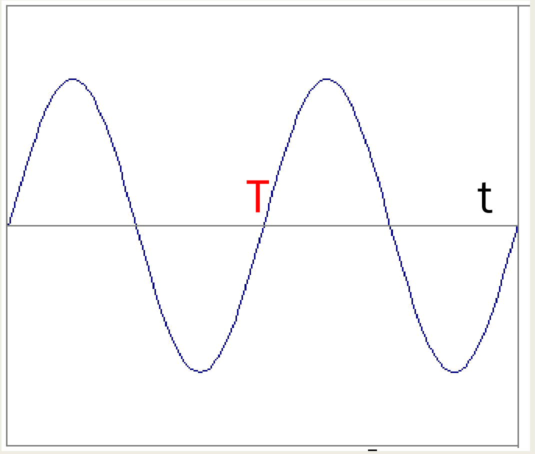
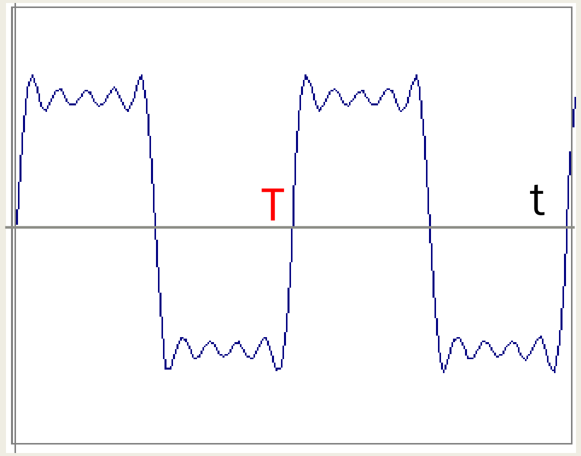
FREKVENCIJKI DOMEN

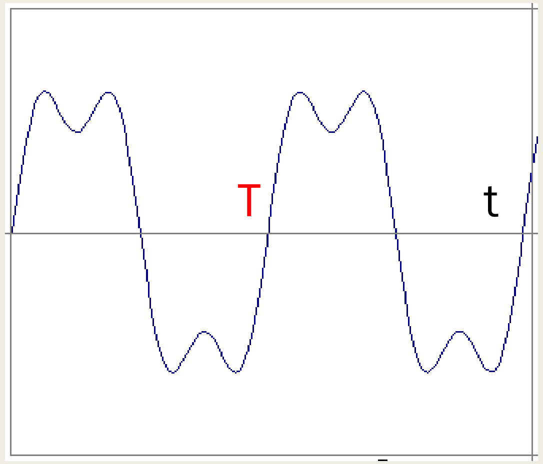
Primena DSP u upravljanju

Signali u vremenskom domenu

- Analiza složenih signala
- Frekvencija signala jedna od najbitnijih karakteristika
- Primer analize zvuka
- Frekvencijski domen







Kolika je frekvencija
Prikazanih signala?

Frekvencijska analiza signala

- Metoda koja bi dala odgovor na pitanje frekvencija složenih signala
- Složeni signali su svi osim prostoperiodičnih (sinus, odnosno kosinus)
- Ideja je da se složeni signal (funkcija) razloži na jednostavnije komponente
- Signal, odnosno funkcija f(x) može se predstaviti Tajlorovim redom:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

- Prikaz signala u obliku polinoma nam ipak ne daje odgovor na pitanja vezana za frekvencije signala i nema neki fizički smisao

Jean Baptiste Joseph Fourier (1768-1830)

