Supplementary Material

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This document provides additional information for the paper: Recommend me first: how the order of recommendations might impact fairness for creators.

CCS Concepts: • **Do Not Use This Code → Generate the Correct Terms for Your Paper**; *Generate the Correct Terms for Your Paper*; Generate the Correct Terms for Your Paper; Generate the Correct Terms for Your Paper.

Additional Key Words and Phrases: Do, Not, Us, This, Code, Put, the, Correct, Terms, for, Your, Paper

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1 THEORETICAL RESULTS

Theorem 1.1. *Using popularity-based RS, if we have at a fair state,meaning the number of followers is ordered by the quality, we can show that there is a over* 50% *of maintaining fairness in the next state.*

Proof. Let n be the number of content creators, and a_i^t represent the number of followers of content creator CC_i at time t. Being at a fairstate means that we have $a_1^t \geq a_2^t \geq \cdots \geq a_n^t$. The popularity-based function is given by the probability of being recommended CC_i to a user u at time $t+1$:

$$\mathbb{P}(R^{t+1}(u) = i) = \frac{1 + a_i}{\sum\limits_{j=1}^{n}(1 + a_j)}.$$

Note that the probability does not depends on the user and the denominator is common for every content creators. Hence, by simplicity we note this probability $P^{t+1}(i) = \frac{1+a_i^t}{S}$, where $S := \sum\limits_{j=1}^{n}(1 + a_j)$. Since we are in a fair state, $P^{t+1}(1) \geq P^{t+1}(2) \geq \cdots \geq P^{t+1}(n)$ We introduce for $i = 1, \ldots, n$, Δ_i^{t+1}, corresponding to the number of followers gained at time $t+1$, thus, $a_i^{t+1} = a_i^t + \Delta_i^{t+1}$, and, G_i corresponding to the users that follows i and i is

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their highest-quality they follow.

$$
\begin{aligned}
\mathbb{E}[\Delta_i^{t+1}] &= \sum_{j=i+1}^{n} |G_j| \cdot P^{t+1}(i), \\
&= |G_{i+1}| \cdot P^{t+1}(i) + \sum_{j=i+2}^{n} |G_j| \cdot P^{t+1}(i), \\
&\geq |G_{i+1}| \cdot P^{t+1}(i) + \sum_{j=i+2}^{n} |G_j| \cdot P^{t+1}(i+1), \\
&\geq |G_{i+1}| \cdot P^{t+1}(i) + \mathbb{E}[\Delta_{i+1}^{t+1}], \\
&\geq \mathbb{E}[\Delta_{i+1}^{t+1}].
\end{aligned}
$$

By induction this means that $\mathbb{E}[\Delta_i^{t+1}] \geq \mathbb{E}[\Delta_j^{t+1}] \forall j = i+1, \ldots, n$. This means that for any $i, j \in [1, \ldots, n]$, where $i < j$, we have $\mathbb{P}(a_i^{t+1} < a_j^{t+1}) = \mathbb{P}(\Delta_i^{t+1} + a_i^{t+} < \Delta_j^{t+1} + a_j^{t}) \leq \mathbb{P}(\Delta_j^{t+1} > \Delta_i^{t+1}) \leq 0.5$. To conclude, we obtain

$$
\begin{aligned}
\mathbb{P}(a_1^{t+1} \geq a_2^{t+1} \geq \cdots \geq a_n^{t+1}) &= 1 - \mathbb{P}(\exists i < j \;\; a_i^{t+1} > a_j^{t+1}), \\
&\geq 0.5.
\end{aligned}
$$

□

Theorem 1.2. *Suppose the platform has n content creators, $k \cdot n(n-1)$ users, where $k \in \mathbb{N}$ and $p \in [0,1]$ the noise in user behavior. Then, if $k \geq \frac{20(1-p)(3+4p^2)}{p(2p-1)^2}$ then there is at least 95% chance the best quality content creator is the most followed one during a single pairwise comparison.*

Proof. Let (i, j) be 2 CCs, with loss of generality we can assume that i have higher quality, meaning $i < j$. From our intervention, we show the pair of CCs to 2 different group of size k in the two different ordering. We note the random variable corresponding to number of followers of respectively i and j at time t: $X_i^{(t)}$ and $X_j^{(t)}$. Notice that regardless of their quality, $X_i^{(1)} \sim Bin(k,p)$ and $X_j^{(1)} \sim Bin(k,p)$. Now at time t_2, we have $X_i^{(2)} \sim Bin(k,p) + Bin(k,p)$ and $X_j^{(2)} \sim Bin(k,p) + Bin(X_i^{(1)}, 1-p) + Bin(k - X_i^{(1)}, p)$. The two last term describe how users follow j in the second group. Users who already followed i will follow j with probability $(1-p)$. Users who didn't follow i will follow j with probability p, as j is perceived as the highest quality CC they haven't yet followed.

