Musimetrics

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Can the arts be analyzed in a quantitative manner? We propose a methodology to study music development by applying multivariate statistics on composers characteristics. Seven representative composers were considered in terms of eight main musical features. Grades were assigned to each characteristic and their correlations were analyzed. A bootstrap method was applied to simulate hundreds of artificial composers influenced by the seven representatives chosen. Applying dimensionality reduction we obtained a planar space used to quantify non-numeric relations like dialectics, opposition and innovation. Composers differences on style and technique were represented as geometrical distances in the planar space, making it possible to quantify, for example, how much Bach and Stockhausen differ from other composers or how much Beethoven influenced Brahms. In addition, we compared the results with a prior investigation on philosophy[1]. The influence of dialectics, strong on philosophy, was not remarkable on music. Instead, supporting an observation already considered by music theorists, strong influences were identified between subsequent composers, implying inheritance and suggesting a stronger master-disciple evolution when compared to the philosophy analysis. e

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I. INTRODUCTION

Along the history of music, composers developed their own styles along a continuous search for coherence or unity. In the words of Anton Webern[2], "[...] ever since music has been written most great artists have striven to make this unity ever clearer. Everything that has happened aims at this [...]". Along this process we can identify a constantly heritage of style from one composer to another as "a gradual development from its predecessor", contrasting with the necessity for innovation: "It is only by experiment that progress is possible; it is the man with the forward-looking type of mind [...] who forces man out of the rut of 'what was good enough for my father is good enough for me'."[3]. Development in music follows a dichotomy: while composers aims on innovation, creating their own styles, their technique is based on the works of their predecessors, in a master-apprentice tradition.

Other fields – like philosophy – demonstrate a well-defined trend when considering innovation: unlike music, the quest for difference seems to drive philosophical changes[4]. Recently, this observation became more evident with the application of a quantitative method[1] where multivariate statistics was used to measure non-numeric relations and to represent the historical development as time-series. More specifically, the method consists of scoring memorable philosophers based on some relevant characteristics. The group of philosophers was chosen based on historical relevance. The scores assigned to each philosopher characteristics define a state vector in a feature space. Correlations between these characteristic vectors were identified and principal component analysis (PCA) was applied to represent the philosophical history as a planar space where we could identify interesting properties. On this space, concepts like dialects can be modeled as mathematical relations between the philosophical states. We expanded that analysis to music.

The application of statistical analysis to music is not recent. On musicology, statistical methods have been used to identify many musical characteristics. Simonton[5,6] used time-series analysis to measure the creative productivity of composers based on their music and popularity. Kozbelt[7,8] also analyzed the productivity, but based on the measure of performance time of the compositions and investigated the relation between productivity and versatility. More recent works[9,10] use machine-learning algorithms to recognize musical styles of selected compositions.

Differently from these works, we are not interested in applying statistical analysis to music but on characterizing composers. Eight characteristics were described and scored by the authors, based on the recurrent appearance of these attributes in music pieces. We chose seven representative composers from different periods of music history. This group was chosen purposely to model their influence on contemporaries, represented as a group of "artificial composers", sampled by a bootstrap method. The same statistical method used in philosophy[1] was applied to this set of composers and their characteristics, allowing us to compare the results from both fields. The results present contrasting historical facts, recognized along the history of music, quantified by application of distance metrics which allowed us formalize concepts like dialectics, innovation and opposition, resulting in interpreta-

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tions of music development which are compatible with perspectives from musicians and theorists[2,3].

II. MATHEMATICAL DESCRIPTION

A sequence S of P music composers was chosen based on their relevance as representative of each period of the classical music history. As done for philosophers[1], the set of C measurements define a $C-$dimensional space henceforth referred as the *musical space*. The characteristic vector $\vec{v_i}$ of each composer i defines a respective *composer state* in the musical space.

For the set of P composers, we used the same elements defined for philosophers [1]: *average state at time i*, named $\vec{a_i}$; the *opposite state* of a given composer state $\vec{v_i}$, named $\vec{r_i}$; the *opposition vector* of composer state $\vec{v_i}$, named $\vec{D_i}$; and the *opposition amplitude* of that same state, $||\vec{D_i}||$. The dialectics is quantified between a triple of successive composers i, j and k of the given set P.

