- **Дискретизација PID-а:**

$$u(t) = K \cdot \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau)\, d\tau + T_d \cdot \frac{de}{dt} \right]$$

- **Позициони PID:**

$$u(k) = K \cdot \left[e(k) + \frac{T}{T_i} \sum_{i=0}^{k-1} e(i) + \frac{T_d}{T} \cdot \left(e(k) - e(k-1)\right) \right]$$

- **Брзински PID:**

$$u(k) = u(k-1) + \Delta u(k)$$

$$u(k-1) = K \cdot \left[e(k-1) + \frac{T}{T_i} \sum_{i=0}^{k-2} e(i) + \frac{T_d}{T} \cdot \left(e(k-1) - e(k-2)\right) \right]$$

$$\Delta u(k) = u(k) - u(k-1) = g_0 \cdot e(k) + g_1 \cdot e(k-1) + g_2 \cdot e(k-2)$$

$$g_0 = K \cdot \left[1 + \frac{T_d}{T}\right] \qquad g_1 = -K \cdot \left[1 + 2\frac{T_d}{T} - \frac{T}{T_i}\right] \qquad g_2 = K \frac{T_d}{T}$$

$$u(k) = u(k-1) + \underbrace{g_0 \cdot e(k) + g_1 \cdot e(k-1) + g_2 \cdot e(k-2)}_{\Delta u}$$

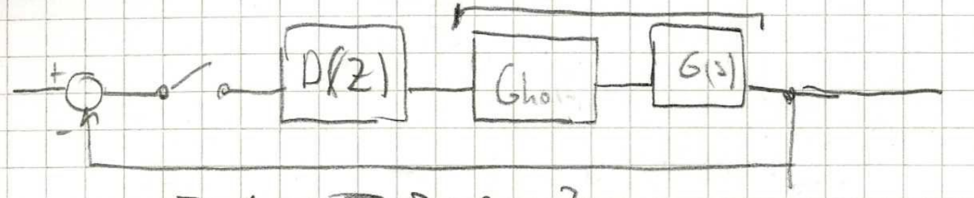
Брзински мора имати I дејство. Ауто → ручно bumpless)

- **Дискретни облици:**

- P: $P(t_k) = K \cdot \left[b \cdot r(t_k) - y(t_k) \right] \qquad b \in [0, 1]$
- I: $I(t_{k+1}) = I(t_k) + \frac{KT}{T_i} \cdot e(t_k)$
- D (ојлер 1): $D(t_{k+1}) = \left[1 - \frac{N \cdot T}{T_d}\right] \cdot D(t_k) - K \cdot N \left[y(t_{k+1}) - y(t_k) \right]$ $\qquad \boxed{T_d > \frac{NT}{2}}$
- D (ојлер 2): $D(t_k) = \left[\frac{T_d}{T_d + NT}\right] \cdot D(t_{k-1}) - \frac{K \cdot N \cdot T_d}{T_d + N \cdot T} \cdot \left[y(t_k) - y(t_{k-1}) \right]$
- D (тустин): $\boxed{T_d > \frac{NT}{2}}$

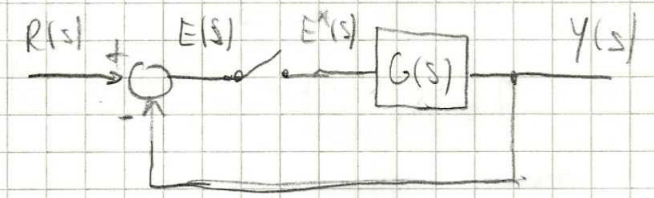
$$D(t_k) = \left[\frac{2T_d - NT}{2T_d + NT}\right] D(t_{k-1}) - \frac{2 \cdot K \cdot N \cdot T_d}{2T_d + NT} \cdot \left[y(t_k) - y(t_{k-1}) \right]$$

