

Aufgabe 1) $\{true\} \Rightarrow \{7+3=10\} x:=7 \{x+3=10\} y:=x+3 \{y=10\}$

Aufgabe 2)
$$C \left\{ \begin{array}{l} \text{read } x; \\ \text{if } x > 0 \text{ output } 1 \text{ else} \\ \text{if } x = 0 \text{ output } 0 \text{ else output } -1 \end{array} \right\} C_1$$

Behauptung: $\{input = (n), output = ()\} \subset \{output = (signum\ n)\}$

Beweis: $\{input = (n), output = ()\} \Rightarrow$

$\{hd(input) = n, output = ()\} \text{ read } x \{x = n, output = ()\}$

N1 $\{x > 0, x = n, output = ()\} \Rightarrow$

$\{n > 0, output.1 = ()\} \text{ output } 1 \{n > 0, output = (1)\} \Rightarrow$
 $\{output = (signum\ n)\}$

N2 $\{\neg x > 0, x = 0, x = n, output = ()\} \Rightarrow$

$\{n = 0, output.0 = ()\} \text{ output } 0 \{n = 0, output = (0)\} \Rightarrow$
 $\{output = (signum\ n)\}$

N3 $\{\neg x > 0, \neg x = 0, x = n, output = ()\} \Rightarrow$

$\{n < 0, output.-1 = (-1)\} \text{ output } -1 \{n < 0, output = (-1)\} \Rightarrow$
 $\{output = (signum\ n)\}$

da die Schlussregel für die bed. Anweisung folgt:

$\{x = n, output = ()\} \subset \{output = (signum\ n)\}$

Q.E.D.

Aufgabe 3

$$C \left\{ \begin{array}{l} S := 0; \\ \text{while not eof do} \\ \quad \text{read } x; \\ \quad S := S + x; \end{array} \right\}_{C_2} \quad C_1$$

output s

Behauptung:

$$\{\text{input} = (a_1, \dots, a_k), \text{output} = ()\} \subset \{\text{output} = (\sum_{i=1}^k a_i)\}$$

$$\{\text{input} = (a_1, \dots, a_k), \text{output} = ()\} \Rightarrow \{\text{input} = (a_1, \dots, a_k), \text{output} = (), 0 = 0\}$$

$$S := 0 \quad \{\text{input} = (a_1, \dots, a_k), \text{output} = (), S = 0\} \quad C_1$$

$$\{Q \wedge \text{eof}\} \Rightarrow \{S = \sum_{i=1}^n a_i, \text{output} = ()\} \Rightarrow$$

$$\{\text{output} \cdot S = (\sum_{i=1}^n a_i)\} \quad \text{output } s \quad \{\text{output} = (\sum_{i=1}^n a_i)\}$$

Q.E.D.

N: Invariante $Q = \sum \text{input} + S = \sum_{i=1}^n a_i, \text{output} = ()$

zu
 C_1 $\{\neg \text{eof} \wedge Q\} \Rightarrow \{\text{output} = (), \sum \text{tl}(\text{input}) + S + \text{hd}(\text{input}) = \sum_{i=1}^n a_i\}$

$$\text{read } x \quad \left\{ \begin{array}{l} \text{output} = (), \\ \sum \text{input} + S + x = \sum_{i=1}^n a_i \end{array} \right\} S := S + x$$

$$\{\sum \text{input} + S = \sum_{i=1}^n a_i, \text{output} = ()\} = \{Q\}$$

Aufgabe 4) Sei $z = (s, e, a) \in \text{ZUSTAND}$ mit

$$\mathcal{C}[\text{output } T] z = z' \in \text{ZUSTAND}.$$

Daraus folgt, dass $\mathcal{C}[\text{output } T] z \neq \text{Fehler}$
und $T[T] z = (n, z)$ mit $n \in \mathbb{N}$.

Es gilt:

$$\mathcal{C}[\text{output } T] z = (s, e, a.n) = z'$$

Sei $Q[\text{output}.T/\text{output}]$ wahr bzgl. z , dann
gilt Q bzgl. z' , da die Auswertung von T
ohne Nebenwirkung n erfolgt.