III. MUSICAL CHARACTERISTICS

To create the musical space we derived eight variables corresponding to distinct characteristics commonly found in music compositions. The characteristics are related with the basic elements of music – melody, harmony, rhythm, timbre, form and tessitura[11] – and non-musical issues like historical events that have influenced the compositions, for example, the presence of Church. All the eight characteristics are listed below:

Sacred - Secular (*S-P*): the sacred or religious music is composed through religious influence or used for its purposes. *Masses*, *motets* and hymns, dedicated to the Christian liturgy, are well known examples[3]. Secular music has no relation with religion including popular songs like Italian madrigals and German *lieds*[11].

Short duration - Long duration (*S-L*): compositions are quantified having short duration when they do not have more than few minutes of execution. Long duration compositions have at least 20 minutes of execution or more. The same consideration was adopted by Kozbelt[7,8] in his analysis of time execution.

Harmony - Counterpoint (*H-C*): harmony regards the vertical combination of notes, while counterpoint focuses on horizontal combinations[11].

Vocal - Instrumental (*V-I*): compositions using just vocals (e.g. *cantata*) or exclusively instruments (e.g. *sonata*). It is interesting to note the use of vocals over instruments on Sacred compositions[3].

Non-discursive - Discursive (*N-D*): compositions based or not on verbal discourse, like programmatic music or Baroque rhetoric, where the composer wants to "tell a history" invoking images to the listeners mind[11]. Its contrary part is known as *absolute music* where the music is written to be appreciated simply by what it is.

Motivic Stability - Motivic Variety (*M-V*): motivic pieces presents equilibrium between repetition, reuse and variation of melodic motives. Bach is noticeable by his *development by variation* of motives, contrasting with the constantly inventive use of new materials by Mozart[2].

Rhythmic Simplicity - Rhythmic Complexity (*R-P*): presence or not of polyrhythms, the use of independent rhythms at the same time – also known as *rhythmic counterpoint*[11] – a characteristic constantly found in Romanticism and the works of 20th-century composers like Stravinsky.

Harmonic Stability - Harmonic Variety (*T-M*): rate of tonality change along a piece or its stability. After the highly polyphonic development in Renaissance, Beethoven is credited as the composer who returned to the maximum exploration of harmonic variety[2].

IV. RESULTS AND DISCUSSION

Memorable composers were chosen as key representatives of musical development. This group was chosen purposely to model their influence over contemporaries, creating a concise parallel with music history. We modeled this group of influenced composers as new artificial samples generated by a bootstrap method, better explained in this section.

The sequence is ordered chronologically and presented on Table I with each composer related with its historical period.

TABLE I. The sequence of music composers ordered chronologically with the outstanding period their represents.

Composers	Eras
Monteverdi	Renaissance
Bach	Baroque
Mozart	Classical
Beethoven	Classical → Romantic
Brahms	Romantic
Stravinsky	20th-century
Stockhausen	Contemporary

The quantification of the eight musical characteristics was performed jointly by the authors of this article and is shown in Table II. The scores were numerical values between 1 and 9. Values more close of 1 reveals the composer tended to the first element of each characteristic pair, and vice versa.

This data set defines an 8-dimensional musical space, each dimension corresponding of one characteristic considering the group of 7 composers. Such small data set is not adequate to statistical analysis. The analysis of this set would be highly biased by the small sample.

TABLE II. Quantification of the eight music characteristics for each of the seven composers.

Composers	S-P	S-L	H-C	V-I	N-D	M-V	R-P	T-M
Monteverdi	3.0	8.0	5.0	3.0	7.0	5.0	3.0	7.0
Bach	2.0	6.0	9.0	2.0	8.0	2.0	1.0	5.0
Mozart	6.0	4.0	1.0	6.0	6.0	7.0	2.0	2.0
Beethoven	7.0	8.0	2.5	8.0	5.0	4.0	4.0	7.0
Brahms	6.0	6.0	4.0	7.0	4.5	6.5	5.0	7.0
Stravinsky	8.0	7.0	6.0	7.0	8.0	5.0	8.0	5.0
Stockhausen	7.0	4.0	8.0	7.0	5.0	8.0	9.0	6.0

A. Bootstrap method for sampling artificial composers

To simulate a more realistic musical trajectory, we used a bootstrap method for generating *artificial composers* contemporaries of the seven chosen.

The bootstrap routine generated randomized scores $\vec{r}$. The values are not totally random, following a probability distribution that models the original $n = 7$ scores, given by $p(\vec{r}) = \sum_{i=1}^{n} e^{\frac{d_i}{2\sigma^2}}$ where d_i is the distance between a random score $\vec{r}$ and the original scores. For each step a value $p(\vec{r})$ is generated and compared with a random normalized value, characterizing the Monte Carlo[12] method to choose a set of samples. This samples simulates new randomized composers scores – while respecting the historical influence of the main 7 original exponents. Higher the value of $p(\vec{r})$, greater the influence of the original scores over $\vec{r}$. For the analysis we used 500 bootstrap samples obtained by the bootstrap process together with the original scores, considering $\sigma = 1.4$.