Note $S := X_i^{(2)} - X_j^{(2)}$, we want to determine the minimum group size k such that $P(S > 0) > 0.95$. We have

$$
\begin{aligned}
\mathbb{E}[S] = \mathbb{E}[X_i^{(2)}] - \mathbb{E}[X_j^{(2)}] &= 2kp - \mathbb{E}[\mathbb{E}[X_j^{(2)} | X_i^{(1)}]], \\
&= 2kp - (kp + \mathbb{E}[X_i^{(1)}](1-p) + (k - \mathbb{E}[X_i^{(1)}])p), \\
&= 2kp - (kp + kp(1-p) + (k - kp)p), \\
&= kp - 2kp(1-p) = kp(2p-1).
\end{aligned}
$$

We have

$$
\mathbb{V}ar[S] = \mathbb{V}ar[X_i^{(2)}] + \mathbb{V}ar[X_j^{(2)}] - 2\mathbb{C}ov(X_i^{(2)}, X_j^{(2)}).
$$

$$\begin{aligned}
\mathbb{V}ar[X_j^{(2)}] &= \mathbb{E}[\mathbb{V}ar[X_j^{(2)}|X_i^{(1)}]] + \mathbb{V}ar[\mathbb{E}[X_j^{(2)}|X_i^{(1)}]],\\
&= \mathbb{E}[\mathbb{V}ar[Bin(k,p)|X_i^{(1)}]] + \mathbb{E}[\mathbb{V}ar[Bin(X_i^{(1)},1-p)|X_i^{(1)}]] + \mathbb{E}[\mathbb{V}ar[Bin(k-X_i^{(1)},p)|X_i^{(1)}]],\\
&\quad + \mathbb{V}ar[kp + X_i^{(1)}(1-p) + (k-X_i^{(1)})p],\\
&= kp(1-p) + \mathbb{E}[X_i^{(1)}](1-p)p + (k-\mathbb{E}[X_i^{(1)}])p(1-p) + \mathbb{V}ar[2kp + X_i^{(1)}(1-2p)],\\
&= 2kp(1-p) + (1-2p)^2kp(1-p),
\end{aligned}$$

$$\begin{aligned}
\mathbb{C}ov(X_i^{(2)}, X_j^{(2)}) &= \mathbb{C}ov(X_i^{(1)}, X_j^{(2)}),\\
&= \mathbb{C}ov(X_i^{(1)}, Bin(X_i^{(1)},1-p)) + \mathbb{C}ov(X_i^{(1)}, Bin(k-X_i^{(1)},p)),\\
&= (1-p)\mathbb{V}ar[X_i^{(1)}] - p\mathbb{V}ar[X_i^{(1)}],\\
&= (1-p)^2kp - p^2k(1-p) = kp(1-p)(1-2p).
\end{aligned}$$

$$\begin{aligned}
\text{Hence we have, } \mathbb{V}ar[S] &= \mathbb{V}ar[X_i^{(2)}] + \mathbb{V}ar[X_j^{(2)}] - 2\mathbb{C}ov(X_i^{(2)}, X_j^{(2)}),\\
&= 2kp(p-1) + 2kp(p-1) + (1-2p)^2kp(1-p) - 2kp(1-p)(1-2p),\\
&= kp(1-p)(4 + (1-2p)^2 - 2(1-2p)),\\
&= kp(1-p)(4 + 1 - 4p - 4p^2 - 2 + 4p),\\
&= kp(1-p)(3+4p^2).
\end{aligned}$$

Hence , to ensure $\mathbb{P}(S \leq 0) \leq 0.05$ we use Chebyshev's inequality:

$$\mathbb{P}(S \leq 0) = \mathbb{P}\left(\frac{(S-\mathbb{E}[S])}{\sqrt{\mathbb{V}ar[S]}} \leq -\frac{\mathbb{E}[S]}{\sqrt{\mathbb{V}ar[S]}}\right) \leq \frac{\mathbb{V}ar[S]}{\mathbb{E}[S]^2}.$$

