and is indifferent between trading and not when an equality holds. Similarly, a noise trader with private value shock n_t^d buys if $\mathbb{E}(V^d | h_t^d, n_t^d) > a_t^d(1 + \rho)$, sells if $\mathbb{E}(V^d | h_t^d, n_t^d) < b_t^d(1 - \rho)$, chooses not to trade if $b_t^d(1 - \rho) < \mathbb{E}(V^d | h_t^d, n_t^d) < a_t^d(1 + \rho)$, and is indifferent between buying (or selling) and no trading if an equality holds.²¹

¹⁹ Recall that the ask is the minimum value satisfying the market maker’s zero-profit condition (see equation 1). The equilibrium threshold is the smallest solution to the quadratic equation because, given such a solution, the ask is indeed the smallest price satisfying the zero-profit condition.

²⁰ The nature of the solutions guarantees that the equilibrium sell threshold is smaller than the equilibrium buy threshold, that is, $\sigma_t^d \leq \beta_t^d$.

²¹ Note that in countries like France and Italy an FTT is levied on purchases only. In our model, however, this is equivalent to taxing both purchases and sales; the reason is that each trader trades only once and this trade determines his payoff. A seller’s payoff, for instance, is the difference between the

In the presence of an FTT, the equilibrium can still be characterized in terms of buy and sell thresholds for informed and noise traders. Of course, such thresholds differ from those without tax, since the introduction of an FTT makes trading unprofitable for some informed and noise traders who would otherwise trade. Specifically, the buy thresholds for informed and noise traders are implicitly given by the following system of equations:

$$\begin{aligned} E(V^d|h_t^d, \beta_t^d) &= a_t^d(1 + \rho), \\ E(V^d|h_t^d, \kappa_t^d) &= a_t^d(1 + \rho). \end{aligned} \tag{14}$$

From this system of equations, one can derive (10) and an expression similar to (11), as in the previous analysis. After equating $E(V^d|h_t^d, \beta_t^d)$ to the ask price and some manipulations, one obtains

$$\begin{aligned} (1 + \rho) \Pr(v_H^d|h_t^d, \beta_t^d) - \Pr(v_H^d|h_t^d, \text{buy}_t^d) + \frac{\rho}{\lambda_H} = \\ \left(\frac{\delta}{1 - \delta} \right) [(1 + \rho) \Pr(v_L^d|h_t^d, \beta_t^d) - \Pr(v_L^d|h_t^d, \text{buy}_t^d)], \end{aligned} \tag{15}$$

which is a cubic equation in β_t^d ; its smallest root in $[0, 1]$ gives the equilibrium buy threshold for informed traders. From this and from an expression analogous to (11), one obtains the equilibrium buy threshold for the noise trader. An identical analysis finds the equilibrium sell thresholds.

This analysis highlights two phenomena: informed traders who receive less informative signals and noise traders who receive weaker shocks may find it optimal to abstain from trade to avoid paying the tax; therefore, the tax modifies the equilibrium bid and ask prices, since the measure of informed and noise traders on both sides of the market is different.

Before we do so, let us observe that if the tax is high enough, traders may be unwilling to participate in the market altogether. If noise traders are unwilling to participate at the beginning of the day, the market does not even open, since the market maker would not be able to trade with informed traders only and make zero expected profits. With a tax rate

price he receives (after paying the tax) and the realized asset value. In actual markets, to obtain the same payoff, after selling an asset, a seller must repurchase it, at which point he would pay the tax. By taxing also sales, we (approximately) have the same tax burden as in an actual markets where only purchases are taxes. In other words, in actual markets, each transaction is taxed (and the tax paid by the buyer); in our model, however, if we only applied the tax to purchases, all transactions classified as a sale would be tax exempt.

such that noise traders do participate, informed traders may still be unwilling to trade. The next proposition states these results formally.

Proposition 1. *There exists a tax rate*

$$\bar{\rho}^N = \left(\frac{2\delta}{1 - |2\delta - 1|} \right) \lambda_H \quad (16)$$

such that if $\rho > \bar{\rho}^N$, in equilibrium, traders do not trade at time $t = 1$ (for any signal realization), that is, the market does not open. Suppose $\rho < \bar{\rho}^N$, then there exists a tax rate

$$\bar{\rho}^I = \left(\frac{2 \min\{\tau, 1\} \delta}{1 - \min\{\tau, 1\} |2\delta - 1|} \right) \lambda_H \leq \bar{\rho}^N, \quad (17)$$

such that if $\rho > \bar{\rho}^I$, in equilibrium, informed traders do not trade at any time t (for any signal realization).

We refer the reader to the Appendix for the proof of this and all following propositions. Here we note that, in the proofs, the thresholds $\bar{\rho}^N$ and $\bar{\rho}^I$ are computed considering the first trading time in a day. If traders do not find it profitable to trade at that time, they never find it optimal, since no information is revealed by the trading history and the prices set by the market maker remain constant. Note that if $\tau \geq 1$, $\bar{\rho}^I = \bar{\rho}^N$, since after an extreme signal (shock) both informed traders and noise traders learn the value of the asset; in contrast, for $\tau < 1$, $\bar{\rho}^I$ is increasing in τ , that is, in how strongly the signal affects informed traders' evaluations.²² Note also that the levels of the tax rates $\bar{\rho}^I$ and $\bar{\rho}^N$ increase with λ_H and δ : higher λ_H and δ (that is, higher λ_H and $|\lambda_L| = \frac{\delta}{1-\delta} \lambda_H$) mean that there are higher gains from trade and only a higher tax rate prevents traders from trading.

Finally, we can compute a threshold for the tax rate, such that both types of traders want to participate and are active on both sides of the market at time $t = 1$. We state this level in the following corollary:

Corollary 1. *There exists a tax rate*

²² For $\tau \geq 1$ there is a measure of informed traders receiving a perfectly revealing signal (or at least, a signal overwhelming any history of trades, when $\tau = 1$). Therefore, for a tax to shut informed traders out of the market, the tax payment must be higher than the gains from trade for a trader who knows the realization of the asset value. The same argument applies to noise traders, since $\nu \geq 1$.

$$\bar{\rho} = \left(\frac{2 \min\{\tau, 1\} \delta}{1 + \min\{\tau, 1\} |2\delta - 1|} \right) \lambda_H < \bar{\rho}^I, \quad (18)$$

such that if $\rho < \bar{\rho}$, in equilibrium, at time $t = 1$, both noise traders and informed traders buy and sell for some signal realizations.

Note that $\bar{\rho}$ is smaller than $\bar{\rho}^I$ because it assures that, when $\rho < \bar{\rho}$, informed traders both buy and sell (depending on their signal realization). When studying the asymptotic impact of an FTT we will assume that $\rho < \bar{\rho}$, that is, that the tax rate is low enough that, at least at time $t = 1$, informed traders both buy and sell.

4.1 The Asymptotic Impact of an FTT

We know that in the absence of a tax, bid and ask prices and agents' beliefs converge almost surely to the true asset value (Cipriani and Guarino, 2014). With an FTT this is no longer true. In this section, we describe the asymptotic behavior of prices and beliefs when a tax is levied on transactions. As a first step in the analysis, we prove that, as the number of trading periods in a day goes to infinity, the market maker learns whether an information event has occurred or not.

Proposition 2. *Consider a tax rate $\rho < \bar{\rho}$. In equilibrium, the market maker's posterior belief that an event has occurred on day d , $\alpha_t^d =: \Pr(V^d \neq v^{d-1} | h_t^d)$, converges almost surely to $1_{V^d \neq v^{d-1}}$ as $t \rightarrow \infty$.*

Intuitively, since $\nu \geq 1$, noise traders always trade (either buy or sell) for some signal realizations even in the presence of a tax.²³ Therefore, at any time t , the probability of a buy or sell order is different on an event day and a no-event day, which allows the market maker to update his belief α_t^d .

Although the market always learns the occurrence of an event, it is unable to learn whether the event is good or bad with probability 1 as long as informed traders have bounded beliefs, that is, as long as $\tau < 1$. The reason is that, with probability 1, there will be a time

²³ Recall that the assumption $\rho < \bar{\rho}$ means that at time 1 a positive measure of noise traders finds it optimal to buy and a positive measure finds it optimal to sell given their signal realizations. Since $\nu > 1$, a positive measure of noise traders does not change his evaluation of the asset depending on the history of trades. Therefore, even when the prices change, a measure of noise traders continues to buy or to sell.

after which all informed traders find it optimal not to trade. Nothing is learned about the event from that time onwards. As a result, the price may remain stuck at a high level even if the fundamental is low (or vice versa). To establish this result, we first prove, in the following lemma, that when α_t^d is sufficiently high, there exist thresholds for the probability of a good event (conditional on an event having occurred) such that, once they are reached, informed traders do not trade:

Lemma 1. *Consider a tax rate $\rho < \bar{\rho}$. Let $\delta_t^d =: \Pr(V^d = v_h^d | V^d \neq v^{d-1}, h_t^d)$. If $\tau < 1$ and $\delta\lambda_H/(1 - \delta) < 1$, in equilibrium, there exist an event probability $\bar{a} < 1$ and functions $0 < \delta_l(\alpha_t^d) < \delta_h(\alpha_t^d) < 1$ such that for $\alpha_t^d > \bar{a}$ informed traders do not trade after history h_t^d whenever either $\delta_t^d < \delta_l(\alpha_t^d)$ or $\delta_t^d > \delta_h(\alpha_t^d)$.*

Intuitively, when after a history of trading the market's belief on the likelihood of a good event is very high or very low, the asymmetry of information between the market maker and informed traders becomes small. In this circumstance, the informational content of a private signal is of little importance with respect to that of the history of trades. There will be a point when the valuations of all informed traders (irrespective of the signal they receive) become so close to the bid and ask prices that the expected gain from trading upon private information becomes smaller than the tax. At this point, all informed traders choose not to trade.

Note that the thresholds for which informed traders stop trading are not constant during the day; they change as a function of the market maker's belief about an information event having occurred.²⁴ Therefore, it is possible that after informed traders have stopped trading, they resume doing so as the market maker's updates his belief on the likelihood of an information event. Nevertheless, we can prove that eventually, there will be a time t when all informed traders' activity ceases.²⁵ We state this in the next proposition:

Proposition 3. *Consider a tax rate $\rho < \bar{\rho}$. If $\tau < 1$, in equilibrium, on an event day, for almost all histories there exists a time T , such that, for any $t > T$, informed traders decide*

²⁴ The reason is that the market maker's belief on whether the event has occurred affects the market maker's expectation and therefore the bid and ask prices that he sets.

²⁵ The idea that a transaction tax eventually stops all trading activity is developed by Cipriani and Guarino (2008a) in a Glosten and Milgrom model and, in a different setup, by Lee (1998).

not to trade for any signal realization, that is, as $t \rightarrow \infty$,

$$\Pr(X_t^d = \text{no trade} | h_t^d, s_t^d) = 1, \quad (19)$$

for any s_t^d .

In the social learning literature, the term “informational cascade” refers to a situation in which agents stop learning from market outcomes (be that the prices or other agent’s actions). A proper informational cascade never occurs in our model: as shown in Proposition 2, a trading activity continues the market makers learn whether an event has occurred. Nevertheless, something similar to an informational cascade does occur. As Proposition 3 shows, although learning on whether an event has occurred continues, learning on whether such an event is good or bad eventually stops. After a long enough history of trades, informed traders stop acting upon their signal and, as a result, the market maker’s belief on whether the event was good never converges to either 0 or 1. Moreover, the market maker’s belief on the event being good may converge to a level below δ on a good-event day; analogously, it may converge to a level above δ on a bad-event day. That is, not only learning on the event type is never complete, it may actually be quite wrong.

In the following proposition we provide bounds on the probabilities that the market maker’s belief on a good event remains stuck above 0.5 on a bad-event day, or stuck below 0.5 on a good-event day.

Proposition 4. *Consider a tax rate $\rho < \bar{\rho}$. If $\tau < 1$, in equilibrium, the probability that the belief $\Pr(V^d = v_H^d | h_t^d)$ converges to a value lower than $\delta_l(1)$ on a good-event day ($V^d = v_H^d$) is bounded above by $\frac{(1-\delta_u(1))(\delta-\delta_l(1))}{(1-\delta)(\delta_u(1)-\delta_l(1))}$ and the probability that the belief $\Pr(V^d = v_H^d | h_t^d)$ converges to a value higher than $\delta_u(1)$ on a bad-event day ($V^d = v_L^d$) is bounded above by $\frac{\delta_l(1)(\delta_u(1)-\delta)}{\delta(\delta_u(1)-\delta_l(1))}$.*

The logic of the proof is that, e.g., conditional on a bad event, the likelihood ratio $\frac{\Pr(V^d = v_H^d | h_t^d)}{\Pr(V^d = v_L^d | h_t^d)}$ is a martingale and, therefore, equal in expectation to its unconditional value $(\frac{\delta}{1-\delta})$. This fact, along with the observation that trading ceases after δ_t^d has reached either the high or the low threshold (Lemma 1), allows us to pin down the probability of correct and wrong convergence of the market makers’ belief, $\Pr(V^d = v_H^d | h_t^d)$. To understand why the formulas in Proposition 4 are only probability bounds, it is useful to focus first on the case of no event uncertainty (that is, $\alpha_t^d = \alpha = 1$ for any t). In this case, the market maker stops updating $\Pr(V^d = v_H^d | h_t^d)$ as soon as the thresholds are reached. Moreover, in this case, the probabilities that $\Pr(V^d = v_H^d | h_t^d)$ converges to the low threshold when $V^d = v_H^d$ or to the

high threshold when $V^d = v_L^d$ are equal to $\frac{(1-\delta_u(1))(\delta-\delta_l(1))}{(1-\delta)(\delta_u(1)-\delta_l(1))}$ and $\frac{\delta_l(1)(\delta_u(1)-\delta)}{\delta(\delta_u(1)-\delta_l(1))}$ respectively. In contrast, when there is event uncertainty, the market maker, in principle, may stop learning about whether the event is good or bad before the belief on whether an information event has occurred has converged to 1. This may occur when $\Pr(V^d = v_H^d | h_t^d)$ is above $\delta_u(1)$ or below $\delta_l(1)$. Since the levels at which the belief may be stuck are different, so are the probabilities of these events. That is the reason the probabilities that the price is misaligned with the fundamental value, indicated in Proposition 4, are only bounds. Simulation results, though, show that for typical parameter values the occurrence of the event is usually completely learned ($\alpha_t^d \rightarrow 1$) before the market maker learns the type of event. Therefore, the formulas in Proposition 4 are very close to the true probabilities.

5 The Likelihood Function

In our empirical analysis, we will estimate the model's parameters through maximum likelihood using financial market data for a period with no transaction tax. In this section, we characterize the model's likelihood function. The derivation of the likelihood function follows a similar logic as in Cipriani and Guarino (2014).

In Section 3, we described the equilibrium behavior of market participants for any history of trades. Remember, that the bid and ask prices are uniquely determined by the trade sequence and thus do not contain any additional information once we condition on the order flow. Therefore, we can write the likelihood function for the history of trades only, disregarding bid and ask prices.

