Musimetrics

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(Dated: 29 April 2011)

Based on a quantitative philosophical analysis, we verified the application of same method to another field of knowledge: music. Seven composers of classical music were analyzed with respect of eight main musical characteristics present on their life-work. In addition, both philosophical and musical analysis were compared, revealing some differences on the development of each of these fields, specially with respect to innovation.

PACS numbers: 89.75.Fb,05.65.+b
Keywords: music, musicology, pattern recognition, statistics

I. INTRODUCTION

Along the history of music, composers developed their own styles along a continuous search for coherence or unity. In the words of Anton Webern[1], "[...] ever since music has been written most great artists have striven to make this unity ever clearer. Everything that has happened aims at this [...]". On this process we could argue that inheritance of style from one composer to another is constantly present, as noted by William Lovelock[2]: "[...] it [a new style of writing] is a gradual development from its predecessor, in due course reaching its culmination, and germinating in its life the seeds of its successor", contrasting with the necessity of innovation, also cited by the same author: "It is only by experiment that progress is possible; it is the man with the forward-looking type of mind [...] who forces man out of the rut of 'what was good enough for my father is good enough for me'." How innovation is shown in this master-apprentice context? How the quest for coherence is developed by composers considering this dichotomy?

Other fields of knowledge – like philosophy – demonstrate a well-defined trend when considering innovation: the quest for difference seems to drive philosophical changes[3]. Recently, this observation became more evident with the application of quantitative analysis[4]: a series of statistical and pattern recognition steps that create a formal representation of the philosophical movements along the history. More specifically, the method consists of scoring memorable philosophers based on some dualistic characteristics. The group of philosophers was choose based on the historical relevance of each philosopher. The scores assigned to each philosopher compose a state vector in complex space. The application of Pearson correlation reveals the most important relations between the issues and principal component analysis (PCA) makes possible to represent the philosophical history as a planar graph where we could identify interesting properties. Even argument based methods like dialectics seem to be well modeled as mathematical relations between the philosophical states. Following this work we expanded the analysis in order to better understand how innovation evolved in music and how it is compared to philosophy where opposition was identified as a strong factor concerning innovation and dialectics .

The application of statistical analysis to classical music is not recent. On musicology, statistical methods have been used to identify many musical characteristics. Simonton[5,6], beyond several works, used time-series analysis to measure the creative productivity of composers based on its produced musics and popularity. Kozbelt[7,8] also analysed the productivity, but based on the measure of performance time of the compositions and investigated the relation between productivity and versatility. More recent works[9,10] uses machine-learning algorithms to recognize musical styles of selected compositions.

Differently from these works, we are not interested in applying statistical analysis to music but on characterizing concerning composers. We chose seven memorable composers from different periods of classical music. Eight characteristics were described and scored by the authors, based on the recurrent appearance of these issues in music pieces composed by the selected musicians. The same statistical method developed for philosophy was applied to this set of composers and their characteristics, making possible to compare the results on both areas.

We start by describing how the original method was adapted to the analysis of composers. The eight musical characteristics are discussed as the composers and their respective musical period, followed by results and comparisons between the application of the method on philosophy and music.

II. MATHEMATICAL DESCRIPTION

A set of music composers was chosen based on their relevance as representatives of each period of the classical music history. A sequence S of P composers include the P most visible composers in that time interval. Because this time interval vary on each period, the sequence S

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does not necessarily corresponds to an uniform time series.

As done for philosophers[4], the set of C measurements define a $C-$dimensional space referred as the *musical space*. The characteristic vector $\vec{v_i}$ of each composer i defines a respective *composer state* in the musical space.

For the set of P composers, we used the same elements defined for philosophers [4]: *average state at time i*, named $\vec{a_i}$; the *opposite state* of a given composer state $\vec{v_i}$, named $\vec{r_i}$; the *opposition vector* of composer state $\vec{v_i}$, named $\vec{D_i}$; and the *opposition amplitude* of that same state, $||\vec{D_i}||$.

