Philosometrics

Renato Fabbri,[a] Osvaldo N. Oliveira Jr.,[b] and Luciano da Fontoura Costa[c]
Instituto de Física de São Carlos, Universidade de São Paulo (IFSC/USP)

(Dated: 29 February 2012)

The hypothesis that philosophy is driven by difference/innovation is checked in a quantitative manner. This was performed by assigning grades to eight main philosophical features with respect to seven prominent philosophers, which allowed sound concepts from multivariate statistics and pattern recognition to be applied. A number of insights could then be inferred about the way in which philosophy has been developed. For instance, the evolution of philosophy can be represented as a trajectory on a plane (two instead of the eight original dimensions) whose axes are defined by a linear combination of the philosophical characteristics considered. In addition, all the philosophical moves have been verified to oppose the prevailing philosophical state. We also identified an intense and progressive trend toward dialectics.

PACS numbers: 89.75.Fb,05.65.+b
Keywords: philosophy, pattern recognition, statistics

'One cannot conceive anything so strange and so implausible that it has not already been said by one philosopher or another.'
- *R. Descartes*

I. INTRODUCTION

We wished to start this work in a different way. So, we chose to begin it by stating this objective. By doing so, we immediately incurred into a paradox, for most authors want to be different from the outset and that objective would ultimately imply in being no different at all. In other ways, many authors have tried to distinguish themselves by being different. We believe philosophers are no different in this respect, i.e. they inherently want not to be equal and therefore tend to develop new philosophical views and paradigms. This was the original motivation for the present work.

The emphasis on the quest for difference in philosophy has been expressed by Gilles Deleuze[1], but can also be immediately related to other previous approaches such as Ferdinand de Saussurre's principle[2], in which concepts (words) tend to be different in the sense of meaning distinct things. The paradigm of difference is particularly promising because it is immediately related to the own dynamics of philosophical evolution along time. In other words, what are the forces that drive philosophical changes and innovation? Could dialectics account for some of the main philosophical moves?

In order to try and answer these questions in a more objective way, we resorted to a quantitative approach which turned out to provide a number of remarkable spin-offs and insights. More specifically, we started by identifying prominent philosophers along the history of western philosophy, a set of main philosophical issues, and then assign grades to each of these issues for the chosen philosophers. It should be stressed that we chose a reduced set of philosophers for the sake of simplicity and clarity. Though limited by the arbitrary procedure with which the grades were assigned, the quantification of the main philosophers (and consequently of their views) characteristics paved the way to the application of sound concepts and methods from multivariate statistics[3–5] and pattern recognition[6,7]. Considering the small number of philosophers – which could bias the statistical analysis – we developed a bootstrap method to generate *artificial philosophers*, representing the contemporaries of the representatives chosen. Those individuals were historically influenced by the representatives and this influence was modeled by a distribution probability. In this way, a larger data set was generated, considering 500 new philosophers directly influenced by the originals samples. We checked for correlations between the characteristics, and then applied principal component analysis (PCA)[7] to investigate the dispersion of the philosophical principles. We also considered the time sequence of the philosophers, which allowed us to define respective trajectories of philosophical evolution. By proposing indices to quantify the degree of opposition, skewness and dialectics along the dynamics of philosophical evolution, we observed surprising results.

This work starts by presenting the methodology adopted and then presents the definition of the eight philosophical characteristics chosen, followed by the description of the results and discussion.

[a] `www.estudiolivre.org/el-user.php?view\_user=gk`; Electronic mail: renato.fabbri@gmail.com
[b] `www.polimeros.ifsc.usp.br/professors/professor.php?id=4`; Electronic mail: chu@ifsc.usp.br; Also at IFSC-USP
[c] `http://cyvision.ifsc.usp.br/~luciano/`; Electronic mail: ldfcosta@gmail.com; Also at IFSC-USP

II. MATHEMATICAL DESCRIPTION

The choice of philosophers is inherently important for the time-evolution analysis, as each philosophical move and its characteristics (i.e. opposition and skewness) are defined by each pair of subsequent philosophers along time. We decided to choose the philosophers taking into account their historical importance and visibility. A sequence S of P philosophers along a given period of time would incorporate the P most prominent and visible philosophers in that interval. The use of such a criterion to build the time-sequence for the philosophers implies in not necessarily uniform time-intervals between each pair of subsequent entries.

