A quantitative approach to humanities evolution and two case studies

Vilson Vieira,[a] Renato Fabbri,[b] Gonzalo Travieso,[c] and Luciano da Fontoura Costa[d]
Instituto de Física de São Carlos, Universidade de São Paulo (IFSC/USP)

(Dated: 28 March 2012)

We propose a methodology to study humanities development by applying multivariate statistics on two case studies: composers and philosophers. Seven representative composers and philosophers were considered in terms of eight main musical and philosophical features. Grades were assigned to each characteristic and their correlations were analyzed. A bootstrap method was applied to avoid statistical biasing caused by the small sample data set, simulating hundreds of artificial composers and philosophers influenced by the seven representatives chosen. Afterwards we quantify non-numeric relations like dialectics, opposition and innovation on both fields. Composers and philosophers differences on style and technique were represented as geometrical distances in the feature space, making it possible to quantify, for example, how much Bach and Stockhausen differ from other composers, how much Beethoven influenced Brahms or how much was Spinoza in opposition of their contemporaries. In addition, we compared the results on both fields. Opposition, strong on philosophy, was not remarkable on music. Supporting an observation already considered by music theorists, strong influences were identified between composers by the quantification of dialectics, implying inheritance and suggesting a stronger master-disciple evolution when compared to the philosophy analysis.

PACS numbers: 89.75.Fb,05.65.+b
Keywords: music, musicology, pattern recognition, statistics

I. INTRODUCTION

In the history of music, composers developed their own styles along a continuous search for coherence or unity. In the words of Anton Webern[1], "[...] ever since music has been written most great artists have striven to make this unity ever clearer. Everything that has happened aims at this [...]". Along this process we can identify a constant heritage of style from one composer to another as a gradual development from its predecessor, contrasting with the necessity for innovation. Quoting Lovelock: "[...] by experiment that progress is possible; it is the man with the forward-looking type of mind [...] who forces man out of the rut of 'what was good enough for my father is good enough for me'."[2]. Thus, development in music follows a dichotomy: while composers aims on innovation, creating their own styles, their technique is based on the works of their predecessors, in a master-apprentice tradition.

Other fields like philosophy demonstrate a well-defined trend when considering innovation: unlike music, the quest for difference seems to drive philosophical changes as expressed by Gilles Deleuze[3]. In other approaches such as Ferdinand de Saussurre's principle[4], concepts (words) tend to be different in the sense of meaning distinct things. The paradigm of difference is particularly promising because it is immediately related to the own dynamics of philosophical evolution along time. In other words, what are the forces that drive philosophical changes and innovation? Could dialectics account for some of the main philosophical moves? Are these concepts present on other fields like music?

In order to try and answer these questions, on both humanities fields in a more objective way, we resorted to a quantitative approach which turned out to provide a number of remarkable spin-offs and insights. More specifically, we started by identifying prominent music composers and philosophers along the history, a set of main musical and philosophical issues, and then assign grades to each of these issues for the chosen composers and philosophers. It is important to argue that the grades assignment was not arbitrary. The scores was not assigned on a random way but based on a historical research about techniques and styles used by both composers and philosophers. The grades reveals a *tendency index* about particular characteristics of each composer or philosopher. For example, to say that Bach is more contrapuntist than the others remarkable composers is the same to give a bigger grade – around 8.0 or 9.0 – to the Barroque master and smaller grades to the others.

It should be stressed that we chose a reduced set of philosophers for the sake of simplicity and clarity. Considering this small number of philosophers and composers we developed a bootstrap method[5] to avoid a statistical biasing caused by the small number of samples. In this way we generated a larger data set of 1000 new *artificial composers and philosophers* directly influenced by the originals samples, representing the contemporaries of the representatives chosen.

Those scores assigned to each composer - and philosopher - characteristics define a state vector in its feature space. This quantification of composers and philosophers

[a] `http://automata.cc`; Electronic mail: vilson@void.cc
[b] `http://www.estudiolivre.org/el-user.php?view\_user=gk`; Electronic mail: renato.fabbri@gmail.com
[c] Electronic mail: gonzalo@ifsc.usp.br
[d] `http://cyvision.ifsc.usp.br/~luciano/`; Electronic mail: ldfcosta@gmail.com

characteristics paved the way to the application of sound concepts and methods from multivariate statistics[6–8] and pattern recognition[9,10]. Correlations between these characteristic vectors were identified and principal component analysis (PCA)[10] is applied to represent the musical - and philosophical - history as a planar space where we can follow the historical development as vectorial movements. On this planar space, concepts like dialectics, innovation and opposition, originally non-quantitative, can be modeled as mathematical relations between the individual states. By proposing indices to quantify the degree of opposition, skewness and dialectics along the dynamics of musical and philosophical individual evolutions, we observed surprising results.

It is important to note that application of statistical analysis to music is not recent. On musicology, statistical methods have been used to identify many musical characteristics. Simonton[11,12] used time-series analysis to measure the creative productivity of composers based on their music and popularity. Kozbelt[13,14] also analyzed the productivity, but based on the measure of performance time of the compositions and investigated the relation between productivity and versatility. More recent works[15,16] use machine-learning algorithms to recognize musical styles of selected compositions.

Differently from these works, we are not interested in applying statistical analysis to music but on characterizing composers by the identification of scores based on their recognized styles. On the other hand, automatic information retrieval, common on music, is not common on philosophy. The method proposed here is a way to analyze both fields indenpendently of the nature of their final products – e.g. music pieces and textual documents – but based on a well-formed opinion of reviewers on these fields.

This work starts by presenting the methodology adopted and then presents the seven composers and philosphers followed by the definition of the eight musical and philosophical characteristics chosen. The proposed method was applied on these data sets, allowing us to compare both fields and discuss the results.

II. MATHEMATICAL DESCRIPTION

The choice of composers and philosophers is inherently important for the time-evolution analysis, as each musical and philosophical move and its characteristics (i.e. opposition and skewness) are defined by each pair of subsequent composer or philosophers along time. We decided to choose the composers and philosophers taking into account their historical importance and visibility.

