Can the evolution of music and philosophy be analyzed in a quantitative manner?

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We propose a methodology to study music development by applying multivariate statistics on composers characteristics. Seven representative composers were considered in terms of eight main musical features. Grades were assigned to each characteristic and their correlations were analyzed. A bootstrap method was applied to simulate hundreds of artificial composers influenced by the seven representatives chosen. Afterwards we quantify non-numeric relations like dialectics, opposition and innovation. Composers differences on style and technique were represented as geometrical distances in the feature space, making it possible to quantify, for example, how much Bach and Stockhausen differ from other composers or how much Beethoven influenced Brahms. In addition, we compared the results with a prior investigation on philosophy[1]. Opposition, strong on philosophy, was not remarkable on music. Supporting an observation already considered by music theorists, strong influences were identified between composers by the quantification of dialectics, implying inheritance and suggesting a stronger master-disciple evolution when compared to the philosophy analysis.

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I. INTRODUCTION

In the history of music, composers developed their own styles along a continuous search for coherence or unity. In the words of Anton Webern[2], "[...] ever since music has been written most great artists have striven to make this unity ever clearer. Everything that has happened aims at this [...]". Along this process we can identify a constant heritage of style from one composer to another as a gradual development from its predecessor, contrasting with the necessity for innovation. Quoting Lovelock: "[...] by experiment that progress is possible; it is the man with the forward-looking type of mind [...] who forces man out of the rut of 'what was good enough for my father is good enough for me'."[3]. Thus, development in music follows a dichotomy: while composers aims on innovation, creating their own styles, their technique is based on the works of their predecessors, in a master-apprentice tradition.

Other fields like philosophy demonstrate a well-defined trend when considering innovation: unlike music, the quest for difference seems to drive philosophical changes[4]. Recently, this observation became more evident with the application of a quantitative method[1] where multivariate statistics was used to measure non-numeric relations and to represent the historical development as time-series. More specifically, the method consists of scoring memorable philosophers based on some relevant characteristics. The group of philosophers was chosen based on historical relevance. The scores assigned to each philosopher characteristics define a state vector in a feature space. Correlations between these characteristic vectors were identified and principal component analysis (PCA) was applied to represent the philosophical history as a planar space where we could identify interesting properties. Furthermore, concepts like dialects can be modeled as mathematical relations between the philosophical states. Here, we extend that analysis to music.

The application of statistical analysis to music is not recent. On musicology, statistical methods have been used to identify many musical characteristics. Simonton[5,6] used time-series analysis to measure the creative productivity of composers based on their music and popularity. Kozbelt[7,8] also analyzed the productivity, but based on the measure of performance time of the compositions and investigated the relation between productivity and versatility. More recent works[9,10] use machine-learning algorithms to recognize musical styles of selected compositions.

Differently from these works, we are not interested in applying statistical analysis to music but on characterizing composers. Eight characteristics were described and scored by the authors, based on the recurrent appearance of these attributes in music pieces. We chose seven representative composers from different periods of music history. This group was chosen purposely to model their influence on contemporaries, represented as a group of "artificial composers", sampled by a bootstrap method[11]. The same statistical method used in philosophy[1] was applied to this set of composers and their characteristics, allowing us to compare the results from both fields. The results present contrasting historical facts, recognized along the history of music, quantified by application of distance

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TABLE I. Description of mathematical relations defined for each composer i, j and k given a set of C composers as a time-sequence S.

Average state	$\vec{a}_i = \frac{1}{i}\sum_{k=1}^{i}\vec{v}_k$.
Opposite state	$\vec{r}_i = \vec{v}_i + 2(\vec{a}_i - \vec{v}_i)$
Opposition vector	$\vec{D}_i = \vec{r}_i - \vec{v}_i$
Musical move	$\vec{M}_{i,j} = \vec{v}_j - \vec{v}_i$
Opposition index	$W_{i,j} = \frac{\langle \vec{M}_{i,j}, \vec{D}_i \rangle}{\|\vec{D}_i\|^2}$
Skewness index	$s_{i,j} = \sqrt{\frac{\lvert\vec{v}_i-\vec{v}_j\rvert^2 \lvert\vec{a}_i-\vec{v}_i\rvert^2 - [(\vec{v}_i-\vec{v}_j).(\vec{a}_i-\vec{v}_i)]^2}{\lvert\vec{a}_i-\vec{v}_i\rvert^2}}$
Counter-dialectics index	$d_{i\to k} = \frac{\lvert\langle \vec{v}_j-\vec{v}_i, \vec{v}_k \rangle + \frac{1}{2}\langle \vec{v}_i-\vec{v}_j, \vec{v}_i+\vec{v}_j \rangle\rvert}{\lvert\vec{v}_j-\vec{v}_i\rvert}$

