$$G_{Ho}G^*(s) = \mathcal{Z}\left\{\left(\mathcal{L}^{-1}\left\{G_{Ho}(s)G(s)\right\}\right)^*\right\} = \frac{1-e^{-T}}{z-e^{-T}}$$

$$G_{Ho}H^*(s) = \mathcal{Z}\left\{\left(\mathcal{L}^{-1}\left\{G_{Ho}(s)H(s)\right\}\right)^*\right\}$$

$$= \mathcal{Z}\left\{\left(\mathcal{L}^{-1}\left\{\frac{1-e^{-Ts}}{s}\cdot 2\right\}\right)^*\right\}$$

$$= (1-z^{-1})\,\mathcal{Z}\left\{\left(\mathcal{L}^{-1}\left\{\frac{2}{s}\right\}\right)^*\right\}$$

$$= \frac{\cancel{z-1}}{\cancel{z}}\frac{2\cancel{z}}{\cancel{z-1}} = 2$$

$$G_{DE}(z) = \frac{K\frac{1-e^{-T}}{z-e^{-T}}}{1+2K\frac{1-e^{-T}}{z-e^{-T}}}$$

$$= \frac{K(1-e^{-T})}{z-e^{-T}+2K(1-e^{-T})}$$

Вежбе 04.12.2018.

Грешка у устаљеном стању

- Устаљено стање постоји за стабилне системе

$$\lim_{k\to\infty} f(kT) = \lim_{z\to 1}(1-z^{-1})F(z)$$

1) Наћи одзив система $G(z) = \frac{z}{z-a}$ у устаљеном стању на импулсну побуду.

а) Без граничне теореме

$$G(z) = \frac{Y(z)}{U(z)} \longrightarrow Y(z) = G(z)U(z)$$

$$= \frac{z}{z-a}\,1 \quad /\mathcal{Z}^{-1}$$

$$y(k) = a^k u(k)$$

1° $|a|<1$

$$y_{ss} = \lim_{k\to\infty} y(k) = 0$$

2° $a = 1$

$$y_{ss} = \lim_{k\to\infty} y(k) = 1$$

3° $|a|>1$

$$y_{ss} = \lim_{k\to\infty} y(k) = \infty$$

нема устаљено стање!

б) Са применом граничне теореме

$$Y(z) = \frac{z}{z-a}$$

1° $|a| \geq 1 \rightarrow$ нестабилан

дакле, нема устаљено стање

2° $|a|<1$

$$y_{ss} = \lim_{z\to 1}(1-z^{-1})\frac{z}{z-a}$$

$$= \lim_{z\to 1}\frac{z-1}{\cancel{z}}\cdot\frac{\cancel{z}}{z-a} = 0$$

2) Наћи одзив система $G(z) = \frac{z}{z-0,5}$ у устаљеном стању на степ побуду

$$u(k) \longrightarrow \boxed{G(z)} \longrightarrow y(k)$$

$$G(z) = \frac{Y(z)}{U(z)} \Rightarrow Y(z) = G(z)\,U(z)$$

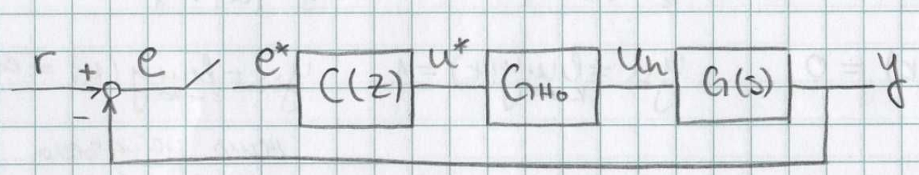
$$= \frac{z}{z-0,5}\,\frac{z}{z-1}$$

$$= \frac{z^2}{(z-0,5)(z-1)}$$

$$y_{ss} = \lim_{z\to 1}(1-z^{-1})\,\frac{z^2}{(z-0,5)(z-1)}$$

$$= \lim_{z\to 1}\frac{\cancel{z-1}}{\cancel{z}}\,\frac{z^{\cancel{2}}}{(\cancel{z-1})(z-0,5)}$$

$$= \frac{1}{0,5} = \underline{\underline{2}}$$



$$e = r - y$$

3) Израчунати грешку у устаљеном стању ако је

$$G(s) = \frac{1}{s+1} \qquad r(t) = u(t)$$

$$C(z) = K$$

$$E^*(s) = R^*(s) - Y^*(s)$$

$$= R^*(s) - U^*(s)\,G_{H0}G^*(s)$$

$$= R^*(s) - E^*(s)\,C(z)\,G_{H0}G^*(s)$$

$$E^*(s)\big(1 + C(z)\,G_{H0}G^*(s)\big) = R^*(s)$$

$$G_{RE}(z) = \frac{E^*(s)}{R^*(s)} = \frac{1}{1 + C(z)\,G_{H0}G^*(s)}$$

$$G_{H0}G^*(s) = \mathcal{Z}\left\{\frac{1-e^{-sT}}{s}\cdot\frac{1}{s+1}\right\}$$

$$= \mathcal{Z}\left\{\frac{1-e^{-sT}}{s(s+1)}\right\}$$

$$= (1-z^{-1})\,\mathcal{Z}\left\{\frac{K_1}{s} + \frac{K_2}{s+1}\right\}$$

$$= (1-z^{-1})\,\mathcal{Z}\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$$

$$= (1-z^{-1})\left[\frac{z}{z-1} - \frac{z}{z-e^{-T}}\right]$$

$$= \frac{\cancel{z-1}}{\cancel{z}}\,\frac{\cancel{z^2} - ze^{-T} \cancel{- z^2} + z}{(\cancel{z-1})(z-e^{-T})}$$