A sequence S of C music composers and P philosophers was chosen based on their relevance at each period of the classical music history and western philosophy history, respectivelly. The set of C measurements define a C-dimensional space henceforth referred as the *musical space*. By the same way we defined a P-dimensional *philosophical space* based on P measurements.

TABLE I. Description of mathematical relations defined for each composer or philosopher i, j and k given a set of C composers or P philosophers as a time-sequence S.

Average state	$\vec{a}_i = \frac{1}{i}\sum_{k=1}^{i}\vec{v}_k.$
Opposite state	$\vec{r}_i = \vec{v}_i + 2(\vec{a}_i - \vec{v}_i)$
Opposition vector	$\vec{D}_i = \vec{r}_i - \vec{v}_i$
Musical or philosophical move	$\vec{M}_{i,j} = \vec{v}_j - \vec{v}_i$
Opposition index	$W_{i,j} = \frac{\langle \vec{M}_{i,j}, \vec{D}_i \rangle}{\vert\vert\vec{D}_i\vert\vert^2}$
Skewness index	$s_{i,j} = \sqrt{\frac{\vert\vec{v}_i-\vec{v}_j\vert^2\vert\vec{a}_i-\vec{v}_i\vert^2-[(\vec{v}_i-\vec{v}_j).(\vec{a}_i-\vec{v}_i)]^2}{\vert\vec{a}_i-\vec{v}_i\vert^2}}$
Counter-dialectics index	$d_{i\to k} = \frac{\vert\langle \vec{v}_j-\vec{v}_i, \vec{v}_k\rangle + \frac{1}{2}\langle \vec{v}_i-\vec{v}_j, \vec{v}_i+\vec{v}_j\rangle\vert}{\vert\vec{v}_j-\vec{v}_i\vert}$

The characteristic vector $\vec{v}_i$ of each composer or philosopher i defines a respective *composer state* in the musical space or *philosopher state* in the philosophical space. For the set of C composers, we defined the same relations adapted for the set of P philosophers, sumarized in Table I.

It is important to note some details about these relations. Here we are focusing on composers, but the same is valid for philosophers or characteristic vectors defined from other fields. Given a set of C composers as a time-sequence S, the *average state* at time i is defined. The *opposite state* is defined as the "counterpoint" of a musical state $\vec{v}_i$, considering its average state: everything running along the opposite direction of $\vec{v}_i$ are understood as opposition. In other words, any displacement from $\vec{v}_i$ along the direction $\vec{r}_i$ is a *contrary move*, and any displacement from $\vec{v}_i$ along the direction $-\vec{r}_i$ is an *emphasis move*. Given a musical state $\vec{v}_i$ and its opposite state $\vec{r}_i$, we can define the *opposition vector* $\vec{D}_i$. These details are better understood analyzing Figure 1.

Considering the time-sequence S we defined relations between pairs of composers. The *musical move* implied by two successive composers at time i and j corresponds to the $\vec{M}_{i,j}$ vector extending from $\vec{v}_i$ to $\vec{v}_j$. Given the musical move we can quantify the intensity of opposition by the projection of $\vec{M}_{i,j}$ along the opposition vector $\vec{D}_i$, normalized, yelding the *opposition index*. Considering the same musical move, the *skewness index* is the distance between $\vec{v}_j$ and the line L_i defined by the vector $\vec{D}_i$, and therefore quantifies how much the new musical state departs from the respective opposition move.

A relationship between a triple of successive composers can also be defined. Considering i, j and k being respectively understood as the *thesis*, *antithesis* and *synthesis*,

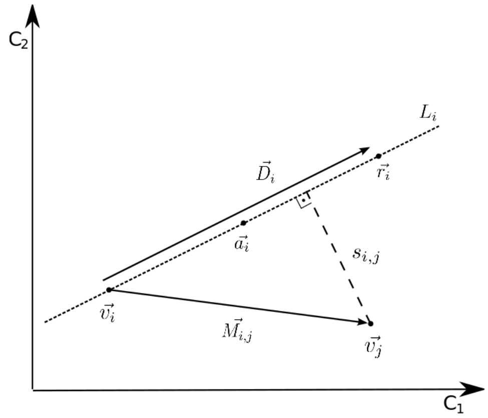

FIG. 1. *Graphical representation of the measures derived from a* musical move.

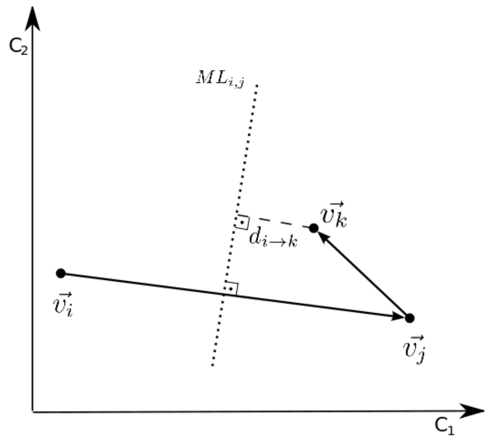

FIG. 2. *Graphical representation of the quantification of dialectics.*

we defined the *counter-dialectics index* by the distance between the musical state $\vec{v}_k$ and the middle line $ML_{i,j}$ defined by the thesis and antithesis, as shown in Figure 2. In higher dimensional philosophical spaces, the middle-hyperplane defined by the points which are at equal distances to both $\vec{v}_i$ and $\vec{v}_j$ should be used instead of the middle line $ML_{i,j}$. The proposed equation for counter-dialectics scales to hyperplanes.

The counter-dialectics index is suggested and used instead of dialectics index to maintain compatibility with the use of a distance from point to line as adopted for the definition of skewness.

III. CHARACTERISTICS

To create the musical and philosophical space we derived eight variables corresponding to distinct characteristics commonly found in music compositions and philosophers works.