metrics which allowed us to formalize concepts like dialectics, innovation and opposition, resulting in interpretations of music development which are compatible with perspectives from musicians and theorists[2,3].

II. MATHEMATICAL DESCRIPTION

A sequence S of C music composers was chosen based on their relevance at each period of the classical music history. As done for philosophers[1], the set of C measurements define a C-dimensional space henceforth referred as the *musical space*. The characteristic vector $\vec{v}_i$ of each composer i defines a respective *composer state* in the musical space. For the set of C composers, we defined the same relations adapted for philosophers[1], sumarized in Table I.

It is important to note some details about these relations. Given a set of C composers as a time-sequence S, the *average state* at time i is defined. The *opposite state* is defined as the "counterpoint" of a musical state $\vec{v}_i$, considering its average state: everything running along the opposite direction of $\vec{v}_i$ are understood as opposition. In other words, any displacement from $\vec{v}_i$ along the direction $\vec{r}_i$ is a *contrary move*, and any displacement from $\vec{v}_i$ along the direction $-\vec{r}_i$ is an *emphasis move.* Given a musical state $\vec{v}_i$ and its opposite state $\vec{r}_i$, we can define the *opposition vector* $\vec{D}_i$. These details are better understood analyzing Figure 1.

Considering the time-sequence S we defined relations between pairs of composers. The *musical move* implied by two successive composers at time i and j corresponds to the $\vec{M}_{i,j}$ vector extending from $\vec{v}_i$ to $\vec{v}_j$. Given the musical move we can quantify the intensity of opposition by the projection of $\vec{M}_{i,j}$ along the opposition vector $\vec{D}_i$, normalized, yelding the *opposition index.* Considering the same musical move, the *skewness index* is the distance between $\vec{v}_j$ and the line L_i defined by the vector $\vec{D}_i$, and therefore quantifies how much the new musical state departs from the respective opposition move.

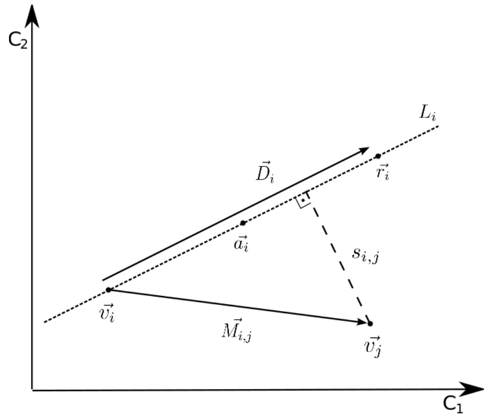

FIG. 1. *Graphical representation of the measures derived from a* musical move[1].

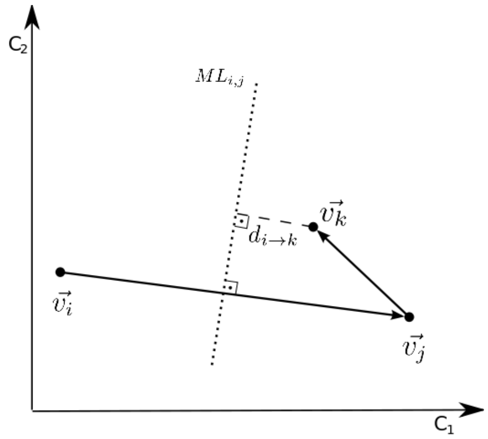

FIG. 2. *Graphical representation of the quantification of dialectics*[1].