$$= \frac{1-e^{-T}}{(z-e^{-T})}$$

$$G_{RE}(z) = \frac{1}{1 + K\,\frac{1-e^{-T}}{z-e^{-T}}}$$

$$= \frac{1}{\frac{z-e^{-T}+K-Ke^{-T}}{z-e^{-T}}}$$

$$= \frac{z-e^{-T}}{z-e^{-T}+K-Ke^{-T}}$$

$$G_{RE}(z) = \frac{E(z)}{R(z)} \longrightarrow E(z) = G_{RE}(z)\,R(z)$$

$$= \frac{z-e^{-T}}{z-e^{-T}+K-Ke^{-T}}\;\frac{z}{z-1}$$

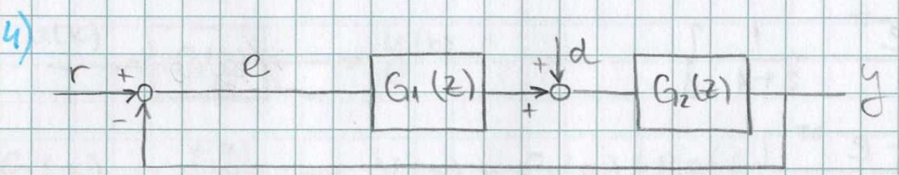
$$\lim_{z\to 1}(1-z^{-1})\,E(z) = \lim_{z\to 1}\left(\frac{\cancel{z-1}}{\cancel{z}}\;\frac{z-e^{-T}}{z-e^{-T}+K-Ke^{-T}}\;\frac{\cancel{z}}{\cancel{z-1}}\right.$$

$$= \frac{1-e^{-T}}{1-e^{-T}+K-Ke^{-T}}$$

Ако систем има више улаза

$$e_{ss} = e_{ss1} + e_{ss2} + \dots + e_{ssn}$$

4)



Пронаћи грешку у устаљеном стању за систем са слике ако је

a) $r(t) = u(t)$

$d(k) = u(k)$

$G_1(z) = 1$

$G_2(z) = \dfrac{1}{z - 0{,}5}$

$e_{ss} = e_{ssr} + e_{ssd}$

e_{ssr}:

$$E(z) = R(z) - Y(z)$$

$$= R(z) - E(z)G_1(z)G_2(z)$$

$$E(z)\left[1 + G_1(z)G_2(z)\right] = R(z)$$

$$G_{RE}(z) = \frac{1}{1 + G_1(z)G_2(z)}$$

$$= \frac{1}{1 + 1 \cdot \frac{1}{z - 0{,}5}}$$

$$= \frac{z - 0{,}5}{z - 0{,}5 + 1}$$

$$= \frac{z - 0{,}5}{z + 0{,}5}$$

$$G_{RE}(z) = \frac{E(z)}{R(z)} \rightarrow E(z) = G_{RE}(z)R(z)$$

$$= \frac{z - 0{,}5}{z + 0{,}5} \cdot \frac{z}{z - 1}$$

$$= \frac{z(z - 0{,}5)}{(z + 0{,}5)(z - 1)}$$

$$e_{ssr} = \lim_{z \to 1}(1 - z^{-1})E(z)$$

$$= \lim_{z \to 1} \frac{\cancel{z - 1}}{\cancel{z}} \cdot \frac{\cancel{z}(z - 0{,}5)}{(z + 0{,}5)\cancel{(z - 1)}}$$

$$= \frac{0{,}5}{1{,}5} = \frac{1}{3}$$

e_{ssd}:

$$E(z) = -Y(z)$$

$$= -G_2(z)\left(E(z)G_1(z) + D(z)\right)$$

$$= -G_2(z)E(z)G_1(z) - G_2(z)D(z)$$

$$E(z)\left[1 + G_2(z)G_1(z)\right] = -G_2(z)D(z)$$

$$G_{DE}(z) = \frac{-G_2(z)}{1 + G_2(z)G_1(z)} \qquad G_{DE}(z) = \frac{E(z)}{D(z)} \rightarrow E(z) = G_{DE}(z)D(z)$$

$$E(z) = \frac{-\frac{1}{z - 0{,}5}}{1 + \frac{1}{z - 0{,}5}} \cdot \frac{z}{z - 1}$$

$$= \frac{\frac{-1}{\cancel{z - 0{,}5}}}{\frac{z - 0{,}5 + 1}{\cancel{z - 0{,}5}}} \cdot \frac{z}{z - 1}$$

$$= \frac{-z}{(z + 0{,}5)(z - 1)}$$

$$e_{ssd} = \lim_{z \to 1}(1 - z^{-1})E(z)$$

$$= \lim_{z \to 1} \frac{\cancel{z - 1}}{\cancel{z}} \cdot \frac{-\cancel{z}}{(z + 0{,}5)\cancel{(z - 1)}}$$

$$= \frac{-1}{1{,}5} = -\frac{2}{3}$$

$$e_{ss} = e_{ssr} + e_{ssd}$$

$$= \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$$

f) $r(k) = u(k)$

$d(k) = u(k)$

$G_1(z) = 1$

$G_2(z) = \frac{1}{z-1}$

$e_{ss} = e_{ssr} + e_{ssd}$

e_{ssr}:

$$G_{RE}(z) = \frac{1}{1+G_1(z)G_2(z)}$$

$$= \frac{1}{1+1\,\frac{1}{z-1}}$$

$$= \frac{z-1}{z\cancel{-1}\cancel{+1}}$$

$$= \frac{z-1}{z}$$

$$G_{RE}(z) = \frac{E(z)}{R(z)} \rightarrow E(z) = G_{RE}(z)R(z)$$

$$= \frac{\cancel{z-1}}{\cancel{z}}\,\frac{\cancel{z}}{\cancel{z-1}} = 1$$

$$e_{ssr} = \lim_{z\to 1}(1-z^{-1})E(z)$$

$$= \lim_{z\to 1}\frac{z-1}{z}\,1 = 0$$

e_{ssd}:

$$G_{DE}(z) = \frac{-G_2(z)}{1+G_1(z)G_2(z)}$$

$$= \frac{-\frac{1}{\cancel{z-1}}}{1+\frac{1}{\cancel{z-1}}}$$

$$= \frac{-1}{z\cancel{-1}\cancel{+1}}$$

$$= -\frac{1}{z}$$

$$G_{DE}(z) = \frac{E(z)}{D(z)} \rightarrow E(z) = G_{DE}(z)D(z)$$

$$= \frac{-1}{\cancel{z}}\,\frac{\cancel{z}}{z-1}$$

$$e_{ssd} = \lim_{z\to 1}(1-z^{-1})E(z)$$

$$= \lim_{z\to 1}\frac{\cancel{z-1}}{z}\,\frac{-1}{\cancel{z-1}} = -1$$

$$e_{ss} = e_{ssr} + e_{ssd} = -1$$

g) $r(k) = u(k)$

$d(k) = u(k)$

$G_1(z) = \frac{1}{z-1}$

$G_2(z) = 1$

e_{ssr}:

$$G_{RE}(z) = \frac{1}{1+G_1(z)G_2(z)}$$

$$= \frac{1}{1+\frac{1}{z-1}\,1}$$

$$= \frac{z-1}{z}$$

$$G_{RE}(z) = \frac{E(z)}{R(z)} \rightarrow E(z) = G_{RE}(z)R(z)$$

$$= \frac{\cancel{z-1}}{\cancel{z}}\,\frac{\cancel{z}}{\cancel{z-1}} = 1$$

$$e_{ssr} = \lim_{z\to 1}(1-z^{-1})E(z)$$

$$= \lim_{z\to 1}\frac{z-1}{z}\,1 = 0$$

e_{ssd}:

$$G_{DE}(z) = \frac{-G_2(z)}{1+G_1(z)G_2(z)}$$

$$= \frac{-1}{1+\frac{1}{z-1}}$$

$$= \frac{-z+1}{z}$$

$$G_{DE}(z) = \frac{E(z)}{D(z)} \rightarrow E(z) = G_{DE}(z)D(z)$$

$$= \frac{-\cancel{(z-1)}}{\cancel{z}} \frac{\cancel{z}}{\cancel{z-1}} = -1$$

$$e_{ssd} = \lim_{z\to 1}(1-z^{-1})E(z)$$

$$= \lim_{z\to 1}\frac{z-1}{z}(-1) = 0$$

$$e_{ss} = 0+0 = 0$$

g) $r(k) = k h(k)$

$d(k) = k h(k)$

$G_1(z) = 1$

$G_2(z) = \frac{1}{z-1}$

e_{ssr}: $G_{RE}(z) = \frac{1}{1+G_1(z)G_2(z)}$

$$= \frac{1}{1+\frac{1}{z-1}}$$

$$= \frac{z-1}{z}$$

$$G_{RE}(z) = \frac{E(z)}{R(z)} \rightarrow E(z) = G_{RE}(z)R(z)$$

$$= \frac{\cancel{z-1}}{\cancel{z}} \frac{\cancel{z}}{(z-1)^2}$$

$$= \frac{1}{z-1}$$

$$e_{ssr} = \lim_{z\to 1}(1-z^{-1})E(z)$$

$$= \lim_{z\to 1}\frac{\cancel{z-1}}{z}\frac{1}{\cancel{z-1}} = 1$$

$N \backslash r$	$h(k)$	$kh(k)$	$\frac{k^2}{2}h(k)$
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{T}{K_v}$	∞
2	0	0	$\frac{T}{K_a}$

$N \backslash r$	$h(k)$	$kh(k)$	$\frac{k^2}{2}h(k)$
0	const.	∞	∞
1	0	const.	∞
2	0	0	const

- Табела се односи на случај када немамо поремећај у директној грани

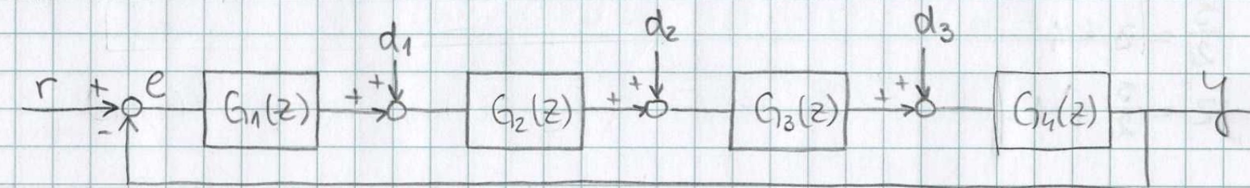
N - ред астатизма у директној грани пре поремећаја

$$K_p = \lim_{s\to 0} W(s)$$

$$K_v = \lim_{s\to 0} sW(s)$$

$$K_a = \lim_{s\to 0} s^2 W(s)$$

ДОМАЋИ



$G_1(z) = \frac{1}{z-0{,}5}$

$G_2(z) = \frac{z}{z-1}$

$G_3(z) = 2$

$G_4(z) = \frac{1}{z-1}$

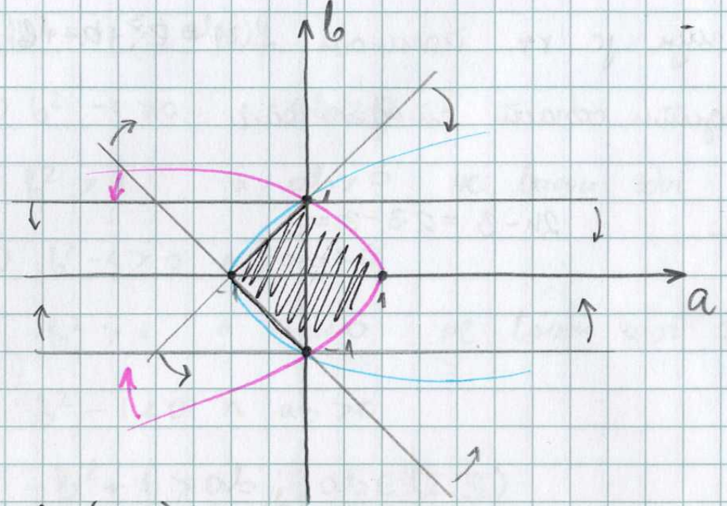
Наћи:

$r(k)$	e_{ssr}
$\delta(k)$	
$h(k)$	
$kh(k)$	

d_1	e_{ssd_1}
$\delta(k)$	
$h(k)$	
$kh(k)$	

d_2	e_{ssd_2}
$\delta(k)$	
$h(k)$	
$kh(k)$	

d_3	e_{ssd_3}
$\delta(k)$	
$h(k)$	
$kh(k)$	



$b > -a-1$

$b < a+1$

$-1 < b < 1$

$1-b^2 > -a$

$1-b^2 > a$

ако $b \in (-1,1)$ тада $a=0$

ако $b=0$: $1-0=-a$, $\boxed{a=-1}$

ако $b \in (1 \vee -1)$: $a=0$

ако $b=0$: $+1-0=a$, $\boxed{a=1}$

Вежбе 20.11.2018.

