Tehnička sredstva automatike - zadaci

1. Za sledeće rezultate mjerenja izračunati
   (a) srednju vrijednost mjerenja
   (b) varijansu
   (c) srednju kvadratnu grešku
   (d) 68% -tni interval povjerenja i
   (e) 95% -tni interval povjerenja

   $R(\Omega) = 51, 51, 49, 50, 48, 49, 50, 50, 51$

   Rješenje:

   (a) $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{9}(3 \cdot 51 + 3 \cdot 50 + 2 \cdot 49 + 48) = 49.9$

   (b) $s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \overline{x})^2 = \frac{1}{8} \cdot 8.89 = 1.11$

   (c) $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} (x_i - \overline{x})^2} = 1.05\Omega$

   (d)

   $$s_{\overline{x}} = \frac{s}{\sqrt{n}} = \frac{s}{3} = 0.35\Omega$$
   $$\overline{x} - s_{\overline{x}} \leq x_0 \leq \overline{x} + s_{\overline{x}}$$
   $$\overline{x} - s_{\overline{x}} = 49.9 - 0.35 = 49.55$$
   $$\overline{x} + s_{\overline{x}} = 49.9 + 0.35 = 50.25$$
   $$49.55 \leq x_0 \leq 50.25$$

   (e)

   $$\overline{x} - 2 \cdot s_{\overline{x}} \leq x_0 \leq \overline{x} + 2 \cdot s_{\overline{x}}$$
   $$\overline{x} - 2 \cdot s_{\overline{x}} = 49.9 - 2 \cdot 0.35 = 49.2$$
   $$\overline{x} + 2 \cdot s_{\overline{x}} = 49.9 + 2 \cdot 0.35 = 50.6$$
   $$49.2 \leq x_0 \leq 50.6$$

$\overline{x}$ - aritmetička sredina rezultata mjerenja

s^2 - očekivani kvadrat odstupanja od srednje vrijednosti (sample variance - kada se procjenjuje varijansa na osnovu n vrijednosti iz beskonačne populacije)

s - srednja kvadratna greška (standardna devijacija ili standardno odstupanje)

s^2 i s predstavljaju mjeru ponovljivosti (preciznosti)

σ^2 - varijansa populacije kada je populacija konačna

σ - standardna devijacija kada je populacija konačna

$s_{\overline{x}}$ - standardna devijacija aritmetičke sredine

99%-tni interval povjerenja:

$[\overline{x} - 3 \cdot S_{\overline{x}}, \overline{x} + 3 \cdot S_{\overline{x}}]$

2. Potenciometar sa 1200 namotaja žice koristi se za mjerenje pozicije vratila. Ulazni opseg je od $-175°$ do $175°$. Izlazni opseg je od 0 do 10 V. Odrediti osjetljivost u Voltima po stepenu, prosječnu rezoluciju u Voltima i u procentima.

   Rješenje:

Osjetljivost = $\frac{10-0}{175°-(-175°)} = 0.0286\frac{V}{°}$

Prosječna rezolucija (%) = $\frac{100}{1200} = 0.0833\%$

Prosječna rezolucija (V) = $\frac{10-0}{1200} = 0.00833V$

3. Kalibracijom mjernog pretvarača za mjerenje sile dobijeni su sledeći podaci:

Ulazna sila (N)	Izlazni napon (V)
0	0.06
2	0.63
4	1.20
6	1.77
8	2.35
10	2.94
12	3.55
14	4.17
16	4.80
18	5.43
20	6.06

Odrediti osjetljivost u Voltima po Njutnu na 20% i 80% punog opsega.

Rješenje:

$$0-20N \Rightarrow \quad ul_{20\%} = 4N$$
$$ul_{80\%} = 16N$$

$$Osjetljivost_{20\%} = \frac{1.77V-0.63V}{6N-2N} = 0.285\frac{V}{N}$$

$$Osjetljivost_{80\%} = \frac{5.43V-4.17V}{18N-14N} = 0.315\frac{V}{N}$$

4. Na 70°C ulaz od 0 bara na mjernom pretvaraču za mjerenje pritiska proizvodi izlaz od $4mA$, a ulaz od 10 bara proizvodi izlaz od $20mA$. Mjerni pretvarač ima termičko pomjeranje nule (Thermal Zero Shift) od $0.02mA/°C$ i termičko pomjeranje osjetljivosti (Thermal Sensitivity Shift) od $0.004(mA/bar)/°C$. Odrediti:

(a) Osjetljivost mjernog pretvarača na $70°C$

(b) Osjetljivost mjernog pretvarača na $20°C$ i $120°C$

(c) Izlaz iz mjernog pretvarača za obar na $20°C$ i $120°C$

(d) Izlaz iz mjernog pretvarača za 10bar na $20°C$ i $120°C$

Rješenje:

(a)

$$Osjetljivost_{70°C} = \frac{20mA - 4mA}{10bar - 0bar} = 1.6\frac{mA}{bar}$$

(b)

$$Osjetljivost_{20°C} = Osjetljivost_{70°C} + (20°C - 70°C) \cdot 0.004\frac{mA/bar}{°C}$$
$$Osjetljivost_{20°C} = 1.6 - 0.004 \cdot 50 = 1.4\frac{mA}{bar}$$
$$Osjetljivost_{120°C} = Osjetljivost_{70°C} + (120°C - 70°C) \cdot 0.004\frac{mA/bar}{°C}$$
$$Osjetljivost_{120°C} = 1.6 + 0.004 \cdot 50 = 1.8\frac{mA}{bar}$$

(c)

$$Izlaz(0bar)_{20°C} = Izlaz(0bar)_{70°C} + (20°C - 70°C) \cdot 0.02\frac{mA}{°C}$$
$$Izlaz(0bar)_{20°C} = 4mA - 50 \cdot 0.02 = 3mA$$
$$Izlaz(0bar)_{120°C} = Izlaz(0bar)_{70°C} + (120°C - 70°C) \cdot 0.02\frac{mA}{°C}$$
$$Izlaz(0bar)_{120°C} = 4mA + 50 \cdot 0.02 = 5mA$$

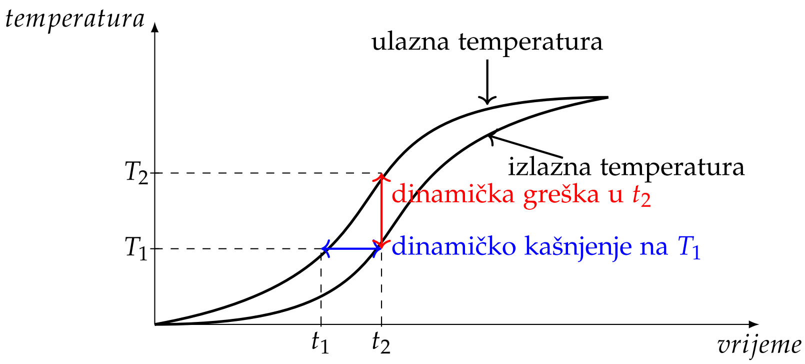
(d)

$$Izlaz(10bar)_{20°C} = Izlaz(0bar)_{20°C} + Osjetljivost_{20°C} \cdot 10bar$$
$$Izlaz(10bar)_{20°C} = 3mA + 1.4\frac{mA}{bar} \cdot 10bar = 17mA$$
$$Izlaz(10bar)_{120°C} = Izlaz(0bar)_{120°C} + Osjetljivost_{120°C} \cdot 10bar$$
$$Izlaz(10bar)_{120°C} = 5mA + 1.8\frac{mA}{bar} \cdot 10bar = 23mA$$

5. Temperaturni senzor se koristi za mjerenje temperature ulja. Ulazna temperatura se povećava konstantnom brzinom od $1.2\frac{°C}{min}$, a dinamičko kašnjenje je $2.3min$.

(a) Kolika je dinamička greška u °C?

(b) Ako je opseg temperaturnog senzora od 75°C do 125°C, kolika je dinamička greška u %?



Rješenje:

(a)

$$\begin{aligned} ulaz(t_1) &= T_1 \\ t_2 &= t_1 + 2.3min \\ ulaz(t_2) &= T_2 = T_1 + 1.2\frac{°C}{min} \cdot 2.3min = T_1 + 2.76°C \\ \text{Dinamička greška} &= ulaz(t_2) - izlaz(t_2) = T_1 + 2.76°C - T_1 = 2.76°C \end{aligned}$$

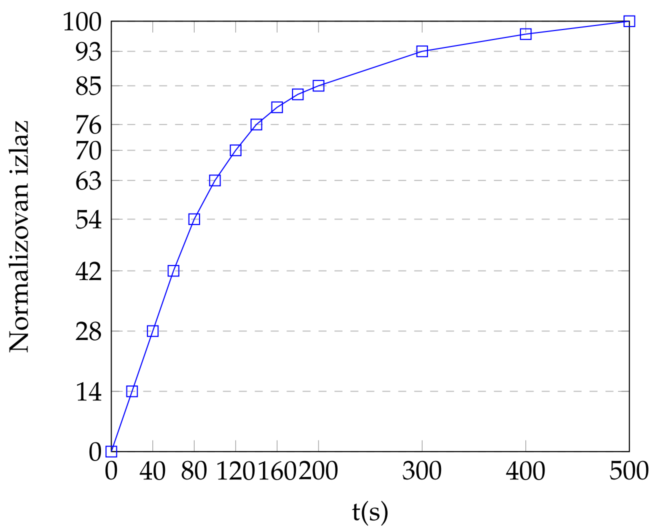
(b)

$$\text{Dinamička greška(\%)} = \frac{2.76°C}{125°C - 75°C}100\% = 5.52\%$$

6. Termometar je držan na temperaturi od 50°C, a zatim uronjen u tečnost temperature 150°C. Mjerenjem su dobijene sledeće vrijednosti:

t (s)	T (°C)	Normalizovan izlaz
0	50	0
20	64	14
40	78	28
60	92	42
80	104	54
100	113	63
120	120	70
140	126	76
160	130	80
180	133	83
200	135	85
300	143	93
400	147	97
500	150	100

Odrediti vrijeme uspona t_u, vrijeme smirenja $T_{S_{5\%}}$ i vremensku konstantu τ.



Rješenje:

$$
\begin{aligned}
t_{10\%} &= \frac{20-0}{14-0}(10-0) \\
t_{10\%} &= 14s \\
t_{90\%} &= 200 + \frac{300-200}{93-85}(90-85) \\
t_{90\%} &= 263s \\
t_u &= t_{90\%} - t_{10\%} = 263 - 14 = 249s \\
t_{S_{5\%}} &= t_{95\%} = 300 + \frac{400-300}{97-93}(95-93) \\
t_{S_{5\%}} &= 350s \\
\tau &= t_{63\%} = 100s
\end{aligned}
$$

7. Otpornost termootpornika od platine je aproksimirana sledećom jednačinom

$$R = R_0(1 + a_1 T + a_2 T^2)$$

gdje je:

$T -$ temperatura u °C

$R -$ otpornost na temperaturi T u Ω

$R_0 -$ otpornost na temperaturi od 0°C u Ω

a_1 i $a_2 -$ konstante

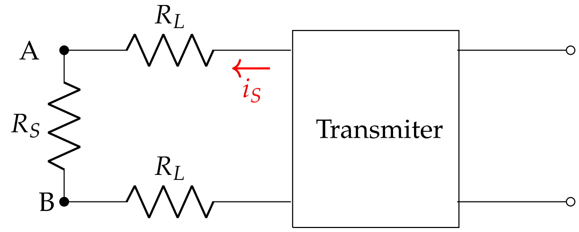
Odrediti konstante R_0, a_1 i a_2.

T (°C)	R(Ω)
0	100.0
25	109.9
50	119.8
75	129.6
100	139.3

Rješenje:

$$100.0 = R_0(1 + 0 \cdot a_1 + 0^2 \cdot a_2)$$
$$119.8 = R_0(1 + 50 \cdot a_1 + 50^2 \cdot a_2)$$
$$139.3 = R_0(1 + 100 \cdot a_1 + 100^2 \cdot a_2)$$
$$R_0 = 100\Omega$$
$$\left.\begin{array}{l} 50 \cdot a_1 + 2500 \cdot a_2 = 0.198 \\ 100 \cdot a_1 + 10000 \cdot a_2 = 0.393 \end{array}\right\} \Rightarrow a_1 = 0.0039, a_2 = -6 \cdot 10^{-7}$$
$$R = 100 \cdot (1 + 0.00399T - 6 \cdot 10^{-7}T^2)$$
$$R = 100 \cdot (1 + 0.00399 \cdot 25 - 6 \cdot 10^{-7} \cdot 25^2) = 109.9\Omega$$

8. Za termootpornik od platine iz prethodnog zadatka odrediti prosječnu osjetljivost na opsegu od 0-100°C. Pretpostaviti da se koristi dvožična veza za povezivanje transmitera i termootpornika koji su udaljeni 2m. Izračunati kolika je greška kada se povezivanje vrši sa svakim od provodnika čiji su prečnici dati u tabeli.