A relationship between a triple of successive composers can also be defined. Considering i, j and k being respectively understood as the *thesis*, *antithesis* and *synthesis*, we defined the *counter-dialectics index* by the distance between the musical state $\vec{v}_k$ and the middle line $ML_{i,j}$ defined by the thesis and antithesis, as shown in Figure 2. In higher dimensional philosophical spaces, the middle-hyperplane defined by the points which are at equal distances to both $\vec{v}_i$ and $\vec{v}_j$ should be used instead of the middle line $ML_{i,j}$. The proposed equation for counter-dialectics scales to hyperplanes.

The counter-dialectics index is suggested and used instead of dialectics index to maintain compatibility with the use of a distance from point to line as adopted for the definition of skewness.

III. MUSICAL CHARACTERISTICS

To create the musical space we derived eight variables corresponding to distinct characteristics commonly found in music compositions. The characteristics are related with the basic elements of music — melody, harmony, rhythm, timbre, form and tessitura[12] — and non-musical issues like historical events that have influenced the compositions, for example, the presence of Church. All the eight characteristics are listed below:

Sacred - Secular (S-S_c): the sacred or religious music is composed through religious influence or used for its purposes. *Masses*, *motets* and hymns, dedicated to the Christian liturgy, are well known examples[3]. Secular music has no or little relation with religion and includes popular songs like Italian madrigals and German *lieds*[12].

Short duration - Long duration (D_s-D_l): compositions are quantified having short duration when they do not have more than few minutes of execution. Long duration compositions have at least 20 minutes of execution or more. The same consideration was adopted by Kozbelt[7,8] in his analysis of time execution.

Harmony - Counterpoint (H-C): harmony regards the vertical combination of notes, while counterpoint focuses on horizontal combinations[12].

Vocal - Instrumental (V-I): compositions using just vocals (e.g. *cantata*) or exclusively instruments (e.g. *sonata*). It is interesting to note the use of vocals over instruments on Sacred compositions[3].

Non-discursive - Discursive (D_n-D): compositions based or not on verbal discourse, like programmatic music or Baroque rhetoric, where the composer wants to "tell a history" invoking images to the listeners mind[12]. Its contrary part is known as *absolute music* where the music is written to be appreciated simply by what it is.

Motivic Stability - Motivic Variety (M_s-M_v): motivic pieces presents equilibrium between repetition, reuse and variation of melodic motives. Bach is noticeable by his *development by variation* of motives, contrasting with the constantly inventive use of new materials by Mozart[2].

Rhythmic Simplicity - Rhythmic Complexity (R_s-R_c): presence or not of polyrhythms, the use of independent rhythms at the same time — also known as *rhythmic counterpoint*[12] — a characteristic constantly found in Romanticism and the works of 20th-century composers like Stravinsky.

Harmonic Stability - Harmonic Variety (H_s-H_v): rate of tonality change along a piece or its stability. After the highly polyphonic development in Renaissance, Webern regarded Beethoven as the composer who returned to the maximum exploration of harmonic variety[2].

IV. RESULTS AND DISCUSSION

Memorable composers were chosen as key representatives of musical development. This group was chosen purposely to model their influence over contemporaries, creating a concise parallel with music history. We modeled this group of influenced composers as new artificial samples generated by a bootstrap method, better explained in this section.

The sequence is ordered chronologically and presented on Table II with each composer related with its historical period.

TABLE II. The sequence of music composers ordered chronologically with the period each represent.

Composer	Movement
Monteverdi	Renaissance
Bach	Baroque
Mozart	Classical
Beethoven	Classical $\rightarrow$ Romantic
Brahms	Romantic
Stravinsky	20th-century
Stockhausen	Contemporary

The quantification of the eight musical characteristics was performed jointly by the authors of this article and is shown in Table III. The scores were numerical values between 1 and 9. Values more close of 1 reveals the composer tended to the first element of each characteristic pair, and vice versa. We emphasize that the focus of this work is not on the specific characteristics used or their attributed numerical values, which can be disputed, but on the techniques employed for the quantitative analysis.

TABLE III. Quantification of the eight music characteristics for each of the seven composers.