A. Musical Characteristics

The characteristics are related with the basic elements of music — melody, harmony, rhythm, timbre, form and tessitura[17] — and non-musical issues like historical events that have influenced the compositions, for example, the presence of Church. All the eight characteristics are listed below:

Sacred - Secular (S-S_c): the sacred or religious music is composed through religious influence or used for its purposes. *Masses*, *motets* and hymns, dedicated to the Christian liturgy, are well known examples[2]. Secular music has no or little relation with religion and includes popular songs like Italian madrigals and German *lieds*[17].

Short duration - Long duration (D_s-D_l): compositions are quantified having short duration when they do not have more than few minutes of execution. Long duration compositions have at least 20 minutes of execution or more. The same consideration was adopted by Kozbelt[13,14] in his analysis of time execution.

Harmony - Counterpoint (H-C): harmony regards the vertical combination of notes, while counterpoint focuses on horizontal combinations[17].

Vocal - Instrumental (V-I): compositions using just vocals (e.g. *cantata*) or exclusively instruments (e.g. *sonata*). It is interesting to note the use of vocals over instruments on Sacred compositions[2].

Non-discursive - Discursive (D_n-D): compositions based or not on verbal discourse, like programmatic music or Baroque rhetoric, where the composer wants to "tell a history" invoking images to the listeners mind[17]. Its contrary part is known as *absolute music* where the music is written to be appreciated simply by what it is.

Motivic Stability - Motivic Variety (M_s-M_v): motivic pieces presents equilibrium between repetition, reuse and variation of melodic motives. Bach is noticeable by his *development by variation* of motives, contrasting with the constantly inventive use of new materials by Mozart[1].

Rhythmic Simplicity - Rhythmic Complexity (R_s-R_c): presence or not of polyrhythms, the use of independent rhythms at the same time — also known as *rhythmic counterpoint*[17] — a characteristic constantly found in Romanticism and the works of 20th-century composers like Stravinsky.

Harmonic Stability - Harmonic Variety (H_s-H_v): rate of tonality change along a piece or its stability. After the highly polyphonic development in Renaissance, Webern regarded Beethoven as the composer who returned to the maximum exploration of harmonic variety[1].

B. Philosophical Characteristics

We derived eight variables corresponding to some of the most recurrent philosophical issues[18–20]. Each of these variables, which define a respective axis in the philosophical space, are briefly described in the following.

Rationalism - Empirism (*R-E*): the rationalists claim that the human acquaintance of knowledge/concepts is significantly independent of sense experience. Empiricists understand sense experience as the main way to gain knowledge/concepts. Frequently, rationalists take the view that the world is affected by intrinsic properties of the human brain, in contrast to the empiricist approach where the world would imprint itself onto our minds.

Essence - Existence (*E-E*): An existence-based understanding of the world has its basis on the fact that things are as an existent unit. Essence focuses on a substance (e.g. intellectual) that precedes existence itself.

Monism - Dualism (*M-D*): Dualism requires the division of the human person into two or more domains, such as matter and soul. Monism is based on a unique "category of being".

Theocentrism - Anthropocentrism (*T-A*): In theocentrism, God is the most important thing in the universe. The anthropocentric view has man as prevalent.

Holism - Reductionism (*H-R*): Reductionism attempts to explain the world in terms of simple components and its emerging properties. Holists focus on the fact that the whole is more than its constitutive parts.

Deductionism - Phenomenology (*D-P*): Phenomenology relies on systematic reflection of consciousness and what happens in conscious acts. Deductionism is based on deriving conclusions from axiomatic systems.

Determinism - Free Will (*D-F*): Free will assumes that humans make choices and these are not predetermined. Determinism understands that every event is fatidic, e.g., perfectly determined by prior states.

Naturalism - Mechanism (*N-M*): Methodological naturalism is the thinking basis of modern science, i.e. hypotheses must be argued and tested in terms of natural laws. Mechanism attempts to build explanation using logic-mathematical processes.

IV. RESULTS AND DISCUSSION

Memorable composers were chosen as key representatives of musical development. This group was chosen purposely to model their influence over contemporaries, creating a concise parallel with music history. We modeled this group of influenced composers as new artificial samples generated by a bootstrap method, better explained in this section. The sequence is ordered chronologically and presented on Table II with each composer related with its historical period. The same was did for philosophy where a set of seven philosophers were chosen spanning the period from Classical Greece until contemporary times, and ordered chronologically as: Plato, Aristotle, Descartes, Espinoza, Kant, Nietzsche, and Deleuze.

The quantification of the eight musical and philosophical characteristics was performed jointly by three of the authors of this article, based on a research of history of music and western philosophy. The scores are shown in Tables III and IV for philosophers and composers, respectivelly. The scores were numerical values between 1 and 9. Values more close of 1 reveals the composer tended to the first element of each characteristic pair, and vice versa. We emphasize that the focus of this work is not on the specific characteristics used or their attributed numerical values, which can be disputed, but on the techniques employed for the quantitative analysis.

TABLE II. The sequence of music composers ordered chronologically with the period each represent.

Composer	Movement
Monteverdi	Renaissance
Bach	Baroque
Mozart	Classical
Beethoven	Classical $\rightarrow$ Romantic
Brahms	Romantic
Stravinsky	20th-century
Stockhausen	Contemporary

TABLE III. Quantification of the eight music characteristics for each of the seven composers.

Composers	S-S_c	D_s-D_l	H-C	V-I	D_n-D	M_s-M_v	R_s-R_c	H_s-H_v
Monteverdi	3.0	8.0	5.0	3.0	7.0	5.0	3.0	7.0
Bach	2.0	6.0	9.0	2.0	8.0	2.0	1.0	5.0
Mozart	6.0	4.0	1.0	6.0	6.0	7.0	2.0	2.0
Beethoven	7.0	8.0	2.5	8.0	5.0	4.0	4.0	7.0
Brahms	6.0	6.0	4.0	7.0	4.5	6.5	5.0	7.0
Stravinsky	8.0	7.0	6.0	7.0	8.0	5.0	8.0	5.0
Stockhausen	7.0	4.0	8.0	7.0	5.0	8.0	9.0	6.0

This data set defines an 8-dimensional musical space where each dimension corresponds to a characteristic that aplies to all 7 composers and philosophers. Such small data sets are not adequate for statistical analysis and the imediate analysis of these sets would be highly biased by the small sample.