The Pearson correlation coefficients between the eight musical characteristics chosen are presented in Table III. The coefficients with absolute value larger than 0.5 are emphasized.

TABLE III. Pearson correlation coefficients between the eight musical characteristics.

-	S-P	S-L	H-C	V-I	N-D	M-V	R-P	T-M
S-P	-	-0.15	-0.03	**0.63**	-0.14	0.10	**0.52**	-0.1
S-L	-	-	-0.05	-0.15	0.14	-0.39	-0.09	0.26
H-C	-	-	-	-0.16	0.23	-0.02	0.41	0.09
V-I	-	-	-	-	-0.3	0.17	0.4	-0.04
N-D	-	-	-	-	-	-0.25	0.01	-0.24
M-V	-	-	-	-	-	-	0.14	-0.04
R-P	-	-	-	-	-	-	-	0.03
T-M	-	-	-	-	-	-	-	-

We can identify some interesting relations between the pairs of characteristics that reflect important facts in music history. For instance, the Pearson correlation coefficient of 0.63 was obtained for the pairs S-P (Sacred or Secular) and V-I (Vocal or Instrumental), which indicate that sacred music tends to be more vocal than instrumental. The coefficient of 0.52 also shows it does not commonly use polyrhythms as we can see analysing the pairs S-P and R-P (Rhythmic Simplicity or Complexity). Negative coefficients of -0.3 for the pairs V-I and N-D (Non-discursive or Discursive) indicated that composers who used just voices on their compositions also preferred to use programmatic musics techniques like baroque rhetoric.

PCA was applied to this set of data, yielding the new variances given in Table IV in terms of percentages of total variance. We can note the concentration of variance along the four first PCA axes, a common effect also observed while analyzing philosophers characteristics[1]. This means we could consider just four dimensions.

TABLE IV. New variances after PCA, in percentages for scores on III.

Eigenvalue	Value
λ_1	29 %
λ_2	19 %
λ_3	17 %
λ_4	14 %
λ_5	7 %
λ_6	6 %
λ_7	4 %
λ_8	4 %

B. Perturbations robustness of the original scores

As done for philosophers analysis, we performed 1000 perturbations of the original scores by adding the values -2, -1, 0, 1 and 2 with uniform probability. In other words, we wanted to test if scoring errors could be sufficient to cause relevant effects on the PCA projections. Interestingly, the values of average and standard deviation for both original and perturbed positions listed in Table V show relatively small changes. It is therefore reasonable to say small errors in the values assigned as scores of composers characteristics did not affected too much its quantification.

C. Results

Table VI shows the normalized weights of the contributions of each original property on the four new main axes. Most of the characteristics contribute almost equally in defining the first two main axes.

Figure 1 presents a 2-dimensional space considering the first two main axes. The arrows follows the time sequence along with the seven composers. Each of these arrows corresponds to a musical move from one composer state to another – for clarity, just the lines of the arrows are preserved. The bootstrap samples define clusters around the original composers.

Bach is positioned far from the rest of composers, which suggests his key role admitted by other great composers like Beethoven and Webern[2]: "In fact Bach com-

TABLE V. Average and standard deviation of the deviations for each composer and for the first 4 eigenvalues.

Composers	μ_Δ	σ_Δ
Monteverdi	2.7627	1.0403
Bach	3.5934	1.0795
Mozart	2.1883	0.9748
Beethoven	1.4753	0.7161
Brahms	1.4911	0.7379
Stravinsky	1.6808	0.8159
Stockhausen	2.7166	1.0387
Eigenvalues	μ_Δ	σ_Δ
λ_1	-0.1327	0.0057
λ_2	-0.0461	0.0040
λ_3	-0.0315	0.0035
λ_4	-0.0169	0.0030

TABLE VI. Percentages of the contributions from each musical characteristic on the four new main axes.