Composer	S-S_c	D_s-D_l	H-C	V-I	D_n-D	M_s-M_v	R_s-R_c	H_s-H_v
Monteverdi	3.0	8.0	5.0	3.0	7.0	5.0	3.0	7.0
Bach	2.0	6.0	9.0	2.0	8.0	2.0	1.0	5.0
Mozart	6.0	4.0	1.0	6.0	6.0	7.0	2.0	2.0
Beethoven	7.0	8.0	2.5	8.0	5.0	4.0	4.0	7.0
Brahms	6.0	6.0	4.0	7.0	4.5	6.5	5.0	7.0
Stravinsky	8.0	7.0	6.0	7.0	8.0	5.0	8.0	5.0
Stockhausen	7.0	4.0	8.0	7.0	5.0	8.0	9.0	6.0

This data set defines an 8-dimensional musical space where each dimension corresponds to a characteristic that aplies to all 7 composers. Such small data set is not adequate for statistical analysis and the imediate analysis of this set would be highly biased by the small sample.

A. Bootstrap method for sampling artificial composers

To simulate a more realistic musical trajectory, we used a bootstrap method for generating *artificial composers* contemporaries of those seven chosen.

The bootstrap routine generated randomized scores $\vec{r}$. The values are not totally random, following a probability distribution that models the original $n = 7$ scores, given by $p(\vec{r}) = \sum_{i=1}^{n} e^{\frac{d_i}{2\sigma^2}}$ where d_i is the distance between a random score $\vec{r}$ and the original score chart. For each step a value $p(\vec{r})$ is generated and compared with a random normalized value, characterizing the Monte Carlo[13] method to choose a set of samples. This samples simulates new randomized composers score charts — while respecting the historical influence of the main 7 original exponents. Higher values of $p(\vec{r})$ imply a stronger influence of the original scores over $\vec{r}$. For the analysis we used 1000 bootstrap samples obtained by the bootstrap process together with the original scores, considering $\sigma = 1.1$. Other values for σ were used yelding distributions with bootstrap samples closer to or further from the original musical states, which does not affected the musical space substantially.

Pearson correlation coefficients between the eight musical characteristics chosen are presented in Table IV. Emphasized coefficients have absolute values larger than 0.5.

TABLE IV. Pearson correlation coefficients between the eight musical characteristics.

-	S-S_c	D_s-D_l	H-C	V-I	D_n-D	M_s-M_v	R_s-R_c	H_s-H_v
S-S_c	-	-0.2	-0.06	**0.69**	-0.18	0.19	**0.56**	-0.16
D_s-D_l	-	-	-0.14	-0.13	0.2	-0.48	-0.2	0.37
H-C	-	-	-	-0.23	0.26	0.05	0.46	0.03
V-I	-	-	-	-	-0.33	0.17	0.42	-0.06
D_n-D	-	-	-	-	-	-0.3	0.02	-0.22
M_s-M_v	-	-	-	-	-	-	0.26	-0.15
R_s-R_c	-	-	-	-	-	-	-	-0.02
H_s-H_v	-	-	-	-	-	-	-	-

We can identify some interesting relations between the pairs of characteristics that reflect important facts in music history. For instance, the Pearson correlation coefficient of 0.69 was obtained for the pairs S-S_c (Sacred or Secular) and V-I (Vocal or Instrumental), which indicate that sacred music tends to be more vocal than instrumental. The coefficient of 0.56 also shows it does not commonly use polyrhythms as we can see analysing the pairs S-S_c and R_s-R_c (Rhythmic Simplicity or Complexity). Negative coefficients of -0.33 for the pairs V-I and D_n-D (Non-discursive or Discursive) indicated that composers who used just voices on their compositions also preferred to use programmatic musics techniques like baroque rhetoric.

PCA was applied to this set of data, yielding the new variances given in Table V in terms of percentages of total variance. We can note the concentration of variance along the four first PCA axes, a common effect also observed while analyzing philosophers characteristics[1]. This would usualy mean that we could consider just four dimensions but as we will see below our measurements differs considerably with the inclusion of all eight components.

TABLE V. New variances after PCA, in percentages for scores on IV.