TABLE IV. Quantification of the eight philosophical characteristics for each of the seven philosophers.

Philosophers	R-E	E-E	M-D	T-A	H-R	D-P	D-F	N-M
Plato	3.0	3.5	9.0	5.	4.5	3.5	5.0	4.5
Aristotle	8.0	7.5	7.0	5.5	7.5	8.0	2.5	2.5
Descartes	1.5	2.5	9.0	6.5	7.0	2.5	7.5	7.5
Espinoza	8.0	2.0	1.0	5.0	2.0	3.0	1.0	1.0
Kant	7.0	2.5	8.5	6.5	4.5	3.5	7.5	5.0
Nietzsche	7.5	9.0	1.0	9.0	5.0	8.0	1.0	1.5
Deleuze	5.5	7.5	1.0	8.	2.5	5.5	5.0	6.0

A. Bootstrap method for sampling artificial composers

To simulate a more realistic musical and philosophical trajectory, we used a bootstrap method for generating *artificial composers and philosophers* contemporaries of those seven chosen. The bootstrap routine generated randomized scores $\vec{r}$. The values are not totally random, following a probability distribution that models the original $n = 7$ scores, given by $p(\vec{r}) = \sum_{i=1}^{n} e^{\frac{d_i}{2\sigma^2}}$ where d_i is the distance between a random score $\vec{r}$ and the original score chart. For each step a value $p(\vec{r})$ is generated and compared with a random normalized value, characterizing the Monte Carlo[21] method to choose a set of samples. This samples simulates new randomized composers and philosophers score charts — while respecting the historical influence of the main 7 original exponents of each field. Higher values of $p(\vec{r})$ imply a stronger influence of the original scores over $\vec{r}$. For the analysis we used 1000 bootstrap samples obtained by the bootstrap process together with the original scores, considering $\sigma = 1.1$. Other values for σ were used yelding distributions with bootstrap samples closer to or further from the original musical and philosophical states, which does not affected the musical or philosophical space substantially. Pearson correlation coefficients between the eight characteristics chosen are presented in Table V for composers and in Table VI for philosophers. Emphasized coefficients have absolute values larger than 0.5.

TABLE V. Pearson correlation coefficients between the eight musical characteristics.

-	S-S_c	D_s-D_l	H-C	V-I	D_n-D	M_s-M_v	R_s-R_c	H_s-H_v
S-S_c	-	-0.2	-0.06	**0.69**	-0.18	0.19	**0.56**	-0.16
D_s-D_l	-	-	-0.14	-0.13	0.2	-0.48	-0.2	0.37
H-C	-	-	-	-0.23	0.26	0.05	0.46	0.03
V-I	-	-	-	-	-0.33	0.17	0.42	-0.06
D_n-D	-	-	-	-	-	-0.3	0.02	-0.22
M_s-M_v	-	-	-	-	-	-	0.26	-0.15
R_s-R_c	-	-	-	-	-	-	-	-0.02
H_s-H_v	-	-	-	-	-	-	-	-

We can identify some interesting relations between the pairs of characteristics that reflect important facts in music and philosophy history. For instance, while considering composers, the Pearson correlation coefficient of 0.69 was obtained for the pairs S-S_c (Sacred or Secular) and V-I (Vocal or Instrumental), which indicate that sacred music tends to be more vocal than instrumental. The coefficient of 0.56 also shows it does not commonly use polyrhythms as we can see analysing the pairs S-S_c and R_s-R_c (Rhythmic Simplicity or Complexity). Negative coefficients of -0.33 for the pairs V-I and D_n-D (Non-discursive or Discursive) indicated that composers who used just voices on their compositions also preferred to use programmatic musics techniques like baroque rhetoric.

TABLE VI. Pearson correlation coefficients between the eight philosophical characteristics. The entries with absolute value larger or equal than 0.35 have been emphasized.

-	R-E	E-E	M-D	T-A	H-R	D-P	D-F	N-M
R-E	1.00	0.37	-0.23	0.15	0.1	***0.46***	-0.27	***-0.46***
E-E	-	1.00	***-0.53***	0.19	0.15	***0.74***	***-0.61***	-0.3
M-D	-	-	1.00	***-0.43***	***0.41***	-0.3	***0.35***	0.01
T-A	-	-	-	1.00	-0.21	0.06	0.19	0.26
H-R	-	-	-	-	1.00	0.32	-0.22	-0.25
D-P	-	-	-	-	-	1.00	***-0.63***	***-0.47***
D-F	-	-	-	-	-	-	1.00	***0.61***
N-M	-	-	-	-	-	-	-	1.00

For philosophers we also observed strong correlations. For instance, the fact that a Pearson correlation coefficient of -0.46 was obtained for the pair of characteristics *R-E* and *N-M* indicates that philosophers who are rationalists strongly tend to be also mechanicists. An even stronger correlation of 0.74, now positive, is observed between *E-E* and *D-P*, suggesting that existentialists also tend to be phenomenologists, as could be expected. Other strong correlations were observed, including a Pearson coefficient of 0.61 between *D-F* and *N-M*. Also interesting is the relatively high correlation between *M-D* and *D-F*, which seems to be directly implied by religious background.

PCA was applied to both data sets, yielding the new variances given in Table VII and VIII, for both music and philosophy, in terms of percentages of total variance. We can note the concentration of variance along the four first PCA axes on composers data set, a common effect also observed while analyzing philosophers characteristics. This would usualy mean that we could consider just four dimensions but as we will see below our measurements differs considerably with the inclusion of all eight components.

TABLE VII. New variances after PCA, in percentages, for musicians data set.

Eigenvalue	Value
λ_1	32 %
λ_2	20 %
λ_3	17 %
λ_4	14 %
λ_5	7 %
λ_6	5 %
λ_7	3 %
λ_8	3 %

B. Robustness to perturbation of the original scores

In order to investigate the effect from the unavoidable errors in the quantification of the characteristics we performed 1000 perturbations of the original scores by adding to each score the values -2, -1, 0, 1 or 2 with uniform probability. In other words, we wanted to test

TABLE VIII. New variances after PCA, in percentages, for philosophers data set.