Имплементација дискретних система

деф. ф-ју преноса

$$u(k) \longrightarrow \boxed{G(z)} \longrightarrow y(k) \qquad G(z) = \frac{Y(z)}{U(z)}$$

пример.

$$G(z) = \frac{1}{z-0{,}5}$$

а) Одзив на импулс

$$u(k) = \begin{cases} 1, & k=0 \\ 0, & k \neq 0 \end{cases} \qquad U(z) = 1$$

$$G(z) = \frac{Y(z)}{U(z)} \longrightarrow Y(z) = G(z)\,U(z)$$

$$= \frac{1}{z-0{,}5}\,1 \quad / \mathcal{Z}^{-1}$$

$$y(k) = \mathcal{Z}^{-1}\left\{\frac{1}{z-0{,}5}\right\}$$

$$= \mathcal{Z}^{-1}\left\{z^{-1}\frac{z}{z-0{,}5}\right\}$$

$$= 0{,}5^{k-1}\,h(k-1)$$

б) Одзив на степ

$$u(k) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases} \qquad U(z) = \frac{z}{z-1}$$

$$G(z) = \frac{Y(z)}{U(z)} \longrightarrow Y(z) = G(z)\,U(z)$$

$$= \frac{1}{z-0{,}5}\,\frac{z}{z-1}$$

$$= \frac{z}{(z-0{,}5)(z-1)} \quad / \mathcal{Z}^{-1}$$

$$y(k) = \mathcal{Z}^{-1}\left\{\frac{z}{(z-0{,}5)(z-1)}\right\}$$

$$= \mathcal{Z}^{-1}\left\{\frac{k_1}{z-0{,}5} + \frac{k_2}{z-1}\right\}$$

$$= \mathcal{Z}^{-1}\left\{-\frac{1}{z-0{,}5} + \frac{2}{z-1}\right\}$$

$$= -\mathcal{Z}^{-1}\left\{\frac{z}{z-0{,}5}z^{-1}\right\} + 2\mathcal{Z}^{-1}\left\{\frac{z}{z-1}z^{-1}\right\}$$

$$= -0{,}5^{k-1}h(k-1) + 2h(k-1)$$

Имплементирати дати систем на рачунару, односно за дате вредности одбирака $u(k)$ и на основу $G(z)$ одредити вр. одбирака $y(k)$:

$$G(z) = \frac{1}{z-0{,}5}$$

$$\frac{Y(z)}{U(z)} = \frac{1}{z-0{,}5}$$

$$Y(z)[z-0{,}5] = U(z)$$

$$U(z) = zY(z) - 0{,}5Y(z) \quad / z^{-1}$$

$$z^{-1}U(z) = Y(z) - 0{,}5z^{-1}Y(z)$$

$$Y(z) = z^{-1}U(z) + 0{,}5z^{-1}Y(z) \quad / \mathcal{Z}^{-1}$$

$$y(k) = u(k-1) + 0{,}5y(k-1)$$

Псеудо код за имплементацију

```
y_pethodno = 0
u_prethodno = 0
while (1)
u_trenutno = pročitaj_u()          → очитавање нових вредности
y_trenutno = 0,5 y_pethodno + u_prethodno   → рачунање нових излаза
y_pethodno = y_trenutno
u_prethodno = y_prethodno          } ажурирање прошењивих
end
```

Дискретизација филтара

$$G(s) = \frac{1}{s(s+1)}$$

- полови: $p_1 = 0 \quad p_2 = -1$
- стабилност: гранично стабилан
- ред система: II

* Импулсно - инваријантна трансформација

$$\mathcal{Z}^{-1}\{G(z)\cdot 1\} = \left(\mathcal{L}^{-1}\{G(s)\cdot 1\}\right)^*$$

$$G(z) = \mathcal{Z}\left\{\left(\mathcal{L}^{-1}\{G(s)\,1\}\right)^*\right\}$$

$$\mathcal{L}^{-1}\{G(s)\,1\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{k_1}{s} + \frac{k_2}{s+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$f(t) = h(t) - e^{-t}h(t) \xrightarrow{*} f(kT) = h(kT) - e^{-kT}h(kT)$$

$$G(z) = \mathcal{Z}\left\{h(kT) - e^{-kT}h(kT)\right\}$$

$$= \frac{z}{z-1} - \frac{z}{z-e^{-T}}$$

$$= \frac{\cancel{z^2} - ze^{-T} - \cancel{z^2} + z}{(z-1)(z-e^{-T})}$$

$$= \frac{z(1-e^{-T})}{(z-1)(z-e^{-T})}$$

$$G(z)=\frac{z(1-e^{-T})}{(z-1)(z-e^{-T})}$$

- полови: $z_1=1 \quad z_2=e^{-T}<1 \quad (T>0)$
- стабилност: гр. стабилан
- ред система: II
- кашњење: $d=\underset{\text{ред имениоца}}{n}-\underset{\text{ред бројиоца}}{m}=2-1=1$ → кашњење од једне периоде

Имплементирати и упоредити импулсне одзиве система G(s) и G(z)

$$G(z)=\frac{z-ze^{-T}}{z^2+z(-1-e^{-T})+e^{-T}}$$

$$\frac{Y(z)}{U(z)}=\frac{z-ze^{-T}}{z^2+z(-1-e^{-T})+e^{-T}}$$

$$z^2Y(z)-zY(z)-e^{-T}zY(z)+e^{-T}Y(z)=zU(z)-ze^{-T}U(z)\ /z^{-2}$$

$$y(k)=(1+e^{-T})y(k-1)+e^{-T}y(k-2)+(1-e^{-T})u(k-1)$$

* Степ-инваријантна трансформација

$$\mathcal{Z}^{-1}\left\{\frac{z}{z-1}G(z)\right\}=\left(\mathcal{L}^{-1}\left\{\frac{1}{s}G(s)\right\}\right)^*$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}G(s)\right\}=\mathcal{L}^{-1}\left\{\frac{1}{s}\frac{1}{s(s+1)}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{K_1}{s^2}+\frac{K_2}{s}+\frac{K_3}{s+1}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}-\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$f(t)=th(t)-h(t)+e^{-t}h(t)$$

$$f(kT)=Th(kT)-h(kT)+e^{-kT}h(kT)$$

$$\frac{z}{z-1}G(z)=\mathcal{Z}\left\{\left(\mathcal{L}^{-1}\left\{\frac{1}{s}G(s)\right\}\right)^*\right\}$$