Let us denote the history of trades at the end of a trading day by $h^d := h_{T_d}^d$, where T_d is the number of trading times on day d . We denote the likelihood function by

$$\mathcal{L}(\Phi; \{h^d\}_{d=1}^D) = \Pr(\{h^d\}_{d=1}^D | \Phi) \quad (20)$$

where $\Phi := \{\alpha, \delta, \mu, \tau, \nu, \psi\}$ is the vector of parameters.

Next recall that on day d all market participants know v^{d-1} , and the occurrence of information events is independent across days. Thus the sequence of trades on day d only depends on the realization of V^d and not on any trading data from days other than d . We can therefore write the likelihood function as the product of the likelihoods of daily trading sequences

$$\mathcal{L}(\Phi; \{h^d\}_{d=1}^D) = \Pr(\{h^d\}_{d=1}^D | \Phi) = \prod_{d=1}^D \Pr(h^d | \Phi). \quad (21)$$

Now consider the likelihood of a sequence of trades for a given day. Unlike in the standard market microstructure model of Easley and O'Hara (1987) where only the total number of buys and sells matter for the probability of a given history of trades, in our model, the sequence of trades is important. Informed and price-elastic noise traders update their valuations depending on the trading sequence, and, thus, their probability of trading depends on the observed history of trades up to the time in which they act. Therefore, we compute the likelihood function for the history of trades on day d starting at time 1 and, then, recursively up to time T_d . At trading time t the probability of a given action x_t^d depends on the sequence of previous trades h_t^d , and we have that

$$\Pr(h_{t+1}^d | \Phi) = \Pr(x_t^d | h_t^d, \Phi) \Pr(h_t^d | \Phi). \quad (22)$$

To compute $\Pr(x_t^d | h_t^d, \Phi)$, we express it in terms of the value-contingent trading probabilities

$$\begin{aligned} \Pr(x_t^d | h_t^d, \Phi) &= \Pr(x_t^d | h_t^d, v_H^d, \Phi) \Pr(v_H^d | h_t^d, \Phi) + \\ &\Pr(x_t^d | h_t^d, v_L^d, \Phi) \Pr(v_L^d | h_t^d, \Phi) + \Pr(x_t^d | h_t^d, v^{d-1}, \Phi) \Pr(v^{d-1} | h_t^d, \Phi). \end{aligned} \quad (23)$$

We now illustrate how to compute the value-contingent probabilities of a trade x_t^d . Consider period $t = 1$ and suppose, for instance, that there was a buy order, $x_1^d = \text{buy}$. The probability of such an order for a given asset value depends on the buy thresholds for informed and price-elastic noise traders, β_1^d and κ_1^d , which are functions of the model's parameters. Having obtained the value-contingent probabilities of a buy order in period 1, we can then update the market makers' beliefs using Bayes' rule. Hence, consider period t and suppose again that x_t^d is a buy order. The equilibrium buy thresholds, β_t^d and κ_t^d , will be functions of the market maker's beliefs given the trading history up to (but not including) period t , as well as the parameters of the model.

Once we have solved for β_t^d and κ_t^d , we can compute the probability of a buy order on a good-event day:

$$\begin{aligned} \Pr(x_t^d = \text{buy} | h_t^d, v_H^d, \Phi) &= \\ \mu [1 - F^H(\beta_t^d | v_H^d)] + (1 - \mu) \psi(1 - \kappa_t^d), \end{aligned} \quad (24)$$

where $F^H(\cdot | v_H^d)$ is the cumulative distribution function of $f^H(\cdot | v_H^d)$. Recall that a trader active at time t is an informed trader with probability μ and a noise trader with probability $1 - \mu$. An informed trader buys if his signal is above the buy threshold β_t^d , which happens

with probability $1 - F^H(\beta_t^d | v_H^d)$. A noise trader receives a private value shock with probability ψ , in which case he buys if his shock is larger than κ_t^d , which happens with probability $1 - \kappa_t^d$ (as these shocks are uniformly distributed). Similarly, on a bad-event day, we have that

$$\begin{aligned} \Pr(x_t^d = \text{buy} | h_t^d, v_L^d, \Phi) = \\ \mu [1 - F^L(\beta_t^d | v_L^d)] + (1 - \mu)\psi(1 - \kappa_t^d). \end{aligned} \quad (25)$$

On a no-event day ($V^d = v^{d-1}$), a market order can only come from a noise trader. Therefore,

$$\Pr(x_t^d = \text{buy} | h_t^d, v^{d-1}, \Phi) = \psi(1 - \kappa_t^d). \quad (26)$$

By the same logic, we obtain the value-contingent probabilities for a sell order at t by computing σ_t^d and γ_t^d . The probability of a sell order on a good event day, for instance, is

$$\begin{aligned} \Pr(x_t^d = \text{sell} | h_t^d, v_H^d, \Phi) = \\ \mu F^H(\sigma_t^d | v_H^d) + (1 - \mu)\psi\gamma_t^d. \end{aligned} \quad (27)$$

The probability of a no-trade is just the complement to the probabilities of a buy and of a sell. Finally, to compute $\Pr(x_t^d | h_t^d, \Phi)$, we need the conditional probabilities of V^d given the history until time t ; that is, $\Pr(V^d = v | h_t^d, \Phi)$ for $v \in \{v_L^d, v^{d-1}, v_H^d\}$. These can also be computed recursively by using Bayes's rule.

To conclude, let us provide some intuition regarding the model's identification. Similarly to analogous structural models of market microstructure, our model classifies days into high-volume days with a prevalence of buys (good event days), high-volume days with a prevalence of sells (bad event days), and low-volume days (no event days). The parameter α denotes the probability that there is an event at the beginning of a trading day. We use data across trading days to identify it. The imbalance between buys and sells on event days identifies δ . No-event days allow us to identify ψ and ν , since on no-event days only noise traders trade. ψ is identified from trading volume on no-event days. We use the sequence of intraday trades to identify ν ; different sequences of trades produce different histories of bid-ask spreads. Consequently, the implied probabilities of trade along this sequence depend on the price-elasticity of noise traders as determined by ν . Finally, on good-event days, the ratio between buys and sells is determined by the proportion of traders who trade in the right direction (i.e., buy when there is a good event), which depends on μ and τ . An analogous argument

holds for bad-event days. For any given estimate of μ and τ there is only one predicted ratio between buy and sell orders on the two types of days.

6 Data

We estimate our model with a group of stocks traded on the Italian Stock Exchange (“Borsa Italiana”) from June 2012 until February 2013. Specifically, the dataset consists of 15 stocks belonging to “STAR,” the segment of Borsa Italiana’s equity market dedicated to midsize companies with a market capitalization of less than one billion euros.²⁶

We use data for the trading occurring during the continuous auction, from 9:00am until 5:25pm; we exclude the opening call auction and the closing call auction. From Borsa Italiana we obtained data on the posted best bid and ask prices, the prices at which the transactions occurred, and the time when the quotes were posted and when the transactions occurred at millisecond precision.²⁷ To classify a trade as a sell or a buy order, we use the standard algorithm proposed by Lee and Ready (1991). We compare the transaction price with the prevailing quotes before a trade occurred. Every trade above the midpoint is classified as a buy order, and every trade below the midpoint is classified as a sell order; trades at the midpoint are classified as buy or sell orders according to whether the transaction price had increased (uptick) or decreased (downtick) with respect to the previous one. If there was no change in the transaction price, we look at the previous price movements.^{28 29}

The dataset does not contain any information on no-trades. We use the established

²⁶ In Italy, an FTT was introduced in March 2013, but was only applied to transactions of shares of companies with market capitalization higher than 500 million euros. The companies in our dataset all had a capitalization below this threshold.

²⁷ The Italian stock exchange (Borsa Italiana), which includes the STAR segment, is an order-driven market, operating with an electronic limit-order book called Mercato Telematico Azionario (MTA). Estimating a model of a quote driven market with data from order-driven markets is common in the literature (see, e.g., Atkas *et al.*, 2007 and Easley *et al.*, 2010). Indeed, the theoretical model of Easley *et al.* (1997), also a quote-driven market, has already been estimated with STAR data by Perotti and Rindi (2010) to compute the Probability of Informed Trading (PIN). The rationale for estimating a model of a quote-driven market with data from an order-driven market is that the best bid and ask in the limit-order book can be thought of as representing bid and ask prices set by a competitive market maker who makes zero expected profits.

²⁸ As a matter of fact, trades at the midpoint were almost non existent for any stock.

²⁹ Additionally, as standard practice with transaction data: 1) if more than one quote occurred at the same millisecond, we only consider the last quote; 2) if more than one trade occurred at the same millisecond, we treat them as one trade at the average price; 3) we eliminate crossed quotes (bid greater than the ask).

convention of inserting no-trades between two transactions if the elapsed time between them exceeds a particular time interval (see, e.g., Easley *et al.*, 1997). We choose an interval of five minutes; if there is no trading activity for more than five minutes, we insert a number of no-trades equal the number of minutes without trading activity divided by five. As a robustness check, we also repeat our estimation with different no-trade intervals (one and three minutes).

Table 1 provides some summary statistics for daily trading activity in our dataset. Our sample consists of 173 trading days. For each stock, we exclude days without any trading activity and outlier days, defined as days in which the number of trades exceeds the average daily number of trades by more than four standard deviations.³⁰ The median number of daily trades across stocks is 21; the median buy-sell ratio across stocks is one, that is, trading activity is, overall, balanced.

Table 1: **Trading activity.**

	buys	sells	trades	no trades	buy sell ratio	actions
Mean	12	12	24	92	1.06	116
Median	11	9	21	93	1.00	114

Summary statistics for average daily trading activity across stocks.

7 Estimation Results

In this section, we present the parameter estimates for all the 15 stocks in our sample. The parameters are obtained by estimating the model presented in Section 2 (i.e., with no FTT) for each stock in our dataset through maximum likelihood.³¹ Table 2 presents the parameter estimates and their standard errors for each stock.³² Table 3 reports a series of Wald tests for

³⁰ As a robustness check, we also re-estimate our model including outlier days. The results are in Table C.1 of the Appendix.

³¹ In the estimation, we use the Numerical Algorithms Group’s (NAG), Matlab’s implementation of the Nelder-Mead algorithm, to maximize the log-likelihood. In order to gain comfort in our estimation strategy, before estimating the model with actual transaction data, we simulated data for several sets of parameters and verified that we would be able to recover them with our estimation algorithm. The optimization routine converges to the same parameter set starting from a large set of initial conditions.

³² Standard errors are computed numerically with the BHHH estimator (see Berndt *et al.*, 1974). We indicate a standard error of 0.00 when it is smaller than 0.005).

several relevant nulls that we will discuss along with the parameter estimates; it also reports two likelihood-ratio tests comparing our model with two restricted versions: Cipriani and Guarino (2014), in which $\nu = 2$ and Easley *et al.* (1997) in which $\tau = \nu = 2$.

The probability of an information event (α) ranges between 0.30 and 0.80; for the median stock it is 0.40. Since the highest standard deviation across stock is 0.04, for all stocks there is significant evidence of information events. Stocks vary substantially on the probability of a good event (δ), which ranges from 0.17 to 1, with a median of 0.56. Because of relatively large standard errors, we can reject the null that $\delta = 0.5$, that is, that good and bad events are equally probable, for six stocks only (at the 5% significance level).

On event days, the proportion of informed traders (μ) has a median (and mean) of 0.22 and ranges from 0.10 to 0.35. The probability that a noise trader trades (ψ) ranges between 0.05 and 0.68, for a median value of 0.15; the value of ψ clearly reflects the choice of no-trade interval.³³ Given that, on average, in our dataset there are 116 decisions per day, our estimates indicate that, on the median no-event day, the number of trades should be (approximately) $116\psi_{Med} = 17$ and on the median event day the same number should be (approximately) $116[\psi_{Med}(1 - \mu_{Med}) + \mu_{Med}] = 38$. Overall, the expected number of daily trades in a median day should be $38\alpha_{Med} + 17(1 - \alpha_{Med}) = 38(0.40) + 17(1 - 0.40) = 25$, compared to 21 in our data.³⁴

The informativeness of the private signal (τ) equals 0.37 for the median stock, ranging from 0 (Ternienergia) to 1.76 (Landi);³⁵ this implies that, for the median stock, the probability of receiving an “incorrect signal” (e.g., higher than 0.5 on a bad-event day) is 0.41.³⁶ With the exception of one stock (Landi), as Table 3 shows, τ is significantly lower than one, that is, beliefs are bounded; given that beliefs are bounded, we can also reject the null that signals are perfectly informative, $\tau = 2$ (as assumed in Easley *et al.*, 1997).³⁷

³³ Table C.2 in the Appendix reports the estimates for a 1-minute and a 3-minute no-trade interval.

³⁴ In these computations, we have assumed that both noise traders receiving a shock and informed traders always trade (i.e., we have disregarded no trading due to the presence of the bid-ask spread). This is why these statistics are approximations (indeed, they are upper bounds).

³⁵ For Ternienergia the value of τ almost equal to 0 indicates that informed traders are aware of the event but, essentially, not of its direction.

³⁶ One can show that the probability is equal to $0.5 - 0.25\tau$.

³⁷ The estimate of 1.76 for Landi means that there is a positive measure of signals that perfectly reveal the value of the asset; the estimate close to 0 for Ternienergia means that informed traders know that there has been an event, but have no information about whether it was good or bad (i.e., they are informed about the realized volatility but not about the directional change). Hypothesis test for the individual stocks are reported in Table C.3 of the Appendix.

Table 2: **Estimation Results.**

	α	δ	μ	τ	ν	ψ
Acotel	0.42 (0.01)	0.50 (0.13)	0.25 (0.01)	0.29 (0.02)	1.23 (0.04)	0.12 (0.00)
B&C Speakers	0.30 (0.00)	1.00 (0.00)	0.16 (0.00)	0.64 (0.00)	1.08 (0.00)	0.08 (0.00)
Bolzoni	0.38 (0.03)	0.88 (0.09)	0.28 (0.01)	0.23 (0.02)	1.22 (0.05)	0.15 (0.00)
Cembre	0.45 (0.01)	0.37 (0.09)	0.16 (0.01)	0.49 (0.01)	1.15 (0.00)	0.07 (0.00)
Elen	0.34 (0.03)	0.68 (0.1)	0.32 (0.01)	0.24 (0.01)	1.32 (0.02)	0.21 (0.00)
Elica	0.48 (0.03)	0.56 (0.09)	0.23 (0.01)	0.42 (0.02)	1.25 (0.03)	0.22 (0.00)
Exprivia	0.33 (0.01)	0.91 (0.01)	0.22 (0.01)	0.30 (0.01)	1.26 (0.02)	0.17 (0.00)
Gefran	0.37 (0.04)	0.50 (0.7)	0.10 (0.01)	0.07 (0.03)	1.60 (0.12)	0.05 (0.00)
Landi	0.77 (0.01)	0.47 (0.01)	0.17 (0.00)	1.76 (0.01)	1.00 (0.00)	0.68 (0.00)
Nice	0.51 (0.03)	0.38 (0.08)	0.14 (0.01)	0.62 (0.03)	1.01 (0.06)	0.07 (0.00)
Panaria	0.40 (0.03)	0.84 (0.08)	0.15 (0.00)	0.39 (0.03)	1.18 (0.04)	0.12 (0.00)
Poltrona	0.56 (0.01)	0.59 (0.07)	0.28 (0.00)	0.58 (0.02)	1.08 (0.01)	0.30 (0.00)
Prima	0.36 (0.04)	0.17 (0.37)	0.14 (0.01)	0.09 (0.02)	1.52 (0.07)	0.09 (0.00)
Sabaf	0.44 (0.03)	0.41 (0.08)	0.35 (0.01)	0.37 (0.01)	1.14 (0.03)	0.20 (0.00)
Ternienergia	0.31 (0.00)	0.99 (0.00)	0.33 (0.00)	0.00 (0.00)	1.65 (0.00)	0.17 (0.00)
Mean	0.43	0.62	0.22	0.43	1.25	0.18
Median	0.4	0.56	0.22	0.37	1.22	0.15
Standard Deviation	0.12	0.25	0.08	0.42	0.20	0.16

Estimates for the parameters of the model for all stocks (BHHH standard errors in parentheses).