Prečnik provodnika (mm)	Otpornost($\frac{\Omega}{km}$)
2	5.20864
0.8	33.292
0.2	538.248

Rješenje:

$$\text{Osjetljivost} = \frac{139.3\Omega - 100.0\Omega}{100°C - 0°C} = 0.393\frac{\Omega}{°C}$$
$$\text{Greška}[\Omega] = R_m - R_s = (R_S + 2R_L) - R_S = 2R_L$$
$$\text{Greška}[°C] = \frac{\text{Greška}[\Omega]}{\text{Osjetljivost}\frac{\Omega}{°C}}$$
$$2mm \Rightarrow \text{Greška}[°C] = \frac{2(2\cdot 10^{-3})km \cdot 5.20864\frac{\Omega}{km}}{0.393\frac{\Omega}{°C}} = 0.053°C$$
$$0.8mm \Rightarrow \text{Greška}[°C] = \frac{2(2\cdot 10^{-3})km \cdot 33.292\frac{\Omega}{km}}{0.393\frac{\Omega}{°C}} = 0.339°C$$
$$0.2mm \Rightarrow \text{Greška}[°C] = \frac{2(2\cdot 10^{-3})km \cdot 538.248\frac{\Omega}{km}}{0.393\frac{\Omega}{°C}} = 5.48°C$$

9. Termootpornik se koristi za mjerenje temperature vazduha koji struji kroz cijev brzinom od $1m/s$. Disipaciona konstanta za takvu sredinu je $20mW/°C$. Temperatura vazduha je $50°C$, a otpornost termootpornika na toj temperaturi je 200Ω. Odrediti grešku usljed samozagrijavanja za sledeće struje senzora: 1mA, 10mA, 20mA.

Rješenje:

$$i_s = 1mA \Rightarrow \Delta T = \frac{(1\cdot 10^{-3}A)^2 \cdot 200\Omega}{20\cdot 10^{-3}\frac{W}{°C}} = 0.01°C$$
$$i_s = 10mA \Rightarrow \Delta T = \frac{(10\cdot 10^{-3}A)^2 \cdot 200\Omega}{20\cdot 10^{-3}\frac{W}{°C}} = 1°C$$
$$i_s = 20mA \Rightarrow \Delta T = \frac{(20\cdot 10^{-3}A)^2 \cdot 200\Omega}{20\cdot 10^{-3}\frac{W}{°C}} = 4°C$$

D (disipaciona konstanta) - snaga koja je potrebna da bi se povećala temperatura na senzoru za 1°C. Definiše se za određeni komercijalni senzor za određene uslove.

$D = h \cdot s,$ gdje je

$h[\frac{W}{°Cm^2}]$ - koeficijent prelaza toplote na površini pretvarač-fluid, a s - površina pretvarača koja je u kontaktu sa fluidom

T_2 - temperatura termootpornika
T_1 - temperatura okoline
$T_2 = T_1 + \frac{P}{D}$
$\Delta T = T_2 - T_1 = \frac{P}{D} = \frac{{i_s}^2 R_S}{D}$

10. Termopar tipa J (gvožđe - konstantan) koristi se za mjerenje temperature u peći. Referentni spoj se nalazi na temperaturi od 0°C, a mjerni spoj (topli spoj) se nalazi u peći.

(a) Odrediti mjerenu temperaturu ako je izmjeren napon od 28.3mV

(b) Odrediti mjerenu temperaturu ukoliko se koristi termopar tipa K i izmjeren je napon od 34.2mV.

	mjereni napon (mV)	
T (°C)	J	K
0	0	0
100	5.27	4.10
200	10.78	8.13
300	16.32	12.21
400	21.85	16.40
500	27.39	20.65
600	33.11	24.91
700	39.15	29.14
800	45.53	33.30
900		37.36
1000		41.31

Rješenje:

(a)

$$\begin{aligned} 28.3mV &\Rightarrow 27.39 < 28.3 < 33.11 \\ & 500°C < T < 600°C \end{aligned}$$

$$T = T_L + (U_m - U_L) \cdot \frac{T_H - T_L}{U_H - U_L}$$

$$T = 500°C + (28.3mV - 27.39mV) \cdot \frac{600°C - 500°C}{33.11mV - 27.39mv}$$

$$T = 500°C + 0.91mV \cdot \frac{100°C}{5.72mv} = 515.9°C$$

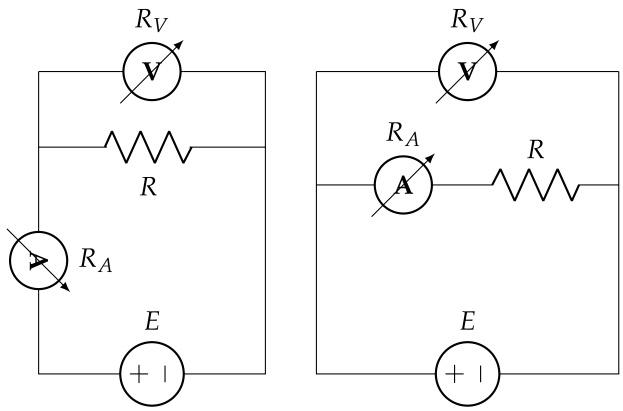
(b)

$$\begin{aligned} 34.2mV &\Rightarrow 33.3 < 34.2 < 37.6 \\ & 800°C < T < 900°C \end{aligned}$$

$$T = 800°C + (34.2mV - 33.3mV) \cdot \frac{900°C - 800°C}{37.36mV - 33.3mv}$$

$$T = 822.2°C$$

11. Izračunati apsolutnu i relativnu grešku pri mjerenju otpora pomoću ampermetra i voltmetra.



(a) (b)

Rješenje:

(a)

$$I_A = \frac{E}{R_A + R_V \parallel R} = \frac{E}{R_A + \frac{R_V R}{R_V + R}} = \frac{E}{\frac{R_A R_V + R_A R + R_V R}{R_V + R}}$$

$$I_A = \frac{E(R_V + R)}{R_A R_V + R_A R + R_V R}$$

$$U_V = I_A \cdot (R \parallel R_V) = \frac{E\cancel{(R_V + R)}}{R_A R_V + R_A R + R_V R} \cdot \frac{R R_V}{\cancel{R + R_V}}$$

$$U_V = E \frac{R_V R}{R_A R_V + R_A R + R_V R}$$

$$R_m = \frac{U_V}{I_A} = \cancel{E} \frac{R R_v}{\cancel{R_A R_V + R_A R + R_V R}} \frac{\cancel{R_A R_V + R_A R + R_V R}}{\cancel{E}(R_v + R)}$$

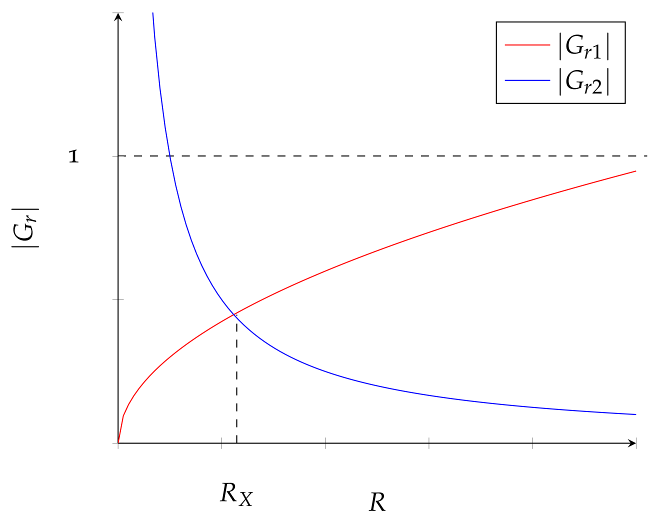
$$R_m = \frac{R R_V}{R_V + R}$$

$$G_{a1} = R_m - R = \frac{R R_V}{R + R_V} - \frac{R(R + R_v)}{R + R_V} = -\frac{R^2}{R + R_V}$$

$$G_{r1} = \frac{G_{a1}}{R} = -\frac{R}{R + R_V} \Rightarrow |G_{r1}| = \frac{R}{R + R_V}$$

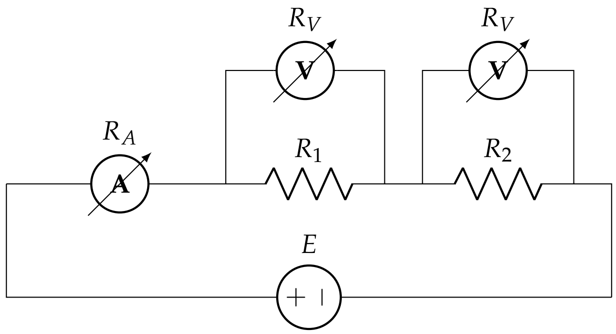
(b)

$$I = \frac{E}{R_V \parallel (R + R_A)} = \frac{E(R_V + R + R_A)}{R_V R + R_V R_A}$$
$$I_A(R_A + R) = (I - I_A)R_V$$
$$I_A = \frac{R_V}{R_A + R + R_V} I = \frac{\cancel{R_V}}{\cancel{R_A + R + R_V}} \frac{E(\cancel{R_V + R + R_A})}{\cancel{R_V}(R + R_A)}$$
$$I_A = \frac{E}{R + R_A}$$
$$U_V = E$$
$$R_m = \frac{U_V}{I_A} = R + R_A$$
$$G_{a2} = R_m - R = R_A$$
$$G_{r2} = \frac{G_{a2}}{R} = \frac{R_A}{R}$$



$$\frac{R_A}{R_X} = \frac{R_X}{R_X + R_V}$$
$$R_X^2 = R_X R_A + R_V R_A$$
$$R_X^2 + R_X R_A + R_V R_A = 0$$
$$R_{X_{1,2}} = \frac{-R_A \pm \sqrt{R_A^2 + 4R_A R_V}}{2}$$
$$R_A = 1\Omega, R_V = 1M\Omega$$
$$R_{X_{1,2}} = \frac{-1 \pm \sqrt{1^2 + 4 \cdot 10^6}}{2} = \frac{-1 \pm 2000}{2} \Rightarrow R_X = 999,5\Omega$$

12. Izračunati apsolutnu i relativnu grešku mjerenja.
$R_1 = 100\Omega \quad R_2 = 20k\Omega \quad E = 20V \quad E_A = 1\Omega R_V = 100k\Omega$



Rješenje:

$$U_{V1} = \frac{R_V \parallel R_1}{R_A + R_V \parallel R_1 + R_V \parallel R_2} \cdot E \qquad I_A = \frac{E}{R_A + R_V \parallel R_1 + R_V \parallel R_2}$$

$$U_{V2} = \frac{R_V \parallel R_2}{R_A + R_V \parallel R_1 + R_V \parallel R_2} \cdot E$$

$$R_{m1} = \frac{U_{V1}}{I_A} = R_V \parallel R_1 \qquad R_{m2} = \frac{U_{V2}}{I_A} = R_V \parallel R_2$$

$$G_{a1} = R_{m1} - R_1 = \frac{R_V R_1}{R_V + R_1} - \frac{R_V R_1 + {R_1}^2}{R_V + R_1} = -\frac{{R_1}^2}{R_V + R_1} = -\frac{100^2}{100000 + 100}$$

$$G_{a1} = -0.01\Omega$$

$$G_{r1} = -\frac{0.01}{100} = -1 \cdot 10^{-4}$$

$$G_{a2} = -\frac{{R_2}^2}{R_V + R_2} = \frac{20000^2}{100000 + 20000} = 3333\Omega$$

$$G_{r2} = \frac{3333}{20000} = 0.167$$

13. Mjerenje sile vrši se pomoću mjerne trake kao na slici. Za sledeće parametre
Konzola:

$$E = 2 \cdot 10^{11} \frac{N}{m^2}$$

$$b = 1.25cm$$

$$h = 0.25cm$$

$$L = 6cm$$

$$\sigma_m(\text{maksimalni dozvoljeni napon}) = 5 \cdot 10^8 \frac{N}{m^2}$$

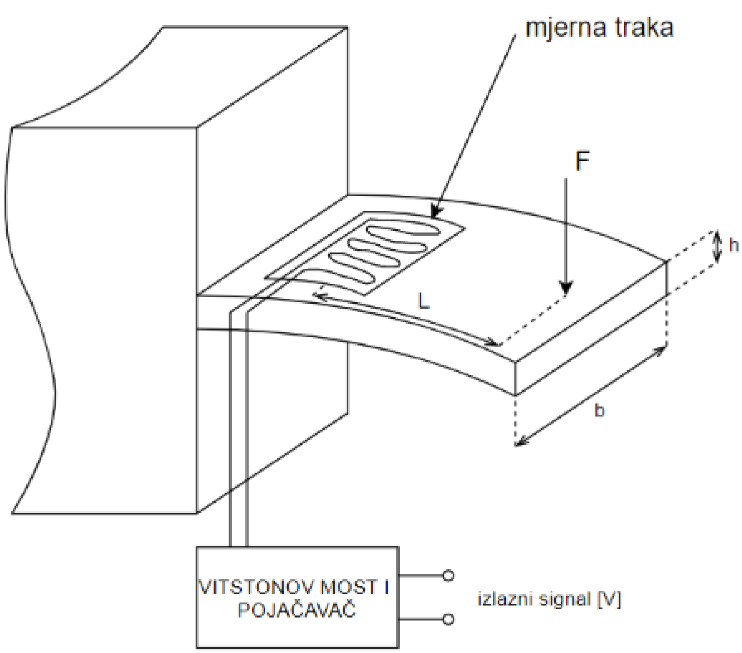
Mjerna traka:

$$K = 2$$

$$R_0 = 200\Omega$$

(a) Odrediti maksimalnu silu koja može da se mjeri

(b) Odrediti kolika je promjena otpornosti mjerne trake za maksimalnu silu.