$$\rightarrow G(z)=\frac{z-1}{z}\mathcal{Z}\left\{\left(\mathcal{L}^{-1}\left\{\frac{1}{s}G(s)\right\}\right)^*\right\}$$

$$G(z)=\frac{z-1}{z}\mathcal{Z}\left\{Th(kT)-h(kT)+e^{-kT}h(kT)\right\}$$

$$=\frac{z-1}{z}\left(\frac{Tz}{(z-1)^2}-\frac{z}{z-1}+\frac{z}{z-e^{-T}}\right)$$

$$=\frac{\cancel{z-1}}{\cancel{z}}\frac{T\cancel{z}}{(z-1)^{\cancel{2}}}-\frac{\cancel{z-1}}{\cancel{z}}\frac{\cancel{z}}{\cancel{z-1}}+\frac{z-1}{\cancel{z}}\frac{\cancel{z}}{z-e^{-T}}$$

$$=\frac{T}{(z-1)}-1+\frac{z-1}{z-e^{-T}}$$

$$=\frac{z(T+e^{-T}-1)+(-Te^{-T}+1)}{(z-1)(z-e^{-T})}$$

- полови: $z_1=1 \quad z_2=e^{-T}$
- стаб: гр. стабилан
- ред система: II
- кашњење: $d=n-m=2-1=1$

* Прва Ојлерова тр-ја

- Диференцирање унапред

$$y'(t)=u(t)\ /\mathcal{L}$$

$$sY(s)=U(s)$$

$$\frac{y(k+1)-y(k)}{T}=u(k)\ /\mathcal{Z}$$

$$\frac{zY(z)-Y(z)}{T}=U(z)$$

$$\left.\begin{array}{l}\dfrac{Y(z)(z-1)}{T}=U(z)\\[2ex] Y(s)\cdot s=U(s)\end{array}\right\}\rightarrow s=\frac{z-1}{T}$$

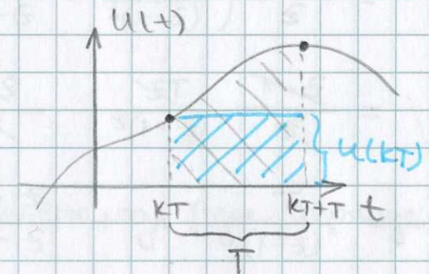
$$sT=z-1$$

$$z=sT+1$$

• Интеграција уназад

$$\dot{y}(t) = u(t)$$

$$\int_{kT}^{kT+T} \dot{y}(t)\,dt = \int_{kT}^{kT+T} u(t)\,dt$$



$$y(kT+T) - y(kT) = u(kT)T \quad / Z$$

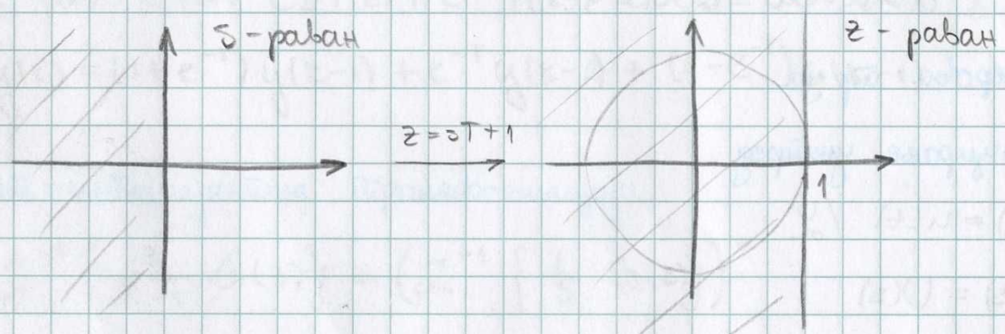
$$zY(z) - Y(z) = T\,U(z)$$

$$Y(z)(z-1) = T\,U(z)$$

$$Y(z)\,\frac{z-1}{T} = U(z) \qquad s = \frac{z-1}{T}$$

$$Y(s)\cdot s = U(s) \qquad z = sT + 1$$

• Пресликавање стабилних полова из s у z



$$G(s) = \frac{1}{s(s+1)}$$

$$s = \frac{z-1}{T}$$

$$G(z) = \frac{1}{\frac{z-1}{T}\left(\frac{z-1}{T}+1\right)} = \frac{1}{\frac{z-1}{T}\left(\frac{z-1+T}{T}\right)} = \frac{T^2}{(z-1)(z-1+T)}$$

• полови: $z_1 = 1 \quad z_2 = 1-T$

• стабилност: $|1-T| < 1$

$T \in (0,2]$ гр. стаб.

$T > 2$ нестабилан

• ред система: II

• кашњење: $d = n - m = 2 - 0 = 2$ периоде

* Друга Ојлерова трансформација

• Диференцирање уназад

$$\dot{y}(t) = u(t) \quad / L$$

$$sY(s) = U(s)$$

$$\frac{y(k) - y(k-1)}{T} = u(k) \quad / Z$$

$$\frac{Y(z) - z^{-1}Y(z)}{T} = U(z)$$

$$\left.\begin{aligned} Y(z)\left(\frac{1-z^{-1}}{T}\right) &= U(z) \\ Y(s) \quad s \quad &= U(s) \end{aligned}\right\} \quad s = \frac{1-z^{-1}}{T}$$

$$sT = 1 - z^{-1}$$

$$z^{-1} = sT - 1$$

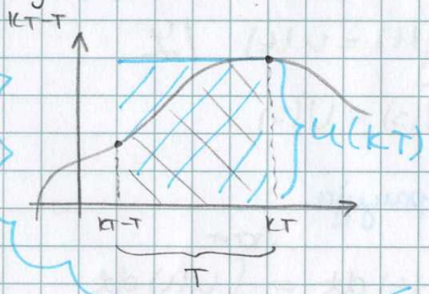
$$z = \frac{1}{1-sT}$$

• Интеграција унапред

$$\dot{y}(t) = u(t)$$

$$\int_{kT-T}^{kT} \dot{y}(t)\,dt = \int_{kT-T}^{kT} u(t)\,dt$$

$$\int_{kT-T}^{kT} u(t)\,dt = ?$$



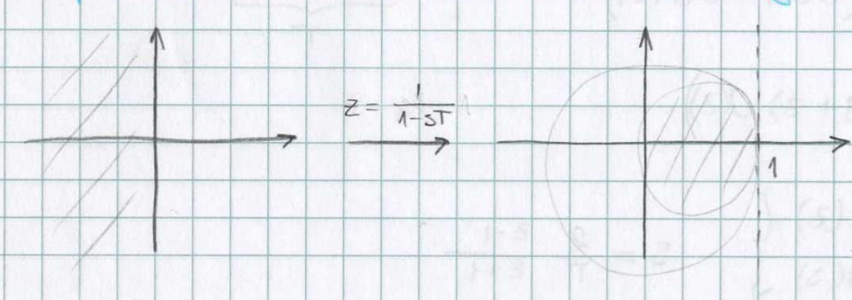
$$y(kT) - y(kT-T) = T\,u(kT) \quad / Z$$