The set of C measurements used to characterize the philosophers define a $C-$dimensional feature space which will be henceforth referred to as the *philosophical space*. The characteristic vector $\vec{v_i}$ of each philosopher i defines a respective *philosophical state* in the philosophical space. Given a set of P philosophers, the *average state at time* i, $i \leq P$, is defined as

$$\vec{a_i} = \frac{1}{i}\sum_{k=1}^{i} \vec{v_k}. \tag{1}$$

The *opposite state* of a given philosophical state $\vec{v_i}$ is defined as:

$$\vec{r_i} = \vec{v_i} + 2(\vec{a_i} - \vec{v_i}) = 2\vec{a_i} - \vec{v_i} \tag{2}$$

The *opposition vector* of philosophical state $\vec{v_i}$ is given us by $\vec{D_i} = \vec{r_i} - \vec{v_i}$. The *opposition amplitude* of that same state is defined as $||\vec{D_i}||$.

An *emphasis move* taking place from the philosophical state $\vec{v_i}$ is any displacement from $\vec{v_i}$ along the direction $-\vec{r_i}$. A *contrary move* from the philosophical state $\vec{v_i}$ is any displacement from $\vec{v_i}$ along the direction $\vec{r_i}$.

Given a time-sequence S of P philosophers, the *philosophical move* implied by two successive philosophers i and j corresponds to the $\vec{M_{i,j}}$ vector extending from $\vec{v_i}$ to $\vec{v_j}$, i.e.

$$\vec{M_{i,j}} = \vec{v_j} - \vec{v_i} \tag{3}$$

In principle, an innovative or differentiated philosophical move would be such that it departs substantially from the current philosophical state $\vec{v_i}$. We decided to decompose innovation moves into two main subtypes: opposition and skewness.

The *opposition index* $W_{i,j}$ of a given philosophical move $\vec{M_{(i,j)}}$ is defined as

$$W_{i,j} = \frac{\left\langle \vec{M_{i,j}}, \vec{D_i} \right\rangle}{||\vec{D_i}||^2}. \tag{4}$$

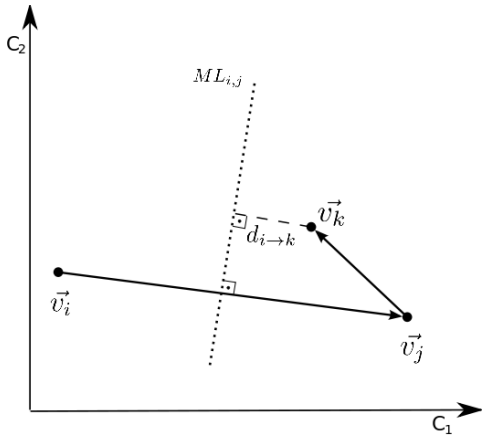

FIG. 1. *Graphical representation of the quantification of dialectics.*

This index quantifies the intensity of opposition of that respective philosophical move, in the sense of having a large projection along the vector $\vec{D_i}$. It should also be noticed that the repetition of opposition moves lead to little innovation, as it would imply in an oscillation around the average state.

The *skewness index* $s_{i,j}$ of that same philosophical move is the distance beetwen $\vec{v_j}$ and the line L_i defined by the vector $\vec{D_i}$, and therefore quantifies how much the new philosophical state departs from the respective opposition move. Actually, a sequence of moves with zero skewness would represent more trivial oscillations within the opposition line L_i.