The dialectics is quantified between a triple of successive composers i, j and k of the given set P. This makes possible to examine whether the dialectics remains an interesting relation between music composers like it was for philosophers.

III. MUSICAL CHARACTERISTICS

To create the musical space we derived eight variables corresponding to distinct characteristics commonly found on music compositions. The characteristics are related with the basic elements of music – melody, harmony, rhythm, timbre, form and tessitura[11] – and non-musical issues like historical events that have influenced the compositions, for example, the presence of Church along the early development of music. All the eight characteristics are listed below:

Sacred - Secular (*S-P*): the sacred or religious music is composed through religious influence or used for its purposes. *Masses*, *Motets* and hymns, dedicated to the Catholic liturgy, are well known examples[2]. Secular music is its opposite, having no relation with religion, also known as popular songs like Italian madrigals and German *Lieds*[11].

Short duration - Long duration (*S-L*): compositions are quantified having short duration when it does not have more than few minutes of execution. Long duration compositions have at least 20 minutes of execution or more. The same consideration was did by Kozbelt[7,8] with his analysis of time execution.

Harmony - Counterpoint (*H-C*): harmony emphasizes the harmonic discourse, considering the priority use of just one single melody, contrasting with counterpoint, which Bach's fugues are the better examples[11].

Vocal - Instrumental (*V-I*): compositions using just vocals (e.g. *cantata*) or exclusively instruments (e.g. *sonata*). It is interesting to note the use supreme of vocals over instruments on Sacred compositions[2].

Non-discursive - Discursive (*N-D*): compositions based or not on verbal discourse, like programmatic music or Baroque rhetoric, where the composer wants to "tell a history" invoking images to the listeners mind[11]. Its contrary part is known as *absolute music* where the music is written to be appreciated simply by what it is.

Motivic Stability - Motivic Variety (*M-V*): motivic pieces presents equilibrium between repetition, reuse and variation of melodic motives. The other vary the use of new materials along the composition. Bach is noticeable by his *development by variation* of motives, contrasting with the constantly inventive use of new materials by Mozart[1].

Rhythmic Simplicity - Rhythmic Complexity (*R-P*): presence or not of polyrhythms, the use of independent rhythms at the same time – also known as *rhythmic counterpoint*[11] – a characteristic constantly found on the works of 20th-century composers like Stravinsky.

Harmonic Stability - Harmonic Variety (*T-M*): rate of tonality change on a piece or its stability. After the highly polyphony development in Renaissance, Beethoven is credited as the composer who returned to the maximum exploration of harmonic variety[1].

IV. RESULTS AND DISCUSSION

Memorable composers were chosen as key representatives of the musical development. The set is ordered chronologically and presented on Table I with each composer related with its historical period.

TABLE I. The set of music composers ordered chronologically with the outstanding period their represents.

Composers	Eras
Monteverdi	Renaissance
Bach	Baroque
Mozart	Classical
Beethoven	Classical → Romantic
Brahms	Romantic
Stravinsky	20th-century
Stockhausen	Contemporary

The quantification of the eight musical characteristics was performed jointly by the authors of this article and is shown in Table II. The scores were numerical values between 1 and 9. Values more close of 1 reveals the composer tended to the first element of each characteristic pair and vice versa. It is important to note the efficiency of this method concerning possible errors due to a subsequent perturbation of original scores. If this initial step of analysis was susceptible to errors, all the proposed work would be bound to failure. This perturbation method is better explained in this section.

This data set defines an 8-dimensional musical space, each dimension corresponding of one characteristic. The Pearson correlation coefficients between the eight musical characteristics chosen are presented in Table III. The coefficients with absolute value larger than 0.5 are emphasized.

We can identify some interesting relations between the pairs of characteristics that reflect important facts found on music history. For instance, the Pearson correlation coefficient of 0.95 was obtained for the pairs S-P (Sacred or Secular) and V-I (Vocal or Instrumental), which indicates sacred music is more vocal then instrumental. The

TABLE II. Quantification of the eight music characteristics for each of the seven composers.