Eigenvalue	Value
λ_1	32 %
λ_2	20 %
λ_3	17 %
λ_4	14 %
λ_5	7 %
λ_6	5 %
λ_7	3 %
λ_8	3 %

TABLE VI. Averages and standard deviations of the deviations for each composer and for the 8 eigenvalues.

Composers	μ_Δ	σ_Δ
Monteverdi	3.7347	0.8503
Bach	5.3561	0.9379
Mozart	4.4319	0.8911
Beethoven	3.4987	0.7851
Brahms	3.0449	0.6996
Stravinsky	3.6339	0.7960
Stockhausen	4.2143	0.9029
Eigenvalues	μ_Δ	σ_Δ
λ_1	-0.1759	0.0045
λ_2	-0.0638	0.0026
λ_3	-0.0411	0.0021
λ_4	-0.0144	0.0019
λ_5	0.0578	0.0021
λ_6	0.0736	0.0023
λ_7	0.0080	0.0027
λ_8	0.0835	0.0030

B. Robustness to perturbation of the original scores

As done for philosophers analysis, we performed 1000 perturbations of the original scores by adding to each score the values -2, -1, 0, 1 or 2 with uniform probability. In other words, we wanted to test if scoring errors could be sufficient to cause relevant effects on the PCA projections. Interestingly, the values of average and standard deviation for both original and perturbed positions listed in Table VI show relatively small changes. It is therefore reasonable to say that the small errors in the values assigned as scores of composers characteristics do not affected too much its quantification.

C. Results

Table VII shows the normalized weights of the contributions of each original property on the eight axes. Most of the characteristics contribute almost equally in defining the axes.

Figure 3 presents a 2-dimensional space considering the first two main axes. The arrows follows the time sequence along with the seven composers. Each of these arrows corresponds to a musical move from one composer state

TABLE VII. Percentages of the contributions from each musical characteristic on the eight new main axes.

Musical Charac.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
S-S_c	19.78	4.04	10.38	10.60	17.55	36.60	4.41	0.63
D_s-D_l	13.63	9.21	19.17	3.55	3.13	1.65	25.55	24.05
H-C	1.44	26.62	8.26	13.97	21.71	7.76	13.98	12.20
V-I	18.35	12.82	9.29	8.02	9.37	40.95	2.12	2.03
D_n-D	6.31	10.73	15.48	26.29	4.04	1.86	25.29	2.35
M_s-M_v	16.94	13.28	15.03	4.84	32.25	1.70	2.62	4.37
R_s-R_c	14.13	3.26	15.58	13.80	7.48	1.88	1.36	35.99
H_s-H_v	9.38	20.00	6.75	18.88	4.45	7.56	24.62	18.36

to another – for clarity, just the lines of the arrows are preserved. The bootstrap samples define clusters around the original composers.