$$Y(z) - z^{-1}Y(z) = T\,U(z)$$

$$Y(z)(1 - z^{-1}) = T\,U(z)$$

$$\left.\begin{aligned} Y(z)\left(\frac{1-z^{-1}}{T}\right) &= U(z) \\ Y(s) \quad s \quad &= u(s) \end{aligned}\right\} \quad s = \frac{1-z^{-1}}{T}$$

• Пресликавање стабилних полова из s у z



$$\frac{1}{1-(-\infty)} = \frac{1}{\infty} = 0$$

$$\frac{1}{1-0} = 1$$

Нула се слика у 1

$$G(s) = \frac{1}{s(s+1)} \qquad s = \frac{z-1}{zT}$$

$$G(s) = \frac{1}{\frac{z-1}{zT}\left(\frac{z-1}{zT}+1\right)}$$

$$= \frac{1}{\frac{z-1}{zT}\;\frac{z-1+zT}{zT}}$$

$$= \frac{(zT)^2}{(z-1)(z(1+T)-1)}$$

$$= \frac{(zT)^2}{(z-1)(1+T)\left(1-\frac{1}{1+T}\right)}$$

$$= \frac{(zT)^2}{(z-1)\left(z-\frac{1}{1+T}\right)}\;\frac{1}{1+T}$$

- полови: $z_1 = 1 \quad z_2 = \frac{1}{1+T}$
- стабилност: гр. стаб.
- ред система: II
- кашњење: $d = n - m = 2 - 2 = 0$

* Тустинова апроксимација

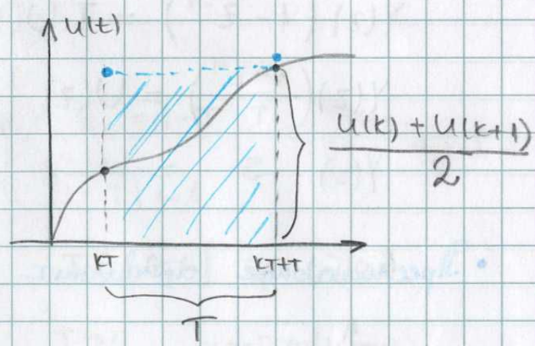
$$\dot{y}(t) = u(t) \quad /\mathcal{L}$$

$$sY(s) = U(s)$$

- Интеграција

$$\int_{kT}^{kT+T} \dot{y}(t)\,dt = \int_{kT}^{kT+T} u(t)\,dt$$

$$y(kT+T) - y(kT) = \frac{T(u(k)+u(k+1))}{2} \quad /\mathcal{Z}$$



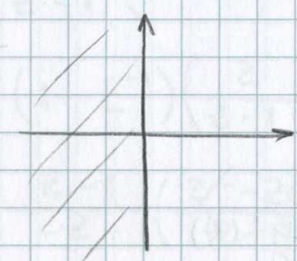
$$zY(z) - Y(z) = \frac{T}{2}\left(U(z) + zU(z)\right)$$

$$Y(z)(z-1) = \frac{T}{2}\left((1+z)U(z)\right)$$

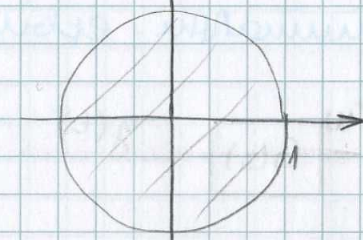
$$\left.\begin{aligned} Y(z)\,\frac{2}{T}\,\frac{z-1}{z+1} &= U(z) \\ Y(s)\quad s\quad &= U(s) \end{aligned}\right\} \qquad s = \frac{2}{T}\,\frac{z-1}{z+1}$$

$$z = 1 + \frac{sT}{2} \Big/ 1 - \frac{sT}{2}$$

- Пресликавање стабилних полова из s у z



$$z = \frac{1+\frac{sT}{2}}{1-\frac{sT}{2}}$$



$$G(s) = \frac{1}{s(s+1)} \qquad s = \frac{2}{T}\,\frac{z-1}{z+1}$$

$$G(s) = \frac{1}{\frac{2}{T}\frac{z-1}{z+1}\left(\frac{2}{T}\frac{z-1}{z+1}+1\right)}$$

$$= \frac{1}{\frac{2}{T}\frac{z-1}{z+1}\left(\frac{2z-2+zT+T}{T(z+1)}\right)}$$

$$= \frac{T^2(z+1)^2}{2(z-1)(z(2+T)-2+T)}$$

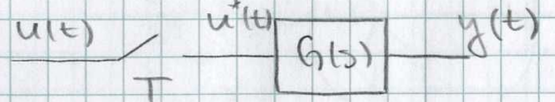
$$= \frac{T^2(z+1)^2}{2(z-1)(2+T)\left(z-\frac{2-T}{2+T}\right)}$$

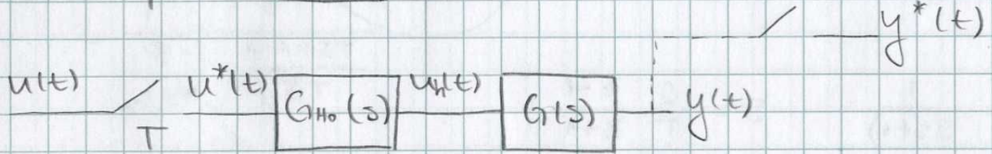
$$= \frac{T^2(z+1)^2}{2(z-1)\left(z+\frac{T-2}{T+2}\right)}\;\frac{1}{2+T}$$

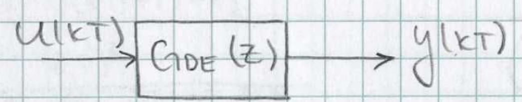
- полови: $z_1 = 1 \quad z_2 = -\frac{T-2}{T+2}$
- стабилност: гр. стабилан
- ред система: II
- кашњење: $d = n - m = 2 - 2 = 0$

Вежбе 27.11.2018.