Composers	S-P	S-L	H-C	V-I	N-D	M-V	R-P	T-M
Monteverdi	3.0	8.0	5.0	3.0	7.0	5.0	3.0	7.0
Bach	2.0	6.0	9.0	2.0	8.0	2.0	1.0	5.0
Mozart	6.0	4.0	1.0	6.0	6.0	7.0	2.0	2.0
Beethoven	7.0	8.0	2.5	8.0	5.0	4.0	4.0	7.0
Brahms	6.0	6.0	4.0	7.0	4.5	6.5	5.0	7.0
Stravinsky	8.0	7.0	6.0	7.0	8.0	5.0	8.0	5.0
Stockhausen	7.0	4.0	8.0	7.0	5.0	8.0	9.0	6.0

TABLE III. Pearson correlation coefficients between the eight musical characteristics.

-	S-P	S-L	H-C	V-I	N-D	M-V	R-P	T-M
S-P	-	-0.16	-0.35	**0.95**	-0.43	**0.56**	**0.74**	-0.05
S-L	-	-	-0.07	-0.12	0.26	**-0.61**	-0.15	**0.63**
H-C	-	-	-	-0.45	0.43	-0.29	0.27	0.25
V-I	-	-	-	-	**-0.65**	**0.55**	**0.64**	0.09
N-D	-	-	-	-	-	**-0.61**	-0.24	-0.3
M-V	-	-	-	-	-	-	**0.55**	-0.16
R-P	-	-	-	-	-	-	-	0.27
T-M	-	-	-	-	-	-	-	-

coefficient of 0.74 also shows it does not commonly use polyrhythms as we can see analysing the pairs S-P and R-P (Rhythmic Simplicity or Complexity). Negative coefficients indicate the contrary direction like the value of -0.65 of the pairs V-I and N-D (Non-discursive or Discursive) indicating composers that used just voices on their compositions also preferred to use programmatic musics techniques like baroque rhetoric. To reduce the number of dimensions – making possible to visualize the musical space as a planar graph, filtering just the relevant characteristics – PCA was applied to this set of data, yielding the new variances given in Table IV in terms of percentages of total variance. We can note the concentration of variance along the two first PCA axes, a common effect also examined while analysing philosophers characteristics[4]. This means we could consider just two dimensions, yielding a planar musical space.

TABLE IV. New variances after PCA, in percentages for scores on III.

Eigenvalue	Value
λ_1	45.62 %
λ_2	22.32 %
λ_3	17.32 %
λ_4	11.09 %
λ_5	3.62 %
λ_6	0.26 %
λ_7	0. %

As done for philosophers analysis, we performed 1000 perturbations of the original scores by adding the values -2, -1, 0, 1 and 2 with uniform probability. In other words, we wanted to test if scoring errors could be sufficient to cause relevant effects on the PCA projections. Interestingly, the values of average and standard deviation for both original and perturbed positions listed on Table V show relatively small perturbations. It is therefore reasonable to say small errors in the values assigned as scores of composers characteristics did not affected too much its quantification.

TABLE V. Average and standard deviation of the deviations for each composer and for the first 4 eigenvalues.

Composers	μ_Δ	σ_Δ
Monteverdi	1.5398	0.8598
Bach	1.2971	0.8858
Mozart	3.2419	1.9023
Beethoven	1.8649	1.0310
Brahms	1.2971	0.7041
Stravinsky	1.3389	0.6496
Stockhausen	1.7097	0.9447
Eigenvalues	μ_Δ	σ_Δ
λ_1	-0.0391	0.0519
λ_2	0.0336	0.0343
λ_3	-0.0034	0.0283
λ_4	-0.0114	0.0261

Table VI shows the normalized weights of the contributions of each original property on the two new main axes. Most of the characteristics contribute almost equally in defining the two main axes, letting us to consider a 2-dimensional space presented in Figure 1.