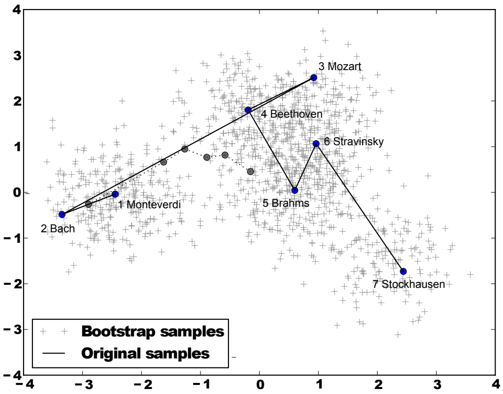

FIG. 3. *2-dimensional projected musical space.*

Bach is found far from the rest of composers, which suggests his key role acknowledged by other great composers like Beethoven and Webern[2]: “In fact Bach composed everything, concerned himself with everything that gives food for thought!”. The greatest subsequent change takes place from Bach to Mozart, reflecting a substantial difference in style. We can identify a strong relationship between Beethoven and Brahms, supporting the belief by the *virtuosi* Hans von Bülow[14] when he stated the 1^{st} Symphony of Brahms as, in reality, being the 10^{th} *Symphony of Beethoven*, clamming Brahms as the true successor of Beethoven. Stravinsky is near to Beethoven and Brahms, presumably due to his heterogeneity[3,12]. Beethoven is also near to Mozart who deeply influenced Beethoven, mainly in his early works. For Webern, Beethoven was the unique classicist who really came close to the coherence found in the pieces of the Burgundian School: “Not even in Haydn and Mozart do we see these two forms as clearly as in Beethoven. The period and the eight-bar sentence are at their purest in Beethoven; in his predecessors we find only traces of them”[2]. It could explain the proximity of Beethoven to the Renaissance Monteverdi. Stockhausen is a deviating point when compared with the others and it could present even more detachment if we had considered vanguard characteristics — e.g. timbre exploration by using electronic devices[3] — not shared by his precursors.

To complement the analysis, Table VIII gives the opposition and skewness indices for each of the six musical moves, showing the movements are driven by rather small opposition and strong skewness. In other words, most musical moves seems to seek more innovation than opposition. Dialectics is also shown in Table IX and will play a key role in the next section.

TABLE VIII. Opposition and skewness indices for each of the six musical moves.

Musical Move	$W_{i,j}$	$s_{i,j}$
Monteverdi → Bach	1.0	0.
Bach → Mozart	1.0196	1.9042
Mozart → Beethoven	0.4991	2.8665
Beethoven → Brahms	0.2669	1.7495
Brahms → Stravinsky	0.4582	2.6844
Stravinsky → Stockhausen	0.2516	3.1348

TABLE IX. Counter-dialectics index for each of the five subsequent pairs of musical moves considering the 8 components.

Musical Triple	$d_{i \to k}$
Monteverdi → Bach → Mozart	2.0586
Bach → Mozart → Beethoven	1.2020
Mozart → Beethoven → Brahms	1.0769
Beethoven → Brahms → Stravinsky	0.2518
Brahms → Stravinsky → Stockhausen	0.2549

We performed Wards hierarchical clustering[15] to complement the analysis. This algorithm clusters the original scores taking into account their distance. The generated dendrogram in Figure 4 shows the composers considering their similarity. The representation supports the observations discussed previously. It is interesting to note the cluster formed by Beethoven and Brahms, reflecting their heritage. Stravinsky and Stockhausen forms another cluster and Mozart remains in isolation, as like Bach and Monteverdi. Both relations were also present in the planar space shown in Figure 3.

V. COMPARISONS WITH PHILOSOPHERS ANALYSIS

The results of composers analysis are surprising when compared with philosophers[1]. It is important to note that we preserved the number of characteristics and performed the same bootstrap method to generate a larger set of samples, making possible this comparison. The variances after PCA (Table X) concentrates in the four first new axis, similar to the variances for composers shown at Table V. If we compare the discussed musical space with the philosophical one in Figure 5 we identify

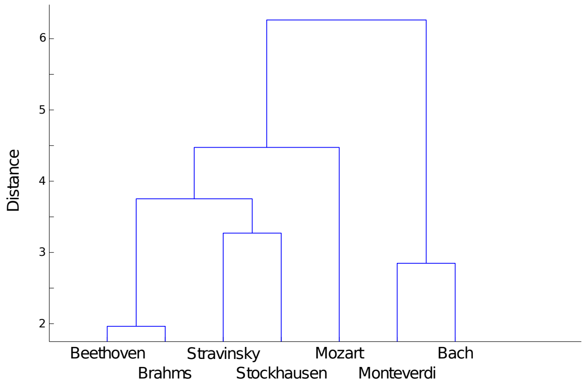

FIG. 4. *Wards hierarchical clustering of the seven composers.*

opposite movements along all the philosophy history in contrast to music. This reveals a notorious characteristic of the way philosophers seem to have evolved their ideas, driven by opposition ($W_{i,j}$), as shown in Table XI, while composers tend to be more influenced by their predecessors as far as their dialectics measures are concerned ($1/d_{i\to k}$).