Table 3: **Test of hypotheses.**

H_0	$\tau = 2$	$\tau = 1$	$\nu = 2$	$\nu = 1$	$\delta = 0.5$	$\alpha = 1$	<i>CG</i>	<i>EKO</i>
H_1	<	<	<	>	\neq	<	\neq	\neq
Rejections of H_0	15	14	15	14	6	15	15	15

Results of various hypothesis tests. The first six columns are Wald tests, the last two columns are LR tests. CG refers to Cipriani and Guarino (2014) and EKO refers to Easley *et al.* (1997). H_0 specifies the null hypothesis, H_1 the alternative. The number of rejections (i.e. number of stocks for which H_0 is rejected) is based on a confidence level of 1%.

The parameter ν (which measures the impact of noise traders' shocks on their valuations) for the median stock is 1.22; this estimate implies that, for the median stock, 64% of noise traders who receive a shock are price elastic.³⁸ For all the 15 stocks we can reject the null that $\nu = 1$ at conventional significance levels: that is, there is a strictly positive measure of price inelastic noise traders (who buy or sell independently of the price).

Finally, recall that our model is equivalent to Cipriani and Guarino (2014) when $\nu = 2$ and to Easley *et al.* (1997) when $\tau = \nu = 2$. Through a likelihood-ratio test, for all stocks we can reject the restriction that $\nu = 2$, that is, that all traders are price-inelastic as usually assumed in the empirical market microstructure literature; we can also reject the joint restriction $\tau = \nu = 2$, that is, that both noise and informed traders are price inelastic.

8 The Impact of an FTT

We now move to illustrate the impact of an FTT under different tax rates. Given the parameter estimates of each stock, we study the tax impact in three ways: i) we compute $\bar{\rho}^N$, the tax rate above which the market does not open, and $\bar{\rho}^I$, the tax rate above which all trading is noise; ii) we compute the probability of severe mispricing, that is, that the market maker's belief about the occurrence of a good event remains stuck at a high (low) level on a bad (good)-event day; and iii) we simulate the price path and the trading flow and compute measures of trading volume, price volatility, and informational inefficiency.

³⁸ Noise traders are price elastic when they receive signals belonging to the intersection of the supports of g^H and g^L . The proportion of price-elastic noise traders is given by the area corresponding to such intersection, measured with the (true) uniform distribution, that is, with $\frac{2-\nu}{\nu}$.

To conduct this analysis, we need an estimate of λ^H , the percentage change in the asset value on a good-event day.³⁹ Indeed, when a tax is present, λ^H is needed to calculate the equilibrium trading thresholds for our informed and price-elastic noise traders: the size of the tax in relation to the possible gains and losses from holding the asset (λ^H) determines whether traders buy or sell (recall expression (15)). We did not have to estimate λ^H as part of the maximum likelihood estimation, since we estimated the model in the absence of a tax. In the following section, we estimate λ^H through a different method.

8.1 Public Information and the Estimation of λ^H

Let us denote by p^d the closing price on day d for a given stock and by Δp^d the price change on day d (i.e., the difference between closing price and opening price). From our model, the standard deviation of the daily price change (in percentage terms) is given by⁴⁰

$$\sigma\left(\frac{\Delta p^d}{p_1^d}\right) = \left[\left(\frac{\alpha\delta}{1-\delta}\right)\lambda^H\right]^{\frac{1}{2}}. \quad (28)$$

Hence, for each stock, we can estimate λ^H from the parameter estimates of α and δ and from $\sigma(\frac{\Delta p^d}{p_1^d})$, which we can estimate from the stock daily closing and opening prices. Unlike in our theoretical model, however, observed price changes may be due to two kinds of events: i) events that were private information to traders and were revealed to the market by the order flow; and ii) events that became public information during or after the trading day. The parameter λ^H in our model only refers to the first kind of events; in contrast, the observed variance of the price is the result of both events. Since λ_H should only reflect the movement in the value of the asset due to private information, we need to decompose the price variance. If we did not do so, our estimates of λ^H would be inflated and as a result, the impact of a tax would be underestimated. The reason is that only such events determine the gains from trade of informed traders that may be (partially or completely) offset by the tax.

To this purpose, we use the variance decomposition method developed by Hasbrouck

³⁹ Recall that λ^L , the percentage change in asset value on a bad-event day, is related to λ^H by the martingale condition.

⁴⁰ Since without an FTT, the asset price converges to the fundamental value, with probability $(1 - \alpha)$, $\frac{\Delta p^d}{p_1^d} = 0$; with probability $\alpha\delta$, $\frac{\Delta p^d}{p_1^d} = \lambda^H$; and with probability $\alpha(1 - \delta)$, $\frac{\Delta p^d}{p_1^d} = \lambda^L = \frac{\delta}{(1-\delta)}\lambda^H$; hence, equation (28).

(1991): we decompose stock price percentage changes into a trade-correlated component, interpreted as the component driven by private information, and a trade-uncorrelated component, interpreted as the component driven by public information. We discuss this procedure in more detail in the Appendix.

Table 4: **Hasbrouck decomposition.**

	σ	σ^{PI}	R_w	λ^H	λ^L
Acotel	0.0186	0.004	0.2175	0.0063	-0.0062
B&C Speakers	0.017	0.004	0.2367	0.0005	-0.1096
Bolzoni	0.0211	0.0061	0.2872	0.0036	-0.0266
Cembre	0.0119	0.0019	0.1611	0.0037	-0.0022
Elen	0.0167	0.0041	0.2456	0.0048	-0.0102
Elica	0.0251	0.0108	0.432	0.014	-0.0175
Exprivia	0.0202	0.0016	0.0777	0.0009	-0.0086
Gefran	0.0152	0.0087	0.5715	0.0143	-0.0143
Landi	0.0201	0.0042	0.2104	0.0051	-0.0046
Nice	0.0119	0.0033	0.2751	0.0059	-0.0036
Panaria	0.0234	0.0095	0.4071	0.0065	-0.0351
Poltrona	0.0133	0.001	0.0715	0.0011	-0.0015
Prima	0.0163	0.0068	0.4208	0.025	-0.0052
Sabaf	0.0154	0.0039	0.2543	0.0071	-0.0049
Ternienergia	0.0126	0.0039	0.3087	0.0006	-0.0813
Mean	0.0173	0.0049	0.2785	0.0066	-0.0221
Median	0.0167	0.004	0.2543	0.0051	-0.0086
Standard Deviation	0.0041	0.0029	0.1352	0.0066	0.0317

Results for Hasbrouck (1991)’s variance decomposition and estimates for λ^H and λ^L for all stocks. R_w refers to the square root of Hasbrouck’s R^2 statistic, σ is the standard deviation of daily log-price changes, σ^{PI} is standard deviation of daily log-price changes due to private information.

Table 4 reports the result of Hasbrouck (1991)’s decomposition (in particular, the square root of Hasbrouck R_w^2 statistics) along with the standard deviation of daily log-price changes (σ), the standard deviation of daily log-price changes due to private information (σ^{PI}) and the estimates for λ^H .⁴¹ We find that for the median stock 25% of the standard deviation of the log-price changes is due to private information. Across stocks, the median standard

⁴¹ We prefer to report the square root of Hasbrouck’s statistics because we are decomposing the standard deviation of percentage changes (whereas Hasbrouck decomposes their variance).

deviation of daily log-price changes is 1.67%; the median standard deviation of daily log-price changes due to private information is 0.40%. Given these results and our parameter estimates, across all stocks the median estimate of λ^H is 0.5% (and that of λ^L is -0.9%).⁴² Therefore, although the proportion of the daily standard deviation due to private information is just above 20%, the percentage daily changes in the asset values due to private information are substantial.

8.2 Threshold Tax Rates

Given the estimates for λ^H , we can now start our analysis of the tax impact. Proposition 1 defines two tax rate thresholds, $\bar{\rho}^N$ and $\bar{\rho}^I$ such that for any tax rate $\rho > \bar{\rho}^N$ no trader would ever trade, and for any tax rate $\rho \in (\bar{\rho}^I, \bar{\rho}^N)$ only noise traders would ever trade. These thresholds for our stocks are reported in Table 5. Across stocks, the tax rate that would shut down all trading activity ranges between 15 bps and 11%, with a median of 1% and an average of 2.4%; a 0.5% would shut down the market for only 2 out of 15 stocks. This is a high tax rate, an order of magnitude higher than the transaction taxes actually implemented (e.g., 20 – 30 bps in France and 10 – 12 bps in Italy). In contrast, relatively low tax levels are enough to shut down any information-based activity in the market: for the median stock, a tax rate of 17 bps is enough to eliminate all information-based trading activity. As Table 5 shows, a tax rate of 5 bps shuts down all informed trading for only one stock; a tax of 10 bps, for 5 stocks; and a tax of 20 bps for 10 stocks.

The thresholds $\bar{\rho}^N$ and $\bar{\rho}^I$ are higher for stocks with a higher daily change in asset value (λ^H) and with a higher δ (recall expression (16)). The difference between the levels of $\bar{\rho}^I$ and $\bar{\rho}^N$ is due to the fact that, for all but one stock (Landi), τ is lower than 1: the lower informativeness of informed traders' signal decreases the maximum tax rate that can be levied without shutting down their activity (recall expression (17)).⁴³

Overall, our analysis indicates that tax rates in the range usually implemented (i.e., between 10 and 20 bps) would not lead to complete market breakdown, as some noise traders would continue to trade even with a tax. It would, however, severely impact price discovery,

⁴² The average estimate of λ^L is higher (in absolute value) than that of λ^H because across stocks the median δ is greater than 0.5 and, by the martingale assumption, this implies a larger downward movement.

⁴³ Note that in the table, for Landi we report $\bar{\rho}^I = \bar{\rho}^N$. The reason is that the threshold for which informed traders would stop trading would be higher than $\bar{\rho}^N$. At $\bar{\rho}^N$, however, noise traders stop trading, and so do informed traders, given that the market breaks down.

Table 5: The Tax Rate Thresholds $\bar{\rho}^I$ and $\bar{\rho}^N$.

	$\bar{\rho}^I$	$\bar{\rho}^N$
Acotel	0.0018	0.0063
B&C Speakers	0.0017	0.1096
Bolzoni	0.0018	0.0266
Cembre	0.0016	0.0037
Elen	0.0017	0.0102
Elica	0.0068	0.0175
Exprivia	0.0006	0.0086
Gefran	0.0009	0.0143
Landi	0.0051	0.0051
Nice	0.0033	0.0059
Panaria	0.0058	0.0351
Poltrona	0.0008	0.0015
Prima	0.0008	0.025
Sabaf	0.0023	0.0071
Ternienergia	0	0.0813
Mean	0.0023	0.0239
Median	0.0017	0.0102
Standard Deviation	0.0020	0.0311

Tax rate threshold $\bar{\rho}^I$ and $\bar{\rho}^N$ given in Proposition 1. For tax rates above $\bar{\rho}^I$ all informed trading activity ceases, and for tax rates above $\bar{\rho}^N$ the market shuts down (0.01 corresponds to 1% or 100bps).

with a large number of stocks in which aggregation of private information by the market price would cease.

8.3 Asymptotic Impact of the FTT

Let us now study the asymptotic impact of the FTT. Lemma 1 gives us analytical expressions for two functions, $\delta_l(\alpha_t^d)$ and $\delta_u(\alpha_t^d)$; they define two thresholds such that, in the presence of a tax, informed traders stop trading whenever their belief on the good event (δ_t^d) is either below the lower threshold or above the upper threshold. As we know from Proposition 4, given these thresholds, we can compute bounds on the probability that the market maker's belief on whether the event is good (and, therefore, the price) remains stuck at a high level when the day is bad, or at a low level when the day is good. We refer to these cases as cases of “wrong convergence,” since the belief (and the price) converge to a high level whereas the

asset value is low or vice versa.

In Table 6, we report the median and mean thresholds evaluated at $\alpha_t^d = 1$, for a 5 bps, 10 bps, and 20 bps FTT, along with the probabilities of wrong convergence on a good-event and on a bad-event day. The thresholds and probabilities for individual stocks are reported in the Appendix.

Table 6: **Belief thresholds and probabilities of wrong convergence.**

		5bps	10bps	20bps
$\delta_l(1)$	Mean	0.05	0.06	0.08
	Median	0.04	0.05	0.05
	Standard Deviation	0.04	0.05	0.09
$\delta_u(1)$	Mean	0.95	0.94	0.91
	Median	0.96	0.95	0.95
	Standard Deviation	0.04	0.05	0.09
$\Pr(\delta_\infty^d = \delta_l(1) v_H^d)$	Mean	0.07	0.06	0.12
	Median	0.03	0.03	0.04
	Standard Deviation	0.13	0.07	0.05
$\Pr(\delta_\infty^d = \delta_u(1) v_L^d)$	Mean	0.13	0.18	0.10
	Median	0.05	0.09	0.06
	Standard Deviation	0.20	0.21	0.15

Thresholds $\delta_l(1)$ and $\delta_u(1)$ as well as the probabilities of wrong convergence as given in Proposition 4.

For a 5 bps FTT, the median lower and upper thresholds are 0.04 and 0.96. On a good-event day, the median probability of wrong convergence (i.e., with the belief stuck at 0.04) is (approximately) 3%; it is 5% on a bad event-day. The slightly higher probability on a bad-event day stems from the fact that δ is on average (across all stocks) higher than 0.5; that is, traders' and the market maker's priors are closer to the true asset value on a good-event day, making the likelihood of a wrong convergence lower.

When the tax rate is 10 bps, the median thresholds become 0.05 and 0.95; the probabilities of wrong convergence become 3% and 9%. At a first glance, one may be surprised that the probability of wrong convergence on a good-event day does not increase. It must be noted, though, that the mean and median in the table refer to a different number of stocks as we move from 5 bps to 10 bps, since for 10 bps for some stocks informed trading activity is shut down and the thresholds cannot be computed. In Table C.4 of the Appendix, we report the same statistics as in Table 6, estimated only on those stocks for which the thresholds can be computed for all three tax rates; the thresholds move monotonically away from the extremes

and, as a result, the probabilities of severe mispricing increase monotonically.