TABLE VI. Percentages of the contributions from each musical characteristic on the two new main axes.

Musical Characteristics	C_1	C_2
S-P	18.46	4.30
S-L	7.83	25.56
H-C	8.32	7.72
V-I	19.05	6.57
N-D	14.92	3.38
M-V	16.97	9.58
R-P	14.32	12.28
T-M	0.12	30.60

The arrows follow the time sequence along with the seven composers. Each of these arrows corresponds to a musical move from one composer state to another – for clarity, just the lines of the arrows are preserved. The graph shows interesting results.

Bach, as could be expected, is positioned far from the rest of composers. It is related of his incomparable genius, notoriously admitted by other great composers like Beethoven and Webern[1]: "In fact Bach composed everything, concerned himself with everything that gives food for thought!". We can identify a strong relationship between Beethoven and Brahms, reflecting the belief of the *virtuosi* Hans von Bülow[12] when he stated the 1^{st} Symphony of Brahms as, in reality, being the 10^{th} *Symphony of Beethoven*, clamming Brahms as the true successor of Beethoven. Stravinsky is at the cen-

ter of all relations, presumably due to his heterogeneity[2,11]. For Webern, Beethoven was the unique classicist who really came close to the coherence found in the pieces of the Burgundian School: "Not even in Haydn and Mozart do we see these two forms as clearly as in Beethoven. The period and the eight-bar sentence are at their purest in Beethoven; in his predecessors we find only traces of them"[1]. It could explain the proximity of Beethoven to the Renaissance Monteverdi. Mozart is the most deviating point in the space, even when compared with Beethoven who was deeply influenced by Mozart, mainly in his early works. This is possibly explained by the overly complexity of Mozart works, just influenced by his contemporary Haydn[2]. Maybe Stockhausen could present similar detachment if we considered more vanguard characteristics – e.g. timbre exploration by using electronic devices[2] – not shared by his precursors. In general, the musical movements have minor opposition, and remembering the beginning of this work, it reflects the apprentice-master tradition present on music: the composers tend to build their own works based on its precursor. This reveals a crucial difference considering the *memory state* along the development of philosophy and music: while a philosopher was influenced by the opposition of ideas of its two predecessors, composers were commonly influenced by their direct predecessor. Therefore, we can argue that philosophy presents a *memory-2* state, while music presents *memory-1*, considering *memory-N* being the number N of past generations that influenced a philosopher or composer.

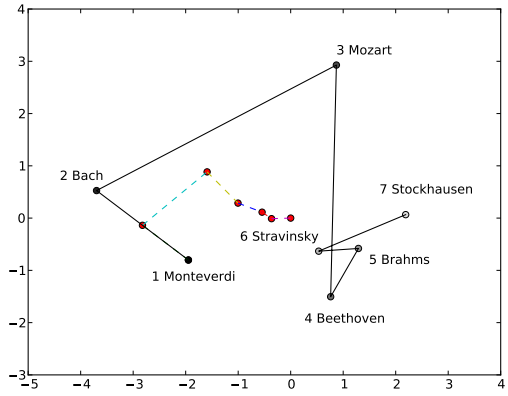

FIG. 1. *2-dimensional projected musical space.*

To complement the analysis, Table VII gives the opposition and skewness indices for each of the six musical moves, showing the movements are driven by rather small opposition and strong skewness. In other words, most musical moves do not benefit from opposition as far as innovation is concerned. Dialectics is also analyzed on Table VIII where we identified an alternation of values along the pairs of subsequent musical movements: the first value of counter-dialectics is greater than the second, that is lesser than the third and so on. There is no strong dialectics, but a continuous variation.