We need to find k such that

$$\begin{aligned}
\frac{\mathbb{V}ar[S]}{\mathbb{E}[S]^2} \leq 0.05 &\iff \frac{kp(1-p)(3+4p^2)}{k^2p^2(2p-1)^2} \leq 0.05m\\
&\iff \frac{(1-p)(3+4p^2)}{kp(2p-1)^2} \leq 0.05,\\
&\iff k \geq \frac{20(1-p)(3+4p^2)}{p(2p-1)^2}.
\end{aligned}$$

□

We now extend to one community of satisficiers and maximizers with equal distributions in the groups and satisficiers having the same threshold.

Theorem 1.3. *If the platform has n content creators and $k \cdot n!$ users (where $k \in \mathbb{N}$) we can achieve fairness for all CCs by showing a unique permutation to k users.*

Proof. During the first n time-steps, we recommend each unique permutation of content creators to exactly one group of k users. We suppose that for each group, there are m maximizers and s satisficers, where $m+s = k$. We also suppose that the satisficers have the same level of satisficing threshold $q \in [1, \ldots, n]$. Denote a_i the number of followers of CC_i for $i = 1, \ldots, n$. For maximizers, they decide to follow CC_i if and only if i is recommended before $\{1,\ldots,i-1\}$. For satisficers, we need to differenciate two cases: (i) if $i > q$ they decide to follow CC_i if and only if i is recommended before $\{1,\ldots,i-1\}$, (ii) if $i \leq q$ they decide to follow CC_i if and only if i is recommended before $\{1,\ldots,q\}$.

Hence (i)if $i > q$, we have $a_i^n = k\frac{n!}{i}$, and (ii) if $i \leq q$ notice that the set of permutations of n where i is placed before $\{1,\ldots,q\}$ is included in the set of permutations of n where i is placed before $\{1,\ldots,i-1\}$. So we have that $a_i^n = k\frac{n!}{q} + \sum_{g\in G'} s_g$ where G' corresponds to the groups assigned to the permutations where i is before $\{1,\ldots,i-1\}$ but after $\{i+1,\ldots,q\}$. The number of permutations verifying these conditions is equal to $n!(\frac{1}{i}-\frac{1}{q})$. Since we assume that the number of satisficers is the same for all groups, we have $a_i^n = k\frac{n!}{q} + sn!(\frac{1}{i}-\frac{1}{q}) = m\frac{n!}{q} + s\frac{n!}{i}$.

In both cases, the sequence is decreasing $\{a_i^n\}$ for $i > q$ and $i \leq q$. At$i = q$, we have $a_i^n = k\frac{n!}{q}$ in both sequences, thus, the overall sequence is decreasing. After n time steps, we reach an absorbing state where users won't follow any other creators. We have $a_1^t \geq a_2^t \geq \cdots \geq a_n^t$ at $t \geq n$ at $t \geq n$

□

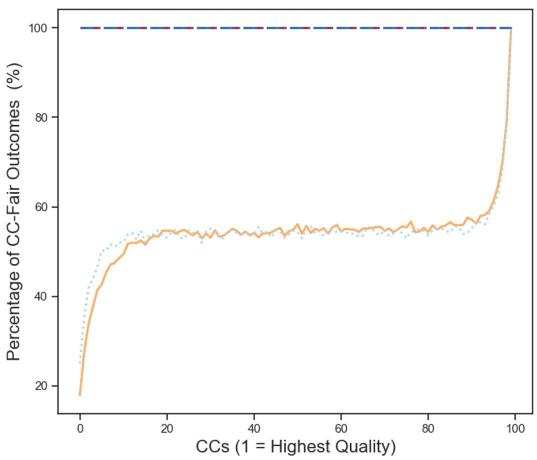
Theorem 1.4. *Suppose the platform has n content creators and $k \cdot n(n-1)$ users, where $k \in \mathbb{N}$. After the ordered pairwise comparison in the first two rounds, the outcome is IF for all CCs.*