Finally, note that the bounds computed analytically according to Proposition 4 are virtually identical to the proportions of wrong convergence we obtain by simulating the model with our parameter estimates. Recall that the reason Proposition 4 provides bounds and not the precise probabilities is that it can happen that the thresholds are reached before the occurrence of the event is completely learned, that is, before α_t^d has converged to 1. This, however, in practice does not seem to occur, at least for our parameter estimates.

Our analysis shows that even when an FTT does not prevent all informed traders from trading, the price may still end up not reflecting the information held by these traders; indeed, even when the tax rate is small, there is a sizeable probability of wrong convergence (and, therefore, of the price being stuck far away from the asset value).

8.4 Within Day Effects of an FTT

Whereas one can derive the asymptotic effects of an FTT analytically, it is difficult to obtain analytical results for the impact of the tax on the intra-day trading activity. This is a difficulty that our model shares with any Glosten and Milgrom (1985) type of model. Even the directional impact of a tax on the informativeness of each trade is ambiguous from a theoretical standpoint, since the tax affects the behavior of both informed and noise traders.⁴⁴

Since the impact of a tax is hard to tease out analytically, we proceed by simulating the model. In particular, for each stock, we first simulate the model using our parameter estimates in the absence of a tax. Then, we simulate the same economy with an FTT and a tax rate of 5 bps, 10 bps and 20 bps. We aim to quantify the effect of the FTT on the trading volume, volatility, liquidity (the bid-ask spread) and informational efficiency. For each stock, the simulations are run for 100,000 days, each consisting of 190 trading times (decisions).⁴⁵ All the results that we report are means across these simulated days.

⁴⁴ Indeed, because of this difficulty, Dupont and Lee (2007) and Sørensen (2017) study the impact of an FTT restricting their analysis to a static (one-period) version of the Glosten and Milgrom (1985) model.

⁴⁵ We illustrate our results with 190 decisions, the median of the maximum number of decisions across stocks. With other number of decisions, we obtain very similar results. Note also that each day is simulated as if it were the first day of trading activity; additionally, we set $v^0 = \frac{100(1-\delta)}{\lambda^H}$, which guarantees that each day the possible price range (i.e., $v_H - v_L$) equals 100. We chose to do so in order to be able to interpret many of the simulation results (e.g., the bid and ask spread or the average distance of the

8.4.1 Impact of an FTT on Volume

We start by considering the impact of the FTT on trading volume. Since in our model the trade size is one share, we define the volume of trade as the number of daily transactions. Table 7 reports the simulated number of trades (buys or sells) for different tax rates. The results are reported both for all days and for event and no-event days separately. We only report the aggregate statistics here and refer the reader to the Appendix for those on individual stocks.

Table 7: **Trade volume.**

		0bps	5bps	10bps	20bps
All Days	Mean	36	14	11	8
	Median	34	13	10	8
	Standard Deviation	23	8	7	7
Event Days	Mean	47	19	13	8
	Median	47	14	10	7
	Standard Deviation	23	13	11	8
No-Event Days	Mean	27	9	9	8
	Median	25	8	8	8
	Standard Deviation	21	6	5	6

Daily volume of trade (number of daily transactions) by type of day and tax rate.

On no-event days, for the median stock, a 5 bps FTT reduces the volume of trade by approximately 68% (from 25 to 8 trades). This is obviously due to price elastic noise traders who prefer not to trade in the presence of a tax. On event days, the drop in trading volume is even higher, equal to 71% (from 47 to 14 trades): the tax affects not only noise traders but also informed traders. These results are in line with the strong negative impact on trading volume found in the empirical literature on transaction taxes (see e.g. Umlauf (1993), Jones and Seguin (1997)). The effect is typically stronger on illiquid assets such as the stocks considered here.

Increasing the tax rate reduces the trading volume even further, but not by much. A simple way to understand this result is to note that the process of learning is quite fast and, as a result, the market maker's and traders' beliefs converge very quickly to each other, which makes even a small tax become binding very soon. Increasing the tax rate makes the tax binding even earlier, but not by a lot.

price from the fundamental) as percentages.

It is interesting to study how the market composition changes with the tax rate. Table 8 reports the mean and median proportion of trades by informed traders on event days for different tax rates. Without a tax, informed traders comprise 62% of trading activity for the median stock. This proportion decreases to 45% with a 5 bps tax, and to 33% with a 10 bps tax; when the FTT is 20 bps there is no informed trading activity in the median stock, and across all stocks the average proportion is 11%. Consistent with the fact that for most stocks informed traders are more elastic than noise traders (because $\tau < 1$), the tax impacts informed traders' activity to a greater extent than noise traders' activity. An exception is Landi, for which $\tau = 1.76 > \nu$: since there is a higher fraction of inelastic informed traders than inelastic noise traders, informed trading activity actually increases with the tax (see Table C.5 in the Appendix).

Table 8: **Informed Trading.**

	0bps	5bps	10bps	20bps
Mean	0.58	0.36	0.25	0.11
Median	0.62	0.45	0.33	0
Standard Deviation	0.09	0.17	0.19	0.17

Proportion of trades on event days that are due to informed traders by tax rate.

8.4.2 Impact of an FTT on the Bid-Ask Spread

Let us now study the impact of an FTT on the bid-ask spread, a standard measure of liquidity. Theory does not give us a clear prediction on the effect of the FTT on the spread. Suppose, for the sake of an example, that only informed traders are price elastic, as in Cipriani and Guarino (2014). In this case, the FTT crowds out a fraction of informed traders and thus reduces the adverse selection problem. Everything else being constant, the market maker needs to protect himself less against private information, which leads to a lower spread. However, with fewer informed traders, learning is slower and, therefore, the spread converges to zero at a slower pace; as a result, the average daily spread the day may actually increase with the tax. Additionally, in our model, also noise traders are price elastic, which makes the effect of the tax even harder to pin down and, as we will show through the simulation, even non-monotonic in the tax rate.

In Table 9, we show the simulated impact of the FTT on the average daily bid-ask spread

by type of day.⁴⁶ Remember that, in the simulations, the daily price range is always set at 100; therefore, a spread of, e.g., 5 can be read as a spread of 5% of the maximum possible daily change in asset value due to private information.⁴⁷ For all tax rates, the median spread is higher on event days than on no-event days, as one would expect. This reflects the fact that on event days the market maker learns not only that an event has occurred but also whether it is good or bad.

Table 9: **Liquidity.**

		0bps	5bps	10bps	20bps
All Days	Mean	4	6	5	2
	Median	4	5	4	0
	Standard Deviation	3	8	8	5
Event Days	Mean	6	6	5	2
	Median	6	5	4	0
	Standard Deviation	4	7	7	4
No-Event Days	Mean	2	6	5	3
	Median	1	3	2	0
	Standard Deviation	2	10	9	7

Daily bid-ask spread by type of day and tax rate.

Interestingly, both the median and the mean spread change non-monotonically with the tax rate. Indeed, the median spread is 3.7% with no tax, increases to 5% with a 5 bps tax and then decreases for higher rates. The non-monotonicity is mainly driven by the behavior of the spread on no-event days (e.g., the mean spread on no-event days goes from 2, with no tax, to 6, with a 5 bps tax). One way to think about this is that the tax, by reducing the trading activity of informed traders, increases the proportion of no trades on event days, making it harder to learn if no event has occurred, thus keeping the spread away from zero for longer time.

8.4.3 Impact of an FTT on Volatility

It is also interesting to look at the impact of the FTT on price volatility. We consider the average standard deviation of intraday price changes (i.e., changes between trading times

⁴⁶ Specifically, for each simulated day, we compute the average spread for all the decisions (trading times) in the day.

⁴⁷ The price change due to private information is only a fraction of the overall price change, which includes public information (see Section 8.1).

during a day).⁴⁸ The results are reported in Table 10.

Not surprisingly, volatility is higher on event days than on no-event days, for all levels of the tax. On event days, and for all days, volatility is monotonically decreasing in the tax rate for all stocks but one: since the tax reduces the proportion of informed trading, it also lowers price volatility. The exception is again Landi: as the tax affects noise traders more than informed traders, it increases the informativeness of a trade and therefore price volatility.

Table 10: **Volatility.**

		0bps	5bps	10bps	20bps
All Days	Mean	5	4	3	1
	Median	5	4	3	0
	Standard Deviation	4	4	3	3
Event Days	Mean	9	7	4	2
	Median	11	7	5	0
	Standard Deviation	6	5	5	4
No-Event Days	Mean	2	2	1	1
	Median	1	2	1	0
	Standard Deviation	3	2	2	1

Intraday volatility by type of day and tax rate.

On no event days, volatility is very low. For the median stock, it moves non-monotonically with the tax rate, in particular, it is higher for a 5 bps FTT than in the absence of a tax. As we already explained for the bid-ask spread, the tax makes it harder to infer that no event has occurred, with the result that volatility can be higher. For rates higher than 5 bps, the impact on informed traders' activity is so large that overall volatility decreases.

8.4.4 Impact of an FTT on Informational Efficiency

Finally, we investigate how the FTT influences the ability of the market to aggregate private information into the price. We focus on the market maker's expected value of the asset value, $p_t^d = \mathbb{E}(V^d | h_t^d)$ (which we have defined as the price of the asset) over the course of the trading day. For each stock, we analyze to what extent this price deviates from the true asset

⁴⁸ Remember that, in Section 2, we defined the price at time t of day d as $p_t^d = E(V^d | h_t^d)$.

value v^d as in the following expression (where $InIn$ stands for Informational Inefficiency):

$$InIn =: \frac{1}{D} \frac{1}{T} \sum_{t=1}^D \sum_{t=1}^T |p_t^d - v^d|. \quad (31)$$

A higher value of $InIn$ indicates a higher informational inefficiency of the market. Table 11 reports the results. A 5 bps tax rate reduces the informational efficiency of the market compared to the situation of no tax for all stocks but one (Landi). Higher tax rates, have, on average (and in median), the effect of lowering the informational efficiency further. As we know, for all stocks but one, the tax has a higher impact on informed traders than on noise traders (since, typically, $\tau < 1$). A tax reduces informed trading activity, thus lowering market efficiency.⁴⁹

Notice that the effect of the tax is different for Landi, as one would expect. For this stock, the tax actually has a positive impact on informational efficiency, clearly due to the impact on the composition of the market (discussed above) that becomes more informed, as the tax rate increases.

Table 11: **Informational Efficiency.**

		0bps	5bps	10bps	20bps
All Days	Mean	11	13	14	14
	Median	12	13	15	15
	Standard Deviation	5	8	8	9
Event Days	Mean	23	27	30	32
	Median	25	32	33	37
	Standard Deviation	13	15	17	18
No-Event Days	Mean	1	2	1	1
	Median	1	1	1	0
	Standard Deviation	3	2	1	2

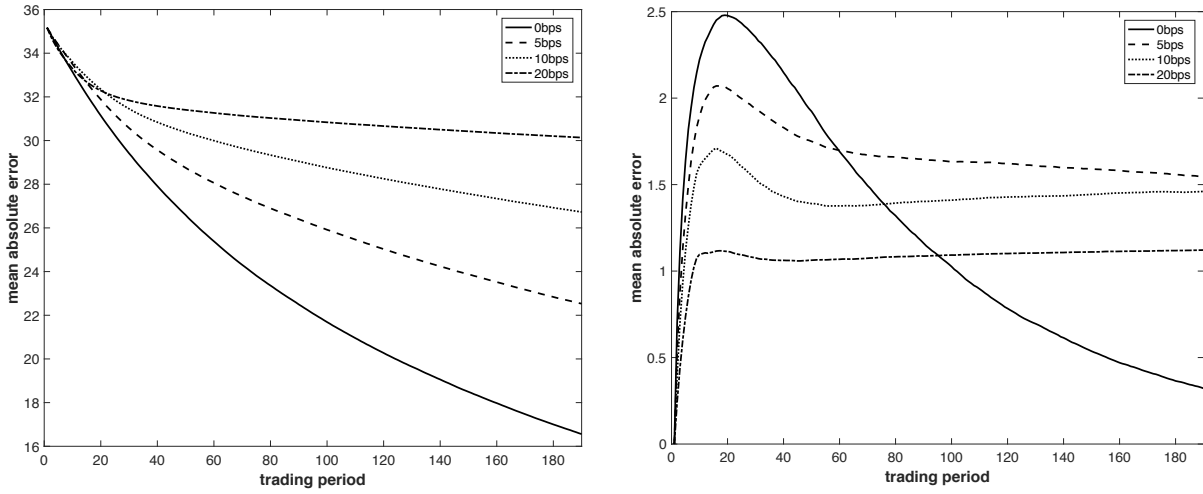
Mean absolute distance of the price from the fundamental value by type of day and tax rates.

It is interesting to look at the evolution of information efficiency over the course of the day. In Figure 4 we plot the mean (across stocks) distance between price and fundamental value $|p_t^d - v^d|$ by trading time, separately for event and no-event days, and for different

⁴⁹ The effect is non monotonic for some stocks. The reason is analogous to that explained in the previous sections.

tax rates. Without a tax, on an event day, on average the price monotonically converges towards the fundamental value. With a 5 bps tax, the distance decreases at a much lower rate. For 20 bps the distance almost remains constant and close to the beginning of the day's level. The reason is that for many stocks, for a 20 bps tax rate, informed traders do not trade, and no information is aggregated by the price. On no-event days, without a tax, in the early periods the distance increases (since the price incorporates a positive probability that an event occurred); eventually, though, the market realizes that no event has occurred and the distance goes to 0. With a tax, instead, since event and no-event days are more similar, the price moves less from the unconditional expected value, which is also the asset value realization. For this reason, the distance between price and fundamental is lower than without a tax. Over time, the market learns very slowly and the distance is almost constant.⁵⁰

Figure 4: **Informational efficiency by type of day, trading period and tax.**



The left figure displays the mean absolute error $|p_t^d - v^d|$ for event days, the right figure displays the mean absolute error for no event days.

⁵⁰ This is just in the time span we have used in our chart. Of course, asymptotically, the market learns that no event has occurred and the orange line in the plot converges to 0 as well, as the number of trading times grows large.

9 Conclusion

We have presented a novel methodology to quantify the impact of an FTT on the informational efficiency of financial markets. We built a sequential trading model in which both informed traders and noise traders are price-elastic. Informed traders receive private information of heterogeneous quality; noise traders receive liquidity shocks of different size. These are modelling innovations, compared to previous work in empirical market microstructure, in which noise trading is assumed to be completely price inelastic. Our model encompasses the case of price inelastic traders but does not restrict to it. We think these innovations are crucial to estimate the impact of an FTT as of any other transaction cost. We estimated the model through maximum likelihood for a sample of Italian mid-sized stocks, using data relative to a period before an FTT was introduced in Italy. The structural estimation allowed us to estimate the probability of (good or bad) informational events, the composition of the market in terms of informed and noise trading as well as the traders' price elasticity. For our stocks, noise traders are typically price elastic, although less so than informed traders. As a result, the introduction of an FTT typically changes the composition of the market making the order flow less informative, that is, having a negative effect on informational efficiency. We characterized the impact of an FTT on the asymptotic convergence of the price to the asset's fundamental value theoretically, and quantified it using our estimates. We found that in a sizeable frequency of cases an FTT may impede price convergence by the end of a trading day.