We also suggest an index to quantify the dialectics between a triple of successive philosophers i, j and k. More specifically, the philosophical state $\vec{v_i}$ is understood as the *thesis*, the state j is taken as the *antithesis*, with the synthesis being associated to the state $\vec{v_k}$[8]. The hypothesis that k is the consequence, among other forces, of a dialectics between the views $\vec{v_i}$ and $\vec{v_j}$ can be expressed by the fact that the philosophical state $\vec{v_k}$ be located near the middle line $ML_{i,j}$ defined by the thesis and antithesis (i.e. the points which are at an equal distance to both $\vec{v_i}$ and $\vec{v_j}$) relatively to the opposition amplitude $||\vec{D_i}||$.

Therefore, the *counter-dialectic index* is defined as

$$\rho_{i\to k} = d_{i\to k}/||\vec{M_{i,j}}||, \tag{5}$$

where $d_{i\to k}$ is the distance between the philosophical state $\vec{v_k}$ and the middle-line $ML_{i,j}$ between $\vec{v_i}$ and $\vec{v_j}$[9]. Note that $0 \leq d_{i\to k} \leq 1$. The choice of counter-dialectics instead of dialectics is justified to maintain compatibility with the use of a distance from point to line as adopted for the definition of skewness.

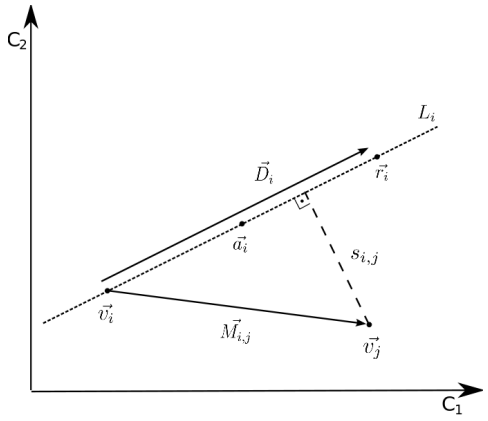

FIG. 2. *Graphical representation of the measures derived from a* philosophical move.

III. PHILOSOPHICAL AXES

We derived eight variables corresponding to some of the most recurrent philosophical issues[8,10,11]. Each of these variables, which define a respective axis in the philosophical space, are briefly described in the following.

Rationalism - Empirism (*R-E*): the rationalists claim that the human acquaintance of knowledge/concepts is significantly independent of sense experience. Empiricists understand sense experience as the main way to gain knowledge/concepts. Frequently, rationalists take the view that the world is affected by intrinsic properties of the human brain, in contrast to the empiricist approach where the world would imprint itself onto our minds.

Essence - Existence (*E-E*): An existence-based understanding of the world has its basis on the fact that things are as an existent unit. Essence focuses on a substance (e.g. intellectual) that precedes existence itself.

Monism - Dualism (*M-D*): Dualism requires the division of the human person into two or more domains, such as matter and soul. Monism is based on a unique "category of being".

Theocentrism - Anthropocentrism (*T-A*): In theocentrism, God is the most important thing in the universe. The anthropocentric view has man as prevalent.

Holism - Reductionism (*H-R*): Reductionism attempts to explain the world in terms of simple components and its emerging properties. Holists focus on the fact that the whole is more than its constitutive parts.

Deductionism - Phenomenology (*D-P*): Phenomenology relies on systematic reflection of consciousness and what happens in conscious acts. Deductionism is based on deriving conclusions from axiomatic systems.

Determinism - Free Will (*D-F*): Free will assumes that humans make choices and these are not predetermined. Determinism understands that every event is fatidic, e.g., perfectly determined by prior states.

TABLE I. Quantification of the eight philosophical characteristics for each of the seven philosophers.

Philosophers	R-E	E-E	M-D	T-A	H-R	D-P	D-F	N-M
Plato	3.0	3.5	9.0	5.	4.5	3.5	5.0	4.5
Aristotle	8.0	7.5	7.0	5.5	7.5	8.0	2.5	2.5
Descartes	1.5	2.5	9.0	6.5	7.0	2.5	7.5	7.5
Espinoza	8.0	2.0	1.0	5.0	2.0	3.0	1.0	1.0
Kant	7.0	2.5	8.5	6.5	4.5	3.5	7.5	5.0
Nietzsche	7.5	9.0	1.0	9.0	5.0	8.0	1.0	1.5
Deleuze	5.5	7.5	1.0	8.	2.5	5.5	5.0	6.0

Naturalism - Mechanism (*N-M*): Methodological naturalism is the thinking basis of modern science, i.e. hypotheses must be argued and tested in terms of natural laws. Mechanism attempts to build explanation using logic-mathematical processes.