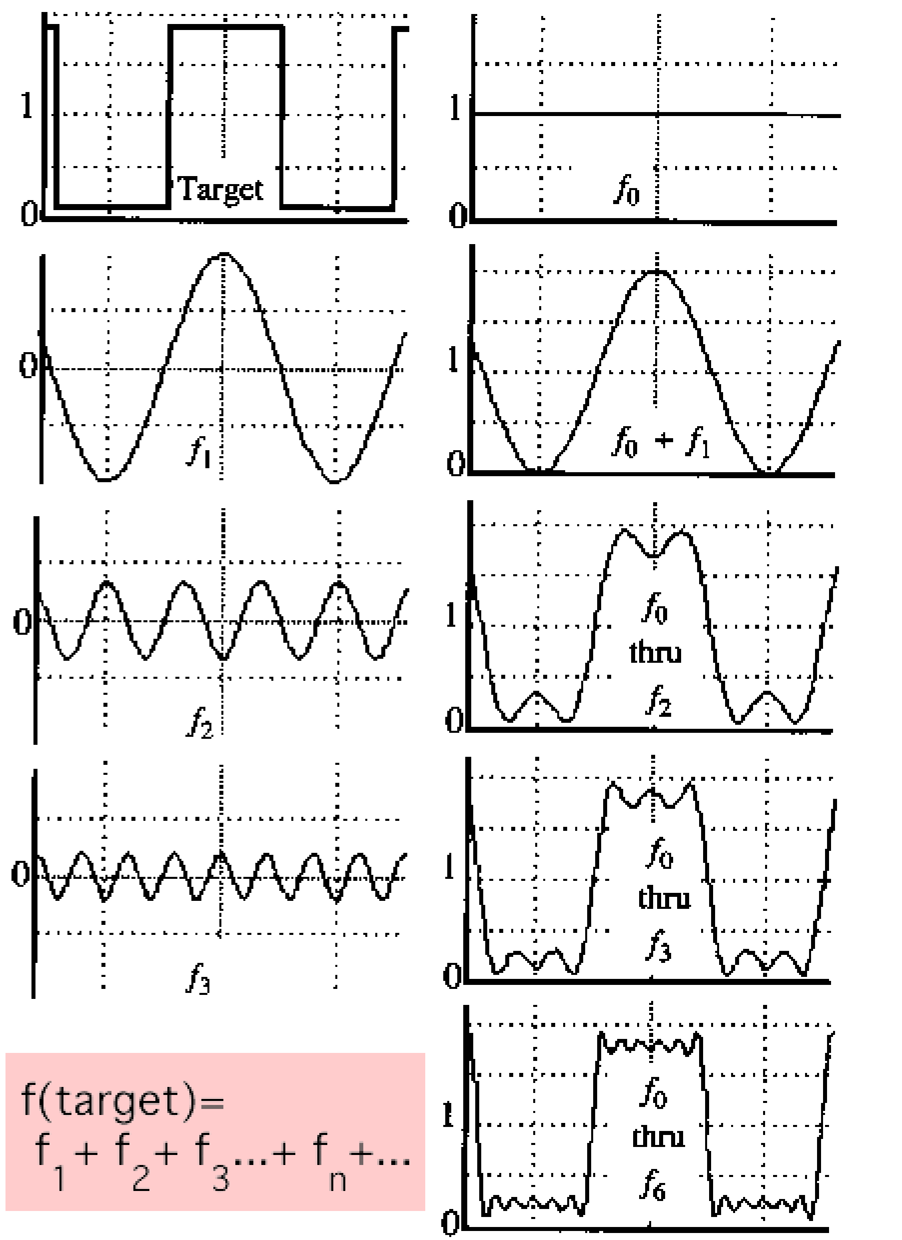
- 1807. godine je tvrdio:
  - *Svaka periodična funkcija se može predstaviti linearnom kombinacijom sinusa i kosinusa na različitim frekvencijama.*
- Teško za poverovati?
  - *Nisu poverovali ni Lagrange, Laplace, Poisson and ostali savremenici*
  - *Furijeovi radovi čak nisu prevedeni na engleski sve do 1878!*
- Ali je istina!
  - *Danas poznato kao Furijeov red*
  - *Verovatno jedan od najznačajnijih alata koje danas upotrebljavamo u inženjerstvu*

Suma prostoperiodičnih funkcija

- Osnovna komponenta:

$$A\sin(\omega x + \phi)$$

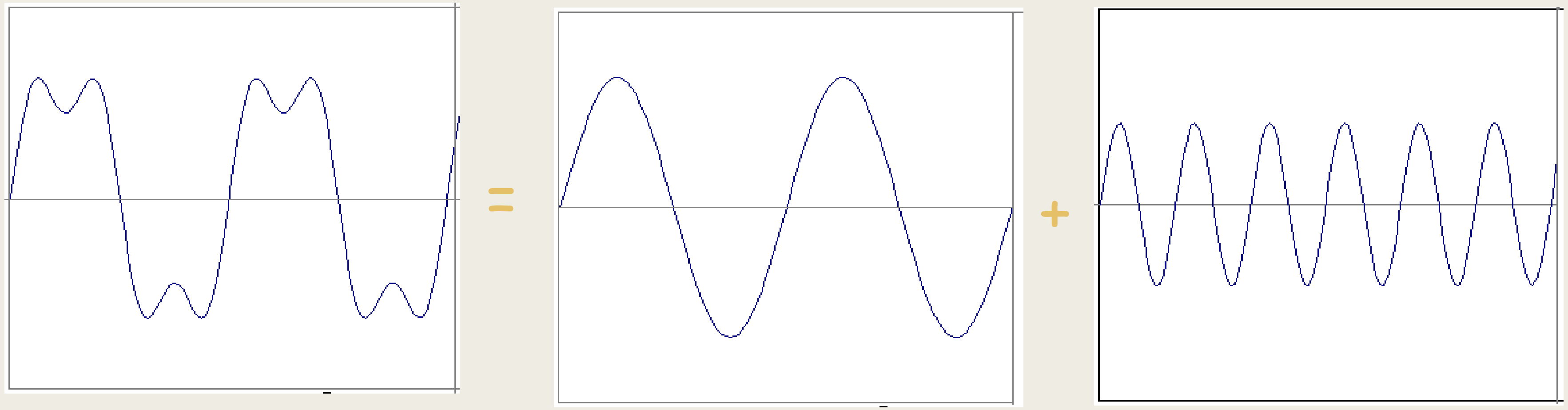
- Sabiranjem dovoljnog broja osnovnih komponenti možemo dobiti bilo koji signal *f(x)*
- Tri podešljiva parametra svake komponente:
  - *Frekvencija* ω
  - *Amplituda A*
  - *Faza* ϕ