We have illustrated our methodology for a sample of stocks, but of course it can be applied much more generally, both to the stock and to the bond market. Independently of the specific results we obtain for our stocks, we believe our analysis helps to understand the intricate ways in which an FTT affects traders' behavior and, as a result, the informational efficiency of financial markets.

References

- Aktas, N., E. De Bodt, F. Declerck, and H. Van Oppens, 2007. "The PIN anomaly around M&A announcements," *Journal of Financial Markets*, 10(2), pp. 169-191.
- Avery, C. and P. Zemsky, 1998. "Multidimensional uncertainty and herd behavior in financial markets," *American Economic Review*, pp. 724-748.

- Berndt, E., B. Hall, and R. Hall, 1974. "Estimation and inference in nonlinear structural models," *Annals of Economic and Social Measurement*, 3(4) NBER, pp. 103-116.
- Bikhchandani, S., D. Hirshleifer, and I. Welch, 1992. "A theory of fads, fashion, custom, and cultural change as informational cascades," *Journal of Political Economy*, 100(5), pp. 992-1026.
- Cipriani, M. and A. Guarino, 2014. "Estimating a structural model of herd behavior in financial markets," *American Economic Review*, 104(1), pp. 224-251.
- Cipriani, M., and A. Guarino, 2008. "Herd Behavior and Contagion in financial markets," *The B.E. Journal of Theoretical Economics*, 8, Contributions, Article 24.
- Cipriani, M., and A. Guarino, 2008a. "Transaction costs and informational cascades in financial markets," *Journal of Economic Behavior & Organization*, 68(3), pp. 581-592.
- Dávila, E. and C. Parlato, 2019. "Trading Costs and Informational Efficiency," mimeo.
- Deng, Y., X. Liu, and S.J. Wei, 2018. "One fundamental and two taxes: When does a Tobin tax reduce financial price volatility?," *Journal of Financial Economics*, 130(3), pp. 663-692.
- Dupont, D. and G. Lee, 2007. "Effects of securities transaction taxes on depth and bid-ask spread," *Economic Theory* 31(2), pp. 393-400.
- Easley, D., N. Kiefer, M. O'Hara and J. Paperman, 1996. "Liquidity, information, and infrequently traded stocks," *The Journal of Finance*, 51(4), pp.1405-1436.
- Easley, D., N. Kiefer, and M. O'Hara, 1997. "One day in the life of a very common stock," *Review of Financial Studies*, 10(3), pp. 805-835.
- Easley, D. and M. O'Hara, 1987. "Price, trade size, and information in securities markets," *Journal of Financial Economics*, 19(1), pp. 69-90.
- Edwards, F.R., 1993. "Taxing transactions in futures markets: objectives and effects," *Journal of Financial Services Research*, 7(1), pp. 75-91.
- Gale, D., 1996. "What have we learned from social learning?," *European Economic Review*, 40(3), pp.617-628.

Gervais, S., 1997. "Market Microstructure with Uncertain Information Precision: A Multi-Period Analysis," mimeo.

Glosten L. and P. Milgrom, 1985. "Bid, ask and transaction prices in a specialist market with heterogeneously informed traders," *Journal of Financial Economics*, 14(1), pp. 71-100.

Glosten, L. and T. Putnins, 2016. "Welfare costs and informed trade," mimeo.

Habermeier K.F. and A. Kirilenko, 2003. "Securities transaction taxes and financial markets," IMF Staff Paper, 50(1), pp. 165-180.

Hasbrouck, J., 1991. "The summary informativeness of stock trades: An econometric analysis," *Review of Financial Studies*, 4(3), pp. 571-595.

Hirshleifer, D. and S. Hong Teoh, 2003. "Herd behaviour and cascading in capital markets: A review and synthesis," *European Financial Management*, 9(1), pp.25-66.

Jones, C.M. and P.J. Seguin, 1997. "Transaction costs and price volatility: evidence from commission deregulation," *American Economic Review*, 87(4), pp.728-737.

Kupiec, P.H., 1996. "Noise traders, excess volatility, and a securities transactions tax," *Journal of Financial Services Research*, 10(2), pp. 115-129.

Lee, I.H., 1998. "Market crashes and informational avalanches," *Review of Economic Studies*, 65, pp. 741-760.

Lee, C., and M. Ready, 1991. "Inferring trade direction from intraday data," *Journal of Finance*, 46(2), pp. 733-746.

Mannaro, K., M. Marchesi and A. Setzu, 2008. "Using an artificial financial market for assessing the impact of Tobin-like transaction taxes," *Journal of Economic Behavior & Organization*, 67(2), pp. 445-462.

Nussair, C., S. Robin and B. Ruffieux, 1998. "The effect of transaction costs on double auction markets," *Journal of Economic Behavior & Organization*, 36, pp. 221-233.

Pellizzari, P. and F. Westerhoff, 2009. "Some effects of transaction taxes under different microstructures," *Journal of Economic Behavior & Organization*, 72(3), pp. 850-863.

Schwert, G.W. and P.J. Seguin, 1993. "Securities transaction taxes: an overview of costs, benefits and unresolved questions," *Financial Analysts Journal*, pp. 27-35.

- Smith, L. and P. Sørensen, 2000. "Pathological outcomes of observational learning," *Econometrica*, 68(2), pp. 371-398.
- Sørensen, P., 2017. "The financial transaction tax in markets with adverse selection," mimeo.
- Stiglitz, J.E., 1989. "Using tax policy to curb speculative short-term trading," *Journal of Financial Services Research*, 3(2-3), pp. 101-115.
- Song, F. and J. Zhang, 2005. "Securities transaction tax and market volatility," *Economic Journal*, 115(506), pp. 1103-1120.
- Subrahmanyam, A., 1998. "Transaction taxes and financial market equilibrium," *Journal of Business*, 71(1), pp. 81-118.
- Summers, L.H. and V.P. Summers, 1989. "When financial markets work too well: a cautious case for a securities transactions tax," *Journal of Financial Services Research*, 3(2-3), pp. 261-286.
- Tobin, J., 1978. "A proposal for international monetary reform," *Eastern Economic Journal*, 4(3/4), pp. 153-159.
- Umlauf, S.R., 1993. "Transaction taxes and the behavior of the Swedish stock market," *Journal of Financial Economics*, 33(2), pp. 227-240.
- Welch, I., 1992. "Sequential sales, learning, and cascades," *Journal of Finance*, 47(2), pp. 695-732.

Appendix (for online publication only)

A. Proofs of Analytical Results

Proof of Proposition 1

First, observe that a noise trader with shock $N_t^d = 1$ believes the asset value to be v_H^d with probability 1 and noise traders with shock $N_t^d = 0$ believes the asset value to be v_L^d with probability 1, since $\nu \geq 1$. Hence, if the tax rate ρ is so high that that no noise trader trades, no informed trader will not trade; therefore, to prove the first part of the proposition, we will focus on noise traders. At $t = 1$, no noise trader trades if and only if the following two inequalities hold:

$$\begin{aligned} E(V^d | N_1^d = 1) &< \text{ask}_1^d(1 + \rho), \\ E(V^d | N_1^d = 0) &> \text{bid}_1^d(1 - \rho), \end{aligned}$$

that is, a noise trader with the highest signal ($N_1^d = 1$) does not want to buy and a noise trader with the lowest signal ($N_1^d = 0$) does not want to sell given ask and bid prices and the transaction tax. Taking into account that when, in equilibrium, informed traders do not trade, the bid and ask are both equal to the unconditional expected value of the asset, and that $\mathbb{E}(V^d) = v^{d-1}$, noise traders do not trade if and only if

$$\begin{aligned} v^{d-1} [1 + \Pr(V^d = v_H^d | N_1^d = 1)\lambda_H + \Pr(V^d = v_L^d | N_1^d = 1)\lambda_L] &< v^{d-1}(1 + \rho), \\ v^{d-1} [1 + \Pr(V^d = v_H^d | N_1^d = 0)\lambda_H + \Pr(V^d = v_L^d | N_1^d = 0)\lambda_L] &> v^{d-1}(1 - \rho), \end{aligned}$$

or

$$\begin{aligned} \Pr(V^d = v_H^d | N_1^d = 1)\lambda_H + \Pr(V^d = v_L^d | N_1^d = 1)\lambda_L &< \rho, \\ \Pr(V^d = v_H^d | N_1^d = 0)\lambda_H + \Pr(V^d = v_L^d | N_1^d = 0)\lambda_L &> -\rho, \end{aligned}$$

that is,

$$\begin{aligned} \Pr(V^d = v_H^d | N_1^d = 1)\lambda_H - \Pr(V^d = v_L^d | N_1^d = 1)\frac{\delta}{1 - \delta}\lambda_H &< \rho, \\ \Pr(V^d = v_H^d | N_1^d = 0)\lambda_H - \Pr(V^d = v_L^d | N_1^d = 0)\frac{\delta}{1 - \delta}\lambda_H &> -\rho, \end{aligned}$$

which, since $\Pr(V^d = v_H^d | N_1^d = 1) = 1$ and $\Pr(V^d = v_H^d | N_1^d = 0) = 0$, simplifies into

$$\begin{aligned}\lambda_H &< \rho, \\ \frac{\delta}{1-\delta}\lambda_H &< \rho.\end{aligned}$$

These inequalities are jointly satisfied if and only if

$$\rho > \max \left\{ 1, \frac{\delta}{1-\delta} \right\} \lambda_H \equiv \bar{\rho}^N,$$

or, equivalently,

$$\rho > \frac{2\delta}{1-|2\delta-1|} \lambda_H \equiv \bar{\rho}^N.$$

Lastly, note that as noise traders do not trade, the market does not open.

We now derive conditions under which informed traders do not buy or sell assuming that noise traders continue to be active. We start with the case in which informed traders do not buy. The informed trader with the highest ex-post valuation is the trader with signal $S_t = 1$. The probability he attributes to an increase in the asset value after observing history h_t^d and this signal is

$$q_h(p) \equiv \Pr(V^d = v_h^d | S_t = 1, h_t^d) = \frac{(1+\tau)p}{(1+\tau)p + (1-\tau)(1-p)},$$

where, to simplify notation, we denote $\Pr(V^d = v_h^d | h_t^d)$ by p . Informed traders do not buy whenever their highest possible ex-post valuation is below the current ask price including the transaction tax, that is

$$v^{d-1} + q_h(p)\lambda_H v^{d-1} - \left(\frac{\delta}{1-\delta} \right) (1 - q_h(p))\lambda_H v^{d-1} < (1+\rho)\text{ask}.$$

If informed traders do not buy, the market maker sets the ask price taking into account that any buy will come from a noise trader. In that case, the market maker's posterior $\Pr(V^d = v_h^d | \text{buy}_t^d, h_t^d)$ is

$$\frac{(1-\mu)\psi(1-\kappa)ap}{(1-\mu)\psi(1-\kappa)a + \psi(1-\kappa)(1-a)} = \tilde{a}p,$$

where $a = \Pr(V^d = v^{d-1} | h_t^d)$ and \tilde{a} designates the market maker's posterior probability that

an event has occurred given that a noise trader is active in t , that is,

$$\tilde{a} = \Pr(V^d = v^{d-1} | \text{noise trader active}, h_t^d) = \frac{(1 - \mu)a}{1 - \mu a}.$$

The market maker's valuation of the asset if a noise trader buys after history h_t^d is then given by

$$v^{d-1} + \left[\tilde{a} p - \left(\frac{\delta}{1 - \delta} \right) \tilde{a} (1 - p) \right] \lambda_H v^{d-1} = v^{d-1} + \tilde{a} \left(\frac{p - \delta}{1 - \delta} \right) \lambda_H v^{d-1}.$$

It follows that informed traders do not buy whenever their highest possible valuation is below the market maker's valuation of the asset (plus tax) when only noise traders are active.⁵¹ This condition is met whenever

$$q_h(p) - \delta - (1 + \rho)\tilde{a}(p - \delta) < (1 - \delta) \frac{\rho}{\lambda_H}. \quad (29)$$

Analogous arguments establish the condition under which informed traders do not sell, assuming that noise traders continue to sell. This condition is given by

$$(1 - \rho)\tilde{a}(p - \delta) - (q_l(p) - \delta) < (1 - \delta) \frac{\rho}{\lambda_H}, \quad (30)$$

where $q_l(p)$ is the posterior probability an informed trader with the lowest possible signal, $S_t = 0$, assigns to the asset having increased in value, that is

$$q_l(p) = \frac{(1 - \tau)p}{(1 - \tau)p + (1 + \tau)(1 - p)}.$$

For $\tau \geq 1$ the proof that no informed trader trades is identical to that for noise traders. Now consider $\tau < 1$. At $t = 1$, we have $p = \Pr(V^d = v_H^d | V^d \neq v^{d-1}) = \delta$. Thus, from (29), the condition under which informed traders do not buy is given by

$$q_h(p) - \delta < (1 - \delta) \frac{\rho}{\lambda_H}.$$

⁵¹ Note that this latter valuation yields the lowest possible ask price a market maker could set given his priors. Any valuation that would put positive probability on an informed trader buying would result in a higher posterior valuation of the asset and, hence, a higher ask price.

This can be simplified to yield

$$\rho > \left[\frac{2\delta\tau}{1 - (1 - 2\delta)\tau} \right] \lambda_H.$$

Likewise, from (30) we obtain the condition under which informed traders do not sell at $t = 1$, which is

$$\delta - q_t(p) < (1 - \delta) \frac{\rho}{\lambda_H},$$

which is equivalent to

$$\rho > \left[\frac{2\delta\tau}{1 + (1 - 2\delta)\tau} \right] \lambda_H.$$

It follows that informed traders neither buy nor sell whenever

$$\begin{aligned} \rho > \rho^I &\equiv \max \left\{ \frac{2\delta \min\{\tau, 1\}}{1 - (1 - 2\delta) \min\{\tau, 1\}}, \frac{2\delta \min\{\tau, 1\}}{1 + (1 - 2\delta) \min\{\tau, 1\}} \right\} \lambda_H \\ &= \left(\frac{2\delta \min\{\tau, 1\}}{1 - \min\{\tau, 1\}|1 - 2\delta|} \right) \lambda_H. \end{aligned}$$

Proof of Corollary 1

A similar reasoning to that used for the proof of Proposition 1 shows that informed traders are active on both sides of the market for at least some signal realizations if and only if

$$\left[\min \left\{ \frac{(1 + \tau)\delta}{(1 + \tau)\delta + (1 - \tau)(1 - \delta)}, 1 \right\} \right] \frac{\delta}{1 - \delta} \lambda_H > \rho, \quad (31)$$

$$\left[-\max \left\{ \frac{(1 - \tau)\delta}{(1 - \tau)\delta + (1 + \tau)(1 - \delta)}, 1 \right\} \right] \frac{\delta}{1 - \delta} \lambda_H > \rho. \quad (32)$$

The two inequalities are jointly satisfied if and only if

$$\rho < \left(\frac{2 \min\{\tau, 1\} \delta}{1 + \min\{\tau, 1\}|2\delta - 1|} \right) \lambda_H.$$

Proof of Proposition 2

The process $\{\alpha_t^d\}_{t=1}^\infty$ is a bounded martingale under the filtration generated by the successive trades on day d . Hence, by the Martingale Convergence Theorem, α_t^d converges almost surely to a random variable.