Musical Characteristics	C_1	C_2	C_3	C_4
S-P	21.79	5.60	6.05	11.88
S-L	11.96	7.80	23.99	8.00
H-C	0.85	27.99	5.24	16.80
V-I	21.92	3.41	9.10	9.69
N-D	2.56	0.20	25.73	20.24
M-V	18.07	21.67	3.58	4.04
R-P	12.36	11.50	13.73	17.60
T-M	10.44	21.80	12.54	11.71

posed everything, concerned himself with everything that gives food for thought!". The greatest subsequent change takes place from Bach to Mozart, reflecting a substantial difference in style. We can identify a strong relationship between Beethoven and Brahms, supporting the belief by the *virtuosi* Hans von Bülow[13] when he stated the 1^{st} Symphony of Brahms as, in reality, being the 10^{th} *Symphony of Beethoven*, clamming Brahms as the true successor of Beethoven. Stravinsky is near

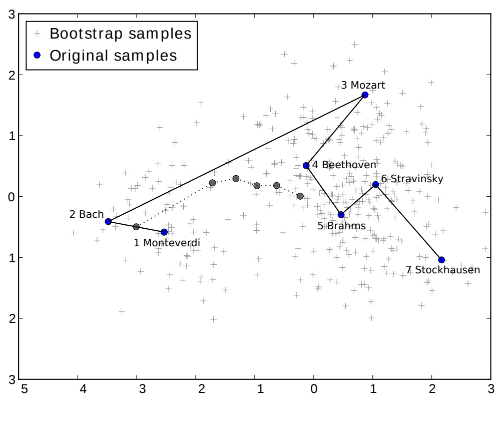

FIG. 1. *2-dimensional projected musical space.*

of Beethoven and Brahms, presumably due to his heterogeneity[3,11]. Beethoven is also near of Mozart who deeply influenced Beethoven, mainly in his early works. For Webern, Beethoven was the unique classicist who really came close to the coherence found in the pieces of the Burgundian School: "Not even in Haydn and Mozart do we see these two forms as clearly as in Beethoven. The period and the eight-bar sentence are at their purest in Beethoven; in his predecessors we find only traces of them"[2]. It could explain the proximity of Beethoven to the Renaissance Monteverdi. Stockhausen is a deviating point when compared with the others and it could present even more detachment if we had considered vanguard characteristics – e.g. timbre exploration by using electronic devices[3] – not shared by his precursors. In general, the musical movements had minor opposition and, remembering the beginning of this work, it reflects the master-apprentice tradition present in music: the composers tend to build their own works confirming their precursors legacy. This reveals a crucial difference considering the *memory treatment* along the development of philosophy and music: while a philosopher was influenced by the opposition of ideas from his two predecessors, composers were commonly influenced by their direct predecessor. Therefore, we can argue that philosophy presents a *memory-2* state, while music presents *memory-1*, considering *memory-N* being the number N of past generations those influenced a philosopher or composer. Considering the linearity of musical movements we can identify the abscissa as a "time axis" representing the development of music along the history, with some composers like Beethoven returning to Monteverdi and others advancing to the modern age like Stravinsky and Stockhausen.

To complement the analysis, Table VII gives the opposition and skewness indices for each of the six musical moves, showing the movements are driven by rather small opposition and strong skewness. In other words, most musical moves do not benefit from opposition as far as innovation is concerned. Dialectics is also analyzed on Table VIII where we identified an alternation of values along the pairs of subsequent musical movements: the first value of counter-dialectics is greater than the second, that is lesser than the third and so on. There is no strong dialectics, but a continuous variation.

TABLE VII. Opposition and skewness indices for each of the six musical moves.

Musical Move	$W_{i,j}$	$s_{i,j}$
Monteverdi → Bach	1.0	0.
Bach → Mozart	1.2780	0.4831
Mozart → Beethoven	0.2827	0.4519
Beethoven → Brahms	-0.1396	0.9676
Brahms → Stravinsky	-0.1421	0.6887
Stravinsky → Stockhausen	-0.3566	1.3891

TABLE VIII. Counter-dialectics index for each of the five subsequent pairs of musical moves.

Musical Triple	$d_{i\to k}$
Monteverdi → Bach → Mozart	3.5573
Bach → Mozart → Beethoven	0.2097
Mozart → Beethoven → Brahms	0.6479
Beethoven → Brahms → Stravinsky	0.4472
Brahms → Stravinsky → Stockhausen	0.5721

V. COMPARISONS WITH PHILOSOPHERS ANALYSIS

The results of composers analysis when compared with philosophers[1] reveals surprising results. It is important to note we preserved the number of characteristics and performed the same bootstrap method to generate a larger set of samples, making possible this comparison. The variances after PCA (Table IX) concentrates in the four first new axis, similar to the variances for composers shown at Table IV. If we compare the discussed musical space with the philosophical one in Figure 2 we identify opposite movements along all the philosophy history in contrast to music. This reveals a notorious characteristic of the way philosophers seem to have evolved their ideas, driven by opposition, while composers tend to be more influenced by their predecessors.