IV. RESULTS AND DISCUSSION

A set of seven philosophers were chosen spanning the period from Classical Greece until contemporary times, and ordered chronologically as: Plato, Aristotle, Descartes, Espinoza, Kant, Nietzsche, and Deleuze. The quantification of the eight philosophical characteristics was performed jointly by two of the authors of this article and is shown in Table I.

This dataset defines an 8-dimensional philosophical space for the group of 7 representative philosophers, a small data set, not suitable for statistical analysis. To increase the number of philosophers we generated random scores. Along the history, a reduced number of representatives influenced their contemporaries. We modeled this influence generating samples that follows a probability distribution based on the original scores. In this way, the generated samples are not totally random values but respected the influence of the 7 representatives chosen. A bootstrap method was used to create each sample, generating a randomized score $\vec{r}$ that follows a probability distribution given by $p(\vec{r}) = \sum_{i=1}^{n} e^{\frac{d_i}{2\sigma^2}}$ where d_i is the distance between a random score $\vec{r}$ and the original scores. Following the Monte Carlo method, we calculated $p(\vec{r})$ for each step and compared it with a random normalized value, selecting just the samples influenced by the originals. We considered $\sigma = 1.1$ for this analysis, generating 1000 samples using the bootstrap method.

Table II presents the Pearson correlation coefficients between the eight philosophical characteristics chosen, already considering the samples obtained by the bootstrap method together with the original scores. The coefficients with absolute value larger than 0.35 are emphasized. Strong positive or negative correlations can be observed for several pairs of characteristics. For instance, the fact that a Pearson correlation coefficient of -0.46 was obtained for the pair of characteristics *R-E* and *N-M* indicates that philosophers who are rationalists strongly

TABLE II. Pearson correlation coefficients between the eight philosophical characteristics. The entries with absolute value larger or equal than 0.35 have been emphasized.

-	R-E	E-E	M-D	T-A	H-R	D-P	D-F	N-M
R-E	1.00	0.37	-0.23	0.15	0.1	***0.46***	-0.27	***-0.46***
E-E	-	1.00	***-0.53***	0.19	0.15	***0.74***	***-0.61***	-0.3
M-D	-	-	1.00	***-0.43***	***0.41***	-0.3	***0.35***	0.01
T-A	-	-	-	1.00	-0.21	0.06	0.19	0.26
H-R	-	-	-	-	1.00	0.32	-0.22	-0.25
D-P	-	-	-	-	-	1.00	***-0.63***	***-0.47***
D-F	-	-	-	-	-	-	1.00	***0.61***
N-M	-	-	-	-	-	-	-	1.00

TABLE III. New variances after PCA, in percentages.

Eigenvalue	Value
λ_1	41 %
λ_2	23 %
λ_3	12 %
λ_4	11 %
λ_5	5 %
λ_6	4 %
λ_7	3 %
λ_8	2 %

tend to be also mechanicists. An even stronger correlation of 0.74, now positive, is observed between *E-E* and *D-P*, suggesting that existentialists also tend to be phenomenologists, as could be expected. Other strong correlations were observed, including a Pearson coefficient of 0.61 between *D-F* and *N-M*. Also interesting is the relatively high correlation between *M-D* and *D-F*, which seems to be directly implied by religious background.

PCA was applied to this set of data, yielding the new variances given in Table III in terms of the percentages of the total variance. It is clear that the four first PCA axes account for 85% of the total variance of the dataset in the original philosophical space. It is therefore reasonable to say that the properties of the philosophers can be well described and analyzed while considering just four dimensions.