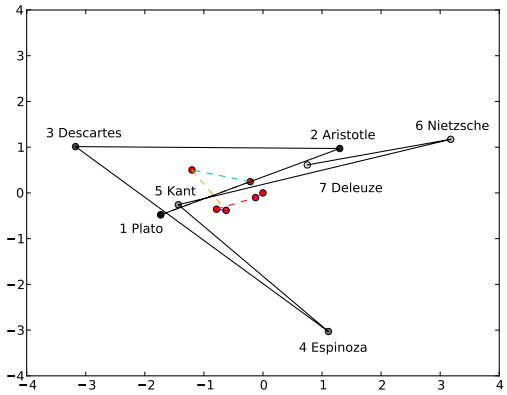

FIG. 2. *2-dimensional projected philosophical space*[4].

TABLE VII. Opposition and skewness indices for each of the six musical moves.

Musical Move	$W_{i,j}$	$s_{i,j}$
Monteverdi → Bach	1.0	0
Bach → Mozart	1.1484	1.6028
Mozart → Beethoven	0.5679	2.4759
Beethoven → Brahms	0.0929	0.9919
Brahms → Stravinsky	0.1996	0.2954
Stravinsky → Stockhausen	-0.3224	1.7169

TABLE VIII. Counter-dialectics index for each of the five subsequent pairs of musical moves.

Musical Triple	$d_{i\to k}$
Monteverdi → Bach → Mozart	0.495
Bach → Mozart → Beethoven	0.082
Mozart → Beethoven → Brahms	0.289
Beethoven → Brahms → Stravinsky	0.104
Brahms → Stravinsky → Stockhausen	1.741

V. COMPARISONS WITH PHILOSOPHERS ANALYSIS

The results of composers analysis when compared with philosophers reveals surprising results. If we compare the discussed musical space with the philosophical one of Figure 2 we can identify opposite movements along all the philosophy history in contrast to music. This reveals a notorious characteristic of the way philosophers seem to have evolved their ideas, driven by opposition, while composers tend to inherit characteristics from the works of their predecessors.

TABLE IX. Opposition and skewness indices for each of the six philosophical moves[4].

Philosophical Move	$W_{i,j}$	$s_{i,j}$
Plato → Aristotle	1.0	0
Aristotle → Descartes	0.8622	0.8656
Descartes → Espinoza	0.9803	1.4930
Espinoza → Kant	0.5693	0.4715
Kant → Nietzsche	0.8021	0.8726
Nietzsche → Deleuze	0.3647	0.3148

TABLE X. Counter-dialectics index for each of the five subsequent pairs of philosophical moves.

Philosophical Triple	$d_{i\to k}$
Plato → Aristotle → Descartes	0.700
Aristotle → Descartes → Espinoza	0.466
Descartes → Espinoza → Kant	0.137
Espinoza → Kant → Nietzsche	0.048
Kant → Nietzsche → Deleuze	0.015

The opposition and skewness indices for philosophers listed in Table IX, endorse the minor role of opposition in composers. We can note strong opposition and rather small skewness on philosophical moves while small opposition and strong skewness on musical moves. A fact that could explain the differences is the more homogeneous distribution of scores on composers, while on philosophers the scores shown significantly quantitative difference, concerning the nature of their characteristics.

When comparing dialectics, other curious facts arise: the dialectics indices on Table X are considerably stronger philosophical moves than for composers. Both indices are also shown in Figure 3 where we can see a constantly decrease of counter-dialectics, contrasting the continuously variation of the indices when considering the composers. This makes possible to argue that dialectics is stronger on philosophy than on music where a constantly return to the origins are clearly visible on some composers. This reveals the nature of the musical development, based on the search for a unity. Using the words of Webern, the search for the "comprehensibility", but always inheriting the teachings of their old masters.