Eigenvalue	Value
λ_1	41 %
λ_2	23 %
λ_3	12 %
λ_4	11 %
λ_5	5 %
λ_6	4 %
λ_7	3 %
λ_8	2 %

TABLE IX. Averages and standard deviations of the deviations for each composer and for the 8 eigenvalues.

Composers	μ_Δ	σ_Δ
Monteverdi	3.7347	0.8503
Bach	5.3561	0.9379
Mozart	4.4319	0.8911
Beethoven	3.4987	0.7851
Brahms	3.0449	0.6996
Stravinsky	3.6339	0.7960
Stockhausen	4.2143	0.9029
Eigenvalues	μ_Δ	σ_Δ
λ_1	-0.1759	0.0045
λ_2	-0.0638	0.0026
λ_3	-0.0411	0.0021
λ_4	-0.0144	0.0019
λ_5	0.0578	0.0021
λ_6	0.0736	0.0023
λ_7	0.0080	0.0027
λ_8	0.0835	0.0030

if scoring errors could be sufficient to cause relevant effects on the PCA projections. Interestingly, the values of average and standard deviation for both original and perturbed positions listed in Tables IX and X show relatively small changes. It is therefore reasonable to say that the small errors in the values assigned as scores of composers characteristics do not affected too much its quantification.

C. Results

Tables XI and XII shows the normalized weights of the contributions of each original property on the eight axes for both composers and philosophers. Most of the characteristics contribute almost equally in defining the axes.

Figures 4 and 3 presents a 2-dimensional space considering the first two main axes. The arrows follows the time sequence along with the seven composers and philosophers. Each of these arrows corresponds to a musical or philosophical move from one composer or philosophers state to another – for clarity, just the lines of the arrows are preserved. The bootstrap samples define clusters around the original composers and philosophers.

Starting by analyzing the musical space, Bach is

TABLE X. Average and standard deviation of the deviations for each philosopher and for the 8 eigenvalues.

Philosophers	μ_Δ	σ_Δ
Plato	3.3263	0.7673
Aristotle	4.0896	0.8930
Descartes	4.3081	0.9225
Espinoza	4.9709	0.9131
Kant	3.2845	0.7749
Nietzsche	5.3195	0.9797
Deleuze	4.0990	0.8970
Eigenvalues	μ_Δ	σ_Δ
λ_1	-0.2618	0.0068
λ_2	-0.0976	0.0035
λ_3	0.0154	0.0025
λ_4	0.0212	0.0024
λ_5	0.0697	0.0026
λ_6	0.0807	0.0030
λ_7	0.0877	0.0032
λ_8	0.0846	0.0036

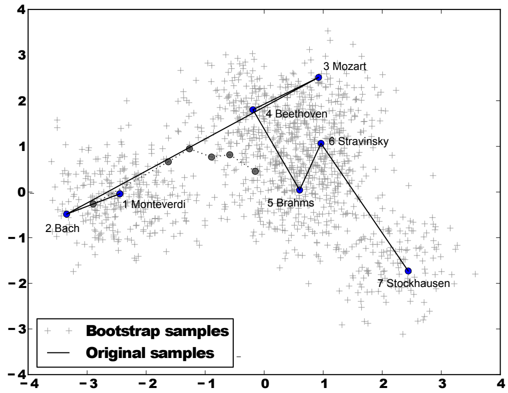

FIG. 3. *2-dimensional projected musical space.*

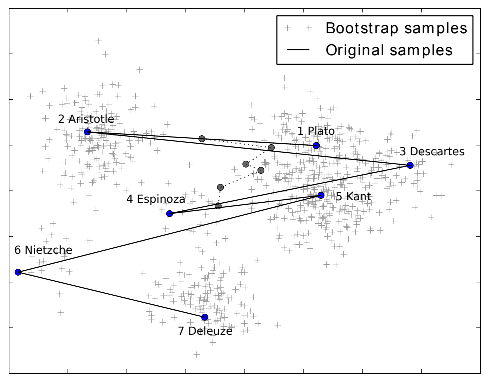

FIG. 4. *2-dimensional projected philosophical space.*

TABLE XI. Percentages of the contributions from each musical characteristic on the eight new main axes.

Musical Charac.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
S-S_c	19.78	4.04	10.38	10.60	17.55	36.60	4.41	0.63
D_s-D_l	13.63	9.21	19.17	3.55	3.13	1.65	25.55	24.05
H-C	1.44	26.62	8.26	13.97	21.71	7.76	13.98	12.20
V-I	18.35	12.82	9.29	8.02	9.37	40.95	2.12	2.03
D_n-D	6.31	10.73	15.48	26.29	4.04	1.86	25.29	2.35
M_s-M_v	16.94	13.28	15.03	4.84	32.25	1.70	2.62	4.37
R_s-R_c	14.13	3.26	15.58	13.80	7.48	1.88	1.36	35.99
H_s-H_v	9.38	20.00	6.75	18.88	4.45	7.56	24.62	18.36

TABLE XII. Percentages of the contributions from each philosophical characteristic on the four new main axes.