Rješenje:

$$F_m = \frac{\sigma_m bh^2}{6L} = \frac{5 \cdot 10^8 \frac{N}{m^2} \cdot 1.25 \cdot 10^{-2} \cdot (0.25 \cdot 10^{-2})^2}{6 \cdot 6 \cdot 10^{-2}} = 108.5N$$

$$\Delta R_m = \frac{6LK}{bh^2E} \cdot F \cdot R_0 = \frac{\sigma_m}{E} K \cdot R_0 = \frac{5 \cdot 10^8 \frac{N}{m^2}}{2 \cdot 10^{11} \frac{N}{m^2}} \cdot 2 \cdot 200\Omega$$

$$\Delta R_m = 1\Omega$$

$K = \frac{\Delta R/R_0}{\Delta L/L_0}$

K - faktor trake

ΔR - promjena otpornosti u Ω

R_0 - otpornost mjerne trake u Ω

ΔL - promjena dužine trake u m

L_0 - dužina trake u m

$\sigma = \frac{6FL}{bh^2}$

σ - napon

$\sigma = \frac{F_N}{A}$

F_N - sila koja djeluje na element

A - poprečni presjek na kome djeluje sila

$E = \frac{\sigma}{\delta}$

E - modul elastičnosti

$\delta = \frac{\Delta L}{L_0}$ - istezanje

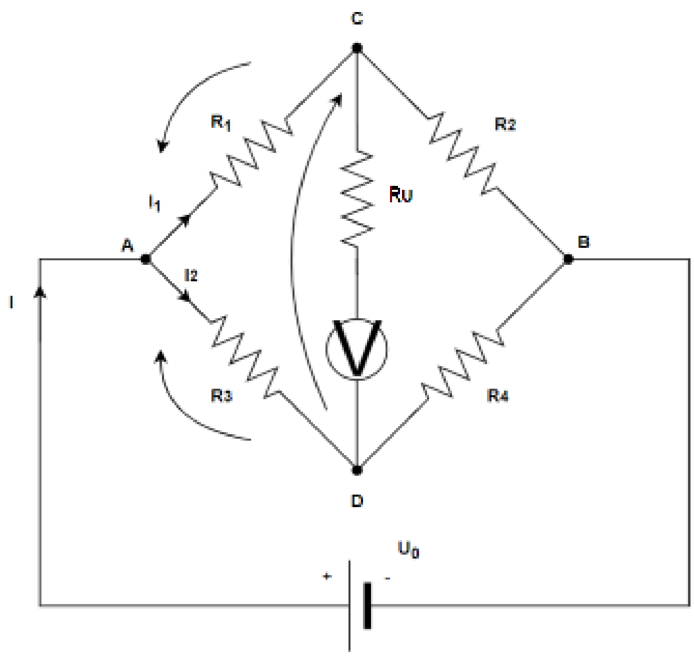
$\delta = \frac{\sigma}{E} = \frac{6FL}{bh^2E} = \frac{\Delta\ L}{L_0}$

$\frac{\Delta R}{R_0} = \frac{6LK}{bh^2E}F$

$\sigma = \frac{6FL}{bh^2}$

$\frac{\Delta L}{L_0} = \frac{\sigma}{E} = \frac{\Delta R}{R_0 K} \Rightarrow \Delta R = \frac{\sigma}{E} R_0 K$

Vitstonov most

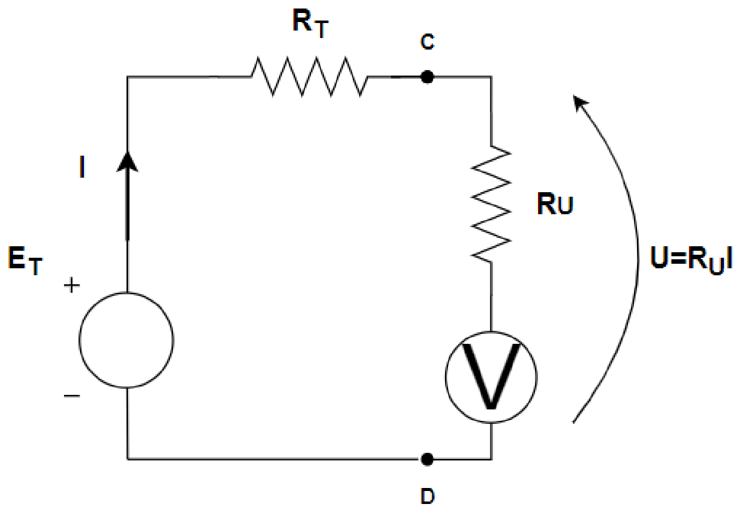


$$I_1 = \frac{U_0}{R_1 + R_2}$$

$$I_2 = \frac{U_0}{R_3 + R_4}$$

$$U_{CD} = I_2 R_3 - I_1 R_1$$

$$U_{CD} = R_3 \frac{U_0}{R_3 + R_4} - R_1 \frac{U_0}{R_1 + R_2}$$



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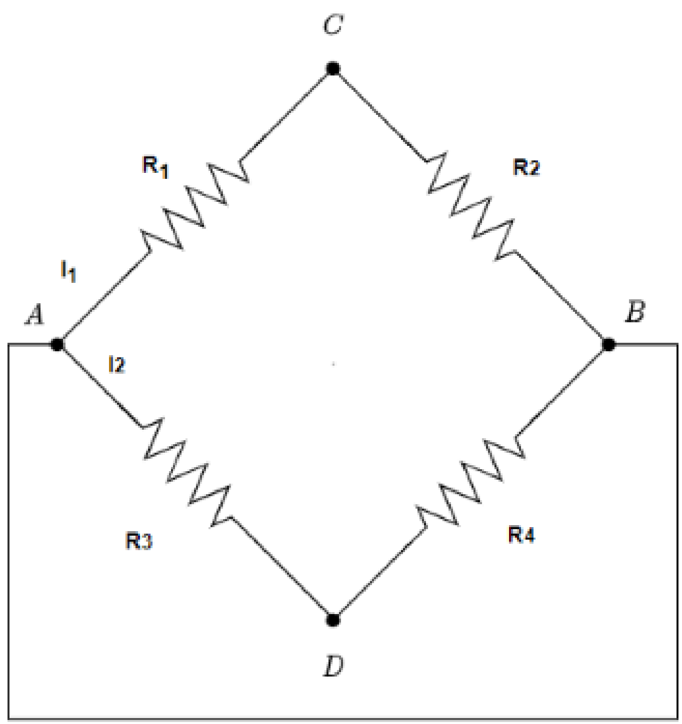
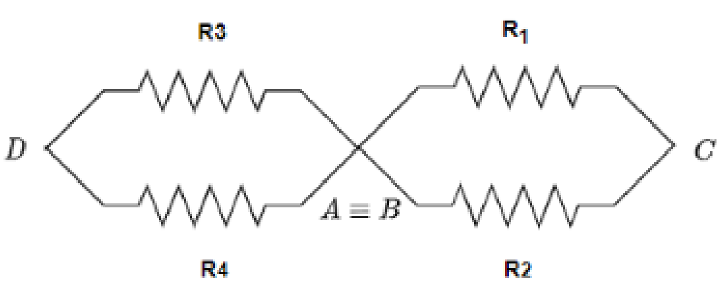
U=R_UI
D



C
R_1
R2
I_1
A
B
I2
R3
R4
D



R3
R_1
D
C
$A \equiv B$
R4
R2

$$E_T = U_{CD} = U_0 \frac{R_3(R_1 + R_2) - R_1(R_3 + R_4)}{(R_3 + R_4)(R_1 + R_2)}$$

$$R_T = ?$$

$$R_T = R_3 \parallel R_4 + R_1 \parallel R_2$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = \frac{R_1 R_2(R_3 + R_4) + R_3 R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4)}$$

$$I = \frac{E_T}{R_T + R_U}$$

$$R_T + R_U = \frac{R_1 R_2(R_3 + R_4) + R_3 R_4(R_1 + R_2) + R_U(R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2)(R_3 + R_4)}$$

$$I = U_0 \frac{R_3(R_1 + R_2) - R_1(R_3 + R_4)}{\cancel{(R_1 + R_2)(R_3 + R_4)}} \frac{\cancel{(R_1 + R_2)(R_3 + R_4)}}{R_1 R_2(R_3 + R_4) + R_3 R_4(R_1 + R_2) + R_U(R_1 + R_2)(R_3 + R_4)}$$

$$I = U_0 \frac{R_2 R_3 - R_1 R_4}{R_1 R_2(R_3 + R_4) + R_3 R_4(R_1 + R_2) + R_U(R_1 + R_2)(R_3 + R_4)}$$

Galvanometar:

$R_U = 0 \Rightarrow I = U_0 \frac{R_2 R_3 - R_1 R_4}{R_1 R_2(R_3+R_4) + R_3 R_4(R_1+R_2)}$

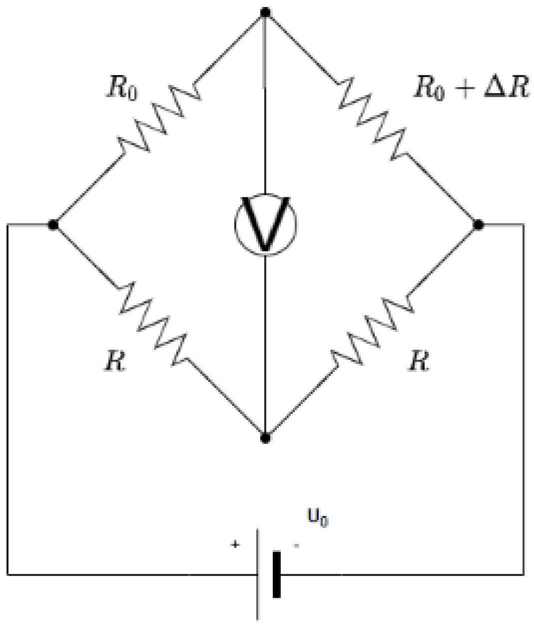
Voltmetar:

$R_U \to \infty$

$$U = R_U I = R_U U_0 \frac{R_2 R_3 - R_1 R_4}{R_1 R_2(R_3 + R_4) + R_3 R_4(R_1 + R_2) + R_U(R_1 + R_2)(R_3 + R_4)}$$

$$U = U_0 \frac{R_2 R_3 - R_1 R_4}{\cancelto{0}{\frac{R_1 R_2(R_3+R_4) + R_3 R_4(R_1+R_2)}{R_U}} + (R_1 + R_2)(R_3 + R_4)}$$

$$U = U_0 \frac{R_2 R_3 - R_1 R_4}{(R_1 + R_2)(R_3 + R_4)}$$



$$\Delta R = 0 \Rightarrow RR_0 - RR_0 = 0 \Rightarrow U = 0$$

$$\Delta R \neq 0 \Rightarrow U = U_0 \frac{(R_0 + \Delta R)R - RR_0}{(R_0 + R_0 + \Delta R)(R + R)}$$

$$U = U_0 \frac{\cancel{R_0 R} + \Delta R R - \cancel{R R_0}}{(2R_0 + \Delta R)2R}$$