Now consider a given day d , and suppose an event has occurred, $V^d \neq v^{d-1}$. First,

suppose there is a set of histories with positive measure for which α_t^d converges to a value in $(0, 1)$. Then for such trading histories, for any arbitrary value η , there exists a time T_η such that for all $t > T_\eta$, after observing any given action (buy, sell or no trade), the change in market maker's belief's about the occurrence of an event is lower than η . This means that the probabilities of observing any given action (buy, sell, or no trade) conditional on whether an event has occurred or not cannot differ by an amount larger than an arbitrary value depending on η otherwise the market maker's beliefs would change by a non-arbitrary amount, contradicting convergence. Hence, for instance, for a sell order in t it must be that

$$|(\mu F(\sigma_t^d|V^d) + (1 - \mu)\psi G(\gamma_t^d)) - \psi G(\gamma_t^d)| < \varepsilon^S(\eta),$$

that is,

$$|F(\sigma_t^d|V^d) - \psi G(\gamma_t^d)| < \frac{\varepsilon^S(\eta)}{\mu}, \quad (A1)$$

for both $V^d = v_L^d$ and $V^d = v_H^d$ and for a small $\varepsilon^S(\eta)$.

Similarly, for a buy order, it must be that

$$|(\mu(1 - F(\beta_t^d|V^d)) + (1 - \mu)\psi(1 - G(\kappa_t^d))) - \psi(1 - G(\kappa_t^d))| < \varepsilon^B(\eta),$$

that is,

$$|(1 - F(\beta_t^d|V^d)) - \psi(1 - G(\kappa_t^d))| < \frac{\varepsilon^B(\eta)}{\mu}, \quad (A2)$$

for both $V^d = v_L^d$ and $V^d = v_H^d$ and for a small $\varepsilon^B(\eta)$.

As the probability of selling by a noise trader cannot depend on the asset value, for equality (A1) to be satisfied, it must be that $|F(\sigma_t|V^d = v_L^d) - F(\sigma_t|V^d = v_H^d)|$ is arbitrarily close to zero. But this is only possible if informed traders sell for almost all signal realizations or for almost no signal realization.⁵² The same considerations apply to the buy order. Since, however, $\psi < 1$, the case in which informed traders sell for almost all signal realizations ($F(\sigma_t^d|V^d) = 1$) cannot satisfy (A1).⁵³ Suppose instead that an informed trader does not sell for almost any signal realization ($F(\sigma_t^d|V^d) = 0$); in this case, equality (A1) could be

⁵² For any σ_t in the interior, $|F(\sigma_t|V^d = v_L^d) - F(\sigma_t|V^d = v_H^d)|$ is bounded away from zero.

⁵³ Note that, even if $\psi = 1$, the same argument would hold. Indeed, since $\nu \geq 1$, there is always a shock such that a noise trader's valuation of the asset is v_H^d or v_L^d , that is, the probability of a noise trader buying (or selling) can never be arbitrarily close to 1.

satisfied if $G(\gamma_t^d) = 0$ (also noise traders do not sell for almost any signal realization). By the same logic, however, for (A2) to be also satisfied, noise traders must also not buy for almost any signal realizations ($G(\kappa_t^d) = 1$). If $G(\gamma_t^d) = 0$ and $G(\kappa_t^d) = 1$ noise traders would never trade (for almost all signal realization). This is, however, impossible, since noise traders find it optimal to buy (or sell) at time 1 for a positive measure of signal realizations ($\rho < \rho^N$) and, therefore, must find it optimal to buy (and to sell) at any time t at least a positive measure of signal realizations.

To conclude the proof, we must show that α_t^d cannot converge to 0. Suppose it did. Then, for instance, on a good-event day, the probability of a sell order would converge to

$$\mu F(\sigma_t^d | V^d = v_H^d) + (1 - \mu)\psi G(0.5),$$

since the market maker would set both bid and ask prices (approximately) equal to the unconditional expected value, v^{d-1} . This probability is different from that of a no-event day, equal to $\psi G(0.5)$. The same argument holds for a buy order and a no trade. Hence, at t goes to infinity, the market maker would learn that $\alpha_t^d \neq 0$. The same argument applies to a bad-event day.

Similar arguments prove the convergence of α_t^d to 0 when $V^d = v^{d-1}$.

Proof of Lemma 1

Treating the condition under which no informed trader buys as an equality yields a quadratic equation in p ,

$$q_h(p) - \delta - (1 + \rho)\tilde{a}(p - \delta) = (1 - \delta)\frac{\rho}{\lambda_H}, \quad (33)$$

where \tilde{a} , the market maker's posterior event probability after having observed a noise trader trade, is as defined before

$$\tilde{a}(a) = \frac{(1 - \mu)a}{1 - \mu a}.$$

If $\rho < \rho^I$, both roots are real and decreasing in the prior event probability a . The original inequality holds whenever p is higher than the larger root, denoted by p_b^1 , or lower than the smaller root, denoted by p_b^2 . Furthermore, for $\tau < 1$, the larger of the two roots tends to $+\infty$ as a goes to zero and is strictly smaller than 1 when $a = 1$. It follows that there exists a value $0 < a_h < 1$ such that for all $a > a_h$ informed traders stop buying whenever p is higher

than $p_b^1(a) < 1$. a_h is given by the solution to $p_b^1(a_h) = 1$, which yields

$$\tilde{a}(a_h) = \left(\frac{1}{1+\rho} \right) \left(\frac{\lambda_H - \rho}{\lambda_H} \right) < 1 \Rightarrow a_h < 1.$$

Similarly, treating the condition under which no informed trader sells as an equality yields a quadratic equation in p ,

$$(1-\rho)\tilde{a}(p-\delta) - (q_l(p) - \delta) = (1-\delta)\frac{\rho}{\lambda_H}. \quad (34)$$

Both roots are real for $\rho < \rho^I$ and increasing in a . Informed traders do not sell whenever p is higher than the larger root, denoted by p_s^1 , or lower than the smaller root, denoted by p_s^2 . The smaller of the two roots tends to $-\infty$ as a goes to zero and, if $\lambda_H < (1-\delta)/\delta$, is strictly above 0 for $a = 1$. Thus, there exists an event uncertainty a_l such that for all $a > a_l$ no informed trader sells whenever p is lower than $p_s^2(a) > 0$. a_l is given by the solution to $p_s^2(a_l) = 0$, which yields

$$\tilde{a}(a_l) = \left(\frac{1}{1-\rho} \right) \left[1 - \frac{(1-\delta)\rho}{\delta\lambda_H} \right] < 1 \Rightarrow a_l < 1.$$

Finally, let

$$\bar{a} = \max\{a_l, a_h\} \quad , \quad \delta_h(a) = \max\{p_b^1(a), p_b^2(a), p_s^1(a)p_s^2(a)\}$$

and

$$\delta_l(a) = \min\{p_b^1(a), p_b^2(a), p_s^1(a)p_s^2(a)\}.$$

The above arguments establish that $\bar{a} < 1$ and $0 < \delta_l(a) < \delta_h(a) < 1$ which concludes the proof.

Proof of Proposition 3

By Proposition 2 we know that on event days α_t^d converges to 1. By Lemma 1, we know that there are two thresholds $\delta_l(1)$ and $\delta_h(1)$ ($0 < \delta_l(1) < \delta_h(1) < 1$) such that informed traders do not trade for $\delta_t^d < \delta_l(1)$ or $\delta_t^d > \delta_h(1)$. Moreover, since δ_t^d is a bounded martingale, it must converge. If it converged to any value in $(\delta_l(1), \delta_h(1))$, informed traders would keep buying or selling for some signal realizations (since for $\alpha_t^d = 1$ there is never herding). Since informed traders trade only when their expected gain is greater than the tax paid, the market maker would update δ_t^d by an amount bounded away from zero, a contradiction.

Proof of Proposition 4

The proof of the proposition follows a similar logic as Cipriani and Guarino (2008). They compute the probability of convergence to a wrong fundamental in a Glosten and Milgrom (1985) model with traders' heterogeneous valuations; here, we extend the logic of their proof to a transaction tax and to a market with information event. We provide the proof for $V^d = v_H^d$. The proof for $V^d = v_L^d$ is analogous.

Step 1. *The likelihood ratio $\frac{\Pr(V^d = v_L^d | h_t^d)}{\Pr(V^d = v_H^d | h_t^d)}$ is a martingale conditional on $V^d = v_H^d$.*

Proof of step 1. Recall that $\delta_t^d =: \Pr(V^d = v_H^d | V^d \neq v^{d-1}, h_t^d)$. We can then write $\Pr(V^d = v_H^d | h_t^d) = \alpha_t^d \delta_t^d$ and $\Pr(V^d = v_L^d | h_t^d) = \alpha_t^d (1 - \delta_t^d)$. *I added footnote⁵⁴* We then have:

$$\frac{\Pr(V^d = v_L^d | h_t^d)}{\Pr(V^d = v_H^d | h_t^d)} = \frac{1 - \delta_t^d}{\delta_t^d}.$$

We must show that

$$\mathbb{E} \left(\frac{1 - \delta_{t+1}^d}{\delta_{t+1}^d} \mid h_t^d, V^d = v_H^d \right) = \frac{1 - \delta_t^d}{\delta_t^d}.$$

By Bayes' rule,

$$\begin{aligned} \mathbb{E} \left(\frac{1 - \delta_{t+1}^d}{\delta_{t+1}^d} \mid h_t^d, V^d = v_H^d \right) &= \mathbb{E} \left(\frac{\Pr(X_{t+1}^d = x_{t+1}^d | V^d = v_L^d, h_t^d)(1 - \delta_t^d)}{\Pr(X_{t+1}^d = x_{t+1}^d | V^d = v_H^d, h_t^d)\delta_t^d} \mid V^d = v_H^d \right) \\ &= \left(\frac{1 - \delta_t^d}{\delta_t^d} \right) \sum_{x_{t+1} \in \{\text{sell}, \text{nt}, \text{buy}\}} \Pr(X_{t+1}^d = x_{t+1}^d | V^d = v_H^d, h_t^d) \frac{\Pr(X_{t+1}^d | V^d = v_L^d, h_t^d)}{\Pr(X_{t+1}^d | V^d = v_H^d, h_t^d)} = \frac{1 - \delta_t^d}{\delta_t^d}. \end{aligned}$$

$$\begin{aligned} \mathbb{E} \left(\frac{1 - \delta_{t+1}^d}{\delta_{t+1}^d} \mid h_t^d, V^d = v_H^d \right) &= \mathbb{E} \left(\frac{\Pr(X_{t+1}^d = x_{t+1}^d | V^d = v_L^d, h_t^d)(1 - \delta_t^d)}{\Pr(X_{t+1}^d = x_{t+1}^d | V^d = v_H^d, h_t^d)\delta_t^d} \mid V^d = v_H^d \right) \\ &= \left(\frac{1 - \delta_t^d}{\delta_t^d} \right) \sum_{x_{t+1} \in \{\text{sell}, \text{nt}, \text{buy}\}} \left[\frac{\Pr(X_{t+1}^d | V^d = v_L^d, h_t^d)}{\Pr(X_{t+1}^d | V^d = v_H^d, h_t^d)} \Pr(X_{t+1}^d = x_{t+1}^d | V^d = v_H^d, h_t^d) \right] = \frac{1 - \delta_t^d}{\delta_t^d}. \end{aligned}$$

⁵⁴ The expressions are true because $\Pr(V^d = v_H^d | V^d = v^{d-1}, h_t^d) = \Pr(V^d = v_H^d | V^d = v^{d-1}, h_t^d) = 0$.

Step 2. *The probability that the belief $\Pr(V^d = v_H^d | h_t^d)$ remains stuck at a value lower than $\delta_l(1)$ on an good event day ($V^d = v_H^d$) is bounded above by*

$$\frac{\delta_l(1)[\delta_h(1) - \delta]}{\delta[\delta_h(1) - \delta_l(1)]}.$$

Proof of step 2. Consider the case in which $\alpha_t^d = 1$. Consider the time T at which informed traders stop trading (as defined in Proposition 3). Since the likelihood ratio $\frac{\Pr(V^d = v_L^d | h_t^d)}{\Pr(V^d = v_H^d | h_t^d)}$ is a martingale conditional on $V^d = v_H^d$, we have that

$$\mathbb{E} \left(\frac{1 - \delta_T^d}{\delta_T^d} \right) = \frac{1 - \delta}{\delta}.$$

As the posterior beliefs δ_t^d have to converge to either $\delta_h(1)$ or $\delta_l(1)$ we have

$$\mathbb{E} \left(\frac{1 - \delta_T^d}{\delta_T^d} \right) \approx (\Pr(\delta_T^d = \delta_h(1) | h_T^d)) \left(\frac{1 - \delta_h(1)}{\delta_h(1)} \right) + (1 - \Pr(\delta_T^d = \delta_h(1) | h_T^d)) \left(\frac{1 - \delta_l(1)}{\delta_l(1)} \right),$$

from which we obtain that

$$\Pr(\delta_T^d = \delta_h(1) | h_T^d) \approx \frac{\delta_h(1)[\delta - \delta_l(1)]}{\delta[\delta_h(1) - \delta_l(1)]},$$

and

$$\Pr(\delta_T^d = \delta_l(1) | h_T^d) \approx \frac{\delta_l(1)[\delta_h(1) - \delta]}{\delta[\delta_h(1) - \delta_l(1)]}.$$

Finally, note that since for $\alpha_t^d < 1$, $\delta_h(\alpha_t^d)$ can be higher than $\delta_h(1)$ and $\delta_l(\alpha_t^d)$ can be lower than $\delta_l(1)$, informed traders may stop trading before α_t^d has converged to 1. Hence $\Pr(\delta_T^d = \delta_l(1) | h_T^d)$ is only bounded from above by the expression derived assuming $\alpha_T^d = 1$.

B. Calibration of the Transaction Tax

In this section we explain how we calibrate the size of the increase of the asset value, λ^H , on good event days. The parameters λ^H and λ^L in our model refer to the upward and downward movements of asset values that are due to private information held by informed traders. However, not all observed stock price movements are due to informed trading. Other possible sources are public news announcements. To calibrate λ^H using stock price data, we need to isolate the variability in stock prices that is due to informed trading. To do

so, we use the variance decomposition proposed by Hasbrouck (1991) that decomposes price changes into a trade-correlated component, interpreted as the component driven by private information, and a trade-uncorrelated component.

We calculate Hasbrouck's variance decomposition using intraday quote updates, expressed as changes in the logarithm of the mid-quote, and all trades that occur during continuous trading. We use a lag length of 500 for the bivariate VAR. Denote the estimate of this variance for stock s by $\hat{\sigma}_p^2(s)$. A fraction $R_w^2(s)$ of this variance is related to trading based on private information, where we obtain the estimate $\hat{R}_w^2(s)$ from the Hasbrouck decomposition for stock s . The values for Hasbrouck's $\hat{R}_w(s)$ measure for all 15 stocks are reported in Table 4 of the main text. For our median stock, for example, the decomposition implies that approximately 6.5% of the variance of intraday changes in the price is due to private information based trading.