In general, the musical movements had minor opposition and, remembering the beginning of this work, it reflects the master-apprentice tradition present in music: the composers tend to build their own works confirming their precursors legacy, resulting in a greater dialectics than the philosophers related measures. This reveals a crucial difference considering the *memory treatment* along the development of philosophy and music: using the same techniques this article does[1], we could verify that a philosopher was influenced by the opposition of ideas from his direct predecessor, while here composers were commonly influenced by their both predecessors. Therefore, we can argue that philosophy presents a *memory-1* state, while music presents *memory-2*, considering *memory-N* being as number N of past generations whose influence on a philosopher or composer is being considered. Considering the linearity of musical movements we can identify the abscissa as a "time axis" representing the development of music along the history, with some composers like Beethoven returning to Monteverdi and others advancing to the modern age like Stravinsky and Stockhausen.

The opposition and skewness indices for philosophers listed in Table XI endorses the minor role of opposition in composers at the period considered. We can observe strong opposition in philosophical moves contrasted to small opposition in musical movements. Also, the dialectics presents a phase difference suggesting knownledge and aesthetics transfer latency between each of these human fields.

When comparing dialectics, other curious facts arise: the dialectics indices for musicians in Table IX are considerably stronger moves than for philosophers in Table XII. Both indices are also shown in Figure 6 where we can see a constant decrease of counter-dialectics. This makes it possible to argue that dialectics is stronger in music where a constantly return to the origins are clearly visible. This reveals the nature of the musical development, based on the search for a unity. Using the words of Webern, the search for the "comprehensibility" but always influenced by their old masters.

TABLE X. New variances after PCA for philosophers scores in percentages.

Eigenvalue	Value
λ_1	40 %
λ_2	23 %
λ_3	13 %
λ_4	10 %
λ_5	5 %
λ_6	4 %
λ_7	3 %
λ_8	2 %

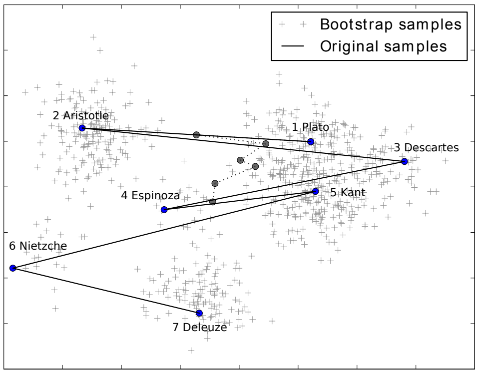

FIG. 5. *2-dimensional projected philosophical space.*

VI. CONCLUDING REMARKS

Motivated by the understanding of how innovation evolves in music history, we extended a quantitative method recently applied to the study of philosophical

TABLE XI. Opposition and skewness indices for each of the six philosophical moves.

Philosophical Move	$W_{i,j}$	$s_{i,j}$
Plato → Aristotle	1.0	0
Aristotle → Descartes	0.8740	1.1205
Descartes → Espinoza	0.9137	2.3856
Espinoza → Kant	0.6014	1.6842
Kant → Nietzsche	1.1102	2.9716
Nietzsche → Deleuze	0.3584	2.4890

TABLE XII. Counter-dialectics index for each of the five subsequent pairs of philosophical moves, considering all components.

Philosophical Triple	$d_{i\to k}$
Plato → Aristotle → Descartes	3.0198
Aristotle → Descartes → Espinoza	1.8916
Descartes → Espinoza → Kant	1.1536
Espinoza → Kant → Nietzsche	1.1530
Kant → Nietzsche → Deleuze	0.2705

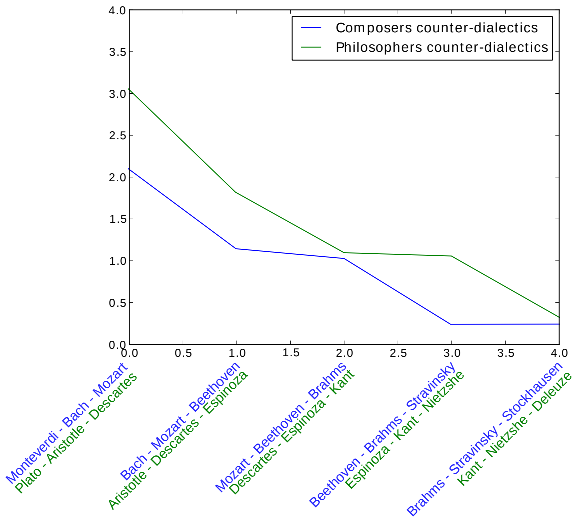

FIG. 6. *Comparison between composers and philosophers counter-dialectics indices*