TABLE IX. New variances after PCA for philosophers scores in percentages.

Eigenvalue	Value
λ_1	39 %
λ_2	19 %
λ_3	17 %
λ_4	8 %
λ_5	6 %
λ_6	4 %
λ_7	3 %
λ_8	3 %

The opposition and skewness indices for philosophers listed in Table X endorses the minor role of opposition in composers. We can observe strong opposition and rather small skewness in philosophical moves contrasted to small opposition and strong skewness in musical movements. Also, the oscillating dialectics of both are out of phase, indicating transfer latency.

When comparing dialectics, other curious facts arise: the dialectics indices in Table XI are considerably stronger philosophical moves than for composers. Both indices are also shown in Figure 3 where we can see a constantly decrease of counter-dialectics, contrasting the continuously variation of the indices when considering the composers. This makes possible to argue that dialectics is stronger in philosophy than in music where a constantly return to the origins are clearly visible on some composers. This reveals the nature of the musical development, based on the search for a unity. Using the words of Webern, the search for the "comprehensibility"

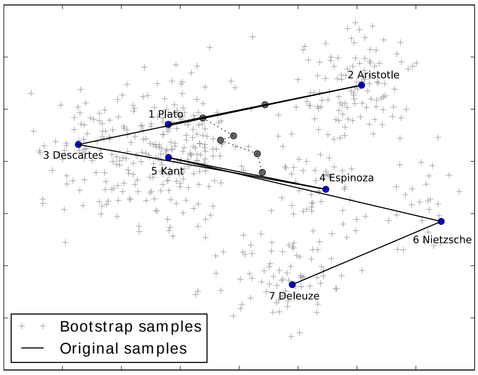

FIG. 2. *2-dimensional projected philosophical space.*

TABLE X. Opposition and skewness indices for each of the six philosophical moves.

Philosophical Move	$W_{i,j}$	$s_{i,j}$
Plato → Aristotle	1.0	0.0
Aristotle → Descartes	0.8922	0.0145
Descartes → Espinoza	0.7838	1.1218
Espinoza → Kant	0.6548	0.7059
Kant → Nietzsche	1.5124	1.4479
Nietzsche → Deleuze	0.3235	1.9040

but always influenced by their old masters.

Complementing the analysis we performed Wards hierarchical clustering. This algorithm clusters the original scores taking into account their distance. The generated dendrogram in Figure 4 shows the composers considering their similarity. The representation supports the observations discussed previously. It is interesting to note the cluster formed by Beethoven and Brahms, reflecting their heritage. Stravinsky and Stockhausen forms another cluster and Mozart remains in isolation, as like Bach and Monteverdi. Both relations were also present in the planar space shown at Figure 1.

TABLE XI. Counter-dialectics index for each of the five subsequent pairs of philosophical moves.

Philosophical Triple	$d_{i\to k}$
Plato → Aristotle → Descartes	0.968
Aristotle → Descartes → Espinoza	0.287
Descartes → Espinoza → Kant	0.138
Espinoza → Kant → Nietzsche	1.247
Kant → Nietzsche → Deleuze	0.054

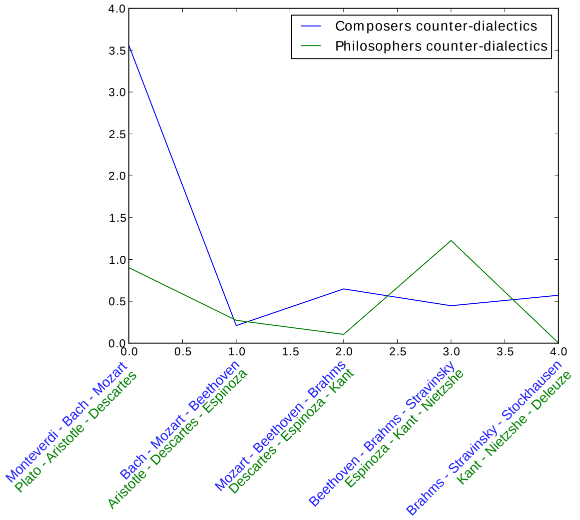

FIG. 3. *Comparison between composers and philosophers counter-dialectics indices*