Next we note that in our model, daily log-price changes can be (approximately) related to λ^H via

$$\ln p_T^d - \ln p_1^d \approx \frac{\Delta p^d}{p_1^d} = \lambda^d \text{ where } \lambda^d \in \{\lambda^L, 0, \lambda^H\} \text{ and } \lambda^L = -\frac{\delta}{1-\delta}\lambda^H,$$

where p_1^d is the opening price on trading day d , p_T^d the closing price and $\Delta p^d = p_T^d - p_1^d$. It follows that the variance of daily log-price changes in our model is approximately

$$Var\left(\ln \frac{p_T^d}{p_1^d}\right) \approx \alpha\delta (\lambda^H)^2 + \alpha(1-\delta) \left(\frac{\delta}{1-\delta}\right)^2 (\lambda^H)^2 = \left(\frac{\alpha\delta}{1-\delta}\right) (\lambda^H)^2.$$

This then allows us to calibrate λ^H by equating the model-implied variance of log-price changes to $R_w^2(s)\sigma_p^2(s)$, that is

$$\left(\frac{\hat{\alpha}(s)\hat{\delta}(s)}{1-\hat{\delta}(s)}\right) \hat{\lambda}^H(s)^2 = \hat{R}_w^2(s)\hat{\sigma}_p^2(s),$$

from which we obtain

$$\hat{\lambda}^H(s) = \hat{\sigma}_p(s) \sqrt{\left(\frac{1-\hat{\delta}(s)}{\hat{\alpha}(s)\hat{\delta}(s)}\right) \hat{R}_w^2(s)}.$$

Table 4 reports the calibrated value for λ_H by stock.

C. Additional Estimation Results

Table C.1: **Parameter estimates with outlier days:** Parameter estimates for estimations in which days classified as outliers due to high trading activity are not removed from the sample.

	α	δ	μ	τ	ν	ψ
Acotel	0.43	0.57	0.24	0.27	1.29	0.12
B&C Speakers	0.06	0.91	0.55	0.64	1.89	0.06
Bolzoni	0.42	0.92	0.29	0.22	1.23	0.16
Cembre	0.52	0.23	0.62	0.42	1.03	0.07
Elen	0.39	0.41	0.31	0.30	1.28	0.23
Elica	0.32	0.60	0.35	0.13	1.45	0.22
Exprivia	0.41	0.83	0.24	0.33	1.19	0.20
Gefran	0.31	0.57	0.12	0.33	1.26	0.06
Landi	0.59	0.66	0.18	1.96	1.52	0.66
Nice	0.48	0.46	0.16	0.58	1.06	0.08
Panaria	0.40	0.67	0.15	0.39	1.23	0.14
Poltrona	1.00	0.88	0.92	0.41	1.92	0.64
Prima	0.38	0.18	0.14	0.12	1.44	0.09
Sabaf	0.42	0.30	0.35	0.28	1.28	0.20
Ternienergia	0.33	0.91	0.35	0.04	1.61	0.18
Mean	0.43	0.61	0.33	0.43	1.38	0.21
Median	0.41	0.60	0.29	0.33	1.28	0.16
Standard deviation	0.20	0.25	0.22	0.45	0.26	0.19

Table C.2: **Parameter estimates for 1 and 3 minutes no-trade interval:** Parameter estimates for estimations which use a time interval of 3 minutes to determine the number of “no trades” in the sample.

NT time	1min						3min					
	α	δ	μ	τ	ν	ψ	α	δ	μ	τ	ν	ψ
Acotel	0.38	0.60	0.05	0.23	1.28	0.02	0.32	0.63	0.15	0.23	1.29	0.07
B&C Speakers	0.30	0.89	0.04	0.54	1.07	0.01	0.35	0.94	0.10	0.57	1.09	0.04
Bolzoni	0.43	0.92	0.08	0.17	1.21	0.03	0.39	0.88	0.20	0.19	1.22	0.09
Cembre	0.48	0.37	0.03	0.50	1.15	0.01	0.49	0.38	0.10	0.53	1.13	0.04
Elen	0.49	0.60	0.07	0.29	1.16	0.04	0.43	0.68	0.21	0.28	1.27	0.13
Elica	0.38	0.53	0.13	0.11	1.11	0.04	0.46	0.78	0.25	0.07	1.46	0.11
Exprivia	0.39	0.96	0.05	0.25	1.32	0.03	0.43	0.88	0.14	0.33	1.23	0.10
Gefran	0.35	0.71	0.02	0.05	1.64	0.01	0.38	0.51	0.06	0.06	1.60	0.03
Landi	0.70	0.66	0.30	0.36	1.19	0.13	0.46	0.34	0.18	1.97	1.99	0.34
Nice	0.49	0.34	0.03	0.63	1.02	0.01	0.53	0.37	0.09	0.64	1.02	0.04
Panaria	0.41	0.82	0.03	0.33	1.21	0.02	0.42	0.83	0.09	0.33	1.21	0.07
Poltrona	0.16	0.70	0.08	2.00	2.00	0.06	0.54	0.62	0.18	0.66	1.00	0.20
Prima	0.37	0.10	0.03	0.08	1.54	0.02	0.37	0.18	0.08	0.09	1.51	0.05
Sabaf	0.37	0.48	0.09	0.34	1.18	0.04	0.47	0.34	0.24	0.38	1.15	0.12
Ternienergia	0.24	0.84	0.08	0.00	1.72	0.03	0.30	0.78	0.24	0.00	1.99	0.10
Mean	0.40	0.64	0.07	0.39	1.32	0.03	0.42	0.61	0.15	0.42	1.34	0.10
Median	0.38	0.66	0.05	0.29	1.21	0.03	0.43	0.63	0.15	0.33	1.23	0.09
StDev	0.12	0.24	0.07	0.48	0.28	0.03	0.07	0.24	0.06	0.48	0.31	0.08

Table C.3: **Test of hypotheses.** Results of various hypothesis tests. The first six columns are Wald tests, the last two columns are LR tests. CG refers to Cipriani and Guarino (2014) and EKO refers to Easley *et al.* (1997). H_0 specifies the null hypothesis, H_1 the alternative. The number of rejections (i.e. number of stocks for which H_0 is rejected) is based on a confidence level of 1%.

H0	$\tau = 2$	$\tau = 1$	$\nu = 2$	$\nu = 1$	$\delta = 0.5$	$\alpha = 1$	CG	EKO
Acotel	0.00	0.00	0.00	0.00	0.48	0.00	0.00	0.00
B&C Speakers	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bolzoni	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cembre	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00
Elen	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00
Elica	0.00	0.00	0.00	0.00	0.26	0.00	0.00	0.00
Exprivia	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Gefran	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.00
Landi	0.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
Nice	0.00	0.00	0.00	0.43	0.07	0.00	0.00	0.00
Panaria	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Poltrona	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00
Prima	0.00	0.00	0.00	0.00	0.19	0.00	0.00	0.00
Sabaf	0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.00
Ternienergia	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H1	<	<	<	>	\neq	<	\neq	\neq
Rejections of H0	15	14	15	14	6	15	15	15

Table C.4: **Belief thresholds and probabilities of wrong convergence.** Thresholds $\delta_l(1)$ and $\delta_u(1)$ as well as the probabilities of wrong convergence as given in Proposition 4 by tax rate.

stock	$\delta_l(1)$	$\delta_u(1)$	$\Pr(\delta_l v_H^d)$	$\Pr(\delta_u v_L^d)$	$\delta_l(1)$	$\delta_u(1)$	$\Pr(\delta_l v_H^d)$	$\Pr(\delta_u v_L^d)$	$\delta_l(1)$	$\delta_u(1)$	$\Pr(\delta_l v_H^d)$	$\Pr(\delta_u v_L^d)$
tax (bps)	5				10				20			
Acotel	0.05	0.95	0.05	0.05	0.12	0.88	0.12	0.12	0.12	0.88	0.12	0.12
BC & Speakers	0.001	1.00	0	0.29	0.002	1.00	0	0.59	0	0.59	0	0.59
Bolzoni	0.03	0.97	0.003	0.22	0.06	0.94	0.004	0.48	0.06	0.94	0.004	0.48
Cembre	0.05	0.95	0.09	0.03	0.12	0.88	0.22	0.06	0.12	0.88	0.22	0.06
Elen	0.06	0.94	0.02	0.13	0.13	0.87	0.05	0.30	0.13	0.87	0.05	0.30
Elica	0.01	0.99	0.01	0.01	0.02	0.98	0.02	0.03	0.05	0.95	0.04	0.06
Exprivia	0.07	0.93	0.002	0.75								
Gefran	0.15	0.85	0.15	0.15								
Landi	0	1	0	0	0	1	0	0	0	1	0	0
Nice	0.02	0.98	0.03	0.01	0.04	0.96	0.06	0.02	0.09	0.91	0.16	0.05
Panaria	0.01	0.99	0.002	0.05	0.02	0.98	0.003	0.11	0.04	0.96	0.01	0.23
Poltrona	0.10	0.90	0.06	0.14								
Prima	0.10	0.90	0.51	0.01								
Sabaf	0.04	0.96	0.06	0.03	0.09	0.91	0.13	0.06	0.24	0.76	0.40	0.14
Ternienergia												
Mean	0.05	0.95	0.07	0.13	0.06	0.94	0.06	0.18	0.08	0.91	0.12	0.10
Median	0.04	0.96	0.03	0.05	0.05	0.95	0.03	0.09	0.05	0.95	0.04	0.06
StDev	0.04	0.04	0.13	0.20	0.05	0.05	0.07	0.21	0.09	0.09	0.17	0.09

The table shows the same statistics as Table 6 at the stock level

Table C.5: **Proportion of informed traders on event days:** Average proportion of trades by informed traders on event days by tax rate. The simulations are based on 100,000 trading days with 190 trading dates each.

	0bps	5bps	10bps	20bps
Acotel	0.65	0.51	0.37	0
B&C Speakers	0.68	0.42	0.28	0
Bolzoni	0.63	0.46	0.35	0
Cembre	0.67	0.48	0.33	0
Elen	0.59	0.45	0.33	0
Elica	0.52	0.47	0.44	0.38
Exprivia	0.55	0.07	0	0
Gefran	0.62	0.31	0	0
Landi	0.30	0.34	0.37	0.35
Nice	0.66	0.53	0.48	0.36
Panaria	0.54	0.48	0.44	0.38
Poltrona	0.53	0.26	0	NA
Prima	0.56	0.20	0	0
Sabaf	0.65	0.49	0.39	0.13
Ternienergia	0.63	0	0	0
Mean	0.58	0.36	0.25	0.11
Median	0.62	0.45	0.33	0

Table C.6: **Trade volume.** Average daily number of trades by type of event and tax rate. Simulated data for 100,000 trading days with 190 trading dates (simulation standard errors in parentheses).

tax (in bps)	all days				event				no event			
	0	5	10	20	0	5	10	20	0	5	10	20
Acotel	28.09 (0.04)	14.32 (0.03)	11.71 (0.02)	7.80 (0.01)	41.02 (0.03)	24.63 (0.03)	14.74 (0.02)	6.70 (0.01)	18.76 (0.02)	6.87 (0.02)	9.53 (0.02)	8.60 (0.01)
B&C Speakers	14.85 (0.02)	1.46 (0.01)	1.17 (0.005)	1.02 (0.003)	24.73 (0.03)	2.17 (0.02)	1.36 (0.01)	0.90 (0.01)	10.55 (0.01)	1.15 (0.005)	1.09 (0.004)	1.08 (0.004)
Bolzoni	34.84 (0.04)	16.92 (0.04)	13.99 (0.02)	10.06 (0.01)	50.01 (0.03)	28.72 (0.04)	16.72 (0.03)	8.55 (0.01)	25.45 (0.02)	9.61 (0.03)	12.31 (0.02)	10.99 (0.01)
Cembre	16.41 (0.03)	6.36 (0.01)	4.12 (0.01)	3.20 (0.01)	24.99 (0.02)	9.45 (0.02)	4.77 (0.01)	2.89 (0.01)	9.44 (0.01)	3.85 (0.01)	3.60 (0.01)	3.45 (0.01)
Elen	45.25 (0.04)	23.23 (0.04)	22.36 (0.02)	18.25 (0.01)	62.39 (0.03)	38.63 (0.04)	25.30 (0.03)	14.74 (0.01)	36.31 (0.02)	15.20 (0.04)	20.83 (0.02)	20.08 (0.02)
Elica	45.59 (0.04)	29.33 (0.06)	26.86 (0.05)	23.73 (0.03)	58.27 (0.03)	47.12 (0.04)	40.83 (0.04)	31.41 (0.03)	33.96 (0.02)	13.01 (0.04)	14.04 (0.04)	16.67 (0.03)
Exprivia	34.21 (0.04)	12.58 (0.01)	6.08 (0.01)	6.08 (0.01)	49.65 (0.03)	11.35 (0.02)	5.14 (0.01)	5.14 (0.01)	26.48 (0.02)	13.20 (0.01)	6.55 (0.01)	6.55 (0.01)
Gefran	12.06 (0.02)	7.69 (0.01)	6.31 (0.01)	6.31 (0.01)	19.55 (0.02)	9.26 (0.02)	5.90 (0.01)	5.89 (0.01)	7.62 (0.01)	6.76 (0.01)	6.56 (0.01)	6.56 (0.01)
Landi	101.86 (0.03)	10.56 (0.03)	7.35 (0.02)	3.96 (0.01)	106.05 (0.02)	11.14 (0.03)	8.27 (0.02)	4.58 (0.01)	88.14 (0.05)	8.67 (0.05)	4.35 (0.03)	1.94 (0.02)
Nice	15.77 (0.02)	3.69 (0.01)	1.90 (0.01)	0.51 (0.003)	22.64 (0.02)	6.49 (0.02)	3.06 (0.01)	0.64 (0.01)	8.73 (0.01)	0.82 (0.01)	0.70 (0.01)	0.37 (0.004)
Panaria	24.32 (0.03)	15.32 (0.04)	13.45 (0.03)	11.08 (0.02)	35.64 (0.03)	26.91 (0.03)	21.68 (0.03)	15.33 (0.03)	16.69 (0.01)	7.51 (0.02)	7.90 (0.02)	8.22 (0.02)
Poltrona	59.56 (0.04)	11.58 (0.01)	7.53 (0.01)	0 (0)	69.65 (0.03)	12.12 (0.02)	6.45 (0.01)	0 (0)	46.77 (0.02)	10.90 (0.02)	8.90 (0.01)	0 (0)
Prima	21.12 (0.03)	12.69 (0.01)	11.06 (0.01)	11.03 (0.01)	31.73 (0.03)	13.67 (0.03)	10.05 (0.02)	10.01 (0.02)	15.03 (0.01)	12.13 (0.01)	11.64 (0.01)	11.62 (0.01)
Sabaf	44.53 (0.04)	17.39 (0.05)	14.56 (0.02)	9.94 (0.01)	58.15 (0.03)	31.16 (0.03)	20.36 (0.02)	9.35 (0.01)	33.79 (0.02)	6.54 (0.03)	9.99 (0.03)	10.41 (0.01)
Ternienergia	39.86 (0.04)	22.61 (0.02)	11.30 (0.01)	11.30 (0.01)	57.87 (0.03)	16.89 (0.02)	8.47 (0.02)	8.47 (0.02)	31.79 (0.02)	25.18 (0.02)	12.57 (0.01)	12.57 (0.01)

The table shows the same statistics as Table 7 at the stock level.