In order to investigate the effect from the unavoidable errors in the quantification of the philosophical characteristics, we performed 1000 perturbations of the original scores by adding the values -2, -1, 0, 1, and 2 with uniform probability. Table IV shows the average and standard deviation of the deviations obtained for each philosopher considering the original and perturbed positions, as well as the average and standard deviation of the first 4 eigenvalues (i.e. the variances of the perturbed configurations). Interestingly, these results show relatively small effects of the perturbations on the PCA projections and variances. In other words, the assignment of the original measurements does not seem to be too critical for the results.

It is particularly striking that the seeming complexity of the philosophical traits quantified in terms of the eight characteristics boiled down to no more than four

TABLE IV. Average and standard deviation of the deviations for each philosopher and for the 8 eigenvalues.

Philosophers	μ_Δ	σ_Δ
Plato	3.3263	0.7673
Aristotle	4.0896	0.8930
Descartes	4.3081	0.9225
Espinoza	4.9709	0.9131
Kant	3.2845	0.7749
Nietzsche	5.3195	0.9797
Deleuze	4.0990	0.8970
Eigenvalues	μ_Δ	σ_Δ
λ_1	-0.2618	0.0068
λ_2	-0.0976	0.0035
λ_3	0.0154	0.0025
λ_4	0.0212	0.0024
λ_5	0.0697	0.0026
λ_6	0.0807	0.0030
λ_7	0.0877	0.0032
λ_8	0.0846	0.0036

TABLE V. Percentages of the contributions from each philosophical characteristic on the four new main axes.

Philosophical Characteristics	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
R-E	13.35	1.81	13.11	31.12	15.06	10.99	8.20	3.56
E-E	18.13	7.88	2.10	14.68	11.41	5.31	7.98	33.50
M-D	10.07	24.59	10.71	2.91	0.69	19.39	20.95	2.91
T-A	1.08	24.18	21.51	0.68	23.72	6.31	7,27	2.06
H-R	5.90	21.49	22.29	14.90	6.35	18.81	8.69	2.05
D-P	18.99	1.29	6.85	8.41	9.67	20.50	11.47	24.93
D-F	14.57	13.67	8.37	18.93	21.12	9.06	12.77	14.98
N-M	17.86	5.06	15.03	8.33	11.96	9.59	22.62	15.97

dimensions for their representation. This is a direct consequence of the high Pearson correlation coefficients between several of those characteristics, meaning that most of them tend to be related.

Also shown in Table V are the normalized weights (as percentages) of the contributions of each original property on the four new main axes. Most of the characteristics contribute almost equally in defining the four main axes. Consequently, we now have two possibilities for representing the philosophical space: to consider almost equally all these characteristics or to define four new philosophical features as given by the PCA.

Making possible to represent the philosophical development as a planar space, the 2-dimensional projected space is presented in Figure 3, where the arrows follow the time sequence along with the chosen philosophers. Recall that each of these arrows corresponds to a philosophical move.

Table VI gives the opposition and skewness indices for each of the six philosophical moves.

All the philosophical moves tend to take place according to a well-defined and intense opposition from the average state. Also surprisingly, rather small skewness has been found to underlie most philosophical moves, meaning that most philosophical moves are driven al-

