

Linear Algebra

Linear algebra is a branch of mathematics that deals with linear equations, matrices, vectors, and their properties. It is concerned with the study of vector spaces and linear transformations between them.

Linear Equation

A linear equation is an algebraic equation that describes a straight line on a coordinate plane.

It typically takes the form of $y = mx + b$,

where “ y ” and “ x ” are variables and “ m ” and “ b ” are constants. “ m ” represents the slope of the line, or how steeply it rises or falls, while “ b ” represents the y-intercept, or where the line crosses the vertical y-axis.

What is Algebra

Algebra is a type of math that uses letters and symbols (called variables) to represent numbers and teaches us how to work with these letters and symbols using certain rules and equations.

What is an Equation

An equation is all about balancing stuffs on both sides of the equal sign.(very informal)

An equation is a mathematical Statement that check weather two expression on both sides of the equal sign is equal or not.(little formal wrong because it's not a conditional if statement)

An equation is a mathematical statement that express equality of two-expressions by connecting them with the equal sign.(little formal)

An equation is a way of expressing the idea that two things are the same. which means that if you perform the same operations on both sides, they will still be equal.(ChatGPT)

$$2 * 5 = 10$$

$$2x + 5 = 7$$

What is an Expression

An expression is a mathematical statement that involves numbers, symbols and arithmetic operators (+, -, *, /). But it does not involves an equal sign.

$$2 * 5$$

$$2x + 5$$

What is a Statement

A statement is a sentence that is either True or False.

A statement is a declarative sentence that expresses an idea, act, opinion, or fact that is either True or False.

Example:

“The sun rises in the east.”

What is a Proposition?

A proposition is a statement that is either True or False.

A statement is a declarative sentence that expresses an idea, act, opinion, or fact that is either True or False.

Ex: “*The sun rises in the east.*”

What’s the difference between a statement and proposition?

an example sentence can be a question:

“*What time is it?*”

“*What time is it?*” is not a statement; it is a question. It is an interrogative sentence that asks for information about the current time.

What is a Question?

A question is an interrogative sentence that requires an answer.

So by definition questions are not statements.

Answer: *Statements can and do not have to be true or false, whereas propositions do need to meet that requirement.*

So sentence which cannot be evaluated as True or False are Statements.

So sentence which have to be evaluated as True or False are Propositions.

Every statement makes a **claim** that can be evaluated as true or false based on scientific evidence are propositions.

Here’s an example of a proposition: “The Earth orbits around the Sun.” This statement makes a claim that can be evaluated as true or false based on scientific evidence.

On the other hand, not all statements are propositions. For example, questions, commands, and exclamations are types of statements that do not express a complete thought and therefore are not propositions.

Here's an example of a non-propositional statement: "Please pass me the salt." This statement is a command that does not express a complete thought that can be evaluated as true or false. It is simply a request for action.

Declarative and Imperative Sentence

A declarative sentence is a sentence that makes a declaration or statement of fact. It provides information or describes a situation without giving any commands or instructions. Example: "The sun rises in the east."

An imperative sentence, on the other hand, is a sentence that gives a command, direction or instruction. It is used to tell someone what to do or how to do something. Example: "Close the door, please."

In programming, a declarative sentence is a type of code that describes what should be done and leaves it up to the computer to figure out how to do it. This is commonly used in functional programming languages, where programs are structured as a series of functions that take inputs and produce outputs.

On the other hand, an imperative statement is a type of code that gives step-by-step instructions for how a task should be performed. This is commonly used in procedural programming languages, where programs are structured as a sequence of instructions that execute one after another.

What is an Algebraic Equation

Definition 1:

It's a Mathematical Statement that involves "*variables*" and that describes the relationship between those variables using arithmetics.

Definition 2:

An algebraic equation is a mathematical statement that contains one or more variables and describes a relationship between those variables using mathematical operations such as addition, subtraction, multiplication, and division.

The purpose of an algebraic equation is to find the values of the variables that make the equation true.

For example, the equation $2x + 3 = 7$ is an algebraic equation. It contains one variable " x " and requires finding the value of " x " that makes the equation true.