Philos. Charac.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
R-E	13.35	1.81	13.11	31.12	15.06	10.99	8.20	3.56
E-E	18.13	7.88	2.10	14.68	11.41	5.31	7.98	33.50
M-D	10.07	24.59	10.71	2.91	0.69	19.39	20.95	2.91
T-A	1.08	24.18	21.51	0.68	23.72	6.31	7,27	2.06
H-R	5.90	21.49	22.29	14.90	6.35	18.81	8.69	2.05
D-P	18.99	1.29	6.85	8.41	9.67	20.50	11.47	24.93
D-F	14.57	13.67	8.37	18.93	21.12	9.06	12.77	14.98
N-M	17.86	5.06	15.03	8.33	11.96	9.59	22.62	15.97

found far from the rest of composers, which suggests his key role acknowledged by other great composers like Beethoven and Webern[1]: "In fact Bach composed everything, concerned himself with everything that gives food for thought!". The greatest subsequent change takes place from Bach to Mozart, reflecting a substantial difference in style. We can identify a strong relationship between Beethoven and Brahms, supporting the belief by the *virtuosi* Hans von Bülow[22] when he stated the 1^{st} Symphony of Brahms as, in reality, being the 10^{th} *Symphony of Beethoven*, clamming Brahms as the true successor of Beethoven. Stravinsky is near to Beethoven and Brahms, presumably due to his heterogeneity[2,17]. Beethoven is also near to Mozart who deeply influenced Beethoven, mainly in his early works. For Webern, Beethoven was the unique classicist who really came close to the coherence found in the pieces of the Burgundian School: "Not even in Haydn and Mozart do we see these two forms as clearly as in Beethoven. The period and the eight-bar sentence are at their purest in Beethoven; in his predecessors we find only traces of them"[1]. It could explain the proximity of Beethoven to the Renaissance Monteverdi. Stockhausen is a deviating point when compared with the others and it could present even more detachment if we had considered vanguard characteristics — e.g. timbre exploration by using electronic devices[2] — not shared by his precursors.

To complement the analysis of musicians, Table XIII gives the opposition and skewness indices for each of the six musical moves, showing the movements are driven by rather small opposition and strong skewness. In other words, most musical moves seems to seek more innovation than opposition. Dialectics is also shown in Table XIV and will play a key role in the next section.

TABLE XIII. Opposition and skewness indices for each of the six musical moves.

Musical Move	$W_{i,j}$	$s_{i,j}$
Monteverdi → Bach	1.0	0.
Bach → Mozart	1.0196	1.9042
Mozart → Beethoven	0.4991	2.8665
Beethoven → Brahms	0.2669	1.7495
Brahms → Stravinsky	0.4582	2.6844
Stravinsky → Stockhausen	0.2516	3.1348

TABLE XIV. Counter-dialectics index for each of the five subsequent pairs of musical moves considering the 8 components.

Musical Triple	$d_{i\to k}$
Monteverdi → Bach → Mozart	2.0586
Bach → Mozart → Beethoven	1.2020
Mozart → Beethoven → Brahms	1.0769
Beethoven → Brahms → Stravinsky	0.2518
Brahms → Stravinsky → Stockhausen	0.2549

While analyzing philosophy by opposition and skewness indices, shown in Table XV, all the philosophical moves tend to take place according to a well-defined and intense opposition from the average state. Also surprisingly, rather small skewness has been found to underlie most philosophical moves, meaning that most philosophical moves are driven almost exclusively by opposition to the current philosophical state. Remarkable results have been obtained also for dialectics shown in Table XVI. We identified progressively stronger dialectics trends among subsequent pairs of philosophical moves.

In order to complement our analysis of the relationship between the composers and philosophers pairs and triples, we performed Wards hierarchical clustering[23] to complement the analysis. This algorithm clusters the original scores taking into account their distance. The generated dendrogram in Figure 5 shows the composers considering their similarity, the same to philosophers is presented on Figure 6. The representation supports the observations discussed previously. It is interesting to note the cluster formed by Beethoven and Brahms, reflecting their heritage. Stravinsky and Stockhausen forms

TABLE XV. Opposition and skewness indices for each of the six philosophical moves.

Philosophical Move	$W_{i,j}$	$s_{i,j}$
Plato → Aristotle	1.0	0
Aristotle → Descartes	0.8740	1.1205
Descartes → Espinoza	0.9137	2.3856
Espinoza → Kant	0.6014	1.6842
Kant → Nietzsche	1.1102	2.9716
Nietzsche → Deleuze	0.3584	2.4890

TABLE XVI. Counter-dialectics index for each of the five subsequent pairs of philosophical moves.

Philosophical Triple	$d_{i \to k}$
Plato $\to$ Aristotle $\to$ Descartes	3.0198
Aristotle $\to$ Descartes $\to$ Espinoza	1.8916
Descartes $\to$ Espinoza $\to$ Kant	1.1536
Espinoza $\to$ Kant $\to$ Nietzsche	1.1530
Kant $\to$ Nietzsche $\to$ Deleuze	0.2705

another cluster and Mozart remains in isolation, as like Bach and Monteverdi. Both relations were also present in the planar space shown in Figure 3.

FIG. 5. *Wards hierarchical clustering of the seven composers.*

V. COMPARISONS WITH PHILOSOPHERS ANALYSIS

The results of composers analysis are surprising when compared with philosophers. It is important to note that

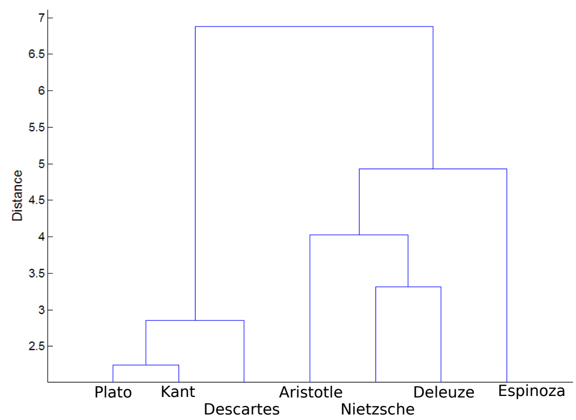

FIG. 6. *Wards hierarchical clustering of the seven philosophers considering all the eight features.*

we preserved the number of characteristics and performed the same bootstrap method to generate a larger set of samples, making possible this comparison. The variances after PCA (Table VIII) concentrates in the four first new axis, similar to the variances for composers shown at Table VIII. If we compare the discussed musical space (Figure 3) with the philosophical one in Figure 4 we identify opposite movements along all the philosophy history in contrast to music. This reveals a notorious characteristic of the way philosophers seem to have evolved their ideas, driven by opposition ($W_{i,j}$), as shown in Table XV, while composers tend to be more influenced by their predecessors as far as their dialectics measures are concerned ($1/d_{i \to k}$).