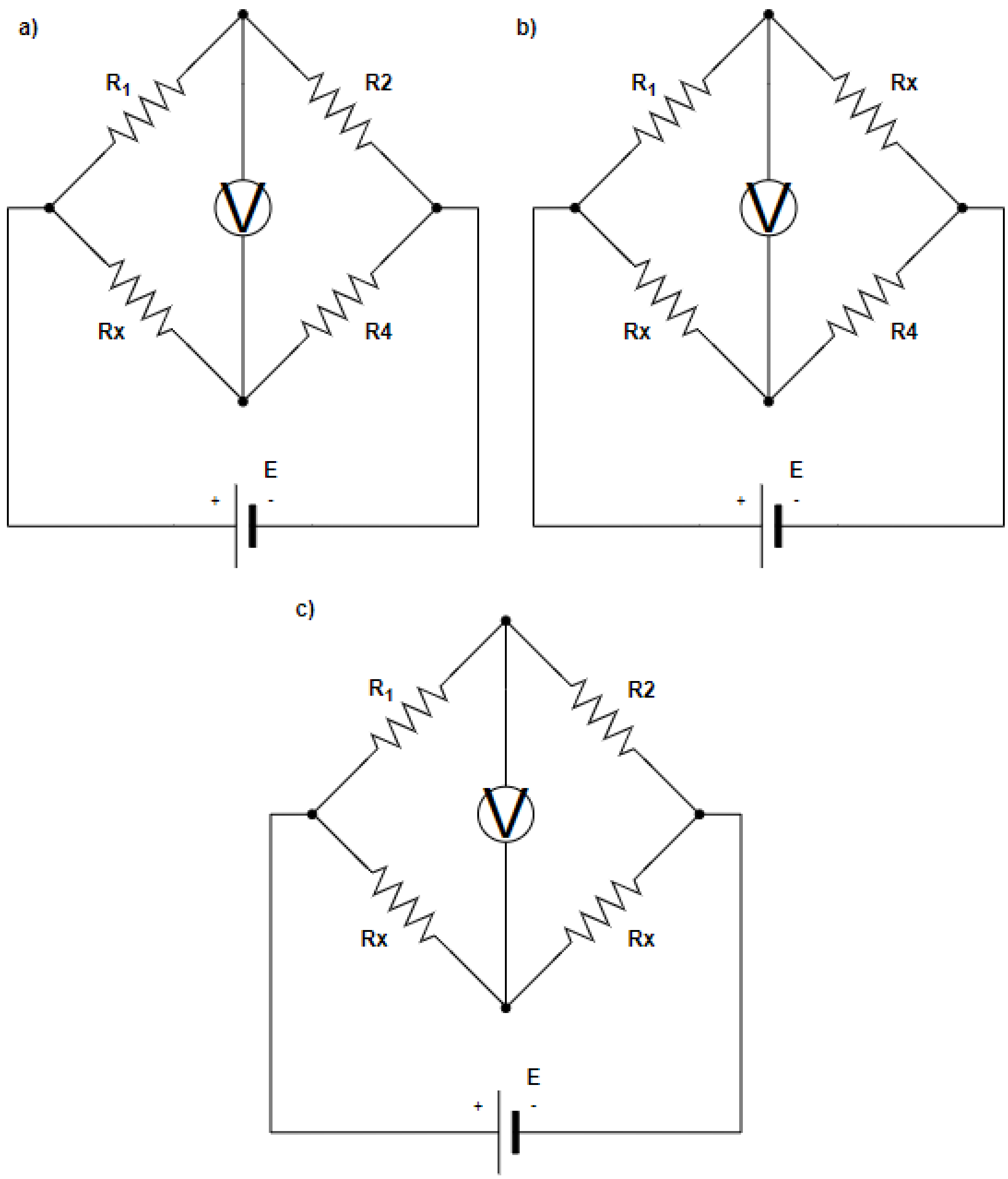
$$= U_0 \frac{R}{(2R_0 + \Delta R)2R} \Delta R$$

$$\Delta R << 2R_0$$

$$U \approx K \Delta R$$

$$K = U_0 \frac{R}{4R_0 R}$$

14. Odrediti osjetljivost statičke karakteristike izlazni napon-otpor senzora za Vitstonove mostove koji su aranžirani kao na slici. Smatrati da je unutrašnja otpornost naponskog izvora jednaka nuli, a da je ulazni otpor voltmentra beskonačan.
$R_1 = R_2 = R_4 = R$


a)
R1
R2
V
Rx
R4
E
+
-
b)
R1
Rx
V
Rx
R4
E
+
-
c)
R1
R2
V
Rx
Rx
E
+
-

Rješenje:

(a)

$$
\begin{aligned}
U &= E\frac{R_2R_X - R_1R_4}{(R_1+R_2)(R_4+R_X)} = E\frac{RR_X - R^2}{2R(R+R_X)} \\
S_U(R_X) &= \frac{\Delta U}{\Delta R_X}\left[\frac{V}{\Omega}\right] \\
S_U &= \lim_{\Delta R_X \to 0}\left(\frac{\Delta U}{\Delta R_X}\right) = \frac{dU}{dR_X} \\
\frac{dU}{dR_X} &= E\frac{R2R(R+R_X) - 2R(RR_X - R^2)}{4R^2(R+R_X)^2} \\
&= \frac{2R^2 + \cancel{2R^2R_X} - \cancel{2R^2R_X} + 2R^3}{4R^2(R+R_X)^2} = \frac{ER}{(R+R_X)^2}
\end{aligned}
$$

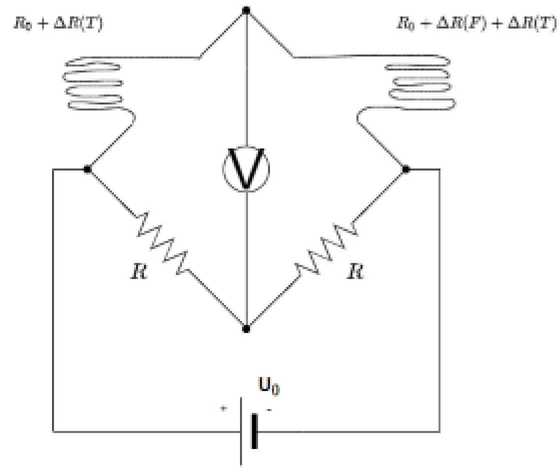
(b)

$$
\begin{aligned}
U &= E\frac{R_X^2 - R_1R_4}{(R_11+R_X)(R_4+R_X)} = E\frac{R_X^2 - R^2}{(R+R_X)^2} \\
U &= E\frac{(R_X-R)(R_X+R)}{(R+R_X)^2} = E\frac{R_X - R}{R+R_X} \\
\frac{dU}{dR_X} &= E\frac{(R_X+R)-(R_X-R)}{(R+R_X)^2} = E\frac{2R}{(R+R_X)^2}
\end{aligned}
$$

(c)

$$
\begin{aligned}
U &= E\frac{R_2R_X - R_1R_X}{(R_1+R_2)(R_X+R_X)} = 0 \\
S_U &= \frac{dU}{dR_X} = 0
\end{aligned}
$$

Temperaturna kompenzacija mjerne trake



$$U = U_0 \frac{(R_0 + \Delta R(F) + \Delta R(T))R - R(R_0 + \Delta R(T))}{(R_0 + \Delta R(T) + R_0 + \Delta R(F) + \Delta R(T))(R + R)} = 0$$

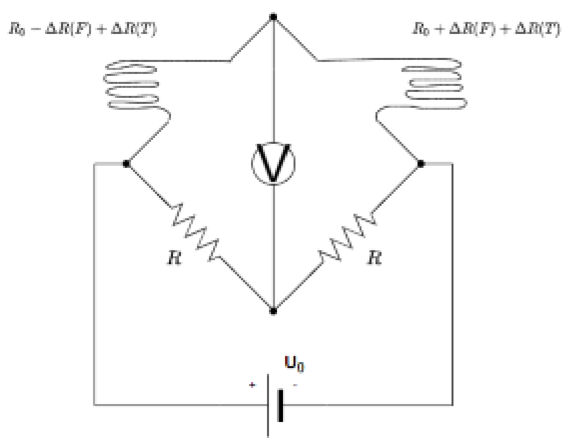
$$U = U_0 \frac{\cancel{R_0 R} + \Delta R(F)R + \cancel{\Delta R(T)R} - \cancel{RR_0} - \cancel{R\Delta R(T)}}{(2R_0 + \Delta R(F) + 2\Delta R(T))2R}$$

$$U = U_0 \frac{R}{(2R_0 + \Delta R(F) + 2\Delta R(T))2R} \Delta R(F)$$

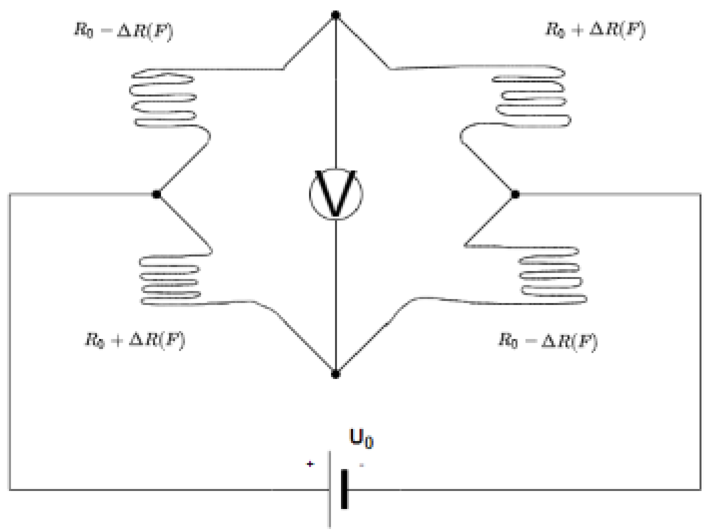
$$\Delta R(F) << 2R_0$$

$$\Delta R(T) << 2R_0$$

$$U = K\Delta R(F)$$



$$U = U_0 \frac{(R_0 + \Delta R(F))R - (R_0 - \Delta R(F))R}{(R_0 - \Delta R(F) + R_0 + \Delta R(F))2R} = U_0 \frac{2\Delta R(F)R}{2R_0 2R}$$

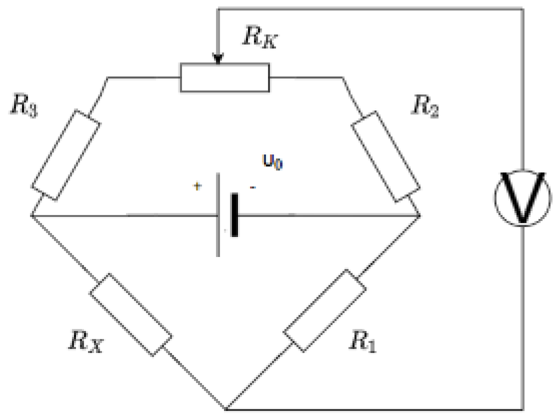


$$U = U_0 \frac{(R_0 + \Delta R(F))(R_0 + \Delta R(F)) - (R_0 - \Delta R(F))(R_0 - \Delta R(F))}{(R_0 - \cancel{\Delta R(F)} + R_0 + \Delta R(F))(R_0 + \cancel{\Delta R(F)} + R_0 - \Delta R(F))}$$

$$U = U_0 \frac{\cancel{R_0^2} + 2R_0\Delta R(F) + \cancel{\Delta R(F)^2} - \cancel{R_0^2} + 2R_0\Delta R(F) - \cancel{\Delta R(F)^2}}{(R_0 + R_0)(R_0 + R_0)}$$

$$U = U_0 \frac{4R_0\Delta R(F)}{4R_0^2}$$

15. Izvesti izraz za položaj klizača x za uravnoteženi Vitstonov most. Smatrati da je $0 < x < 1$. Izvesti izraz za označavanje skale.