Дигитални еквиваленти







$$Y^*(s) = (G_{H0}(s)G(s))^* U^*(s)$$

$$Y^*(s) = G_{H0}G(s)^* U^*(s)$$

$$Y(z) = G_{H0}G(z)\cdot U(z) \qquad G_{H0}G(s)^* = \mathcal{Z}\{(\mathcal{L}^{-1}\{G_{H0}(s)G(s)\})^*\}$$

1. Дата је ф-ја преноса:

$$G(s) = \frac{1}{s+1}$$

$$G_{DE}(z) = \frac{Y(z)}{U(z)} = G_{H0}G(z) = \mathcal{Z}\{(\mathcal{L}^{-1}\{G_{H0}(s)G(s)\})^*\}$$

$$= \mathcal{Z}\left\{\left(\mathcal{L}^{-1}\left\{\frac{1-e^{-sT}}{s}\cdot\frac{1}{s+1}\right\}\right)^*\right\}$$

$$= \mathcal{Z}\left\{\left(\mathcal{L}^{-1}\left\{\frac{1-e^{-sT}}{s(s+1)}\right\}\right)^*\right\}$$

$$= \mathcal{Z}\left\{\left(\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\} - \mathcal{L}^{-1}\left\{\frac{e^{-sT}}{s(s+1)}\right\}\right)^*\right\}$$

$$= \mathcal{Z}\{(h(t) - h(t)e^{-t} - h(t-T) + h(t-T)e^{-(t-T)})^*\}$$

$$= \mathcal{Z}\{h(kT) - h(kT)e^{-kT} - h(kT-T) + h(kT-T)e^{-(kT-T)}\}$$

$$= \frac{z}{z-1} - \frac{z}{z-e^{-T}} - \frac{z}{z-1}z^{-1} + \frac{z}{z-e^{-T}}z^{-1}$$

$$= \frac{z}{z-1}(1-z^{-1}) - \frac{z}{z-e^{-T}}(1-z^{-1})$$

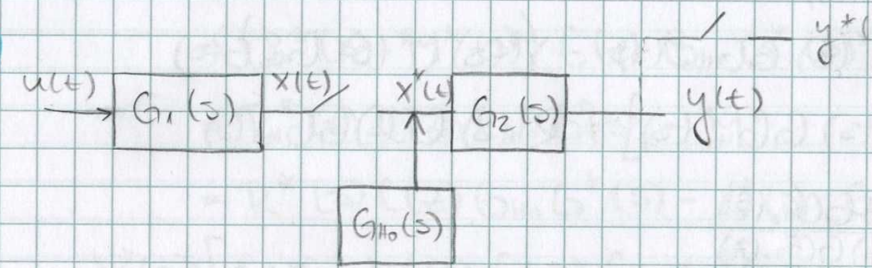
$$= (1-z^{-1})\left(\frac{z}{z-1} - \frac{z}{z-e^{-T}}\right)$$

$$= \frac{\cancel{z-1}}{\cancel{z}}\left(\frac{\cancel{z^2} - ze^{-T} - \cancel{z^2} + z}{\cancel{(z-1)}(z-e^{-T})}\right)$$

$$= \frac{1-e^{-T}}{z-e^{-T}}$$

- Какав је одзив стварног сист. и дигиталног екв.?

2)



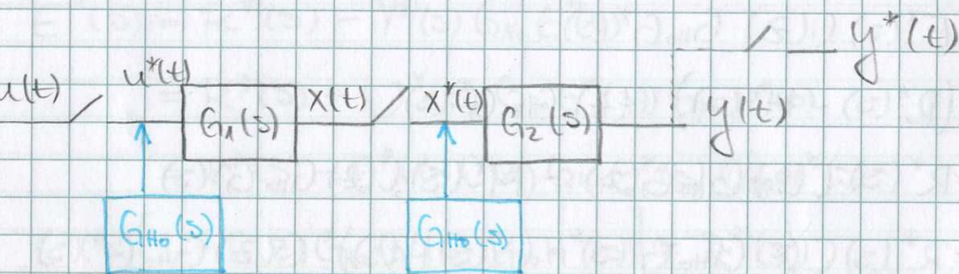
$$Y^*(s) = X^*(s)\, G_{H0}G_2(s)^*$$

$$= (U(s)G_1(s))^*\, G_{H0}G_2(s)^*$$

$$Y(z) = UG_1(z)\cdot G_{H0}G_2(z)$$

$\frac{Y(z)}{U(z)}$ → нема ф-је преноса јер U не зависи од z!

3)

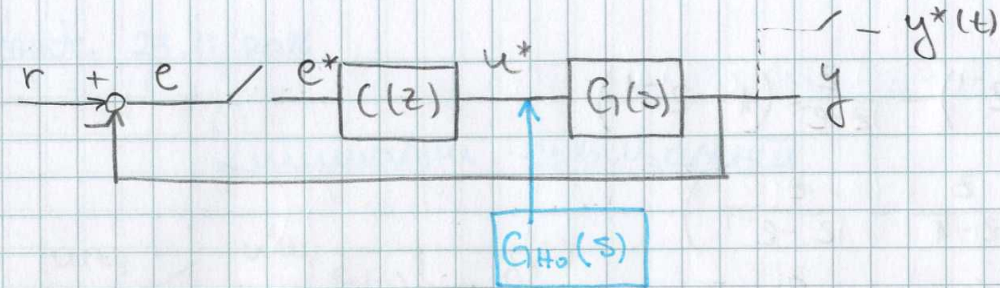


$$Y^*(s) = X^*(s)\cdot G_{H0}G_2(s)^*$$

$$= U^*(s)\, G_{H0}G_1(s)^*\, G_{H0}G_2(s)^*$$

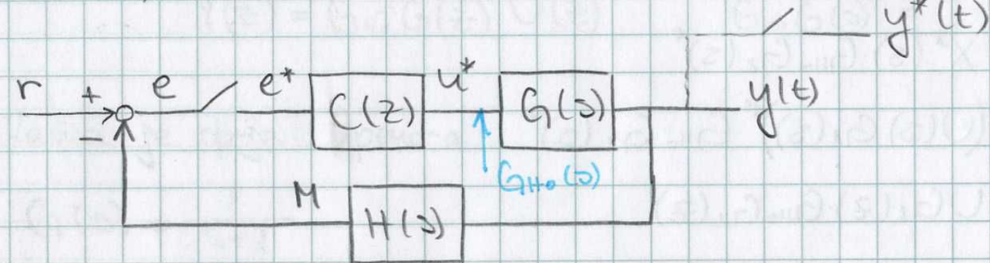
$$\frac{Y(z)}{U(z)} = G_{H0}G_1(z)\, G_{H0}G_1(z)$$