Table C.7: **Liquidity: Bid-Ask Spread.** Average daily bid-ask spread by type of event and tax rate. Simulated data for 100,000 trading days with 190 trading dates (Simulation standard errors in parentheses).

tax	all days						event						no event					
	0	5	10	20	0	5	10	20	0	5	10	20	0	5	10	20	0	20
Acotel	4.88 (0.01)	5.58 (0.01)	4.94 (0.01)	0 (0)	9.40 (0.01)	8.76 (0.01)	5.76 (0.01)	0 (0)	1.62 (0.01)	3.28 (0.01)	4.34 (0.01)	0 (0)	1.62 (0.01)	3.28 (0.01)	4.34 (0.01)	0 (0)	1.62 (0.01)	3.28 (0.01)
B&C Speakers	0.20 (0.002)	0.18 (0.002)	0.13 (0.001)	0 (0)	0.46 (0.01)	0.31 (0.01)	0.20 (0.004)	0 (0)	0.09 (0.001)	0.12 (0.001)	0.10 (0.001)	0 (0)	0.09 (0.001)	0.12 (0.001)	0.10 (0.001)	0 (0)	0.09 (0.001)	0.12 (0.001)
Bolzoni	1.61 (0.01)	1.75 (0.01)	1.50 (0.003)	0 (0)	3.51 (0.01)	2.96 (0.01)	1.86 (0.01)	0 (0)	0.43 (0.002)	1.01 (0.004)	1.27 (0.002)	0 (0)	0.43 (0.002)	1.01 (0.004)	1.27 (0.002)	0 (0)	0.43 (0.002)	1.01 (0.004)
Cembre	7.85 (0.02)	10.74 (0.01)	7.76 (0.01)	0 (0)	12.50 (0.02)	12.46 (0.02)	7.88 (0.01)	0 (0)	4.08 (0.01)	9.34 (0.02)	7.66 (0.01)	0 (0)	4.08 (0.01)	9.34 (0.02)	7.66 (0.01)	0 (0)	4.08 (0.01)	9.34 (0.02)
Elen	2.64 (0.01)	2.89 (0.01)	2.53 (0.004)	0 (0)	6.29 (0.01)	5.37 (0.01)	3.44 (0.01)	0 (0)	0.74 (0.004)	1.60 (0.01)	2.06 (0.004)	0 (0)	0.74 (0.004)	1.60 (0.01)	2.06 (0.004)	0 (0)	0.74 (0.004)	1.60 (0.01)
Elica	5.07 (0.01)	5.41 (0.01)	5.68 (0.01)	5.92 (0.01)	8.18 (0.01)	8.33 (0.01)	8.31 (0.01)	7.63 (0.01)	2.21 (0.01)	2.73 (0.01)	3.27 (0.01)	4.35 (0.01)	2.21 (0.01)	2.73 (0.01)	3.27 (0.01)	4.35 (0.01)	2.21 (0.01)	2.73 (0.01)
Exprivia	1.26 (0.01)	0.24 (0.002)	0 (0)	0 (0)	2.89 (0.01)	0.31 (0.004)	0 (0)	0 (0)	0.45 (0.002)	0.20 (0.001)	0 (0)	0 (0)	0.45 (0.002)	0.20 (0.001)	0 (0)	0 (0)	0.45 (0.002)	0.20 (0.001)
Gefran	1.26 (0.003)	0.69 (0.002)	0 (0)	0 (0)	2.39 (0.002)	0.93 (0.003)	0 (0)	0 (0)	0.60 (0.002)	0.54 (0.002)	0 (0)	0 (0)	0.60 (0.002)	0.54 (0.002)	0 (0)	0 (0)	0.60 (0.002)	0.54 (0.002)
Landi	5.24 (0.01)	8.34 (0.03)	8.45 (0.03)	8.72 (0.03)	4.62 (0.01)	4.01 (0.01)	4.32 (0.01)	4.88 (0.01)	7.28 (0.02)	22.52 (0.03)	21.96 (0.03)	21.32 (0.05)	7.28 (0.02)	22.52 (0.03)	21.96 (0.03)	21.32 (0.05)	7.28 (0.02)	22.52 (0.03)
Nice	9.62 (0.02)	30.82 (0.03)	29.77 (0.02)	15.95 (0.003)	13.30 (0.03)	25.55 (0.04)	27.53 (0.03)	15.95 (0.005)	5.85 (0.02)	36.22 (0.02)	32.06 (0.02)	15.95 (0.004)	5.85 (0.02)	36.22 (0.02)	32.06 (0.02)	15.95 (0.004)	5.85 (0.02)	36.22 (0.02)
Panaria	2.99 (0.01)	3.32 (0.01)	3.54 (0.01)	3.62 (0.01)	5.27 (0.01)	5.35 (0.01)	5.21 (0.01)	4.52 (0.01)	1.44 (0.01)	1.96 (0.01)	2.41 (0.01)	3.01 (0.01)	1.44 (0.01)	1.96 (0.01)	2.41 (0.01)	3.01 (0.01)	1.44 (0.01)	1.96 (0.01)
Poltrona	5.38 (0.01)	8.84 (0.01)	0 (0)	0 (0)	7.33 (0.02)	8.45 (0.01)	0 (0)	0 (0)	2.90 (0.01)	9.34 (0.01)	0 (0)	0 (0)	2.90 (0.01)	9.34 (0.01)	0 (0)	0 (0)	2.90 (0.01)	9.34 (0.01)
Prima	0.79 (0.002)	0.34 (0.001)	0 (0)	0 (0)	1.59 (0.002)	0.43 (0.002)	0 (0)	0 (0)	0.33 (0.001)	0.28 (0.001)	0 (0)	0 (0)	0.33 (0.001)	0.28 (0.001)	0 (0)	0 (0)	0.33 (0.001)	0.28 (0.001)
Sabaf	4.59 (0.01)	5.71 (0.01)	6.32 (0.01)	2.07 (0.004)	8.54 (0.02)	8.79 (0.02)	7.73 (0.01)	2.02 (0.01)	1.48 (0.01)	3.28 (0.01)	5.20 (0.01)	2.11 (0.01)	1.48 (0.01)	3.28 (0.01)	5.20 (0.01)	2.11 (0.01)	1.48 (0.01)	3.28 (0.01)
Ternienergia	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)

The table shows the same statistics as Table 9 at the stock level.

Table C.8: **Price Volatility.** Average standard deviation of prices (intraday) by type of event and tax rate. Simulated data for 100,000 trading days with 190 trading dates (simulation standard errors in parentheses).

tax	all days					event					no event				
	0	5	10	20		0	5	10	20		0	5	10	20	
Acotel	6.10 (0.02)	5.31 (0.02)	4.06 (0.01)	0		13.19 (0.02)	10.32 (0.02)	6.43 (0.01)	0		0.99 (0.01)	1.70 (0.01)	2.35 (0.01)	0	
B&C Speakers	0.15 (0.003)	0.08 (0.002)	0.05 (0.002)	0		0.44 (0.01)	0.19 (0.01)	0.10 (0.01)	0		0.03 (0.001)	0.03 (0.001)	0.03 (0.0005)	0	
Bolzoni	2.15 (0.01)	1.80 (0.01)	1.24 (0.01)	0		5.10 (0.02)	3.75 (0.02)	2.06 (0.01)	0		0.33 (0.003)	0.59 (0.004)	0.73 (0.003)	0	
Cembre	7.31 (0.02)	5.92 (0.02)	3.17 (0.01)	0		14.54 (0.02)	9.64 (0.02)	3.94 (0.01)	0		1.44 (0.01)	2.90 (0.01)	2.54 (0.01)	0	
Elen	4.14 (0.02)	3.66 (0.01)	2.94 (0.01)	0		10.78 (0.02)	8.33 (0.02)	5.44 (0.02)	0		0.67 (0.01)	1.23 (0.01)	1.63 (0.01)	0	
Elica	8.02 (0.02)	7.91 (0.02)	7.72 (0.02)	7.13		14.67 (0.02)	14.17 (0.02)	13.46 (0.02)	11.43		1.93 (0.01)	2.17 (0.01)	2.46 (0.01)	3.18	
Exprivia	1.67 (0.01)	0.28 (0.003)	0	0		4.32 (0.02)	0.43 (0.01)	0	0		0.35 (0.003)	0.20 (0.002)	0	0	
Gefran	0.96 (0.004)	0.45 (0.002)	0	0		2.15 (0.01)	0.71 (0.004)	0	0		0.25 (0.001)	0.29 (0.001)	0	0	
Landi	13.88 (0.02)	11.32 (0.02)	10.34 (0.02)	8.23		14.72 (0.02)	13.17 (0.02)	11.93 (0.02)	9.76		11.11 (0.03)	5.27 (0.02)	5.13 (0.02)	3.20	
Nice	8.80 (0.03)	7.14 (0.02)	4.57 (0.02)	1.46		15.39 (0.03)	11.83 (0.02)	6.69 (0.02)	1.71		2.05 (0.01)	2.35 (0.02)	2.39 (0.02)	1.21	
Panaria	3.27 (0.01)	3.19 (0.01)	3.02 (0.01)	2.58		6.84 (0.02)	6.46 (0.02)	5.82 (0.02)	4.37		0.87 (0.01)	0.99 (0.01)	1.14 (0.01)	1.37	
Poltrona	10.49 (0.03)	6.28 (0.01)	0	0		16.43 (0.02)	7.01 (0.01)	0	0		2.97 (0.02)	5.35 (0.01)	0	0	
Prima	0.80 (0.003)	0.30 (0.001)	0	0		1.83 (0.005)	0.46 (0.003)	0	0		0.20 (0.001)	0.21 (0.001)	0	0	
Sabaf	7.55 (0.03)	6.81 (0.02)	5.99 (0.02)	1.48		15.59 (0.02)	12.94 (0.02)	9.84 (0.02)	1.75		1.21 (0.01)	1.97 (0.01)	2.96 (0.01)	1.27	
Temiennergia	0 (0)	0 (0)	0 (0)	0		0 (0)	0 (0)	0 (0)	0		0 (0)	0 (0)	0 (0)	0	

The table shows the same statistics as Table 10 at the stock level.

Table C.9: **Informational Efficiency.** Average daily mean absolute error of prices, $|p_t^d - v^d|$, by type of event and tax rate. Simulated data for 100,000 trading days with 190 trading dates (simulation standard errors in parentheses).

tax	all days				event				no event			
	0	5	10	20	0	5	10	20	0	5	10	20
Acotel	15.72 (0.02)	18.10 (0.02)	20.79 (0.01)	20.96 0	36.77 (0.02)	41.13 (0.02)	46.14 (0.01)	50 (0)	0.53 (0.01)	1.48 (0.01)	2.49 (0.01)	0 (0)
B&C Speakers	0.22 (0.003)	0.31 (0.002)	0.29 (0.002)	0.26 (0)	0.69 (0.01)	0.88 (0.01)	0.86 (0.01)	0.85 (0)	0.02 (0.001)	0.06 (0.001)	0.04 (0.0005)	0 (0)
Bolzoni	7.25 (0.01)	7.83 (0.01)	8.41 (0.01)	8.06 (0)	18.68 (0.02)	19.67 (0.02)	20.82 (0.01)	21.10 (0)	0.17 (0.003)	0.51 (0.004)	0.73 (0.003)	0 (0)
Cembre	13.03 (0.02)	18.66 (0.02)	21.92 (0.01)	20.90 (0)	27.94 (0.02)	37.43 (0.02)	44.70 (0.01)	46.65 (0)	0.92 (0.01)	3.41 (0.01)	3.42 (0.01)	0 (0)
Elen	11.94 (0.02)	13.41 (0.01)	14.88 (0.01)	14.86 (0)	34.18 (0.02)	37.24 (0.02)	40.65 (0.02)	43.36 (0)	0.34 (0.01)	0.99 (0.01)	1.45 (0.01)	0 (0)
Elica	15.81 (0.02)	16.26 (0.02)	17.02 (0.02)	19.31 (0.02)	31.77 (0.02)	32.34 (0.02)	33.45 (0.02)	36.92 (0.02)	1.17 (0.01)	1.51 (0.01)	1.95 (0.01)	3.15 (0.02)
Exprivia	4.94 (0.01)	5.65 (0.003)	5.48 (0)	5.48 (0)	14.41 (0.02)	16.44 (0.01)	16.43 (0)	16.43 (0)	0.20 (0.003)	0.26 (0.002)	0 (0)	0 (0)
Gefran	18.61 (0.004)	18.84 (0.002)	18.60 (0)	18.60 (0)	49.66 (0.01)	49.92 (0.004)	50 (0)	50 (0)	0.22 (0.001)	0.43 (0.001)	0 (0)	0 (0)
Landi	11.19 (0.02)	4.96 (0.02)	4.83 (0.02)	5.96 (0.02)	11.40 (0.02)	5.57 (0.02)	5.12 (0.02)	5.30 (0.02)	10.50 (0.03)	2.94 (0.02)	3.86 (0.02)	8.15 (0.03)
Nice	13.10 (0.03)	19.50 (0.02)	23.07 (0.02)	24.40 (0.01)	24.57 (0.03)	35.37 (0.02)	42.13 (0.02)	46.49 (0.01)	1.35 (0.01)	3.26 (0.02)	3.55 (0.02)	1.77 (0.01)
Panaria	9.06 (0.01)	9.29 (0.01)	9.72 (0.01)	10.63 (0.01)	21.61 (0.02)	21.89 (0.02)	22.57 (0.02)	24.15 (0.02)	0.61 (0.01)	0.80 (0.01)	1.06 (0.01)	1.53 (0.01)
Poltrona	12.17 (0.03)	27.12 (0.01)	27.07 (0)	27.07 (0)	20.46 (0.02)	42.62 (0.01)	48.42 (0.02)	48.42 (0.02)	1.67 (0.01)	7.45 (0.01)	0 (0.01)	0 (0.01)
Prima	10.34 (0.003)	10.55 (0.001)	10.33 0	10.33 0	28.08 (0.005)	28.41 (0.003)	28.33 (0)	28.33 (0)	0.15 (0.001)	0.30 (0.001)	0 (0)	0 (0)
Sabaf	12.02 (0.03)	15.13 (0.02)	18.89 (0.02)	21.82 (0.01)	26.55 (0.02)	32.20 (0.02)	39.05 (0.02)	47.74 (0.01)	0.57 (0.01)	1.67 (0.01)	3 (0.01)	1.38 (0.01)
Temiennergia	0.43 (0)	0.43 (0)	0.43 (0)	0.43 (0)	1.39 (0)	1.39 (0)	1.39 (0)	1.39 (0)	0 (0)	0 (0)	0 (0)	0 (0)

The table shows the same statistics as Table 11 at the stock level.