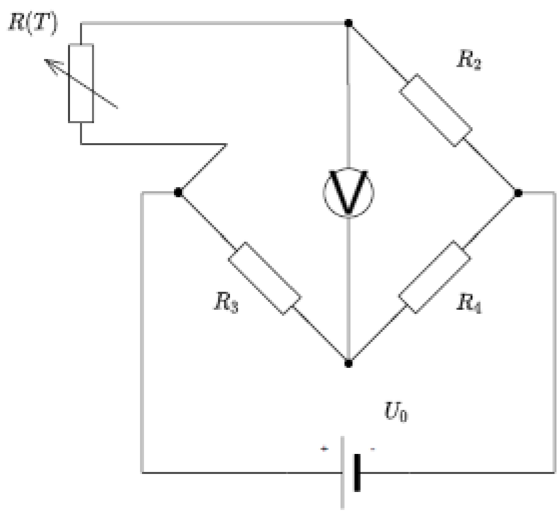
Rješenje:

$$U = 0 \Leftrightarrow (R_3 + R_K x)R_1 - (R_2 + (1-x)R_K)R_X = 0$$
$$R_3R_1 + R_KR_1x - R_2R_X - (1-x)R_KR_X = 0$$

$$R_KR_1x + R_KR_Xx = -R_3R_1 + R_2R_X + R_KR_X$$
$$x(R_KR_1 + R_KR_X) = R_2R_X + R_KR_X - R_3R_1$$
$$x = \frac{R_2R_X + R_KR_X - R_3R_1}{R_K(R_1 + R_X)}$$
$$R_X = \frac{(R_3 + xR_K)R_1}{R_2 + (1-x)R_K}$$
$$x = 0 \Rightarrow \frac{R_3R_1}{R_2 + R_K} = R_{X_{min}}$$
$$x = 1 \Rightarrow \frac{(R_3 + R_K)R_1}{R_2} = R_{X_{max}}$$

16. Termootpornik sa karakteristikom $R(T) = R_0(1+\alpha T)$ priključen je na most. Pretvarač se prvo nalazi u vazduhu na temperaturi $Tvazd = 20°C$, a zatim se spušta u vodu koja je na temperaturi $T_{vode} = 100°C$. Dimenzionisati elemente mosta koji je uravnotežen na T_{vazd}, ako je najveće dozvoljeno samozagrijavanje pretvarača $T_m = 0.2°C$.
$R_0 = 100\Omega, \alpha = 3.9 \cdot 10^{-3} °C^{-1}, Dvazduh = 1.5\frac{mW}{°C}, Dvoda = 5\frac{mW}{°C}, R_4 = 10k\Omega, U_0 = 10V$

$D = hs$, s - površina pretvarača
$h\left[\frac{W}{°Cm^2}\right]$- koeficijent prelaza toplote na površini kontakta pretvarač-fluid
T_2-temperatura otpornika
T_1-temperatura okoline
$T_2 - T_1 = \frac{P}{D}$
$T_2 - T_1 \le T_m$
$\frac{P}{D} \le T_m$
$P = I^2R(T)$
$I = \frac{U_0}{R(T)+R_2} \Rightarrow P = \frac{U_0^2}{(R(T)+R_2)^2}R(T)$
$\frac{U_0^2}{D}\frac{R(T)}{(R(T)+R_2)^2} \le T_m$
$\frac{U_0^2}{D}\frac{R(T)}{T_m} \le (R(T)+R_2)^2 \Rightarrow R_2 \ge U_0\sqrt{\frac{R(T)}{DT_m}} - R(T)$



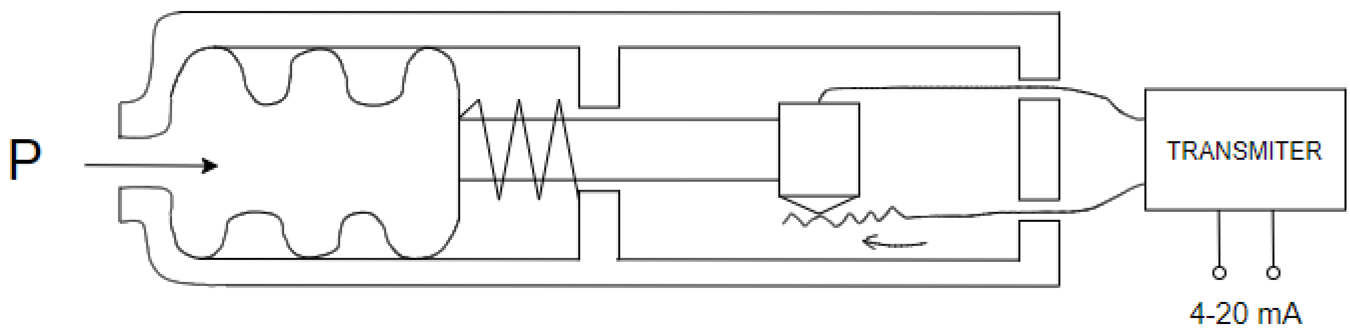
Rješenje:

$$R(20°C) = 100(1 + 20 \cdot 3.9 \cdot 10^{-3}) = 107.8\Omega$$
$$R(100°C) = 100(1 + 100 \cdot 3.9 \cdot 10^{-3}) = 139\Omega$$
$$R_2 \ge U_0\sqrt{\frac{R(20°C)}{DvazduhT_m}} - R(20°C) = 10\sqrt{\frac{107.8}{1.5 \cdot 10^{-3} \cdot 0.2}} - 107.8\Omega$$
$$R_2 \ge 5886.6\Omega$$
$$R_2 \ge U_0\sqrt{\frac{R(100°C)}{DvodaT_m}} - R(100°C) = 10\sqrt{\frac{139}{5 \cdot 10^{-3} \cdot 0.2}} - 139\Omega$$
$$R_2 \ge 3589\Omega$$
$$R_2 = 6k\Omega$$
$$R(20°C)R_4 = R_2R_3$$
$$R_3 = \frac{R(20°C)R_4}{R_2} = \frac{107.8 \cdot 10 \cdot 10^3}{6 \cdot 10^3} = 179.7\Omega$$

17. Senzor sa mijehom za mjerenje pritiska koji je prikazan na slici ima sledeće karakteristike

$$A_{mijeha} = 12.9cm^2$$
$$K_{opruge} = 80\frac{N}{cm}$$
$$K_{mijeha} = 6\frac{N}{cm}$$

Koliki je mjerni opseg senzora ako je maksimalno pomjeranje mijeha 1.5cm?

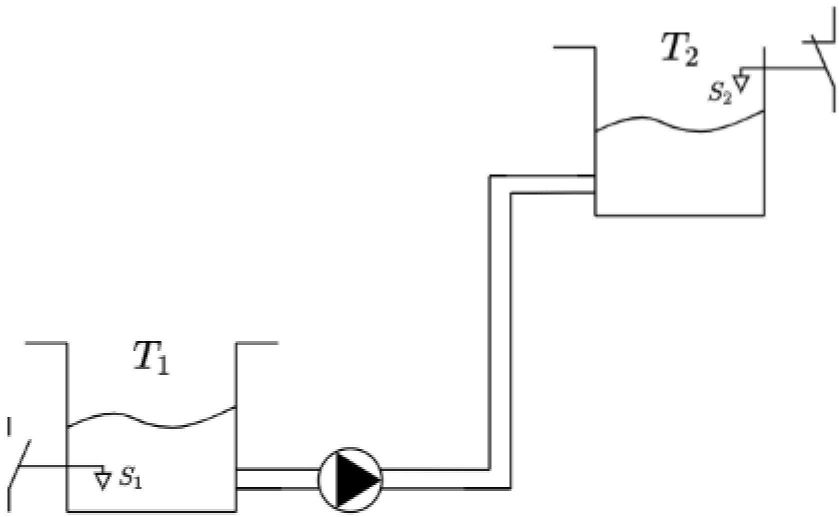


Rješenje:

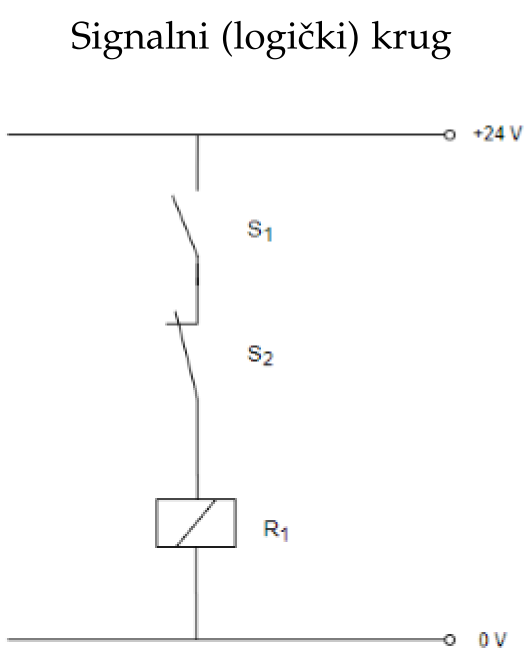
$$F = F_{mijeh} + F_{opruga} = K_{mijeh} \cdot x + K_{opruga} \cdot x$$
$$F = (K_{mijeh} + K_{opruga}) \cdot x = (80 + 6)\frac{N}{cm} \cdot 1.5cm$$
$$F = 129N$$
$$p = \frac{F}{A_{mijeh}} = \frac{129N}{12.9cm^2} = 10\frac{N}{cm^2} = 100\frac{kN}{m^2}$$
$$p = 100kPa = 1bar$$

Mjerni opseg senzora je od 0 do 1 bar.

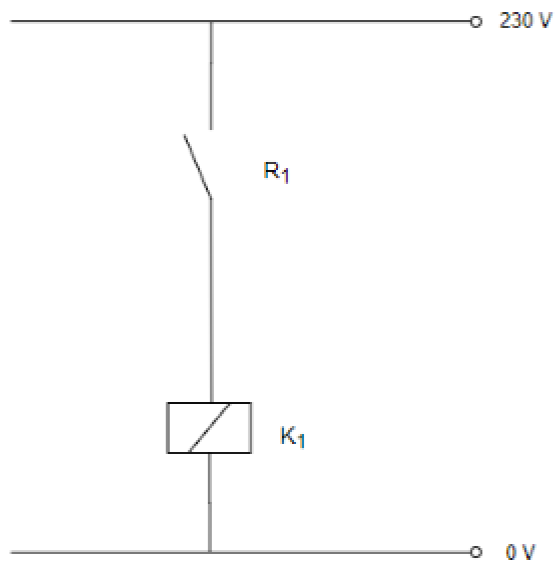
18. Za sistem sa slike u kome se crpi voda iz rezervoara T_1 i puni rezervoar T_2 pomoću pumpe P_1, nacrtati signalni, upravljački i energetski krug koji će automatski da upravlja sistemom, vodeći računa o zaštiti pumpe od rada na suvo i prelivanja rezervoara T_2.