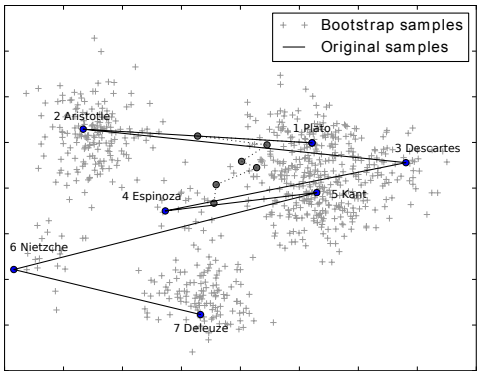

FIG. 3. *2-dimensional projected philosophical space.*

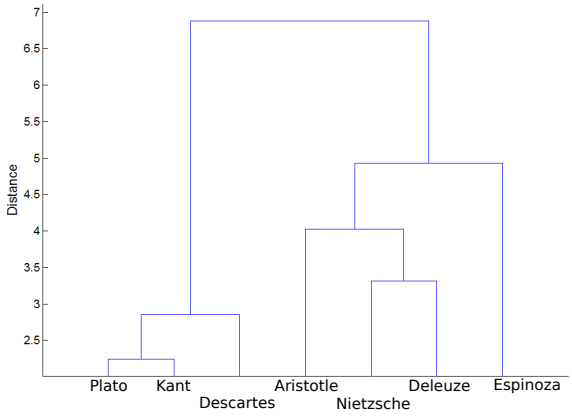

FIG. 4. *Wards hierarchical clustering of the seven philosophers considering all the eight features.*

TABLE VI. Opposition and skewness indices for each of the six philosophical moves.

Philosophical Move	$W_{i,j}$	$s_{i,j}$
Plato → Aristotle	1.0	0
Aristotle → Descartes	0.8740	1.1205
Descartes → Espinoza	0.9137	2.3856
Espinoza → Kant	0.6014	1.6842
Kant → Nietzsche	1.1102	2.9716
Nietzsche → Deleuze	0.3584	2.4890

most exclusively by opposition to the current philosophical state. Remarkable results have been obtained also for dialectics. We identified progressively stronger dialectics trends among subsequent pairs of philosophical moves.

In order to complement our analysis of the relationship between the philosophers, we applied Wards hierarchical clustering algorithm[6,7] considering the eight original features. This methodology clusters the individuals progressively taking into account their intra-cluster dispersion, so that the obtained dendrogram reflects the similarity between the philosophers. Two main groups can be identified, one including Plato, Descartes and Kant, and the other containing the remainder philosophers. The former group, which can also be identified in the left-hand side of the PCA diagram in Figure 3.

TABLE VII. Counter-dialectics index for each of the five subsequent pairs of philosophical moves.

Philosophical Triple	$d_{i\to k}$
Plato → Aristotle → Descartes	3.0198
Aristotle → Descartes → Espinoza	1.8916
Descartes → Espinoza → Kant	1.1536
Espinoza → Kant → Nietzsche	1.1530
Kant → Nietzsche → Deleuze	0.2705

V. CONCLUDING REMARKS

Though science and philosophy share the same objective of explaining, modeling, understanding and predicting nature, an essential difference between them remains in that the latter has not relied systematically on quantification of the natural world. Several of the ambiguities of the philosophical debate can be ultimately identified as consequences of such a lack of quantitative rigour.

In this work, we reported a quantitative approach to characterize and analyze philosophical thought based on the assignment of scores to several philosophers regarding eight principal attributes that have characterized philosophical thought. In doing so, it became possible to apply several concepts from multivariate statistics and pattern recognition areas, and derive insights on the nature of philosophical thinking along its historical evolution. A bootstrap method was applied to create a better representation of the true history: not just the seven philosophers chosen were responsible to development of the whole philosophy history, other contemporaries gave their contribution but influenced by some representatives. The bootstrap method generated samples representing those contemporaries philosophers and the influence was modeled as a probability distribution. The high Pearson correlation coefficients between the eight selected philosophical features revealed strong interrelationships among them, which means that they tend to go together, possibly even implying one another. Further studies would be however required to try to identify causality relationships between these characteristics.

The application of Principal Components Analysis yielded additional insights. Confirming the high level of correlations between the chosen philosophical characteristics, we found that the dispersion of the philosophical states can be effectively projected into a 2-dimensional space, with several implications. The apparent complex-

ity of philosophy as suggested by the eight original axes seems to be largely illusory, being a consequence of the intense correlations between the quantified philosophical characteristics. Furthermore, the evolution of philosophy appears to be constrained as a consequence of this mainly planar organization of the philosophical space, in the sense that the philosophical moves are mostly restricted to a plane region. Therefore, the hypothesis that philosophers seek difference and innovation seems to be fundamentally constrained by the relatively narrow space where philosophical moves have taken place.