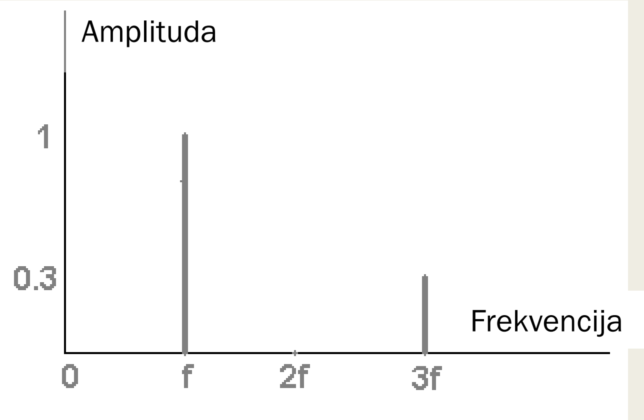


f(target)=
$f_1 + f_2 + f_3 ... + f_n + ...$

Suma prostoperiodičnih funkcija

- Primer : $g(t) = \sin(2pi\, f\, t) + (1/3)\sin(2pi\, (3f)\, t)$





Furijeov red
Frekvencijski domen periodičnih signala

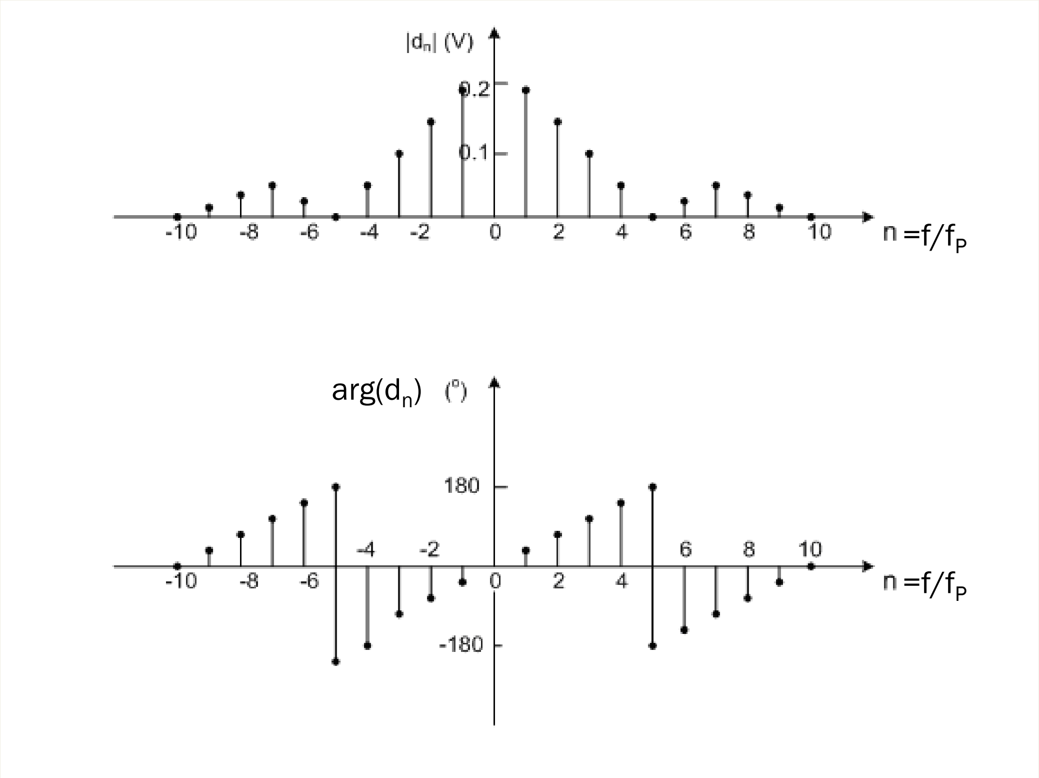
- Perioda periodičnog signala je najmanji pozitivan realan broj T_P za koji važi: $f(t) = f(t + kT_P),\;\; k = 1,2,3,\ldots$
- Periodični signal *f(t)*, periode T_P može se prikazati Furijeovim redom:
- $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_P t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_P t)\,; \cdots \omega_P = \frac{2\pi}{T_P}$
- $a_0 = \frac{1}{T_P}\int_{-T_P/2}^{T_P/2} f(t)dt$
- $a_n = \frac{2}{T_P}\int_{-T_P/2}^{T_P/2} f(t)\cos(n\omega_P t)\,dt\,; \cdots n = 1, 2,3, \ldots$
- $b_n = \frac{2}{T_P}\int_{-T_P/2}^{T_P/2} f(t)\sin(n\omega_P t)\,dt; \cdots n = 1, 2,3, \ldots$

Furijeov red

- Ako primenimo Ojlerove formule Furijeov red možemo napisati kao sumu kompleksnih prostoperiodičnih funkcija:
- $f(t) = \sum_{n=-\infty}^{\infty} d_n e^{jn\omega_P t}$
- $d_n = \frac{1}{T_P} \int_{-T_P/2}^{T_P/2} f(t) e^{-jn\omega_P t} dt$
- Ili na sledeći način:
- $f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_P t + \varphi_n)$
- $A_0 = |d_0|$
- $A_n = 2|d_n|$
- $\varphi_n = arg(d_n)$

Spektar signala

- Najčešće se grafički prikazuje na dva grafika, nezavisna promenljiva je frekvencija:
  - *Amplitudski spektar: prikazuje vrednosti koeficijenta* $|d_n|$ *ili* A_n
  - *Fazni spektar: prikazuje vrednosti koeficijenta* $\arg(d_n)$ *ili* φ_n
- Harmonik = prostoperiodična komponenta
- *n* je redni broj harmonika
- Nulti harmonik = srednja vrednost signala
- Amplitudski spektar je parna funkcija
- Fazni spektar je neparna funkcija