In general, the musical movements had minor opposition and, remembering the beginning of this work, it reflects the master-apprentice tradition present in music: the composers tend to build their own works confirming their precursors legacy, resulting in a greater dialectics than the philosophers related measures. This reveals a crucial difference considering the *memory treatment* along the development of philosophy and music: using the same techniques, we could verify that a philosopher was influenced by the opposition of ideas from his direct predecessor, while here composers were commonly influenced by their both predecessors. Therefore, we can argue that philosophy presents a *memory-1* state, while music presents *memory-2*, considering *memory-N* being as number N of past generations whose influence on a philosopher or composer is being considered.

Considering the linearity of musical movements we can identify the abscissa as a "time axis" representing the development of music along the history, with some composers like Beethoven returning to Monteverdi and others advancing to the modern age like Stravinsky and Stockhausen.

The opposition and skewness indices for philosophers listed in Table XV endorses the minor role of opposition in composers at the period considered. We can observe strong opposition in philosophical moves contrasted to small opposition in musical movements. Also, the dialectics presents a phase difference suggesting knownledge and aesthetics transfer latency between each of these human fields.

When comparing dialectics, other curious facts arise: the dialectics indices for musicians in Table XIV are considerably stronger moves than for philosophers in Table XVI. Both indices are also shown in Figure 7 where we can see a constant decrease of counter-dialectics. This makes it possible to argue that dialectics is stronger in music where a constantly return to the origins are clearly visible. This reveals the nature of the musical development, based on the search for a unity. Using the words of Webern, the search for the "comprehensibility" but always influenced by their old masters.

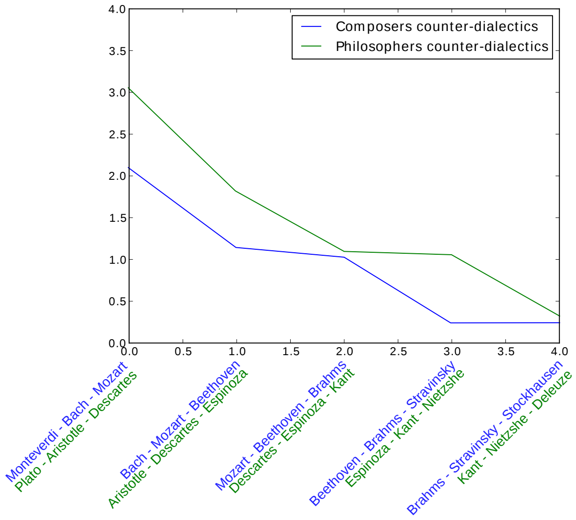

FIG. 7. *Comparison between composers and philosophers counter-dialectics indices*

VI. CONCLUDING REMARKS

Motivated by the understanding of how humanities evolves, we proposed a quantitative method and applied that in two case studies: music and philosophy. Statistical methods is nowadays commonly used for the study of music features and composers productivity, but analysis of composers characteristics modification along the music history has been less explored. Automatic information retrieval techniques while common in music analysis are fewer on philosophy considering the nature of final products of both fields. The method proposed here also has a high level database, making it possible where other methods require corpora of musical scores and texts for Natural Language Processing. In this method, scores are assigned to each feature present in each work life production. This required both bootstrap and robust statistical analysis.

It should be acknowledged that the scores and choice of main musical or philosophical characteristics adopted in the current work are largely arbitrary and could be substantially improved. However, the perturbation analysis performed in this work suggests that the effect of non-systematic errors in assigning the scores does not seem to be critical and has little overall impact on the conclusions we have derived. In addition, whatever the effects of the scores and choice of main features, it should be emphasized that the proposed methodology can be readily applied to expanded and enhanced sets of scores or musical and philosophical characteristics. As a matter of fact, this is perhaps the main contribution of this work, i.e. the proposal of a sound, formal quantitative methodology which can lead to comprehensive, objective insights about how humanities fields has evolved since its earliest origins. It is also worth observing that this methodology is not restricted to individual composers or philosophers, and it can be adapted to the investigation of musical and philosophical schools, individual pieces (e.g. music suites or books), or even of the works of the same composers or philosophers along distinct periods of time. It is also possible to apply this methodology to other areas such as poetry, cinema and science. These scores reveal not the exact profile of composers, but a tendency of how their techniques are usually present. Information retrieval techniques could be considered to make the study stronger, but the current method gives an *initial guess* based on reviewers opinion.