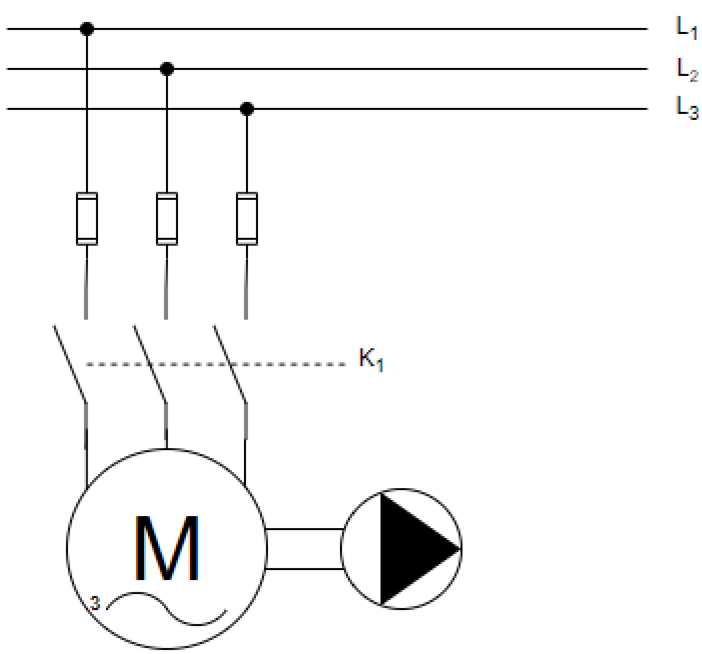
Rješenje:



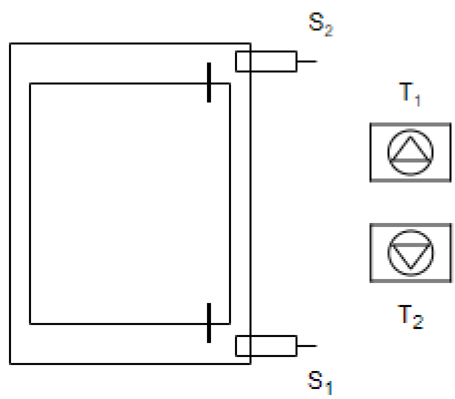
Upravljački krug



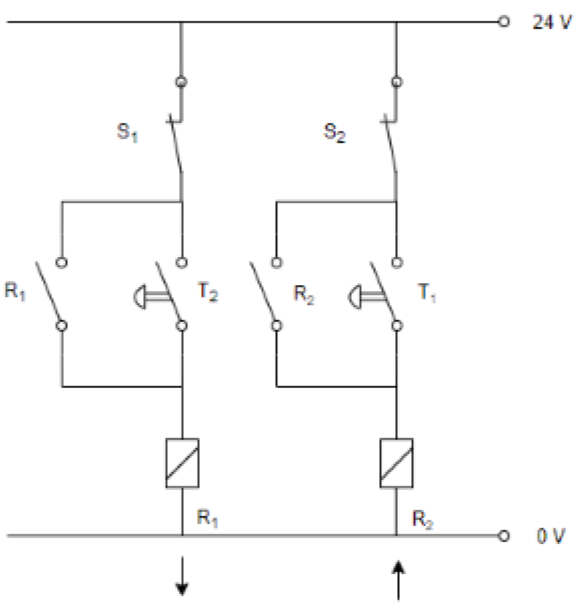
Energetski krug

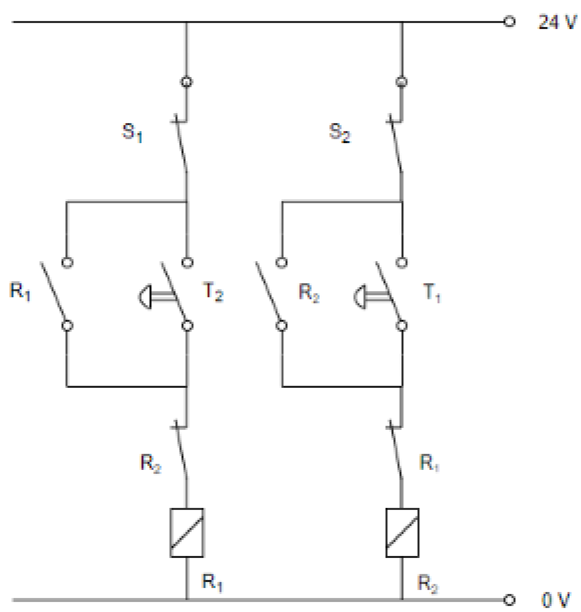


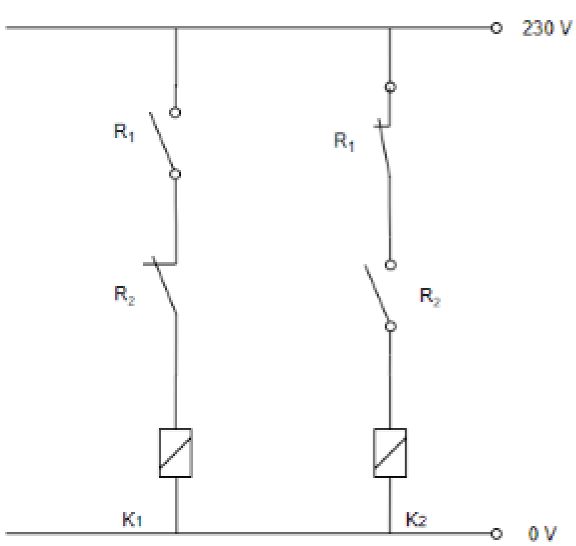
19. Kod sistema predstavljenog na slici, malim teretnim liftom se upravlja pomoću tastera T_1 i T_2, a krajnje pozicije se kontrolišu pomoću dva induktivna davača S_1 i S_2. Nacrtati signalni, upravljački i energetski krug za upravljanje liftom.

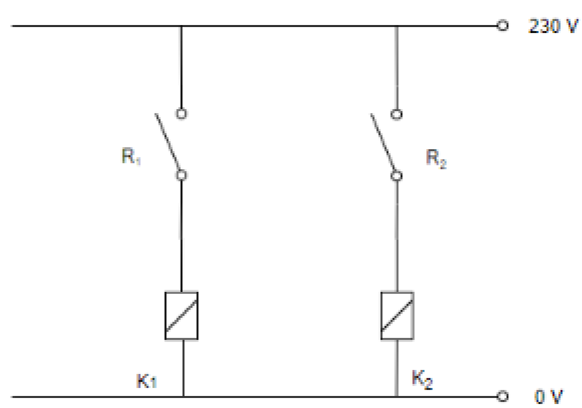


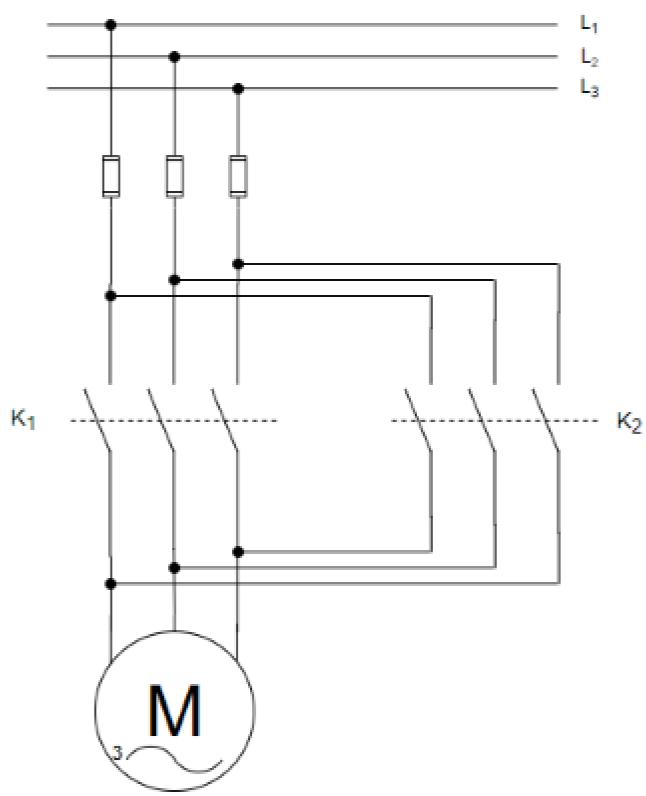
Rješenje:



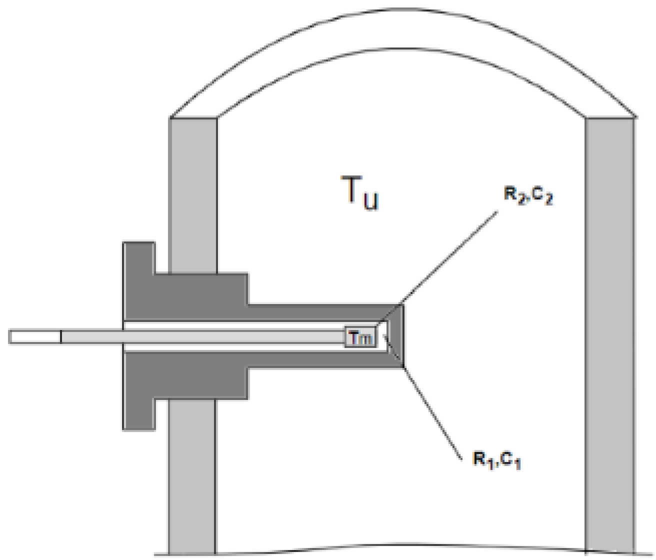



230 V
R1
R2
R1
R2
K1
K2
0 V


230 V
R1
R2
K1
K2
0 V


L1
L2
L3
K1
K2
M
3

20. Odrediti funkciju prenosa senzora sa slike.



$R_1 -$ Termička otpornost između mjerenog fluida i fluida unutar čaure

$R_2 -$ Termička otpornost između fluida unutar čaure i termometra

$C_1 -$ Termički kapacitet čaure

$C_2 -$ Termički kapacitet termometra

$\tau_1 = R_1 C_1 -$ Vremenska konstanta čaure

$\tau_2 = R_2 C_2 -$ Vremenska konstanta termometra

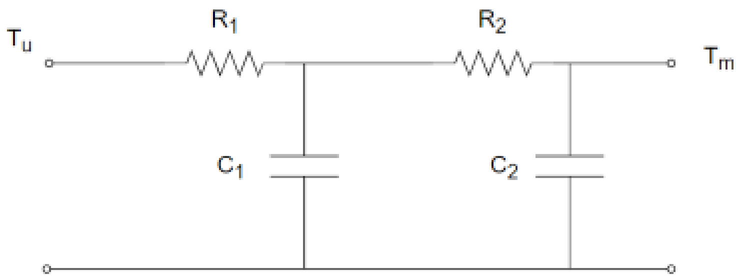
$R_1 = 125 \frac{K}{W}$

$C_1 = 0.45 \frac{J}{K}$

$R_2 = 65 \frac{K}{W}$

$C_2 = 0.72 \frac{J}{K}$

Rješenje:



$$
\begin{aligned}
T_X &= T_u \frac{Z_{C1}}{Z_{C1} + Z_{R1}} \\
Z_X &= \frac{Z_{R1} Z_{C1}}{Z_{R1} + Z_{C1}} \\
T_m &= T_X \frac{Z_{C2}}{Z_X + Z_{C2} + Z_{R2}} \\
T_m &= T_u \frac{Z_{C1} Z_{C2}}{(Z_X + Z_{C2} + Z_{R2})(Z_{C1} + Z_{R1})} \\
\frac{T_m}{T_u} &= \frac{Z_{C1} Z_{C2}}{Z_{R1} Z_{C1} + Z_{R1} Z_{R2} + Z_{R1} Z_{C2} + Z_{C1} Z_{R2} + Z_{C1} Z_{C2}} \\
G(s) &= \frac{\frac{1}{C_1 s} \frac{1}{C_2 s}}{R_1 \frac{1}{C_1 s} + R_1 R_2 + R_1 \frac{1}{C_2 s} + R_2 \frac{1}{C_1 s} + \frac{1}{C_1 s} \frac{1}{C_2 s}} \\
G(s) &= \frac{1}{1 + (R_1 C_2 + R_1 C_1 + R_2 C_2) s + R_1 R_2 C_1 C_2 s^2} \\
G(s) &= \frac{1}{1 + (R_1 C_2 + \tau_1 + \tau_2) s + \tau_1 \tau_2 s^2} \\
\tau_1 &= R_1 C_1 = 56.25 \\
\tau_2 &= R_2 C_2 = 46.8 \\
G(s) &= \frac{1}{2632.5 s^2 + 193.05 s + 1}
\end{aligned}
$$

21. Turbinski mjerač protoka ima karakteristiku $K = 12.2 \frac{cm^3}{impuls}$. Odrediti zapreminu tečnosti koja je protekla za sledeće brojeve impulsa.

   (a) 220

   (b) 1200

   (c) 470

Odrediti prosječan protok za date brojeve impulsa, ako su se desili u toku od 140s.