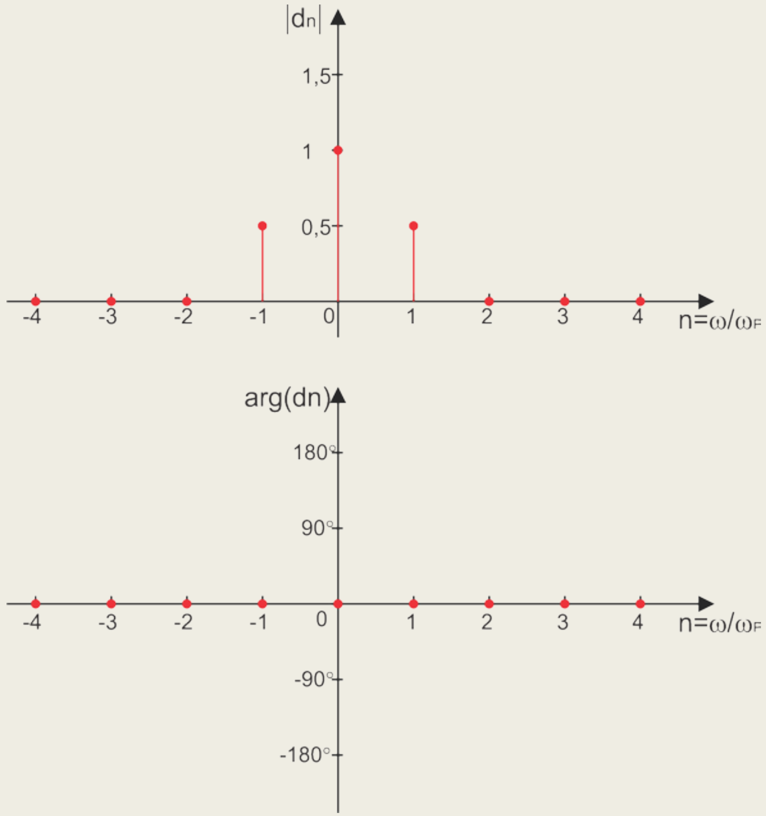
Snaga

- Snaga nultog harmonika: $P_0 = A_0^2 = |d_0|^2$
- Snaga ostalih harmonika: $P_n = ½ A_n^2 = 2|d_n|^2$
- Ukupna snaga signala: $P = A_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2$
- Odnosno: $P = \frac{1}{T}\int_{-T/2}^{T/2} f^2(t)dt$

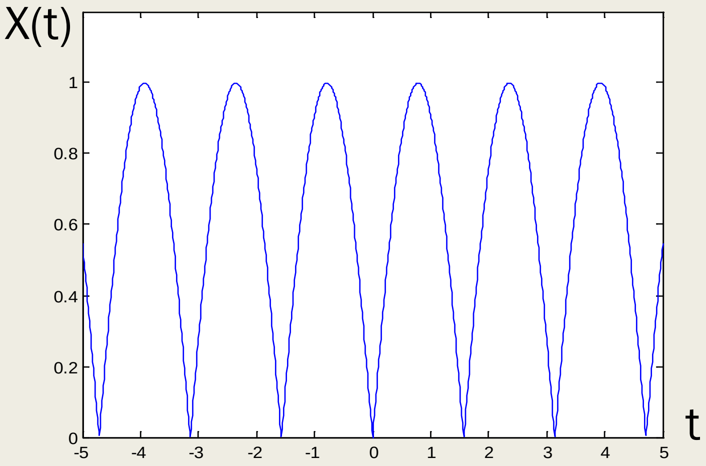
Primer 1: $f(t) = 1 + \cos(\omega_P t)$

$$f(t) = 1 + \frac{e^{-j\omega_P t} + e^{j\omega_P t}}{2} = 1 + \frac{1}{2}e^{-j\omega_P t} + \frac{1}{2}e^{j\omega_P t}$$

$d_0 = 1$ $\quad d_{-1} = \frac{1}{2}$ $\quad d_1 = \frac{1}{2}$ $\quad d_n = 0\ za\ |n| > 1$



Primer 2: $x(t) = |\sin 2t|$



- Perioda funkcije sin2t = sin(2 πt/T_P)?
- Perioda je $T_P = \pi$
- Perioda x(t) je upola manja
- T = (π/2)s i ω_0 = 4rad/s

$$d_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt = \frac{1}{T}\int_0^T \sin 2t \cdot e^{-j4nt}dt = \frac{1}{T}\int_0^T \frac{e^{j2t} - e^{-j2t}}{2j}\cdot e^{-j4nt}dt =$$

$$= \frac{1}{2jT}\int_0^T e^{j2(1-2n)t}dt - \frac{1}{2jT}\int_0^T e^{-j2(1+2n)t}dt =$$

$$= \frac{1}{2jT}\cdot\frac{1}{2(1-2n)}e^{j2(1-2n)t}\Big|_0^T + \frac{1}{2jT}\cdot\frac{1}{2(1+2n)}e^{-j2(1+2n)t}\Big|_0^T =$$

$$= \frac{1}{4T(1-2n)}\cdot\left(1 - e^{j2(1-2n)T}\right) + \frac{1}{4T(1+2n)}\cdot\left(1 - e^{-j2(1+2n)T}\right)$$

$$d_n = \frac{(1+2n)\left(1-e^{j2(1-2n)T}\right)+(1-2n)\left(1-e^{-j2(1+2n)T}\right)}{4T(1-4n^2)} =$$

$$\frac{\left(1-e^{j2(1-2n)T}+1-e^{-j2(1+2n)T}\right)+2n\left(1-e^{j2(1-2n)T}-1+e^{-j2(1+2n)T}\right)}{4T(1-4n^2)} =$$