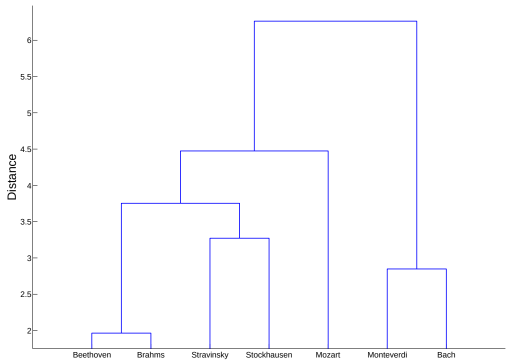

FIG. 4. *Wards hierarchical clustering of the seven composers.*

VI. CONCLUDING REMARKS

Motivated by the understanding of how innovation evolves in music history, we extended a quantitative method recently applied to the study of philosophical characteristics[1] and compared the results. Statistical methods have been commonly applied to the study of music features and composers productivity, but analysis of composers characteristics along the music history has been less explored. The method differs on the aspect of how the characteristics concerning composers are treated: scores are assigned to each feature common in musical works. These scores reveal not the exact profile of composers, but a tendency of how their techniques relate one another. To make the simulation more realistic, we considered not just the small number of 7 composers, but derived other 500 new "artificial composers" through a bootstrap method. A larger data set made possible the statistical analysis, considering not just the original scored composers, but other samples those respect the historical tendency of the formers, modeled by a probabilistic distribution. In order to investigate the relationship between this scoring we applied Pearson correlation analysis. The results demonstrated a strong correlation between some characteristics, which allows us to group this values, creating a reduced number of features that summarizes the most important characteristics. PCA was also applied to these components, reducing the complex space to a planar graph where the most interesting properties were identified.

Historical landmarks in music are well-defined in the planar space, like the isolation of Bach, Mozart and Stockhausen, the proximity between Beethoven and Brahms and the distance from Bach and Mozart, the heterogeneity of Stravinsky and the vanguard of contemporary composers like Stockhausen. Even not so visible relations, like the trend to return to the maximum domain of polyphony – present on Renaissance – by Beethoven could also be clearly observable, demonstrating the time nature of the space.

The dichotomy between master-apprentice tradition on music and the quest for innovation that opened this discussion could be visualized quantitatively. Each composer demonstrated his own style, differing considerably from his predecessor – clearly shown when analyzing pairs of subsequent composers like Bach and Mozart, Mozart and Beethoven or Stravinsky and Stockhausen. Otherwise, the inheritance of predecessors styles is also present when analyzing the direct relations between Mozart and Beethoven or Beethoven and Brahms, or indirect ones between Bach and Beethoven or Beethoven and Monteverdi. The entire scenario presented a "continual pattern" between composers – motivated by the influence of theirs predecessors – but also showed a force repelling both of them: the innovation, or in the words of William Lovelock[3], the "experimentation" that makes progress possible.

Along the analysis we noticed interesting differences when comparing composers with philosophers. While on philosophy the innovation is notably marked by opposition of each philosophers ideas, it is less present for music composers. The lack of strong opposition movements in musical space indicates the music innovation is driven by a constant heritage of each composer from his predecessor. We represented this characteristic referring to a *memory state* where philosophers shows *memory-2* – each philosopher was influenced by the opposite ideas of its two predecessors – while composers shows *memory-1* – inheriting the style of their direct predecessor. The analysis of both dialectics values also shown surprising results: while on philosophy the dialectics indices are arranged on a increasing series – showing a strong influence of dialectics to philosophy development – the same dialectics indices on music exhibits a constantly variation. This behavior presumably indicates a constantly quest for coherence by the composers, a fact notably observed by the studies of Anton Webern[2].

The quantitative methodology initially applied to the analysis of philosophy[1] proved to be extensible to other fields of knowledge like music, reflecting with considerable efficiency, specific details concerning each field.

Computational analysis of music scores could be applied to automate the quantification of composers characteristics, like identification of melodic and harmonic patterns or the presence or not of polyrhythms, motivic and harmonic stability[14]. More composers could be inserted in the set for the analysis of a wider time-line, possibly including more representatives of each music periods.

While taking the first steps on the direction of a quantitative approach to arts and philosophy we believe that an understanding of the creative process could also be eventually quantified. We want to end this work going back to Webern, who early envisioned these relations: "It is clear that where relatedness and unity are omnipresent, comprehensibility is also guaranteed. And all the rest is dilettantism, nothing else, for all time, and always has been. That's so not only in music but everywhere."

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