(a)

$$V = KN = 12.2\frac{cm^3}{impuls} \cdot 220 = 2684cm^3$$
$$Q = \frac{V}{\Delta t} = \frac{2684cm^3}{140s} = 19.2\frac{cm^3}{s}$$

(b)

$$V = 12.2\frac{cm^3}{impuls} \cdot 1200 = 14640cm^3$$
$$Q = \frac{14640cm^3}{140s} = 104.6\frac{cm^3}{s}$$

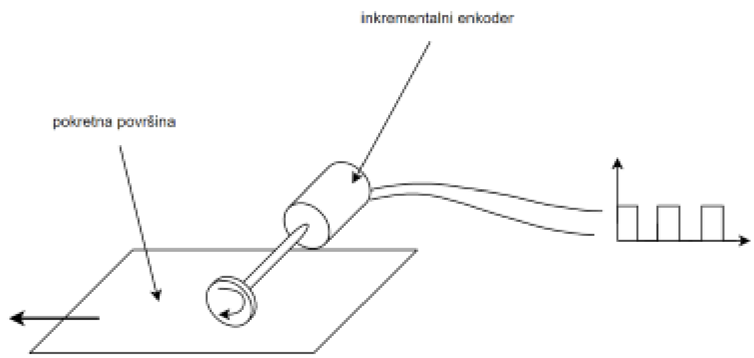
(c)

$$V = 12.2\frac{cm^3}{impuls} \cdot 470 = 5734cm^3$$
$$Q = \frac{5734cm^3}{140s} = 41\frac{cm^3}{s}$$

$$q_{prosjecno} = \frac{KN}{\Delta t} \Rightarrow q = Kf$$
$$f = 10Hz \Rightarrow q = 12.2 \cdot 10 = 122\frac{cm^3}{s}$$

22. Inkrementalni enkoder se koristi za mjerenje pravolinijskog pomjeranja kao na slici. Prečnik točka je 5.91cm, a rezolucija enkodera je $180\frac{impulsa}{obrtaj}$. Odrediti koliko je pomjeranje pokretne površine po impulsu enkodera i koliko se ukupno pomjerila pokretna površina ako je sa enkodera dobijeno 700 impulsa.

$$N = 180 \quad \text{Pomjeranje po impulsu} = \frac{\pi d}{N} = \frac{\pi \cdot 0.0591}{180} = 0.00103m = 0.103cm$$
$$\text{Ukupno pomjeranje} = 0.00103 \cdot 700 = 0.722m$$



23. Apsolutni enkoder se koristi za mjerenje koje zahtjeva rezoluciju od najmanje 1′. Odrediti broj bita koji je potreban za traženu rezoluciju.

$$\begin{aligned} \text{Broj minuta u krugu} &= 60 \cdot 360 = 21600'/krug \\ 2^{14} &= 16384 \\ 2^{15} &= 37768 \Rightarrow \text{Potrebno je 15 bita} \end{aligned}$$

24. Inkrementalni enkoder ima rezoluciju $2000\frac{impulsa}{obrtaj}$.

    (a) Odrediti broj impulsa koji će generisati enkoder u toku 5 ms ako se osovina čiju brzinu mjeri obrće brzinom od $1200\frac{obrtaja}{min}$.

    (b) Odrediti brzinu osovine usljed koje je generisano 224 impulsa u intervalu od 5 ms.

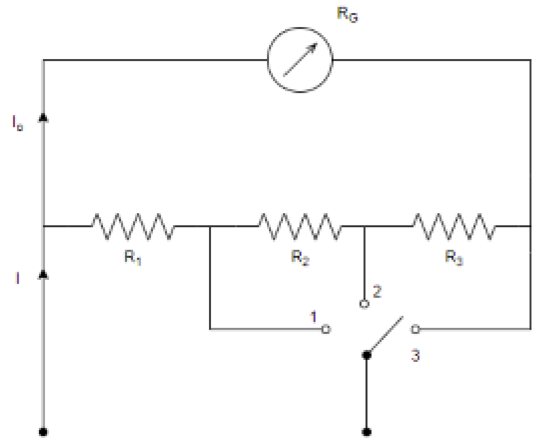
(a)

$$\text{Broj impulsa} = \frac{0.005s}{60} \cdot 1200\frac{obrtaja}{min} \cdot 2000\frac{impulsa}{obrtaj} = 200\text{impulsa}$$

(b)

$$\begin{aligned} f &= \frac{224}{0.005} \\ \text{brzina} &= \frac{f \cdot 60}{rezolucija} = \frac{224 \cdot 60}{0.005 \cdot 2000} = 1344\frac{obrtaja}{min} \end{aligned}$$

25. Dat je ampermetar sa pokretnim kalemom unutrašnje otpornosti $R_G = 100\Omega$. Maksimalna struja koja odgovara punom otklonu kazaljke je $I_{omax} = 1mA$. Primjenom Ejrtonovog šanta proširiti mjerni opseg instrumenta na 10 mA, 20 mA i 30 mA.



$$I_o = \frac{R_1}{R_G + R_1 + R_2 + R_3} \cdot I \tag{1}$$

$$I_o = \frac{R_1 + R_2}{R_G + R_1 + R_2 + R_3} \cdot I \tag{2}$$

$$I_o = \frac{R_1 + R_2 + R_3}{R_G + R_1 + R_2 + R_3} \cdot I \tag{3}$$

$$
\begin{aligned}
I_{max1} &= \left(\frac{R_2+R_3+R_G}{R_1}+1\right)\cdot I_{o\,max} \\
I_{max2} &= \left(\frac{R_3+R_G}{R_1+R_2}+1\right)\cdot I_{o\,max} \\
I_{max3} &= \left(\frac{R_G}{R_1+R_2+R_3}+1\right)\cdot I_{o\,max} \\
R_S &= R_1+R_2+R_3 \\
I_{max3} &= 10mA \\
10 &= \left(\frac{R_G}{R_S}+1\right)\cdot 1 \Rightarrow 10\cdot R_S = 100+R_S \\
R_S &= \frac{100}{9}\Omega \\
I_{max1} &= 30mA \\
R_S &= \frac{100}{9} \Rightarrow (R_2+R_3) = \frac{100}{9}-R_1 \\
30 &= \frac{\frac{100}{9}+100-R_1}{R_1}+1 \Rightarrow 30\cdot R_1 = \frac{100}{9}+100-R_1+R_1 = \frac{1000}{9} \\
R_1 &= \frac{1000}{9}\cdot\frac{1}{30} = \frac{100}{27}\Omega \\
R_3 &= \frac{100}{9}-\frac{100}{27}-R_2 = \frac{200}{27}-R_2 \\
I_{max2} &= 20mA \\
20 &= \frac{100+\frac{200}{27}-R_2}{R_2+\frac{100}{27}}+1 \\
20\cdot\left(R_2+\frac{100}{27}\right) &= 100+\frac{200}{27}-R_2+R_2+\frac{100}{27} \\
R_2+\frac{100}{27} &= 5+\frac{10}{27}+\frac{5}{27} \\
R_2 &= \frac{10+135+5-100}{27} = \frac{50}{27}\Omega \\
R_3 &= \frac{150}{27}\Omega
\end{aligned}
$$

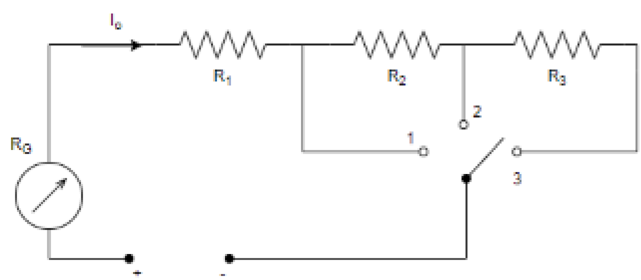
26. Neka je dat voltmetar sa pokretnim kalemom karakteristične otpornosti $R_k = 1\frac{k\Omega}{V}$. Unutrašnja otpornost uređaja je $R_G = 10k\Omega$. Koliki maksimalni napon uređaj može da mjeri? Kolika je maksimalna struja kroz uređaj? Upotrebom predotpornika

proširiti mjerni opseg na 20V, 50V i 100V.

$$R_k = 1\frac{k\Omega}{V} = \frac{R_G}{U_{omax}} = \frac{10k\Omega}{U_{omax}} \Rightarrow U_{omax} = 10V$$

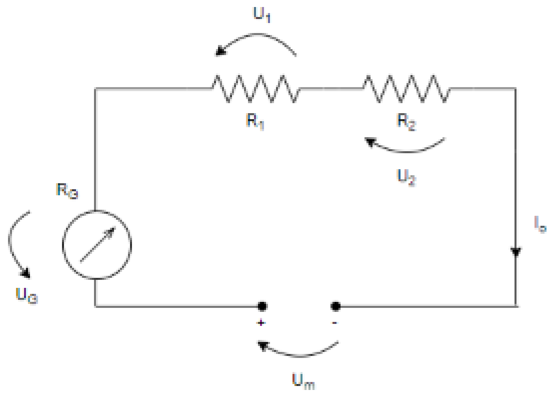
$$U_{omax} = R_G \cdot I_{omax} \Rightarrow I_{omax} = \frac{U_{omax}}{R_G} = \frac{1}{\frac{R_G}{U_{omax}}} = \frac{1}{R_k}$$

$$I_{omax} = \frac{1}{R_k} = 1mA$$



$$U_{max3} = (R_1 + R_2 + R_3 + R_G)I_{omax}$$
$$U_{max2} = (R_1 + R_2 + R_G)I_{omax}$$
$$U_{max1} = (R_1 + R_G)I_{omax}$$
$$20V = (R_1 + 10k\Omega) \cdot 1mA \Rightarrow R_1 = 10k\Omega$$
$$U_{max2} = 50V = (20k\Omega + R_2) \cdot 1mA$$
$$R_2 = 30k\Omega \quad R_3 = 50k\Omega$$

27. Odrediti mjereni napon ako instrument iz prethodnog zadatka pokazuje 8V, pri čemu se prekidač nalazi u položaju broj 2. Kolika je maksimalna greška mjerenja ako je otpornik R_2 tolerancije $\pm 0.1\%$.



$$U_M = U_G + (R_1 + R_2) \cdot I_o = U_G + (R_1 + R_2)\frac{U_G}{R_G} = U_G(1 + \frac{R_1 + R_2}{R_G}) = K \cdot U_G \quad K = 1 + \frac{R_1 + R_2}{R_G}$$

$$K = 1 + \frac{10 + 30}{10} = 5$$

$$U_M = 40V$$

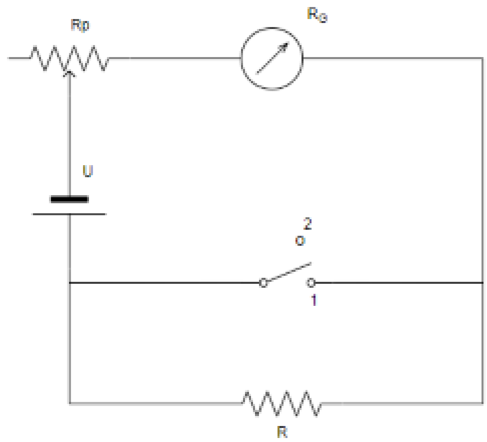
$$R_2 = 30k\Omega \pm 30\Omega \Rightarrow \Delta R = \pm 30\Omega$$

$$\Delta U = I_o \Delta R = \frac{U_G}{R_G}\Delta R$$

$$\Delta U = U_G(\pm\frac{30}{10000}) = U_G(\pm 0.3\%)$$

$$U_G = 8V \Rightarrow \Delta U = \pm 0.024V$$

28. Otpornost se mjeri instrumentom sa pokretnim kalemom. Napon laboratorijskog izvora je $10V$. Struja maksimalnog otklona ampermetra je $I_o = 10mA$. U položaju 2 instrument pokazuje vrijednost $I = 6mA$. Kolika je vrijednost mjerenog otpornika?



$$I_o = \frac{U}{R_G + R_p} \quad (1)$$
$$I = \frac{U}{R_G + R_p + R} \quad (2)$$

$$\alpha = \frac{I}{I_o}$$
$$\frac{I}{I_o} = \frac{\frac{U}{R_G+R_p+R}}{\frac{U}{R_G+R_p}} = \frac{R_G + R_p}{R_G + R_p + R}$$
$$R_G + R_p = \frac{U}{I_o}$$
$$\frac{I}{I_o} = \frac{\frac{U}{I_o}}{R + \frac{U}{I_o}}$$
$$R\alpha + \alpha\frac{U}{I_o} = \frac{U}{I_o} \Rightarrow R = \frac{U}{I_o}\frac{1-\alpha}{\alpha}$$
$$\alpha = \frac{6}{10} = \frac{3}{5}$$
$$R = \frac{10}{10 \cdot 10^{-3}}\frac{1-\frac{3}{5}}{\frac{3}{5}} = 1000\frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}k\Omega$$

29. Odrediti srednju i efektivnu vrijednost dvostrano ispravljenog
   (a) sinusnog signala
   (b) trougaonog talasnog oblika
   (c) pravougaonog talasnog oblika faktora ispune 50%

$$U_{sr} = \frac{1}{T}\int_0^T U(t)\,dt \qquad U_{eff} = \sqrt{\frac{1}{T}\int_0^T U^2(t)\,dt}$$