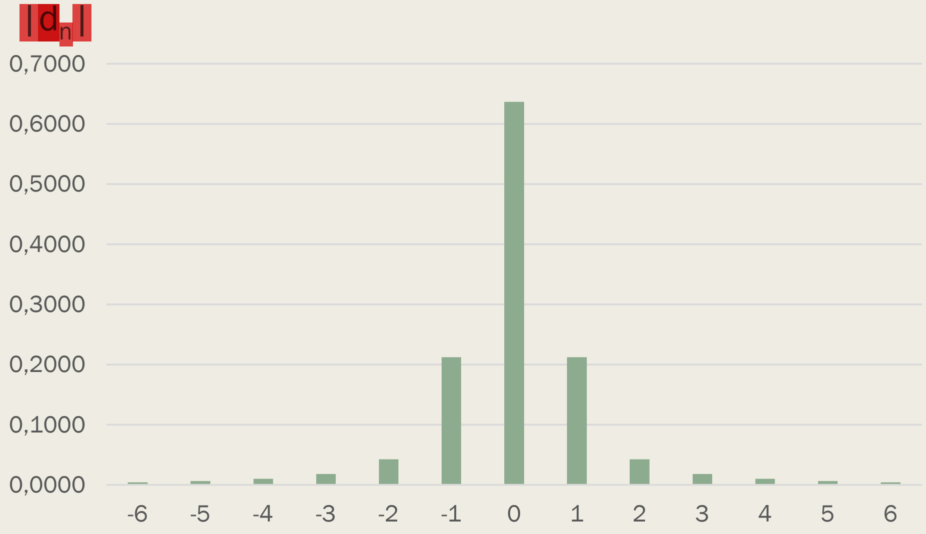
$$\frac{\left(2-e^{j2(1-2n)T}-e^{-j2(1+2n)T}\right)+2n\left(e^{-j2(1+2n)T}-e^{j2(1-2n)T}\right)}{4T(1-4n^2)} =$$

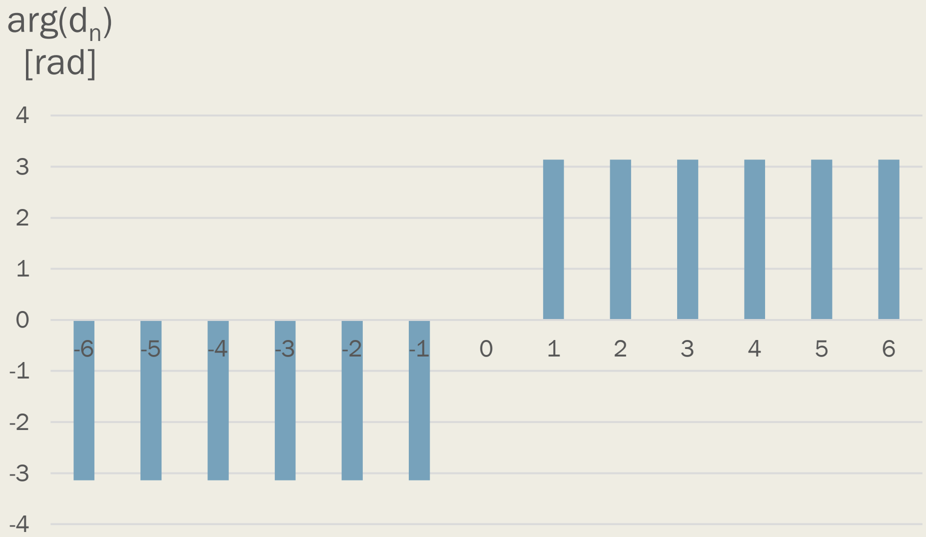
$$\frac{\left(2-e^{j2(1-2n)T}-e^{-j2(1+2n)T}\right)+2n\left(e^{-j2(1+2n)T}-e^{j2(1-2n)T}\right)}{4T(1-4n^2)} =$$

$$\frac{\left(2-e^{j2T}e^{-j4nT}-e^{-j2T}e^{-j4nT}\right)+2n\left(e^{-j2T}e^{-j4nT}-e^{j2T}e^{-j4nT}\right)}{4T(1-4n^2)} =$$

$$\frac{\left(2-e^{j\pi}e^{-j2n\pi}-e^{-j\pi}e^{-j2n\pi}\right)+2n\left(e^{-j\pi}e^{-j2n\pi}-e^{j\pi}e^{-j2n\pi}\right)}{4T(1-4n^2)} =$$