To make the simulation more realistic, we considered not just the small number of 7 original samples, but derived other 1000 new “artificial samples” through a bootstrap method. A larger data set made possible the statistical analysis, considering not just the original scored composers or philosophers, but other samples that respect the historical presence of the formers. This other thousand entities were modeled by a probabilistic distribution, and avoided a biasing caused by the use of only 7 samples. In order to investigate the relationship between this scorings we applied Pearson correlation analysis. The results demonstrated a strong correlation between some characteristics, which allows us to group this values, creating a reduced number of features that summarizes the most important characteristics. PCA was also applied to these components, reducing the complex space to a planar graph where some of the most interesting properties can be visualized. Historical landmarks in both music and philosophy are well-defined in the planar space, like the isolation of Bach, Mozart and Stockhausen, the proximity between Beethoven and Brahms and the distance from Bach and Mozart, the heterogeneity of Stravinsky and the vanguard of contemporary composers like Stockhausen. Even not so visible relations, like the trend to return to the maximum domain of polyphony – present on Renaissance – by Beethoven could also be clearly observable, demonstrating the chronological nature of the space. The dichotomy between master-apprentice tradition on music and the quest for innovation that opened this discussion could be visualized quantitatively. Each composer demonstrated his own style, differing considerably from his predecessor – clearly shown when analyzing pairs of subsequent composers like Bach and Mozart, Mozart and Beethoven or Stravinsky and Stockhausen. Otherwise, the inheritance of predecessors styles is also present when analyzing the direct relations between Mozart and Beethoven or Beethoven and Brahms, or indirect ones between Bach and Beethoven or Beethoven and Monteverdi. The entire scenario presented a “continual pattern” between composers – motivated by the influence of theirs predecessors – but also showed a force repelling both of them: the innovation, or in the words of William Lovelock[2], the “experimentation” that makes progress possible. Along the analysis we noticed interesting differences when comparing composers with philosophers. While on philosophy the innovation is notably marked by opposition of each philosophers ideas, it is less present for music composers. The lack of strong opposition movements and proeminent presence of dialectics in musical space indicates the music innovation is driven by a constant heritage of each composer from his predecessors. We represented this characteristic referring to a *memory state* where philosophers shows *memory-1* – each philosopher was influenced by opposite ideas of its direct predecessor – while composers shows *memory-2* – inheriting the style of their both direct predecessors. The analysis of both dialectics values also shown surprising results: on philosophy the dialectics indices are arranged on a increasing series – showing a strong influence of dialectics to philosophy development – the dialectics indices on music exhibits the same pattern, but with an offset. This behavior presumably indicates a constant quest for coherence by the composers, a fact notably observed by the studies of Anton Webern[1] should have somewhat the same kernel and a lattency between the effects. Another result is that the quantitative methodology initially applied to the analysis of philosophy[24] proved to be extensible to other fields of knowledge – in this case music – reflecting with considerable efficiency details concerning the specific field. Computational analysis of music scores could be applied to automate the quantification of composers characteristics, like identification of melodic and harmonic patterns or the presence or not of polyrhythms, motivic and harmonic stability[25]. More composers could be inserted in the set for the analysis of a wider time-line, possibly including more representatives of each music period. We want to end this work going back to Webern, who early envisioned these relations: "It is clear that where relatedness and unity are omnipresent, comprehensibility is also guaranteed. And all the rest is dilettantism, nothing else, for all time, and always has been. That's so not only in music but everywhere."

ACKNOWLEDGMENTS

Luciano da F. Costa thanks CNPq (308231/03-1) and FAPESP (05/00587-5) for sponsorship. Gonzalo Travieso thanks CNPq (308118/2010-3) for sponsorship. Vilson Vieira and Renato Fabbri is grateful to CAPES and the Postgrad Committee of the IFSC.

Appendix A: A Brief Explanantion of Principal Component Analysis (PCA)

In plain words, PCA is a dimensionality reduction procedure performed through axes rotation. It operates by concentrating dispersion/variance along the first new axes, which are denominated the principal components.

The technique consists in finding the eigenvectors and eigenvalues of the covariance matrix of the respective random vectors (i.e. the vectors associated with each philosophical state). The eigenvalues correspond to the variances of the new variables. When multiplied by the original feature matrix, the eigenvectors yield the new random variables which are fully uncorrelated.

For a more extensive explanation of PCA, please refer to[10] and references therein.

[1] Anton Webern, *The Path To The New Music* (Theodore Presser Company, 1963).
[2] William Lovelock, *A Concise History of Music* (Hammond Textbooks, 1962).
[3] G. Deleuze, *Difference and Repetition* (Continuum, 1968).
[4] F. de Saussure, *Course in General Linguistics* (Books LLC, 1916).
[5] Hal Varian, "Bootstrap tutorial," The Mathematica Journal **9** (2005).
[6] A. Papoulis S. U. Pillai, *Probability, Random Variables and Stochastic Processes* (McGraw Hill Higher Education, 2002).
[7] R. A. Johnson D. W. Wichern, *Applied Multivariate Statistical Analysis* (Prentice Hall, 2007).
[8] C. W. Therrien, *Discrete Random Signals and Statistical Signal Processing* (Prentice Hall, 1992).
[9] R. O. Duda P. E. Hart D. G. Stork, *Pattern Classification* (Wiley-Interscience, 2000).
[10] L. da F. Costa R. M. C. Jr., *Shape Analysis and Classification: Theory and Practice (Image Processing Series)* (CRC Press, 2000).
[11] Dean Keith Simonton, "Emergence and realization of genius: The lives and works of 120 classical composers," Journal of Personality and Social Psychology **61**, 829 – 840 (1991), ISSN 0022-3514.
[12] Dean K. Simonton, "Creative productivity, age, and stress: A biographical time-series analysis of 10 classical composers," Journal of Personality and Social Psychology **35**, 791 – 804 (1977), ISSN 0022-3514.
[13] Aaron Kozbelt, "Performance time productivity and versatility estimates for 102 classical composers," Psychology of Music **37**, 25–46 (2009).
[14] Aaron Kozbelt, "A quantitative analysis of Beethoven as self-critic: implications for psychological theories of musical creativity," Psychology of Music **35**, 144–168 (2007).
[15] Peter van Kranenburg, "Musical style recognition – a quantitative approach," in *Proceedings of the Conference on Interdisciplinary Musicology (CIM04)* (2004).
[16] Peter van Kranenburg, "On measuring musical style – the case of some disputed organ fugues in the J.S. Bach (BWV) catalogue," Computing In Musicology **15** (2007-8).
[17] Roy Bennett, *History of Music* (Cambridge University Press, 1982).
[18] B. Russel, *A History of Western Philosophy* (Simon and Schuster Touchstone, 1967).
[19] D. Papineau, *Philosophy* (Oxford University Press, 2009).
[20] F. G. G. Deleuze, *What Is Philosophy?* (Simon and Schuster Touchstone, 1991).
[21] Christian P. Robert, "Simulation in statistics," in *Proceedings of the 2011 Winter Simulation Conference* (2011) arXiv:1105.4823.
[22] Alan Walker, *Hans von Bülow: a life and times* (Oxford University Press, 2010).
[23] Joe H Ward Jr, "Hierarchical grouping to optimize an objective function," Journal of the American Statistical Association **58**, 236–244 (1963).
[24] Luciano da Fontoura Costa Renato Fabbri, Osvaldo N. Oliveira Jr., "Philosometrics," (October 2010), arXiv:1010.1880v2.
[25] Debora C Correa, Jose H Saito, and Luciano da F Costa, "Musical genres: beating to the rhythms of different drums," New Journal of Physics **12** (2010).