Special attention was also given to formalizing the analysis of the time evolution of philosophy, which was performed thanks to the proposal of the opposition, skewness and counter-dialectics indices. We believe these indices can be used to quantify innovation along the development of philosophy. To our great surprise, we found that innovation in most philosophical moves have been mostly a consequence of opposition to the current philosophical state, with rather small skewness. Also surprising was the identification of strongly dialectics component in most pairs of subsequent philosophical moves. This suggests that dialectics has played indeed a key role in the development of philosophical thought.

It should be acknowledged that the scores and choice of main philosophical characteristics adopted in the current work are largely arbitrary and could be substantially improved. However, the perturbation analysis performed in this work suggests that the effect of non-systematic errors in assigning the scores does not seem to be critical and has little overall impact on the conclusions we have derived. In addition, whatever the effects of the scores and choice of main features, it should be emphasized that the proposed methodology can be readily applied to expanded and enhanced sets of scores or philosophical characteristics. As a matter of fact, this is perhaps the main contribution of this work, i.e. the proposal of a sound, formal quantitative methodology which can lead to comprehensive, objective insights about how philosophy has evolved since its earliest origins. It is also worth observing that this methodology is not restricted to individual philosophers, and it can be adapted to the investigation of philosophical schools, individual pieces (e.g. books), or even of the works of the same philosophers along distinct periods of time. It is also possible to apply this methodology to other areas such as arts and science.

Going back to the beginning of this work, we conclude that innovation/difference in philosophical development, at least as represented by the chosen eight measurements and seven philosophers, may have been driven mostly by opposition moves and dialectics in a rather narrow effective philosophical space. These results beg the question of how the philosophical space could be expanded, so as to favor innovations. We are hopeful that quantitative approaches such as the one reported here may represent some first steps in that direction.

ACKNOWLEDGMENTS

Luciano da F. Costa thanks CNPq (308231/03-1) and FAPESP (05/00587-5) for sponsorship. Renato Fabbri is grateful to CAPES and the Postgrad Committee of the IFSC. The authors thanks Debora Correa for providing the Ward's hierarchical clustering of the philosophers.

Appendix A: A Brief Explanantion of Principal Component Analysis (PCA)

In plain words, PCA is a dimensionality reduction procedure performed through axes rotation. It operates by concentrating dispersion/variance along the first new axes, which are denominated the principal components.

The technique consists in finding the eigenvectors and eigenvalues of the covariance matrix of the respective random vectors (i.e. the vectors associated with each philosophical state). The eigenvalues correspond to the variances of the new variables. When multiplied by the original feature matrix, the eigenvectors yield the new random variables which are fully uncorrelated.

For a more extensive explanation of PCA, please refer to[7] and references therein.

[1] G. Deleuze, *Difference and Repetition* (Continuum, 1968).
[2] F. de Saussure, *Course in General Linguistics* (Books LLC, 1916).
[3] A. Papoulis S. U. Pillai, *Probability, Random Variables and Stochastic Processes* (McGraw Hill Higher Education, 2002).
[4] R. A. Johnson D. W. Wichern, *Applied Multivariate Statistical Analysis* (Prentice Hall, 2007).
[5] C. W. Therrien, *Discrete Random Signals and Statistical Signal Processing* (Prentice Hall, 1992).
[6] R. O. Duda P. E. Hart D. G. Stork, *Pattern Classification* (Wiley-Interscience, 2000).
[7] L. da F. Costa R. M. C. Jr., *Shape Analysis and Classification: Theory and Practice (Image Processing Series)* (CRC Press, 2000).
[8] D. Papineau, *Philosophy* (Oxford University Press, 2009).
[9] In higher dimensional philosophical spaces, the middle-hyperplane defined by the points which are at equal distances to both $\vec{v_i}$ and $\vec{v_j}$ should be used instead of the middle-line.
[10] B. Russel, *A History of Western Philosophy* (Simon and Schuster Touchstone, 1967).
[11] F. G. G. Deleuze, *What Is Philosophy?* (Simon and Schuster Touchstone, 1991).