Algebraic equations can be linear or non-linear depending on the degree of variables in the equation. A linear equation has variables raised to the power of 1 only, while a non-linear equation has variables raised to any other power than 1.

Example:

The algebraic equation for the cost of a pizza with two variables, size and toppings, can be written as:

Cost of pizza = (Base price + (Price per topping * Number of toppings)) * Size factor

where,

Base price is the cost of a plain pizza without any toppings Price per topping is the cost of each topping added to the pizza Number of toppings is the total number of toppings added to the pizza Size factor is the factor by which the base price is multiplied based on the size of the pizza In symbols,

$$C = (B + (T * N)) * S$$

where,

- C represents the cost of the pizza
- B represents the base price
- T represents the price per topping
- N represents the number of toppings
- S represents the size factor.

Let's say the base price of a 10-inch pizza is \$8, the price per topping is \$1, and the size factor for a 14-inch pizza is 1.5. If you order a 14-inch pizza with four toppings, the cost of the pizza would be calculated as follows:

$$B = \$8, T = \$1, N = 4, S = 1.5$$

$$\text{Cost of pizza} = (\$8 + (\$1 * 4)) * 1.5 = \$12$$

Conclusion

Algebraic Equation is all about finding the unknown with the knowns.

How do we know if an Equation is Linear or not?

A linear equation will always be of the form:

$$y = mx + b$$

where:

- x and y are variables
- m is the slope or gradient of the line
- b is the y-intercept

If an equation is not in this form, it is not a linear equation.

- The degree of a variable is $\neq 1$. (such as x^2 or y^3)
- Equation involves functions (such as $\sin(x)$ or e^x) is non linear.

Methods of solving an Equation:

There are several methods for solving equations to find the value of x , depending on the type and complexity of the equation. Some common methods include:

Simplification and Isolation: This method is used for simple linear equations in one variable. You can simplify the equation by combining like terms, then isolate the variable on one side of the equation. For example, in the equation $2x + 5 = 17$, you can simplify by subtracting 5 from both sides of the equation to get $2x = 12$, then divide by 2 to get $x = 6$.

Factoring: This method is used for quadratic equations, where the equation can be written in the form $ax^2 + bx + c = 0$. You can factor the equation into two brackets, set each bracket equal to zero, and solve for x . For example, in the equation $3x^2 - 9x = 0$, you can factor out x to get $x(3x - 9) = 0$, then set each bracket equal to zero to get $x = 0$ or $x = 3$.

Substitution: This method is used for systems of equations with multiple variables. You can solve for one variable in terms of another variable in one equation, and substitute that expression into the other equation to eliminate one variable. For example, in the system of equations $2x + y = 7$ and $x - y = 1$, you can solve for y in terms of x in the second equation to get $y = x - 1$, then substitute that expression into the first equation to get $2x + (x - 1) = 7$, which simplifies to $3x = 8$. Solving for x gives $x = 8/3$, and substituting back into the equation $y = x - 1$ gives $y = 5/3$.

Graphing: This method is used to find approximate solutions to equations by plotting the equation on a graph and finding where it intersects with the x -axis. For example, in the equation $y = x^2 - 3x + 2$, you can plot the equation on a graph and estimate that the solution is around $x = 1$ or $x = 2$.

Types of Equation

Linear equations: These are equations in which the highest power of the variable is 1, such as $y = mx + b$.

Quadratic equations: These are equations in which the highest power of the variable is 2, such as $ax^2 + bx + c = 0$.

Cubic equations: These are equations in which the highest power of the variable is 3, such as $ax^3 + bx^2 + cx + d = 0$.

Exponential equations: These are equations in which the variable appears in an exponent, such as $y = a^x$.

Logarithmic equations: These are equations in which the variable appears inside a logarithm, such as $\log_a x = b$.

Trigonometric equations: These are equations that involve trigonometric functions such as \sin , \cos , and \tan .

Polynomial equations: These are equations in which the variable can have any non-negative integer power, such as $ax^n + bx(n-1) + \dots + k = 0$.

Rational equations: These are equations in which the variables are in fractions or ratios, such as $(x+1)/(x-2) = 3$.