- Дигитални упр. системи: → у $G(z)$



$$G(z)=\frac{z-1}{z}\cdot Z\left[\frac{G(s)}{s}\right]$$

- Грешке у устаљеном стању:



$$E(s)=R(s)-Y(s) \quad /^*$$
$$Y(s)=G(s)\cdot E^*(s) \quad /^*$$

$$E^*(s)=R^*(s)-G^*(s)\cdot E^*(s)$$
$$E^*(s)=\frac{R(s)}{1+G^*(s)}$$
$$\boxed{E(z)=\frac{R(z)}{1+G(z)}}$$

- Способност система да прати жељену вредност - e_{ss}

дискретан систем

$$e_{ss}(kT)\equiv e_{ss}=\lim_{k\to\infty}e(kT)=\lim_{z\to1}(1-z^{-1})\cdot E(z)=\lim_{z\to1}\left(\frac{z-1}{z}\right)\cdot E(z)$$

$$\boxed{e_{ss}=\lim_{z\to1}(z-1)\cdot E(z)}$$

$$G(z)=\frac{K\cdot\prod_{j=1}^{m}(z-z_j)}{(z-1)^N\cdot\prod_{i=1}^{n}(z-p_i)}$$

N - ред астатизма

$N+n \geq m$

$$K_{dc}=\left.\frac{K\cdot\prod_{j=1}^{m}(z-p_j)}{\prod_{i=1}^{n}(z-z_i)}\right|_{z_i\neq1}$$

- Dead beat рег.

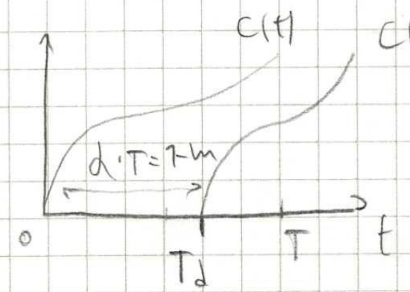
k - разлика степена имениоца и бројиоца

- Идеалан одзив који захтева упр. сигнала велике амплитуде

$$\boxed{W(z)=z^{-k}}\ ;\ D(z)=\frac{1}{G(z)}\cdot\frac{W(z)}{1-W(z)} \Rightarrow \boxed{D(z)=\frac{1}{G(z)}\left(\frac{1}{z^k-1}\right)}$$

- Модификована Z трансф.

$0\leq m\leq 1$



$$c_d(t)=c(t-T_d) \qquad T_d<t$$
$$c_d(t)=c(t-\alpha T) \qquad T_d=\alpha T$$
$$c_d(t)=c(t-(1-m)T) \qquad \alpha=\frac{T_d}{T} \qquad m=1-\alpha$$

$$C(z,m)=Z[c_d(t)]=Z_m[c(t)]=\sum_{k=0}^{\infty}c(t-(1-m)\cdot T)\cdot z^{-k}$$

$$D(z)=\frac{G_s}{1-G_s}\cdot\frac{1}{G} \qquad G_s=z^{-d}$$

– Дискретизація мат. моделі у просторі станів:

$$u(t_k) \longrightarrow \boxed{D/A} \longrightarrow \boxed{\begin{array}{l} \dot{x} = Ax + B\cdot u \\ y = Cx + Du \end{array}} \longrightarrow \boxed{A/D} \longrightarrow y(t_k)$$

$$\begin{array}{l} x(t_{k+1}) = \phi\, x(t_k) + \Gamma \cdot u(t_k) \\ y(t_k) = C \cdot x(t_k) + D \cdot u(t_k) \end{array}$$

$$G(z) = C \cdot (zI - \Phi)^{-1} \cdot \Gamma + D$$

$$\bar{\Phi} = Z^{-1}\left\{(sI - A)^{-1}\right\} \qquad \phi = \bar{\Phi}(T)$$

$$\Gamma = \int_0^T \bar{\Phi}(\tau) \cdot B \cdot d\tau \qquad (sI - A)^{-1} = \frac{\operatorname{adj}(sI - A)}{\det(sI - A)}$$

$$f(s) = \det(sI - A) \qquad f(z) = \det(zI - A)$$