4)



$$Y^*(s) = U^*(s)\big(G(s)G_{H0}(s)\big)^*$$
$$= U^*(s)\cdot GG_{H0}^*(s)$$
$$= E^*(s)\,C^*(s)\cdot GG_{H0}^*(s)$$
$$= \big(R^*(s) - Y^*(s)\big)\,C^*(s)\,G_{H0}G^*(s)$$
$$Y^*(s) = R^*(s)\,C^*(s)\,GG_{H0}^*(s) - Y^*(s)\,C^*(s)\,G_{H0}G^*(s)$$
$$Y^*(s)\left[1 + C^*(s)\,GG_{H0}^*(s)\right] = R^*(s)\,C^*(s)\,GG_{H0}(s)$$
$$\frac{Y(z)}{R(z)} = \frac{C(z)\,GG_{H0}(z)}{1 + C(z)\,GG_{H0}(z)}$$

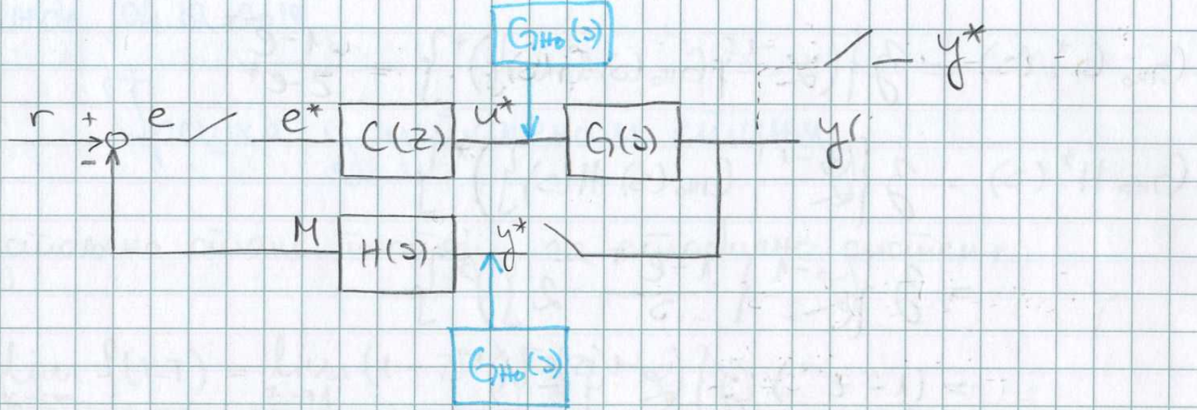
5)



$$Y^*(s) = U^*(s)\cdot G_{H0}G^*(s)$$
$$= E^*(s)\,C(z)\,G_{H0}G^*(s)$$
$$= \big(R^*(s) - M^*(s)\big)\,C(z)\,G_{H0}G^*(s)$$
$$= R^*(s)\,C(z)\,G_{H0}G^*(s) - M^*(s)\,C(z)\,G_{H0}G^*(s)$$
$$= R^*(s)\,C(z)\,G_{H0}G^*(s) - \big(Y(s)H(s)\big)^*\,C(z)\,G_{H0}G^*(s)$$
$$= R^*(s)\,C(z)\,G_{H0}G^*(s) - YH^*(s)\,C(z)\,G_{H0}G^*(s)$$

фја преноса не постоји

6)



$$Y^*(s) = U^*(s)\,G_{H0}G^*(s)$$
$$= E^*(s)\,C(z)\,G_{H0}G^*(s)$$
$$= \big(R^*(s) - M^*(s)\big)\,C(z)\,G_{H0}G^*(s)$$
$$= R^*(s)\,C(z)\,G_{H0}G^*(s) - M^*$$
$$= R^*(s)\,C(z)\,G_{H0}G^*(s) - Y^*(s)\,G_{H0}H^*(s)\,C(z)\,G_{H0}G^*(s)$$
$$Y^*(s)\left[1 + G_{H0}H^*(s)\,C(z)\,G_{H0}G^*(s)\right] = R^*(s)\,C(z)\,G_{H0}G^*(s)$$
$$\frac{Y^*(s)}{R^*(s)} = \frac{C(z)\,G_{H0}G^*(s)}{1 + G_{H0}H^*(s)\,C(z)\,G_{H0}G^*(s)} = G_{DE}(z)$$

Која је фја преноса од r до e?

$$E^*(s) = R^*(s) - M^*(s)$$
$$E^*(s) = R^*(s) - Y^*(s)\,G_{H0}G^*(s)$$
$$= R^*(s) - U^*(s)\,G_{H0}G^*(s)\,G_{H0}H^*(s)$$
$$= R^*(s) - E^*(s)\,C(z)\,G_{H0}G^*(s)\,G_{H0}H^*(s)$$
$$E^*(s)\left[1 + C(z)\,G_{H0}G^*(s)\,G_{H0}H^*(s)\right] = R^*(s)$$
$$\frac{E^*(s)}{R^*(s)} = \frac{1}{1 + C(z)\,G_{H0}G^*(s)\,G_{H0}H^*(s)}$$

Ако је $C(z) = K$, $G(s) = \frac{1}{s+1}$, $H(s) = 2$. Наћи $G_{DE}(z)$ из примера 6

Имплементирати у матлабу и упоредити са реалним системом

$$G_{DE}(z) = \frac{C(z)\,G_{H0}G^*(s)}{1 + G_{H0}G^*(s)\,G_{H0}H^*(s)\,C(z)}$$