characteristics[1] and compared the results. Statistical methods is nowadays commonly used for the study of music features and composers productivity, but analysis of composers characteristics modification along the music history has been less explored. The method differs on the aspect of how the characteristics concerning composers are treated: scores are assigned to each feature common in musical works. These scores reveal not the exact profile of composers, but a tendency of how their techniques are usually present. To make the simulation more realistic, we considered not just the small number of 7 composers, but derived other 1000 new "artificial composers" through a bootstrap method. A larger data set made possible the statistical analysis, considering not just the original scored composers, but other samples that respect the historical presence of the formers. This other thousand composers were modeled by a probabilistic distribution, and avoided a biasing caused by the use of only 7 composers. In order to investigate the relationship between this scorings we applied Pearson correlation analysis. The results demonstrated a strong correlation between some characteristics, which allows us to group this values, creating a reduced number of features that summarizes the most important characteristics. PCA was also applied to these components, reducing the complex space to a planar graph where some of the most interesting properties can be visualized.

Historical landmarks in music are well-defined in the planar space, like the isolation of Bach, Mozart and Stockhausen, the proximity between Beethoven and Brahms and the distance from Bach and Mozart, the heterogeneity of Stravinsky and the vanguard of contemporary composers like Stockhausen. Even not so visible relations, like the trend to return to the maximum domain of polyphony – present on Renaissance – by Beethoven could also be clearly observable, demonstrating the chronological nature of the space.

The dichotomy between master-apprentice tradition on music and the quest for innovation that opened this discussion could be visualized quantitatively. Each composer demonstrated his own style, differing considerably from his predecessor – clearly shown when analyzing pairs of subsequent composers like Bach and Mozart, Mozart and Beethoven or Stravinsky and Stockhausen. Otherwise, the inheritance of predecessors styles is also present when analyzing the direct relations between Mozart and Beethoven or Beethoven and Brahms, or indirect ones between Bach and Beethoven or Beethoven and Monteverdi. The entire scenario presented a "continual pattern" between composers – motivated by the influence of theirs predecessors – but also showed a force repelling both of them: the innovation, or in the words of William Lovelock[3], the "experimentation" that makes progress possible.

Along the analysis we noticed interesting differences when comparing composers with philosophers. While on philosophy the innovation is notably marked by opposition of each philosophers ideas, it is less present for music composers. The lack of strong opposition movements and proeminent presence of dialectics in musical space indicates the music innovation is driven by a constant heritage of each composer from his predecessors. We represented this characteristic referring to a *memory state* where philosophers shows *memory-1* – each philosopher was influenced by opposite ideas of its direct predecessor – while composers shows *memory-2* – inheriting the style of their both direct predecessors. The analysis of both dialectics values also shown surprising results: on philosophy the dialectics indices are arranged on a increasing series – showing a strong influence of dialectics to philosophy development – the dialectics indices on music exhibits the same pattern, but with an offset. This behavior presumably indicates a constant quest for coherence by the composers, a fact notably observed by the studies of Anton Webern[2] should have somewhat the same kernel and a lattency between the effects.

Another result is that the quantitative methodology initially applied to the analysis of philosophy[1] proved to be extensible to other fields of knowledge – in this case music – reflecting with considerable efficiency details concerning the specific field.

Computational analysis of music scores could be applied to automate the quantification of composers characteristics, like identification of melodic and harmonic patterns or the presence or not of polyrhythms, motivic and harmonic stability[16]. More composers could be inserted in the set for the analysis of a wider time-line, possibly including more representatives of each music period.

We want to end this work going back to Webern, who early envisioned these relations: "It is clear that where relatedness and unity are omnipresent, comprehensibility is also guaranteed. And all the rest is dilettantism, nothing else, for all time, and always has been. That's so not only in music but everywhere."

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