VI. CONCLUDING REMARKS

Motivated by the understanding of how innovation on music history evolves compared with philosophy, we extended a quantitative method recently applied to the study of philosophical characteristics. Statistics methods have been historically applied to the study of music features and composers productivity, but not for the analysis of composers characteristics. The method differs from the others on the aspect of how the characteristics concerning composers are treated: scores are attributed for each feature commonly noticeable on the works of renowned composers. These scores reveals not the exact

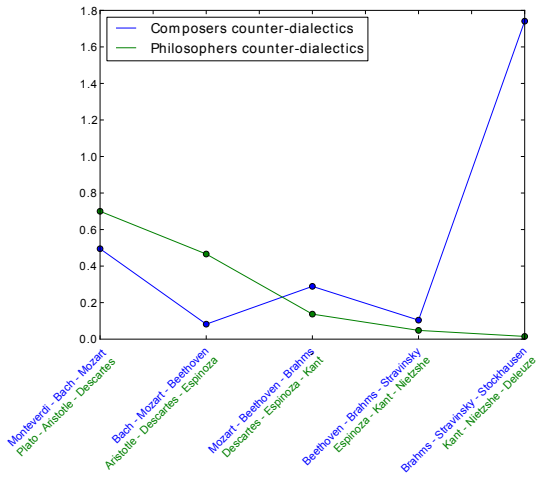

FIG. 3. *Comparison between composers and philosophers counter-dialectics indices*

profile of composers but a tendency of how their composition techniques relate with each other. In order to investigate the relationship between this scoring we applied Pearson correlation analysis. The results demonstrated a strong correlation between the characteristics, which allows us to group this values, creating a reduced number of features that summarizes the most important characteristics. PCA was also applied to these components, reducing the complex space to a planar graph where the most interesting properties were identified.

Historical landmarks in music are well-defined on the graph, like the isolated geniuses of Bach and Mozart, the proximity between Beethoven and Brahms, the heterogeneity of Stravinsky or the vanguard of contemporary composers like Stockhausen. Even not so visible relations, like the return to the maximum domain of polyphony – present on Renaissance – by Beethoven could also be clearly observable.

The dichotomy between master-apprentice tradition on music and the quest for innovation that opened this discussion could be visualized quantitatively. Each composer demonstrated his own style, differing considerably from his predecessor – clearly shown when analyzing pairs of subsequent composers like Bach and Mozart, Mozart and Beethoven or Stravinsky and Stockhausen. Otherwise, the inheritance of predecessors styles is also present when analyzing the direct relations between Beethoven and Brahms, or indirect ones between Bach and Beethoven or Beethoven and Monteverdi. The entire scenario presented a "cyclical pattern" between composers – motivated by the influence of theirs predecessors – but also showed a force repelling both of them: the innovation, or in the words of William Lovelock, the "experimentation" that makes progress possible.

Along the analysis we noticed interesting differences

when comparing composers with philosophers. While on philosophy the innovation is notably marked by opposition of each philosophers ideas, it is less present for music composers. The lack of strong opposition movements in musical space indicates the music innovation is driven by a constant inheritance of each composer from his predecessors. We represented this characteristic referring to a *memory state* where philosophers shows *memory-2* – each philosopher was influenced by the opposite ideas of its two predecessors – while composers shows *memory-1* – inheriting the style of their direct predecessor. The analysis of both dialectics values also shown surprising results: while on philosophy the dialectics indices are arranged on a increasing series – showing a strong influence of dialectics to philosophy development – the same dialectics indices on music exhibits a constantly variation. This behavior presumably indicates a constantly quest for coherence by the composers, a fact previously observed by the studies of Anton Webern.

The quantitative methodology initially applied to the analysis of philosophy proved to be extensible to other fields of knowledge like music, reflecting with considerable efficiency, specific details concerning each field. Computational analysis of music scores could be applied to automate the quantification of composers characteristics, like identification of melodic and harmonic patterns or the presence or not of polyrhythms, motivic and harmonic stability[13]. More composers could be inserted on the set for the analysis of a wider time-line, possibly including more representatives of each music periods.

While taking the first steps on the direction of a quantitative approach to arts and philosophy we believe that an understanding of the creative process could also be eventually quantified. We want to end this work quoting Webern again, who early envisioned these relations: "It is clear that where relatedness and unity are omnipresent, comprehensibility is also guaranteed. And all the rest is dilettantism, nothing else, for all time, and always has been. That's so not only in music but everywhere."

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