(a)

$$U(t) = |U_m \sin \omega t| \qquad \omega = \frac{2\pi}{T}$$

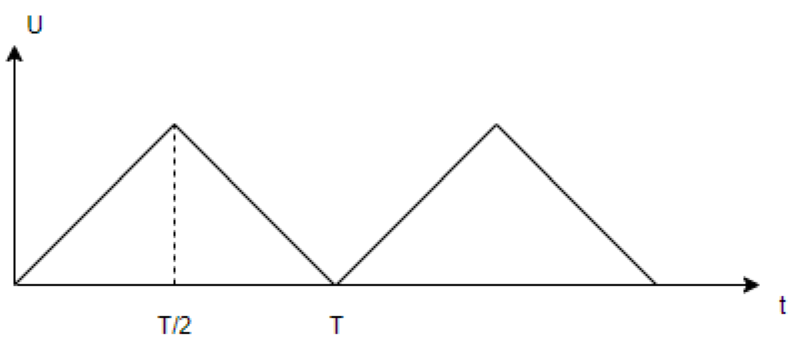
$$U_{sr} = \frac{1}{T}\int_0^T |U_m \sin \omega t|\,dt = \frac{2}{T}\int_0^{\frac{T}{2}} U_m \sin \omega t\,dt$$

$$= \frac{2}{T}U_m \int_0^{\frac{T}{2}} \sin \frac{2\pi}{T} t\,dt = \frac{2}{T}U_m \frac{T}{2\pi}\left(-\cos \frac{2\pi}{T} t\right)\Bigg|_0^{\frac{T}{2}} = \frac{U_m \cdot 2}{\pi}$$

$$U_{eff} = \sqrt{\frac{1}{T}\int_0^T {U_m}^2 \sin^2 \omega t\,dt} = \sqrt{\frac{2}{T}\int_0^{\frac{T}{2}} {U_m}^2 \sin^2 \omega t\,dt}$$

$$= \sqrt{\frac{2}{T}{U_m}^2 \int_0^{\frac{T}{2}} \left(\frac{1}{2} - \frac{1}{2}\cos 2\omega t\right)\,dt} = \frac{\sqrt{2}U_m}{2}$$

(b)



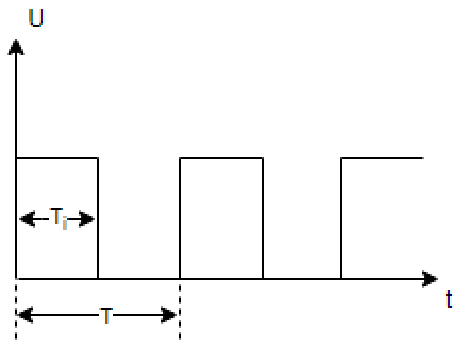
$$U_{sr} = \frac{1}{T}\int_0^T U(t)\,dt = \frac{2}{T}\int_0^{\frac{T}{2}} \frac{U_m}{\frac{T}{2}} t\,dt$$

$$= \frac{2}{T}\frac{U_m}{\frac{T}{2}}\frac{t^2}{2}\Bigg|_0^{\frac{T}{2}} = \frac{4U_m}{T^2}\cdot\frac{T^2}{8} = \frac{U_m}{2}$$

$$U_{eff} = \sqrt{\frac{2}{T}\int_0^{\frac{T}{2}} \frac{{U_m}^2}{(\frac{T}{2})^2} t^2\,dt} = \sqrt{\frac{2}{T}{U_m}^2 \frac{4}{T^2}\cdot\frac{t^3}{3}\Bigg|_0^{\frac{T}{2}}}$$

$$= \sqrt{\frac{2}{T}{U_m}^2 \frac{4}{T^2}\cdot\frac{T^3}{24}} = \frac{U_m}{\sqrt{3}}$$

(c)



$$T = 2 \cdot T_i$$
$$U_{sr} = \frac{1}{T}\int_0^T U(t)\,dt = \frac{1}{T}\int_0^{T_i} U_m\,dt = U_m\frac{T_i}{T}$$
$$U_{sr} = \frac{U_m}{2}$$
$$U_{eff} = \sqrt{\frac{1}{T}\int_0^{T_i} {U_m}^2\,dt} = U_m\sqrt{\frac{T_i}{T}} = \frac{U_m}{\sqrt{2}}$$

30. Izračunati faktor oblika za signale iz prethodnog zadatka. Koli- ka sistematska greška se pravi ako se mjeri efektivna vrijednost trougaonog i pravougaonog talasnog oblika instrumentom sa pokretnim kalemom koji je baždaren za mjerenje efektivne vrijednosti sinusnog signala?

$$F_o = \frac{U_{eff}}{U_{sr}}$$
$$F_{os} = \frac{\sqrt{2}U_m}{2} \cdot \frac{\pi}{U_m \cdot 2} = \frac{\pi}{2\sqrt{2}} = 1.11$$
$$F_{ot} = \frac{U_m}{\sqrt{3}} \cdot \frac{2}{U_m} = \frac{2}{\sqrt{3}} = 1.16$$
$$F_{op} = \frac{U_m}{\sqrt{2}} \cdot \frac{2}{U_m} = \sqrt{2} = 1.41$$
$$U_m = F_o \cdot U_{sr}$$

(a) trougaoni talasni oblik

$$U_m = F_{os} \cdot U_{sr}$$
$$U_o = F_{ot} \cdot U_{sr}$$
$$G_r = \frac{U_m - U_o}{U_o} = \frac{F_{os} - F_{ot}}{F_{ot}} \cdot 100\% = \frac{1.11 - 1.16}{1.16} \cdot 100\% = -4.3\%$$

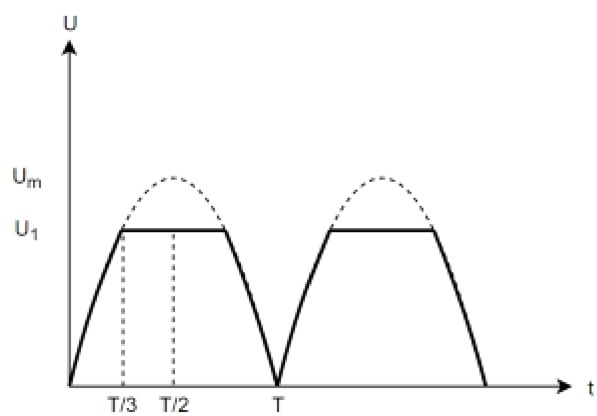
(b) pravougaoni talasni oblik

$$U_m = F_{os} \cdot U_{sr}$$
$$U_o = F_{op} \cdot U_{sr}$$
$$G_r = \frac{U_m - U_o}{U_o} = \frac{F_{os} - F_{op}}{F_{op}} \cdot 100\% = \frac{1.11 - 1.41}{1.41} \cdot 100\% = -21.27\%$$

31. Voltmetrom se mjeri efektivna vrijednost napona trougaonog talasnog oblika. Ako voltmetar pokazuje 10 V, kolika je stvarna efektivna vrijednost mjerenog napona?

$$U_m = F_{os} \cdot U_{sr} \Rightarrow U_{sr} = \frac{10}{1.11} = 9V$$
$$U_{eff} = F_{ot} \cdot U_{sr} = 1.16 \cdot 9 = 10.45V$$

32. Izračunati faktor oblika signala sa slike. Kolika sistematska greška se pravi ako se mjeri efektivna vrijednost signala sa slike instrumentom sa pokretnim kalemom koji je baždaren za mjerenje efektivne vrijednosti dvostrano ispravljenog sinusnog signala?



$$
\begin{aligned}
\omega &= \frac{2\pi}{2T} = \frac{\pi}{T} \\
U(t) &= U_m \sin\left(\frac{\pi}{T} t\right) \\
U_1 &= U\left(\frac{T}{3}\right) = U_m \sin\left(\frac{\pi}{T}\frac{T}{3}\right) = U_m \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} U_m \\
U_{sr} &= \frac{1}{T}\int_0^T U(t)\,dt = \frac{2}{T}\int_0^{\frac{T}{2}} U(t)\,dt = \frac{2}{T}\left[\int_0^{\frac{T}{3}} U(t)\,dt + \int_{\frac{T}{3}}^{\frac{T}{2}} U(t)\,dt\right] \\
U_{sr} &= \frac{2}{T}\left[\int_0^{\frac{T}{3}} U_m \sin\left(\frac{\pi}{T} t\right) dt + \int_{\frac{T}{3}}^{\frac{T}{2}} \frac{\sqrt{3}}{2} U_m\,dt\right] \\
U_{sr} &= \frac{2U_m}{T}\left[\int_0^{\frac{T}{3}} \sin\left(\frac{\pi}{T} t\right) dt + \frac{\sqrt{3}}{2}\int_{\frac{T}{3}}^{\frac{T}{2}} dt\right] \\
U_{sr} &= \frac{2U_m}{T}\left[\frac{T}{\pi}\left(-\cos\left(\frac{\pi}{T} t\right)\right)\Bigg|_0^{\frac{T}{3}} + \frac{\sqrt{3}}{2} t\Bigg|_{\frac{T}{3}}^{\frac{T}{2}}\right] \\
U_{sr} &= \frac{2U_m}{T}\left[-\frac{T}{\pi}\left(\cos\left(\frac{\pi}{3}\right) - \cos(0)\right) + \frac{\sqrt{3}}{2}\left(\frac{T}{2} - \frac{T}{3}\right)\right] \\
U_{sr} &= \frac{2U_m}{\cancel{T}}\left[\frac{\cancel{T}}{\pi}\left(1 - \frac{1}{2}\right) + \frac{\sqrt{3}}{2}\frac{\cancel{T}}{6}\right] \\
U_{sr} &= 2U_m\left[\frac{1}{2\pi} + \frac{\sqrt{3}}{12}\right] = U_m\left[\frac{1}{\pi} + \frac{\sqrt{3}}{6}\right] \approx 0.607 U_m
\end{aligned}
$$

$$U_{eff} = \sqrt{\frac{1}{T}\int_0^T U^2(t)\,dt} = \sqrt{\frac{2}{T}\int_0^{\frac{T}{2}} U^2(t)\,dt}$$

$$U_{eff} = \sqrt{\frac{2}{T}\left[\int_0^{\frac{T}{3}} {U_m}^2 \sin^2(\frac{\pi}{T}t)\,dt + \int_{\frac{T}{3}}^{\frac{T}{2}} \left(\frac{\sqrt{3}}{2}U_m\right)^2 dt\right]}$$

$$U_{eff} = U_m\sqrt{\frac{2}{T}\left[\int_0^{\frac{T}{3}} \sin^2(\frac{\pi}{T}t)\,dt + \frac{3}{4}\int_{\frac{T}{3}}^{\frac{T}{2}} dt\right]}$$

$$U_{eff} = U_m\sqrt{\frac{2}{T}\left[\int_0^{\frac{T}{3}} \frac{1-\cos(\frac{2\pi}{T}t)}{2}\,dt + \frac{3}{4}t\Big|_{\frac{T}{3}}^{\frac{T}{2}}\right]}$$

$$U_{eff} = U_m\sqrt{\frac{\cancel{2}}{T}\left[\frac{1}{\cancel{2}}\int_0^{\frac{T}{3}} dt - \frac{1}{\cancel{2}}\int_0^{\frac{T}{3}} \cos(\frac{2\pi}{T}t)\,dt + \frac{T}{\overset{4}{\cancel{8}}}\right]}$$

$$U_{eff} = U_m\sqrt{\frac{1}{T}\left[t\Big|_0^{\frac{T}{3}} - \frac{T}{2\pi}\sin(\frac{2\pi}{T}t)\Big|_0^{\frac{T}{3}} + \frac{T}{4}\right]}$$

$$U_{eff} = U_m\sqrt{\frac{1}{T}\left[t\Big|_0^{\frac{T}{3}} - \frac{T}{2\pi}\sin(\frac{2\pi}{T}t)\Big|_0^{\frac{T}{3}} + \frac{T}{4}\right]}$$

$$U_{eff} = U_m\sqrt{\frac{1}{\cancel{T}}\left[\frac{7}{12}\cancel{T} - \frac{\cancel{T}}{2\pi}\sin(\frac{2\pi}{3})\right]} = U_m\sqrt{\frac{7}{12} - \frac{1}{2\pi}\frac{\sqrt{3}}{2}} = U_m\sqrt{\frac{7}{12} - \frac{\sqrt{3}}{4\pi}}$$

$$U_{eff} = 0.667U_m$$

$$F_o = \frac{U_{eff}}{U_{sr}} = \frac{0.667U_m}{0.607U_m} = 1.099$$

$$G_r = \frac{U_m - U_o}{U_o} = \frac{F_{os}\cancel{U_{sr}} - F_o\cancel{U_{sr}}}{F_o\cancel{U_{sr}}} = \frac{F_{os} - F_o}{F_o} = \frac{1.11 - 1.099}{1.099} = 0.0101$$

$$G_r(\%) = 1.01\%$$