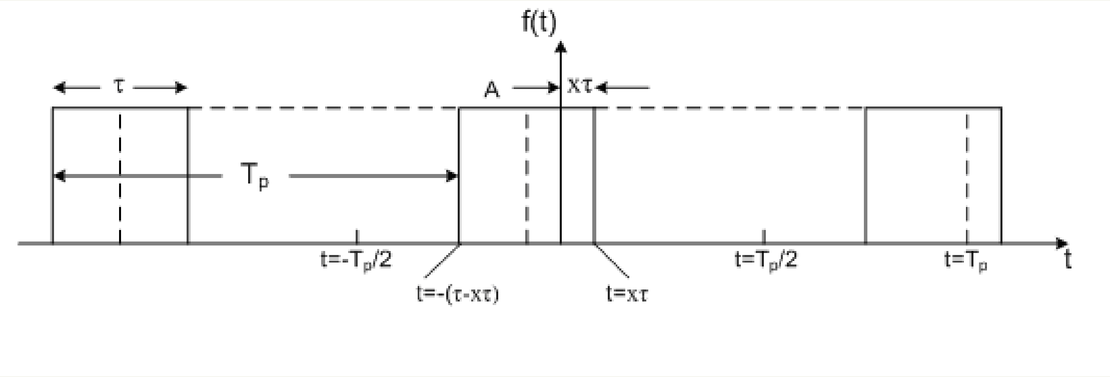
$$\frac{(2+1+1)+2n(-1+1)}{4T(1-4n^2)} = \frac{4}{4T(1-4n^2)} = \frac{2}{\pi(1-4n^2)}$$





n	$\|d_n\|$	arg(dn)
-6	0.0045	-3.14
-5	0.0064	-3.14
-4	0.0101	-3.14
-3	0.0182	-3.14
-2	0.0425	-3.14
-1	0.2123	-3.14
0	0.6369	0
1	0.2123	3.14
2	0.0425	3.14
3	0.0182	3.14
4	0.0101	3.14
5	0.0064	3.14
6	0.0045	3.14

Primer 3:



$$d_n = \frac{1}{T_p}\int_{x\tau-\tau}^{x\tau} Ae^{-jn\omega_0 t}dt = \frac{A}{-jn\omega_0 T_p} e^{-jn\omega_0 t}\Big|_{x\tau-\tau}^{x\tau} = \frac{A}{jn\omega_0 T_p} e^{-jn\omega_0 x\tau}(e^{jn\omega_0\tau}-1) =$$