Proof. Again, we suppose that for each group, there are m maximizers and s satisficers, where $m + s = k$. We also suppose that the satisficers have the same level of satisficing threshold $q \in [1,\ldots,n]$. At time-step $t = 1$, we recommend each content creator to exactly $(n-1)$ groups of k users, meaning $a_i^1 = k(n-1)$. At time-step $t = 2$, maximizers would follow the recommended creator if and only if this second CC has better quality than the first one. Satisficers would also follow this behavior except if the first CC has better quality than the threshold. Hence, if $i > q$, both maximizers and satisficers will follow i at t_2 if and only if they are shown a worst quality CC at t_1. There are $(n-i)$ pairs in the following form (j,i) where $j > i$. Hence $k(n-i)$ users will follow CC_i. If $i \leq q$, satisficers will follow i at t_2 if and only if they are shown a creator with quality worse than the threshold. Hence $m(n-i) + s(n-q)$ users will follow CC_i.we have $a_i^2 = \begin{cases} k(2n-i-1) \text{ if } i \geq q \\ k(2n-1) - mi - sq \text{ if } i \leq q \end{cases}$,which decreases as i increases.

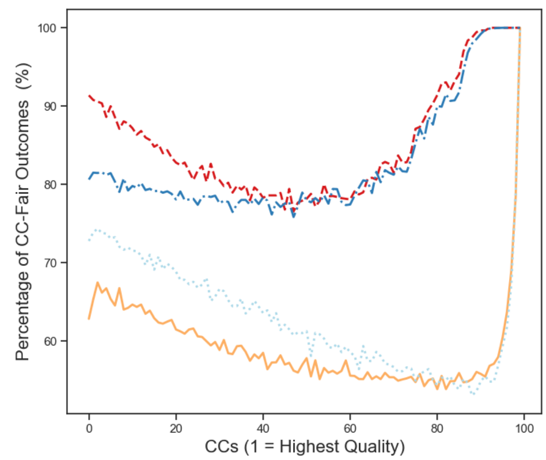
□

2 ADDITIONAL RESULTS

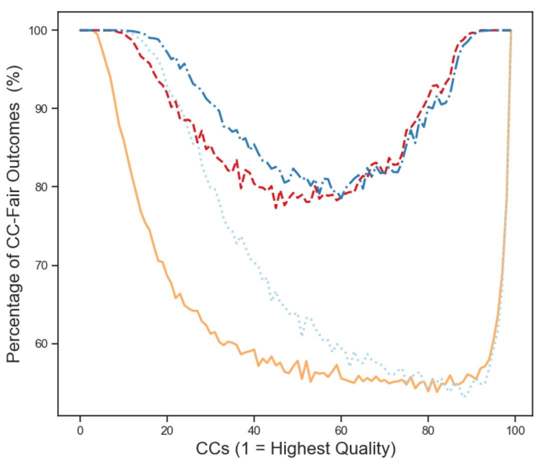
Ordered pairwise comparison improves fairness throughout the process



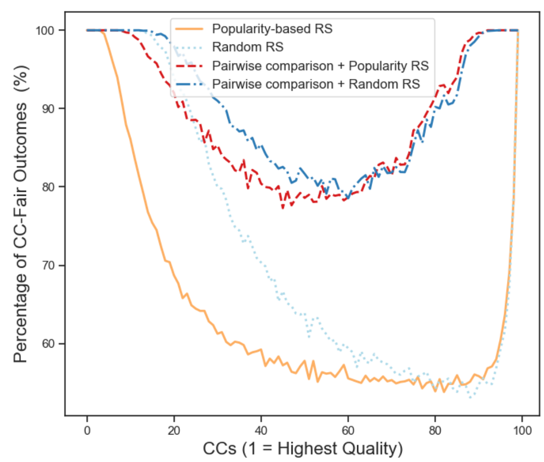
(a) Fairness Distribution by CC Rank at time-step 2



(b) Fairness Distribution by CC Rank at time-step 5



(c) Fairness Distribution by CC Rank at time-step 20



(d) Fairness Distribution by CC Rank at time-step 100

Fig. 1. Comparative analysis of recommendation systems with 100 content creators and 49500 users: random (Light Blue), popularity-based (Orange), ordered pairwise comparison followed by random (Dark Blue) and ordered pairwise comparison followed by popularity-based (Red) recommendations. The creators are ranked based on their quality (CC_1 highest quality CC to CC_{100} lowest). Plots show the percentage of fair outcomes per content creator across simulations at different time-steps: (a) $t = 2$(b) $t = 5$(c) $t = 20$ (d) $t = 100$.