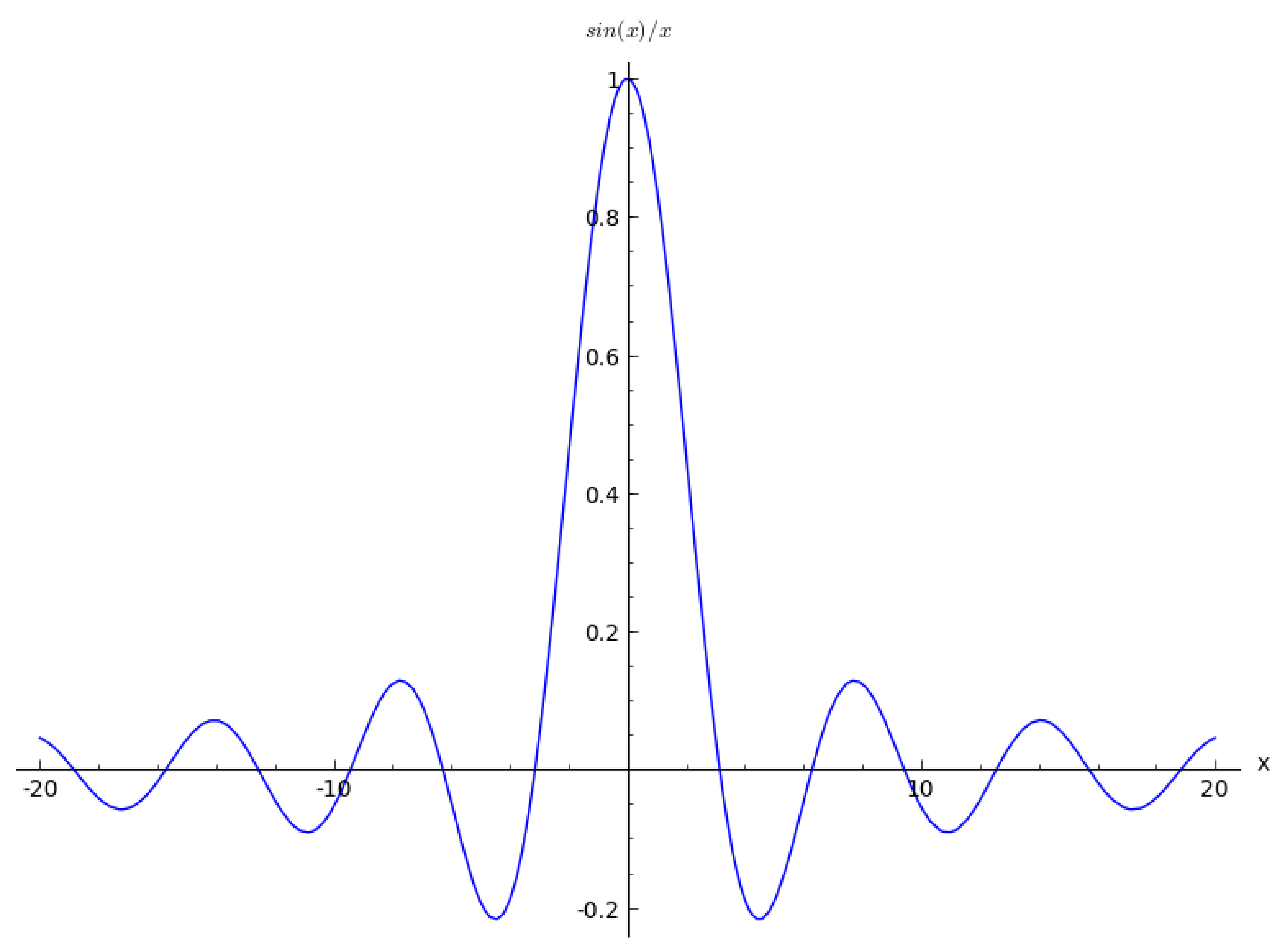
$$= \frac{A}{jn\omega_0 T_p} e^{-jn\omega_0 x\tau} e^{jn\omega_0\tau/2} 2j\sin(n\omega_0\tau/2) = \frac{A\tau}{T_p}\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2} e^{jn\omega_0(0.5-x)\tau}$$

$$(e^{jn\omega_0\tau}-1) = e^{jn\omega_0\tau/2}\cdot 2j\frac{e^{jn\omega_0\tau/2}-e^{-jn\omega_0\tau/2}}{2j} = e^{jn\omega_0\tau/2}\cdot 2j\cdot\sin(\frac{n\omega_0\tau}{2})$$

$$|d_n| = \frac{A\tau}{T_p}\left|\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}\right|$$

$$\phi_n = n\omega_0(0.5-x)\tau \quad \pm\pi$$

pri promeni znaka: $\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}$



$sin(x)/x$
1
0.8
0.6
0.4
0.2
-0.2
-20
-10
10
20
x

Za $\tau = T_P/5$ i $x=0$

Harmonici sa nultom amplitudom:

$$|d_n| = \frac{A\tau}{T_p}\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2} = 0$$

$$\frac{n\omega_0\tau}{2} = k\pi \Rightarrow n = \frac{2k\pi}{\omega_0\tau} = \frac{kT_p}{\tau}$$

$$\omega_0 = \frac{2\pi}{T_p} \quad \text{i} \quad \tau = \frac{T_p}{5} \Rightarrow n = 5k$$

Za $\tau = T_P/2$ $\quad n=2k$
svi parni harmonici imaju nultu amplitudu